The Carbon Bubble:
Climate Policy in a Fire-Sale Model of Deleveraging[*]

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Comments/Corrections Welcome

Abstract

Committed and credible implementation of climate change policy, consistent with the usual 2°C limit, is thought to require large fossil fuel asset write-offs. This issue, termed the Carbon Bubble, is usually presented as having implications for investors but, for the first time, this paper discusses its implications for macroeconomic policy and for climate policy itself. We embed the Carbon Bubble in a macroeconomic model exhibiting a financial accelerator: if investors are leveraged, the Carbon Bubble may precipitate a fire-sale as investors rush for the exits, and generate a large and persistent fall in output and investment. We investigate policy responses which can accompany the writing-off of fossil fuel assets, like debt transfers, investment subsidies, government guarantees, or even deception about the true scale of the required write-off of fossil fuel assets. We find a role for policy in mitigating the Carbon Bubble.

Keywords: Carbon Bubble, fire-sale, Kiyotaki and Moore's (1997) model, deleveraging, carbon tax, resource substitution, 2°C target.

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1   Introduction

In 1996, EU Governments set a global temperature target of two degree Celsius (°C) above pre-industrial level which was made international policy at the 2009 United Nations Climate Change Conference in Copenhagen.[1] A global mean temperature increase of 2°C is considered as a threshold separating safety from extreme events: significant extinctions of species, reductions in water availability and food production, catastrophic ice sheet disintegration and sea level rise (EU Climate Change Expert Group, 2008). The Potsdam Climate Institute has calculated that if we want to reduce the probability of exceeding 2°C warming to 20%, then only one-fifth of the Earth's proven fossil fuel reserves can be burned unabated[2] (Carbon Tracker Initiative, 2011).[3] As a consequence, there is a global "carbon budget" of allowable emissions, whilst the rest is "unburnable carbon". The Carbon Tracker Initiative's (2011) report warns that, analogously to the subprime mortgage problem that precipitated the 2008-09 Financial Crisis, the global economy is once again mis-pricing assets as markets overlook this "unburnable carbon" problem. This issue is termed the "Carbon Bubble" because the imposition of climate policy consistent with the Potsdam Climate Institute's calculations would mean that the fundamental value of many fossil fuel assets must be zero as they cannot be used. Their current market value must therefore be made up of a zero fundamental value, and a "bubble" component: the Carbon Bubble.[4]

Despite climate-science based claims that not even all existing fossil fuel assets can be used, capital markets place a positive value on fossil fuel reserves. Investors use the reserves that companies claim to own as an indicator of future revenues, and the share price of fossil fuel companies is heavily influenced by the reserves on their books. Fossil fuel companies still have an incentive to invest to find new reserves, and to invest in new technology that will allow the exploitation of currently unprofitable resources, even though the exploitation of these deposits is inconsistent with the climate change targets

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[1]See Jaeger and Jaeger (2011) for a summary of how the target emerged and evolved.

[2]By "unabated" we mean without the use of, for example, Carbon Capture and Storage (CCS) technology in which the fossil fuels are burned, but the carbon dioxide does not reach the atmosphere.

[3]Another recent estimate is from McGlade and Ekins (2015) who suggest that "globally, a third of oil reserves, half of gas reserves and over 80 per cent of current coal reserves should remain unused ... in order to meet the target of 2°C".

[4]Note that a "bubble" of this form is not consistent with bubbles as described in the economics literature that follows from Tirole (1985). What has been termed the Carbon Bubble is a real asset which has positive fundamental value in one state of the world (no regulation) but not in another (with regulation) - it is not a bubble at all in the economic sense. However, this is the terminology that has been adopted. One of the first appearance of the term in a popular media article, and one of the most cited news item on this topic, is "Global Warming's Terrifying New Math" by Bill McKibben, published in Rolling Stone in June 2012. According to Google Trends, web search on the term "Carbon Bubble" reached a high around May 2015.

that the world's governments have signed up to.[5] If policymakers enforce compliance with the 2°C target, markets will begin to recognise that the values of the reserves on these companies' books are untenable, and the value of the companies will fall considerably as a consequence of these *stranded assets*. Also, the values of companies using cheap fossil fuels as an input are also likely to fall. It is not only the equity of companies that is exposed to this, the quality of the debt they have issued is also exposed and there will be defaults and ratings downgrades.

The severity of the financial crisis has proven that a financial market disruption, induced by a problem in a small portion of the economy, can cause a deep recession. The deleveraging of the financial sector results in declining asset prices and consequent decreases in the debt capacity of the non financial sector, which must then reduce the level of leveraged investment. As economic activity worsens, the asset price drop fuels further debt capacity reductions in a downward spiral. This is the so called "financial accelerator" mechanism of feedback between the financial and non financial sectors. Credible climate policy implementation will lead to the write-off of fossil fuel assets. Then, if fossil fuel companies are using their balance sheets as collateral, or if investors are using their holdings of exposed financial assets as collateral, these write-offs could lead to a breakdown of credit relationships and a general decline in the amount of total credit supplied to the economy. If a limitation in the total carbon budget was imposed suddenly, this could cause a "sudden stop" akin to, or worse than, the 2008 Financial Crisis (Mendoza, 2010): the Carbon Bubble could burst.

A recessionary response is particularly damaging with respect to the implementation of the climate policy itself, as one of its aims is to provide the incentives for investments in alternative energy capital, in order to replace the current fossil fuel based energy infrastructure. A substantial stock of zero carbon productive capacity will need to be in place at the point at which the carbon budget is exhausted, but the "bursting of the Carbon Bubble" could throw the economy into a deep recession, thus depriving green technology of investment funds when they are most needed. Even if the fossil fuels assets really should be written-off to avoid disastrous global warming, the implementation of such a policy must pay cognisance to the impact that it will have upon investment.

This paper models the consequences of a major write-off of energy capital that would follow the implementation of such climate policy. We assume a binding cumulative emissions allowance (a carbon budget), and incorporate a financial accelerator effect by using the credit amplification mechanism of Kiyotaki and Moore (1997), where entrepreneurs borrow from savers using their current asset holdings as collateral. This framework allows us to go beyond the discussions of the Carbon Bubble that have appeared to date,

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[5]The Carbon Tracker Initiative's (2015) report estimates that up to $2.2tn of new and existing investments is in danger of being wasted over the coming decade.

and to start considering the link from the impact of policies upon financial markets and the macroeconomy, back to the appropriate climate policies itself. Alongside the credible implementation of climate policy consistent with the 2°C limit, we consider policies that transfer investors' debts to the government, subsidise investment, and provide government guarantees on investors' borrowings. We show that these macroeconomic policies, by mitigating the impact of the Carbon Bubble upon the balance sheets of investors, can be welfare enhancing (though not necessarily Pareto improving), even if such policies are welfare destroying under normal circumstances.

The main contribution of this article is to link, for the first time, the issue of the Carbon Bubble with the financial accelerator mechanism, and to analyse the interaction between climate policy and macroeconomic stabilisation.[6] Our conclusions provide an indication of which types of policies are effective in raising welfare and investment by mitigating the macroeconomic impact of implementing policy consistent with the 2°C limit. We chose a tractable model to create this first meeting between macro-financial and climate-economy models, and with which to form initial conclusions, but our aim is to start the conversation and to provoke further research. We do not attempt to provide a precise quantitative macroeconomic forecast of the consequences of implementing climate policy consistent with the 2°C limit.

The remainder of the paper is organized as follows: Section 2 reviews some of the literature on the financial accelerator and on a cumulative carbon emissions constraint that are relevant to the issue outlined in this paper; Section 3 outlines the technical details of the model; Section 4 describes the process we used to calibrate the model, so that it broadly replicates the outcomes seen over the 2008-09 Financial Crisis; Section 5 sets out the Carbon Bubble scenario in which the planner bans investment in new high carbon energy capital, and implements restrictions on the usage of existing carbon assets. This section examines policies which the planner can additionally implement, in order to minimise the business cycle response, boost alternative energy capital investment, and boost welfare. Section 6 concludes.

2   Relevant Literature

This paper uses the financial accelerator model of Kiyotaki and Moore (1997). In this model, relatively patient savers lend to relatively impatient entrepreneurs. There is a financial friction because it is possible for the entrepreneurs to repudiate their debt by walking away from their capital. The savers therefore require that the credit that they advance is fully collateralised by the value of the (fixed) capital. This leads to a financial accelerator which exaggerates the fluctuations in output and investment following a relat-

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[6]Though the Bank of England has also signalled that it is investigating this issue - see Carney (2014).

ively small temporary shock to the economy. When the economy experiences a negative shock, there is a dynamic feedback process between the value of capital and the level of borrowing. Capital values fall, which means entrepreneurs have lower debt carrying capacity, since the capital value was used as collateral for debt. They therefore have to sell capital to repay debt, which lowers the price of capital further: a fall in the value of capital precipitates forced sales to ensure borrowing and collateral requirements are aligned, but this forced sale causes prices to fall again which causes further forced sales, and further price falls, and so on. These are the dynamics of a "fire-sale".

Gerke et al. (2013) show that most models of the financial accelerator share qualitatively similar features. We choose to work with the model of Kiyotaki and Moore (1997) because it has several features that are attractive given our exercise. Firstly, we require a tractable formal model to examine the interaction between climate policy induced energy capital write-offs and the macroeconomy, and Kiyotaki and Moore (1997) is a tractable model with fire-sale dynamics that we can work with. Secondly, the model presented in Kiyotaki and Moore (1997) does not return to steady state following a very large negative shock. Entrepreneurs need some positive net worth in order to support their borrowing. For large negative shocks the net worth of entrepreneurs is negative. In the basic Kiyotaki and Moore's (1997) model there will then be no fixed capital allocated to the entrepreneurs in the period following the shock, and for all periods thereafter. This allows us to make the rhetorical point that such a shock can wipe out the entrepreneurial sector, and that other mechanisms or interventions are necessary to restart leveraged investment. We follow Cordoba and Ripoll (2004) and introduce a debt renegotiation process that ensures the model can always return to the interior steady state.[7] This requires a more complex timing of production decisions, which mitigates the price response to large negative shocks.

In Cordoba and Ripoll (2004) it is assumed that markets are open during the day, shocks occur at dusk, and then there is a window of opportunity for debt renegotiation to take place, before production occurs overnight. If entrepreneurs want to, they can default on the debt, crucially, before production takes place: the lender gets the ownership of the fixed capital but loses the outstanding value of the debt. They may be able to do better by renegotiating the outstanding value of the debt down to the new value of the collateral and incentivising the entrepreneurs to engage in production. This shares the burden of the fall in fixed capital values with the lenders and ultimately limits the decrease in fixed capital prices and output with respect to Kiyotaki and Moore (1997). Following a positive

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[7]See Footnote 29 for a discussion of the other steady states of the Kiyotaki and Moore's (1997) model. Cordoba and Ripoll (2004) actually introduce debt renegotiation in the basic version, Kiyotaki and Moore (1997, Chapter II), whereas we use the full version, Kiyotaki and Moore (1997, Chapter III). The debt renegotiation mechanism is therefore not identical, but we base our approach on Cordoba and Ripoll (2004).

shock, entrepreneurs do not have the incentive to default and so no renegotiation of the debt occurs.

Thirdly, another feature of the Kiyotaki and Moore's (1997) model which makes it attractive for our exercise is the Leontief combination of fixed capital and uncollateralisable, idiosyncratic capital (or "trees" in the notation of Kiyotaki and Moore (1997)) that the entrepreneurs use. Hassler et al. (2012) measure a very low elasticity of substitution between energy and other inputs to production, at least in the short-run. Therefore a Leontief specification is a convenient modelling device to capture this fact. The idiosyncratic capital stock in our exercise has the interpretation of energy related capital, while the fixed capital is all other capital in the economy. So for example, the idiosyncratic capital includes coal mines, power stations, wind farms and a balanced electricity grid, while the fixed capital includes the factory which uses the electricity, but does not directly care how this electricity has been produced.[8] Our innovation here is to introduce two flavours of idiosyncratic capital that entrepreneurs can develop: a more productive high carbon variety and a less productive zero carbon variety. Carbon based production is assumed to cause a global externality that the infinitesimal entrepreneurs will take as given: in the absence of policy they will therefore choose to produce using the high carbon variety. Policy (in the form of taxes and subsidies) can however induce the entrepreneurs to use zero carbon production. This framework allows us to model the Carbon Bubble, which has hitherto not been considered as part of the literature on the economics of climate change.

The standard approach to the economics of climate change, Nordhaus's (2008) Integrated Assessment Model (IAM), considers climate change in an optimal economic growth framework which includes damages from climate change. Typically IAMs balance the economic benefits of fossil fuel emissions for production against the economic damages from climate change, to produce some optimal timepath for emissions reduction which is implemented with a timepath of carbon taxes. The scientific literature, on the other hand, suggests that the first order impact of emissions in any given period is related to their contribution to the overall cumulative emissions, which is the main driver behind climate change (Allen et al., 2009; IPCC, 2014). This is also consistent with the headlines from the Carbon Tracker Initiative's (2011) report which talked of a "carbon budget". In this paper, we will use this idea of a cumulative emissions constraint, or carbon budget, which makes the modelling exercise easier: we model a cumulative emissions limit separating non catastrophic damages, which are broadly undetectable in the social welfare function, from catastrophic damages which cause infinitely negative social welfare[9] and so must be

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[8]Though such a factory does care indirectly about the production techniques used in the creation of its electricity inputs, since the productivity of this production will affect electricity prices and thus affect the factory's cost base.

[9]E.g. human extinction.

avoided at all costs.

One way to think about imposing such a cumulative emissions constraint that embeds it within the standard approach is to say that we are arguing probabilistically, and invoke Weitzman (2009). Perhaps the damages associated with climate change have an uncertainty that grows with their median size. With low emissions, within our allowed carbon budget, we have low median damages and further, the uncertainty on these damages has a thin-tailed distribution: the product of the infinitely negative impact of catastrophic damages with the zero chance of them occurring is zero. The expected impact of such emissions is close to the medium impact and it is almost undetectable in terms of overall social welfare. Our carbon budget represents some threshold between a thin-tailed and a fat-tailed distribution for damages from emissions. With a fat-tailed distribution of damages, the product of the infinitely negative impact of catastrophic damages with the zero chance of them occurring is infinitely negative. Therefore, for emissions greater than the carbon budget, although the median impact is smoothly increasing in emission levels, the expected value tends to infinity across this threshold. Therefore, treating climate damages as approximately zero within the carbon budget and infinite beyond the carbon budget can be rationalised, and it simplifies the modelling substantially.

3 The Model

We develop a two-agent closed economy model which extends the "full version" of Kiyotaki and Moore (1997, Chapter III) by allowing entrepreneurs to choose between two types of investment good (which we label as "energy capital") with different productivity.[10] We also introduce a simple government or policymaker.

Time is discrete and indexed by $t = 0, 1, 2, ..., \infty$. There are two types of infinitely lived agents: a continuum of entrepreneurs of mass m_e, and a continuum of savers of mass m_s.[11] For simplicity, m_e is normalised to unity, and m_s is referred to as m. Entrepreneurs and savers have the following preferences

$$\max_{\{x_s\}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} x_s \right] \qquad \text{and} \qquad \max_{\{x_s'\}} E_t \left[\sum_{s=t}^{\infty} (\beta')^{s-t} x_s' \right] \tag{1}$$

i.e. they both maximize the expected discounted utilities from consumption: x_t and x_t' represent consumption at date t of the entrepreneur and the saver respectively; $0 < \beta < 1$ and $0 < \beta' < 1$ indicate the discount factors; and E_t indicates expectations formed at date

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[10]In the terminology of the original paper, "farmers" can choose between two types of "trees".

[11]Variables regarding the savers are identified by the prime. Aggregate variables will be capitalized. Steady state variables will be starred. For a list of variables and parameters, and their definitions, see Appendix A.1.

t. Both types of agents are risk neutral but they differ in their rates of time preference: entrepreneurs are more impatient i.e. they have a lower discount factor than savers.

Assumption A $\qquad \beta < \beta'$.

Exogenous ex-ante heterogeneity on the subjective discount factors not only allows us to keep the model tractable but also ensures the model simultaneously has borrowers and lenders.[12]

There are three types of goods: fixed capital (K), energy capital (Z) and non durable commodity. The energy capital has two flavours: high carbon and zero carbon, indexed by H and L respectively. The non durable commodity cannot be stored but can be consumed or invested in energy capital. The fixed capital does not depreciate and is available in a fixed aggregate amount, given by $\bar{K}$, while both types of energy capital depreciate at rate $1 - \lambda$ per period.

The government can levy a tax on the output of an entrepreneur who uses high carbon energy capital i.e. a carbon tax, and provide a green subsidy to entrepreneurs using zero carbon energy capital. The net position of the government is either financed through a lump-sum tax or distributed through a lump-sum transfer on a per capita basis i.e. the government runs a balanced budget.

At the end of each time period $t-1$, there is a competitive asset market and a competitive one-period credit market. In the former, one unit of the fixed capital is exchanged for q_{t-1} units of the commodity; in the second, one unit of the commodity at date $t-1$ is exchanged for R_{t-1} units of the commodity at date t. The commodity is assumed to be the numeraire, so that its price is normalised to unity. Then q_t represents the price per unit of fixed capital, and R_t is the gross interest rate. At the start of a new period t, markets are closed (although there is a window of opportunity for debt renegotiation): stocks of fixed capital, energy capital, and debt holdings are state variables. Production then takes place over period t.

3.1 Entrepreneurs

An entrepreneur produces a quantity of the commodity, y, with a one-period Leontief production function: fixed capital, k, is combined with energy capital, z, in $1 : 1$ proportion.

This period's decisions affect next period's production. The entrepreneur can choose between two technologies. Choosing the first, k_{t-1} units of fixed capital are combined with z_{t-1}^H units of the high carbon energy capital, producing y_t units of the commodity. However, this choice implies that the after tax output available to the entrepreneur will

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[12]This is in line with many dynamic (stochastic) general equilibrium models of financial friction e.g. Kiyotaki and Moore (1997), Iacoviello (2005), Iacoviello and Neri (2010), Devereux and Yetman (2010), Pariès et al. (2011), and Liu et al. (2013).

be reduced by any proportional carbon tax implemented, $\tilde{\tau}_t$:

$$
\begin{aligned}
y_t = F_H(k_{t-1}, z_{t-1}^H) &= \left(a^H + c\right) \times \min\left(k_{t-1}, z_{t-1}^H\right) \quad (2) \\
(1 - \tilde{\tau}_t)\, y_t &= \left(a^H - \tau_t + c\right) \times \min\left(k_{t-1}, z_{t-1}^H\right).
\end{aligned}
$$

Choosing the second, the entrepreneur combines k_{t-1} units of fixed capital with z_{t-1}^L units of the zero carbon energy capital and benefits from a proportional subsidy, $\tilde{\varsigma}_t$. The output available to the entrepreneur, however, will be increased only by a fraction $\delta \in [0,1]$ of the subsidy implemented, where δ is a structural parameter representing the effectiveness of the subsidy.[13],[14]

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\begin{aligned}
y_t = F_L\left(k_{t-1}, z_{t-1}^L\right) &= \left(a^L - (1 - \delta)\varsigma_t + c\right) \times \min\left(k_{t-1}, z_{t-1}^L\right) \quad (3) \\
(1 + \tilde{\varsigma}_t)\, y_t &= \left(a^L + \delta\varsigma_t + c\right) \times \min\left(k_{t-1}, z_{t-1}^L\right).
\end{aligned}
$$

No matter the technology used, ck_{t-1} units of the y_t units of output produced at date t are not tradable and must be consumed by the entrepreneurs (who therefore must pay any carbon tax levied out of tradable output).[15]

The dichotomous variable $a_t^i + c$ represents the net productivity of capital in the hands of entrepreneurs, and is given by $a^H - \tau_t + c$ if the high carbon energy capital is used in production, and by $a^L + \delta\varsigma_t + c$ if zero carbon energy capital is used. We assume that zero carbon energy capital is intrinsically less productive than high carbon energy capital:[16]

Assumption B $\qquad a^H > a^L$.

The commodity can be consumed or invested. For that portion of their output which

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[13]When $\delta = 0$, the subsidy is completely ineffective in raising net private productivity, while with $\delta = 1$ there is no cost in terms of productivity associated with the subsidy. Being credit constrained, the entrepreneurial sector will use a sub-optimally low quantity of capital in equilibrium: a subsidy would therefore move the economy towards first best by mitigating against the credit frictions. We want to look at policies that mitigate the problem of the Carbon Bubble, but we do not want to eliminate credit constraints in steady state. Therefore we introduce this productivity destroying distortion associated with the subsidy, which will be calibrated so that the policymaker does not want to use the subsidy in steady state. If there are any benefits (measured using the policymaker's objective function) in applying a subsidy, these must therefore be due to the Carbon Bubble issue. More details on the subsidy can be found in Appendix A.2.

[14]In the remainder of the paper we discuss only $\tau_t = \tilde{\tau}_t(a^H + c)$ and $\varsigma_t = \tilde{\varsigma}_t\left(a^L - (1 - \delta)\varsigma_t + c\right)$, positive bijective transformations of the proportional tax rate and subsidy rate into units that can be compared to the productivities of the two alternative technologies.

[15]The ratio $a^i/(a^i + c)$ represents an upper bound on the entrepreneur's savings rate. This non-tradable quantity of output is introduced in Kiyotaki and Moore (1997) to avoid the possibility that the entrepreneur keeps postponing consumption. Indeed, since preferences are linear, entrepreneurs would like to not consume and increase investment. While this assumption and the presence of linear preferences but different discount factors can be considered as unorthodox modelling choices, Kiyotaki and Moore (1997, Appendix) show that the same qualitative results can be obtained using an overlapping generations model with standard concave preferences and conventional saving/consumption decisions.

[16]This is in line with Acemoglu et al. (2012) where the carbon sector is assumed to have an "initial productivity advantage" over the clean sector.

is invested, the entrepreneur converts ϕ units of the commodity into one unit of energy capital: ϕ is the output cost of investing in one unit of energy capital.[17]

Two critical assumptions in Kiyotaki and Moore (1997) are imposed here. Firstly, the entrepreneur cannot pre-commit to work and can freely decide to withdraw their labour: Hart and Moore (1994) refer to this option as "inalienability of human capital". Secondly, the entrepreneur's technology and energy capital are idiosyncratic. Thus, if they decide to withdraw their labour between dates t and $t+1$, there would be only the fixed capital k_t and no output at $t+1$. Given these assumptions, a constraint arises limiting the debt of an entrepreneur. An entrepreneur may want to repudiate their contract when their debt becomes too onerous. The lender knows this possibility and asks the entrepreneur to back the loan with collateral. Rather than the amount of collateral depending upon the relative bargaining power of the agents, Hart and Moore (1994) suggest that the lender will require the full value of their counterpart's assets as collateral. Thus, for an amount of debt b_t and current fixed capital holdings k_t, the entrepreneur must repay $R_{t+1}b_t$ next period, at which time their collateral will be worth $q_{t+1}k_t$. Entrepreneurs are therefore subject to the following borrowing constraint:

$$b_t \leq \frac{q_{t+1}k_t}{R_{t+1}}. \tag{4}$$

Consider an entrepreneur who holds k_{t-1} units of fixed capital, $z_{t-1} = k_{t-1}$ units of energy capital, and has gross debt b_{t-1} at the end of period $t-1$. At date t they receive net income from production of $a_t^i k_{t-1}$ units of tradable output (depending on the technology used), they incur a new loan b_t and acquire more fixed capital, $k_t - k_{t-1}$. Having experienced depreciation and having increased their fixed capital holdings, the entrepreneur will have to convert part of the tradable output to energy capital. In general, they will have to invest $\phi(k_t - \lambda k_{t-1})$ in order to have enough energy capital to cover depreciation and new fixed capital acquisition. They then repay the accumulated debt, $R_t b_{t-1}$, and choose how much to consume in excess of the amount of non-tradable output, $(x_t - ck_{t-1})$. In addition, they receive a per capita transfer from the government or pay the per capita tax, g_t, depending on the net position of the government. Thus, the entrepreneur's flow-of-funds constraint, as at the end of period t, is given by

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - ck_{t-1}) + \tau_t k_{t-1} = a^H k_{t-1} + b_t + g_t \tag{5a}$$

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - ck_{t-1}) = a^L k_{t-1} + \delta \varsigma k_{t-1} + b_t + g_t. \tag{5b}$$

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[17]Note that, instead of writing the model in terms of differing productivities of high and zero carbon technologies, $\{a^H, a^L\}$, we reach qualitatively the same results by writing the model in terms of differing output costs of investing in energy capital, $\{\phi^H, \phi^L\}$, as in van der Zwaan et al. (2002). Since results are qualitatively similar, we do not present this alternative model here.

The first line refers to an entrepreneur who uses the high carbon energy capital, while the second relates to the use of the zero carbon energy capital.

Each period only a fraction, $0 \leq \pi \leq 1$, of entrepreneurs have an investment opportunity.[18] Thus, with probability $1 - \pi$, the entrepreneur cannot invest and must downsize its scale of operation, since the depreciation of their energy capital implies $z_t^i = \lambda z_{t-1}^i$. This probabilistic investment assumption,[19] when combined with Leontief production, means that with probability $1 - \pi$ the entrepreneur also faces the constraint

$$k_t \leq \lambda k_{t-1}. \tag{6}$$

3.2 Representative Saver

Savers are willing to lend commodities to entrepreneurs in return for debt contracts, and they also produce commodities by means of a decreasing return to scale technology which uses the fixed capital as an input and takes one period, according to

$$y_t' = \Psi(k_{t-1}') \qquad \text{with } \Psi' > 0, \quad \Psi'' < 0. \tag{7}$$

Savers are never credit constrained because they can trade all their output and no particular skill is required in their production process (there are no idosyncratic technologies or capital goods used). Savers solve the maximization problem in (1), subject to their budget constraint

$$q_t(k_t' - k_{t-1}') + R_t b_{t-1}' + x_t' = \Psi(k_{t-1}') + b_t' + g_t. \tag{8}$$

Equation (8) should be read as follows: a saver who produces $\Psi(k_{t-1}')$ units of the commodity, incurs (issues) new debt, b_t', and receives (pays) the per capita government expenditure (tax), g_t, (right-hand side) can cover the cost of buying fixed capital, $q_t(k_t' - k_{t-1}')$, repaying (collecting on) the previous debt (including interest), $R_t b_{t-1}'$, and consuming, x_t', (left-hand side). Note that b_{t-1}' and b_t' can (and will in equilibrium) be negative.

3.3 Competitive Equilibrium

In general, an equilibrium consists of a sequence of prices $\{(q_t, R_t, \tau_t, \varsigma_t)\}$, allocations for the entrepreneur $\{(x_t, k_t, z_t, b_t)\}$ and the saver $\{(x_t', k_t', b_t')\}$ such that, taking the prices as given, each entrepreneur solves the maximization problem in (1) subject to the technological constraints in either (2) or (3) and, if appropriate, (6), the borrowing constraint

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[18]The arrival rate of the investment opportunity is independent through time and across agents.

[19]This assumption is introduced by Kiyotaki and Moore (1997, page 229 - 230) to capture "the idea that ... investment in fixed assets is typically occasional and lumpy".

in (4) and the flow-of-funds constraint in (5a) or (5b); each saver maximizes (1) subject to the technological constraint in (7) and the budget constraint in (8); the government always runs a balanced budget; and the goods, asset and credit markets clear.

Using γ_t to indicate the share of aggregate entrepreneurs' fixed capital holdings which are combined with high carbon energy capital at time t, let I_t^H, I_t^L, B_t, $mb_t' \equiv B_t'$, K_t, $mk_t' \equiv K_t'$, X_t, $mx_t' \equiv X_t'$, Y_t, $my_t' \equiv Y_t'$, $\tau_t \gamma_t K_t \equiv T_t$, $\varsigma_t(1 - \gamma_t)K_t \equiv P_t$, $(1 + m)g_t = G_t$ be aggregate investment flows, aggregate borrowing, aggregate fixed capital holdings, aggregate consumption, aggregate output, aggregate carbon tax, aggregate green subsidy, and aggregate lump-sum transfer ($+ve$) or tax ($-ve$). Then the government budget constraint and the market clearing conditions for assets, credit and goods are, respectively,

$$T_t - P_t = G_t \tag{9a}$$

$$K_t + K_t' = \bar{K} \tag{9b}$$

$$B_t + B_t' = 0 \tag{9c}$$

$$I_t^H + I_t^L + X_t + X_t' + G_t - T_t + P_t = Y_t + Y_t'. \tag{9d}$$

Note that, given assumption A, the impatient entrepreneurs borrow from the patient savers in equilibrium. Moreover, given that savers are risk neutral and there is no uncertainty, the rate of interest, R_t, is constant and determined by the patient saver's rate of time preference i.e. $R_t = 1/\beta' \equiv R$.

To characterize equilibrium, we start with the savers since their maximization problem is not affected by the carbon tax nor the green subsidy. Since the savers are not credit constrained, their fixed capital holdings are such that they are indifferent between buying and selling this capital. This is the case if the rate of return from buying fixed capital is equal to the rate of return of selling[20]

$$\frac{\Psi'(k_t')}{u_t} = R \tag{10}$$

where

$$u_t \equiv q_t - \frac{q_{t+1}}{R} \tag{11}$$

has a dual role. This "user cost of capital" is defined from the point of view of the

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[20]Equivalently, savers' fixed capital purchases are such that they equate the marginal product of fixed capital, $(1/R)\Psi'(k_t')$, obtained by using fixed capital to produce, and the opportunity cost of not selling the fixed capital this period at price q_t and waiting until the next when, from the point of view of today, they will be worth $(1/R)q_{t+1}$.

entrepreneur as the down payment required to purchase one unit of the fixed capital[21] but it is also the opportunity cost of holding fixed capital for savers.

Using (9b) together with (10), the following asset market equilibrium condition is obtained:

$$u_t = \frac{1}{R}\Psi'\left(\frac{\bar{K} - K_t}{m}\right) \equiv u(K_t). \tag{12}$$

The ratio $(\bar{K} - K_t)/m$ is the representative saver's fixed capital holdings. An increase in the saver' demand for fixed capital causes the middle term of Equation (12) to decrease, given the assumption of decreasing marginal productivity in (7). Equivalently, an increase in entrepreneurs' demand for fixed capital needs a decrease in savers' demand for the market to clear: this is achieved by a rise in the user cost, u_t. Thus, $u' > 0$.

Now consider a carbon tax rate and green subsidy rate, τ_t and ς_t, such that the after tax productivity of the high carbon technology is equal to the after subsidy and distortion productivity of the zero carbon technology i.e. $a_t \equiv a^H - \tau_t = a^L + \delta\varsigma_t$. In this scenario, the entrepreneur is indifferent between the two technologies. To characterize the equilibrium, we thus indicate with $\gamma \in [0,1]$ the share of aggregate entrepreneurs' fixed capital holdings used with high carbon energy capital in equilibrium.

Entrepreneurs who can invest at date t will prefer borrowing up to the limit and investing, rather than saving or consuming, hence limiting their consumption to the current non-tradable output ($x_t = ck_{t-1}$). Thus, the credit constraint in (4) is binding and the flow-of-funds constraint in (5) can be rearranged as[22]

$$k_t = \frac{1}{q_t + \phi - \frac{q_{t+1}}{R}}\left[(q_t + \lambda\phi + a_t)k_{t-1} - Rb_{t-1} + g_t\right]. \tag{13}$$

At the end of period t, the net worth of an entrepreneur is given by the expression in the square brackets, and consists of the value of the tradable output, plus the value of the fixed capital and remaining energy capital, less the debt repayment, Rb_{t-1}, plus (minus) the lump-sum transfer (tax) from the government. This net worth is used by the entrepreneur to cover that part of total investment, $k_t(q_t + \phi)$, exceeding the amount they can borrow using their fixed capital as collateral, $k_t q_{t+1}/R$.

An entrepreneur who cannot invest at t, given that they will not want to waste their remaining stock of energy capital, will adjust their levels of debt and fixed capital such

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[21]It represents the amount an agent has to provide when buying fixed capital, and it is given by the difference between the price of one unit of fixed capital and the amount the entrepreneur can borrow using that unit as collateral.

[22]The following relationship is derived by noticing that (5a) applies to a share γ of entrepreneur's fixed capital holdings, while (5b) to the remaining $1 - \gamma$, and by using $b_t = q_{t+1}k_t/R$ from (4).

that Equation (6) will hold with equality i.e.

$$k_t = \lambda k_{t-1}. \tag{14}$$

Since the previous equations are all linear in k_{t-1} and b_{t-1}, we can derive the equations of motion for the entrepreneurs' aggregate fixed capital holdings[23]

$$K_t = (1-\pi)\lambda K_{t-1} + \frac{\pi}{q_t + \phi - \frac{q_{t+1}}{R}}\left[\left(q_t + \phi\lambda + a\right)K_{t-1} - RB_{t-1} + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}K_{t-1}\right] \tag{15}$$

and borrowing[24]

$$B_t = q_t(K_t - K_{t-1}) + \phi(K_t - \lambda K_{t-1}) + RB_{t-1} - aK_{t-1} - \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}K_{t-1}. \tag{16}$$

One interesting implication of Equation (15) is that demand for fixed capital from the entrepreneurial sector increases given an increase, in equal proportion, of both today's and tomorrow's fixed capital prices. A rise in the current price increases entrepreneur's net worth and a rise in the future prices strengthens the value of the collateral (thus allowing the entrepreneurs to borrow more) and this more than compensates for the price-increase induced reduction in demand.

We are now able to characterize, for given K_{t-1} and B_{t-1}, the perfect foresight competitive equilibrium from date t onward as the paths of aggregate entrepreneurs' fixed capital holdings and debts, and fixed capital prices, $\{K_{t+s}, B_{t+s}, q_{t+s}\}_{s=0}^{\infty}$, such that Equations (12), (15), and (16) are satisfied for all t.[25,26]

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[23]This is obtained by noticing that Equation (13) refers to a fraction π of investors, while Equation (14) applies to the remaining $1-\pi$. Moreover, we express the total transfers from the government to the entrepreneurs as the fraction $1/(1+m)$ of the net position of the government, $[\gamma\tau - (1-\gamma)\varsigma]K_{t-1}$.

[24]This is obtained by solving for b_t the flow-of-funds constraint in (5), where (5a) applies to γ entrepreneurs and (5b) to $1-\gamma$, with $x_t = ck_{t-1}$.

[25]Note that Equations (15) and (16) are very similar to the equations of motion derived in Kiyotaki and Moore (1997). Our addition of a debt renegotiation mechanism, based on Cordoba and Ripoll (2004), does not affect the equations of motion under perfect foresight - since adverse shocks and hence debt renegotiation do not occur under perfect foresight. We return to the debt renegotiation mechanism in Sections 4 and 5 where its incorporation will allow the economy to recover from very large exogenous shocks imposed at time t, through an instantaneous adjustment at time t^+, with the economy thereafter following the perfect foresight, risk free path.

[26]We refer the interested reader to Kiyotaki and Moore (1997, footnote 22) for the full proof of the claims on the behaviour of investing and non-investing entrepreneurs. They show that by Assumption A investment strictly dominates saving while Assumption G in Appendix A.1 ensures that an entrepreneur prefers to invest (if they can) or save (if they cannot invest) rather than consuming the marginal unit of tradable output.

3.4 Steady State

In this subsection we consider only the interesting case in which policy has been used to make entrepreneurs indifferent over which technology they use.[27]

Proposition 1 *Given constant $a \equiv a^H - \tau = a^L + \delta\varsigma$, there exists a continuum of steady state equilibria, $(q^\star, K^\star, B^\star)$, with associated $u^\star$, indexed by $\gamma \in [0, 1]$, where*

$$\left(\frac{B}{K}\right)^\star = \frac{\phi\lambda - \phi + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}}{R - 1} \tag{17a}$$

$$u^\star = \frac{1}{R}\Psi'\left(\frac{\bar{K} - K^\star}{m}\right) = \frac{R-1}{R}q^\star \tag{17b}$$

$$u^\star = \frac{\pi\left[a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right] - \phi(1-\lambda)(1 - R + R\pi)}{\pi\lambda + (1-\lambda)(1 - R + R\pi)}. \tag{17c}$$

Given Assumptions E and F,[28] the values for $(B/K)^\star$ and $u^\star$ in Equations (17a) and (17c) are positive. For any combination of values of γ and a, this steady state is unique: the assumptions on the savers' production function make the middle term of Equation (17b) decreasing (and continuous) in K, while the expression for $u^\star$ in the right hand side of Equation (17c) is given by a constant. Thus, given Assumption D, the two expressions for $u^\star$ cross only once.[29,30]

Once the government has effectively set a private "productivity target" for the entrepreneurial sector, a, through τ and ς, Equation (17a) says that in steady state the entrepreneur uses the amount of tradable output, $aK^\star$, together with (net of) the transfer (tax) from the government, $\frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}K^\star$, to repay the interest on the debt, $(R-1)B^\star$, and to replace the amount of energy capital that has depreciated in the period, $\phi(1-\lambda)K^\star$. As a result, the scale of operation of the entrepreneurial sector neither increases nor decreases.

Figure 1 provides a visual representation of different scenarios. The horizontal axis shows demand for fixed capital from the entrepreneurs from left to right and from the savers from right to left. Since the market for fixed capital clears, the sum of the two

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[27]If this were not the case then entrepreneurs would use either 100% low or high carbon energy capital and the steady state would feature only one of these technologies. The steady state would therefore be exactly that of Kiyotaki and Moore (1997).

[28]See Appendix A.1 for further assumptions used in obtaining the model's equilibrium.

[29]As Kiyotaki and Moore (1997), we focus only on these interior steady state equilibria. Note, however, that like in Kiyotaki and Moore (1997), there are another two steady states: (1) fixed capital price below savers' marginal product when using $\bar{K}$, so all fixed capital in hands of the savers; (2) fixed capital price and debt holdings both tending to infinity, all the fixed capital is in the hands of the entrepreneurs, and the price growth is such that next period collateral value is always sufficient to take on the required debt levels this period. We use debt renegotiation to ensure that the economy can converge back to the interior steady state, and we avoid consideration of these other steady states.

[30]In Appendix A.3, we show that we can refer the interested reader to Kiyotaki and Moore (1995) for the analysis of the stability of the system.

demands is equal to $\bar{K}$. The vertical axis consists of the net marginal product of fixed capital, which is constant at $a + c$ for entrepreneurs but decreasing with fixed capital usage for savers.

Were the debt enforcement problem absent, and absent any government policy (so no carbon taxes or green subsidies), the economy would be able to reach the first best allocation, E_{FB}, in which the entirety of the aggregate entrepreneurs' fixed capital holdings are used with high carbon energy capital. In this scenario, entrepreneurs are not constrained in the amount they can borrow. Thus, the marginal products of the two sectors are identical. In contrast, in the constrained economy too much of fixed capital is left in the hands of the savers and entrepreneurs have a higher marginal product than savers.

Consider two particular equilibria. In a world which only uses the high carbon energy capital (i.e. $\gamma = 1$), and with $a = a^H$ (i.e. no carbon tax), the equilibrium is given by $E_H^\star$, where the aggregate entrepreneurs' fixed capital holding is $K_H^\star$. On the contrary, the fully decarbonised equilibrium (i.e. $\gamma = 0$) with $a = a^L$ (i.e. no green subsidy), is $E_L^\star$, with corresponding $K_L^\star$.[31] It easy to show that the former equilibrium provides a larger share of fixed capital to the entrepreneurs, $K_H^\star$, compared to the latter. As a consequence, output, investment, borrowing and consumption are higher. Intuitively, having the government set a lower private productivity target for the entrepreneurial sector, this not only earns less revenue with respect to a higher private productivity target, but also has lower net worth. Thus, in general, entrepreneurs can borrow, invest and produce less. To clear the market, the demand for fixed capital by the savers must be higher in the decarbonised world, which requires a lower user cost. But a lower user cost is associated with a lower fixed capital price and thus with a lower net worth of the constrained sector, which translates into less collateral. Less collateral means lower investment and production, and so on in a vicious circle.

The amount of fixed capital used by the entrepreneurs, $K^\star(a, \gamma)$, for any $a \in [a^L, a^H]$, $\gamma \in [0, 1]$, is within the interval $[K_L^\star, K_H^\star]$ and is a monotonically increasing function of both the private productivity of the entrepreneurs' technology, a, and the share of fixed capital used in conjunction with high carbon energy capital, $\gamma \in [0, 1]$. Increasing either a or γ results in an higher average productivity of the fixed capital, and consequently in an higher net worth of the entrepreneurial sector. As a consequence, the representative entrepreneur can afford higher fixed capital holdings. The area of the triangle $HE_H^\star E_{FB}$ gives the minimum output loss of the constrained equilibrium relative to first best, while the remaining shaded area indicates the further maximum output loss caused by reducing the target private productivity, $a \in [a^L, a^H]$ (by increasing the carbon tax and decreasing the green subsidy), and reducing the share, $\gamma \in [0, 1]$, of aggregate entrepreneurs' fixed

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[31]These equilibria can be considered as the most extreme ones, as they correspond to the floor and ceiling values of $K^\star$ for the continuum of equilibria such that $a \in [a^L, a^H]$ and $\gamma \in [0, 1]$.
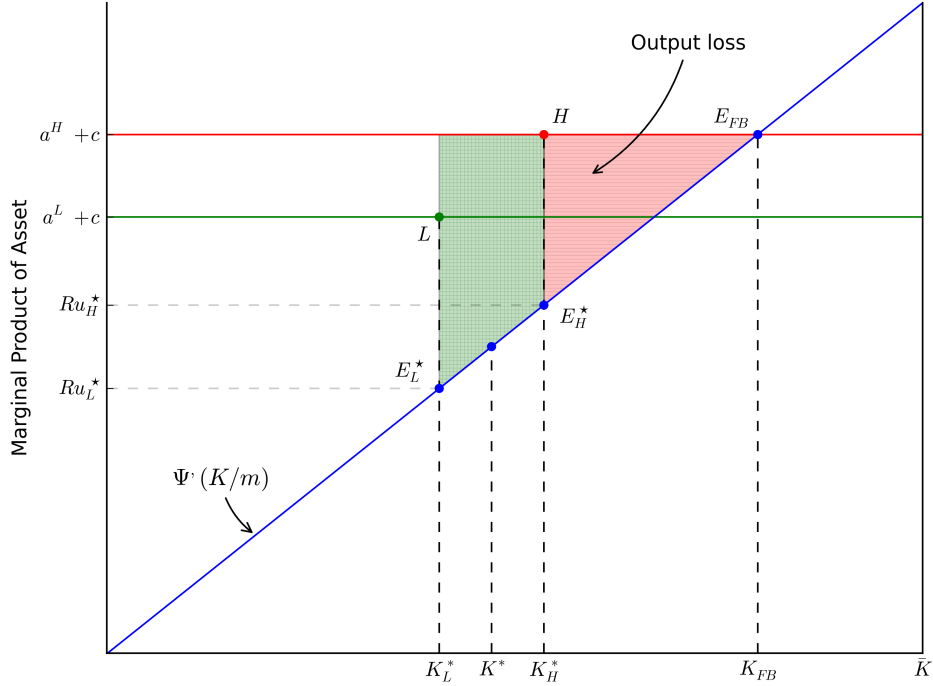
Figure 1: Comparison of steady states

capital holdings used with high carbon energy capital.[32]

4   Calibration

Kiyotaki and Moore (1997, Chapter III) present some simulations of a full Kiyotaki and Moore's (1997) economy under a certain parametrisation. Their "calibration" though is not suitable for our exercise because, in the absence of debt renegotiation, the economy can only return to steady state for extremely small negative shocks. And it turns out that relying too heavily upon debt renegotiation ruins the story that models of this sort tell: debt renegotiation makes the world increasingly "classical" in that it eliminates the debt overhang and the persistent negative effects from the shock; all the adjustment happens at

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[32]Appendix A.4 presents an interesting result: it is possible that a higher steady state investment flow in zero carbon energy capital can be achieved by raising the proportion, γ, of high carbon energy capital used i.e. we may see higher absolute investment levels in zero carbon energy if there is also some investment in high carbon energy capital. The relationship can be non-monotone. This is because the higher the share γ of entrepreneurs using high carbon production and investing in high carbon energy capital, the higher is the net productivity of the fixed capital, and the higher are tax revenues and so the per capita transfer. Entrepreneurs have higher net worth and so can hold more of the fixed capital. This potentially allows the entrepreneurs who are using zero carbon production and investing in zero carbon energy capital to borrow more, invest more and produce more. Appendix A.4 also shows that this non-monotonic relationship is due to the presence of credit frictions. However, this result does not hold given the calibration we use (see Section 4), under which the highest steady state level of zero carbon investment is achieved with $\gamma = 0$.

$t = 0$, and after this the economy is able to ramp up investment and return to steady state in a similar manner to a neoclassical growth model exhibiting conditional convergence.

Therefore we need an alternative calibration strategy, which is developed in this section. We define several parameters based on definitional convenience, try to match the energy share of the economy, broadly match the experience of the 2008-09 Financial Crisis, and then ensure that only shocks approaching the severity of the Carbon Bubble itself need to use the debt renegotiation process. We check that this calibration satisfies all the assumptions made on parameter restrictions from Section 3 and from Appendix A.1. More details and analysis of the simulations run to work out this calibration can be found in Appendix A.5.[33]

4.1 Savers production function, definition of welfare, and time in the model

We start by following Kiyotaki and Moore (1997) and impose the following linear structure for the user cost,

Assumption C $u(K) = \frac{1}{R}\Psi'\left(\frac{\bar{K}-K}{m}\right) \equiv K - \nu.$

Integrating the savers' production function up, means that we have some constant production flow, independent of the level of fixed capital used by the savers, that must be calibrated in order to look at aggregate production. For definitional convenience we assume that this constant is such that steady state consumption flow is the same for both individual savers and entrepreneurs.[34]

We assume $\beta = \beta' - \epsilon$ for infinitesimally small $\epsilon > 0$. This means that, although the savers are more patient than the entrepreneurs, for any practical calculation, their discount factors are the same. The utilitarian social welfare function maximised by the policymaker at $t = 0$ is then

$$\sum_{t=0}^{\infty}\left(\beta^t x_t + m(\beta')^t x_t'\right) \approx \sum_{t=0}^{\infty}\left(\frac{1}{R}\right)^t (X_t + X_t').$$

Therefore, policy is chosen by the policymaker to maximise the present discounted value of all future "Net National Income" flows in the model.

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[33]We tried many different parameter combinations in generating this calibration. Quantitatively these choices clearly have an impact, and we determined the marginal impact of changing each parameter upon various aspects of the solution. However, qualitatively there is very little dependence upon the calibration: our general conclusions about policy effectiveness are robust to the particulars of this calibration.

[34]This requires that we choose which steady state that we mean: either the initial steady state prior to the Carbon Bubble which involves both high carbon and zero carbon energy capital, or the decarbonised steady state which the economy converges to after the Carbon Bubble announcement. For convenience, we use the decarbonised steady state.

We follow Kiyotaki and Moore (1997), and set the depreciation rate of energy capital, $\lambda = 0.975$, and the interest rate, $R = 1.01$, so that time periods can be interpreted as quarter years. This corresponds to a depreciation rate of 10% per annum for energy capital, and an annual interest rate on debt of 4%.

Finally, we normalise productivity, $a^H = 1$, and we set the non-tradable output share equal to the tradable output share using zero carbon energy capital, i.e. $c = a^L$.

4.2 The energy sector

According to Newell et al. (2016), and to energy mix figures from EIA (2016, Table 1.2), fossil fuels represent around 80% of energy generation. This gives us a calibrated value of $\gamma = 0.8$.

The EIA (2015, Table 1) provide figures on the "total system levelized costs of electricity", which we apply to their energy mix figures to estimate that fossil fuel generation costs around 10% less per unit of energy supplied. This allows us to set $a^L = 0.9 \times a^H = 0.9$.

Both fossil fuels and alternative energy generating capacity exist in the data, and we can only replicate this in the model if their net private productivities, after taxes and subsidies, have been equalised. We choose the subsidy induced distortion parameter, δ, such that the optimal subsidy rate from the planner's perspective in the initial steady state is $\varsigma = 0$.[35] This means that net private productivity is $a = a^L$, and carbon tax $\tau = a^H - a^L$.

We use the steady state value of high carbon energy capital as a percentage of total capital, $\gamma \phi K_F^\star / \left(\phi K_F^\star + q_F^\star \bar{K} \right)$, as a calibration target. Averaging and rounding figures from Dietz et al. (2016)[36] and EIA (2016, Table 1.7)[37] gives us a target steady state value of high carbon energy capital as a percentage of total capital value of 4.5%. This is achieved, conditional on the other parameters of the model, by adjusting $\bar{K}$.

As discussed in Section 1, there is a carbon budget of allowable future emissions. The

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[35]This means that the optimal subsidy from the planner's perspective in the decarbonised steady state is actually negative if we allow negative distortions because the distortion is so large. This is because this optimal $\varsigma = 0$ calibration target requires a distortion large enough to offset the benefits of a higher private productivity applied to the 80% of output that is produced using the undistorted high carbon energy capital. However, we do not allow negative distortions and so the optimal subsidy from the planner's perspective in the decarbonised steady state is again zero. See Appendix A.2 for more information on the subsidy.

[36]Dietz et al. (2016, page 3) say that "the total stock market capitalization today of fossil fuel companies has been estimated at US$5 trillion", and that the "Financial Stability Board ... puts the value of global non-bank financial assets at US$143.3 trillion in 2013": 5/143 = 3.5%.

[37]The 20-year average for Energy Expenditures as Share of GDP, in EIA (2016, Table 1.7), is 7.4%, so the value of fossil energy assets should represent 80% × 7.4% = 5.9% if the US figures are a good guide to the global figures, and if these assets have the same term as the average of other assets (less if they are shorter duration). The calibrated value is therefore 0.5 × (3.5% + 5.9%) = 4.7%, rounded to 4.5%.

current stock of energy capital is estimated to embody future emissions that exceed this carbon budget. Carbon Tracker Initiative (2013, page 15) says "an estimated 65-80% of listed companies' current reserves cannot be burnt unmitigated", while IEA (2012, page 3) says "No more than one-third of proven reserves of fossil fuels can be consumed", if we are to remain within the 2°C limit for climate change (with some probability). Consistent with these estimates (since the financial value of the lower quality reserves that should be "stranded" first will be less than the financial value of high quality reserves, per unit carbon), let us assume that the total value that must be written-off is 50% of the value of high carbon energy capital. Since we are assuming that the value of high carbon energy capital is 4.5% of total capital value, this means that the Carbon Bubble scenario will involve a write-off of productive capital which has a value of 2.25% of total capital value.

4.3 The financial crisis

The financial crisis began with the realisation that the fundamental value of subprime mortgages (and the CDOs into which they were bundled) was much lower than had previously been recognised. Hellwig (2009) estimated that the total value of subprime mortgages outstanding was $1.1tn in the second quarter of 2008. Comparing to the energy sector estimates above, the subprime mortgage sector was approximately equal in value to 20% of the fossil fuel sector. In order to calibrate our model, in some sense, to the experience of the financial crisis, we alter the model slightly to remove the productivity differential between the high and zero carbon energy technologies,[38] and assume that a "financial crisis" can be precipitated by writing-off 20% of the high carbon energy capital (or equivalently since productivities have been equalised, by writing-off 16% of total energy capital). In terms of a percentage of the value of total capital in the economy, our financial crisis simulation is precipitated by writing-off 0.9% of total capital value. The Carbon Bubble is therefore assumed to be two and a half times the size of the financial crisis.

We calibrate to the impact of the financial crisis on output and upon asset values. Data from FRED[39] suggests that annual percentage changes in "Constant GDP per capita for the World" were consistently just below 3% prior to the financial crisis, but fell to less than -3% when the crisis struck. The Kiyotaki and Moore's (1997) model is a steady state model, with no growth in per capita incomes, so this data suggests that the financial crisis scenario in the model should involve a fall in output of around 6%. The asset impact of the financial crisis was large relative to the approximate 0.9% fall which precipitated it. The loss of 0.9% should represent a relatively mild adverse event to a well diversified

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[38]Both technologies are assumed to have productivity equal to a^L, and we apply no carbon taxes or zero carbon subsidies, so net private productivity is $a = a^L$, and per capita transfer is zero, $g = 0$.

[39]Accessible at https://research.stlouisfed.org/fred2/series/NYGDPPCAPKDWLD.

investor. Instead we saw the S&P500 decline by 40%[40] and corporate bond spreads rise.[41] On the other hand, the effective value of public assets, inferred from government bond prices, rose as interest rates fell (at least in non-Eurozone periphery countries). A back of the envelope calculation[42] suggests that a well diversified investor experienced a fall in asset values of around 20%.

As well as calibrating the parameters m, ϕ, π and ν such that an energy capital write-off of 0.9% of the total steady state value of all the capital in the economy precipitates dynamics that see capital values fall by 20% and output fall by 6%, we also insist that only shocks approaching 90% of the required Carbon Bubble write-off require debt renegotiation.[43]

5 Dynamic Simulations

Now that we have developed the analytic framework, and calibrated the model, in this section we turn to the issue of the Carbon Bubble. Here we imagine a scenario loosely modelled upon the current state of the global economy's capital stock: efforts have been made to provide incentives to develop and deploy zero carbon energy capital, but at the global level, the stock of high carbon energy capital is not falling; global reserves of fossil fuels are more than sufficient to exceed some carbon budget; and energy capital investments that lock the economy into high carbon patterns of use are still being made. Therefore, as described in Section 4.2, in the periods prior to the start of our dynamic simulations, we consider the global economy to be in a steady state in which the private returns from investment in both high and zero carbon energy capital are equalised via the imposition of a carbon tax, but that we are in a steady state characterised by $\gamma = 0.8$. The values for the fixed capital used by, and the debt holdings of, entrepreneurs in this steady state are $K_F^\star$ and $B_F^\star$ respectively. Therefore, the steady state high carbon energy capital stock is $Z^H = \gamma K_F^\star$.

At the start of our simulation, the planner makes an announcement: future investment in high carbon energy capital is banned, and, as explained in Section 4.2, the total future

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[40]Percent change from 1 year ago, viewed in quarterly timesteps, from https://research.stlouisfed.org/fred2/series/SP500

[41]For example, "Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity" rose from just over 1.5% prior to the financial crisis, to more than 5.5% at the height of the crisis, see https://research.stlouisfed.org/fred2/series/BAA10YM.

[42]Assumes public assets are 20% of total assets, and private assets are funded 50:50 debt and equity. Public assets grow in value by around 10% (calculated as 4% income, plus an interest rate fall of 2% on a bond portfolio with a discounted mean term of 5 years). Equity falls by 40% (with no income - the S&P500 is a total return index), and corporate debt falls by 10% (calculated as income of 4%, plus an interest rate rise of 2% on a bond portfolio with a discounted mean term of around 5 years).

[43]Since more than one combination of parameters delivered the required calibration, we chose the one with the shortest cycle length.

use of high carbon energy capital is limited to 50% of the total use implied by the value of the current stock.[44] The carbon production in period t is linear in the amount of high carbon investment energy capital, Z_t^H, used in production at t. Since we can choose units, let this amount of carbon production also equal Z_t^H for simplicity. Effectively, the policymaker announces a carbon budget, $\bar{S}$, which satisfies

$$\bar{S} = 50\% \times \sum_{t=0}^{\infty} \lambda^t Z^H = 50\% \times \frac{\gamma K_F^\star}{1 - \lambda}. \tag{18}$$

In the first period, $t = 0$, timing is as described in Figure 2. At the time of the announcement (the very start of the $t = 0$ period), asset and credit markets have just closed, so that $K_0 = K_F^\star$ and $B_0 = B_F^\star$ are state variables. The announcement affects prices which hugely impair entrepreneurs' balance sheets. However, after the announcement there is a debt renegotiation opportunity which changes B_0 to $B_{0+} \leq B_F^\star$, but cannot alter K_0: savers and entrepreneurs adjust their credit positions given the entrepreneurs' net worth implied by their fixed real holdings of the asset. Renegotiation does not take place if the economy can converge back to the steady state given $B_{0+} = B_F^\star$, because savers have no incentive to renegotiate. However, if the economy cannot converge back to its steady state then both parties have the incentive to renegotiate the outstanding value of the debt.[45] The debt level retained by the entrepreneurs is reduced to $B_{0+} < B_F^\star$, where this B_{0+} value is the maximum value for debt levels consistent with the economy being able to reach steady state. Production then takes place with entrepreneurs using $Z_0^H = \gamma K_F^\star$ high carbon energy capital.[46]
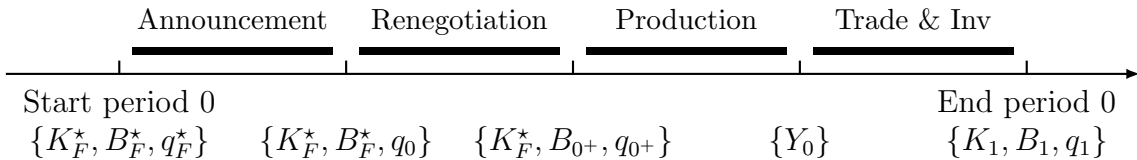

| Announcement | Renegotiation | Production | Trade & Inv |

Start period 0
$\{K_F^\star, B_F^\star, q_F^\star\}$ $\{K_F^\star, B_F^\star, q_0\}$ $\{K_F^\star, B_{0+}, q_{0+}\}$ $\{Y_0\}$ End period 0 $\{K_1, B_1, q_1\}$

Figure 2: Timing

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[44]If no further investment is made, and the current stock is fully used, depreciation is at rate $1 - \lambda$.

[45]Entrepreneurs would always like to reduce their debt. If the economy cannot converge back to the interior steady state, then the outside option for the representative saver is to accept the economy converging to the steady state with no fixed capital in the hands of the entrepreneurs. Savers prefer to reduce their debt rather than accept this, and so engage in renegotiation. They write-off the minimum quantity of debt such that the interior steady state can be reached. We refer an interested reader to Appendix A.5 for how renegotiation influences the dynamics following a shock, and to Appendix A.6 for more information about the renegotiation process.

[46]Note that alternatively, entrepreneurs could leave some of the fixed asset unused in this first period, since markets are closed and it cannot be traded back to the savers. We checked and verified that this option is not optimal for the entrepreneurs, as the reduced consumption in the first period (and the lower net worth) is enough to offset the positive effect of an increased remaining carbon budget.

At the end of the period, savers and entrepreneurs receive their output, the asset and credit markets open, and agents make consumption and investment decisions. Entrepreneurs must also decide the share $\rho \in [0,1]$ of the remaining $\lambda \gamma K_F^\star$ high carbon energy capital they will use, and the rate, $1 - \lambda_H \in [1 - \lambda, 1]$, at which they will retire these goods. These choices are a function of prices, q_1, and determine the values of the state variables for the next period: K_1 and B_1. The choices of (ρ, λ_H) are clearly not independent, and must satisfy the carbon budget announced by the social planner:

$$\bar{S} = \gamma K_F^\star + \sum_{t=0}^{\infty} \rho \lambda \gamma K_F^\star \lambda_H^t$$

$$\text{i.e. } \lambda_H = 1 - \frac{\lambda \rho (1 - \lambda)}{\lambda - 0.5}.$$

While λ is a structural parameter which determines the depreciation rate of energy capital, $\lambda_H \in [0, \lambda]$ is a choice by the entrepreneurs: they can choose to retire their high carbon goods at a rate faster than the depreciation rate in order to allow them to use more high carbon goods initially. Entrepreneurs make this choice optimally, and it turns out that they always choose $\rho = 1$ i.e. they produce at the maximum rate that they are able to in the initial periods, and accept a high depreciation rate for their high carbon energy capital.[47]

Figure 3 gives an overview of the responses of the economy to implementing $\bar{S}$ at $t = 0$.[48] It shows movement in $K/K^\star$, $Y/Y^\star$, $B/B^\star$, $q/q^\star$, and $I/I^\star$ i.e. the ratios of entrepreneurs' fixed capital, total output, investors' debt, price of fixed capital, and aggregate investment flow, to their respective decarbonised steady state values.

As soon as the carbon budget is announced, the price of fixed capital collapses by approximately 39%. Without the debt renegotiation mechanism, the economy would collapse following a shock of this magnitude, and never return to the interior steady state. With this mechanism, at $t = 0^+$ the entrepreneurs are able to renegotiate the value of their remaining debt down to approximately 95% of the debt in the high carbon steady state at $t = 0$,[49] and the economy can return to the interior steady state.

The shock and the renegotiation have opposite effects on the net worth of an entre-

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[47]This implies that the remaining stock is retired at rate $1 - \lambda_H \approx 5.1\%$ per period, more than twice as fast as the depreciation rate.

[48]The dynamics of the model are solved for using numerical simulation of the forward shooting method. Details of the algorithm are given in Appendix A.6 but the rough approach is to guess the discontinuous change in the fixed capital price following the shock and iterate the economy forward through time to see if it converges back to steady state. If the price eventually explodes (tends to zero), the initial guess is revised downward (upward). This "guess and check" procedure is repeated until the fixed capital price is within some tolerance level of its steady state value at the end of the projection.

[49]In the middle panel of Figure 3, $B/B^\star$ starts from the post renegotiation value at $t = 0^+$, while the bullet point represents the pre-shock value, i.e. the ratio of the steady state value of debt before the write-off of high carbon energy capital, $B_F^\star$, to the decarbonised steady state value, $B^\star$.
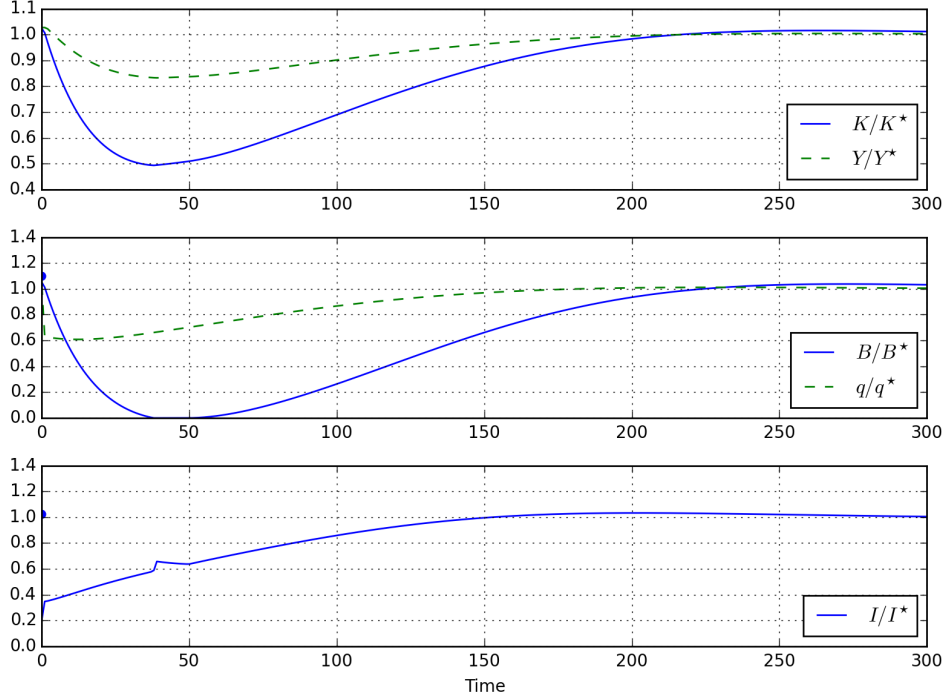
Figure 3: The burst of the bubble

preneur. The former pushes the value of the collateral (and in future the tradable output available to the entrepreneur) down, but the latter decreases the debt repayments required. The net effect though is that entrepreneurs repair their balance sheets rather than increase investment in replacement productive capacity. Note that, since the entrepreneurs use the full fossil fuel steady state level of high carbon energy capital in production in the first period, that they receive the full fossil fuel steady state level of income at the end of the first period. Further, due to debt renegotiation, their required debt repayment at the end of the first period is lower than in the previous steady state. So the cashflow position of the entrepreneurs is improved relative to the previous fossil fuel steady state period, and the fixed capital price is lower because of the shock, so it is conceivable that entrepreneurs would use their improved cashflow position to increase their holdings of the now cheaper fixed capital. However the price fall has damaged the entrepreneurs' balance sheets such that, even with their improved cashflow position, they choose to sell fixed capital back to the savers and repay debt i.e. as discussed in Section 2, the fall in asset values precipitates forced sales to ensure borrowing and collateral requirements are aligned, but this forced sale causes prices to fall again which causes further forced sales, and further price falls, and so on: we see fire-sale dynamics. The process stops when fixed capital becomes so unproductive in the hands of the savers that the entrepreneurs can once again afford the lowered price, and the economy recovers towards the new steady state.

In the dynamics associated with the announcement of the Carbon Bubble, the entre-

preneurial sector deleverages, reducing both assets and debt, until around period 40 (10 years after the announcement if we interpret periods as quarters). At this point debt levels reach zero,[50] and the holdings of fixed capital in the hands of the entrepreneurs are around 48% of steady state levels. Since a large share of fixed capital is employed in the low productivity sector, output collapses. Output bottoms out at almost 19% below the previous steady state value, and at more than 16% below the new steady state value. Investment levels also fall markedly, by around 60% - even though the economy is in short supply of energy capital.[51] When debt is constrained to stay greater than or equal to zero, we see that there is a spike in investment levels: this is because entrepreneurs would really like to continue deleveraging, reducing fixed capital holdings and debt levels further, but they cannot. Since they cannot pay down any more debt, the impatient entrepreneurs do not have anything else to do with their cashflow, other than to increase investment. The economy takes approximately 200 periods to fully recover from the announcement of the Carbon Bubble and stabilise around the new decarbonised steady state.

In the next subsections, we consider four possible additional actions for the planner that mitigate some of the welfare loss associated with writing-off the high carbon energy capital. The first consists of a tax funded transfer of entrepreneurs' debt; the second of a tax funded investment subsidy; the third is a government guarantee which relaxes credit constraints; and the last involves deceiving the market about the amount of high carbon energy capital that is permitted to be used.

5.1 Tax Funded Transfer of Investors' Debt

The entrepreneurial sector is credit constrained, and following the imposition of climate policy, it is burdened with excessive debt relative to its assets. Perhaps the planner can achieve a better outcome if the burden of this debt is shifted to an economic actor who is not credit constrained. We suppose that the planner first announces the carbon budget $\bar{S}$, and that renegotiation takes place exactly as in no-policy scenario. After renegotiation takes place, the social planner takes over some share $\omega \in [0, 1]$ of the remaining entrepreneurs' debt, B_{0+}, and funds the debt repayments through lump-sum taxes.

The social planner repays debt, $B_0^G = \omega B_{0+}$, by raising a constant per capita tax, τ^G,

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[50]We impose a constraint requiring debt to not fall below zero, which is binding in this simulation. This constraint, when it turns out to be binding, lowers the fixed capital price over the whole simulation, relative to allowing negative debt.

[51]In the lower panel of Figure 3, the line represents investment in zero carbon energy capital weighted over its decarbonised steady state level. However, the scatter represents the ratio of the steady state level of investment at $t = 0$, which consists of both low and high carbon energy capital, to the decarbonised steady state level, which by definition is composed by investment in zero carbon energy capital alone.

over $T = 100$ periods.[52] This implies

$$B_0^G = \sum_{t=1}^{T} (1+m)\,\beta^t \tau^G = (1+m)\,\tau^G \frac{\beta}{1-\beta}\left(1-\beta^T\right).$$

Then, in all periods $t \in [1, T]$, the social planner receives tax income of $(1+m)\tau^G$ from entrepreneurs and savers, repays the accumulated debt to the savers, RB_{t-1}^G, and raises new debt from the savers, B_t^G, according to

$$B_t^G = (1+m)\,\tau^G \frac{\beta}{1-\beta}\left(1-\beta^{T-t}\right).$$

The social planner chooses the value of B_0^G (or equivalently, the share ω or the value of τ^G) to maximise our measure of social welfare. Welfare is increasing in ω up to $\omega = 90\%$. The welfare gain over 200 periods induced by implementing this optimal $\omega = 90\%$ policy is $+5.2\%$. The same analysis is conducted in the decarbonised steady state and we find that the optimal debt transfer policy is $\omega = 60\%$, and the welfare increase is only 0.6%. Thus, although the social planner would wish to implement a debt transfer policy in normal times anyway, we can conclude that the Carbon Bubble makes the planner want to implement more debt transfer, and that this debt transfer policy provides greater benefit. There is more need for such a policy in the Carbon Bubble scenario. Figure 4 gives an overview of the responses of the economy to implementing the optimal policy, $\omega = 90\%$, at $t = 0$.[53]

Following the policymaker's actions, the price of fixed capital increases by around 17%. As a consequence of the debt transfer, entrepreneurs' borrowing starts at 10% of the steady state value, and increases to 18% over the first period as the entrepreneurs use their cashflow to balance their fixed capital holdings with their energy capital stocks. This involves increasing their fixed capital holdings by around 2% and increasing investment levels to twice the steady state level. The entrepreneurs start to take on debt, to operate with more of the fixed asset than in steady state, and to build up stocks of zero carbon energy capital eventually to a level above their steady state value. Prior to $t = 100$, the economy is heading towards a "steady state" with taxes, which is characterised by lower: output; entrepreneur fixed asset holdings; debt; asset values; and investment levels, than in the true steady state. Once the taxes and government intervention in the debt markets ceases at $t = 100$ then the economy converges to the model's true steady state. Over

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[52]The choice of $T = 100$ periods is relatively arbitrary. A less arbitrary choice would have been the issue of perpetuities, but this would have changed the steady state, which is problematic since we are running a numerical rather than analytic analysis. This length was chosen because 25 years is a common term for new issues of government debt.

[53]See Appendix A.7 for the equivalent figure following the optimal policy of $\omega = 60\%$ from the decarbonised steady state.
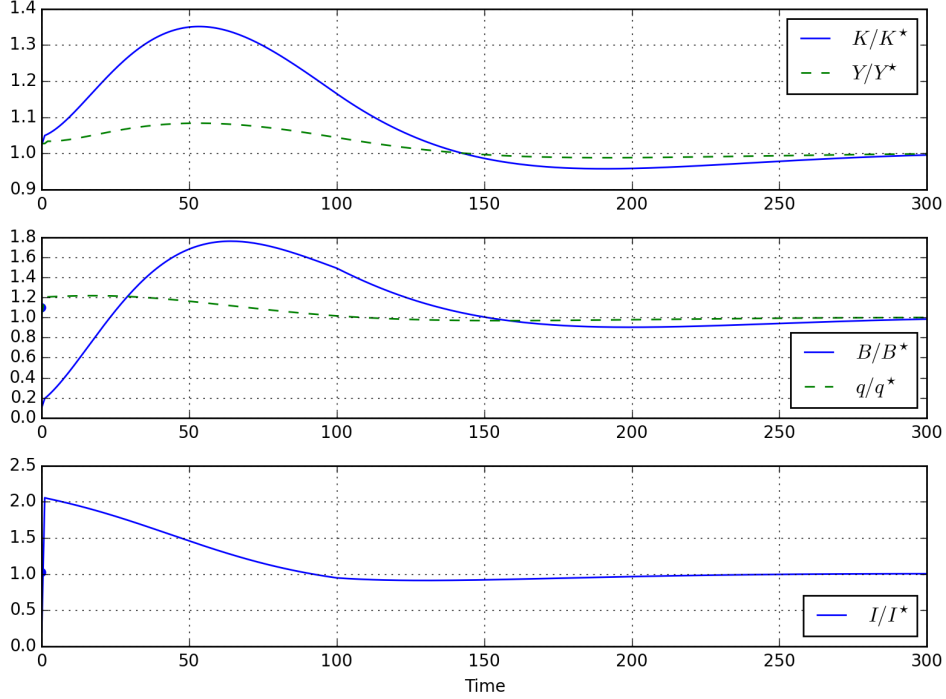
Figure 4: Transferring entrepreneurs' debt

the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 50% higher than in the no-policy scenario, and welfare is more than 5% higher.

Implementing this debt transfer policy does not however represent a Pareto improvement over the Carbon Bubble with no-policy scenario. The welfare improvement is composed of $+73\%$ for entrepreneurs, and -11% for savers. Savers have limited upside from this policy, but they still pay taxes to fund it.

5.2 Subsidy

After banning new high carbon investment and announcing the carbon budget that constrains the use of existing high carbon energy capital, in this policy scenario the social planner also announces an increased level of subsidy paid to entrepreneurs to boost the private productivity of their production. This subsidy, for simplicity, will linearly decrease back to its optimal level over 100 periods.[54]

As we described in Footnote 13, subsidies in a pure Kiyotaki and Moore's (1997) economy serve to relax credit constraints and could be used to achieve the first best. In order to avoid this, we introduced a black-box "distortion" such that the optimal subsidy in steady state was zero. Therefore in the absence of some issue like the Carbon Bubble,

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[54]This length was chosen for comparability with the debt transfer policy.

a subsidy is welfare destroying because of this distortion.

Since new investment in high carbon energy capital is banned, there is no incentive problem with simply paying a subsidy, independent of the technology used, that increases the private productivity of all production in the entrepreneurial sector. Alternatively the extra subsidy could be targeted only to output produced using zero carbon energy capital. Targeting the subsidy would mean that it was more privately profitable to use zero carbon energy capital than the remaining permitted stocks of high carbon energy capital. The only difference between these policies is that, for a given level of subsidy, the targeted subsidy provides a lower boost to entrepreneurs' incomes.[55] Subsidies, targeted and untargeted, both serve to relax credit constraints in a very similar way. Given the similarity, we only show results for the untargeted subsidy policy.

We find that with the Carbon Bubble, there is a clear optimal subsidy which boosts private productivity by, initially, around 45%. Figure 5 shows the dynamics following the carbon budget announcement, when the planner implements this optimal subsidy program.[56]

The decreases in the price of fixed capital is now approximately 2%. We still see fire-sale dynamics, but they are much less severe. In particular, investment is quickly at levels greater than its steady state level. The entrepreneurs' fixed capital holdings and hence collateral are higher than in the no-policy scenario, therefore debt does not need to decrease as much as before, and entrepreneurs have more funds to invest to replace the lost high carbon energy capital. Over the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 40% higher, and welfare is more than 3% higher, than in the no-policy scenario.

Again, this is not a Pareto improvement with respect to the Carbon Bubble with no-policy scenario. Savers are worse off (-7%) because of the increased tax they have to pay to fund the subsidy, whereas the entrepreneurs, who benefit from the subsidy, have a welfare increase of $+49\%$.

5.3 Government Guarantee

In this policy scenario, we model a government guarantee which reassures lenders and relaxes credit constraints. Specifically, we imagine a guarantee that effectively multiplies

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[55]It is possible that entrepreneurs would not want to use high carbon energy capital given their lower private productivity under the targeted subsidy, and in which case they would accept lower levels of output and sell fixed capital back to the savers. However, this is not optimal behaviour given our parameters and scenario.

[56]See Appendix A.7 for the equivalent figure following the implementation of this +45% subsidy from the decarbonised steady state. This shows that implementing the subsidy from steady state also creates an economic boom, but such a boom is not welfare enhancing relative to steady state, because it involves a small increase in output and a larger increase in investment, with a fall in aggregate consumption. In contrast, in the Carbon Bubble scenario, this policy is welfare enhancing relative to no policy.
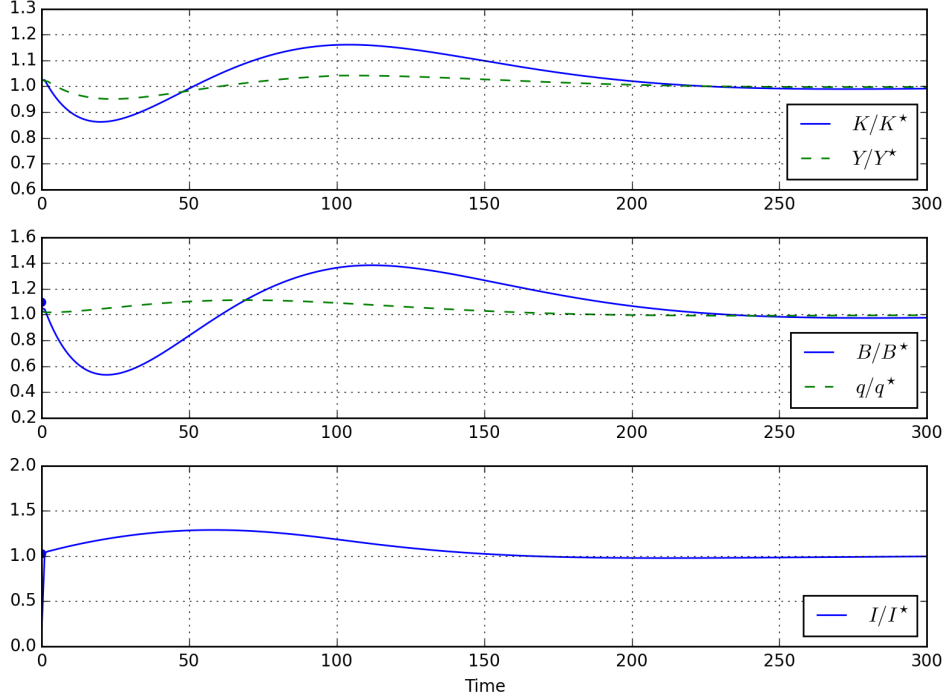
Figure 5: Subsidising entrepreneurs

an entrepreneur's collateral: for a given quantity of collateral, the entrepreneurs can borrow more. Analytically, Equation (4) is modified to

$$b_t \leq \frac{q_{t+1}k_t}{R} \times (1 + gtee_t).$$

Along the perfect foresight equilibrium path, the government never needs to pay out anything related to this guarantee. It therefore does not alter the flow of funds equation, Equation (16). Further, it does not alter the equations from the savers' optimality conditions, Equations (11) and (12). The only equation that changes is the equation of motion for capital, Equation (15), which becomes

$$K_t = (1 - \pi)\lambda K_{t-1}$$
$$+ \frac{\pi}{q_t + \phi - \frac{q_{t+1}}{R}(1 + gtee_t)} \left[(q_t + \phi\lambda + a)K_{t-1} - RB_{t-1} + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}K_{t-1} \right].$$

Immediately after the same debt renegotiation as in the no-policy scenario, the planner announces a linearly reducing guarantee which reaches zero after 100 periods.[57] The steady state to which the economy is converging is therefore unchanged. Figure 6 shows

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[57]Again, 100 periods is chosen for consistency with the previous two policies. This implies $gtee_t = \max\{0, (100 - t)/100\} \times gtee_0$.

the dynamics following the carbon budget announcement, when the planner implements a guarantee starting at $gtee_0 = 20\%$, which is approximately optimal given our parameters.
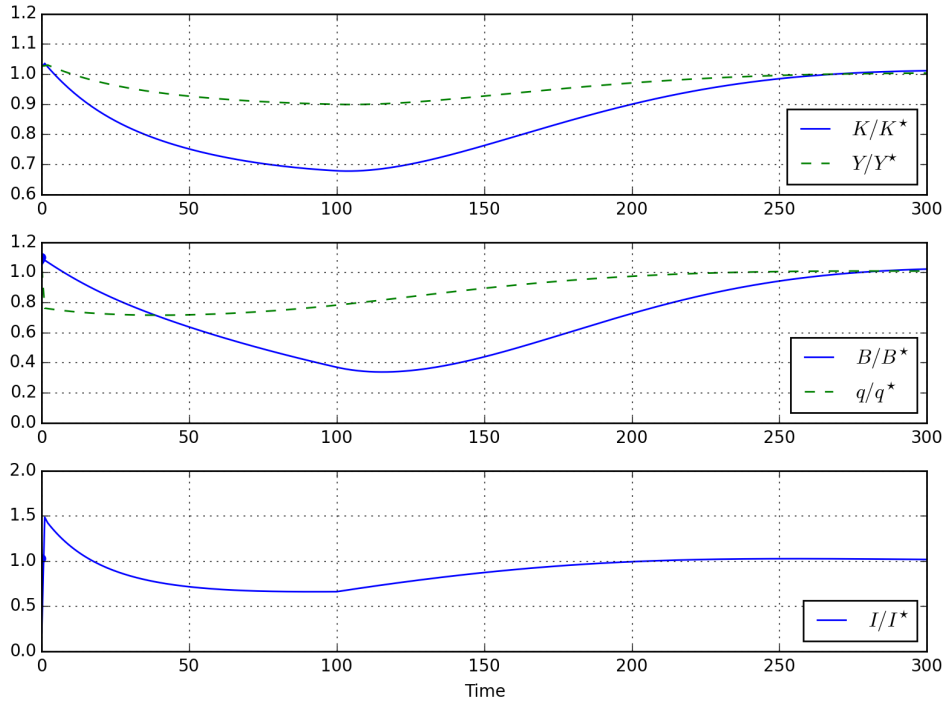

Figure 6: Providing a government guarantee

The decreases in the price of fixed capital is now approximately 25%. The government guarantee allows entrepreneurs to have access to more debt, and essentially they re-borrow all the debt which was written off in the renegotiation such that debt levels are 1% higher in the first period than in the initial steady state. The entrepreneurs use the proceeds of this borrowing to increase their fixed capital holdings by 1%, and increase their investment levels by almost 50%. Thereafter, while we still see fire-sale dynamics, they are much less severe, with investment levels staying relatively high. Over the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 8% higher, and welfare is almost 3% higher, than in the no-policy scenario.[58]

Contrary to the first two policies we have examined, this policy does produce a Pareto improvement with respect to the Carbon Bubble with no-policy scenario. Savers are basically indifferent (+0%) and entrepreneurs (+19%) are better off. This is because this policy relaxes credit constraints at no cost. This may of course be unrealistic.

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[58]Appendix A.7 shows the equivalent figure following the implementation of the optimal guarantee from the decarbonised steady state. This optimal guarantee starts from only 1.5% (rather than the 20% optimum in the Carbon Bubble scenario), and provides an aggregate welfare benefit of only 0.005% (rather than the 3% gain in the Carbon Bubble scenario). So again, like the debt transfer policy, the planner wants to implement this guarantee policy to some degree in normal times, but wants to do much more, and sees much greater benefit, in scenarios like the Carbon Bubble.

5.4 Deception

Here we consider a different possible action for the planner: the planner can vary the amount of current high carbon energy capital that it tells the market is allowed to be used, $\hat{S}$. The planner announces $\hat{S} \neq \bar{S}$; the entrepreneurs respond by choosing the optimal proportion of high carbon energy capital to use initially, $\rho = 1$ as before, and hence a depreciation rate for the remaining high carbon energy capital of

$$\lambda_H = 1 - \frac{\lambda(1 - \lambda)}{\lambda - (1 - \hat{S}/S)}$$

where $(1 - \bar{S}/S) = 0.5$ under an honest planner. For $\hat{S} > \bar{S}$, and thus $(1 - \hat{S}/S) < 0.5$, the economy's actual carbon budget, $\bar{S}$, is used at some time T. When this happens, $\bar{S}$ is revealed to all agents and the entrepreneurs are compelled to leave unused their remaining high carbon energy capital: high carbon production is abruptly banned in a desperate attempt to avoid catastrophic climate change and consequential societal collapse.

Why would the social planner want to announce $\hat{S} \neq \bar{S}$? In a canonical growth or business cycle model, the social planner does not have any incentive to lie: stating an $\hat{S} > \bar{S}$ would cause a welfare destroying discontinuity in consumption across the period in which $\bar{S}$ is revealed. In this model, on the contrary, overstating the actual carbon budget limits the fall in the price of fixed capital and thus the decrease in the value of the collateral.[59] This allows higher investment in zero carbon technology, and potentially generates enough productive capacity between period 0 and T, when $\bar{S}$ is revealed, to mean that the present value of consumption flows is higher under deception.

There are four qualitatively distinct regions of the solution space as a function of $\hat{S}$:

1. For very small initial write-offs (i.e. the policy maker announces a very large carbon budget, $\hat{S} >> \bar{S}$) there is no need for debt renegotiation on the first announcement. However, the true carbon budget quickly runs out and this causes a large shock at T. Debt renegotiation is needed for the economy to cope with this second shock.

2. For intermediate levels of the announced carbon budget, no debt renegotiation is required on the first announcement. Again, the true carbon budget runs out before the carbon budget that the entrepreneurs thought they were working with, which causes a negative shock at T. However this shock is not as severe as that in the first region, and no debt renegotiation is required at T.

3. For relatively honest carbon budgets, $\hat{S} \gtrsim \bar{S}$, the announcement necessitates debt renegotiation as in the no-policy scenario. The negative shock at T, when the true

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[59]In addition to risk neutrality meaning that consumption discontinuities are not welfare destroying here.

carbon budget runs out, is mild enough to require no further debt renegotiation.

4. For unnecessarily aggressive carbon budgets, $\hat{S} < \bar{S}$, the announcement necessitates debt renegotiation. The true carbon budget is never exhausted in this case.[60]

These four regions are clearly seen in Figure 7, which shows the welfare induced by the policymaker's choice of $\hat{S}$. We see a clear optimum policy in Region 2, and we see that too little written-off means the second announcement is overly costly, whereas too much written-off means the first announcement is too costly.[61]
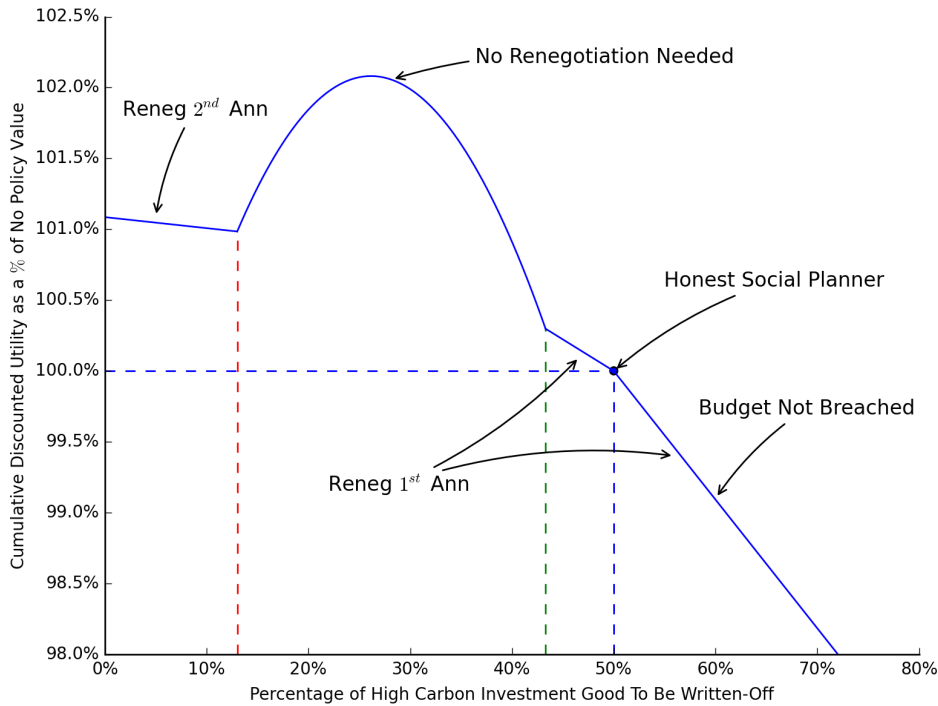
Figure 7: Welfare against percentage of high carbon goods written-off at 0

Figure 8 presents the simulation for the optimal $\hat{S}$, consistent with approximately 28% of the high carbon energy capital being discarded at $t = 0$ (as opposed to $\bar{S}$ which is consistent with 50% being discarded).

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[60]Note that this means that these solutions are fundamentally different from the previous three: in the previous, the total carbon emitted is $\bar{S}$, whereas here the total carbon emitted is less than $\bar{S}$.

[61]Regions 1 and 3 highlight the perverse effects of debt renegotiation upon welfare discussed in Section 4: debt renegotiation eliminates the debt overhang and the persistent negative effects from the shock; this means that more debt renegotiation, conditional on the same overall use of energy capital, is beneficial. For Region 1, increasing the initially announced carbon budget means more debt renegotiation is required at T, which due to these perverse effects increases welfare. Likewise for Region 3, reducing the initially announced carbon budget means more debt renegotiation is required at $t = 0$, which due to these perverse effects means a much reduced rate of decrease in welfare (relative to the slope of the right hand side of region 2). Region 4 is different since reducing the initially announced carbon budget also reduces the total amount of high carbon energy capital used, reducing welfare.
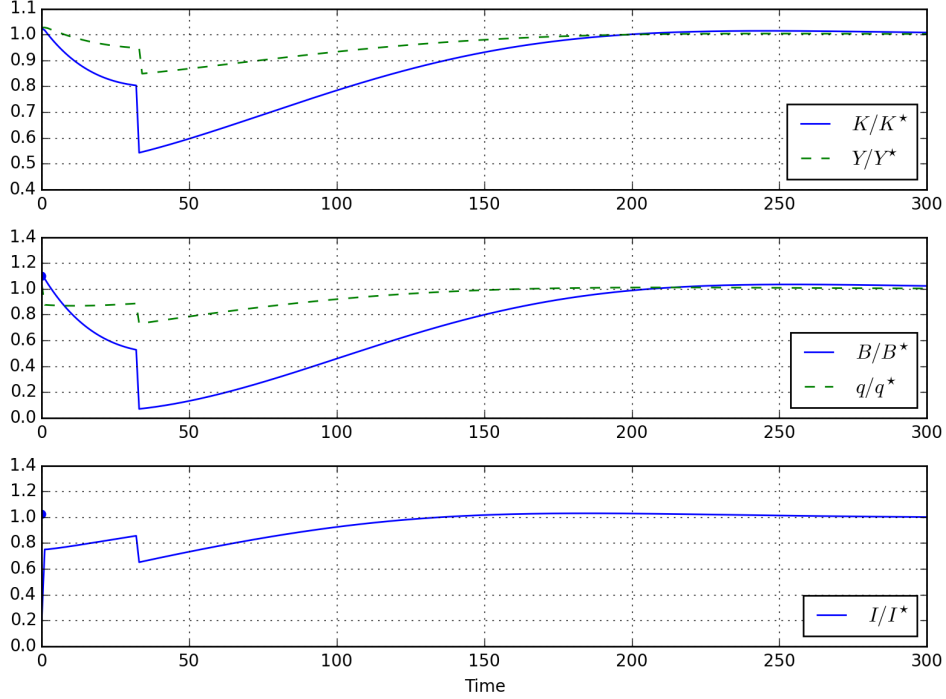
Figure 8: Dishonest social planner

The price of fixed capital falls immediately by approximately 15% and no debt renegotiation is required. Investment does not fall as much as in the no-policy scenario, and when the carbon budget runs out at around $T = 32$, despite a large negative shock which induces large scale deleveraging, investment can soon continue. At T a little more than 67% of aggregate entrepreneurs' asset holdings are already dedicated to zero carbon energy capital so that, when the remaining high carbon resources must be left unused, an alternative productive capacity already exists. This limits the magnitude of the recession which results: although it remains severe, no debt renegotiation is required. Over 200 periods, the cumulative investment flow in zero carbon energy capital is almost 12% higher than in the no-policy scenario; and welfare is more than 2% higher (+17% for entrepreneurs and 0% for savers). This policy is a Pareto improvement on the no-policy scenario.

6   Conclusions

This paper analyses the effects of the credible implementation of climate change targets, which imply that a substantial proportion of fossil fuel assets become "stranded", in an economy characterized by collateral constraints. To do this, we consider a simple extension of the Kiyotaki and Moore's (1997) model. We allow for two investment goods representing high carbon and zero carbon energy capital. This framework allows us to model, for the

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first time in the economics of climate change, the so-called "Carbon Bubble".

The Carbon Bubble, or the enforced abandonment of some proportion of carbon intensive productive capacity and the consequent write-off of carbon intensive assets, is an issue introduced by the Carbon Tracker Initiative's (2011) report as a warning to investors: climate change mandates a policy response, and you, as an investor, should protect your portfolio from this policy response. By incorporating the Carbon Bubble issue within a macro-financial model, we go beyond this warning to investors, and can start the conversation around appropriate macroeconomic policy that should accompany the Carbon Bubble.

We take as given that climate science mandates a severe climate policy response, such that society has a limited "carbon budget" relative to its ability to emit carbon pollution: at $t = 0$, the social planner learns that, to avoid the collapse of civilization, the economy will be able to use only one-half (by value, not by carbon content) of the current stock of high carbon energy capital. Imposing this carbon budget severely damages the balance sheets of investors, and in the presence of financial frictions, this has major macroeconomic implications. We consider the social planner's problem in facilitating the transition from an high carbon economy to the carbon-free era, by choosing policies to maximise welfare defined as the discounted present value of future consumption flows.

The first policy is for the public sector, which is assumed not to be credit constrained, to take over the debt obligations of the credit constrained entrepreneurs, which it repays from lump-sum taxation. We note that the policymaker would want to implement this policy even in normal times, but that following the announcement of a binding carbon budget, the policymaker wants to do more of this policy. The improved net asset position of entrepreneurs allows them to invest more than is the case without debt reallocation, driving the economy out of the recession faster, in spite of the presence of the lump-sum tax. The second policy provides investment subsidies. These subsidies are calibrated under the assumption that they are sub-optimal in steady state, but we see that under the conditions of the Carbon Bubble, it is optimal for a positive subsidy level to be set, since it again improves the balance sheets of entrepreneurs and hence overall macroeconomic performance. However, we see that these policies are not a Pareto improvement relative to no-policy. Despite the improved macroeconomic performance, it is entrepreneurs that capture the major part of the benefits, and savers lose out overall due to the need to pay the taxes to fund these policies.

The third policy we consider is a government guarantee that provides a collateral multiplier to the entrepreneurs: for a given value of collateral, the guarantee increases the amount that savers are willing to lend to the entrepreneurs. By allowing more lending after the Carbon Bubble shock, this policy also allows entrepreneurs to invest more than is the case without policy, with positive macroeconomic consequences. Because this guarantee is never actually used along the perfect foresight equilibrium path, this policy is costless

and serves only to relax credit constraints. It therefore represents a Pareto improvement on the no-policy scenario. However, we caution that this result may be unrealistic since the policy's costs are "out of the model".

The final policy that we model is for the planner to dishonestly announce a larger carbon budget than is really the case. This causes a smaller recession with investment levels holding up better than would be the case under the true carbon budget. When the true carbon budget is exhausted, its existence is revealed and all usage of high carbon energy capital must cease, causing a second recession. Between the announcement of the policy at $t = 0$, and the point at which the true carbon budget is revealed, economic activity is higher than it would have been given an honest announcement, and so investment in replacement zero carbon energy capital has also been greater. When the economy must switch to the zero carbon technology, it has an alternative productive capacity already available which limits the reduction in output and consumption. We find that it is optimal for the planner to behave dishonestly and announce a carbon budget greater than the true carbon budget: so doing results in lower output loss and more investment in zero carbon energy capital. Again, this policy represents a Pareto improvement, but we would caution that such dishonest behaviour perhaps requires an unreasonable disparity in information between the planner and the other agents in the economy, and so the results from this scenario may also be unrealistic.

In all cases we see that welfare enhancing policy leads to higher investment in zero carbon productive capacity over the period in which we still use carbon emitting productive capacity, than under no-policy. These policy experiments show that the balance sheet effects of writing down high carbon assets on investment rates in zero carbon replacement energy capital cannot be ignored in any rational climate policy analysis. The "global balance sheet" will be used to fund the zero carbon infrastructure which must be built to replace our fossil fuel based economy, and the bursting of the Carbon Bubble could throw the economy into a deep recession, depriving green technology of investment funds right when they are most needed. Thus, even if the fossil fuels assets really should be written-off to avoid disastrous global warming, it is likely to be sub-optimal to do this naively. The policy response to the threat of climate change must pay cognisance to the impact that it will have on investors' balance sheets.

Of course, this paper represents a first-order exercise, the purpose of which is to start the technical analysis of the interaction between climate policy and macroeconomic stabilisation, and to provoke further research. We find that policies which mitigate the impact of the Carbon Bubble upon investors' balance sheets can be beneficial. We do not compare the various policies, but simply stress the importance of considering the balance sheet effect.

Perhaps the next step in a research agenda that seeks to accurately model the bursting of the carbon bubble and produce an optimal policy recommendation, should be the

addition of a banking sector.[62] By micro-founding the financial intermediation process, both the economic response to the balance sheet impact of the carbon bubble, and the design of optimal policy to mitigate the shock, can be altered. The perverse effects of debt renegotiation that we found, suggest that the next steps could be to properly model the bankruptcy process, incorporating a fuller description of the capital structure of investors' balance sheets with different priority creditors, and costs of financial distress. Heterogeneous agents may be an important element to add to the model, as in Punzi and Rabitsch (2015). The costless government guarantee suggests that aggregate uncertainty or stochastic noise should be added, so that such a guarantee had an expected cost along the equilibrium path.

On the other hand, it may be felt that rather than increasing the sophistication of the modelling on the macro-financial side, other elements should be improved first. For example, the Pareto sub-optimality of some of the macroeconomically beneficial policies suggests that a political process is important. Perhaps this Pareto sub-optimality problem would be reduced though if savers also supplied labour to the entrepreneurial sector, and there was the possibility of unemployment in recessions. The issue of international spillovers is also important: we have modelled the Carbon Bubble issue as if there is a single global policymaker; but what would be the incentives for a national policymaker enforcing a Carbon Bubble restriction with or without the cooperation of other national policymakers? Another political economy issue that may be desirable to add to the model is some measure of the costs of acting through the planner. In the model as it stands, an obvious optimal policy would be to nationalise investment in energy so that the government, which is not credit constrained, maintains investment in the face of the Carbon Bubble.

There are other areas of the model which could be made more sophisticated in order to fully quantify the impact of the Carbon Bubble and to design optimal policy. The financial accelerator mechanism could be embedded in a standard climate-economy Integrated Assessment Model rather than the reduced form that we have used where you are either within or beyond the carbon budget. The supply side of the model could be made more realistic, with depreciation of, and investment in, the non-energy capital. Endogenous growth is likely an important aspect that should be considered: if learning-by-doing is important then any under-utilisation of capital induced by the Carbon Bubble could be more damaging than in the model presented here.[63] The "black box" distortion associated with the subsidy could also be micro-founded in an endogenous growth framework.

There remains much to do in fully specifying a model which will allow a macroeconomic

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[62]See for example Gersbach and Rochet (2012), Gertler et al. (2012), Caballero and Simsek (2013), and Gertler and Kiyotaki (2015).

[63]Ghisetti et al. (2015) analyse the role of financial barriers behind the adoption of environmental innovations.

forecast of the impact of the Carbon Bubble, and which will allow an optimal policy response to be designed. This paper has started this modelling, and shown that there is a role for policy in mitigating its impact. Policy which protects investors' balance sheets mitigates the macroeconomic downturn, and leads to higher investment in the replacement zero carbon productive capacity over the period in which we still use carbon emitting productive capacity.

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A Appendix

A.1 Glossary of variables and parameters, and model assumptions not given in main text

In the text, lower case letters indicate variables for a representative entrepreneur or for a representative saver if followed by a prime symbol. Upper case letters are aggregate variables. Starred letters represent steady state equilibrium variables.

<div align="center">

Variables and parameters definition

m	relative savers' population size
x	consumption
β	discount factor
k	entrepreneur's fixed capital holdings
$\bar{K}$	total supply of fixed capital
z^H	high carbon energy capital
z^L	zero carbon energy capital
$1 - \lambda$	energy capital depreciation rate
q	relative price of fixed capital
R	gross interest rate
y	output
a^i	tradable proportion of output
c	non-tradable proportion of output
τ	carbon tax rate
ς	green subsidy rate
δ	effectiveness of the green subsidy rate
b	debt
g	per capita government tax or transfer
π	proportion of entrepreneurs with investment opportunity
ϕ	output cost of investing
I	aggregate investment flow
u	user's cost of asset
γ	proportion of entrepreneurs using high carbon technology
a	net private productivity
$1 - \lambda_H$	high carbon energy capital retirement rate
$1 - \rho$	share of entrepreneurs' high carbon good written off at $t = 1$
S	cumulative emissions
$\bar{S}$	actual carbon budget
$\hat{S}$	carbon budget announced by social planner
τ^G	tax funding the transfer of investors' debt
B^G	social planner's debt
ω	share of entrepreneurs' debt taken over by social planner
$gtee$	government guarantee

</div>

We specify below some further model assumptions that are omitted from the main text and which are relevant restrictions in the derivation of the steady state.

Assumption D is included to avoid a corner solution i.e. to ensure that, in the neighbourhood of the steady state, both types of agent produce.

Assumption D
$$\Psi'\left(\tfrac{\bar{K}}{m}\right) < \frac{\pi\left[a+\frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right]-\phi(1-\lambda)(1+R\pi-R)}{\pi\lambda+(1-\lambda)(1-R+R\pi)} < \Psi'(0)$$

Assumption E says that the tradable output is at least enough to substitute the depreciated energy capital,

Assumption E
$$a^L > (1-\lambda)\phi$$

while Assumption F ensures that the probability of investment is not too small

Assumption F
$$\pi > \frac{R-1}{R}$$

Assumptions E and F are also used to ensure that the steady state values $(q^\star, K^\star, B^\star)$ and the associated $u^\star$ are positive.

To guarantee that the entrepreneur will not want to consume more than the non-tradable output, we assume

Assumption G
$$c > \frac{1-\beta R\lambda(1-\pi)}{\beta R[\pi\lambda+(1-\lambda)(1-R+R\pi)]}\left(\tfrac{1}{\beta}-1\right)(a^L+\lambda\phi)$$

Note that, since β and R are close to one, both Assumptions F and G are weak. Finally, we avoid the explosion in asset prices with the following transversality condition

Assumption H
$$\lim_{s\to\infty} E_t(R^{-s}q_{t+s}) = 0$$

A.2 Subsidy induced distortion

In a decarbonised world ($\gamma = 0$), since $K^\star$ is a monotonically increasing function of the productivity target set by the government, the amount of fixed capital used by the entrepreneurs increases with the subsidy. Indeed, an entrepreneur benefits fully from the presence of the subsidy while they only partly contribute to the per capita tax (since this is paid by savers too). However, when there is a cost associated with the subsidy, an increase in the target productivity set by the government has an ambiguous effect on social welfare. Indeed, while entrepreneurs' utility always increases in a (as it is a constant multiple, c, of $K^\star$), the increase in savers' income from increased debt interest as lending increases, may not compensate the decrease due to increasing taxes. We choose the subsidy induced distortion parameter, δ, such that the optimal subsidy rate from the social planner's perspective is $\varsigma = 0$.

Panel (a) of Figure A.1 shows that any positive subsidy is welfare destroying in the neighbourhood of the decarbonised steady state. On the contrary, Panel (b) shows that there is a clear optimal subsidy following the announcement of the carbon budget, because a positive subsidy ameliorates the balance sheet position of the entrepreneurs, and thus allows more investment in alternative productive capacity.

A.3 Stability

In this section we follow Kiyotaki and Moore (1995, Appendix) to linearise the model around the steady state in order to examine the dynamics. The procedure requires using the laws of motion of aggregate entrepreneurs' asset holdings in (15) and borrowing in (16), together with the asset market equilibrium condition in (12), to find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t).

Figure A.1: Welfare effects of the subsidy

(a) Decarbonized Steady State

(b) Carbon Bubble Scenario
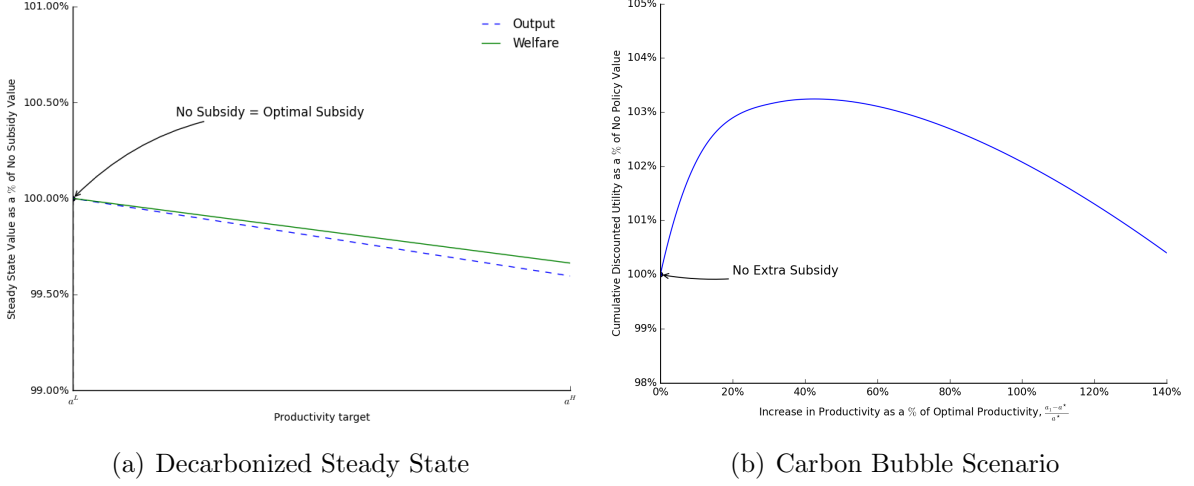
By combining Equations (11) and (12), we find $q_{t+s} = R\big(q_{t+s-1} - u(K_{t+s-1})\big)$ and then substitute this value in Equation (15). Together with (16), we now have the following system of "transition equations" for $s \geq 1$:

$$q_{t+s} = Rq_{t+s-1} - Ru(K_{t+s-1})$$
$$B_{t+s} = q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - \lambda K_{t+s-1}) + RB_{t+s-1} - aK_{t+s-1} +$$
$$- \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}K_{t+s-1}$$
$$K_{t+s} = (1-\pi)\lambda K_{t+s-1} +$$
$$+ \frac{\pi}{\phi + u(K_{t+s})}\left[\left(q_{t+s} + \phi\lambda + a\right)K_{t+s-1} - RB_{t+s-1} + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}K_{t+s-1}\right].$$

Consider taking a first order Taylor series expansion to this system around the steady state,

$$\frac{q_{t+s} - q^\star}{q^\star} \approx \left.\frac{\partial q_{t+s}}{\partial q_{t+s-1}}\right|_{SS}\frac{q^\star}{q^\star}\frac{q_{t+s-1} - q^\star}{q^\star} + \left.\frac{\partial q_{t+s}}{\partial K_{t+s-1}}\right|_{SS}\frac{K^\star}{q^\star}\frac{K_{t+s-1} - K^\star}{K^\star} =$$
$$= R\frac{q_{t+s-1} - q^\star}{q^\star} - Ru'(K^\star)\frac{K^\star}{q^\star}\frac{K_{t+s-1} - K^\star}{K^\star} = \quad \text{using } u^\star = q^\star\left(1 - \frac{1}{R}\right)$$
$$= R\frac{q_{t+s-1} - q^\star}{q^\star} - (R-1)\frac{u'(K^\star)K^\star}{u(K^\star)}\frac{K_{t+s-1} - K^\star}{K^\star}$$

$$\frac{B_{t+s} - B^\star}{B^\star} \approx \left.\frac{\partial B_{t+s}}{\partial B_{t+s-1}}\right|_{SS}\frac{B^\star}{B^\star}\frac{B_{t+s-1} - B^\star}{B^\star} + \left.\frac{\partial B_{t+s}}{\partial q_{t+s-1}}\right|_{SS}\frac{q^\star}{B^\star}\frac{q_{t+s-1} - q^\star}{q^\star} +$$
$$+ \left.\frac{\partial B_{t+s}}{\partial K_{t+s-1}}\right|_{SS}\frac{K^\star}{B^\star}\frac{K_{t+s-1} - K^\star}{K^\star} =$$
$$= \left[R + (q^\star + \phi)\frac{\partial K_{t+s}}{\partial B_{t+s-1}}\right]\frac{B_{t+s-1} - B^\star}{B^\star} + (q^\star + \phi)\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\frac{q^\star}{B^\star}\frac{q_{t+s-1} - q^\star}{q^\star} +$$
$$+ \left[-\left(q^\star + \lambda\phi + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right) + (q^\star + \phi)\frac{\partial K_{t+s}}{\partial K_{t+s-1}}\right]\frac{K^\star}{B^\star}\frac{K_{t+s-1} - K^\star}{K^\star}$$

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$$\frac{K_{t+s} - K^\star}{K^\star} \approx \frac{\partial K_{t+s}}{\partial q_{t+s-1}}\bigg|_{SS} \frac{q^\star}{K^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \frac{\partial K_{t+s}}{\partial B_{t+s-1}}\bigg|_{SS} \frac{B^\star}{K^\star} \frac{B_{t+s-1} - B^\star}{B^\star} +$$

$$+ \frac{\partial K_{t+s}}{\partial K_{t+s-1}}\bigg|_{SS} \frac{K^\star}{K^\star} \frac{K_{t+s-1} - K^\star}{K^\star} =$$

$$= \left[\frac{R\pi K^\star}{\phi + u(K^\star)} - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial q_{t+s-1}} \right] \frac{q^\star}{K^\star} \frac{q_{t+s-1} - q^\star}{q^\star} +$$

$$- \left[\frac{\pi R}{\phi + u(K^\star)} + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial B_{t+s-1}} \right] \frac{B^\star}{K^\star} \frac{B_{t+s-1} - B^\star}{B^\star} +$$

$$+ \left\{ (1 - \pi)\lambda + \frac{\pi \left(q^\star + \phi\lambda + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} \right)}{\phi + u(K^\star)} + \frac{\pi K^\star}{\phi + u(K^\star)} \frac{\partial q_{t+s}}{\partial K_{t+s-1}} \right.$$

$$\left. - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial K_{t+s-1}} \right\} \frac{K_{t+s-1} - K^\star}{K^\star}.$$

From the last approximation, it follows that

$$\frac{\partial K_{t+s}}{\partial q_{t+s-1}} = \frac{R\pi K^\star}{\phi + u(K^\star)} - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial q_{t+s-1}} =$$

$$= \frac{R\pi K^\star}{\phi + u(K^\star)} \left[1 + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \right]^{-1}$$

$$\frac{\partial K_{t+s}}{\partial B_{t+s-1}} = -\frac{\pi R}{\phi + u(K^\star)} - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial B_{t+s-1}} =$$

$$= -\frac{R\pi}{\phi + u(K^\star)} \left[1 + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \right]^{-1}$$

$$\frac{\partial K_{t+s}}{\partial K_{t+s-1}} = (1 - \pi)\lambda + \frac{\pi(q^\star + \phi\lambda + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m})}{\phi + u(K^\star)} + \frac{\pi K^\star}{\phi + u(K^\star)} \frac{\partial q_{t+s}}{\partial K_{t+s-1}} +$$

$$- \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial K_{t+s-1}} =$$

$$= \left[1 + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \right]^{-1}$$

$$\left[(1 - \pi)\lambda + \frac{\pi(q^\star + \phi\lambda + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m})}{\phi + u(K^\star)} + \frac{\pi K^\star}{\phi + u(K^\star)} \frac{\partial q_{t+s}}{\partial K_{t+s-1}} \right].$$

The system can be expressed more compactly as

$$\begin{pmatrix} \hat{q}_{t+s} \\ \hat{B}_{t+s} \\ \hat{K}_{t+s} \end{pmatrix} = J \begin{pmatrix} \hat{q}_{t+s-1} \\ \hat{B}_{t+s-1} \\ \hat{K}_{t+s-1} \end{pmatrix}$$

where an hatted variable indicates proportional deviation from the steady state and J is the Jacobian in elasticity form. An element of the Jacobian is indicated with J_{mn} $m, n = (q, B, K)$, so that $J_{mn} = \frac{\partial m_{t+s}}{\partial n_{t+s-1}} \frac{n^\star}{m^\star}$. More specifically,

$$J_{qq} = R \qquad J_{qB} = 0 \qquad J_{qK} = -(R - 1)\frac{u'(K^\star)K^\star}{u(K^\star)}$$

$$J_{Bq} = (q^\star + \phi)\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\frac{q^\star}{B^\star} = (q^\star + \phi)J_{Kq}\frac{K^\star}{q^\star}\frac{q^\star}{B^\star} = (q^\star + \phi)J_{Kq}\frac{K^\star}{B^\star}$$

$$J_{BK} = \left[-\left(q^\star + \lambda\phi + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right) + (q^\star + \phi)J_{KK}\right]\frac{K^\star}{B^\star}$$

$$J_{BB} = R + (q^\star + \phi)J_{KB}\frac{K^\star}{B^\star} \qquad J_{Kq} = \frac{R^2\pi}{\phi + u(K^\star)}\frac{u^\star}{R-1}\left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}$$

$$J_{KB} = -\frac{R\pi}{\phi + u(K^\star)}\left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}\frac{B^\star}{K^\star}$$

$$J_{KK} = \left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + u(K^\star)}u'(K^\star)\right]^{-1}$$

$$\left[(1-\pi)\lambda + \frac{\pi\left(q^\star + \phi\lambda + a + \frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right)}{\phi + u(K^\star)} - R\pi\frac{u^\star K^\star}{\phi + u(K^\star)}\frac{u'(K^\star)}{u(K^\star)}\right].$$

By renaming the variables accordingly, we can refer the interested reader to Kiyotaki and Moore (1995, Appendix) for the analysis of the stability of the system around the steady state.

A.4 Steady state zero carbon energy investment

One interesting result in the initial steady state is that, in an economy with credit frictions, unlike its frictionless equivalent, the high carbon proportion, γ, that maximises steady state zero carbon investment may be greater than zero. Under certain conditions, the relationship between the proportion of high carbon production, γ, and the absolute value of zero carbon investment is not monotonic: indeed, the higher the share, γ, of entrepreneurs using high carbon production and investing in high carbon energy capital, the higher is the net productivity of the fixed capital, and the higher are tax revenues and so the per capita transfer. This means that entrepreneurs have higher net worth and so can hold more of the fixed capital. Since the fixed capital is more productive in the hands of the entrepreneurs, its value increases. This potentially allows the entrepreneurs who are using zero carbon production and investing in zero carbon energy capital to borrow more, invest more and produce more. Crucially we show that this non-monotonic relationship is due to the presence of credit frictions.

The steady state value of aggregate entrepreneurs' fixed capital holdings is[64]

$$K^\star = \frac{\pi\left[a + \frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right] - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} + \nu.$$

Since zero carbon investment is given by $I_t^L = (1-\gamma)\phi(K_t - \lambda K_{t-1})$, in steady state this value is $I^{L\star} = (1-\gamma)\phi(1-\lambda)K^\star$. Therefore, investment in zero carbon energy capital

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[64]Assumption C implies $K^\star = u^\star + \nu$, where $u^\star$ is given by Equation (17c).

can be expressed as

$$I^{L\star} = (1-\gamma)\phi(1-\lambda)\left\{\frac{\pi\left[a + \frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right] - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} + \nu\right\}.$$

Differentiating it with respect to γ gives

$$\frac{\partial I^{L\star}}{\partial\gamma} = \phi(1-\lambda)\left\{\frac{\frac{\pi(1-\gamma)(\tau+\varsigma)}{1+m} - \pi a + \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} - \nu\right\}.$$

It is then easy to see that under certain conditions (depending on e.g. the difference between the productivities of the two technologies, the fraction of entrepreneurs with respect to savers, the net private productivity), $I^{L\star}$ increases for low levels of γ before starting to decrease, as shown by the solid line in Figure A.2.
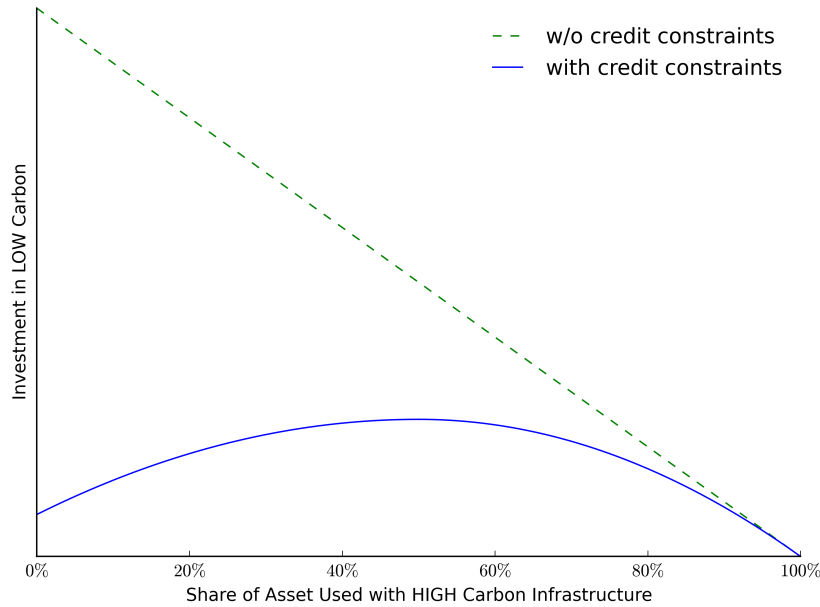
Figure A.2: Absolute investment in zero carbon energy capital as a function of γ

We now want to show that this result is a consequence of the presence of the credit constraint. Consider an economy in which there are no debt enforcement problem so that capital can be optimally allocated. In such an allocation the marginal products of the two technologies would be equalised and the fixed capital price would be given by the discounted gross return from using the entrepreneurs' technology, $q^0 = (a+c)/(R-1)$. It follows that $u^0 = (a+c)/R$ and, given Assumption C, $K^0 = (a+c)/R + v$. Therefore, without the inefficiency caused by the presence of borrowing constraint, investment in zero carbon energy capital would be given by the following relationship

$$I^{L0} = (1-\gamma)\phi(1-\lambda)\left\{\frac{a+c}{R} + v\right\}$$

which is increasing in $1-\gamma$, the proportion of fixed capital used by entrepreneurs in

conjunction with zero carbon energy capital, as shown by the dashed line in Figure A.2. Since the policymaker equalises the private return from using fixed capital with either high or zero carbon energy capital, which is optimally set equal to the returns from the savers' use of fixed capital, it is clear that the proportion γ of high carbon energy capital use cannot affect the amount of fixed capital used overall by the entrepreneurs. Therefore, in steady state, the flow of zero carbon energy capital investment is monotonically decreasing in the high carbon share, γ.

To the extent that the policy target is to maximise investment in zero carbon energy capital, this result shows that the optimal policy may be counter-intuitive: we may get more zero carbon investment if we allow high carbon investment to continue.

A.5 Calibration Strategy

The "calibrations" presented in Kiyotaki and Moore (1997, Chapter III) are not suitable for our exercise because the economy can only return to steady state for extremely small negative shocks: as shown in Figure A.3, the maximum write-off of energy capital that the model can sustain is only approximately 0.2%. Any write-off exceeding this amount requires the introduction of a debt renegotiation mechanism to insure that the economy can converge to the interior steady state.
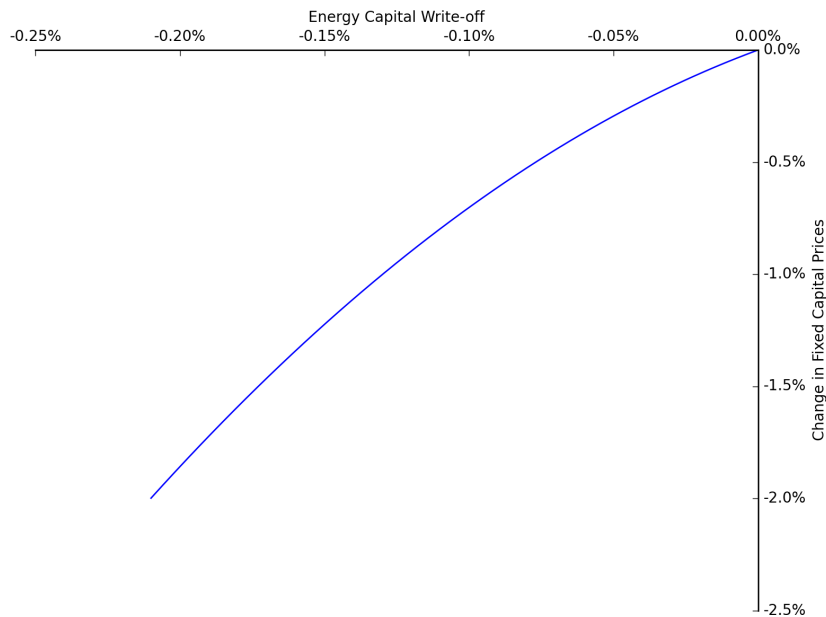
Figure A.3: Maximum shock under original Kiyotaki and Moore's (1997) parametrization

Not only we introduce a debt renegotiation mechanism based on Cordoba and Ripoll (2004), but also we define an alternative calibration strategy. This is explained in Section 4, but it broadly consists of the following steps. Firstly, we define the parameters R, λ, a^H, a_L, a, and c based on definitional convenience, normalization, and to maintain the interpretation of periods as quarter years.

Secondly, we set δ such that the optimal subsidy in the original steady state is zero, and γ using energy data. Thirdly, we try to broadly match the experience of the 2008-09 Financial Crisis. In order to do so, we modify the model slightly and set $\gamma = 0$. This

means that m (and the constant in the savers' production function) do not influence this stage of the calibration since $a = a^L = 0.9$ and $g(m) = [\gamma\tau - (1-\gamma)](1+m) = 0$. We choose ν, ϕ and π such that a write-off of 20% of energy capital, causes a decline of approximately 20% in value of total capital, as one can seen in Figure A.4. This Figure also shows that relying too heavily upon debt renegotiation ruins the story that models of this sort tell, as when debt renegotiation is introduced, this absorbs most of the impact of the shock, while prices change only marginally. As a consequence, we impose that our parametrization must be such that the maximum write-off of energy capital approaches 45% before needing debt renegotiation. Additionally we choose parameters such that we observe that debt holdings are positive in the maximum write-off run. Most calibrations satisfying these conditions produce extremely long economic cycles (when interpreting a period as a quarter). We therefore also try to minimise cycle length subject to satisfying these other conditions. We tried many different parameter combinations in generating this calibration and we assessed the marginal impact of changing each parameter upon various aspects of the solution.
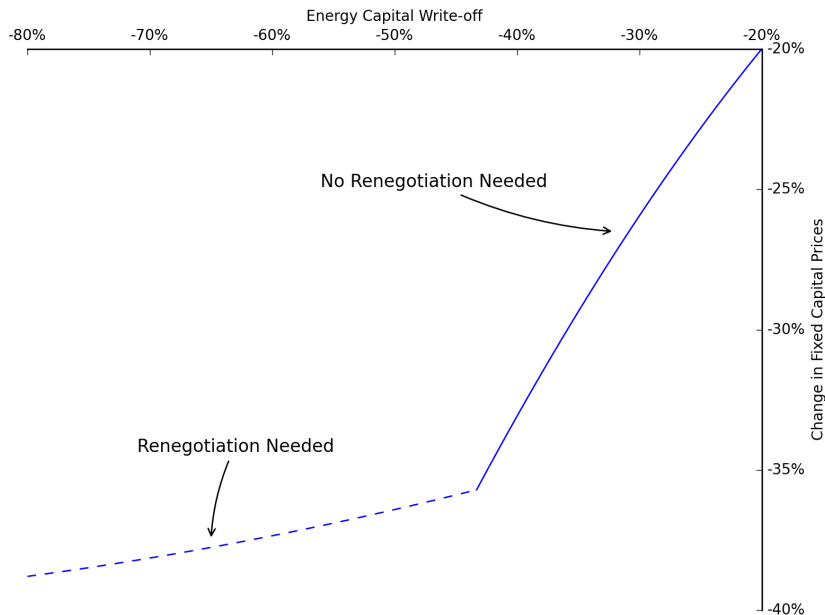
Figure A.4: Current parametrization

Fourthly, we choose $\bar{K}$ such that $\gamma\phi K_F^\star / (\phi K_F^\star + q_F^\star \bar{K}) = 4.5\%$, where $K_F^\star$ is the amount of fixed capital hold by the entrepreneurial sector in the original steady state. Finally, we contemporaneously set m and the constant in the savers' production function so that consumption of individual saver and entrepreneur are equal in the new decarbonised steady state and that the maximum output impact of writing-off 20% of energy capital (i.e. Financial Crisis run) is around 6%.

The calibrated parameters are presented in Table 1. Figure A.5 shows the dynamics on the variable of interest following a shock comparable to the 2008-09 Financial Crisis. Targeted and model values are given in Table 2.

Figure A.6 outlines how much of the macroeconomic fluctuation following the Carbon Bubble are due to the write-off of productive energy asset. In it we see that the effect of moving from the initial steady state, where more productive high energy capital is used, to the decarbonised steady state, without any write-off, is almost undetectable.

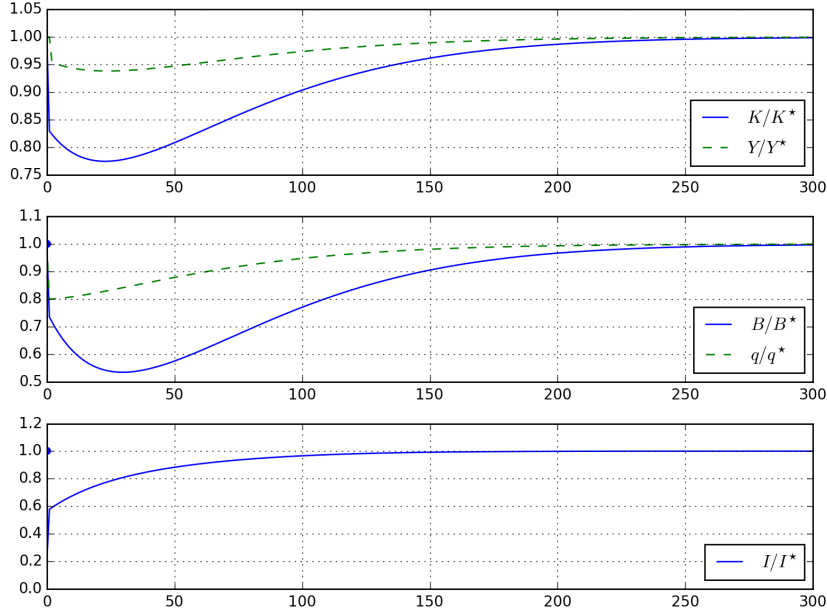| $R = 1.01$	$\lambda = 0.975$	$\pi = 1.5\%$	$\nu = 0.225$
$\phi = 24$	$a^H = 1$	$a^L = a = 0.9$	$c = 1$
$\bar{K} = 5.26$	$\gamma = 0.8$	$m = 2.71$	$const = 3.90$

Table 1: Parameters Values

Figure A.5:   Dynamics induced by the financial crisis shock

	Target	Model
Energy Share	4.50	4.47
Asset Impact of FinCrisis	-20.00%	-19.99%
Output Impact of FinCrisis	-6.00%	-6.00%

Table 2: Calibrated Values

A.6   Shooting Algorithm and Renegotiation Mechanism

The simulations are obtained using the shooting algorithm. By using the laws of motion of aggregate entrepreneurs' asset holdings in (15) and borrowing in (16), together with the asset market equilibrium condition in (12), we can find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t). From Equations (11) and (12), we find $q_{t+1} = R(q_t - u(K_t))$. We now impose $u(K_t) \equiv K_t - \nu$: the previous becomes $q_{t+1} = R(q_t - K_t + \nu)$. The next step is to substitute this value in Equation (15) and solve for K_t. We then have the following
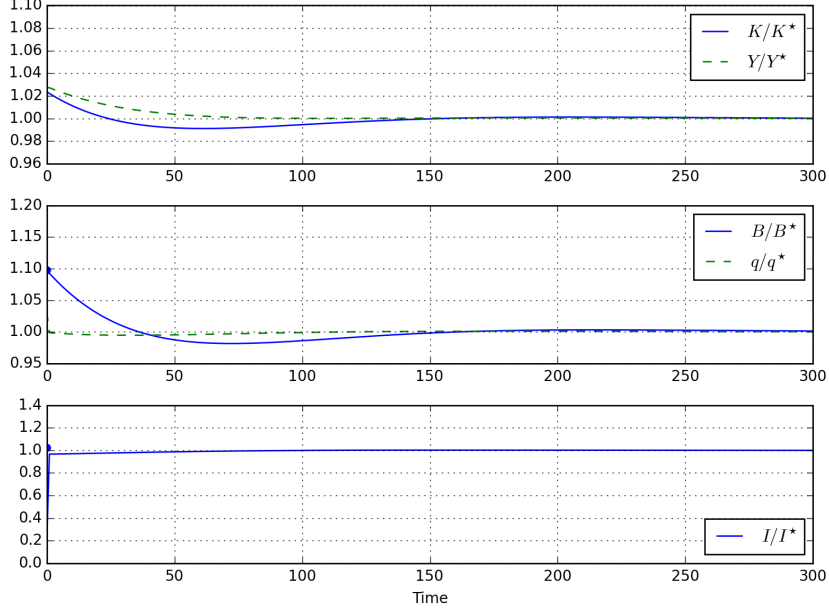
Figure A.6:   Dynamics induced by the productivity differential

system of "transition equations" that we can iterate:

$$q_{t+s} = R(q_{t+s-1} - K_{t+s-1} + \nu) \tag{A.2a}$$

$$B_{t+s} = q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - (\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1}))\,K_{t+s-1}) +$$

$$+ RB_{t+s-1} - aK_{t+s-1} - \frac{\gamma_{t+s-1}\tau_{t+s} - (1 - \gamma_{t+s-1})\varsigma_{t+s}}{1 + m}K_{t+s-1} \tag{A.2b}$$

$$K_{t+s} = \frac{1}{2}\left[\nu - \phi + (1 - \pi)\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1})\right)K_{t+s-1}\right] + \tag{A.2c}$$

$$+ \frac{1}{2}\Bigg\{\left[\phi - \nu - (1 - \pi)\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1})\right)K_{t+s-1}\right]^2 +$$

$$+ 4\Bigg[(\phi - \nu)(1 - \pi)\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1})\right)K_{t+s-1} +$$

$$+ \pi K_{t+s-1}\left[q_{t+s} + \phi\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1 - \gamma_{t+s-1})\right) + a\right] +$$

$$- \pi RB_{t+s-1} + \pi\frac{\gamma_{t+s-1}\tau_{t+s} - (1 - \gamma_{t+s-1})\varsigma_{t+s}}{1 + m}K_{t+s-1}\Bigg]\Bigg\}^{0.5}.$$

When the available carbon budget is announced at t, the amount of entrepreneurs' energy capital, after depreciation, reduces to $[\rho\gamma + (1 - \gamma)]\lambda K_{t+s-1}$, where $(1 - \rho) \in [0, 0.5]$ is the percentage of the stock of high carbon energy capital that it is optimally written off

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by the entrepreneurs at $t = 1$.[65] When this shock hits, Equation (A.2a) does not hold because the asset price jumps in response to the shock and entrepreneurs experience a loss on their asset holdings. In the original Kiyotaki and Moore's (1997) model, a shock of the magnitude we are interested in would throw the economy out of the basin of attraction of the interior steady state. To prevent this, we follow Cordoba and Ripoll (2004) and allow for renegotiation of the debt. Cordoba and Ripoll (2004) introduce renegotiation in the basic Kiyotaki and Moore's (1997, Chapter II) model: analytically, when debt can be renegotiated, debt repayments RB_{t+s-1} are pushed down to the market value $q_{t+s}K_{t+s-1}$ of the collateral. On the contrary, we use the full Kiyotaki and Moore's (1997, Chapter III) model, where the aggregate value of debt is not exactly aligned with the value of the collateral, since a fraction of entrepreneurs cannot invest and thus repay part of their debt obligations. Moreover, rather than allowing renegotiation for any negative shock (as in Cordoba and Ripoll (2004)), we allow for renegotiation only if the economy cannot converge back to the interior steady state following a shock. This is because, while entrepreneurs would always like to reduce their debt, this is not always the case for savers. If the economy cannot converge back to the interior steady state, then the outside option for the representative saver is to accept the economy converging to the steady state with no fixed capital in the hands of the entrepreneurs. Therefore, if the economy cannot converge back to its interior steady state, then both parties have the incentive to renegotiate the outstanding value of the debt. However, for any level of the shock, there are infinite combinations of changes in prices and debt levels such that the economy converges back to its steady state. The debt level retained by the entrepreneurs in our simulations is the maximum value for debt levels consistent with the economy being able to reach its interior steady state, B_{t+s-1}^+, with corresponding prices q_{t+s}^+. This choice makes the downturn less severe and thus reduces the welfare increased induced by our policies. Analytically, when renegotiation takes place, debt repayments RB_{t+s-1} are pushed down to RB_{t+s-1}^+, and prices jump to q_{t+s}^+.

Given the transversality condition in Assumption H, we know that $q_T = q^\star$ for large T. But since Equations (11) and (12) define the asset price variation as a function of K_t, we can project the asset values back from steady state. So the rough ideas is to guess the initial variation in asset price given the shock and then iterate the economy forward through time to see if it converges again to the steady state. If the level of asset price eventually explodes, the initial guess is revised downward; if it is forever smaller then the initial guess is revised upward. This "guess and check" procedure is repeated until the asset price is close to the steady state (i.e. within the arbitrary level of tolerance).

When we allow the social planner to take over a fraction ω of debt from the entrepreneurs, the following additional changes are required in the transition equations. Between

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the period in which the shock is announced and the following period, the value of the entrepreneurs' debt is further reduced to $(1 - \omega)B^+_{t+s-1}$. If the transfer of entrepreneurs' debt is funded with a constant tax τ^G over T periods, for T periods we add τ^G in the right hand side of (A.2b) and subtract $\pi\tau^G$ in the right hand side of (A.2c) (inside the square root). Additionally, the budget constraint of the saver now includes debt repayments and new debt from the social planner, RB^G_{t+s-1} and B^G_{t+s}. While this does not directly influence the transition equations, it changes the consumption of the savers in each period, thus influencing the social welfare level reached by the economy. Finally, at $t = T + 1$, there is no tax any more and the social planner holds no debt, so for $t \geq T + 1$, the system of transition equations in (A.2) holds.
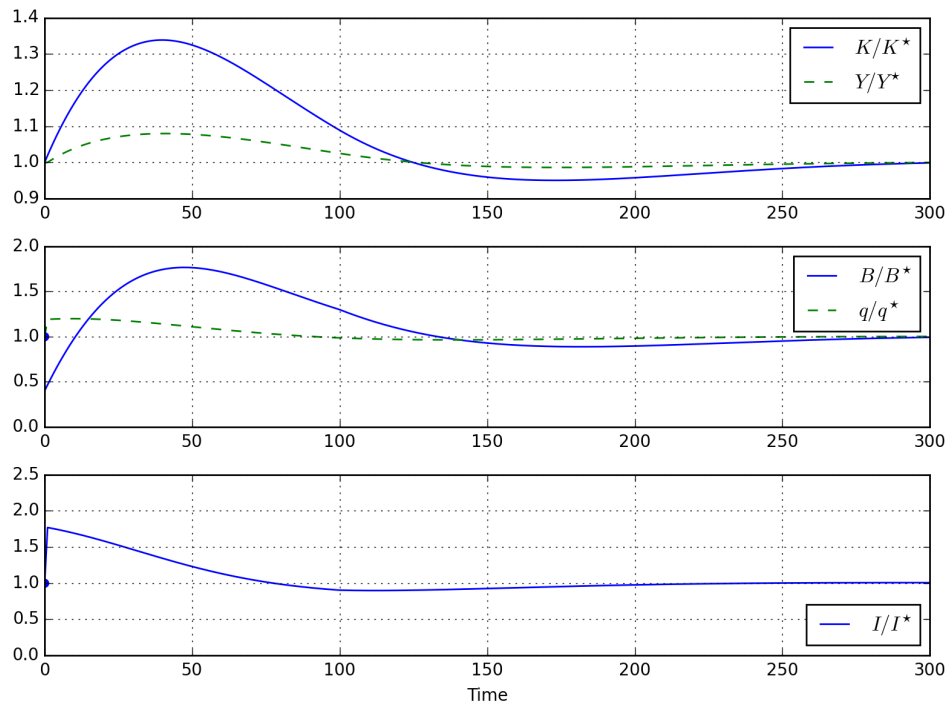
A.7 Further supporting results

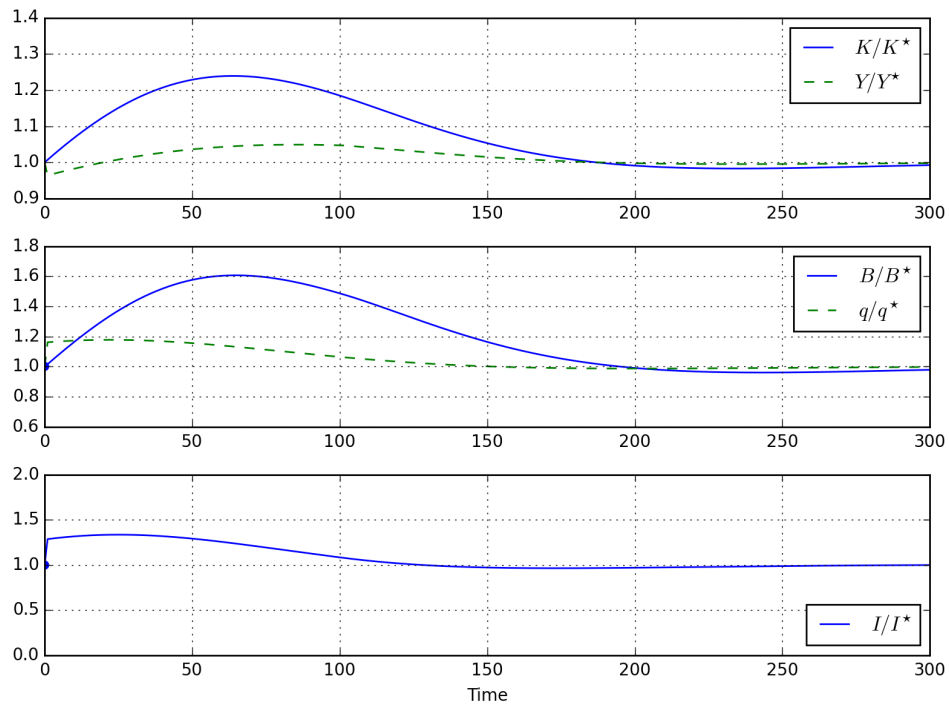Figure A.7: Transferring entrepreneurs' debt, $\omega = 0.6$, in decarbonised steady state

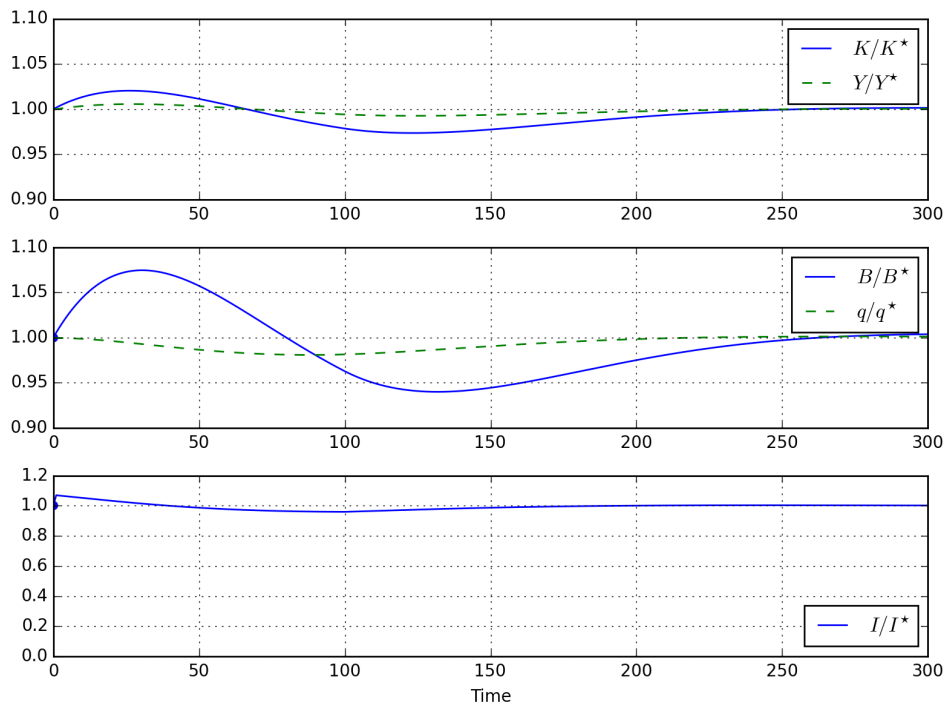Figure A.8: Subsidising entrepreneurs in decarbonised steady state

Figure A.9: Providing a guarantee, $gtee_0 = 1.5\%$, in decarbonised steady state

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