



BANK OF ENGLAND

## External MPC Unit

Discussion Paper No. 35

# Demographics, house prices and mortgage design

David Miles

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David Miles<sup>(1)</sup>

### Abstract

This paper develops a model of the housing market that takes account of population density to assess the impact of population changes on the value and size of the housing stock. The model implies that if population density is on an upward trajectory, rises in population and in incomes increasingly generate price responses and diminishing rises in the stock of housing. This has implications for the optimal structure of housing finance. It makes equity financing of home purchase more desirable. The properties of hybrid debt-equity contracts for financing house purchase are explored.

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## Introduction

Both the size and age structure of the populations in the richer countries of the world will change substantially over the next fifty years. There is nothing new in this – the demographic make-up of countries has changed profoundly over the past fifty years. This has had, and will continue to have, impacts on housing: on the price of homes, on the ownership structure of the stock of residential property, and on the type and number of homes built. There is a large amount of research into the nature of these linkages (see, for example, Di Pasquale and Wheaton (1992); Mankiw and Weil (1989); Engelhardt and Poterba (1991); McFadden (1994); Ermisch (1995); Ermisch (1996); Malpezzi and Maclennan (2001); Levin, Montagnoli and Wright (2009)). Relatively little attention has been paid to the impact of changing population density on housing markets.

The aim of this paper is to explore the impact of demographics on the housing stock and house values and also to assess whether those impacts make alternatives to standard mortgage contracts more attractive. I will develop a model of the housing market which allows for the impact of population density and which suggests that the effects of population change will differ significantly across countries and over time. The model predicts that in a densely populated country like the UK demographic change will generate a rising trajectory of real house prices. If that is an implication of likely demographic changes then it makes even more important an issue which the financial crisis of recent years has already made pressing. That is the question of whether there are better alternatives for new buyers than financing house purchase largely from debt. The issue of whether high debt gearing for new buyers is desirable is more significant the higher are house prices; the housing model developed suggest house prices may follow an upwards trajectory that becomes steeper if population density rises.

In the second part of this paper I consider financing contracts for funding house purchase which are hybrid (part debt, part equity) contracts, contracts that I believe will become more attractive if real house prices are likely to be higher. There is a substantial literature on financing house purchase (for different aspects of this literature see Campbell (2006); Campbell and Viceira (2004); Shiller (2003); Miles (1994,2004); Poterba (1984); Rajan (2010)); but the link between financial structure and demographics is not so well explored.

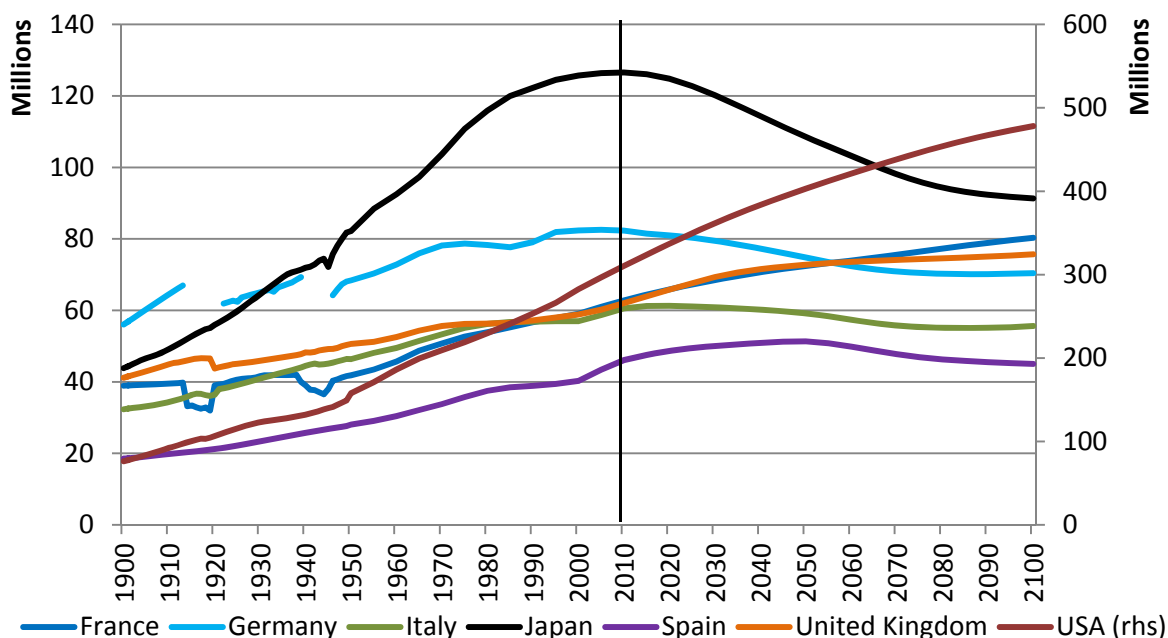
I start by briefly looking at how population structures have changed and at how over the next several decades they may evolve in ways which will affect housing markets in rather different ways across countries (section 1). I then develop the model of the housing market used to analyse the impact of population changes on the price and quantity of housing (section 2). Section 3 analyse the characteristics of hybrid financing contracts for house purchase and whether they become more attractive as a result of impacts of population changes. Section 4 concludes.

### 1. Population trends

Chart 1 shows the evolution of the aggregate population in some of the larger, richer countries over the past 100 years. Populations have risen very substantially in all countries – though at varying rates. United Nations' population projections for the next 100 years are shown in the same chart.

These are the UN medium projections <sup>1</sup>. *If* these projections turn out to be correct the next 100 years will be very different from the past 100 years: in many of these countries population will be roughly flat. In some it will be falling – in Italy, Germany and, especially, in Japan the decline would be marked if the assumptions underlying the UN medium projections turn out to be realised. These countries are projected to be entering a 100 year period of declining population.

**Chart 1: Population trends and forecasts (medium variants) in some major economies<sup>(a)</sup>**



Sources: Liesner (1989), Mitchell (2007) and United Nations.

(a) Data from 2010 are United Nations' medium variant projections.

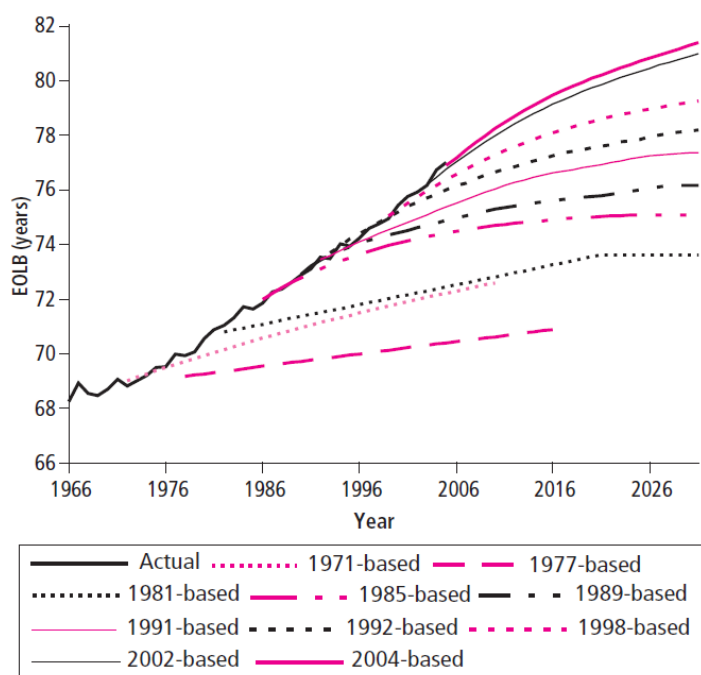
The United Nations' projections use population estimates for 2010 and assumptions regarding future trends in fertility, mortality and migration. The medium variant fertility trends which underpin the population projections in charts 2 and 3 are based on historical trends within each country, but also reflect uncertainty calibrated from historical experience from other countries with similar fertility levels. These result in country-specific probability distributions of future outcomes, the medians of which are used as the medium fertility variants. The mortality assumptions come from models based on recent trends in life expectancy by sex. The models produce smaller gains in life expectancy the higher the life expectancy already reached. Migration rates are assumed to be constant until 2050 and then gradually decline.

The probability of the assumptions underlying the UN medium projections being roughly right is actually rather small. We should treat these projections of sharply slowing rates of population growth with care and scepticism. For many decades in the richer countries the rise in life expectancy has consistently been substantially greater than has been predicted. Chart 2 shows just how substantial and how consistent that under-prediction has been in the UK. About 30 years ago the

<sup>1</sup> For information on the United Nations' projections see [http://esa.un.org/unpd/wpp/Documentation/pdf/WPP2010\\_ASSUMPTIONS\\_AND\\_VARIANTS.pdf](http://esa.un.org/unpd/wpp/Documentation/pdf/WPP2010_ASSUMPTIONS_AND_VARIANTS.pdf).

prediction for life expectancy (at birth) in the UK today was about 10 years lower than today's estimate. If life expectancy can rise by 10 years more than predicted over a period of 30 years the scope to get demographic projections wrong over the next 100 years is enormous. The UN medium projections assume a very gradual – but slowing – increase in life expectancy over the next 100 years. It is far from unlikely that by 2050, when the UN projection for life expectancy at birth (the average for men and women) will be around 85 in most rich countries, life expectancy will be 100 or more. If significant under-prediction in life expectancy continues the projections in Chart 1 probably substantially underestimate the numbers of people alive in the future.

**Chart 2: Actual and projected life expectancy at birth, males, 1966-2031**

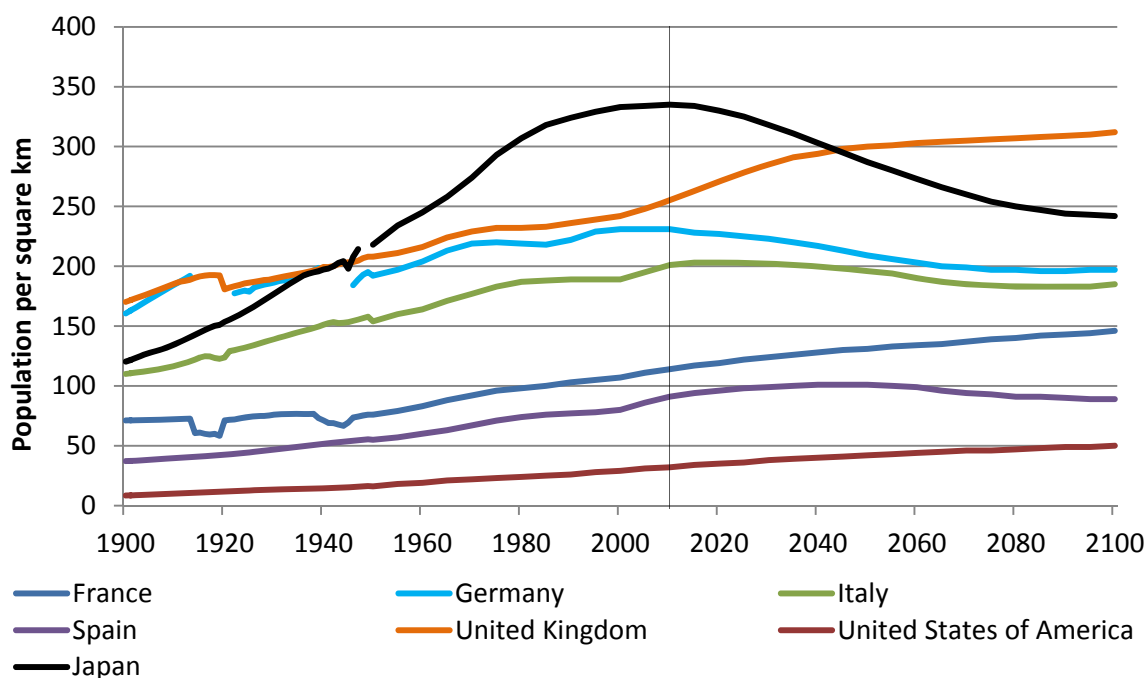


Source: Chris Shaw (2007).

But even if the projections of life expectancy implicit in Chart 1 are right, the projections of net migration might be spectacularly wrong. Projections for world population – unlike those shown for the rich countries in Chart 1 – remain on a significant upward trajectory for several decades. A combination of that with the likelihood of climate change adversely affecting living conditions disproportionately outside the group of richer countries might mean that net immigration to these richer countries will turn out far higher than assumed in the medium projections.

So it may turn out that the population density projections for the rich countries based on the medium UN assumptions (Chart 3) turn out to be substantial underestimates. I think that is very likely. Even if the UN projections are accurate UK population density will rise substantially over the next 50 years. Indeed the UN projections show that amongst the 7 large, rich countries the UK becomes the most densely populated by about mid century – overtaking Japan by about 2040.

Chart 3: Population density (medium variants) <sup>(a)</sup>



Sources: Liesner (1989), Mitchell (2007), United Nations, World Bank, Bank of England calculations.

(a) Pre-1950 figures are estimates based on population figures from Liesner (1989), Mitchell (2007) and land data from the World Bank for 2000. Data from 2010 are United Nations' medium population variants.

## 2. A model of demographics and housing

In analysing the implications of shifting demographics for housing – specifically for the size and value of the housing stock – the elasticity of housing supply is central. That supply elasticity and the type and price of finance available for house purchase are likely to be the two crucial factors that will affect how demographic change generate changes in housing outcomes.

There has been much research on the elasticity of housing supply (for a very good recent review and for careful analysis of the elasticity of supply in the UK see Ball et al (2011). See also Malpezzi and Maclennan(2001)). One thing that emerges clearly in the literature is that differences in supply elasticity across countries are large. Can we expect those differences in supply elasticities to persist, and maybe even become exacerbated as populations change? Might this lead to increasing divergences in real house prices and in housing stocks across countries? That depends on the sensitivity of supply elasticity to population and population density.

The elasticity of supply of housing – that is the responsiveness of the stock housing to changes in the price of housing and to population and income growth – may be a declining function of population density **and** possibly a declining function of real incomes (and therefore likely to be declining over time). I develop a model to explore why this might be true and its implications. But first I should stress that the proposition that supply elasticity might fall with population density and with income is an “other things equal” type of proposition: technological progress in construction (e.g. the ability to build more homes on a given space) can, in principle, offset any trend towards lower supply responsiveness.

One reason that supply elasticity may decline with income is that the economic good of “space” (an amalgam of the benefits to humans of quiet, of absence of crowding, of expansive views and so on) is likely to be a normal good – it has a positive income elasticity of demand. Indeed “space” is plausibly a superior (or luxury) good – one with an income elasticity of demand that exceeds unity. As people become richer the value they place upon having space around them on which houses are **not** built rises, and that raises the price which has to be paid to get land that **can** be built on.

It is also plausible that supply elasticity also depends upon density. The more dense is the population the more valuable is the (ever scarcer) economic good of peace, extensive views and of having space around one on which houses do not stand. That drives up the value of not having people build further houses around one. I suspect the density effect is also non-linear – if we are in a place with very sparse population some rise in density is not likely to drive land values (and so house prices) up much. But as density rises the effect may become stronger.

If both population density and real average incomes are both rising residential land prices will tend to be rising; and it is possible that the impact of further rises in per capita income and in population will be *increasingly* on price.

I believe that allowing for the economic good “space” is important in thinking about housing, and crucial in analysing the impact of population change; but surprisingly few analyses take account of it. I will now develop a model which allows for this economic good of “space”. Following Poterba (1984) let the aggregate quantity of housing services demanded (HSd) depend on the real rental price of a unit of housing (R), and also on population (N) and the level of average incomes (Y),

$$HSd = f(R, Y, N)$$

Assume that the flow of housing services (HSs) is produced by the stock of housing H

$$HSs = g(H)$$

Denote the real price of a unit of housing by P. An equilibrium (arbitrage) condition on an equilibrium growth path where expectations are realised is that the rental price is the real user cost of housing

$$R = P \times (r + \gamma + c + \text{tax} - p)$$

r is the real interest rate (in terms of consumer goods); p is rate of change of real house prices (also in terms of consumer goods),  $\gamma$  is the depreciation rate on the stock of housing, “tax” are taxes due and c are other costs of having a home e.g. insurance costs (both as a percent of house value).

The stock of housing changes by:

$$\dot{H} = I - \gamma H$$

We assume the level of new housing construction (I) depends on the real price of housing:

$$I = \varphi(P, \cdot)$$

The function  $\varphi(P, \cdot)$  reflects the technology of house building and the input costs into a house: materials, wages and, crucially, the price of residential land. The latter we denote  $L_p$ . The higher

the price of land ( $L_p$ ) the greater needs to be the real price of a standard unit of housing to make construction profitable. Thus we write:

$$I = \varphi(P, L_p)$$

$$\varphi(P, L_p)'_P > 0, \varphi(P, L_p)'_{L_p} < 0$$

We assume that the amount of land that could be developed for residential structures is a given quantity  $L$ . It can be used for two purposes – built directly upon (to support a residential structure) or left without a structure on it<sup>2</sup>. The latter category includes its use as part of a house (e.g. a garden) or its use in some other form (a park, a field, a non developed open space). To build a house it is necessary to use some land. The use of land for housing is therefore a positive function of  $H$  (which we denote  $L_h(H)$ ). For simplicity I am going to assume a fixed requirement of land per house so that there is a unit elasticity of demand for land for houses with respect to the housing stock. We also assume that the demand for land for non-housing purposes is related to the density of housing development – the more dense is housing development (the higher is  $H/L$ ) the greater is the demand for, and value placed upon, non-built upon land. But demand for land for non-housing purposes is also a function of land price ( $L_p$ ) and average incomes ( $Y$ ). Denote this demand, which we will call the demand for space,  $L_s$ , by  $L_s(H/L, L_p, Y)$

To clear the land market we need:

$$L = L_h(H) + L_s(H/L, L_p, Y)$$

I assume the elasticity of demand for housing with respect to average income and the income elasticity of demand for “space” are both equal to  $\epsilon$ . Much empirical evidence suggests that the income elasticity of demand for housing is around 1. I think it is plausible that  $\epsilon \geq 1$ .

I assume that the elasticity of housing demand with respect to population ( $N$ ) is unity. I denote the proportionate growth of population by  $n$ .

The elasticity of demand for space with respect to density ( $H/L$ ) is hard to know much about, beyond that it is positive. I denote it by  $\beta$ . I think there is some plausibility to the hypothesis that  $\beta = 1$ . Note that with land fixed  $H/L$  grows by the same as the growth in the housing stock (which is  $\dot{H}/H$  and denoted by  $h$ ) and so the demand for space grows as housing rises at a rate  $\beta h$ . Recall that we assume that any land used as “space” is land that could be developed for housing and that land has a price of  $L_p$ . The user cost of land which is left as space is  $L_p \times (r - l_p)$  where  $l_p$  is the percentage change in land prices. (We assume here that land does not depreciate, has no maintenance cost and is not taxed). Thus the user cost of land is proportional to the land price, in the same way as the user cost of housing is proportional to the real price of a house. I denote the elasticity of demand for space with respect to the price of land (or user cost of land) by  $\alpha$ .

Let us now make the strong assumption that the (non-land) construction costs of housing are constant in real terms. This is roughly the assumption that changes in the wages and other costs of materials paid in the construction sector, net of technological progress in house building, are the same (on average) as for the production of other goods. If that is so then the relative cost of

<sup>2</sup> The amount of land should not be thought of as literally the total land mass of a country. Much of that land (certainly land under water and much of mountain areas) is not suitable for building.



constructing a house (not including the cost of the land) to that of other goods is constant. This has the strong implication that on equilibrium growth paths real house prices only rise because land prices might be rising. This means that the equilibrium path of real land prices is the key to the long run equilibrium trajectory of real house prices (a feature which I believe is consistent with the long run evidence on construction costs, land prices and house prices, see below; it is also consistent with early work by Muth (1960) later confirmed by Follain (1979)). If real construction costs are flat then real house prices will tend to rise at a somewhat slower rate than the rise in real land prices, which are only one element of house value. In the US and in the UK real house prices have indeed tended to rise somewhat more slowly than land values. I assume that on an equilibrium path land prices rise by a multiple ( $\lambda$ ) of house prices; that multiple reflects the total value of a house relative to the land value so will be greater than unity. Thus  $l_p = \lambda p$ , with  $\lambda > 1$ .

Of course land prices are thoroughly endogenous. Equilibrium in the land market requires that the change in the demand for land for housing (which in proportional terms equals the percentage change in the housing stock, denoted  $h$ ) equals the decline in the demand for “space”. The demand for land for “space” depends on incomes (which we assume grow by  $y$  and with an income elasticity of demand of  $\epsilon$ ), and on price (with an assumed absolute price elasticity of  $\alpha$ ). The absolute change in the demand for land for housing is  $h$  (the percentage growth in housing) multiplied by the existing stock of land used for housing,  $L_h(H)$ .

The absolute change in the demand for “space” is the percentage change multiplied by the existing stock of land devoted to space (which is  $L_s(\cdot)$ )

We require that the aggregate change in the demand for land adds to zero:

$$h \times L_h(H) + (y\epsilon + \beta h - \alpha\lambda p) \times L_s(\cdot) = 0$$

Let the proportion of land devoted to housing rather than to “space” at a point in time be denoted  $d$  (for density). Thus  $L_h = dL$  and  $L_s = (1-d)L$

So we can write the previous equation:

$$(h \times d) L = (-y\epsilon - \beta h + \alpha\lambda p) \times (1-d) L$$

In steady state the percentage change in the demand for housing ( $h$ ) equals the growth in population ( $n$ ), plus the growth in incomes ( $y$ ) times the income elasticity ( $\epsilon$ ), plus the rise in prices ( $p$ ) times the price elasticity of demand (denoted  $-\pi$ ). Thus:

$$h = n + y\epsilon - \pi p$$

Combining the previous two equations and solving yields:

$$p = \frac{y\epsilon(1+\beta(1-d))+n(d+\beta(1-d))}{(1-\beta)\pi d + \beta\pi + \alpha\lambda(1-d)}$$

$$h = \frac{y\epsilon(\alpha\lambda(1-d)-\pi(1-d))+n(\alpha\lambda(1-d))}{(1-\beta)\pi d + \beta\pi + \alpha\lambda(1-d)}$$

If the elasticity of demand for space with respect to density is unity,  $\beta = 1$  and the two equations simplify to:

$$p = \frac{(2-d)y\epsilon+n}{\pi+\alpha\lambda(1-d)}$$

$$h = \frac{ye(\alpha\lambda(1-d)-\pi(1-d))+n\alpha\lambda(1-d)}{\pi+\alpha\lambda(1-d)}$$

In terms of the link between demographics and housing the key feature of this model is this: for a given rate of growth of incomes ( $y$ ) and populations ( $n$ ) the rate of increase in real house prices is an increasing function of density ( $d$ ). Furthermore, the response of the growth in the stock of housing to growth in population is a declining function of density – rises in population bring forth ever smaller rises in the housing stock. This is easiest to see in the case where  $\beta = 1$ . I take  $\lambda = 1.25$  as a plausible value. In that case at very low levels of density, and when  $\epsilon$ ,  $\pi$  and  $\alpha$  are all 1, then we have roughly that  $p = 0.89y + 0.44n$ ; and  $h = 0.11y + 0.56n$ . If  $d = 0.50$  we would have  $p = 0.92y + 0.62n$ ; and  $h = 0.08y + 0.38n$ . So the impact of population growth on real house price growth would have risen by about 50% (from 0.44 to 0.62) and the growth of the housing stock would fall from 56% of the proportionate rise in population to 38% of it. With rising population  $d$  is itself rising over time so the elasticity of supply is ever falling.

Table 1 shows what happens to the steady state growth of real house prices in this model for various parameter values. In all cases the rise in house prices for a given rise in population gets larger the greater is density. The impact of population growth on house prices is larger – and the impact on supply of housing smaller – the more land is devoted to housing (the higher is  $d$ ). More densely populated countries should have higher steady state house price inflation, for a given rise in population, and more of the impact of a demographic induced rise in demand for housing shows up as higher land and house values and less as more housing. This is depicted in Figure 1 below – which shows that as demand rises over time (from  $D_0$  to  $D_1..D_2..D_3$ ) the rise in house price is greater, and the rise in housing stock smaller, for the more densely populated country where a greater share of land is already devoted to housing.

Note also that when the two price elasticities of demand ( $\pi$ ,  $\alpha$ ) are 0.5 (rather than 1) the income price elasticity of demand for housing with respect to income doubles and approaches 2 as population density gets larger.

**Table 1: Impact of population density on house price changes**

	$\epsilon, \pi, \alpha = 1; \beta = 0.5$	$\epsilon, \pi, \alpha, \beta = 1$	$\epsilon, \pi, \alpha = 1; \beta = 2$	$\epsilon, \beta = 1; \pi, \alpha = 0.5$
<b>d=1%</b>	$p = .86y + .29n$	$p = .89y + .45n$	$p = .92y + .62n$	$p = 1.78y + 1.89n$
<b>d=25%</b>	$p = .88y + .40n$	$p = .90y + .52n$	$p = .93y + .65n$	$p = 1.81y + 1.03n$
<b>d=50%</b>	$p = .91y + .55n$	$p = .92y + .62n$	$p = .94y + .71n$	$p = 1.85y + 1.23n$
<b>d=75%</b>	$p = .95y + .74n$	$p = .95y + .76n$	$p = .96y + .80n$	$p = 1.90y + 1.52n$
<b>d=90%</b>	$p = .98y + .88n$	$p = .98y + .89n$	$p = .98y + .90n$	$p = 1.96y + 1.78n$

We set  $\lambda = 1.25$

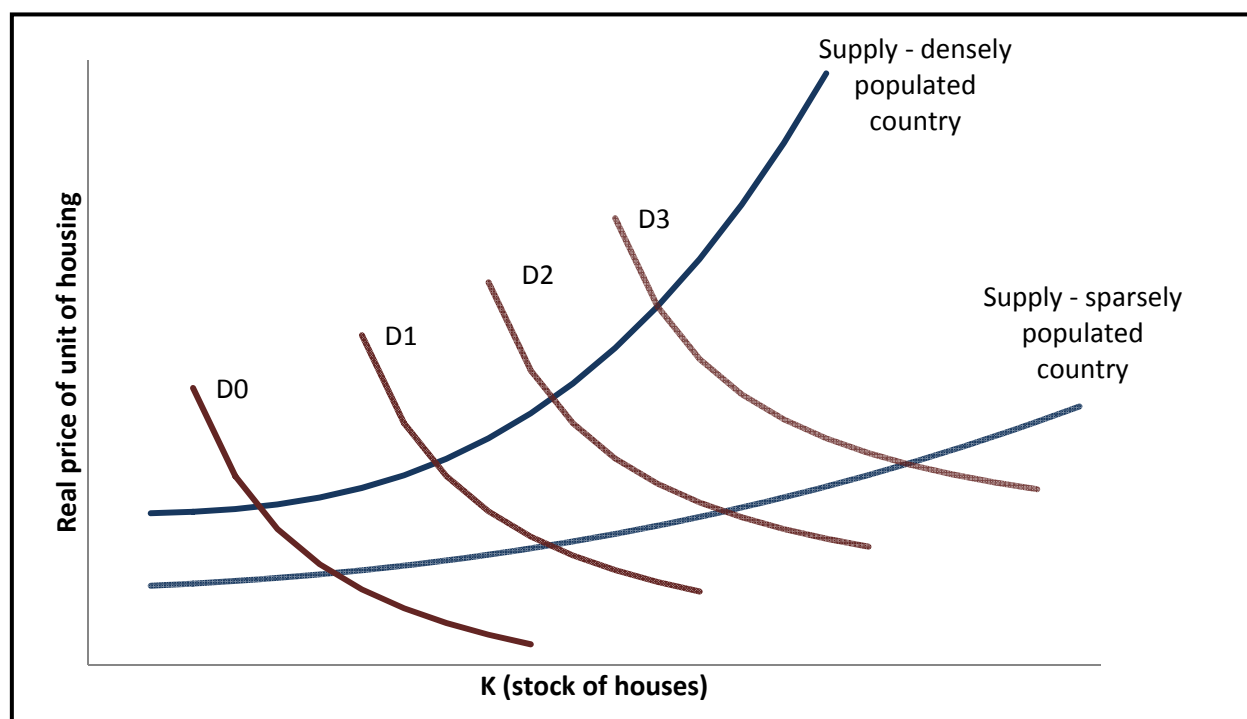
In this model it is likely that as density rises, house prices eventually come to rise faster than incomes. Suppose that  $\epsilon, \pi, \alpha, \beta = 1$ . Then with incomes rising at 2% a year and population at ½% at very low density ( $d=1\%$ ) the house price to income ratio is constant since house prices also rise at

2.0%. At  $d = 50\%$  house prices would be rising at 2.15%; at  $d = 90\%$  house prices are rising a bit more than 2.4% a year, noticeably faster than incomes.

The simple model generates some obvious implications:

1. The real prices of a unit of housing tends to be higher in more densely populated countries.
2. With growing population and incomes the rise in real house prices is faster in densely populated countries.
3. The elasticity of supply of housing is lower in densely populated countries and so is the rise in the stock of housing for a given rise in demand.
4. The amount of housing rises faster in less densely populated country.

**Figure 1: Stylized example of real house prices as a function of population density**



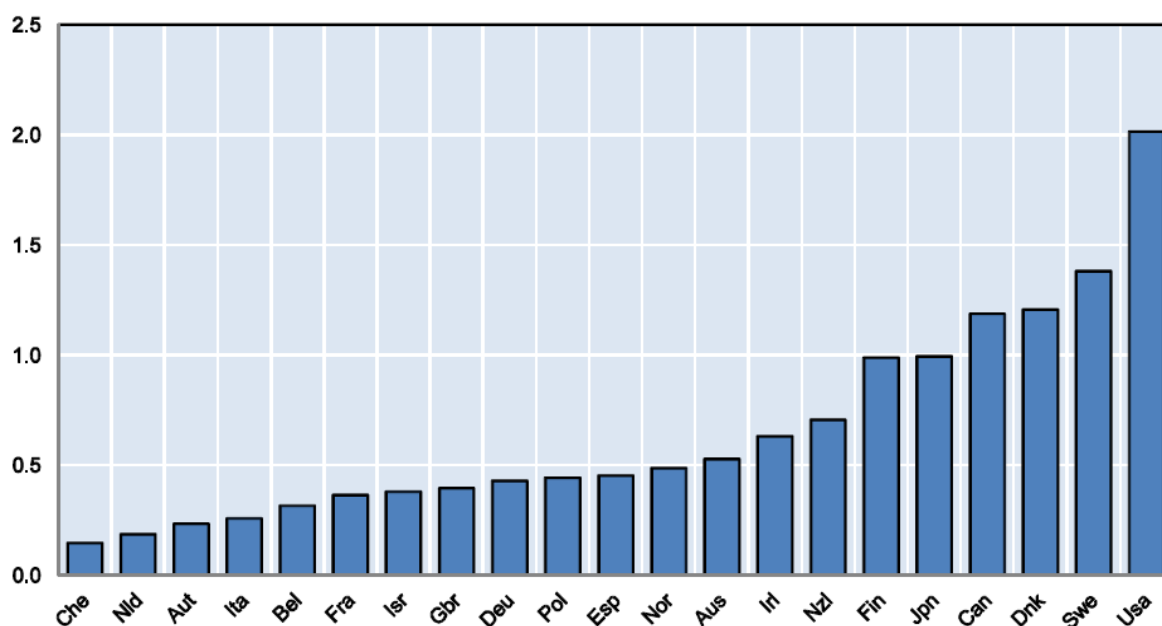
Is the evidence consistent with these simple predictions?

Chart 4 shows recent OECD estimates of the long run elasticity of new housing supply. There is striking diversity in the size of these estimates of supply elasticity across countries. On the whole countries with low density of population have high supply elasticities ( USA, Canada, Finland, Denmark, Sweden); countries which are densely populated (Netherlands, Belgium, UK, Israel, Italy ) have relatively low supply elasticities. The phenomenon also seems to apply within countries; relatively densely populated regions seem to have a lower supply elasticity. Chart 5 shows the cross country picture and also the pattern across US cities.

There are clearly exceptions to this rule: Japan has a relatively high supply elasticity – at least according to the OECD estimates; France has a relatively low elasticity.

Another piece of evidence is consistent with the simple model outlined above: rises in land prices – as opposed to rises in the cost at which residential structures can be created – account for by far the larger part of the rise in real house values over time. Differential technical progress is not likely to be the major factor behind the rise in real house prices that has been common to nearly all the richer countries over the past 40 years. In many countries house prices more than doubled in real terms between 1970 and 2010. If real construction costs were to have more than doubled over the same period then house building productivity would have needed to lag that in other industries by more than 1.7% a year. Since 1.7% a year is probably in excess of general productivity improvements we would need there to have been technological regress in house building.

**Chart 4 – International estimates of the long-run elasticity of new housing supply<sup>(a)</sup>**

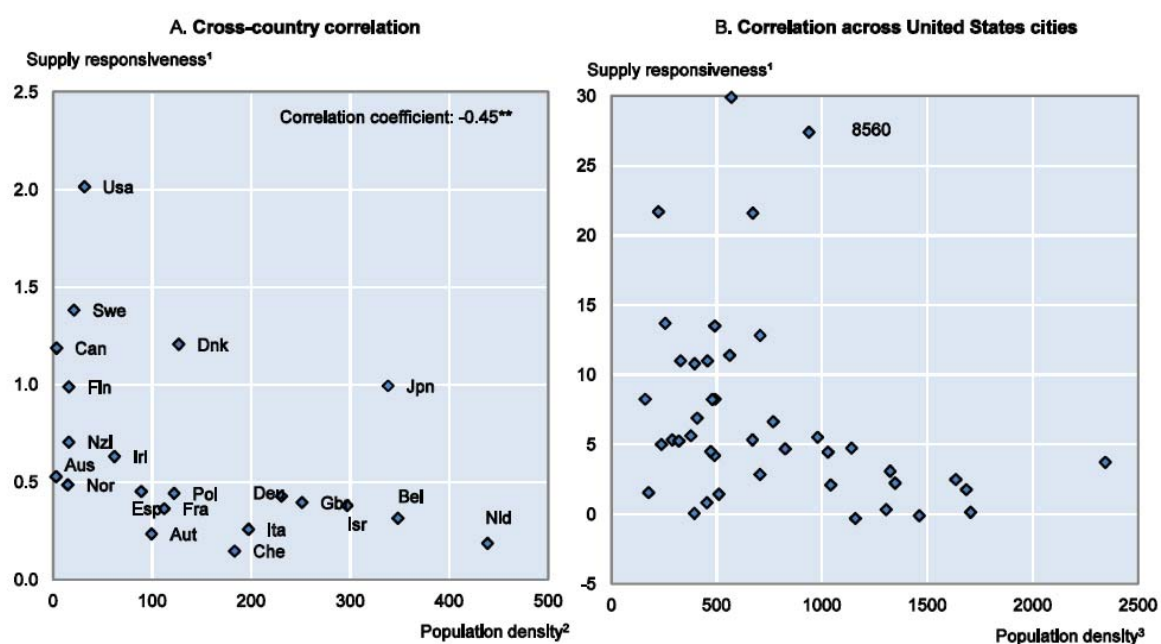


Source: OECD estimates, Andrews, Caldera Sánchez and Johansson (2011).

(a) Estimates of the long-run price elasticity of new housing supply where new supply is measured by residential investments. All elasticities are significant at least at the 10% level. In the case of Spain, restricting the sample to the period 1995-2007, which would reflect recent developments in housing markets (such as the large stock of unsold houses resulting from the construction boom starting in 2000 and peaking in 2007-09), only slightly increases the estimate of the elasticity of housing supply from 0.45 to 0.58. Estimation period early 1980s to mid 2000s.

In the UK residential land prices have risen by much more than house values, though are highly correlated with them (Chart 6). In contrast, an index of real rebuilding costs (that is construction costs) have been relatively flat over the past 35 years. In the US this pattern of land price rises being highly correlated with – but greater than – house price changes is also apparent. Chart 7 shows 10-year changes in real house prices and in real land prices by US states over the past thirty years. The correlation is high but the scales of the axes show just how much more variable land prices have been.

Chart 5: Price responsiveness of supply and scarcity of land



Source: Andrews, Caldera Sánchez and Johansson (2011).

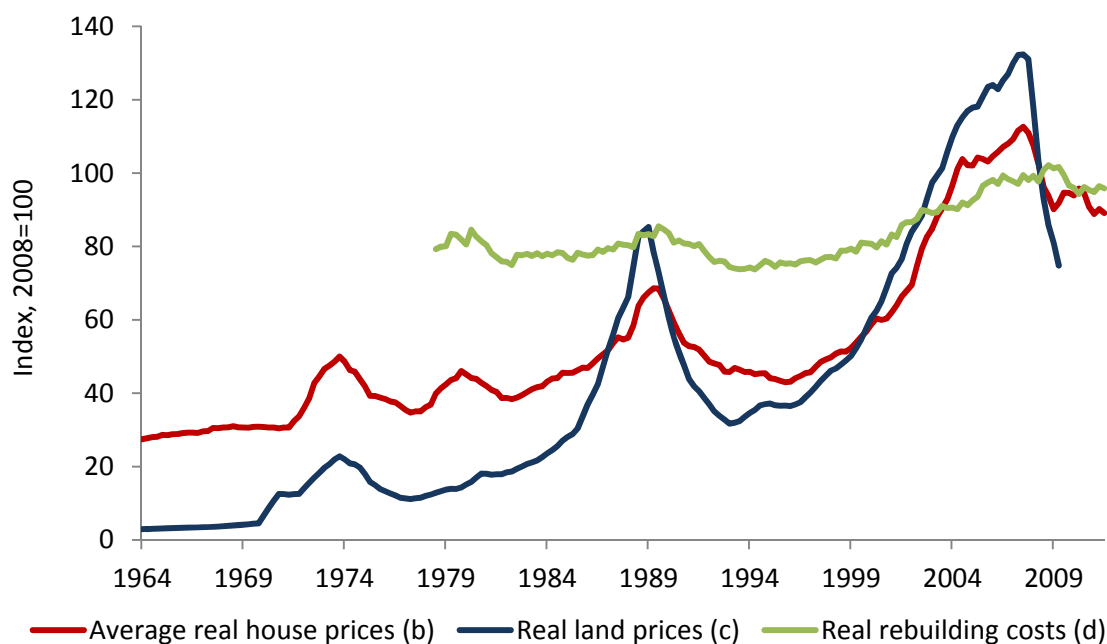
(1) OECD estimates of country-specific supply responsiveness (see Caldera Sánchez and Johansson (2011)) and estimates of supply responsiveness for United States cities taken from Green et al. (2005).

(2) Population density measured as population per km<sup>2</sup>.

(3) Population density measured as population per square mile.

So the long run evidence – on house values, on the elasticity of housing supply, on the path of land prices and of construction costs – seems broadly consistent with the simple model of housing and demographics outlined above. Relatively densely populated countries tend to have high land prices and high house prices – both relative to prices of other goods and relative to incomes. They are also likely to have low supply elasticities. The UK fits this story well: it has high population density, high real land prices, high real house prices, high house prices relative to incomes and a low elasticity of housing supply. It might be argued that these phenomena are a reflection of UK planning rules rather than underlying economic forces. But I find that unconvincing, or at least superficial. It treats planning rules as an exogenous factor. But planning rules reflect the underlying economic forces. It is surely not by chance that restrictions on use of land for residential development have become higher as incomes and population density have risen. The model I developed shows that the value placed upon **not** using land to build housing on will rise with population density and incomes. The planning rules may have a major impact on how the rising value of space is distributed; but it is surely a mistake to see the rise in the value of land as simply a reflection of restrictions that are unrelated to the economic drivers of demand for space that are rooted in preferences and incomes.

**Chart 6: Indices of average house prices, rebuilding costs and residential land prices in the UK<sup>(a)</sup>**



Sources: Building Cost Information Service (RICS), DCLG, Nationwide, ONS, Valuation Office Agency, Bank of England calculations.

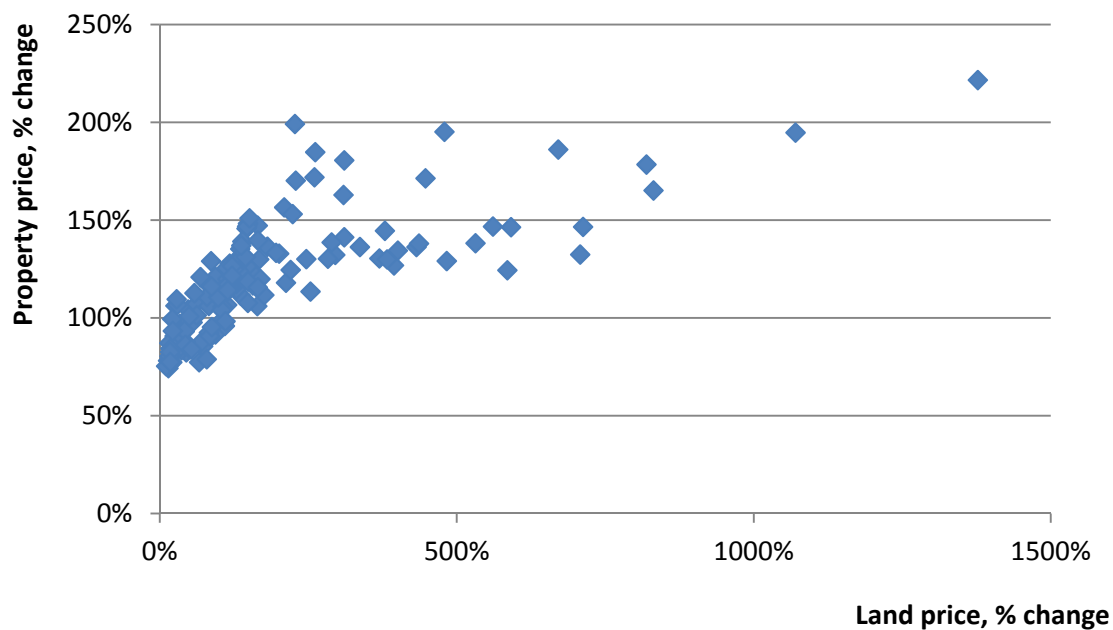
- (a) Past nominal prices converted to present values using the Office for National Statistics Retail Price Index (RPI) and indexed to 2008. Not seasonally adjusted.
- (b) Based on Nationwide nominal house price data.
- (c) Residential land prices per hectare. Regional DCLG and Valuation Office Agency data weighted by population.
- (d) Based on Building Cost Information Service indices.

### 3 House prices and the design of mortgages

The model developed above suggests that the trend of rising real incomes and the likelihood of rising population density mean we should anticipate a rising trajectory for real house prices over the longer term. This is particularly likely in a country like the UK where population density looks set to rise relatively fast. The model also suggests that the upwards trajectory in house values may ultimately become steeper than the rise in real incomes.

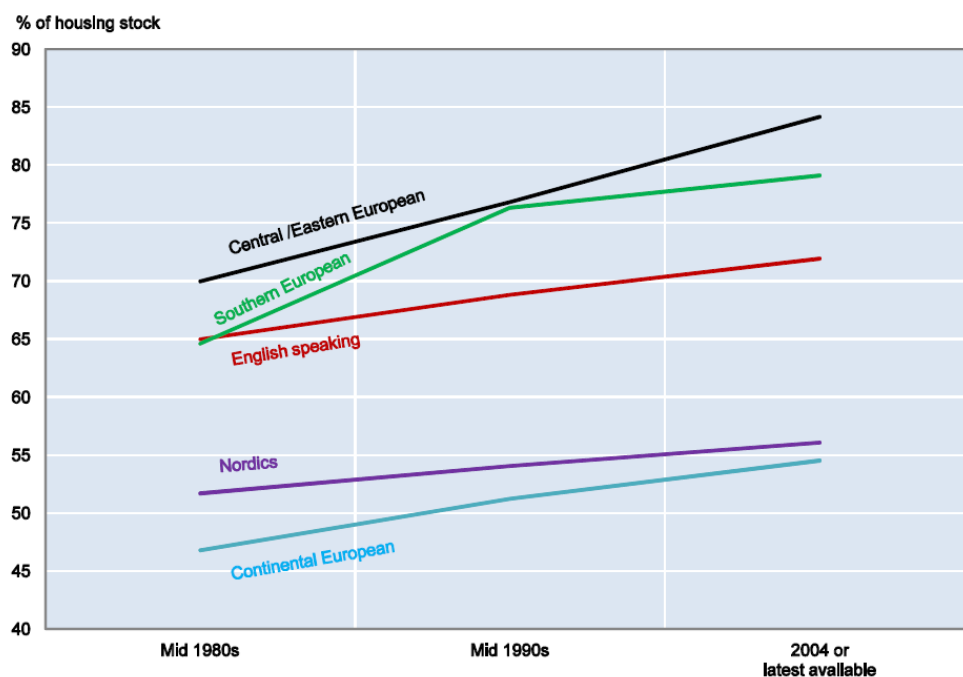
That prompts a question as to the desirability and feasibility of having house purchases largely financed by debt (standard mortgages). That is the issue that I consider in this section. Debt has been the dominant form of financing for many years. Indeed a key factor behind the rise over the past several decades in owner-occupation rates across the richer countries (chart 8) has been the widening availability of mortgage finance, allied with (in most countries) tax advantages for owner-occupation relative to renting.

**Chart 7: 10-year changes in real property and land prices in US states, 1980-2010**



Sources: Lincoln Institute, Bank of England calculations. Nominal series deflated by US CPI index to 1975 prices. Changes in ten-year steps: 1980 to 1990; 1990 to 2000; 2000-2010.

**Chart 8: Owner-occupation rates in various country groups, mid 1980s-2004**

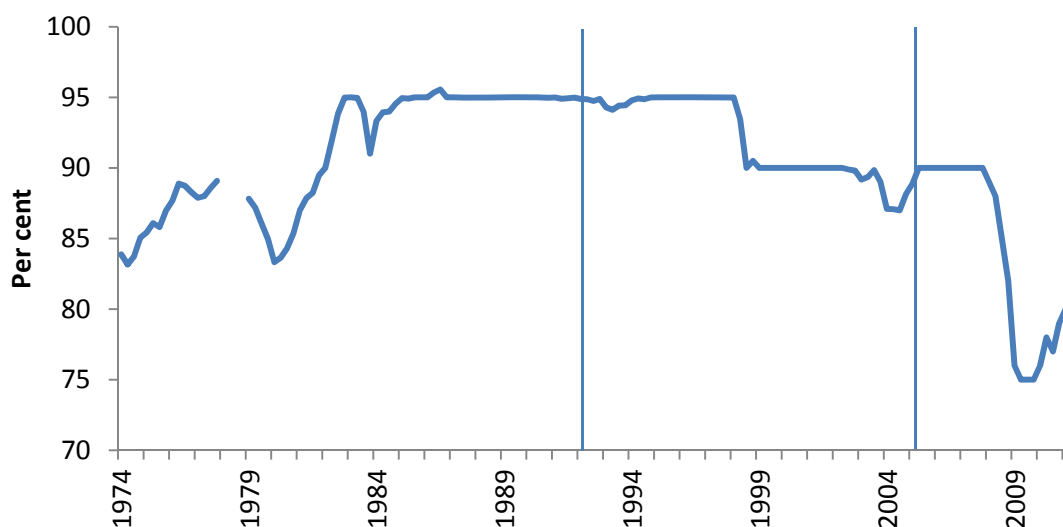


Source: Andrews, Caldera Sánchez and Johansson (2011). Nordics includes Denmark, Norway, Sweden and Finland; English-speaking includes Australia, Canada, the United Kingdom, the United States and Ireland; Continental European includes Austria, Belgium, France, Germany, the Netherlands, Switzerland and Luxembourg; Southern European includes Greece, Spain and Italy; Central/Eastern includes Hungary, Poland and the Russian Federation. The homeownership rates in each group refer to the simple average of the rate in individual countries.

But that greater availability of mortgages has now gone into reverse. Some part of that reversal in availability in mortgage finance may be temporary, but some will be permanent. It is very hard to imagine that the availability and cost of mortgages in the US and the UK will return at any foreseeable point to what it was like in the years just before the financial crisis in 2007. In the UK in the years leading up to 2007 the Financial Services Authority have estimated that around half of new mortgage lending was to individuals who did not verify their income in any meaningful way; 100% (and higher) loan to value mortgages were available to new borrowers, often at interest rates that were below those being charged to existing borrowers with low loan to value ratios and who had established a track record of making their regular mortgage repayments. The laxness in lending standards in the US was also remarkable and clearly (at least in retrospect) completely unsustainable.

The change since then has been very abrupt. Indeed what has happened in the UK and in the US has been stark – particularly for new buyers. In the UK banks and building societies are now requiring that house purchases are financed with more equity – the supply of high loan to value ratios mortgages has dried up. This drying up in high loan to value (LTV) ratios has meant that the median LTV ratio of first time buyers in the UK has increased very sharply (Chart 9). The flow of net new mortgage lending has fallen to around zero and the number of housing market transactions has fallen to around half what was normal before the financial crisis. The lack of availability now of very high LTV loans is clear. But it should not – I think – be seen as a sign of a damaged market, one that is not functioning properly. It probably never made sense for there to be 100% mortgages. There may be no price at which it makes commercial sense for such a loan to be available – a point made thirty years ago by Stiglitz and Weiss (1981).

**Chart 9: Median loan-to-value ratios for first-time buyers**



Source: CML ; . Data are not available for 1978. The vertical lines represent breaks in the series.

So new home owners in the future may need to have more equity than was normal in the years leading up to the financial crisis. This will have an impact – probably permanently – on the pattern of home-ownership and – at least transitionally – on the demand for housing.



The first effect is likely to be for prospective buyers to postpone their purchase, while they save more to accumulate a larger deposit. As a result the average age at which people would buy their first home will rise, and the share of owner-occupied houses will fall. The change in the pattern of home-ownership this generates can be expected to be very large if the required equity is only provided by the prospective home-owners themselves. But saving – the provision of internal equity – is not the only source of equity. We can hardly expect prospective first-time buyers to issue shares on the London Stock Exchange. But there are feasible contracts which are a bit like issuing shares.

One variant is shared ownership schemes, in which the buyer acquires only a share of the home and pays rent on the fraction owned by the (outside) equity provider. This reduces the buyer's exposure to changes in the value of the house because they only own a fraction of it. And that exposure is more extreme the higher are house values. Rising populations in many countries – including the UK – is likely to make exposure to house price risk greater because it will tend to drive up house prices.

Another variant are so-called equity loans. In contrast to shared ownership schemes, the buyer retains the ownership of the entire property<sup>3</sup>. But those who provide equity loans accept some of the risk – both upside and downside – that the value of the house changes. Effectively the interest rate on the loan becomes linked to the evolution of the value of the house. Equity loans are hybrid instruments, with characteristics somewhere between straightforward debt and equity.

An equity loan enables a buyer to take out a smaller traditional mortgage. They might obtain this mortgage at a lower interest rate because the mortgage bank's risk has fallen: the bank is protected against house price falls not only by the homeowner's deposit, but also by the equity loan.

Of course, the provider of the equity loan requires a return. For investors in hybrid instruments in firms, this compensation usually comes in the form of some contractual payments during and at the end of the maturity of the investment. These contractual payments can be quite low if investors also expect that their stake might gain in value over time. Similarly, the provider of an equity loan might require regular payments from the homeowner. But such regular payments may not be necessary so long as the final payment is adjusted up. In the case of equity loans, this final payment typically depends on the terminal value of the property. So in exchange for taking a higher share of a capital gain on a property a provider of an equity loan might agree to receive no payments until the property is sold (or at least up until some set date).

Consider the following example. An investor provides an equity loan of £10,000 for a house worth £100,000. In exchange for paying no annual return on this loan the homeowner agrees to give up 20% of any gain in the value of the home and the loan provider agrees to take 10% of any loss in the value of the house. Five years later the homeowner decides to sell the house. If the house at that point is worth £135,000, they repay the investor the initial £10,000 plus 20% of the £35,000 gain in the value of his house, that is, £17,000 in total. If, instead, the house is worth only £80,000, they repay the investor the initial £10,000 less 10% of the £20,000 loss in the value of his house, a total repayment of £8,000. Were both price movements equally likely, the investor would expect to receive  $\frac{1}{2} * (£17,000) + \frac{1}{2} * (£8,000) = £12,500$  on an initial investment of £10,000. This is an expected return of 25% over five years.

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<sup>3</sup> Shared equity schemes and equity loans are offered as part of the UK government's affordable housing schemes.

Figuring out what share in the upside (rises in house values) and the downside (falls in house values) the provider of an equity loan would need to take to make it a reasonable deal – given that the loan pays no regular interest – is not straightforward. It depends on the probabilities of house price changes over long periods. And there are a continuum of contracts that could be acceptable to lenders, some of which have smaller shares of the upside but also smaller shares of the downside. In the example above – and if we assume that we somehow know that the only two, equally probably outcomes are the house falls to £80,000 or rises to £135,000 in value – we can work out what such contracts look like. If, as in the example above, we assume people need to earn an expected return of 25% on a loan of £10,000 any pair of values of x and y that satisfy this equation work:

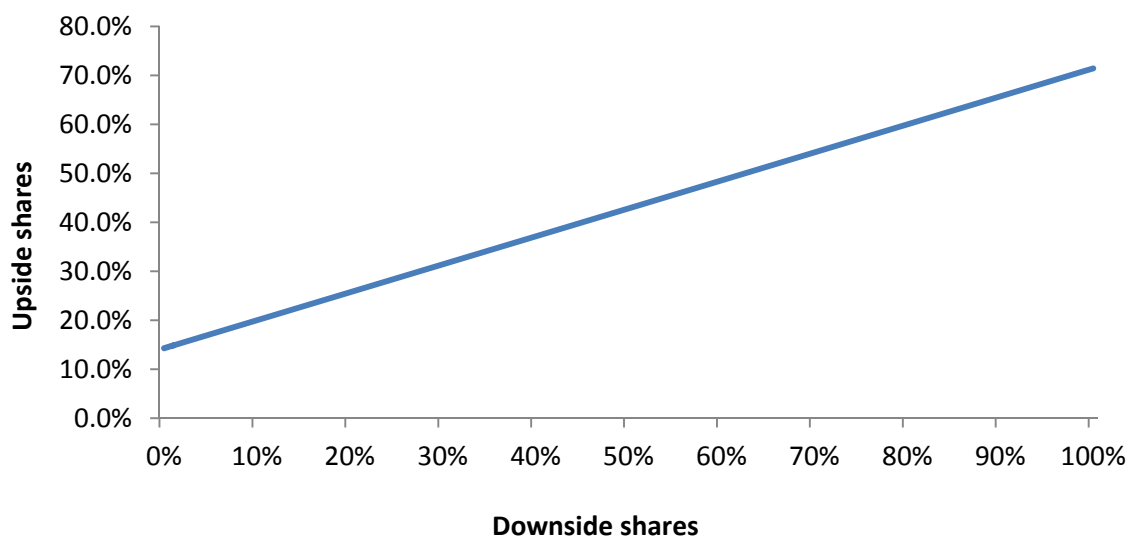
$$\frac{1}{2} * y * £35,000 - \frac{1}{2} * x * £20,000 = £2,500$$

$$\text{So that: } y = [2 * 2,500 + x * 20,000] / 35,000$$

where y is the share of any upside that the provider of the equity loan takes and x is the share of any downside. Chart 10 shows combinations of possible upside and downside shares, which generate the required return to the provider of the equity loan.

We have seen that y= 20% and x = 10% work. So do y = 33% and x = 33% or y=40% and x = 45%. It is possible in this example to share upside and downside risks equally, but also for home-owners to hand more of any losses over than they hand over of any gains.

**Chart 10: Simple example of equity loan – neutral upside and downside shares of investor<sup>(a)</sup>**



(a) Assumptions: Equity loan is 10% of house value. Equal probability of 20% fall and 35% rise in the value of property. Required rate of return on equity loans over a 5 year period is 25%.

A more realistic example would be one where house price changes follow a log normal distribution. I will now explore the nature of contracts that are sustainable in a world where house prices follow such a distribution, using what I think are reasonable calibrations.

We assume that house prices are generated by:

$$\ln(P)_t = a + b \ln(P)_{t-1} + e$$

$e$  is a random shock that follows a normal distribution. One can think of the trend in house prices (reflected in the constant  $a$ ) as coming from the solution to the long run trajectory for house prices generated by the model developed above and where the driving factors are the growth in population and in average incomes. That model had no stochastic factors and was used to analyse long run trends in average house prices. But mortgage design is very much about handling the risks for individual households and the volatility of their incomes and of the values of the specific properties they buy. That is why the stochastic factor in house prices ( $e$ ) is central to this analysis of financing contracts.

If (as seems likely)  $b$  is very close to unity then  $a$  is the expected rate of house price inflation. We will assume from here that  $b$  is 1.

As before, denote the proportion of any house value appreciation that is paid to the provider of an equity loan by  $y$ . Let the proportion of any house price fall that is taken by the equity loan be  $x$ . I denote the percentage change in house prices by  $p$  and the level of house prices by  $P$ . Let the required expected rate of return on the equity loan be  $R_e$ ; and let the value of the equity loan as a proportion of the value of the house value be denoted by  $g$

The rate of return (ex post) on an equity loan is then given by:

$$\begin{aligned} & py/g && \text{if } p > 0 \\ & px/g && \text{if } p < 0 \end{aligned}$$

The equilibrium condition is that:

$$R_e = E(py/g | p > 0) \cdot \text{prob}(p > 0) + E(px/g | p < 0) \cdot \text{prob}(p < 0)$$

where  $E(py/g | p > 0)$  is the expectation of  $py/g$  conditional on  $p > 0$

and  $E(px/g | p < 0)$  is the expectation of  $px/g$  conditional on  $p < 0$

Based on quarterly data of regional house price indices<sup>4</sup> for the UK over the period 1990-2011 the average (across regions) of the mean and standard deviation of quarterly changes in log (nominal) house prices are 1.1% and 3.4%.

The mean changes aggregate easily for different time periods – the average return over 5 years is simply 20 times the average quarterly return. This is not true for standard deviations – only under the assumption that the quarterly random shocks ( $e$ ) are independent and of constant volatility will the five year return variance be 20 times the quarterly variance. (In fact the data suggest that the 5 year variance is rather more than 20 times the quarterly variance.)

We initially will assume that 5 years is the relevant horizon for contracts – though as we shall show below the key parameters  $x$  and  $y$  are relatively insensitive to varying the investment horizon.

Table 2 shows combinations of upside ( $y$ ) and downside ( $x$ ) shares due to an equity lender where the 5 year percentage change in the value of a house follows a normal distribution with an assumed mean of 10% and a standard deviation of 20%. This assumed mean is lower than the realised

<sup>4</sup> The Nationwide regional house price indices.

average across regions over 5 year horizons since 1990. But it is quite probably higher than current (2012) expectations with many housing analysts predicting roughly flat prices for several years. The standard deviation of 20% is a little lower than the sample standard deviation from the past. We initially assume that the required expected (or average) nominal return is 25% over five years. We consider a loan worth 20% of the value of a property ( $g=0.2$ ):

**Table 2: Stylized equity loan – equilibrium combinations of upside and downside shares of investor<sup>(a)</sup>**

<b>Share of upside</b>	36%	39%	42%	45%	48%	52%	55%	58%	61%	64%
<b>Share of downside</b>	0%	11%	22%	32%	43%	57%	68%	78%	89%	100%

Source: own calculations.

(a) Assumptions: Equity loan is 20% of house value. Percentage change in house price value over 5 years follows a normal distribution with mean of 10% and standard deviation of 20%. Required rate of return on equity loan is 25% over five years.

Table 2 reveals several things: First it is possible for home owners to sell all downside risk – with the base parameters it is possible for providers of an equity loan to insure against 100% of house price falls in exchange for taking about 64% of all gains in house value. With the calibration above the chance that house prices will be lower at the end of 5 years is about 30% so this insurance is of value. Second, there is a big range of risk sharing that is feasible: with home owners taking no insurance against the loss in house value they would give up just over a third of any capital appreciation; if they wanted to insurance against 50% of house price losses they would need to give up about one half of any house price appreciation. Third, providers of equity loans need to receive more of any house price appreciation than the share of funding they provide: even with no downside protection the providers of loans need to get almost twice as much of any appreciation (36%) in exchange for providing 20% of the funding.

Table 3-6 below show how the characteristics of the equilibrium shares of upside and downside risk taken by the provider of equity loans varies as we use different assumptions for: the volatility of house prices (Table 3); the time horizon of the investment (where we multiply the base line means and variances of house price returns by the time horizon, which is appropriate for the random walk with drift model – Table 4); the required rate of return on the loans (Table 5) and for the mean expected rate of house price appreciation (Table 6). In each of the tables we assume the equity loan is 20% of the house value and that the share taken of any house price fall is also 20%. For the base case (Table 3) that generated an equilibrium share of any upside gain of about 42%.

**Table 3: Varying house price volatility – equilibrium upside shares of investor<sup>(a)</sup>**

<b>Share of upside</b>	50%	45%	42%	40%	38%	36%
<b>Standard deviation house price change</b>	5%	15%	20%	25%	30%	40%

**Table 4: Varying time horizons – equilibrium upside shares of investor<sup>(a)</sup>**

<b>Share of upside</b>	39%	41%	42%	44%	46%	49%
<b>Time horizon</b>	3 years	4 years	5 years	7 years	10 years	20 years

**Table 5: Varying required rates of return on the loan – equilibrium upside share of investor<sup>(a)</sup>**

<b>Share of upside</b>	27%	35%	42%	49%	57%	64%
<b>Required return on the loan</b>	15%	20%	25%	30%	35%	40%

**Table 6: Varying average house price changes – equilibrium upside share of investor<sup>(a)</sup>**

<b>Share of upside</b>	67%	51%	42%	32%	25%	17%
<b>Mean house price change</b>	3%	7%	10%	15%	20%	30%

(a) Assumptions: Equity loan is 20% of house value. Percentage change in house price value over 5 years follows a normal distribution with 20% standard deviation. Required rate of return on equity loan is 25% over five years.

The tables show:

1. Changing the time horizon has a limited impact on the nature of sustainable contracts when log house price follow the random walk model, though the share of the upside needed to be paid to the equity provider does rise gradually with the time horizon
2. Varying the volatility of house prices has a relatively small impact, but as volatility rises the share of the upside that needs to be paid to the provider of the equity falls
3. Varying the expected rate of house price change has a very substantial effect
4. Varying the required rate of return has a substantial effect.

The attractive thing about there being a continuum of contracts is that home-owners could decide how much house price risk they wanted to sell and how asymmetric the contract they were prepared to accept to reduce the monthly servicing cost of the overall debt on a property. But there are practical issues to do with the timing of people moving house and making sure that home-owners understand contracts they have entered to. There is also a question about the supply of

funding for this type of loan. One way in which financial institutions could provide such loans without taking on significant house price risk is to issue savings products with returns linked to house price changes, a saving product which potential home owners are likely to find particularly useful.

Equity type funding of house purchase has major attractions – at both the micro and macro level: at the micro level they have the potential to allow more efficient sharing of house price risk. The purchase of a very expensive asset by means of high leverage is not – to put it mildly – self evidently the optimal contract (Shiller (1998, 2003)). At the macro level a useful feature of equity loans is that effectively the interest rate paid on the loan is linked to the rate of house price inflation – the higher is house price inflation the higher is the effective interest rate on a portion of the funding of houses. That could be a stabilising force.

These potential advantages of equity financing of home-ownership are greater the higher are house values relative to the value of other goods. One implication of demographic changes is that real house values in densely populated countries with low supply elasticity are likely to be relatively high and, on average, rising.

## Conclusion:

This paper has developed a model of the housing market that takes account of population density to assess the impact of population changes on the value and size of the housing stock. The model implies that if population density is on an upward trajectory rises in population and in incomes increasingly generate price responses and diminishing rises in the stock of housing. This has implications for the optimal structure of housing finance. It makes equity financing of home purchase more desirable. Hybrid debt-equity contracts for financing house purchase have micro and macro benefits. At the household level they allow risk sharing to be more effective – which generates larger benefits the higher are house prices. At the macro level hybrid contracts may be a stabilising force because they generate funding costs that increase as house prices accelerate.

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