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Emmanouil Karimalis,[(1)] Ioannis Kosmidis[(2)] and Gareth W Peters[(3)]

Abstract

We develop a new multi-curve modelling framework for the term-structure of interest rates that can generate consistent cross-country stressed scenarios allowing for significant spillover effects between economies. Modern models of the term structure of interest rates typically fail to capture jointly time and cross-curve dependencies and are not used for stress-testing purposes. Our methodology is able to jointly model the temporal and cross-country dependence structure of interest rate curves and associate movements in the interest rates and cross-country spreads with movements in macroeconomic variables as well as market-wide and country-specific measures of liquidity and credit quality. We apply our methodology to generate contemporaneous stressed scenarios to a set of European yield curves. Motivated by the recent eurozone debt crisis, we apply shocks to Italian and Spanish liquidity and credit variables and evaluate the impact of these shocks on several bond portfolio strategies. The empirical findings suggest that both country-specific liquidity and credit measures are important in explaining the dynamic behaviour of European sovereign interest rate curves and their dependence structure. Nevertheless, their importance varies across time, shock types and investment horizons.

Key words: Stress-testing, term structure, yield curve, liquidity risk, credit risk.

JEL classification: G11, G18, G20, G21, G32, G38.

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1 Introduction

Stress testing and scenario analysis methods are becoming essential tools for effective risk management and macro prudential oversight. They form a core element of risk management tools for quantifying the size of potential losses under extreme stress events, and for identifying the scenarios under which such losses might occur. The utility of stress testing arises from the fact that they inform financial institutions and supervisors of the effects of financial conditions, through a set of adverse changes in risk factors corresponding to "exceptional but plausible" events.

In designing a stress testing framework, one typically has several aspects to consider. For instance, it is adopted practice in banking and insurance industries to focus on designing stress tests based around periods of global and local economic conditions which represent both micro and macroeconomic stressed business cycles. One must therefore, decide upon a candidate set of historical periods to act as observed stress economic regimes. In addition, one also has to address what aspect of the economy should be stressed. For instance, one may consider a wide spectrum of different interest rate environments such as domestic interest rates, international interest rates in multiple yield curves and discount curve settings from Libor rates, OIS rates, corporate yields at different credit ratings and government and municipal bond rates. In addition, one could consider other aspects such as economic factors to do with inflation rates, manufacturing and productivity rates, employment rates, financial market liquidity, financial market volatility, equity and commodity index prices and global currency exchange rates. These are just a small selection of a wider universe of possible micro and macro economic drivers that can be influenced in periods of stress and may be impacting on banking and insurer capital adequacy and solvency.

International policy institutions such as the Bank of International Settlement (BIS) have undertaken market wide surveys to assess what is the range of practice in banks and insurers when it comes to stress test designs. For instance, one may consider the range of practice captured by the 2005 survey (Basel Committee on Global Financial System, 2005) which demonstrated that certain key attributes of the economy were systematically included in most stress tests performed by financial institutions. Importantly, this survey revealed that interest rates stress testing is the most popular type of test carried out by the majority of the surveyed institutions globally. In particular, the vast majority of the surveyed institutions (around 94%) report using stress testing on interest rates. Yet, despite this survey highlighting the importance of interest rate stress testing in the vast majority of financial institutions, there is still a long way to go in developing the adequacy of stress tests related to interest rate events.

This was also highlighted in the Basel guidance (Basel Committee on Banking Supervision, 2010b) where systematic weaknesses in stress tests are identified particularly related to areas such as the behaviour of complex structured products under stressed liquidity conditions; basis risk in relation to hedging strategies; pipeline or securitisation risk; contingent risks; and funding liquidity risk. In several of these aspects one can consider interest rate features, which may not just be interest rates in the nominal currency in which the financial institution operates, but may involve multiple interest rate curves. In such cases the inter-relationship between the curves, their dynamics and the effect of liquidity and credit risks is critical to capture.

With this view in mind, we focus on a *multi-curve* setting and develop a new methodology that can generate consistent - not individually calibrated - *cross-country* stress test scenarios allowing for significant spillover effects between the economies. In particular, we model *jointly* the temporal and cross-country dependence structure of several European sovereign yield curves and associate movements in the yields and cross-country spreads with movements in macroeconomic and financial variables as well as market-wide and country-specific measures of liquidity and credit quality in a statistical rigorous way.[1] The framework also allows the study of interaction between macroeconomy and term structure and the assessment of importance of these factors in the evolution of sovereign yields and cross-market spreads. In addition, within the proposed framework one can more readily handle features of real data such as missingness and/or unbalance datasets. The model can also be easily amended and applied to different Rates markets with similar characteristics to generate consistent stress test scenarios. Modern models of the term structure of interest rates do not typically model jointly the temporal and cross-curve interest-rate dynamics and thus when applied to stress-testing they fail to generate scenarios consistent with their implied dependence structure. To the best of our knowledge, we are the first who provide an analytical framework that allows generating consistent stress scenarios across multiple curves though shocks to market-wide and country-specific shocks.

The framework we develop consists of two parts. In the first part, we model the evolution of the yield curve for each particular country under study using the macro-finance Nelson-Siegel model of Diebold et al. (2006) augmented with key macroeconomic and financial variables, as well as European measures of liquidity and credit quality. The inclusion of market-wide liquidity and credit quality variables in the latent factor specification of the model allows for a statistical treatment dynamic interaction between these risks and the yield curve for each particular country. In the second part, we model the covariance structure of European sovereign yields employing the covariance regression model of Hoff and Niu (2012). In this respect, we parameterize the covariance matrix of sovereign yields as a function of country-specific liquidity and credit quality factors and explore their effects on the heteroscedasticity of European sovereign yields.

It is well-documented in the finance literature that liquidity and credit concerns are important components of the yield spreads (see for example, Duffie et al., 2003; Longstaff et al., 2005; Beber et al., 2009, among others).[2] Nevertheless, so far, most studies have related liquidity and credit risks to the level of yields or yield spreads focusing on certain maturities or markets in isolation without taking into account the dynamic interaction of these risks with the term structure of interest rates and the cross-markets dependence. The importance for taking into account shocks to liquidity and credit quality variables when designing interest rates stress scenarios is not only mandated by academic research but is also a regulatory requirement. The recent 2016 EU-wide bank stress testing exercise designed by the European Banking Authority (EBA) calls specifically banks for incorporating market liquidity and country-specific shocks to sovereign credit spreads when conducting stress testing exercises.[3]

We apply our methodology to generate stressed interest rate scenarios for 5 major economies in the EU, namely Italy, Spain, Germany and France and the UK. Motivated by the recent Eurozone debt crisis, we apply contemporaneous shocks to Italian and Spanish liquidity and credit quality variables at various maturities and based on these shocks generate a number of hypothetical movements in the European yield curves. In general, shocks to Italian and Spanish liquidity and credit

[1] Studies that relate macroeconomic variables to the yield curve include, Kozicki and Tinsley (2001), Ang and Piazzesi (2003), Hördahl et al. (2006), Ang et al. (2006), Dewachter and Lyrio (2006), Balfoussia and Wickens (2007) and Rudebusch and Wu (2008).

[2] Studies on bond liquidity also include Balduzzi et al. (2001), Krishnamurthy (2002), Goldreich et al. (2005), Chordia et al. (2005), Liu et al. (2006), Dick-Nielsen et al. (2012), de Jong and Driessen (2012), Kempf et al. (2012) among others.

[3] See "Adverse macro-financial scenario for the EBA 2016 EU-wide bank stress testing exercise", available at www.eba.europa.eu/documents/10180/1383302/2016+EU-wide+stress+test-Adverse+macro-financial+scenario.pdf.

variables may have significant impact on yields and cross-country spreads. However, the impact of these shocks on yield curves under study vary across country, maturity and time horizon. We also evaluate the impact of Italian and Spanish credit and liquidity shocks on several hypothetical bond portfolio strategies and show that they have very different impacts on the portfolios' values and returns.

The reminder of the paper is organised as follows: Section 2 briefly discusses the evolution of regulatory guidance regarding stress testing and the increasingly important role such stress testing frameworks play in risk management for banks and insurers. Section 3 discusses briefly the alternative approaches employed for interest rates stress testing, while Section 4 introduces the macro-finance Nelson-Siegel model of Diebold et al. (2006) and the covariance regression model of Hoff and Niu (2012) also describes in detail the stress testing methodology. Section 5 describes the data we use in the empirical part of this study while Section 6 presents the empirical results and also discusses how the proposed modeling approach could potentially find alternative applications. Finally, Section 7 concludes.

2 Evolution of regulatory guidance on stress testing frameworks

Multiple regulatory institutions have begun to specify guidance on stress testing scenario development and its use and applicability in assessing capital adequacy and solvency, especially after the 2007-2008 financial crisis. There are many different stress test variations that are considered in banking and insurance contexts and each jurisdiction has different approaches. For instance in the UK one has a range of stress tests overseen by the Bank of England and the Prudential Regulatory Authority (PRA) such as the annual industry stress tests as detailed in Prudential Regulation Authority (2015), which includes details of stress tests for banks and general insurers. In Europe there are stress tests undertaken regularly by the European Banking Authority (EU-wide), which includes tests by the European Central Bank (ECB) to undertake analysis such as the Comprehensive Capital Assessment Review (CCAR). In the United States there are the CCAR assessments and the Dodd-Frank Act stress testing (DFAST) - a complementary exercise to CCAR - undertaken regularly by Federal Reserve System.

Stress testings role as a risk management tool has been strengthened further by regulatory frameworks such as the Basel II and Basel III accords (Basel Committee on Banking Supervision, 2006, 2010a) and through insurance groups such as the Insurance Regulation Committee of the International Actuarial Association (IAA) who released guidelines for insurance stress testing best practice (Insurance Regulation Committee, 2013), the stress testing requirements specified by the Bank of England (Bank of England, 2015, 2016) as well as exchange regulations such as MIFID II and EMIR which have components comprising product governance, which involves rigorous stress testing analysis of developments of structured products (Financial Conduct Authority, 2015). To understand the impact such regulatory frameworks have had with regard to stress testing we note that as a result of the Basel II and Basel III accords mentioned, banks are required to perform stress tests to identify scenarios that could result in significant adverse outcomes and thus to determine the capital they would need to satisfy regulatory requirements. This requirement comes up in multiple places in Basel II and III regulations, firstly as part of the Pillar 1 (minimum capital requirements) of the Basel II framework for any banks that are adopting the Internal Models Approach to determine market risk capital. Secondly, banks using the advanced and foundation internal ratings-based (IRB) approaches for credit risk are also required to conduct credit risk stress tests to assess the robustness of their internal capital assessments and the capital buffers above the regulatory minimum. The minimum set of stress tests in this regard should be performed on the credit portfolios in the banking book. Furthermore, under Pillar 2 there are also other more general stress tests that banks must undertake to assess their capital adequacy and capital buffers.

Stress testing has also started to play a critical role in other financial institutions such as Central Counterparties (CCPs) which employ stress tests to determine the size of their default funds.[4] Recently, the European Securities and Markets Authority (ESMA) conducted its first EU-wide stress test exercise regarding Central Counterparties (CCPs). The exercise was aimed at assessing the resilience and safety of the European CCP sector as well as to identify possible vulnerabilities (European Securities and Markets Authority, 2016). Furthermore, brokerage firms and hedge funds conduct stress testing to calculate portfolio sensitivities, set portfolio limits and evaluate risks where Value-at-Risk (VaR) models are of limited use. Central banks also use stress testing, inter alia, to guide policy on the setting of prudential capital buffers or reveal possible vulnerabilities in the financial system. Going forward, stress testing is likely to become even more important as regulators and market participants have set out recommendations to further enhance these frameworks.[5]

Despite the range of regulatory guidance on stress testing continuing to emerge, at present there is no universal definition for all possible types of scenario analysis and stress testing framework. However, one can adopt a general definition for such quantities that gives the essence of the notion of a scenario and stress test that is universally applicable. For instance, a scenario can be defined as offered by the IAA according to the broad definition:

> A stress test is a projection of the financial condition of a firm or economy under a specific set of severely adverse conditions that may be the result of several risk factors over several time periods with severe consequences that can extend over months or years. Alternatively, it might be just one risk factor and be short in duration. The likelihood of the scenario underlying a stress test has been referred to as extreme but plausible (Insurance Regulation Committee, 2013).

In this way one may consider a stress test as providing an assessment of an extreme scenario, usually with a severe impact on the firm, reflecting the inter-relations between its significant risks. The Basel guidelines (Basel Committee on Banking Supervision, 2010b) state that stress testing plays an important role in the following ways: by providing forward-looking assessments of risk; by overcoming limitations of models and historical data; by supporting internal and external communication; by feeding into capital and liquidity planning procedures; through informing the setting of a bank's risk tolerance; and facilitating the development of risk mitigation or contingency plans across a range of stressed conditions.

In addition, we note that it is common that financial regulators define a stress event as an *extreme* but *plausible* scenario in the market that the portfolio is exposed to. But how can we determine what a plausible scenario is ? This question is particularly pertinent given the points made in the Basel III guidelines which identify that:

> the financial crisis has highlighted weaknesses in stress testing practices employed prior to the start of the turmoil in four broad areas: (i) use of stress testing and integration

[4]See for example the "Best practices for CCPs stress tests" EACH paper available at http://www.eachccp.eu/wp-content/uploads/2015/12/Best-practices-for-CCPs-stress-tests.pdf.

[5]See for example the "Report of the Financial Stability Forum on Enhancing Market and Institutional Resilience", available at www.fsb.org/wp-content/uploads/r_0804.pdf?page_moved=1 or the "Final Report of the IIF Committee on Market Best Practices: Principles of Conduct and Best Practice Recommendations", available at www.apec.org.au/docs/11_CON_GFC/IIF_Final_Report_of_the_Committee_on_Market_Best_Practices.pdf.

in risk governance; (ii) stress testing methodologies; (iii) scenario selection; and (iv) stress testing of specific risks and products (Basel Committee on Banking Supervision, 2010b, p.8).

In particular we note that one must not only identify appropriate stress tests and scenarios but also have flexible methods that are capable of capturing the inter-relations of features of the possible stress in each dimensions of the stress scenario and how it may impact on the given stressed assets, banks or sectors.

With regard to the types of scenario that one should consider, they could be either hypothetical or historical scenarios or a mixture of the two. With regards to historical scenarios, should one consider the major historical crisis periods? One may question what aspects of these periods are still relevant to the market practice and current financial environment? This is why creating a motivation, justification and narrative around particular types of stress tests should first be developed. Historical periods that can be considered for such scenario formation can be periods covering extreme market events such as the Asian financial crisis of 1997, the Russian default of 1998, the Lehman Brother's default in 2007 or the period covering the recent 2009-2012 European sovereign debt crisis. The 2005 BIS survey provides a detailed list of the historical periods used by surveyed institutions to calibrate historical stress scenarios.

3 From yield curve modelling to developing stress tests

In this section we briefly comment on different formulations of stress testing. These can be based on hypothetical shocks or historical shocks calibrated to periods involving significant market events. Some stress testing formulations and their ability to capture inter and intra country effects such as contagion, spill overs and systemic risks will largely depend on the coupling between the approach to yield curve modelling and the formulation of the tests. Many model approaches are unable to capture all these effects within a multi-curve framework.

We begin with discussion on approaches that have been proposed in the context of interest rates scenario generation. As noted in Diebold et al. (2008) it is generally believed that the short end of the yield curve is under the direct control of the central bank in the given country, whilst the longer dated yield maturities are considered to be risk-adjusted averages of the expected future short rates and are based on investors perceptions of monetary policy and economic environment. A natural question is which macro-economic, micro-economic, inter-country specific factors and intra-country regional factors should be stress tested and how they can be incorporated into the modelling perturbation. In addition, the types of perturbation need to be determined. For instance, should one use absolute or relative perturbations of the yields themselves, of the yield curve model parameters (drift, volatility components) or of micro and macro economic factors that enter into the drift and volatility dynamics modelled for the rates.

A popular approach to stress testing formulations is to fit models to the yield curves and then to stress directly the model parameters, where the selection of the parameters is decided based on Principal Component Analysis (PCA) or Independent Component Analysis(ICA). Once these model parameter perturbations are obtained, then the resulting yield spreads can be studied. This approach is discussed in several papers. For instance such an approach can often involve the setting of PCA, which is often used as a tractable method for computing risk scenarios (see, for example, Litterman et al., 1991; Cochrane and Piazzesi, 2005, for further details). In particular, Loretan (1997) and Rodrigues (1997) have proposed to combine movements in principal components to generate stress scenarios. However, these methods share several shortcomings. For instance, PCA methods cannot be used to produce yields at tenors other than those observed in the data (Diebold and Li, 2006). Moreover, Fung and Hsieh (1996) have also shown that during periods of large interest rate moves, the change in the shape of the yield curve is correlated to the level of the interest rate itself. Therefore, specifying shocks in each of the directions given by the retained principal component may not be appropriate to generate stress scenarios under these circumstances. In addition, the dimension-reduction methods are not directly interpretable in terms of the effects of individual stresses of particular factors in the term structure, which, in practice, limit significantly the class of tests which can be performed for the computational advantage of parsimony.

There are only a few studies that tried to combine term structure models with dimension-reduction methods to overcome some of the shortcomings listed above. In particular, Diebold et al. (2008) proposed a two-stage procedure to identify a set of representative yield curve shocks and use them for stress testing purposes. In stage one, they fit a factor mode to actual bond yields and estimate the main shape factors of the yield curves while in the second stage they partition the factors into non-overlapping sets of representative yield curve shocks using cluster analysis. In particular they adopt the projection-pursuit approach of Friedman and Tukey (1974) and Friedman (1987) to obtain separation of classes of stress scenarios in terms of the underlying factors to be stressed. This was achieved by either maximizing or minimizing the kurtosis coefficient. In the first instance this would in principle produce bi-modality in the projection samples with two large clusters in the first instance and in the second instance of minimizing the kurtosis coefficient they argue they may obtain outlier classes or projections, see discussions on the specific single curve framework in Diebold et al. (2008). Within this framework, they can provide a wide variety of historical interest rate shocks, including typical, uncommon, and extreme ones.

Similarly, Charpentier and Villa (2010) propose a two-stage procedure to generate yield curve stress test scenarios. Their approach relies on fitting the term-structure model of Diebold and Li (2006) in stage one to calibrate the set of three latent factors, commonly interpreted as *level*, *slope* and *curvature*, and, in stage two, on estimating three statistically independent components, as linear combination of level, slope and curvature factors using ICA methods. The authors combine movements in ICs to produce stress scenarios by specifying separate shocks in each of the direction given by the three independent components.

Although these studies can account for the dependence of interest rates at all available maturities, they limit significantly the class of tests that can be performed for the computational advantage of parsimony. Furthermore, we are interested in stress test scenarios that can also reflect changes in macro, and micro economic factors, and liquidity and credit quality proxies between multiple country curves either resulting from shocks However, in many settings, this lack of direct interpretation can be detrimental to the analysis. More importantly, existing approaches to stress test design, including the studies mentioned above, focus on *single* curve analysis without taking into account the dynamic cross-country dependence of sovereign yield curves and their interaction with key macroeconomic and financial factors that affect their evolution and cross-country spreads.

4 A multi-curve modelling framework

4.1 The dynamic Nelson-Siegel model

In this section we introduce the latent factor model for the yield curve, initially proposed by Nelson and Siegel (1987), and later extended by Diebold and Li (2006) to a dynamic latent factor model that allows time-varying parameters. We also discuss the state-space representation of the model as introduced in Diebold et al. (2006). Denote the set of yields as $y_t(\tau_i)$, where τ_i denotes the maturity of a zero-coupon bond for a set of N different maturities $\tau_1 \leq \ldots \leq \tau_N$. The term structure of yields for $i = 1, \ldots, N$ at any point in time t is described by the three factor model of Nelson and Siegel (1987) as follows[6]

$$y_t(\tau_i) = \beta_1 + \beta_2 \left(\frac{1 - \mathrm{e}^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \beta_3 \left(\frac{1 - \mathrm{e}^{-\lambda \tau_i}}{\lambda \tau_i} - \mathrm{e}^{-\lambda \tau_i} \right) + \varepsilon(\tau_i)\,, \tag{1}$$

where β_1, β_2, β_3 and λ are fixed parameters. The disturbances $\varepsilon(\tau_i), \ldots, \varepsilon_t(\tau_N)$ are assumed to be independent with zero mean and constant variance. The Nelson-Siegel model in Equation (1) was extended by Diebold and Li (2006) to a dynamic latent factor model where β_1, β_2 and β_3 are interpreted as dynamic latent level, slope and curvature factors; the terms multiplied by these factors are factor loadings. In this respect, the dynamic Nelson-Siegel (DNS) model of Diebold and Li (2006) can be rewritten as follows

$$y_t(\tau_i) = L_t + S_t \left(\frac{1 - \mathrm{e}^{-\lambda \tau_i}}{\lambda \tau_i} \right) + C_t \left(\frac{1 - \mathrm{e}^{-\lambda \tau_i}}{\lambda \tau_i} - \mathrm{e}^{-\lambda \tau_i} \right) + \varepsilon_t(\tau_i)\,, \tag{2}$$

where $t = 1, \ldots, T$, $i = 1, \ldots, N$ while L_t, S_t and C_t are the time-varying counterparts of β_1, β_2 and β_3 respectively. The parameter λ determines the exponential decay rate of the slope and curvature factors. The shape and the form of the yield curve are governed by the three latent factors and their corresponding factor loadings. The loading on the first factor takes the value 1 and is called the *level factor* because it affects all yields equally by setting a baseline level of the yield curve. The loading on the second factor is $(1 - \mathrm{e}^{-\lambda \tau_i})/(\lambda \tau_i)$, a function that starts at 1 and converges monotonically to 0 as τ increases. This factor is interpreted as *slope factor* because it affects short rates more heavily than long rates; consequently, it changes the slope of the yield curve. The loading on the third factor is $((1 - \mathrm{e}^{-\lambda \tau_i})/\lambda \tau_i) - \mathrm{e}^{-\lambda \tau_i}$, which is a function that starts at 0, increases, and then decays to 0. This factor is interpreted as *curvature factor* because it loads medium rates more heavily and, therefore, changes the yield curve curvature.[7] Figure 4.1 plots the Nelson-Siegel factor loadings with fixed $\lambda = 0.0609$ as in Diebold and Li (2006).

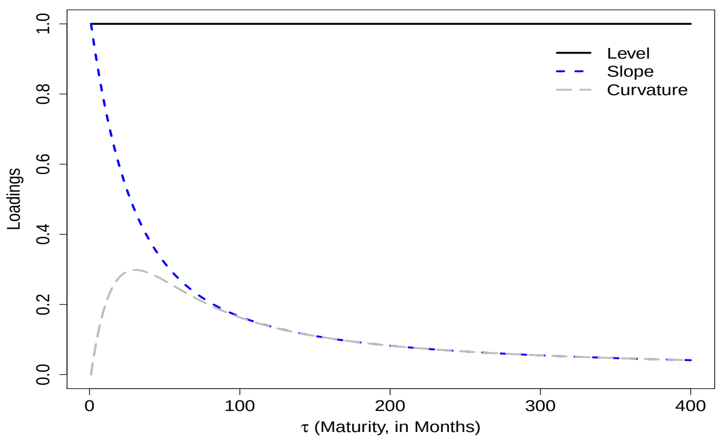


Figure 4.1: Factor loadings of the Nelson-Siegel model with fixed $\lambda = 0.0609$.

[6]The original Nelson-Siegel model representation is slightly different from Equation (1), which has been modified by Diebold and Li (2006) to improve estimation tractability and to facilitate an intuitive interpretation of the factors.

[7]The *level*, *slope* and *curvature* factors are also known as *long-term*, *short-term* and *medium-term* factors respectively because, given their corresponding factor loadings, they affect more heavily long-term, short-term and medium-term interest rates, respectively (see for example, Yu and Zivot (2011), among others).

Diebold and Li (2006) estimate the parameters, $\boldsymbol{\theta}_t = \{L_t, S_t, C_t, \lambda\}$, of the dynamic Nelson-Siegel (DNS) model in Equation (2) by nonlinear least squares for each time period t after fixing λ at a pre-specified value (i.e. $\lambda = 0.0609$). Diebold et al. (2006) go a step further by recognising that the dynamic Nelson-Siegel (DNS) model naturally forms a state-space system when treating $\boldsymbol{\beta}_t = [L_t, S_t, C_t]'$ as a latent vector. The *measurement* equation that relates a set of N yields to the three unobserved factors can be written as

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-\mathrm{e}^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-\mathrm{e}^{-\lambda\tau_1}}{\lambda\tau_1} - \mathrm{e}^{-\lambda\tau_1} \\ 1 & \frac{1-\mathrm{e}^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-\mathrm{e}^{-\lambda\tau_2}}{\lambda\tau_2} - \mathrm{e}^{-\lambda\tau_2} \\ \vdots & & \\ 1 & \frac{1-\mathrm{e}^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-\mathrm{e}^{-\lambda\tau_N}}{\lambda\tau_N} - \mathrm{e}^{-\lambda\tau_N} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{pmatrix}, \tag{3}$$

In a matrix notation, Equation (3) can be written as

$$\boldsymbol{y}_t = \boldsymbol{\Lambda}(\lambda)\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t\,, \tag{4}$$

with observation vector $\boldsymbol{y}_t = [y_t(\tau_1), \ldots, y_t(\tau_N)]'$, latent vector $\boldsymbol{\beta}_t = [L_t, S_t, C_t]'$, disturbance vector $\boldsymbol{\varepsilon}_t = [\varepsilon_t(\tau_1), \ldots, \varepsilon_t(\tau_N)]'$ and the $N \times 3$ factor loadings matrix $\boldsymbol{\Lambda}(\lambda)$, whose (i, l) element is given by

$$\boldsymbol{\Lambda}_{il}(\lambda) = \begin{cases} 1\,, & \text{for} \quad l = 1\,, \\ (1 - e^{-\lambda\tau_i})/\lambda\tau_i\,, & \text{for} \quad l = 2\,, \\ (1 - e^{-\lambda\tau_i} - \lambda\tau_i\, e^{-\lambda\tau_i})/\lambda\tau_i\,, & \text{for} \quad l = 3\,. \end{cases} \tag{5}$$

The factors L_t, S_t and C_t in Diebold et al. (2006) follow a vector autoregressive process of first order, VAR(1).[8] In general, the time-series dynamics for the 3×1 latent vector $\boldsymbol{\beta}_t$ are modelled as a VAR(p)-process, that is

$$\boldsymbol{\beta}_t = \boldsymbol{\mu} + \sum_{j=1}^{p} \boldsymbol{\Phi}_j \boldsymbol{\beta}_{t-j} + \boldsymbol{\upsilon}_t\,, \tag{6}$$

for $t = 1, \ldots, T$, where $\boldsymbol{\mu} = [\mu_s, \mu_l, \mu_c]'$ is a vector of intercepts, $\boldsymbol{\Phi}_j$ is a 3×3 coefficient matrix for $j = 1 \ldots p$ and $\boldsymbol{\upsilon}_t = [\upsilon_{lt}, \upsilon_{st}, \upsilon_{ct}]'$ is the disturbance vector. The system is complete once the covariance structure of the *measurement errors* $\boldsymbol{H}$ and the covariance of *transition errors* $\boldsymbol{Q}$ are specified. Diebold et al. (2006) make the standard assumption that the white noise errors in the *measurement* and *transition* equations are orthogonal to each other and to the initial state, such that

$$\begin{pmatrix} \boldsymbol{\upsilon}_t \\ \boldsymbol{\varepsilon}_t \end{pmatrix} = WN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \boldsymbol{Q} & 0 \\ 0 & \boldsymbol{H} \end{pmatrix} \right), \quad \mathbb{E}(\boldsymbol{\beta}_0 \boldsymbol{\upsilon}_t') = 0 \quad \text{and} \quad \mathbb{E}(\boldsymbol{\beta}_0 \boldsymbol{\varepsilon}_t') = 0\,.$$

In Diebold et al. (2006) the $\boldsymbol{H}$ matrix is assumed to be diagonal whereas the $\boldsymbol{Q}$ matrix can involve non zero covariances. The assumption of a diagonal $\boldsymbol{H}$ matrix, which implies mutually uncorrelated deviations of yields of various maturities from the yield curve, is common. It used for computational tractability. On the other hand, the assumption of an unrestricted $\boldsymbol{Q}$ matrix allows the shocks of the three factors to be correlated. Diebold et al. (2006) extended the dynamic Nelson-Siegel (DNS) model by including observable macroeconomic variables (specifically, real activity, inflation, and the monetary policy instrument) to study the interaction between the

[8] Diebold and Li (2006) employ the VAR(1) assumption only for the sake of transparency and parsimony; however, ARMA state vector dynamics of any order can be easily accommodated in state-space form.

macroeconomy and the yield curve. Therefore, the macroeconomic variables are directly added to the set of space variables, and Equation (6) is replaced by

$$\boldsymbol{\beta}_t = \boldsymbol{\mu} + \sum_{j=1}^{p} \boldsymbol{\Phi}_j \boldsymbol{\beta}_{t-j} + \boldsymbol{A}\boldsymbol{X}_t + \boldsymbol{v}_t\,, \tag{7}$$

where vector $\boldsymbol{X}_t$ is a $r \times 1$ vector of exogenous macroeconomic variables observable at time t and $\boldsymbol{A}$ is a $3 \times r$ matrix of regression coefficients with r representing the number of observable variables. The model is a linear Gaussian state-space model. The vector of latent factors $\boldsymbol{\beta}_t$ is therefore optimally estimated using the Kalman filter given past and current observations up to time t, i.e. $\boldsymbol{Y}_t = \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t\}$.

4.2 The covariance regression model

The covariance regression model of Hoff and Niu (2012) proposes a parsimonious way to parametrise the covariance structure of a multivariate response vector as a function of explanatory variables. Let $\boldsymbol{\eta} \in \mathbb{R}^p$ be a random multivariate response vector and $\boldsymbol{x} \in \mathbb{R}^q$ be a vector of explanatory variables. Hoff and Niu (2012) propose a flexible method for modelling the conditional covariance matrix of $\boldsymbol{\eta}$ given $\boldsymbol{x}$, $\boldsymbol{\Sigma}_x = \mathrm{Cov}[\boldsymbol{\eta}|\boldsymbol{x}]$, where, $\boldsymbol{\Sigma}_x$ is expressed as

$$\boldsymbol{\Sigma}_x = \boldsymbol{\Psi} + \boldsymbol{B}\boldsymbol{x}\boldsymbol{x}^T\boldsymbol{B}^T\,, \tag{8}$$

with $\boldsymbol{\Psi}$ a $p \times p$ positive-definite matrix, and $\mathbf{B}$ a $p \times q$ matrix of coefficients. The resulting covariance matrix is positive-definite for all values of $\boldsymbol{x}$ because the covariance is equal to a "baseline" covariance matrix $\boldsymbol{\Psi}$ plus a $p \times p$ positive-definite matrix that depends on $\boldsymbol{x}$. Hoff and Niu (2012) show that the covariance regression model has a random-effects model representation. The random-effects representation for a covariance regression model is

$$\boldsymbol{\eta}_t = \boldsymbol{\mu}_{x_t} + \gamma_t \times \boldsymbol{B}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t\,, \tag{9}$$

where $\mathrm{E}[\boldsymbol{\varepsilon}_t] = \mathbf{0}, \mathrm{Cov}[\boldsymbol{\varepsilon}_t] = \boldsymbol{\Psi}, \mathrm{E}[\gamma_t] = 0, \mathrm{Var}[\gamma_t] = 1$ and $\mathrm{E}[\gamma_t \times \boldsymbol{\varepsilon}_t] = \mathbf{0}$. The covariance matrix for $\boldsymbol{\eta}_t$ given $\boldsymbol{x}_t$ can then be derived as

$$\begin{aligned}\mathrm{E}\big[(\boldsymbol{\eta}_t - \boldsymbol{\mu}_{x_t})(\boldsymbol{\eta}_t - \boldsymbol{\mu}_{x_t})^T\big] &= \mathrm{E}\big[\gamma_t^2 \boldsymbol{B}\boldsymbol{x}_t\boldsymbol{x}_t^T\boldsymbol{B}^T + \gamma_t(\boldsymbol{B}\boldsymbol{x}_t\boldsymbol{\varepsilon}_t^T + \boldsymbol{\varepsilon}_t^T\boldsymbol{x}_t^T\boldsymbol{B}^T) + \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t^T\big] \\ &= \boldsymbol{B}\boldsymbol{x}_t\boldsymbol{x}_t^T\boldsymbol{B}^T + \boldsymbol{\Psi} \\ &= \boldsymbol{\Sigma}_{x_t}\,.\end{aligned} \tag{10}$$

The model in Equation (9) can also be represented as a factor analysis model and expressed as

$$\begin{pmatrix} \eta_{1,t} - \mu_{x_{1,t}} \\ \vdots \\ \eta_{p,t} - \mu_{x_{p,t}} \end{pmatrix} = \gamma_t \times \begin{pmatrix} \boldsymbol{b}_1^T\boldsymbol{x}_t \\ \vdots \\ \boldsymbol{b}_p^T\boldsymbol{x}_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{p,t} \end{pmatrix}, \tag{11}$$

where $\{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_p\}$ denote the rows of $\boldsymbol{B}$. The latent factor γ_t essentially describes the additional unit-level variability beyond that represented by the error term $\boldsymbol{\varepsilon}_t$, while the vectors $\{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_p\}$ describe how this additional variability is shared across the p different response vectors. For example, large values of $\boldsymbol{b}_j$ indicate large heteroscedasticity in y_j as a function of $\boldsymbol{x}$. Additionally, the direction of vectors $\boldsymbol{b}_j$ and $\boldsymbol{b}_k$ determines the direction of the linear dependence between η_j and η_k , i.e. whether η_j and η_k are positively or negatively correlated. Hoff and Niu (2012) show that maximum likelihood estimation can be performed using an EM-algorithm.[9]

[9] Hoff and Niu (2012) also show that model parameters can be estimated using a Bayesian setting via the Gibbs sampler. In this paper, we employ the maximum likelihood estimation procedure via the EM-algorithm to estimate model parameters.

4.3 Stress testing methodology

Our aim is to develop a modelling strategy that is able to quantify the effects on the sovereign yields from a shock in any of the country-specific liquidity and credit measures. In other words, we wish to assess the impact of country-specific liquidity and credit quality shocks not only on the yield curve of the country which experiences the shock but also on the yield curves of the remaining countries under study. In this way we can effectively study linkages between European sovereign yields and potential spillover effects. In this section, we describe in detail a stress testing procedure that achieves this aim and can generate consistent cross-country stress test scenarios.

Denote the complete set of all country and maturity yields as $\boldsymbol{Y}_t$, where each element $y_{j,t}(\tau_i)$ represents a zero-coupon bond yield at maturity τ_i $(i = 1, \ldots, N)$ for country j $(j = 1, \ldots, d)$ at time t $(t = 1, \ldots, T)$:

$$\boldsymbol{Y}_t = \begin{pmatrix} y_{1,t}(\tau_1) & \ldots & y_{d,t}(\tau_1) \\ \vdots & \vdots & \vdots \\ y_{1,t}(\tau_N) & \ldots & y_{d,t}(\tau_N) \end{pmatrix}.$$

Denote by $\boldsymbol{M}_t$ the set of the Nelson-Siegel term structure regression model of Diebold et al. (2006), where each element $f(\tau_i; \boldsymbol{\beta}_{j,t}, \boldsymbol{\theta}_j)$ is a 1×3 vector of the i-th row of the $\boldsymbol{\Lambda}(\lambda_j)\, \boldsymbol{\beta}_{j,t}$ matrix, with $\boldsymbol{\beta}_{j,t}$ representing the vector of latent factors, $\boldsymbol{\Lambda}(\lambda_j)$ is the $N \times 3$ factor loadings matrix and $\boldsymbol{\theta}_j$ denotes the set of parameters for each country j. Also denote by $\boldsymbol{\Lambda}$ the set of all factor loading matrices $\boldsymbol{\Lambda}(\lambda_j)$. The resulting $N \times 3d$ $\boldsymbol{M}_t$ and $\boldsymbol{\Lambda}$ matrices are given by:

$$\mathbf{M}_t = \begin{pmatrix} f(\tau_1; \boldsymbol{\beta}_{1,t}, \boldsymbol{\theta}_1) & \ldots & f(\tau_1; \boldsymbol{\beta}_{d,t}, \boldsymbol{\theta}_d) \\ \vdots & \vdots & \vdots \\ f(\tau_N; \boldsymbol{\beta}_{1,t}, \boldsymbol{\theta}_1) & \ldots & f(\tau_N; \boldsymbol{\beta}_{d,t}, \boldsymbol{\theta}_d) \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Lambda} = \Big(\boldsymbol{\Lambda}(\lambda_1), \ldots, \boldsymbol{\Lambda}(\lambda_d)\Big).$$

The total observation model with all countries and maturities can be written as follows

$$\begin{pmatrix} y_{1,t}(\tau_1) \\ \vdots \\ y_{1,t}(\tau_N) \\ \vdots \\ y_{d,t}(\tau_1) \\ \vdots \\ y_{d,t}(\tau_N) \end{pmatrix} = \begin{pmatrix} f(\tau_1; \boldsymbol{\beta}_{1,t}, \boldsymbol{\theta}_1) \\ \vdots \\ f(\tau_N; \boldsymbol{\beta}_{1,t}, \boldsymbol{\theta}_1) \\ \vdots \\ f(\tau_1; \boldsymbol{\beta}_{d,t}, \boldsymbol{\theta}_d) \\ \vdots \\ f(\tau_N; \boldsymbol{\beta}_{d,t}, \boldsymbol{\theta}_d) \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t}(\tau_1) \\ \vdots \\ \varepsilon_{1,t}(\tau_N) \\ \vdots \\ \varepsilon_{d,t}(\tau_1) \\ \vdots \\ \varepsilon_{d,t}(\tau_N) \end{pmatrix} + \begin{pmatrix} \zeta_{1,t}(\tau_1) \\ \vdots \\ \zeta_{1,t}(\tau_N) \\ \vdots \\ \zeta_{d,t}(\tau_1) \\ \vdots \\ \zeta_{d,t}(\tau_N) \end{pmatrix}, \tag{12}$$

where $\boldsymbol{\varepsilon}_{j,t} = [\varepsilon_{j,t}(\tau_1), \ldots, \varepsilon_{j,t}(\tau_N)]'$ and $\boldsymbol{\zeta}_{i,t} = [\zeta_{1,t}(\tau_i), \ldots, \zeta_{d,t}(\tau_i)]'$.[10]

We assume that $\mathbb{E}[\boldsymbol{\varepsilon}_{j,t}\, \boldsymbol{\varepsilon}'_{l,t}] = 0\ \forall j, l$ for $j \neq l$, $\mathbb{E}[\boldsymbol{\zeta}_{i,t}\, \boldsymbol{\zeta}'_{l,t}] = 0\ \forall i, l$ for $i \neq l$ and $\mathbb{E}[\varepsilon_{j,t}(\tau_i)\, \zeta_{j,t}(\tau_i)] = 0$

[10] To avoid confusion it should be clarified that Equation (12) assumes that the *measurement errors* in the Nelson-Siegel model of Diebold et al. (2006) decompose into two independent components, one that accounts for the within-country, between maturity dependence (this is the $\boldsymbol{H}$ covariance matrices) and one that accounts for between country dependence and links that to country-specific liquidity and credit quality variables (this is the $\boldsymbol{\Sigma}$ covariance matrices). The total covariance matrix is $\boldsymbol{H} + \boldsymbol{\Sigma}$ by independence. In the paper, we estimate the variance covariance terms in two steps: first estimate the model with only the country-specific error component (i.e. assuming that $\boldsymbol{\Sigma} = 0$), and then base the estimation of $\boldsymbol{\Sigma}$ on the residuals from the previous model fit. Clearly, this is just for practical reasons, and fitting everything jointly would be the best thing to do.

$\forall j, i$. We also assume that

$$\begin{pmatrix} \varepsilon_{j,t}(\tau_1) \\ \vdots \\ \varepsilon_{j,t}(\tau_N) \end{pmatrix} \sim \boldsymbol{N_N}(\mathbf{0}, \boldsymbol{H}_j) \quad \text{and} \quad \begin{pmatrix} \zeta_{1,t}(\tau_i) \\ \vdots \\ \zeta_{d,t}(\tau_i) \end{pmatrix} \sim \boldsymbol{N_d}(\mathbf{0}, \boldsymbol{\Sigma}_{i,t}),$$

where $\boldsymbol{H}_j$ is the covariance matrix of the *measurement errors* in the dynamic Nelson-Siegel model of Diebold et al. (2006). In contrast, $\boldsymbol{\Sigma}_{i,t}$ denotes the cross-country covariance structure of $\boldsymbol{\zeta}_{i,t}$ errors at maturity τ_i. As shown in Section 4.2, $\boldsymbol{\Sigma}_{i,t}$ can be modelled as a quadratic function of explanatory variables $\boldsymbol{x}_{i,t}$, that is

$$\boldsymbol{\Sigma}_{i,t} = \boldsymbol{\Psi}_i + \boldsymbol{B}_i \boldsymbol{x}_{i,t} \boldsymbol{x}_{i,t}^T \boldsymbol{B}_i^T .$$

Under the above assumptions, the distribution of $\boldsymbol{\varepsilon}_{j,t}$ and $\boldsymbol{\zeta}_{i,t}$ for all countries $j = 1, \dots, d$, maturities $\tau_i = \tau_1 \dots, \tau_N$, and time periods $t = 1, \dots, T$ can be written as

$$\begin{pmatrix} \varepsilon_{1,t}(\tau_1) \\ \vdots \\ \varepsilon_{1,t}(\tau_N) \\ \vdots \\ \varepsilon_{d,t}(\tau_1) \\ \vdots \\ \varepsilon_{d,t}(\tau_N) \end{pmatrix} \sim \boldsymbol{N_{N \times d}}(\mathbf{0}, \boldsymbol{H_\varepsilon}) \quad \text{and} \quad \begin{pmatrix} \zeta_{1,t}(\tau_1) \\ \vdots \\ \zeta_{d,t}(\tau_1) \\ \vdots \\ \zeta_{1,t}(\tau_N) \\ \vdots \\ \zeta_{d,t}(\tau_N) \end{pmatrix} \sim \boldsymbol{N_{d \times N}}(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\zeta},t}),$$

where $\boldsymbol{H_\varepsilon} = \bigoplus_{j=1}^{d} \boldsymbol{H}_j$ and $\boldsymbol{\Sigma}_{\boldsymbol{\zeta},t} = \bigoplus_{i=1}^{N} \boldsymbol{\Sigma}_{i,t} = \bigoplus_{i=1}^{N} (\boldsymbol{\Psi}_i + \boldsymbol{B}_i \boldsymbol{x}_{i,t} \boldsymbol{x}_{i,t}^T \boldsymbol{B}_i^T)$. Let $\tilde{\boldsymbol{y}}_t$ be the vector of observable sovereign yields sorted by country at each particular maturity τ_i at time t, that is

$$\tilde{\boldsymbol{y}}_t = \begin{pmatrix} y_{1,t}(\tau_1) \\ \vdots \\ y_{d,t}(\tau_1) \\ \vdots \\ y_{1,t}(\tau_N) \\ \vdots \\ y_{d,t}(\tau_N) \end{pmatrix} = \boldsymbol{R} \operatorname{vec}(\boldsymbol{Y}_t) = \boldsymbol{R} \begin{pmatrix} y_{1,t}(\tau_1) \\ \vdots \\ y_{1,t}(\tau_N) \\ \vdots \\ y_{d,t}(\tau_1) \\ \vdots \\ y_{d,t}(\tau_N) \end{pmatrix}, \tag{13}$$

where $\boldsymbol{R}$ is a permutation matrix that re-orders the vector of observed yields $\operatorname{vec}(\boldsymbol{Y}_t)$ into the new $\tilde{\boldsymbol{y}}_t$ vector. In order to quantify the effects on the sovereign yields from a shock to any of the country-specific liquidity and credit quality measures in vector $\boldsymbol{x}_{i,t}$, we proceed as follows:

Step 1: Estimate the dynamic Nelson-Siegel (DNS) model of Diebold et al. (2006), introduced in Section 4.1, and obtain a vector of estimated parameters $\hat{\boldsymbol{\theta}}_j$ for each country j separately. We model the dynamic movements of L_t, S_t and C_t in Equation (7) employing four alternative autoregressive specifications AR(1), AR(2), VAR(1) and VAR(2) and follow a formal procedure using Akaike's information criterion (Akaike, 1974) and Schwarz's Bayesian information criterion (BIC) (Schwarz, 1978) to select the best fitting model. Further, we model the covariance matrix $\boldsymbol{H}_j$ as a non-diagonal first-order autoregressive covariance structure with heterogenous variances for each country j. We then separate the residuals obtained from the estimated dynamic Nelson-Siegel (DNS) model per maturity i and model the cross-country covariance structure $\boldsymbol{\Sigma}_{i,t}$ as a function of country-specific liquidity and credit quality variables using the covariance regression model of Hoff and Niu (2012) introduced in Section 4.2.

Step 2: Premultiply $\tilde{\boldsymbol{y}}_t$ by $\boldsymbol{\Sigma}_{\zeta,t}^{-1/2}$ to obtain the new vector of transformed yields $\check{\boldsymbol{y}}_t$ in order to account for the cross-country dependence across the maturity spectrum:

$$\check{\boldsymbol{y}}_t = \boldsymbol{\Sigma}_{\zeta,t}^{-1/2}\, \tilde{\boldsymbol{y}}_t = \boldsymbol{\Sigma}_{\zeta,t}^{-1/2}\, \boldsymbol{R}\, \mathrm{vec}(\boldsymbol{Y}_t)\,.$$

Step 3: Given the estimated parameters $\hat{\boldsymbol{\theta}}_1, \ldots, \hat{\boldsymbol{\theta}}_d$, obtained from the Kalman-filter for each particular country j in **Step 1**, calculate $\check{\boldsymbol{\Lambda}}_t$ and $\check{\boldsymbol{H}}_{\boldsymbol{\varepsilon}}$ as

$$\check{\boldsymbol{\Lambda}}_t = \boldsymbol{\Sigma}_{\zeta,t}^{-1/2}\, \tilde{\boldsymbol{\Lambda}} = \boldsymbol{\Sigma}_{\zeta,t}^{-1/2}\, \boldsymbol{R} \begin{pmatrix} \boldsymbol{\Lambda}(\hat{\lambda}_1) \\ \vdots \\ \boldsymbol{\Lambda}(\hat{\lambda}_d) \end{pmatrix} \quad \text{and} \quad \check{\boldsymbol{H}}_{\boldsymbol{\varepsilon}} = \boldsymbol{R}^T\, \boldsymbol{\Sigma}_{\zeta,t}^{-1/2}\, \boldsymbol{R}\, \boldsymbol{H}_{\boldsymbol{\varepsilon}}\, \boldsymbol{R}^T\, (\boldsymbol{\Sigma}_{\zeta,t}^{-1/2})^T\, \boldsymbol{R}\,.$$

Step 4: Split $\check{\boldsymbol{y}}_t$, $\check{\boldsymbol{\Lambda}}_t$ and $\check{\boldsymbol{H}}_{\boldsymbol{\varepsilon}}$ into country specific components $\check{\boldsymbol{y}}_{j,t}$, $\check{\boldsymbol{\Lambda}}_t(\lambda_j)$ and $\check{\boldsymbol{H}}_j$ for country $j = 1, \ldots, d$

$$\check{\boldsymbol{y}}_{j,t} = \begin{pmatrix} \check{y}_{j,t}(\tau_1) \\ \vdots \\ \check{y}_{j,t}(\tau_N) \end{pmatrix}, \quad \begin{pmatrix} \check{\boldsymbol{\Lambda}}_t(\lambda_1) \\ \vdots \\ \check{\boldsymbol{\Lambda}}_t(\lambda_d) \end{pmatrix} = \boldsymbol{R}^T\, \check{\boldsymbol{\Lambda}}_t \quad \text{and} \quad \check{\boldsymbol{H}}_{\boldsymbol{\varepsilon}} = \begin{pmatrix} \check{\boldsymbol{H}}_1 & & \\ & \ddots & \\ & & \check{\boldsymbol{H}}_d \end{pmatrix},$$

where $\check{\boldsymbol{H}}_j$ is the subset of the $\check{\boldsymbol{H}}_{\boldsymbol{\varepsilon}}$ matrix corresponding to country j.

Step 5: Run, separately for each case, the Kalman-filter keeping all static parameters fixed to obtain estimates of latent factors $\check{\boldsymbol{\beta}}_{\boldsymbol{j,t}} = \left[\check{L}_{j,t}, \check{S}_{j,t}, \check{C}_{j,t}\right]'$ for each country.

Step 6: Calculate predicted yields for each country j as

$$\boldsymbol{y}^*_{j,t} = \check{\boldsymbol{\Lambda}}(\lambda_j)\check{\boldsymbol{\beta}}_{j,t}\,.$$

Step 7: Reorganise the predicted yields by maturity and pre-multiply by $\boldsymbol{\Sigma}_{\zeta,t}^{1/2}$ to transform them to their initial scale:

$$\tilde{\boldsymbol{y}}^*_t = \begin{pmatrix} \tilde{y}^*_{1,t}(\tau_1) \\ \vdots \\ \tilde{y}^*_{d,t}(\tau_1) \\ \vdots \\ \tilde{y}^*_{1,t}(\tau_N) \\ \vdots \\ \tilde{y}^*_{d,t}(\tau_N) \end{pmatrix} = \boldsymbol{\Sigma}_{\zeta,t}^{1/2}\, \boldsymbol{R} \begin{pmatrix} y^*_{1,t}(\tau_1) \\ \vdots \\ y^*_{1,t}(\tau_N) \\ \vdots \\ y^*_{d,t}(\tau_1) \\ \vdots \\ y^*_{d,t}(\tau_N) \end{pmatrix} \tag{14}$$

To assess the impact of a shock in any of the $\boldsymbol{x}_t$ variables we repeat **Steps 2-7** twice; Once with covariates $\boldsymbol{x}_t$ to get a set of predicted yields $\tilde{\boldsymbol{y}}^{*(1)}_t$ and once with an appropriately shocked version of $\boldsymbol{x}_t$ to get a new set of predicted yields $\tilde{\boldsymbol{y}}^{*(2)}_t$. The difference of the two sets of predicted yields, i.e. $\tilde{\boldsymbol{y}}^{*(2)}_t - \tilde{\boldsymbol{y}}^{*(1)}_t$, indicates the impact of a shock in the $\boldsymbol{x}_t$ variables on the yield curves of the countries under study and consequently on their corresponding spreads.

5 Data

The data, which will use to demonstrate the empirical performance of our stress testing framework, consists of end-of-day sovereign bond yields, sovereign bid-ask spreads, credit default swap (CDS) spreads, and macroeconomic and financial variables. The data spans security trading in 5 major economies in the European Union: two "peripheral" Eurozone countries (Italy and Spain), two "core" Eurozone countries (Germany and France) and one non-euro European country (United Kingdom).[11] We consider daily end-of-day sovereign yields (midpoints of the quoted daily closing bid and ask yields) with maturities of 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240 and 360 months for each country over the period from November 11, 2008 to February 28, 2014. The full sample consists of 1372 daily yield observations for each maturity and each country, respectively. This period includes a significant number of events, for example the sovereign debt crisis faced by several Eurozone countries such as Greece, Ireland, Portugal, Spain and Cyprus, and a range of policy interventions including the Securities Market Programme (SMP) and the Outright Monetary Transactions (OMT) programme launched by the European Central Bank (ECB) in response to the financial crisis and the liquidity dry-ups in the interbank lending markets. As a result, this is an interesting period to study a variety of effects on the behaviour of the European fixed-income markets and the relative importance of credit and liquidity risks during both calm and stress.

In our analysis, liquidity in the European sovereign bonds is quantified via the quoted bid-ask spreads. Goyenko et al. (2011) argue that the quoted bid-ask spread is a reasonable liquidity proxy and is highly correlated with other liquidity measures in the bond market.[12] The daily end-of-day bid and ask quotes are obtained from Bloomberg for each maturity studied, using the Bloomberg Generic Quote (BGN) pricing source, which reflects consensus quotes among market participants regarding the value of the bond.[13] The measures based on quoted bid-ask spreads from Bloomberg are among the most widely used daily liquidity measures in fixed-income markets (see, e.g., Bao et al., 2011; Longstaff et al., 2005; Chen et al., 2007 amongst others). Schestag et al. (2013) also illustrate that the daily bid-ask quotes from Bloomberg can capture effective transaction costs.

We also use sovereign credit default swap (CDS) spreads to obtain a market estimate of the credit quality for each of the countries in our sample. A credit default swap is an over-the-counter (OTC) derivative contract that provides protection against the risk of a credit event by a particular company or country. The sovereign CDS data used for the analysis are midpoints of the daily closing spreads with maturities of 6, 12, 24, 36, 48, 60, 84, 120, 240 and 360 months from the Thomson Reuters Eikon database, which also consists of market consensus CDS quotes that are published by Thomson Reuters.

The macroeconomic and financial variables consist of inflation data, major exchange rates and proxies for short-term liquidity and credit quality. In particular, the Harmonised Index of Con-

[11]The exclusion of several "peripheral" Eurozone countries, such as Greece, Portugal, and Ireland, and "core" Eurozone countries, such as Austria, Belgium, Finland, and the Netherlands, from the analysis is mainly driven by the lack of sovereign yields, liquidity and credit quality data for the time period and time-to-maturity contracts we want to analyse. However, González-Hermosillo and Johnson (2014) show that Spain and Italy played a more pivotal role in the transmission of financial shocks after 2009. In addition, Alter and Beyer (2014) show that the systemic contributions of Greece, Portugal, and Ireland decreased markedly after the implementation of IMF/EU bailout programs.

[12]For example, Chordia et al. (2001), show that the daily correlations between quoted and effective spread changes are 0.68 in the bond market over their 9-year sample period, while Chordia et al. (2005) show that the correlation between daily quoted spreads and depth is -0.49.

[13]Bloomberg uses BGN (Bloomberg Generic) to construct the yield curve. BGN is the simple average prices, including indicative and executable prices, quoted by high-quality price contributors over a specified time window. BGN prices are re-calculated every day when the market closes. In some cases, bond prices from a specific pricing source are used in lieu of BGN prices (e.g. fixing prices). Outliers (i.e. bonds whose OAS are significantly higher or lower than OAS of comparable bonds) are excluded.

sumer Prices (HICP) is used as a measure of inflation and price stability. The HICP monthly time series for each individual country are obtained from the statistics database of the European Central Bank (ECB) and are subsequently interpolated to daily series using cubic spline techniques. The daily US Dollar (USD), Great Britain Pound (GBP) and Japanese Yen (JPN) exchange rates against the Euro are also obtained from the ECB's database. The spread between the 3-month Euribor rate and the 3-month Eurepo rate, both reported by the European Banking Federation (EBF), is used as a proxy for short-term liquidity. In addition, the 5-year Markit iTraxx Europe index is employed as a credit proxy for the overall credit quality in the European bond market. The Markit iTraxx Europe is a benchmark index comprising 125 equally weighted CDS on investment grade European corporate entities, and the contract with 5 years to maturity is the most actively traded contract. The daily 3-month Euribor and Eurepo rates are obtained from Bloomberg, while the daily 5-year Markit iTraxx Europe index is obtained from the Thomson Reuters Eikon database.

Table A.1 in Appendix A presents descriptive statistics for the sovereign bond yields while Tables A.2 and A.3 present descriptive statistics for the quoted sovereign bid-ask and CDS spreads for each country and maturity, respectively. We note that the typical yield curve for each country is upward sloped, and that the short-term rates are generally more volatile than the long-term rates, especially for Italy and Spain. The German yields are the lowest, on average, across all maturities, followed by the UK and French bond yields.[14] The Spanish and Italian bond yields are the highest across all countries and maturities in our sample highlighting investors' increased credit and liquidity concerns. Table A.2 and A.3 also show that average bid-ask and CDS spreads are much greater in size when compared with the corresponding German, French and UK spreads. The evolution of the 5-year bid-ask and CDS spreads, plotted in Figures 5.1 and 5.2, also confirms investors' risk aversion and negative sentiment toward the sovereign debts of the "peripheral" Eurozone countries. As can be seen in Figures 5.1 and 5.2, both bid-ask and CDS spreads for Italy and Spain peaked between end-2011 and mid-2012. This period corresponds to the peak of the Spanish crisis and the official request of the Spanish government for financial support from Eurozone members.[15]

The turning point of the Eurozone sovereign debt crisis was the July 26, 2012 policy statement by Mario Draghi, president of the European Central Bank (ECB), that "the ECB is ready to do whatever it takes to preserve the euro."[16] This policy statement was followed on September 6, 2012, by the announcement of the Outright Monetary Transactions (OMT) programme.[17] This change in the policy stance triggered a lasting scaling-down in the bond yields of Eurozone countries. The benchmark Spanish 10-year bond yield stayed below 6%, having reached 5% by year's end. Saka et al. (2015) provided empirical evidence regarding the contagion-mitigating effects of the new ECB policy embodied in the OMT programme.

[14] Germany's 12, 24 and 36 month bond yields turned negative between end-2011 and mid-2012 since investors sought refuge in Europe's safest assets over concerns about the solvency of several European economies.

[15] On June 9, 2012, the Eurogroup granted to Spain a financial support package of up to €100 billion in order for the country's financial institutions to be recapitalised. In addition, on June 25, 2012, the Cypriot Government requested financial aid from the euro area members and the International Monetary Fund (IMF) in order to tackle the distress in the country's banking sector and the macroeconomic imbalances.

[16] Mario Draghi, 26 July 2012. See www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html

[17] The Outright Monetary Transactions (OMT) programme is a programme of the European Central Bank (ECB) under which the bank makes purchases of sovereign bonds of Eurozone countries having difficulty issuing debt.

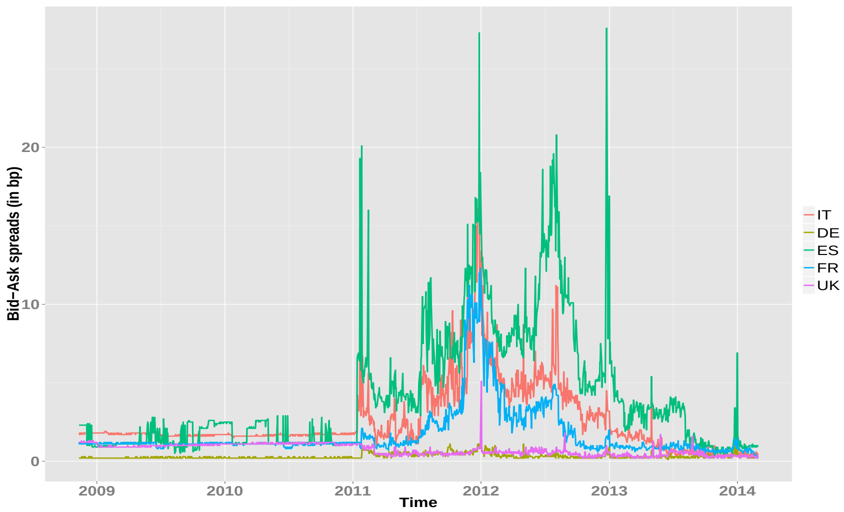


Figure 5.1: Sovereign 5-year bid-ask spreads for Italy, Germany, Spain, France and the United Kingdom.

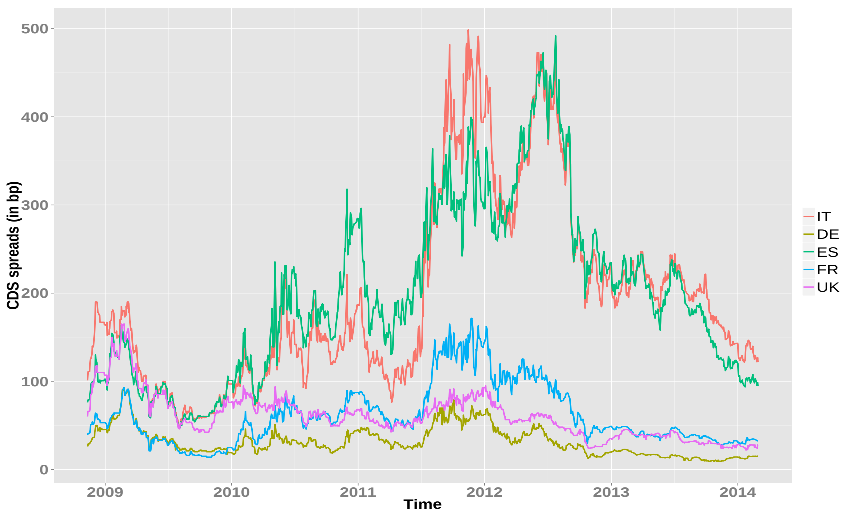


Figure 5.2: Sovereign 5-year credit default swap (CDS) spreads for Italy, Germany, Spain, France and the United Kingdom.

In addition, Table 5.1 presents descriptive statistics for the macroeconomic and financial variables employed in the analysis. The inflation rates for all four Eurozone countries in our sample collectively turned negative over the third quarter of 2009. This period is characterised by severe liquidity dry-ups in the interbank lending markets. The 3-month Euribor-Eurepo spread that measures the difference in interest rates between short-term unsecured and collateralised funding skyrocketed to almost 172 basis points, while the iTraxx Europe index that provides an exogenous credit quality estimate on investment grade European entities, soared to approximately 216 basis points at the end of 2008 illustrating the widespread market concerns about the solvency of several European financial institutions over this period. The volatility for all major exchange rates is also fairly large in our sample period.

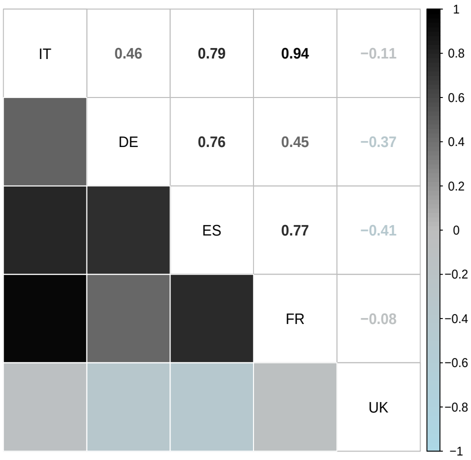
Table 5.1: Summary Statistics: Macroeconomic and Financial variables

Variable	Mean	Sd	Min	Max
HICP.IT	1.97	1.08	−0.14	3.84
HICP.DE	1.39	0.69	−0.50	2.41
HICP.ES	1.69	1.35	−1.40	3.80
HICP.FR	1.30	0.80	−0.72	2.54
HICP.UK	3.07	0.90	1.10	5.23
GBP	0.86	0.03	0.78	0.98
JPY	118.78	12.57	94.63	145.02
USD	1.34	0.06	1.19	1.51
Liquidity	44.22	31.55	12.70	171.70
iTraxx	119.93	33.73	65.30	215.92

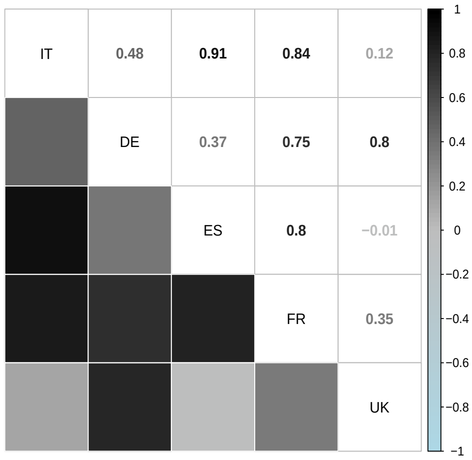
This table reports summary statistics for our sample macroeconomic and financial variables. The Harmonised Index of Consumer Prices (HICP), measured in percentages, is used as an inflation proxy for Italy (HCIP.IT), Germany (HCIP.DE), Spain (HCIP.ES), France (HCIP.FR) and the UK (HCIP.UK). GBP, JPY and USD represent the Great Britain Pound, Japanese Yen and US Dollar exchange rates against the Euro. Liquidity and iTraxx represent liquidity and credit quality variables expressed in basis points.

Figure 5.3 presents average cross-country correlation coefficient estimates for liquidity and credit quality variables. We note that the variability of liquidity correlation estimates is more pronounced when compared with that of credit quality correlation estimates. It can also be noted that there is a strong positive correlation between bond liquidity measures across all Eurozone countries and a negative correlation between all Eurozone countries and the UK. The correlations between credit quality measures are also of great interest. As expected, the correlation between Spanish and Italian credit default swap (CDS) spreads is very strong and positive indicating the widespread market concerns about the sovereign credit default risk of the two "peripheral" Eurozone countries. Interestingly, Spanish and Italian CDS spreads are also strongly and positively correlated with French CDS spreads. Although we lack the statistical power to make more qualitative statements, the increased correlation of the French credit quality measures with those of the "peripheral" Eurozone countries can be partly attributed to the increased concerns over the country's economy and public finances.[18] We also note the weak correlation between the UK credit default swap (CDS) spreads and those of the Eurozone countries, with the exception of the correlation coefficient for Germany, which is strong and positive, and reflects the country's superior credit quality.

[18]France lost its Standard & Poor's top-grade AAA rating in January 2012. In November 2013, Standard & Poor's cut France's credit rating from AA+ to AA, the third tier of credit quality, for the second time in less than two years due to the country's weak economic growth, high unemployment and government spending constraints.



(a) Bid-Ask spread sample correlation estimates.



(b) Credit default swap (CDS) sample correlation estimates.

Figure 5.3: This figure presents average cross-country bond liquidity correlation estimates, calculated using the average quoted bid-ask spreads across maturities for each country, and average cross-country credit quality correlation estimates, calculated using the average credit default swap (CDS) spreads across maturities for each country.

Table 5.2 presents cross-sectional correlation coefficients between our average liquidity and credit quality measures for several maturities. It reports the correlation coefficients of the above measures for all countries in our sample (All countries column) as well as for Eurozone only countries (Eurozone column) to assess the impact of the exclusion of the UK, a non-Eurozone EU country, on the correlation estimates. The correlation estimates between average liquidity and credit quality measures across all maturities in both columns (All countries and Eurozone) are strong and positive suggesting that as liquidity in the sovereign bonds decreases, via the widening of the bid-ask spreads, credit quality also decreases. The qualitative results in Table 5.2 differ from the qualitative findings of Beber et al. (2009) and, more specifically, from the corresponding correlation estimates reported in Table 2 of their paper. Beber et al. (2009), using intraday European government bond quotes and credit default swap (CDS) data, report a negative relation between credit quality and liquidity measures. The discrepancy in the correlation estimates can be attributed to differences in the sample size and in the proxies for measuring liquidity in the European bond markets.[19]

[19] Beber et al. (2009) use intraday inter-dealer European government bond quotes for the period from April 2003 to December 2004. They also consider four alternative measures to capture the liquidity of sovereign bonds: the effective bid-ask spread, the average quoted depth, the cumulative limit-order book depth and the average quoted depth divided by the percentage bid-ask spreads. In contrast, our sample period is more extensive since it includes securities trading from November 11, 2008 to February 28, 2014; however, we are limited to the use of the quoted bid-ask spread for measuring the liquidity of the European sovereign bonds due to lack of limit-order book data.

Table 5.2: Correlation between credit quality and liquidity

Maturity	All countries	Eurozone
12	0.65	0.67
24	0.77	0.78
36	0.74	0.75
48	0.79	0.81
60	0.79	0.80
84	0.79	0.80
120	0.76	0.77
240	0.78	0.79
360	0.80	0.81

This table reports the correlation between the average country credit risk, measured by credit default swap (CDS) spreads quoted for each country/maturity, and the average country bond liquidity, measured by the quoted bid-ask spread for each country/maturity. Column All countries reports the correlation for all countries in the sample, namely Italy, Germany, Spain, France and the UK, while column Eurozone, reports the correlation for all countries in the sample excluding the United Kingdom, which is the only non-euro EU country in our sample.

6 Case Study: Italian and Spanish liquidity and credit shocks

In this section, we demonstrate the empirical performance of our stress testing methodology using as an experimental case a scenario that includes contemporaneous shocks to country-specific liquidity and credit quality variables. Prompted by the recent Eurozone debt crisis we design a stress test scenario in which the "peripheral" Eurozone countries in our sample, namely Italy and Spain, are hit by significant liquidity and credit shocks. Therefore, the scenario formulation includes three different type of shocks: a) a *liquidity shock* only, in which, a shock to relevant bid-ask spreads is applied to each particular tenor separately; b) a *credit shock* only, in which, a shock to relevant cds spreads is applied to each particular tenor separately; and c) a *combined* liquidity and credit shock, in which, a shock to both bid-ask and cds spreads is applied to each particular tenor separately. In practice, each shock type translates into a parallel shift equal to an one standard deviation increase in the country-specific liquidity and credit variables. We apply the methodology described in detail in section 4.3 and, in particular, Steps 1 to 7 to obtain the set of stress scenarios over the sample period for each country and tenor separately and we then evaluate the impact of these shocks on several hypothetical bond portfolios.

Arguably, a number of alternative scenarios can be designed and applied within this framework. For example, macroeconomic shocks would be easily combined with shocks to country-specific liquidity and credit variables for multiple countries and tenors simultaneously. In addition, the magnitude of the shocks can vary across different countries and tenors. For simplicity, in our scenario design, we applied a shock of the same order (i.e. one standard deviation) to relevant liquidity and credit quality variables. The magnitude of these shocks may differ significantly in reality across term structure and thus more sophisticated techniques could be potentially employed.[20]

Figure 6.1 shows the distribution of absolute changes in the European sovereign yields after applying a one standard deviation shock to the Italian and Spanish liquidity and credit variables at the 2 year, 10 year and 30 year tenors separately. For comparison, the figure also displays the distribution of historical daily absolute yield changes (e.g. No shock). Further, Figure 6.2 shows

[20] For instance, a non-linear quantile regression methodology could be fit on liquidity and credit quality variables and the predicted quantiles could be used for generating stress scenarios.

the distribution of absolute changes in the UK yields across term-structure after applying a shock to liquidity and credit variables at the 10 year maturity. Figures 6.1 and 6.2 show that a shock to the Italian and Spanish liquidity and credit variables could have a significant impact not only on the cross-country spreads at the tenors in which the shock is applied to (i.e. 10 year) but also on the cross-maturity spreads for each yield curve. The figures demonstrate the importance of taking into account cross-country and cross-maturity dependencies simultaneously when designing and calibrating a stress scenario and also demonstrate how our modeling approach can effectively capture these forms of dependencies and generate consistent stress scenarios.

In general, it could be argued that shocks to liquidity and credit variables have different impact on the term structure and cross-country spreads. Not surprisingly, the impact of these shocks is more pronounced on the tenors in which the shock is applied to as shown in Figure 6.2 for the 10 year tenor. Further, Figures 6.1 and 6.2 illustrate why the historically calibrated scenarios typically used in stress testing exercises may not be adequate to capture significant spillover effects between economies under certain conditions. For example, it can be shown that there are a number of cases in which shocks to Italian and Spanish liquidity and credit variables could generate significant changes in the European yields, which are much larger in magnitude than the historically observed changes.

Furthermore, Figure 6.3 demonstrates how credit and liquidity shocks to Italian and Spanish variables at various tenors (i.e. 2 year, 10 year and 30 year) affect correlations of European yields. In particular, Figure 6.3 displays smoothed 30-day moving average yield correlation estimates between all possible country pairs under study. The pink lines show the correlation estimates of the historical yield series while red, green and cyan lines show the correlation estimates of the hypothetical yield series obtained from our modelling framework after applying shocks to the Italian and Spanish liquidity and credit variables. It can be noted that the impact of these shocks is more pronounced on cross-country correlation pairs at shorter and medium-term maturities (i.e. 2 year and 10 year) than longer term maturities (i.e. 30 year). Moreover, we note that shocks to credit quality variables have greater impact on cross-country correlation estimates than shocks to liquidity variables of the same magnitude but this impact decays with maturity.

In addition, Figure 6.4 shows the distribution of the European yields after applying shocks to Italian and Spanish credit and liquidity variables at the 2 year, 3 year, 5 year, 10 year, 20 year and 30 year tenors. In general, the distribution of Italian and Spanish hypothetical yields is more disperse when compared to the distribution of hypothetical German, French and the UK yields. In addition, it can be noted that shocks to Italian and Spanish variables can generate negative yields at some particular tenors for the German, French and the UK yields. This is probably due to the negative dependence between the yields and country-specific liquidity and credit factors over specific periods in time that could possible indicate inverstors' "flights" to liquidity or credit quality.

Further, Figure 6.4 is very informative as it illustrates why the historically calibrated scenarios typically used in stress testing exercises may not be adequate to capture significant spillover effects between economies under certain conditions. It can be shown in Figure 6.4 that scenarios calibrated using historical time-series overlap with hypothetical scenarios generated by our modelling framework and thus can provide protection against hypothetical shocks to Italian and Spanish liquidity and credit variables. However, there are a number of cases in which shocks to these variables generate significant changes in the European yields, which are much larger in magnitude than the historically observed changes. For example, it can be seen that shocks at the 3 year or 5 year liquidity and credit quality variables can reduce significantly the French and

UK yields but, on the contrary, the same shocks can materially increase Italian and Spanish yields.

Having generated new hypothetical yield series for each tenor and country under study, we can then calibrate new stress scenarios. We use the 99-th empirical quantile of the hypothetical absolute yield changes for each country and tenor under consideration to calibrate the stress test scenarios.[21] Following Diebold et al. (2008), we use two hypothetical bond portfolios to assess the impact and quantify the magnitude of Italian and Spanish liquidity and credit shocks at portfolio level. The first bond portfolio is an equally-weighted long portfolio that invest an equal amount in six theoretical zero-coupon bonds with maturities equal to: 2 year, 3 year, 5 year, 10 year, 20 year and 30 year. The second bond portfolio is a long-short portfolio that short equal amount in the 2 year, 3 year and 5 year bond and long equal amount in the 10 year, 20 year and 30 year bonds. Diebold et al. (2008) argue, this is a common trading strategy followed by a number of banks, mortgage companies and fixed income trades.

We assume that the curves under study are zero-coupon curves in order to price the portfolios under different stress scenarios. The zero coupon rates are set as the interest rates on the most recent yield curve in our dataset, which is November 2, 2011. Therefore, these bonds are assumed to be priced at par value on November 2, 2011. Using November 2, 2011 baseline rate levels and calibrated stress scenarios we can then obtain a new yield curve for each country to re-value the individual bonds in the portfolio and subsequently calculate portfolios' values and returns. Specifically, we use baseline yields for each country j and maturity τ_i, $y_{j,t}(\tau_i)$, to calculate bond par values. We then apply the calibrated stress scenarios $dy_j^s(\tau_i)$, where $s = \{liquidity, credit, combined\}$, to obtain the yield curve at time $t+1$: $y_{j,t+1}(\tau_i) = y_{j,t}(\tau_i) + dy_j^s(\tau_i)$ and thus to calculate portfolios' new values and returns. We plot the long and long-short portfolios' returns in Figure 6.5.

Each panel in Figure 6.5 displays portfolios' returns when a liquidity only (i.e. Liquidity panel), credit only (i.e. Credit panel) or a contemporaneous liquidity and credit shock is applied (i.e. Combined panel) to relevant Italian and Spanish liquidity and credit quality variables. Furthermore, Figure 6.5 shows the distribution of portfolios' returns when shocks are applied at 2 year, 3 year , 5 year , 10 year , 20 year and 30 year tenors separately. Therefore, it provides a clear way to study how different shock types occurring at different tenors can affect different bond portfolios utilizing different trading strategies. Clearly, the calibrated stress test scenarios have very different impact on the bond portfolios' returns. As expected, shocks to Italian and Spanish liquidity and credit variables have significant impact on the Italian and Spanish bond portfolios. However, these shocks have also significant impact on other European bond portfolios highlighting the strong linkages and spillover effects between European economies.

[21] Stress scenarios can be calibrated in multiple ways. For example, one may consider the worst change in the hypothetical yield series or make use of more extreme quantiles. For consistency, we use the 99-th quantile to calibrate the stress scenarios for each tenor and country under study.

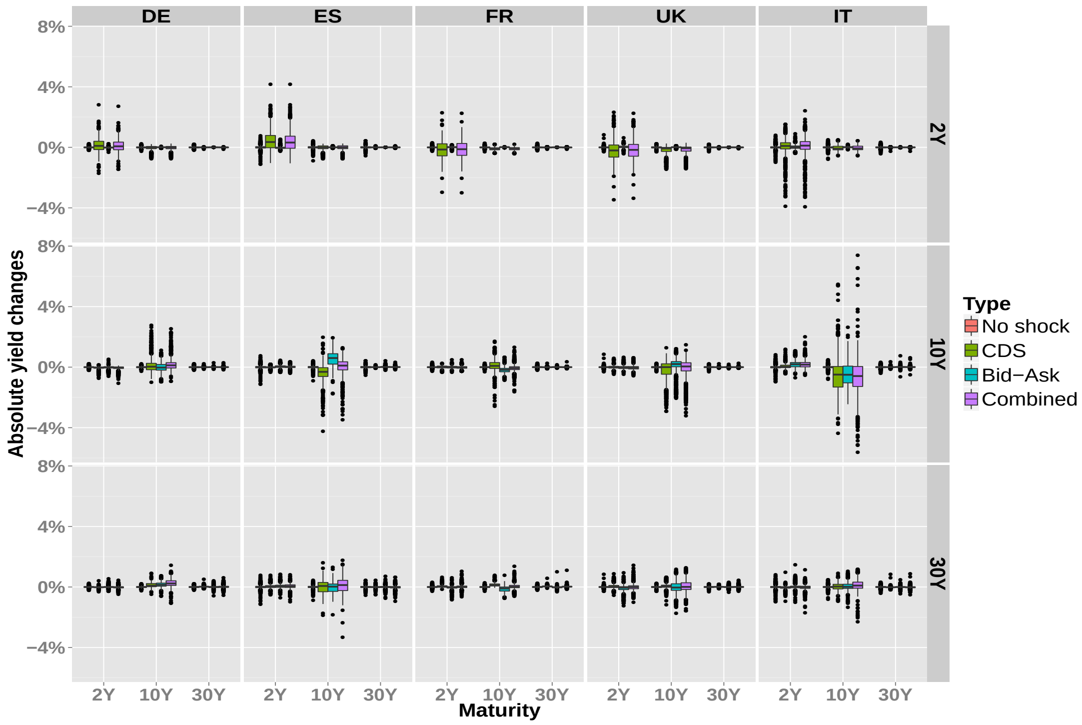


Figure 6.1: This figure shows the distribution of absolute changes in the European bond yields when a credit (CDS), a liquidity (Bid-Ask) or a combined credit and liquidity shock (Combined) is applied to the Italian and Spanish variables at 2 year , 10 year and 30 year maturities as displayed vertically. The distribution of daily absolute changes in the European bond yields is also displayed for comparison (No shock).

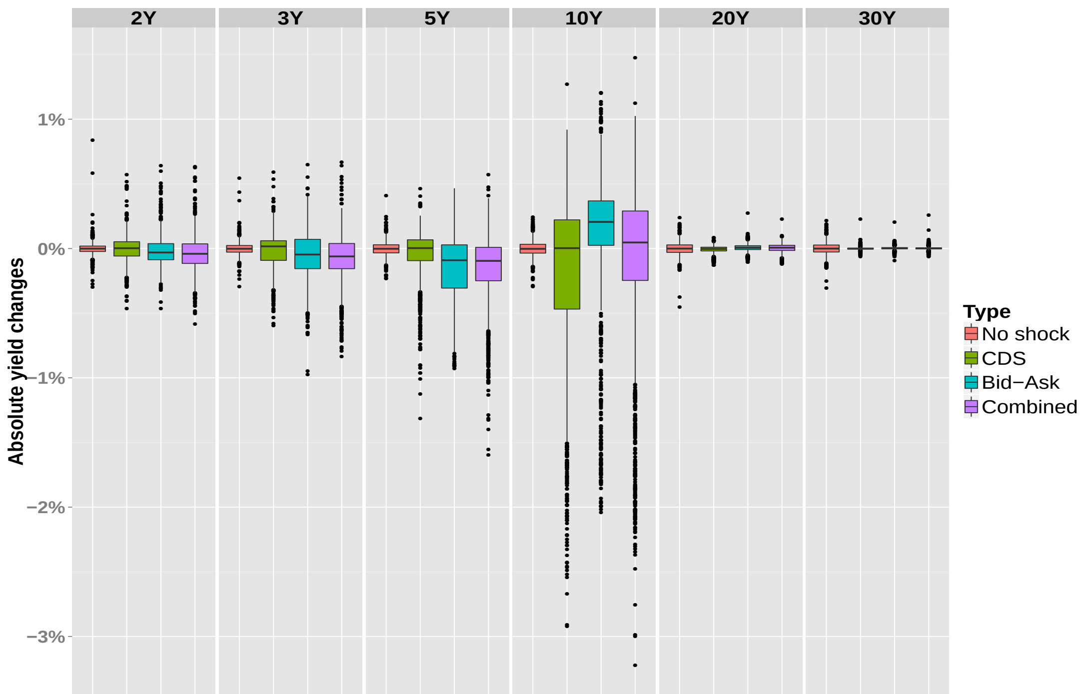


Figure 6.2: This figure shows the distribution of absolute changes in the term-structure of UK bond yields when a credit (CDS), a liquidity (Bid-Ask) or a combined credit and liquidity shock (Combined) is applied to the Italian and Spanish variables at the 10 year maturity. The distribution of daily absolute changes in the European bond yields is also displayed for comparison (No shock).

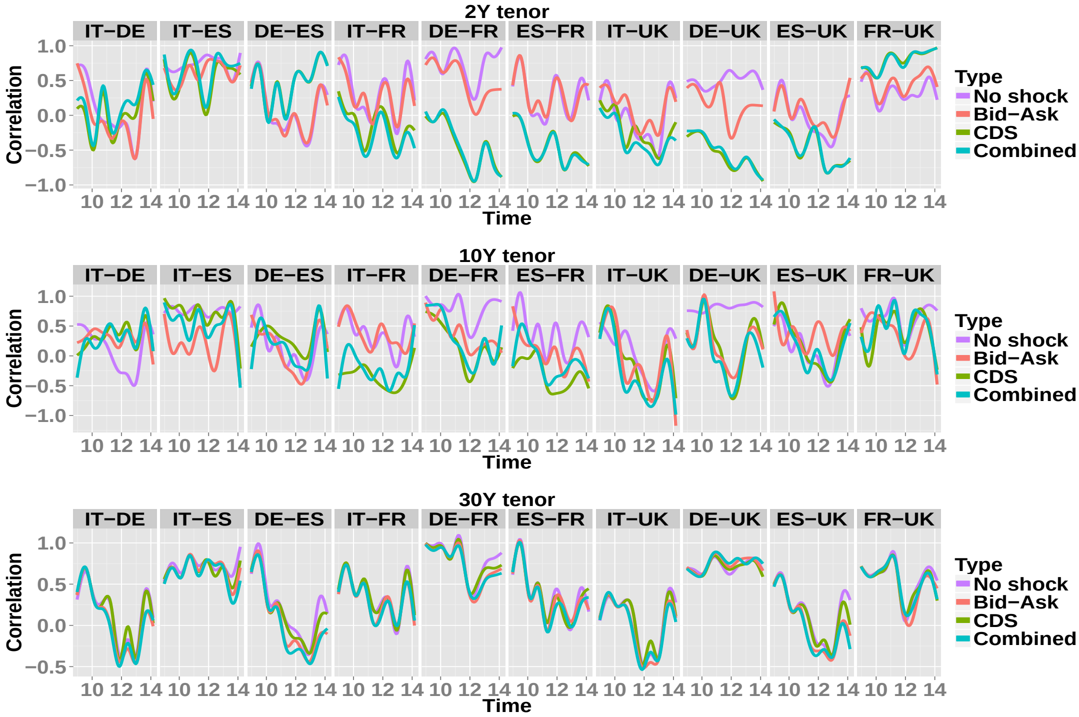


Figure 6.3: This figure shows the impact of shocks to the Italian and Spanish liquidity and credit variables on the correlation of European bond yields at 2 year, 10 year and 30 year maturities. The graphs show smoothed 30-day moving average yield correlation estimates between all possible country pairs under study when a credit (CDS), a liquidity (Bid-Ask) or a combined credit and liquidity shock (Combined) is applied to the Italian and Spanish variables at these particular maturities. The correlations of historical European bond yields is also displayed for comparison (No shock).

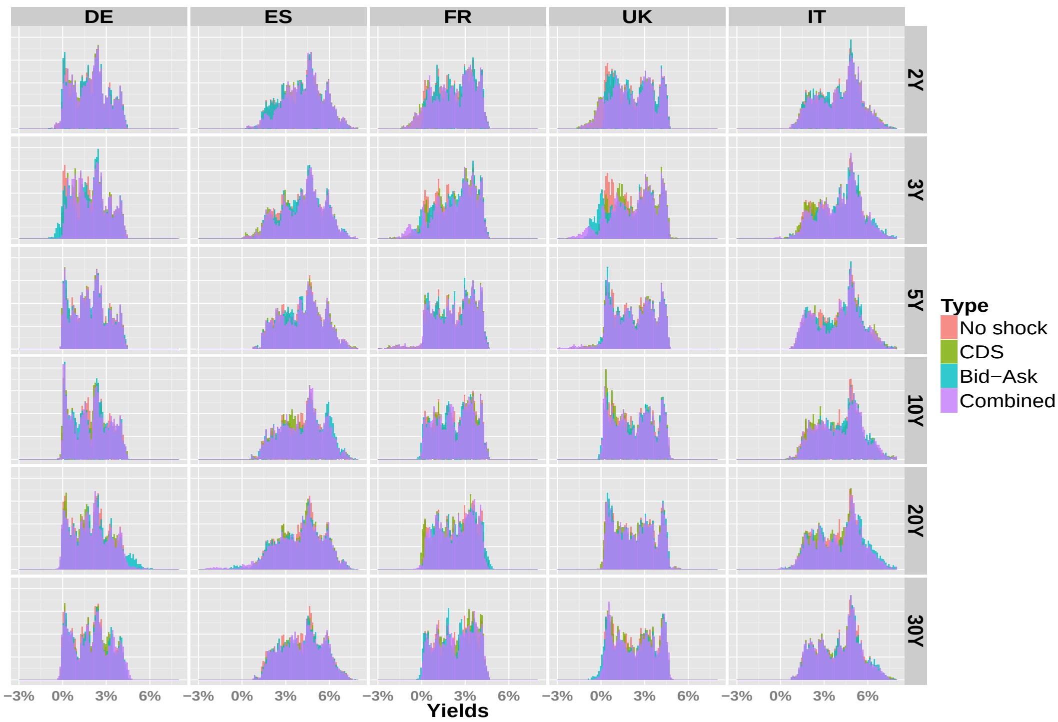
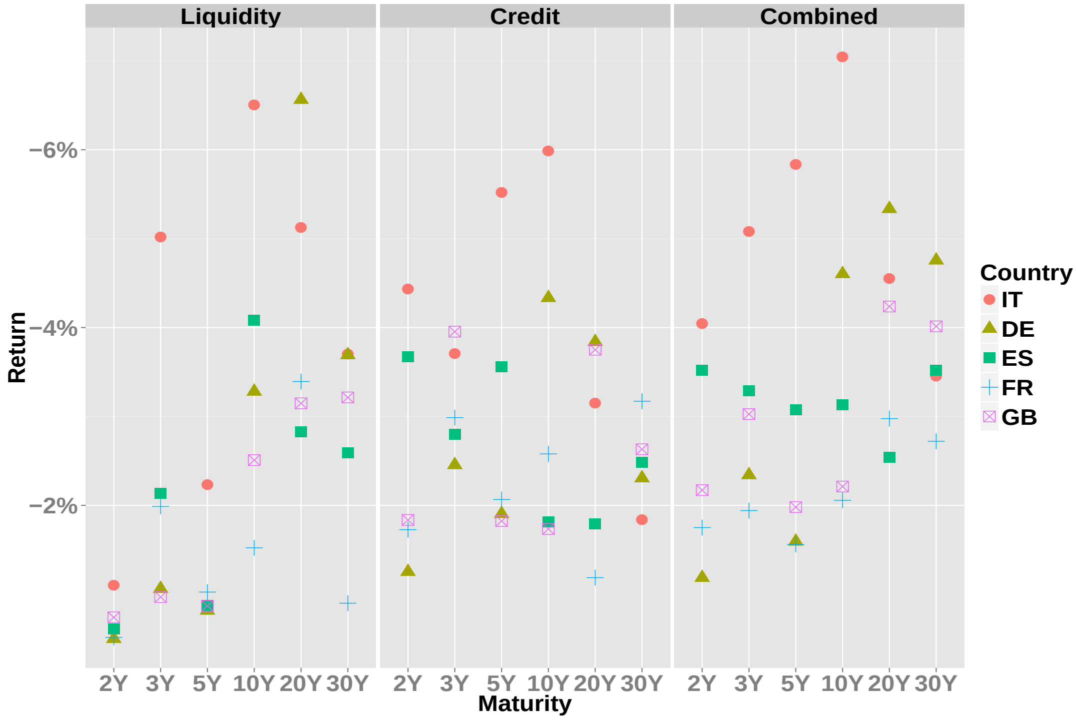
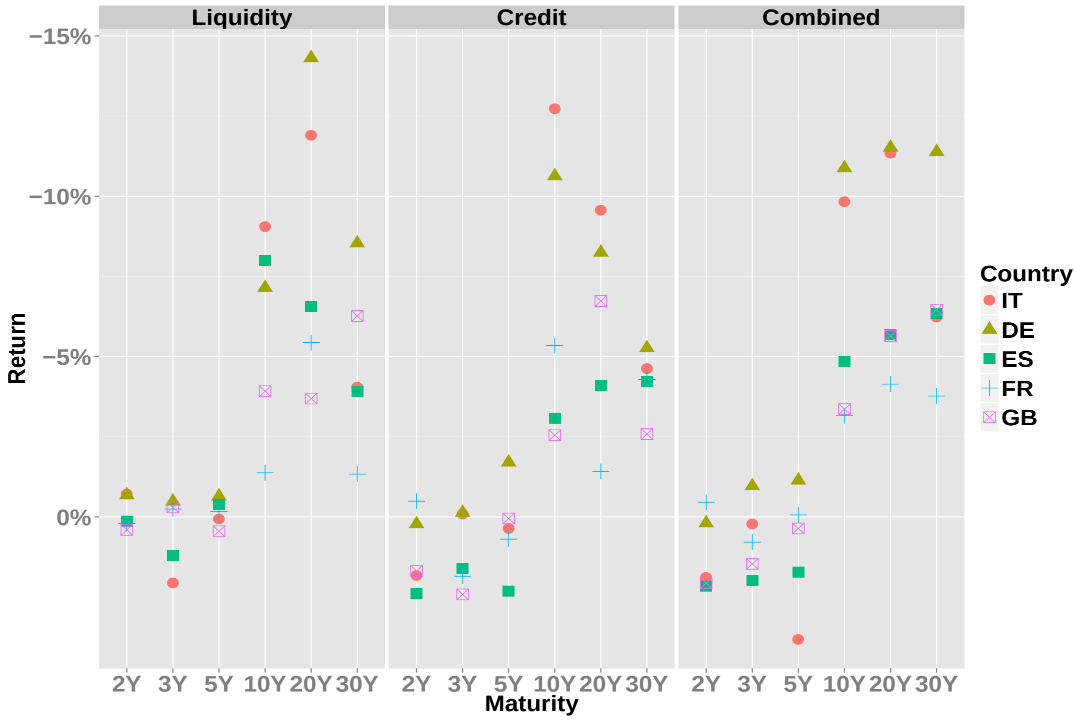


Figure 6.4: This figure shows the distribution of European bond yields when a credit (CDS), a liquidity (Bid-Ask) or a combined credit and liquidity shock (Combined) is applied to the Italian and Spanish variables at 2 year, 3 year, 5 year, 10 year, 20 year and 30 year maturities. The distribution of historical European bond yields is also displayed for comparison (No shock).



(a) Long Bond Portfolio Returns



(b) Long-Short Portfolio Returns

Figure 6.5: This figure shows the impact of shocks in the Italian and Spanish liquidity and credit variables in the returns of Long and Long-Short bond portfolios. Long bond portfolios invest equal amounts in the 2 year, 3 year, 5 year, 10 year, 20 year and 30 year zero coupon bonds for each country. Long-Short bond portfolios short equal amounts in 2 year, 3 year, 5 year and long equal amounts in 10 year, 20 year and 30 year zero coupon bonds for each country.

6.1 Further Applications

The proposed methodology can potentially find a number of alternative risk management applications such as, for example, on collateral haircuts. The motivation for collateralisation is to reduce collateral taker's exposure if the provider fails to perform. Typically, cash or high quality liquid securities such as highly-rated government bonds are used as collateral (see for example Jo Braithwaite and Murphy David (2016) for more details). Usually, a haircut in the value of the collateral is applied to reduce its value. The amount of the haircut essentially reflects the lender's perceived risk of loss from the asset falling in value or being sold in a fire sale.

In general, regulatory guidance on haircuts allows for a wide range of methodological realizations which could lead to potentially very different outcomes. Typically, financial institutions use variants of Value-at-Risk (VaR) methodologies to calibrate their base haircuts. In practice, haircuts consist of a base haircut with further add-ons to account for non-market risks. In order to address risks which are not sufficiently captured by the base haircut calculation, financial institutions need to apply additional measures to manage risks such as FX risk, credit risk, liquidity risk or wrong-way risk. The regulation does not fully specify how these non-market risks could be identified or which tools could be used for their mitigation.[22] Therefore, assessing the conservativeness and the adequacy of these tools to address non-market risks is a non-trivial exercise.

The proposed methodology could be potentially used to quantify the risks identified above and could be also used as a tool to assess the sensitivity of base haircut methodologies to exogenously determined shocks. We present in Figure 6.6 a simple example of how our modeling approach could serve this purpose. We compute daily log returns using Spanish 10 year government bond yields and use a historical 1-day 99% VaR model with a 500-day lookback to calibrate the base haircut over the October 2010 - February 2014 sample period. The red line in Figure 6.6 denotes the level of base haircut in our sample for the 10 year Spanish government bond. It can be shown that the base haircut lies below the 5% level for most of the time in the sample period. In contrast, the green, cyan and violet lines represent the hypothetical haircut levels generated from our modelling approach after applying a one standard deviation shock to 10 year Italian and Spanish liquidity variables only, credit variables only or both liquidity and credit variables.

As shown in Figure 6.6, all hypothetical haircut levels lie above the base haircut levels thus generating more conservative haircut estimates. The maximum haircut level reaches almost up to 25% when a contemporaneous shock is applied to Italian and Spanish credit variables. Interestingly, these haircut levels vary over time and increase significantly in magnitude after Q3 2011, reflecting widespread concerns about the Eurozone debt crisis. On average, it seems that credit shocks have greater impact on haircut levels at this particular tenor than liquidity shocks of the same magnitude. Further, it can be shown that the combined effect of credit and liquidity shocks on Spanish yields can generate lower haircut levels than standalone shocks to liquidity or credit variables. This highlights that the dependence structure between country-specific risks and yields is not constant but rather time-varying. Therefore, shocks of the same type and magnitude may have different impact on the sovereign yields over time and on haircut levels.

[22]For example, CCPs manage non-market risks such as FX, credit, liquidity or wrong way risk (WWR) with distinctive tools. Most CCPs use, among other tools, FX add-ons, concentration limits, or modifications of their base haircut methodologies to address the non-market risks.

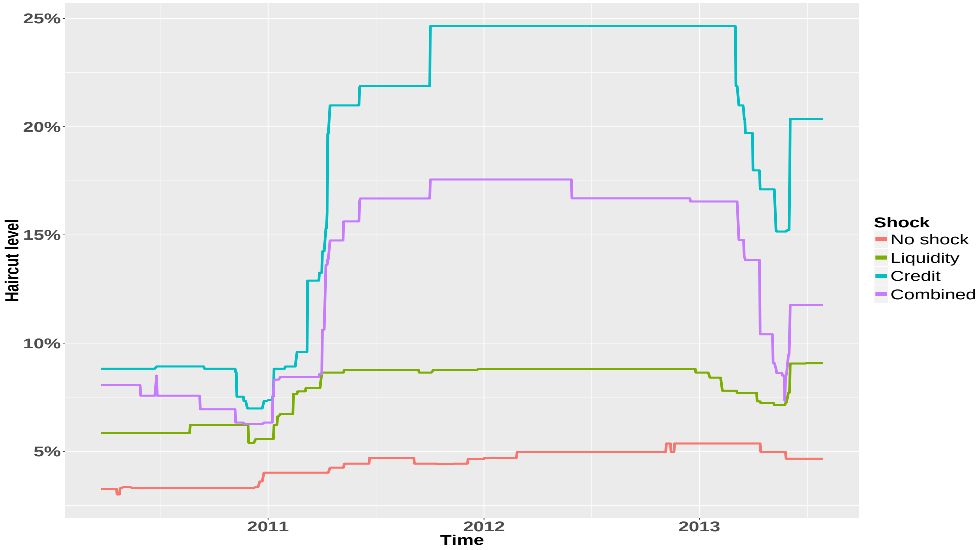


Figure 6.6: This figure shows the 10 year Spanish government bond base and hypothetical haircut levels obtained after applying a one standard deviation shock to 10 year Italian and Spanish liquidity and credit variables. All haircut estimates are computed using a historical simulation 1-day 99% VaR model with a 500-day lookback period.

7 Conclusions

We focus on a multi-curve setting and develop a modeling framework that can generate consistent cross-country stress test scenarios allowing for significant spillover effects between the economies. In particular, we model jointly the temporal and cross-country dependence structure of several European sovereign yield curves and associate movements in the yields and cross-country spreads with movements in macroeconomic and financial variables as well as market-wide and country-specific measures of liquidity and credit quality. The model is flexible enough to accommodate multiple scenarios contemporaneously and thus a large number of consistent scenarios across the curves being modeled can be generated. Moreover, we incorporate observable macroeconomic and financial variables into the modeling specification in a statistical rigorous way. This allows the study of interaction between macroeconomy and term structure and the assessment of importance and impact of these factors in the evolution of sovereign yields and cross-market spreads. Furthermore, within the proposed framework one can more readily handle features of real data such as missingness and/or unbalance datasets. The model can also be easily amended and applied to markets with similar characteristics to generate consistent stress test scenarios.

The analysis is split into two main parts. In the first part, we model the evolution of the yield curve for each particular country under study using the macro-finance Nelson-Siegel model of Diebold et al. (2006) augmented with key macroeconomic and financial variables, as well as European measures of liquidity and credit quality. The inclusion of market-wide liquidity and credit quality variables in the latent factor specification of the model enables the study of the dynamic interaction between these risks and the yield curve for each particular country. In the second part, we model the covariance structure of European sovereign yields employing the covariance regression model of Hoff and Niu (2012). In this respect, we parameterize the covariance matrix of sovereign yields as a function of country-specific liquidity and credit quality factors and explore their effects on the heteroscedasticity of European sovereign yields.

We apply our stress testing framework and generate consistent stress test scenarios to a set of European yield curves. In particular, we design three types of stress scenarios to demonstrate the practical advantage of our stress testing framework. Motivated by the recent Euro-zone debt crisis, we apply shocks to Italian and Spanish liquidity and credit variables at various maturities. We then evaluate impacts of these shocks on several bond portfolio strategies and show that they have very different impacts on the portfolios' values and returns. More specifically, the empirical findings suggest that both country-specific liquidity and credit measures are important in explaining the dynamic behavior of European sovereign yield curves and their dependence structure. Nevertheless, their importance varies across time, shock types and investment horizons. Investors appear to be more concerned with credit quality, while investors' liquidity concerns cannot be discarded, especially during periods of heightened market volatility. We also show how our modeling methodology can find alternative risk management applications such as on collateral haircuts.

Appendices

Appendix A. Descriptive Statistics

Table A.1: Descriptive Statistics: Sovereign yields

	Italy					Germany					Spain					France					UK				
Maturity	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max
12	1368	1.66	1.00	0.57	7.67	1349	0.50	0.49	-0.11	2.19	665	2.06	1.14	0.55	5.47	1356	0.58	0.47	-0.01	2.32	1372	0.52	0.25	0.07	2.29
24	1372	2.34	1.10	0.75	7.58	1366	0.72	0.61	-0.09	2.35	1372	2.51	0.99	0.76	6.57	1372	0.92	0.59	0.02	2.60	1372	0.75	0.44	0.05	2.55
36	1372	2.87	1.10	1.25	7.63	1372	0.91	0.71	-0.05	2.55	1372	3.04	0.98	1.36	7.37	1372	1.19	0.67	0.10	2.82	1372	1.12	0.67	0.08	3.02
48	1372	3.27	1.00	1.71	7.68	1372	1.18	0.80	0.05	2.82	1372	3.35	0.95	1.58	7.40	1372	1.54	0.71	0.29	3.07	1372	1.48	0.77	0.23	3.32
60	1372	3.62	0.99	2.04	7.70	1372	1.45	0.81	0.24	2.90	1372	3.73	0.94	1.99	7.50	1372	1.88	0.71	0.60	3.28	1372	1.78	0.77	0.45	3.49
72	1372	3.81	0.95	2.28	7.66	1372	1.68	0.84	0.41	3.19	1372	3.96	0.95	2.19	7.56	1372	2.10	0.73	0.70	3.45	1241	1.94	0.82	0.62	3.68
84	1372	4.04	0.91	2.59	7.67	1372	1.90	0.84	0.56	3.40	1372	4.21	0.91	2.50	7.53	1372	2.34	0.70	0.91	3.65	1372	2.28	0.82	0.80	3.98
96	1372	4.23	0.83	2.89	7.55	1372	2.09	0.82	0.77	3.60	1372	4.40	0.87	2.84	7.42	1372	2.58	0.68	1.16	3.88	1372	2.52	0.79	1.05	4.06
108	1372	4.43	0.74	3.21	7.28	1371	2.25	0.78	0.98	3.65	1372	4.56	0.87	3.07	7.54	1372	2.77	0.64	1.40	3.94	1249	2.62	0.74	1.35	3.85
120	1372	4.65	0.75	3.46	7.24	1371	2.38	0.75	1.16	3.72	1372	4.77	0.83	3.49	7.57	1372	2.95	0.60	1.66	4.05	1372	2.88	0.77	1.44	4.23
180	1372	5.04	0.70	3.95	7.72	1372	2.82	0.74	1.61	4.27	1329	5.23	0.83	3.94	7.70	1370	3.39	0.54	2.29	4.42	1372	3.40	0.75	2.10	4.85
240	1372	5.32	0.71	4.18	8.04	1372	3.08	0.74	1.75	4.48	1372	5.34	0.79	3.97	7.71	1371	3.56	0.53	2.48	4.64	1372	3.65	0.67	2.46	4.86
360	1372	5.40	0.61	4.45	7.63	1371	3.10	0.70	1.67	4.47	1372	5.43	0.75	3.85	7.54	1371	3.69	0.43	2.77	4.62	1372	3.81	0.53	2.84	4.69

This table reports summary statistics for our sample daily sovereign yields (end-of-day midpoints of the quoted bid and ask yields), expressed in percentages, for various maturities, measured in months, for Italy, Germany, Spain, France and the UK. N is the number of daily observations for each maturity/country.

Table A.2: Descriptive Statistics: Bid-Ask spreads

	Italy					Germany					Spain					France					UK				
Maturity	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max
12	1368	10.72	14.84	1.10	176.70	1349	2.14	2.13	0.80	25.30	665	29.36	28.56	2.40	148.30	1356	4.08	2.13	0.80	17.00	1372	3.33	1.88	0.70	19.80
24	1372	4.78	3.58	0.50	35.90	1366	0.57	0.21	0.20	2.20	1372	7.29	6.10	1.00	40.70	1372	3.04	2.37	0.50	15.90	1372	1.79	0.95	0.50	4.90
36	1372	3.92	3.60	0.30	26.60	1372	0.47	0.24	0.10	1.90	1372	5.65	5.40	0.60	41.00	1372	2.38	1.87	0.50	12.50	1372	1.17	0.60	0.30	2.60
48	1372	3.10	2.59	0.40	20.30	1372	0.36	0.18	0.10	1.70	1372	4.87	4.63	0.70	33.60	1372	2.05	1.83	0.40	11.70	1372	0.90	0.44	0.20	2.00
60	1372	2.58	2.21	0.20	18.10	1372	0.31	0.16	0.10	1.10	1372	4.15	3.97	0.40	27.60	1372	1.72	1.73	0.20	12.30	1372	0.73	0.35	0.20	5.10
72	1372	2.41	1.87	0.30	19.40	1372	0.31	0.15	0.10	1.10	1372	3.81	3.38	0.50	24.50	1372	1.23	1.01	0.20	8.80	1241	0.60	0.25	0.10	1.70
84	1372	2.12	1.56	0.20	16.50	1372	0.37	0.14	0.10	1.60	1372	3.35	2.96	0.30	17.10	1372	1.00	0.70	0.20	6.60	1372	0.55	0.20	0.10	1.60
96	1372	1.89	1.32	0.20	12.40	1372	0.33	0.13	0.10	1.70	1372	3.06	2.84	0.20	16.30	1372	0.96	0.71	0.20	5.40	1372	0.51	0.18	0.10	1.70
108	1372	1.75	1.30	0.30	18.40	1371	0.27	0.12	0.00	0.60	1372	2.73	2.34	0.30	11.60	1372	0.86	0.63	0.20	4.50	1249	0.45	0.19	0.10	1.60
120	1372	1.72	1.28	0.20	15.80	1371	0.34	0.18	0.10	0.80	1372	2.48	2.22	0.20	12.60	1372	0.83	0.59	0.20	5.90	1372	0.46	0.18	0.10	1.80
180	1372	2.06	1.78	0.20	19.10	1372	0.82	0.39	0.40	3.00	1329	2.86	2.41	0.40	16.80	1370	0.94	0.77	0.20	7.00	1372	0.52	0.14	0.20	1.50
240	1372	2.24	2.22	0.50	21.80	1372	0.76	0.40	0.30	2.60	1372	2.93	2.82	0.10	13.30	1371	0.96	0.81	0.20	6.50	1372	0.47	0.11	0.20	1.50
360	1372	1.70	1.56	0.30	14.00	1371	0.59	0.34	0.20	6.00	1372	2.57	2.65	0.00	12.90	1371	0.77	0.62	0.10	5.00	1372	0.43	0.10	0.20	1.40

This table reports summary statistics for our sample sovereign bid-ask spreads (end-of-day quoted bid-ask spreads), expressed in basis points, for various maturities, measured in months, for Italy, Germany, Spain, France and the UK. N is the number of daily observations for each maturity/country.

Table A.3: Descriptive Statistics: Credit Default Swap (CDS) spreads

	Italy					Germany					Spain					France					UK				
Maturity	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max	N	Mean	Sd	Min	Max
12	1367	128.93	113.04	6.00	550.85	1357	12.48	11.43	0.28	50.48	1366	139.18	94.61	14.00	426.63	1343	27.31	24.99	2.02	128.78	1372	27.65	25.53	1.92	140.00
24	1367	155.67	116.55	20.00	542.02	1357	16.26	12.79	1.09	59.23	1366	162.89	101.88	24.00	476.87	1343	35.25	28.16	3.86	142.49	1372	35.40	26.03	4.02	147.50
36	1367	176.13	116.53	34.00	530.17	1357	20.15	14.07	2.53	69.70	1366	180.27	102.82	34.00	494.40	1343	43.80	31.72	6.39	156.98	1372	42.29	26.61	6.11	155.00
48	1367	187.40	111.85	41.00	513.91	1357	25.58	15.14	5.19	81.10	1366	190.23	100.76	40.50	493.25	1343	52.33	32.77	12.01	161.37	1372	51.22	25.94	15.59	160.00
60	1367	196.88	107.76	48.00	498.66	1357	31.47	16.25	9.16	92.50	1370	197.94	98.44	47.00	492.07	1343	61.14	34.23	14.01	171.56	1372	60.25	25.65	22.09	165.00
84	1367	201.46	103.86	49.20	480.66	1356	36.92	15.33	16.95	92.24	1366	202.18	94.28	47.80	468.87	1343	68.84	34.02	15.60	176.03	1372	67.78	21.82	34.58	165.00
120	1367	201.63	98.53	51.00	468.19	1357	41.92	15.01	21.48	91.98	1366	201.27	89.31	49.00	444.51	1343	75.15	34.79	17.00	181.36	1372	74.94	19.80	45.50	165.00
240	1365	195.82	94.21	46.00	463.11	1357	41.85	15.15	20.71	96.02	1364	196.83	83.46	49.00	419.07	1343	75.17	33.69	19.00	182.37	1372	82.54	18.63	45.50	165.00
360	1367	192.90	92.86	41.00	460.04	1357	41.90	15.44	18.18	96.34	1366	194.93	81.12	49.00	408.36	1343	75.83	32.88	25.00	183.86	1372	84.44	18.53	45.50	165.00

This table reports summary statistics for our sample credit default swap (CDS) spreads, expressed in basis points, for various maturities, measured in months, for Italy, Germany, Spain, France and the UK. N is the number of daily observations for each maturity/country.

Appendix B Handling missing data

In our sample there are missing observations related to maturities or time periods. An attractive feature of the state-space framework is its ability to treat time series that have been observed irregularly over time. Suppose, at a given time t, we observe some, but not all, values of observation vector $\boldsymbol{y}_t = [y_t(\tau_1), \ldots, y_t(\tau_N)]'$. We define the partition of the $N \times 1$ observation vector $\boldsymbol{y}_t = [\boldsymbol{y}_t^{(1)\prime}, \boldsymbol{y}_t^{(2)\prime}]'$, where the first $N_t^{(1)} \times 1$ vector $\boldsymbol{y}_t^{(1)}$ is observed and the second $N_t^{(2)} \times 1$ vector $\boldsymbol{y}_t^{(2)}$ is unobserved, where $N_t^{(1)} + N_t^{(2)} = N$. The partitioned observation equation can be given as

$$\begin{pmatrix} \boldsymbol{y}_t^{(1)} \\ \boldsymbol{y}_t^{(2)} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Lambda}^{(1)}(\lambda) \\ \boldsymbol{\Lambda}^{(2)}(\lambda) \end{pmatrix} \boldsymbol{\beta}_t + \begin{pmatrix} \boldsymbol{\varepsilon}_t^{(1)} \\ \boldsymbol{\varepsilon}_t^{(2)} \end{pmatrix}, \tag{15}$$

where $\boldsymbol{\Lambda}^{(1)}(\lambda)$ and $\boldsymbol{\Lambda}^{(2)}(\lambda)$ are partitioned $N_t^{(1)} \times 3$ and $N_t^{(2)} \times 3$ factor loading matrices respectively, while $\boldsymbol{\varepsilon}_t^{(1)}$ and $\boldsymbol{\varepsilon}_t^{(2)}$ are partitioned $N_t^{(1)} \times 1$ and $N_t^{(2)} \times 1$ error vectors, respectively, with the measurement covariance matrix between the observed and unobserved parts being written as follows

$$\mathrm{Cov} \begin{pmatrix} \boldsymbol{\varepsilon}_t^{(1)} \\ \boldsymbol{\varepsilon}_t^{(2)} \end{pmatrix} = \begin{pmatrix} H_t^{(1)} & H_t^{(12)} \\ H_t^{(21)} & H_t^{(2)} \end{pmatrix}.$$

Consequently, at the times of the missing observations, where $\boldsymbol{y}_t^{(2)}$ is not observed, Equation (4) is replaced by

$$\boldsymbol{y}_t^{(1)} = \boldsymbol{\Lambda}^{(1)}(\lambda)\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t^{(1)}, \qquad \boldsymbol{\varepsilon}_t^{(1)} \sim N(0, \boldsymbol{H}^{(1)}), \tag{16}$$

where now the observation equation is $N_t^{(1)}$- dimensional at time t. The Kalman filter proceeds exactly as in the standard case, provided that $\boldsymbol{y}_t$, $\boldsymbol{\Lambda}(\lambda)$ and $\boldsymbol{H}$ are replaced by $\boldsymbol{y}_t^{(1)}$, $\boldsymbol{\Lambda}^{(1)}(\lambda)$ and $\boldsymbol{H}_t^{(1)}$ respectively at relevant time points. It is clear that the dimensionality of the observation equation evolves over time, but this does not affect the validity of the filtering recursion. Once the state-space model parameters are estimated, $\widehat{\boldsymbol{\theta}} = \{\widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Phi}}, \widehat{\boldsymbol{A}}, \boldsymbol{\Lambda}(\widehat{\lambda}), \widehat{\boldsymbol{Q}}, \widehat{\boldsymbol{H}}\}$, we can obtain the missing observations in the vector of missing data $\boldsymbol{y}_t^{(2)}$ at any point in time $t = 1, \ldots, T$. In this respect, each element j in the vector of missing yields $\boldsymbol{y}_t^{(2)}$ can be optimally predicted as follows

$$\boldsymbol{y}_t^{(2,j)} = \boldsymbol{\Lambda}^{(2,j)}(\widehat{\lambda})\widehat{\boldsymbol{b}}_{t|t} + k\sqrt{\left(\boldsymbol{\Lambda}^{(2,j)}(\widehat{\lambda})\ \widehat{\boldsymbol{P}}_{t|t}\ \boldsymbol{\Lambda}^{(2,j)}(\widehat{\lambda})'\right)}, \tag{17}$$

where $\boldsymbol{\Lambda}^{(2,j)}(\widehat{\lambda})$ is the j row of the factor loading coefficient matrix and k is a scale parameter, which is set to a pre-specified value, and controls the deviation of the missing observation from its expected value.

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