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1. INTRODUCTION

The relationships between manufacturing output and the various factors of production is one of the key questions in macro-economics. There has been much work aimed at identifying the own price elasticities of demand of various factors, and following the seminal paper by Christensen et al (1973) [and previously a more restrictive model used by Uzawa (1962)], the cross-elasticities of substitution within a multi-factor model.

In particular, it has been frequently argued that a change in the relative price of a factor of production will lead to substitution of other factors for that factor. The apparent slowdown in the growth of labour productivity from 1973-74 onwards has frequently been attributed to the oil price shock, and subsequent substitution of labour for energy. More recently, the breakdown of most employment and investment equations (estimated on UK data) since 1979 [with labour being over-predicted and investment under-predicted] has been attributed to the rapid growth of labour costs in the three years to 1980 and the second oil price shock of 1979/80 leading to both labour and energy saving investment, and hence substitution of capital for other factors. [This argument is surprising as the series for the user cost of capital used in this paper generally grew faster than other factor costs over the period from 1978.]

This paper attempts to estimate the substitution technologies of UK manufacturing in a model incorporating three factors: capital, labour and energy, to see whether changes in relative prices can explain movements in factor demands in these periods.

The derivation of equations determining demands for factors of production, and hence the technologies of substitution, generally starts from the basis of firms taking prices as given and maximising profits (or minimising costs with given output) subject to a production function relating output to the various factors of production. Early work was frequently based on the functional forms proposed by Cobb and Douglas (1928) (CD) or Arrow, et al (1961) (ACMS). These two forms incorporate two factors of production, usually capital and labour. [See eg, Brechling (1965) or Ball and St Cyr (1966) for employment demand functions and Jorgenson (1963) for fixed capital demand functions.] The CD form imposes a unitary elasticity of substitution between the two factors, whilst the ACMS form allows any constant elasticity of substitution (CES), with the CD form as a special case. Demand equations derived from these forms frequently exclude the direct influence of the other factor, its effect being felt through the net price deflator for output.

A problem with the use of the CD or CES forms in multi-factor models is that they impose a common elasticity of substitution between all factors, as is discussed by Uzawa (1962) and McFadden (1963).

In two recent papers, the OECD (1981, 1982b) have proposed a 'double-CES' function for three factors; two forming a CES function nested within another CES function including the third factor. Whilst not so restrictive as the specification described above, this form still requires two factors to have a common elasticity of substitution with the third. This functional form is also rather complicated to estimate, particularly if more factors are included in a similar fashion.

The limitations of the CD and CES forms motivated research aimed at developing a more general form. Two functional forms resulted, neither imposing any restrictions of common elasticities of substitution. Diewert (1971) proposed a 'Generalized Leontief Production Function', a quadratic of an arbitrary number of inputs reducing to the Leontief fixed input ratios form as a special case. Christensen et al (1973) (CJL), and independently Griliches and Ringstad (1971) and Sargan (1971) following a generalisation of the CES form by Kmenta (1967), proposed the 'Transcendental Logarithmic Production Function' (translog) - a second order approximation to any arbitrary production function - and a cost function dual of similar form. This form has both linear and quadratic terms in an arbitrary number of inputs, reducing to a CD or CES form with several factors as a special case. It allows testing of the theoretically desirable restrictions of constant returns to scale and symmetry.

Section two below discusses the form and properties of the translog production function, a cost function dual of similar form and the implied factor demand functions. Section three describes the data and specification for the estimating demand system for UK manufacturing. Section four presents the results of estimating the system, and section five draws brief conclusions. Details of data are given in appendix.

2. DERIVATION OF FACTOR DEMAND EQUATIONS

The translog production function relating output Q to factor inputs X_i and technological progress T proposed by CJL may be written

$$\ln Q = \ln\lambda + \sum_i \mu_i \ln X_i + \mu_T \ln T + 1/2 \sum_i\sum_j \upsilon_{ij} \ln X_i \ln X_j$$

$$+ 1/2 \sum_i \upsilon_{iT} \ln x_i \ln T + 1/2\, \upsilon_{TT} (\ln T)^2. \qquad (2.1)$$

If this production function has constant returns to scale, the restrictions

$$\sum_i \mu_i = 1,\ \sum_i \upsilon_{ij} = 0,\ \sum_j \upsilon_{ij} = 0,\ \sum_i\sum_j \upsilon_{ij} = 0,\ \sum_i \upsilon_{iT} = 0 \qquad (2.2)$$

hold. If Hicks-neutral technical change holds as well, then

$$\upsilon_{iT} = 0,\ \forall i,\ \upsilon_{TT} = 0. \qquad (2.3)$$

The Slutsky symmetry conditions require

$$\upsilon_{ij} = \upsilon_{ji} \quad \forall i, j \quad i \neq j.$$

If any υ is zero, (2.1) reduces to a simple multi-factor CD form, yet, if any υ is non-zero, the properties are very different.

The marginal productivity of each factor, assuming Hicks-neutral technical change, is given by

$$\frac{\partial \ln Q}{\partial \ln X_i} = \mu_i + \sum_j \upsilon_{ij} \ln X_j. \qquad (2.5)$$

In order to investigate substitution technologies, Samuelson (1953-54), Shephard (1953, 1970), Uzawa (1962) and Diewert (1971) proposed the use of a cost function dual, the existence of which was proved by Samuelson (1953-54) and more completely by Shephard (1953) for any arbitrary production function. [The proof requires the production function to be positive and to exhibit constant returns to scale and non-increasing marginal rates of substitution.] As Diewert (1974) points out, it is likely to be impossible to obtain a cost function and hence factor share equations as explicit functions of the parameters of a production function such as (2.1). The alternative is to write a differentiable second order approximation to any arbitrary cost function, which in the translog form is

$$\ln C = \alpha_0 + \sum_i \alpha_i \ln p_i + 1/2 \sum_i \sum_j \beta_i \ln p_i \ln p_j + \alpha_Q \ln Q$$

$$+ 1/2 \beta_Q (\ln Q)^2 + \sum_i \gamma_i \ln p_i \ln Q + \alpha_T \ln T + 1/2 \beta_T (\ln T)^2$$

$$+ \sum_i \delta_i \ln p_i \ln T + \delta_{QT} \ln Q \ln T \qquad (2.6)$$

where $C = \sum_i p_i X_i$

and P_i is the user cost of factor X_i.

Taking the partial derivative of (2.6) with respect to prices and setting equal to the factor shares [Hotelling's lemma] gives the equilibrium relationships

$$\frac{\partial \ln C}{\partial \ln P_i} = S_i = \frac{p_i X_i}{C} = \alpha_i + \sum_j \beta_{ij} \ln p_j + \gamma_i \ln Q + \delta_i \ln T, \qquad (2.7)$$

for each factor i. It is desirable that additivity be imposed on this system by

$$\sum_i \alpha_i = 1, \ \sum_i \beta_{ij} = 0 \ \forall j, \ \sum_i \gamma_i = 0, \ \sum_i \delta_i = 0 \qquad (2.8)$$

Linear homogenity in prices implies

$$\sum_j \beta_{ij} = 0 \ \forall i \qquad (2.9)$$

and the Slutsky symmetry conditions require

$$\beta_{ij} = \beta_{ji} \quad \forall i, j \quad i \neq j \qquad (2.10)$$

and homotheticity requires

$$\gamma_i = 0 \quad \forall_i \qquad (2.11)$$

Another potentially desirable restriction is that of Hicks-neutral technical change

$$\delta_i = 0 \quad \forall i. \qquad (2.12)$$

Uzawa (1962) showed that the partial elasticity of substitution between factors i and j is given by

$$\sigma_{ij} = C \frac{\partial^2 C}{\partial P_i \partial P_j} \Bigg/ \frac{\partial C}{\partial P_i} \frac{\partial C}{\partial P_j} \tag{2.13}$$

$$= \frac{\beta_{ij}}{S_i S_j} + 1 \quad \forall\ i,j \quad i \neq j \tag{2.14}$$

Unless $S_i S_j$ is constant, these elasticities will vary with relative factor shares. Averages estimated at sample means may be constructed

$$\bar{\sigma}_{ij} = \frac{\hat{\beta}_{ij}}{\bar{S}_i \bar{S}_j} + 1 \tag{2.15}$$

and Humphrey and Moroney (1975) give their variances as

$$\text{var}\ (\bar{\sigma}_{ij}) = \left(\frac{1}{\bar{S}_i \bar{S}_j}\right)^2 \text{var}\ (\beta_{ij}) \tag{2.16}$$

enabling tests of the fixed coefficient or CD forms, or for equality of elasticities between factor groups, to be made.

The own-price elasticity for each factor is given by

$$\bar{\eta}_{ii} = \frac{\beta_{ii}}{\bar{S}_i} + \bar{S}_i - 1 \quad \forall i \tag{2.17}$$

and the ordinary demand cross-elasticities for factors i and j is given by [see, eg, Boyle and Sloane (1982)].

$$\bar{\eta}_{ij} = \frac{\hat{\beta}_{ij}}{\bar{S}_i} + \bar{S}_j \quad \forall\ i,j \quad i \neq j \tag{2.18}$$

with

$$\text{var}\ (\pi_{ij}) = \left(\frac{1}{\bar{S}_i}\right)^2 \text{var}(\hat{\beta}_{ij}) \quad \forall\ i,j. \tag{2.19}$$

The disadvantage of a system of equations such as (2.7) is that the actual factor demands are undefined. Diewert (1974) proposes closing the system with (2.6). This could be estimated simultaneously or by substituting the parameter estimates from (2.6) and using indirect least squares to estimate the remaining parameters. Since the prime concern of this paper is to investigate the technologies of substitution of UK manufacturing, rather than provide a model suitable for forecasting, such an approach awaits further research. [An alternative to the translog model would be the 'Generalized Leontief Function' proposed by Diewert (1971, 1974), which is also a second order approximation to any arbitrary function, and allows derivation of factor demands directly by a system of linear (as opposed to log-linear) equations, although the introduction of the possibility of technological change introduces non-linearities.]

3. EMPIRICAL SPECIFICATION

Recent attempts at identifying factor demand equations have hit several major problems. Firstly, it has frequently been difficult to identify price effects in labour demand equations [see, eg, OECD (1982a) or Hazeldine (1978)] although some authors [eg Peel and Walker (1978) and Nickell (1981)] found some effect, albeit with a long lag. This provides a good argument for a specification which does not impose restrictions which may be rejected by the data, and also for the inclusion of additional factors, to allow free estimation of any price effects. The absence of additional factors is particularly highlighted by the frequent inclusion of productivity trends with a split around the beginning of 1974, which has been rationalised by reference to the oil price shock of the winter of 1973-74 [see, eg, OECD (1981), (1982b)]. The OECD explicitly include 'energy' as a separate factor in their 'double-CES' model to overcome this problem. Energy or 'natural resources' have also been included in many translog models, such as those of Humphrey and Moroney (1975) and McRae and Webster (1980).

Additionally, Berndt and Wood (1979) suggest that capital and energy are complementary, and that after the oil price shock of 1973-74 there was accelerated scrapping and increased obsolescence of the capital stock and consequent substitution of labour for the other two factors of production. This argument receives some strength from the negative conclusions of 'growth accounting' models which suggest that the share of 'energy' in total output is insufficient to have accounted for a productivity slowdown of the magnitude which occurred from 1974. The use of the capital stock series proposed by Baily (1981), discussed below, may make this more apparent.

Another problem has been the apparent breakdown of most factor demand equations [estimated on UK data] since 1979, with sharp movements in relative prices again being advanced as a possible explanation. Estimation of the demand system derived in this note is restricted to the period up to 1979, its forecasting abilities being tested over the subsequent period.

The estimating model has three factors of production: labour (L), fixed capital (K) and energy (E). The utilisation of capital, as used by Nadiri and Rosen (1969), is not included: the dynamic specification of the model allows capacity utilisation to be implicitly included as the difference between the actual level of output and its 'equilibrium' level as defined by the production function using the actual levels of factor inputs. Other measures of capacity utilisation, such as deviations of output from trend, as used by Nadiri and Rosen, are of dubious quality.

The measurement of the capital stock poses considerable problems. The decision taken here is to define the capital stock as a function of an accumulation of constant price investment, using an arbitrary (and possibly undesirable) base year of 1975, and a fixed and arbitrary depreciation rate of 1/2% per quarter. This leads to a fairly close approximation to the capital stock series given in CSO National Income and Expenditure (the 'Blue Book')with the advantages of known depreciation and quarterly figures. An allowance has also been made for finance leasing to manufacturing industry, although the estimates for this prior to 1975 are poor, but fortunately of minor importance. A problem with this measure of capital stock, as pointed out by Baily (1981), is that there may have been an increased rate of scrapping and growing obsolescence, which it would not capture. Baily proposes that stock markets will make some assessment of this, and hence that this capital stock measure should be 'corrected' using the average valuation ratio, or Tobin's 'q' [see Brainard and Tobin (1968); and Jenkinson (1981) for its construction using UK data and its applicability to investment models]. Multiplication of a conventional capital stock series by a function of 'q' [which has to be monotonic increasing with a fixed point at the 'equilibrium' value of 1: in this paper, $\sqrt[2]{q}$] would thus give more information about the flow of capital services.

A refinement not yet considered would be to disaggregate capital into plant and machinery, and new buildings and works [see, eg, Berndt and Christensen (1973)]. However, this still does not get over the problem posed in the capital theory controversy [see, the debate started by Robinson (1953-54) and the comment by Champernowne (1953-54), and ably summarised by Harcourt (1972)].

Similarly to capital, 'energy' could be disaggregated into its different types, as in McRae and Webster (1980), which might alleviate possible problems caused by the increase in the price of oil relative to other energy sources in 1973-74.

One further problem with capital is its 'user cost'. A major problem here is that most measures of real interest rates derived from nominal rates and a measure of expected inflation tend to be negative during much of the late 1970s. The approach used in this paper to combat this problem is to add a premium to the nominal interest rate to allow for the inherent riskiness of industrial investment compared to, say, government stock. The construction of the user cost of capital is discussed in more detail <u>in appendix</u>.

All data are, or proxy, series for UK manufacturing industries, and are quarterly, either by compilation, construction or interpolation. Details are given <u>in appendix</u>.

One drawback of earlier papers estimating 'translog' models (eg Christensen, Jorgenson and Lau, Berndt and Christensen (1973), Humphrey and Moroney (1975), and McRae and Webster (1980)) was their lack of dynamic structure, frequently necessitating an autoregressive parameter, despite the use of annual data. In order to estimate an equation of the form of (2.7) on quarterly data, it is necessary to include some adjustment process with (2.7) as the long-run solution. The seminal paper by Nadiri and Rosen (1969) proposed a model of interrelated adjustment of factors. Using their model to re-write (2.7) in dynamic form, gives:

$$S_{it} = \alpha_i + \sum_{j}\sum_{k=0}^{n} \beta_{ijk} \ln P_{j,t-k} + \sum_{k=0}^{n} \gamma_{ik} \ln Q_{t-k} + \delta_i \ln T$$

$$+ \sum_{j}\sum_{k=0}^{n} \varepsilon_{ijk} S_{j,t-k} + \upsilon_i \qquad (3.1)$$

for each factor i. If $\underline{S}$ is the vector of S_i, $\underline{a}$ the vector of α_i, $\underline{b}_j$ the vector of $b_{ij} = \sum_{k=0} \beta_{ijk}$, $\underline{c}$ the vector of $c_i = \sum_{k=0} \gamma_{ik}$, $\underline{d}$ the vector of δ_i and $\underline{E}$ the matrix of

$e_{ij} = \sum_{k=0}^{n} \varepsilon_{ijk}$, then in static equilibrium where

$\underline{S}_t = \underline{S}_{t-1} = \ldots = \underline{S}_{t-k} = \underline{S}$ the solution of $\underline{S}$ is:

$$\underline{S} = (\underline{I} - \underline{E})^{-1} (\underline{a} + \sum_j \underline{b}_j \ln p_j + \underline{c} \ln Q + \underline{d} \ln T) \qquad (3.2)$$

In practice, a system of equations such as (3.1) is difficult to work with, as the imposition of the restrictions (3.4) - (3.8) below in terms of the static solution (3.2) is complex due to the term $(\underline{I} - \underline{E})^{-1}$. Restricting (3.1) so that;

$$\varepsilon_{ijk} = 0 \quad \forall\ i,j,k \quad i \neq j \qquad (3.3)$$

gives an independent adjustment mechanism, but this restriction was accepted by the data at the 1 per cent level, and it was with this restriction imposed that the estimation reported in the section below was carried out.

The additivity restriction (2.8) is given by

$$\sum_i (\alpha_i + \sum_k \varepsilon_{iik} S_{i,t-k}) = 1, \quad \sum_i \beta_{ijk} = 0 \quad \forall\ j,k, \qquad (3.4)$$

$$\sum_i \gamma_{ik} = 0 \quad \forall\ k, \quad \sum_i \delta_i = 0$$

The first part of this restriction involves terms in the product of the lagged dependent variables and their coefficients. In order to make this tractable, the coefficients of all the dependent variables were restricted to be equal

$$\varepsilon_{iik} = \theta_k \quad \forall\ i, k. \qquad (3.5)$$

As is reported below, this restriction was accepted by the data. Imposition of (3.5) automatically implies additivity, as all the dependent variables sum to unity and have a common set of predetermined variables.

The restrictions (2.9) - (2.12) should be imposed on the steady-state coefficients. Thus homogeneity may be imposed by

$$\sum_{jk}\sum \beta_{ijk} = 0 \quad \forall_i, \tag{3.6}$$

the Slutsky symmetry conditions by;

$$\frac{\sum_k \beta_{ijk}}{1-\sum_k \varepsilon_{iik}} = \frac{\sum_k \beta_{jik}}{1-\sum_k \varepsilon_{jjk}} \qquad \forall\ i,j \quad i \neq j, \tag{3.7}$$

although with (3.5) imposed this becomes

$$\sum_k \beta_{ijk} = \sum_k \beta_{jik} \qquad \forall\ i,j \quad i \neq j \tag{3.7a}$$

homotheticity by

$$\sum_k \gamma_{ik} = 0 \quad \forall\ i \tag{3.8}$$

and Hicks neutrality again by

$$\delta_i = 0 \quad \forall_i \tag{3.9}$$

A further advantage of using (3.5) is that all constraints are now linear in parameters, and so may be estimated using exact methods, rather than inexact and costly non-linear methods.

4. RESULTS OF ESTIMATION

The model used for estimation was (3.1) with the restriction (3.2) imposed, over the period 1964 Q3 - 1979 Q2. A dummy variable was included to remove the effects of the corporation tax regime changes in 1966 Q1 from the valuation of capital. Shortage of degrees of freedom limited the initial lag length to three quarters on all variables. Estimation was initially carried out using OLS. Reduction of the lag length to two quarters was accepted easily by the data, but further general reduction was rejected. Despite some evidence of autocorrelation, it was decided to use these equations, reported as A in tables 1-3, as a basis for further estimation.

In order to allow for cross-correlation of the errors U_i of (3.1), and to allow the testing and possible imposition of the cross-equation constraints (3.5) - (3.9), the system was re-estimated using Three-Stage least squares. The results are given as B in tables 1-3. Several of the steady state price coefficients are significantly different from zero, and when the own-price and cross-price elasticities are calculated, several of these are also significant. Only three elasticities have an unexpected sign; $\bar{\sigma}_{KL}$, $\bar{\eta}_{LE}$ and $\bar{\eta}_{LL}$: none are significant.

In order to test and impose the restrictions (3.5) - (3.9) the following strategy was adopted. First, equality of coefficients on the lagged dependent variable (3.5) was tested and imposed to give additivity. Then homogeneity (3.6) and symmetry (3.7a) were tested and imposed both separately and jointly; next homotheticity (3.8) and finally Hicks-neutrality (3.9) were tested and imposed. The results at each stage are given as C - H in tables 1-3, with the asymptotic test statistics at each stage being reported below.

3SLS TEST STATISTICS

1964 Q3 to 1979 Q2

Stage	Restriction	$\varepsilon'XX'\varepsilon$*	No. of Restrictions: r	Test Statistic ($\sim\chi^2_r$)	$\chi^2_{r,0.05}$
B	Unrestricted	30.3818	-	-	-
C	Additivity	44.6760	4	14.2942	9.4877
D	Additivity and homogeneity	47.4413	3	2.7653	7.8147
E	Additivity and symmetry	47.4179	3	2.7419	7.8147
F	Additivity, homogeneity and symmetry	47.4469	6	2.7709	12.5916
G	Additivity, homogeneity symmetry and homotheticity	54.1995	3	6.7526	7.8147
H	Additivity, homogeneity, symmetry, homotheticity and Hicks-neutral technical change	96.0354	3	41.8359	7.8147

* Sum of squared transformed residuals. The difference between these statistic calculated for both restricted and unrestricted estimates is equal to that calculated by Theil (1971), p.524, eq^n 6.15.

The additivity restriction (3.5) was just rejected by the data at the 1 per cent level, but in view of its importance, particularly in making other restrictions tractable, it was imposed. All further restrictions other than Hicks-neutral technical change were accepted easily by the data. During the imposition of the restrictions (3.5) - (3.9) certain of the less well determined coefficients had highly variable values at intermediate stages. The imposition of Hicks-neutral technical change substantially altered some elasticities, causing some sign changes. The table below gives the steady-state own-price and cross-price elasticities for each of the three factors, with all restrictions except Hicks-neutral technical change imposed.

Matrix of Elasticities

		K	E	L
Capital	K	-0.2011	0.2569	0.2820
		(3.826)	(3.465)	(3.775)
Energy	E		-0.1861	0.1773
			(6.313)	(2.701)
Labour	L			-0.0925
				(4.637)

All elasticities are correctly signed, and fairly well determined. The cross-elasticity between labour and capital, at 0.28, is the highest of the elasticities, all being substantially less than the unitary elasticity implied by the Cobb-Douglas form commonly used to describe UK manufacturing production. The low cross-elasticity between energy and capital, at 0.26, may weaken Berndt and Wood's (1979) argument of complementarity, discussed above.

One very interesting feature is the own-price elasticity for labour, which is near zero. This result supports those of e.g. OECD (1982a), Hazeldine (1977) and Hammond and Asteraki (1983) using more restrictive models, which also fail to identify any own-price effects on the demand for labour.

Full details for each equation at stage G are given in table 4.

The model as at stage G incorporates restrictions on the long-run properties of the equations. It was thought that it might be desirable to test additionally the imposition of homogeneity, symmetry and homotheticity in the short run:

$$\sum_j \beta_{ijk} = 0 \quad \forall\, i,k \qquad - \text{ homogeneity} \qquad (4.1)$$

$$\beta_{ijk} = \beta_{jik} \quad \forall\, i,j,k \quad i \neq j \qquad - \text{ symmetry} \qquad (4.2)$$

$$\gamma_{ik} = \quad \forall\, i,k \qquad - \text{ homotheticity} \qquad (4.3)$$

All three restrictions were conclusively rejected at the 1 per cent level.

Actual values and static predictions over the period 1964Q2 - 1979Q2 for each factor share are shown in charts 1-3.

In order to investigate whether this model is capable of tracking the behaviour of manufacturing industry after the oil price shock of 1973/74, two approaches were used. The first attempted to follow several single equation studies, particularly of the demand for labour, by adding a separate time trend to stage G to proxy an exogenous "productivity slowdown" from 1974 Q1. [See, e.g. Hammond and Asteraki]. The hypothesis that the coefficients on the split trend were zero was easily accepted by the data [$\chi^2_3 = 0.885$ 10% significance level: 6.251]. This result does not deny the possibility of lower productivity growth after 1974 Q1 [this being given by α_T and β_T in equation (2.6), which are not estimated in this system], but does indicate that it did not confine its effects to one factor, namely labour, as has been suggested.

The second approach involved using the equations as estimated over the period 1964 Q3 - 1979 Q2 in a dynamic simulation over the period from 1974 Q1 and examining the tracking performance. The attached charts 4-6 show that for most of the period, all three equations tracked very well. Particular problems were experienced during early 1974, due to the effects of the three-day week, when labour productivity rose dramatically for the duration of the emergency and to the extremely sharp rise in oil prices. Further research might take account of this with a special event dummy. Up to 1979 all three equations followed turning points closely with little error in level. From 1979, however, there was considerable breakdown, which is discussed below.

As a further test, the model was used in a dynamic simulation over the period 1979 Q3 - 1981 Q4. As mentioned above, it was thought that this relatively unrestricted model might help explain the sharp movements in factor shares during the current recession in terms of movements in relative factor prices. In fact over this period and the preceding few quarters, the main movements; a rise in the price of energy relative to labour, and a general rise in the user-cost of capital relative to those of the other two factors, were mainly dominated by sharp oscillations in the growth of the user cost of capital.

Looking at the forecasting behaviour of the model, shown on charts 1-3, it can be readily seen that the share of energy was substantially overpredicted: the response to the price rises of 1979/80 were greater than expected. The share of capital was generally underpredicted, and that of labour overpredicted in common with many single equation models of investment and employment.

The poor forecasting performance of the model over the period from mid-1979 is unsurprising, given that factor shares moved in the same direction as factor prices.

5. CONCLUSIONS

This paper has used a three-factor 'translog' cost function incorporating a dynamic adjustment process and estimated on quarterly data to investigate substitution technologies in UK manufacturing. The data used included a conventional series for capital 'corrected' for accelerated scrapping by the use of the valuation ratio. The model fitted the data well, and the theoretically desirable restrictions of additivity, homogeneity, symmetry and homotheticity were all easily accepted by the data. Hicks-neutral technical change was rejected, however. The estimated own-price and cross-price elasticities were right signed, and generally well determined. In particular, the own-price elasticity of labour was close to zero whilst all other elasticities were substantially less than unity.

When used in a dynamic simulation, the model tracked the period 1974 Q1 - 1979 Q2 well, but broke down from 1979 Q3 onward, in common with many single equation factor demand equations. The simulation from 1974 Q1 suggested that the use of this relatively unrestricted model with energy included as a separate factor removed the problem of a slowdown in the growth of labour productivity (as opposed to all factor productivity, which was not determined) apparent in many single equation studies. This indicates that relative price movements were determinants of the movements in factor shares in the period from 1974 Q1.

The model was less successful in explaining the current recession in terms of relative price movements, particularly as these were generally in the same direction as factor substitution. Thus we are still unable to explain post-1979 behaviour in terms of past relationships.

The estimates of price elasticities in this model, suggest limited possibilities of substitution between factors. This is especially true of labour, where the own-price elasticity is very close to zero. This would appear to indicate that the current and high level of unemployment is not a direct consequence, as some commentators have suggested, of labour 'pricing itself out of work' through the large rise in unit labour costs in the three years to 1980. In fact, between 1978 and the end of 1981, the measure of the user-cost of capital used in this study has generally risen faster than the costs of other factors, and so one might have expected substitution towards labour, if anything, over this period. As it is, the argument that unemployment may be reduced as a direct consequence of reducing the growth of unit labour costs is not supported to any great extent by this work, although the effects of any associated gains in international competitiveness would have favourable effects on output and thus on the demand for labour.

TABLE 1

1964 Q3 to 1979 Q2

CAPITAL

(mean = 0.280194, standard deviation = 11.426%)

Stage	Model	Steady state coefficients *					Price Elasticities *		
		β_{KK}	β_{KE}	β_{KL}	γ_{K}	δ_{K}	η_{KK}	σ_{KE}	σ_{KL}
A	OLS unrestricted	0.1298	0.01360	-0.3016	-0.1595	0.005528	-0.2567	1.6363	-0.6729
		(4.328)	(0.170)	(1.509)	(0.504)	(1.079)	(2.400)	(0.436)	(0.607)
B	3SLS unrestricted	0.1303	0.03694	-0.3815	-0.2962	0.007617	-0.2547	2.7284	-1.1159
		(4.081)	(0.432)	(1.786)	(0.914)	(1.402)	(2.234)	(0.682)	(0.942)
C	Additivity	0.1301	0.04796	-0.4151	-0.3346	0.008364	-0.2555	3.244	-1.3022
		(3.839)	(0.528)	(1.829)	(0.979)	(1.452)	(2.112)	(0.763)	(1.035)
D	Additivity and homogeneity	0.1486	-0.02034	-0.1283	-0.0940	0.000773	-0.1893	-0.0484	0.2884
		(4.751)	(0.268)	(1.357)	(0.326)	(0.422)	(1.695)	(0.014)	(0.550)
E	Additivity and symmetry	0.1498	-0.01467	-0.1351	-0.0866	0.000792	-0.1851	0.3137	0.2506
		(5.489)	(5.456)	(5.385)	(0.323)	(0.435)	(1.900)	(2.494)	(1.801)
F	Additivity, homogeneity and symmetry	0.1498	-0.01467	-0.1351	-0.0861	0.000789	-0.1852	0.3138	0.2056
		(5.510)	(5.464)	(5.423)	(0.322)	(0.436)	(1.909)	(2.499)	(1.814)
G	Additivity, homogeneity, symmetry and homotheticity	0.1453	-0.01588	-0.1295	-	0.000303	-0.2011	0.2569	0.2820
		(9.871)	(10.026)	(9.610)		(0.432)	(3.826)	(3.465)	(3.775)
H	Additivity, homogeneity, symmetry, homotheticity and Hicks-neutral technical change	0.1323	-0.02295	-0.1094	-	-	-0.2475	-0.0737	0.3934
		(8.238)	(14.492)	(7.449)			(4.318)	(0.995)	(4.831)

* Figures in brackets are asymptotically normally distributed test statistics, calculated using the method described by Patterson and Ryding (1982) and equations (2.15) and (2.18), noting that the Wald test statistic is distributed as the square of a normal distribution.

TABLE 2

1964 Q3 to 1979 Q2

ENERGY

(mean = 0.076 2755, standard deviation = 2.121%)

Stage	Model	β_{EK}	β_{EE}	β_{EL}	γ_E	δ_E	σ_{EK}	η_{EE}	σ_{EL}
		Steady state coefficients					Price Elasticities		
A	OLS unrestricted	-0.01301	0.04744	-0.011336	0.001689	-0.000311	0.3915	-0.3017	0.7691
		(4.004)	(5.426)	(0.511)	(0.057)	(0.572)	(2.576)	(2.632)	(1.700)
B	3SLS unrestricted	-0.01277	0.04665	-0.009276	0.001578	-0.000343	0.4025	-0.3121	0.8110
		(4.162)	(5.673)	(0.451)	(0.055)	(0.672)	(2.804)	(2.895)	(1.935)
C	Additivity	-0.012435	0.04566	-0.006392	0.001277	-0.000401	0.4184	-0.3251	0.8698
		(3.716)	(5.091)	(0.285)	(0.041)	(0.720)	(2.673)	(2.764)	(1.900)
D	Additivity and homogeneity	-0.01456	0.05353	-0.038969	-0.025027	0.000453	0.3186	-0.2219	0.2061
		(4.824)	(7.285)	(4.273)	(0.920)	(2.603)	(2.256)	(2.304)	(1.109)
E	Additivity and symmetry	-0.01467	0.05282	-0.037884	-0.025667	0.000443	0.3137	-0.2313	0.2282
		(5.456)	(15.834)	(7.303)	(0.998)	(2.440)	(2.494)	(5.288)	(2.159)
F	Additivity, homogeneity and symmetry	-0.01467	0.05303	-0.038365	-0.025708	0.000452	0.3138	-0.2285	0.2184
		(5.464)	(17.204)	(8.952)	(1.002)	(2.614)	(2.499)	(5.654)	(2.501)
G	Additivity, homogeneity, symmetry and homotheticity	-0.01588	0.05626	-0.04038	-	0.000328	0.2569	-0.1861	0.1773
		(10.026)	(25.023)	(12.529)		(4.537)	(3.465)	(6.313)	(2.701)
H	Additivity, homogeneity, symmetry, homotheticity and Hicks-neutral technical change	-0.02295	0.04541	-0.022466	-	-	-0.07371	-0.3283	0.5423
		(14.492)	(11.791)	(5.029)	-	-	(0.995)	(6.503)	(5.959)

TABLE 3

1964 Q3 to 1979 Q2

LABOUR

(mean = 0.643530, standard deviation = 9.404%)

Stage	Model	β_{LK}	β_{LE}	β_{LL}	γ_L	δ_L	σ_{LK}	σ_{LE}	η_{LL}
		Steady state coefficients					Price Elasticities		
A	OLS unrestricted	-0.1173	-0.05909	0.3074	0.1598	-0.005111	0.3496	-0.2038	0.1212
		(4.232)	(0.799)	(1.675)	(0.536)	(1.089)	(2.275)	(0.135)	(0.425)
B	3SLS unrestricted	-0.1181	-0.08161	0.3853	0.2968	-0.007175	0.3449	-0.6626	0.2423
		(4.018)	(1.038)	(1.966)	(0.984)	(1.438)	(2.115)	(0.414)	(0.795)
C	Additivity	-0.1177	-0.09362	0.4215	0.3333	-0.007962	0.3474	-0.9073	0.2985
		(3.786)	(1.126)	(2.030)	(1.055)	(1.509)	(2.015)	(0.535)	(0.925)
D	Additivity and homogeneity	-0.1341	-0.03319	0.1673	0.1190	-0.001226	0.2564	0.3238	-0.0965
		(4.670)	(0.477)	(1.932)	(0.445)	(0.726)	(1.610)	(0.229)	(0.718)
E	Additivity and symmetry	-0.1351	-0.03788	0.1726	0.1125	-0.001231	0.2506	0.2282	-0.0883
		(5.385)	(7.303)	(7.104)	(0.451)	(0.730)	(1.801)	(2.159)	(2.337)
F	Additivity, homogeneity and symmetry	-0.1351	-0.03836	0.1735	0.1118	-0.001241	0.2506	0.2184	-0.0869
		(5.423)	(8.952)	(7.493)	(0.451)	(0.743)	(1.814)	(2.501)	(2.415)
G	Additivity, homogeneity, symmetry and homotheticity	-0.1295	-0.04038	0.1698	-	-0.000630	0.2820	0.1773	-0.0925
		(9.610)	(12.529)	(13.224)	-	(0.984)	(3.775)	(2.701)	(4.637)
H	Additivity, homogeneity, symmetry and Hicks-neutral technical change	-0.1094	-0.02247	0.1318	-	-	0.3934	0.5423	-0.1516
		(7.449)	(5.029)	(9.328)	-	-	(4.831)	(5.959)	(6.902)

TABLE 4 1964 Q3 to 1979 Q2

Results for model with additivity and long-run homogeneity, symmetry and homotheticity imposed

DEPENDENT VARIABLE: Capital S.E.E. = 6.448% D.W. = 2.888

Coefficients on independent variables*

Lag	S_K	ln p_K	ln p_E	ln p_L	ln Q	ln T	intercept
0		0.1319 (6.667)	0.1072 (1.529)	-0.3424 (1.754)	-0.1688 (1.231)	0.0001365 (0.430)	-0.01781 (0.505)
1	0.9249 (11.599)	-0.1314 (4.039)	-0.3689 (3.007)	0.2865 (0.890)	0.3379 (1.678)		
2	-0.3759 (5.358)	0.0650 (3.003)	0.2546 (3.631)	-0.0025 (0.013)	-0.1691 (1.212)		

DEPENDENT VARIABLE: Energy S.E.E. = 2.118% D.W. = 2.409

Coefficients on independent variables*

Lag	S_E	ln p_K	ln p_E	ln p_L	ln Q	ln T	intercept
0		-0.01516 (7.983)	0.06125 (9.013)	-0.03194 (1.699)	0.005534 (0.424)	0.0001477 (4.175)	-0.1197 (7.750)
1	0.9249 (11.599)	0.01340 (4.143)	-0.05079 (4.035)	0.00746 (0.241)	-0.000498 (0.026)		
2	-0.3759 (5.358)	-0.00541 (2.457)	0.01491 (1.903)	0.00628 (0.346)	-0.005036 (0.375)		

DEPENDENT VARIABLE: Labour S.E.E. = 2.581% D.W. = 2.911

Coefficients on independent variables*

Lag	S_L	ln p_K	ln p_E	ln p_L	ln Q	ln T	intercept
0		-0.1167 (6.443)	-0.1684 (2.625)	0.3743 (2.092)	0.1632 (1.298)	-0.0002843 (0.969)	0.5885 (7.407)
1	0.9249 (11.599)	0.1180 (3.971)	0.4197 (3.725)	-0.2939 (0.997)	-0.3374 (1.825)		
2	-0.3759 (5.358)	-0.0596 (3.020)	-0.2695 (4.179)	-0.0038 (0.0218)	0.1741 (1.362)		

* t statistics in brackets

Data appendix

All data are seasonally adjusted unless specified otherwise. Graphs of the user costs and shares of factors are attached.

Output Q

The quarterly series for net manufacturing output at factor cost is derived from the production index adjusted for sales from stock (source: CSO: Economic Trends p28) based on 1975 value added, to which is added the series for energy consumption, E, described below.

Labour L

This is taken to be the total number of hours worked per quarter: employment, N, multiplied by average hours, H. Employment is derived from the series for GB (source: Dept of Employment: Employment Gazette T 1.2) with an approximate adjustment for Northern Ireland, derived from the Employment Censuses. No adjustment is made for the self employed or for the effects of special employment measures. The index of average hours for operatives in GB (Dept of Employment: Employment Gazette T1.12) multiplied by a base figure for 1962, is used as a proxy for the average hours of all workers in UK manufacturing.

Fixed Capital K

A quarterly series for the net fixed capital stock of manufacturing industry is derived by:

$$K_t = (1 - \delta) K_{t-1} + I_t$$

where δ = depreciation rate, chosen to be 0.5% per quarter to ensure a non-declining capital stock;

I = gross fixed investment at 1975 prices (CSO: Economic Trends Annual Supplement, Table 18) plus finance leasing to manufacturing industry at 1975 prices (source: Bank of England)

using a base figure for capital stock and total leased assets at end 1975 (CSO: National Income and Expenditure, Table 11.12, and Bank of England). This is then multiplied by the square root of the valuation ratio, q, derived after Jenkinson (1981).

Energy Consumption E

The quarterly series is interpolated from an annual series for consumption at 1975 prices based on volume data for final consumption of individual energy types using 1975 expenditure weights (Dept of Energy: Digest of United Kingdom Energy Statistics, Tables 9 and 12). The interpolated series x_i is derived from the annual series X_j by:

$$\min \sum_{i=2}^{4n} (\Delta x_i - \Delta x_{i-1})^2$$

subject to

$$\sum_{i=4j-3}^{4j} x_i = X_j \quad j=1,\ldots,n$$

User Cost of Labour P_L

This is given by

$$P_L = \frac{W.n}{H}$$

where:

W = average earnings per man, obtained from the index of average earnings in manufacturing (Dept of Employment: Employment Gazette T.5.1) and 1975 average wage and salaries in manufacturing (sources: Dept of Employment: Employment Gazette, T.5.1 and CSO: National Income and Expenditure, Table 3.3;

n = proxy for national insurance costs per man,

$$= \frac{YWS + YEC + YECS}{YWS}$$

where:

YWS = income from wages and salaries in the whole economy;

YEC = employers' national insurance and other contributions;

YECS = national insurance surcharge.

(sources: CSO: Economic Trends, p10

User Cost of Capital P_K

This is derived using the formulae proposed by Jorgenson (1963), Nickell (1978) and Jenkinson (1981):

$$P_K = \frac{P_I \, (1-a) \, r}{1 - \tau}$$

where:

$r = \delta + \rho + r^* (1 - \tau) - \hat{\pi}_I$: post-tax real interest rate;

ρ = risk premium, taken to be 3 per cent per quarter. A high value for ρ was necessary in order to ensure positive real interest rates during the period of high inflation and relatively low nominal rates in 1975-77;

r^* = nominal rate of interest, taken to be the rate on five year British government stock (source: CSO: Financial Statistics, Table 13.5;

τ = corporation tax rate;

p_I = price deflator for gross fixed investment in manufacturing and finance leasing to manufacturing (CSO: Economic Trends Annual Supplement, Table 18 and Bank of England, unadjusted for seasonal variation.)

a = present value of investment and depreciation allowances;

$\hat{\pi}_I$ = expected rate of inflation in p_I, taken as being a weighted moving average.

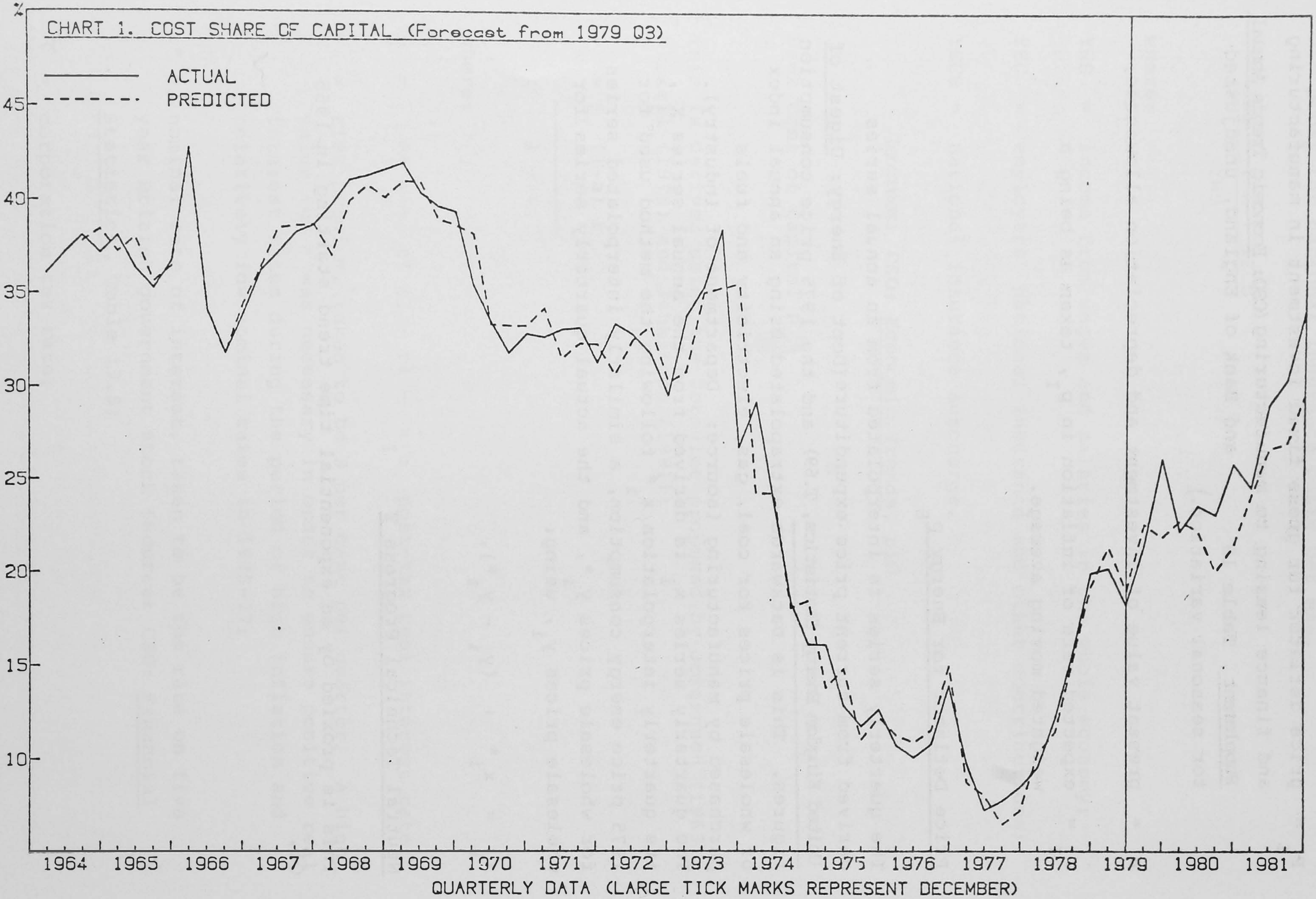
Price Deflator for Energy p_E

The quarterly series is interpolated from an annual series derived from current price expenditure (Dept of Energy: Digest of United Kingdom Energy Statistics, T.69) and the 1975 price consumption figures. This is backwards extrapolated using an annual index of wholesale prices for coal, gas, electricity and fuels purchased by manufacturing (source: Department of Industry). The quarterly series x_i is derived from the annual series X_j, its quarterly interpolation x_i^* following the method used for 1975 price energy consumption, a similarly interpolated series for wholesale prices y_i^*, and the actual quarterly series for wholesale prices y_i, using:

$$x_i = x_i^* + (y_i - y_i^*).$$

Neutral Technical Progress T

This is proxied by an exponential time trend starting in 1955 Q1.


%
CHART 1. COST SHARE OF CAPITAL (Forecast from 1979 Q3)
ACTUAL
PREDICTED
45
40
35
30
25
20
15
10
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
QUARTERLY DATA (LARGE TICK MARKS REPRESENT DECEMBER)

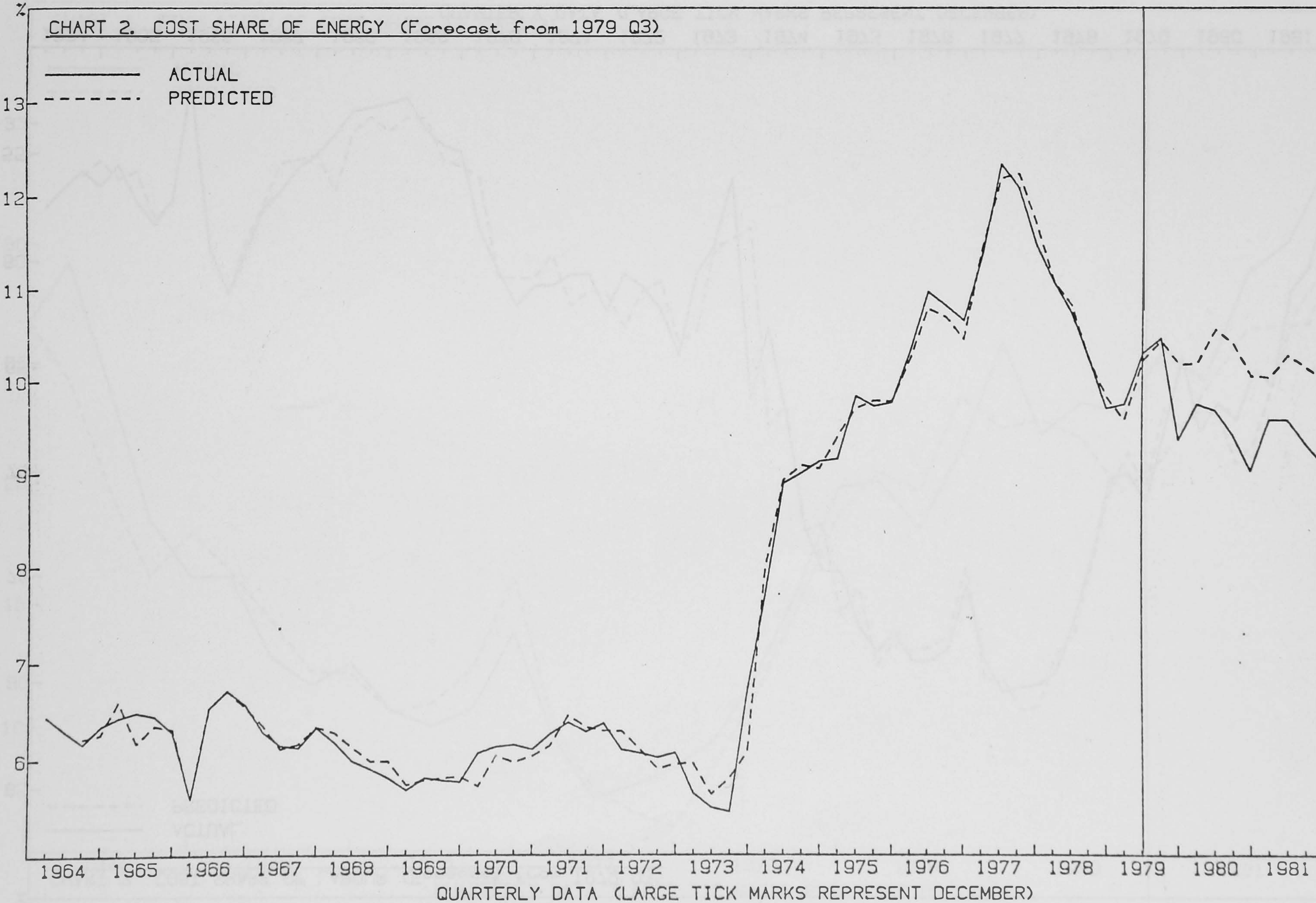
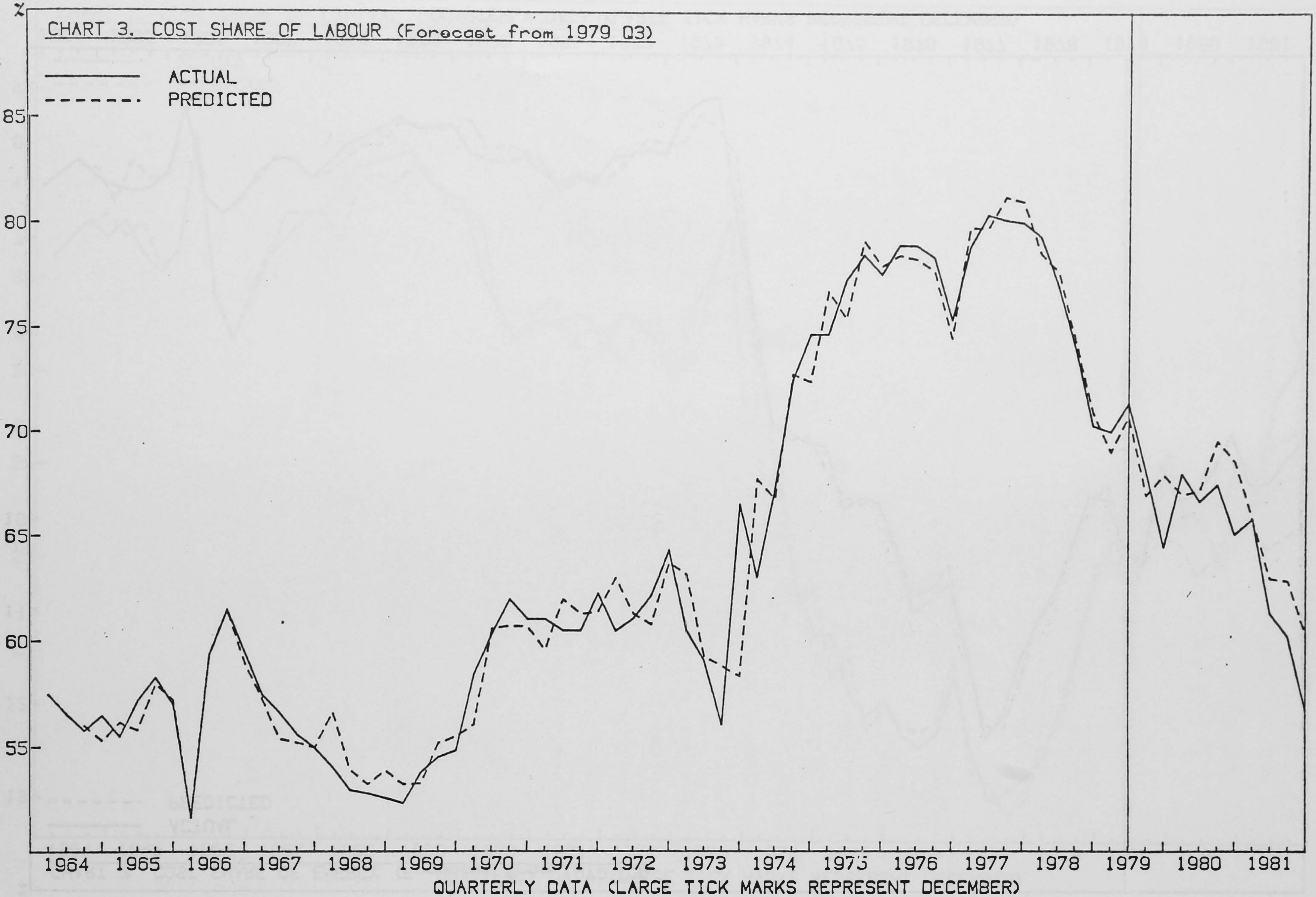


CHART 2. COST SHARE OF ENERGY (Forecast from 1979 Q3)
ACTUAL
PREDICTED
%
13
12
11
10
9
8
7
6
1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981
QUARTERLY DATA (LARGE TICK MARKS REPRESENT DECEMBER)


%
CHART 3. COST SHARE OF LABOUR (Forecast from 1979 Q3)
ACTUAL
PREDICTED
85
80
75
70
65
60
55
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
QUARTERLY DATA (LARGE TICK MARKS REPRESENT DECEMBER)

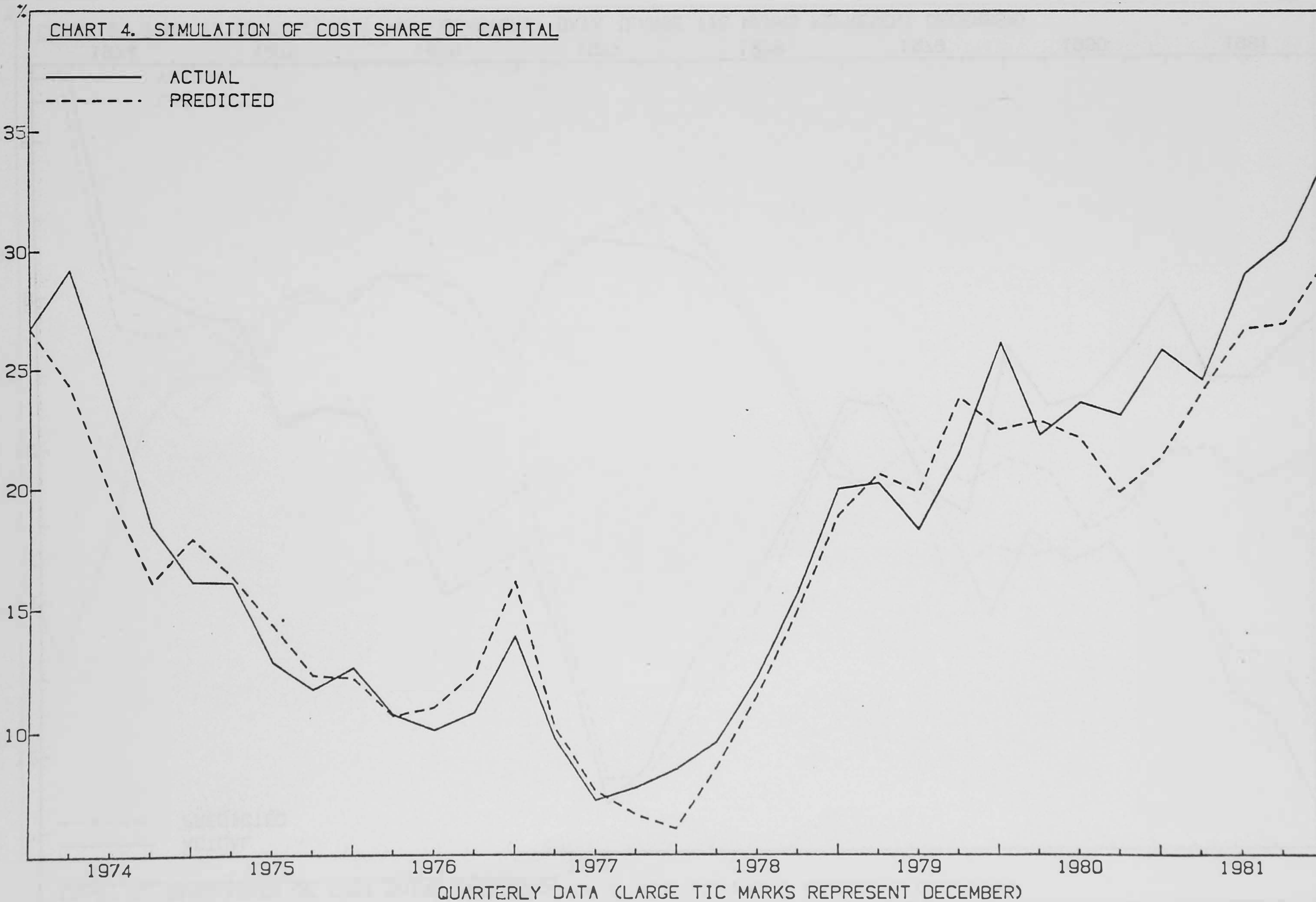


CHART 4. SIMULATION OF COST SHARE OF CAPITAL

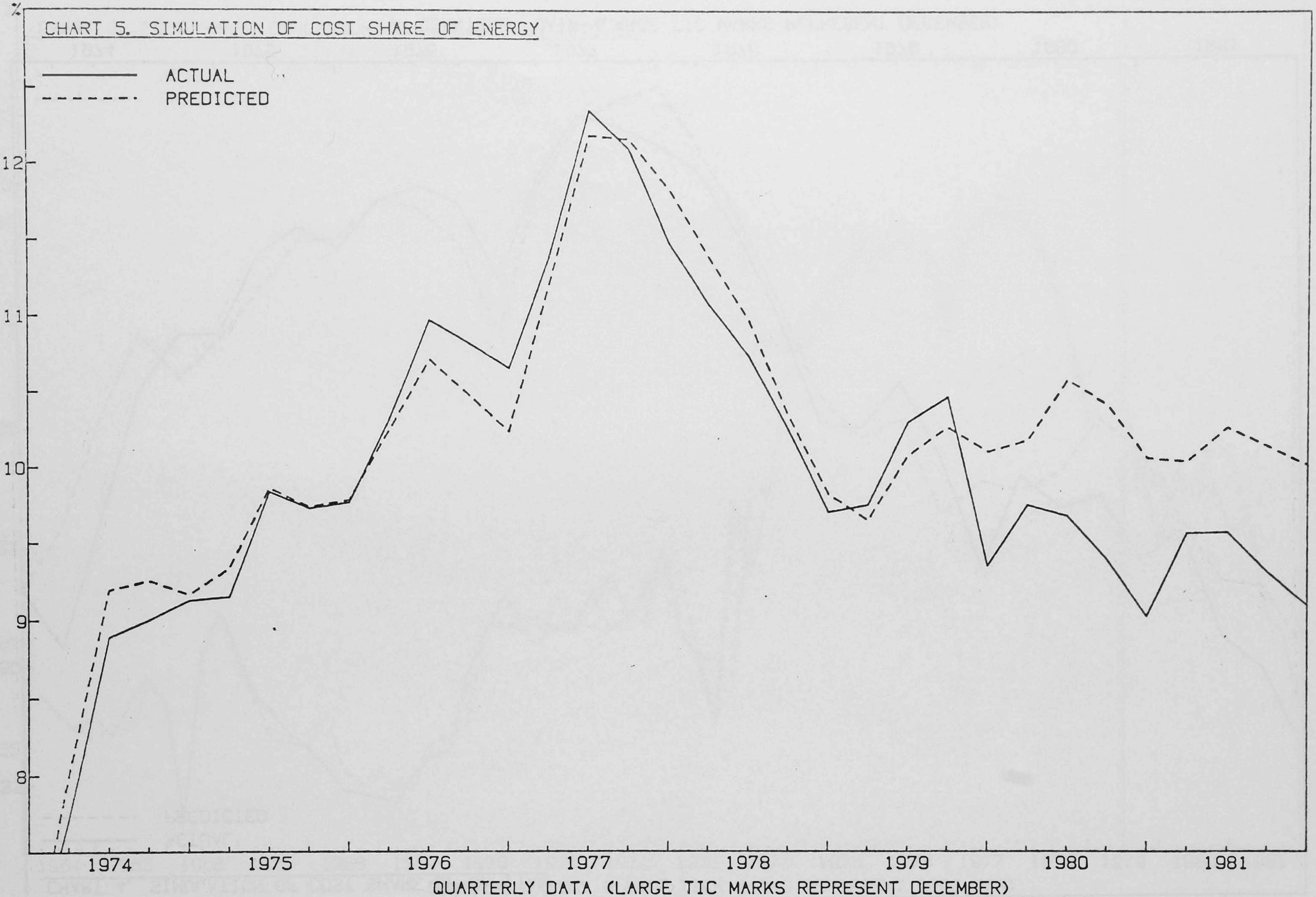

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CHART 5. SIMULATION OF COST SHARE OF ENERGY
ACTUAL
PREDICTED
12
11
10
9
8
1974
1975
1976
1977
1978
1979
1980
1981
QUARTERLY DATA (LARGE TIC MARKS REPRESENT DECEMBER)

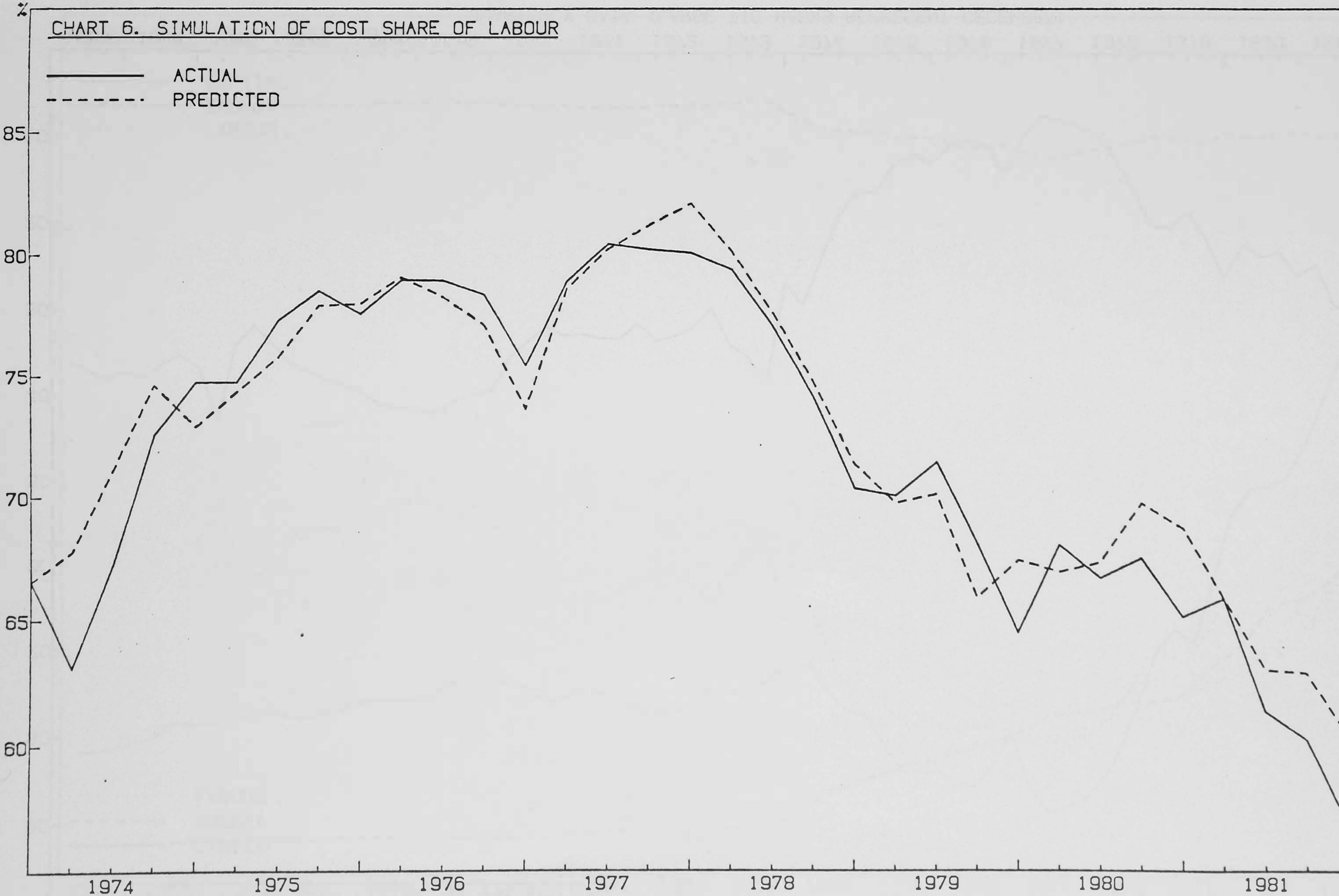
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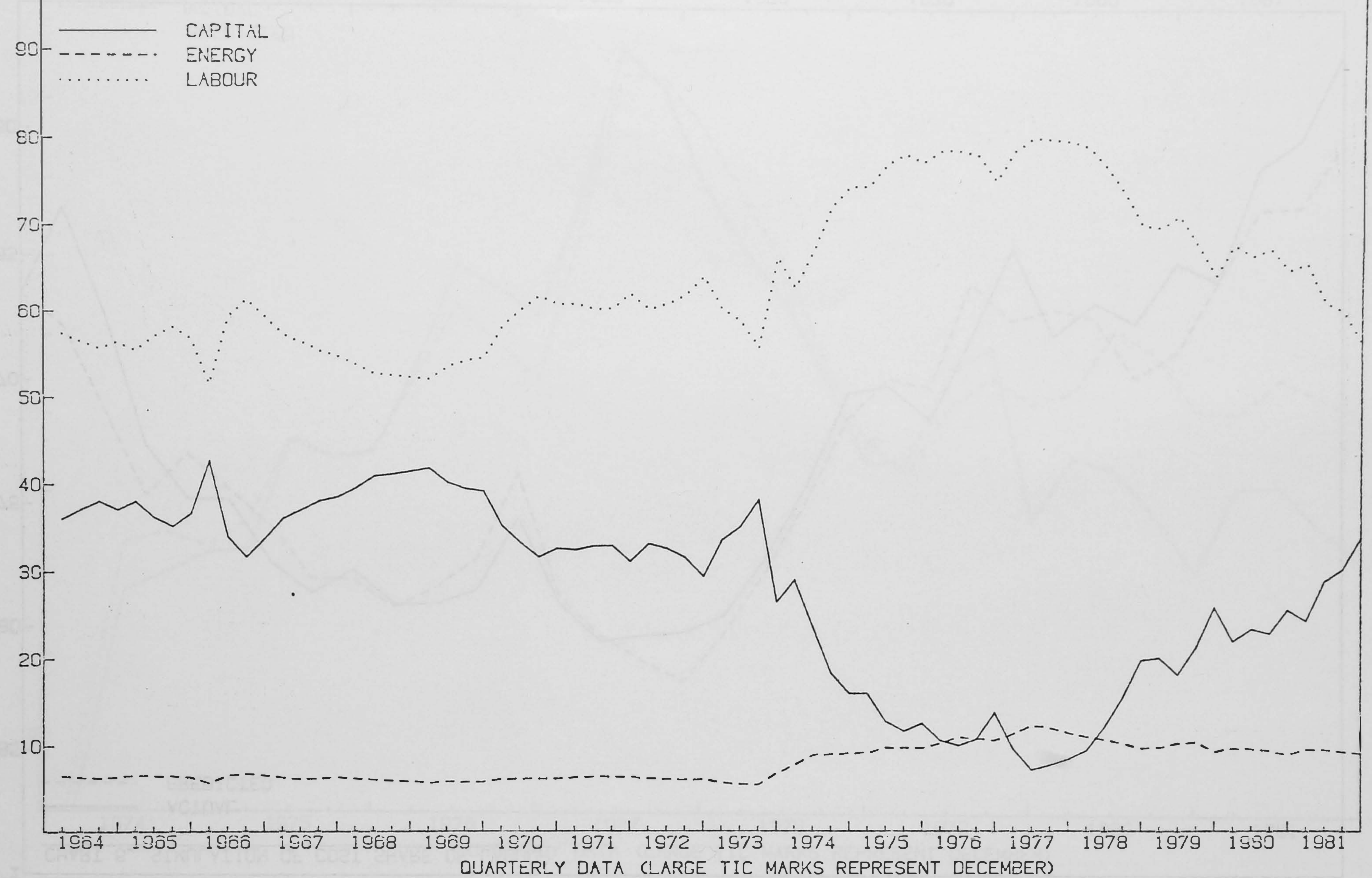
CHART 6. SIMULATION OF COST SHARE OF LABOUR
ACTUAL
PREDICTED
85
80
75
70
65
60
1974
1975
1976
1977
1978
1979
1980
1981
QUARTERLY DATA (LARGE TIC MARKS REPRESENT DECEMBER)

CHART 7. COST SHARES OF FACTORS OF PRODUCTION



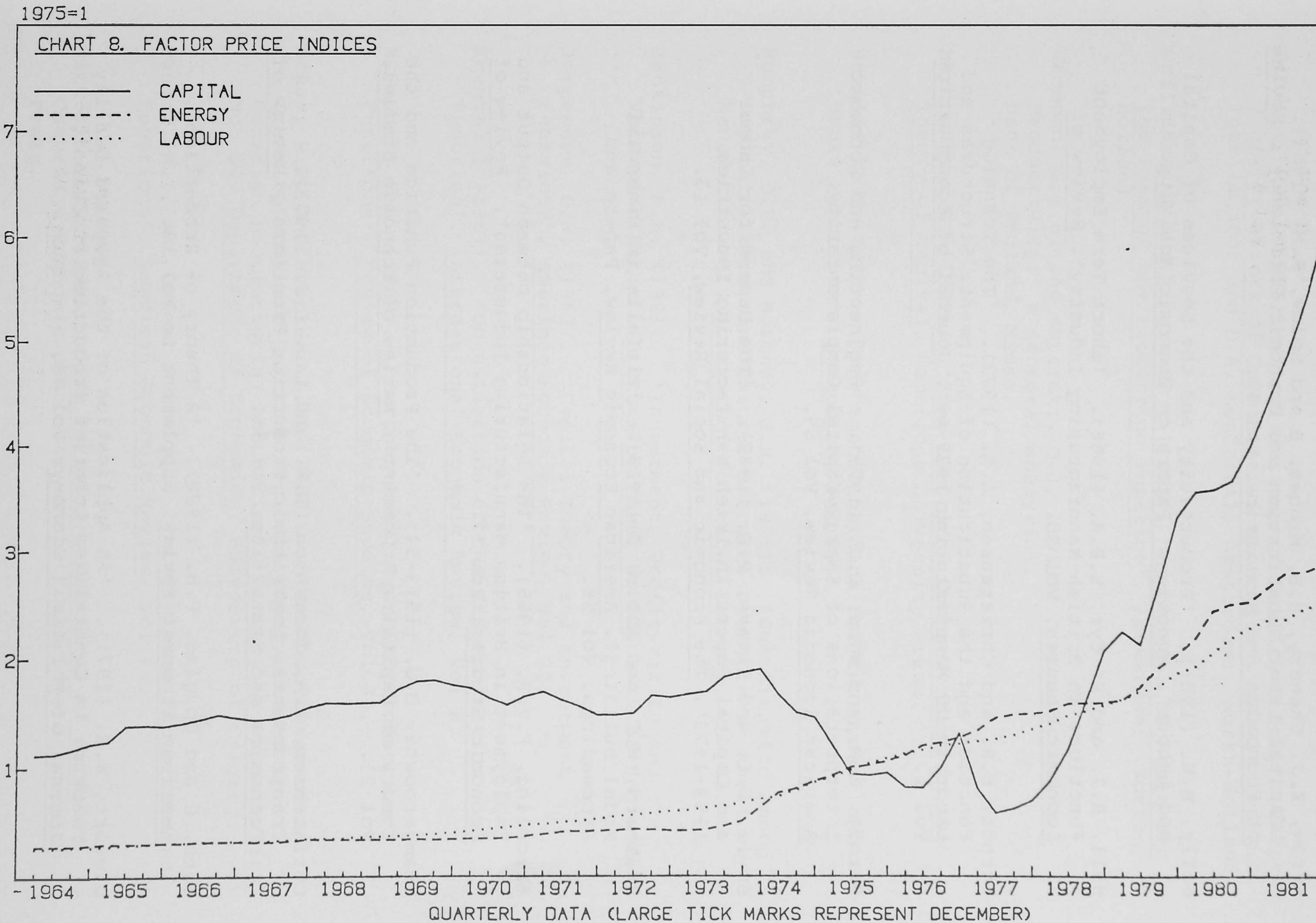


CHART 8. FACTOR PRICE INDICES

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Bank of England Discussion Papers

	Title	Author
1-5, 8, 11-14, 16 & 17	*A list of these papers can be found in the December 1981* Bulletin, *or can be obtained from the Bank. These papers are now out of print, but photocopies can be obtained from University Microfilms International (see below).*	
6	'Real' national saving and its sectoral composition	C T Taylor A R Threadgold
7	The direction of causality between the exchange rate, prices and money	C A Enoch
9	The sterling/dollar rate in the floating rate period: the role of money, prices and intervention	I D Saville
10	Bank lending and the money supply	B J Moore A R Threadgold
15	Influences on the profitability of twenty-two industrial sectors	N P Williams
18	Two studies of commodity price behaviour: Interrelationships between commodity prices Short-run pricing behaviour in commodity markets	 Mrs J L Hedges C A Enoch
19	Unobserved components, signal extraction and relationships between macroeconomic time series	T C Mills
20	A portfolio model of domestic and external financial markets	C B Briault Dr S K Howson
21	Deriving and testing rate of growth and higher order growth effects in dynamic economic models	K D Patterson J Ryding

Technical Series

	Title	Author
1	The consumption function in macroeconomic models: a comparative study*	E P Davis
2	Growth coefficients in error correction and autoregressive distributed lag models	K D Patterson
3	Composite monetary indicators for the United Kingdom; construction and empirical analysis	T C Mills
4	The impact of exchange rate variability on international trade flows	G Justice
5	Trade in manufactures	A C Hotson K L Gardiner
6	A recursive model of personal sector expenditure and accumulation	E P Davis
7	A dynamic 'translog' model of substitution technologies in UK manufacturing industry	D J Asteraki

Papers presented to the Panel of Academic Consultants[a]

	Title	Author
8	International monetary arrangements the limits to planning*	P M Oppenheimer
9	Institutions in the financial markets: questions, and some tentative answers*	M V Posner
10	The arguments for and against protectionism*	M Fg Scott The Hon W A H Godley
14	The usefulness of macroeconomic models*	Prof W H Buiter T F Cripps Prof Angus Deaton Prof A P L Minford M V Posner
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