Bank of England

Discussion Papers

Technical Series

No 32

A note on the estimation of GARCH-M models using the Kalman Filter

by

S G Hall

June 1990

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The object of this Technical Series of Discussion Papers is to give wider circulation to econometric research work predominantly in connection with revising and updating the various Bank models, and to invite comment upon it; any comments should be sent to the author at the address given below.

The views expressed in this paper are those of the author and do not necessarily represent those of the Bank of England. The author would like to thank Robert F Engle, Andrew Harvey and Theo Nijman for helpful comments. All errors remain the author's own.

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ABSTRACT

Following the important work of Engle(1982), which introduced the idea of estimation subject to an error process which had an auto-regressive variance, the usefulness of the Auto-Regressive Conditional Heteroskedastic (ARCH) model has become increasingly apparent. Important further developments in the area include the suggestion of Engle, Lilien and Robins(1987) that some function of the conditional variance could be included in the structural equation being estimated (the ARCH in mean or ARCH-M model) and the generalization of the Arch process proposed by Bollerslev(1986) (the GARCH and GARCH-M models).

The ARCH or GARCH equation is assumed not to be stochastic. This is clearly an extreme assumption, even if the true equation is exact we might suspect that it is at least subject to measurement error. When this assumption is dropped and we consider a stochastic GARCH process the situation becomes much more complex however.This note shows how the stochastic GARCH-M (SGARCH-M) model may be put into state space form and estimated by the Kalman Filter, it shows that conventional GARCH-M estimation is a special case of SGARCH-M estimation when the GARCH equation is non-stochastic and there is no uncertainty about the initial conditions. The Kalman Filter provides a useful way of relaxing these two implausible assumptions.

1. INTRODUCTION

Following the important work of Engle(1982), which introduced the idea of estimation subject to an error process which had an auto-regressive variance, the usefulness of the Auto-Regressive Conditional Heteroskedastic (ARCH) model has become increasingly apparent. Indeed the evidence of ARCH processes in areas such as financial modelling now seems overwhelming. Important further developments in the area include the suggestion of Engle, Lilien and Robins(1987) that some function of the conditional variance could be included in the structural equation being estimated (the ARCH in mean or ARCH-M model) which allows a much more satisfactory treatment of risk terms and the generalization of the Arch process proposed by Bollerslev(1986) which allowed a more parsimonious parameterisation of the model (the GARCH and GARCH-M models).

The ARCH or GARCH equation has a rather interesting status, it is the conditional variance of the structural equation at each point in time yet in the formal exposition it is an exact (non-stochastic) relationship. This is clearly an extreme assumption, even if the true equation is exact we might suspect that it is at least subject to measurement error. When this assumption is dropped and we consider a stochastic GARCH process the situation becomes more complex. In the case of ARCH and GARCH estimation this makes little difference as it is impossible to draw any inference about the conditional variance other than the initial estimate. When the model is extended to an ARCH in mean (ARCH-M) or GARCH in mean (GARCH-M) formulation by explicitly including the conditional variance in the structural equation this is no longer the case. The use of lagged errors which are based on the conditional variance at t-2 or greater means that the conditional variance is not properly conditioned on information at t-1, in the case of the ARCH-M model and the explicit use of the conditional variance at t-2 or greater compounds this problem in the GARCH-M formulation. This arises because when the conditional variance enters the structural equation it is possible to update this estimate of the variance once the full information set at time t becomes available. This updating is exactly analogous to the Kalman Filter updating equations, so that the estimate of the conditional variance at time t+1 is then properly based on the information set at t.

This note shows how the stochastic GARCH-M (SGARCH-M) model may be put into state space form and estimated by the Kalman Filter, it shows that conventional GARCH-M estimation is a special case of SGARCH-M estimation when the GARCH equation is non-stochastic and there is no uncertainty about the initial conditions. The Kalman Filter provides a useful way of relaxing these two implausible assumptions. Most of the analysis will concentrate on the GARCH-M formulation rather than ARCH-M, as the ARCH-M form may be thought of as nested within the more general GARCH-M framework, the results and statement are general to both models.

The next section of this note gives a brief account of the ARCH and GARCH model, section 3 sets out the state space model and the Kalman Filter when the state equation contains some observed components. Section 4 gives the state space representation of the SGARCH-M model and discusses the Kalman Filter estimation and the relationship between the two estimation strategies. Section 5 presents an illustration of the estimation of a simple time series model of inflation using OLS, standard GARCH-M and the new SGARCH-M procedure. Section 6 draws some conclusions.

2. THE GARCH-M MODEL

Engle, Lilien and Robins (1987) suggest an extension of Engle's (1982) ARCH model whereby the conditional first moment of a time series itself becomes a function of the conditional second moment, which follows an ARCH process:

$$y_t = \alpha' x_t + \delta h_t^2 + \epsilon_t \qquad \epsilon_t | \Omega_{t-1} \sim N(0, h_t^2) \qquad (1)$$

$$h_t^2 = \gamma_0 + \sum_{i=1}^{n} \gamma_i \epsilon_{t-i}^2 + \kappa' z_t \qquad (2)$$

where x_t and z_t are vectors of weakly exogenous conditioning variables. Engle, Lilien and Robins(1987) term this kind of model ARCH-in-mean or ARCH-M. Note the h_t^2 is the conditional variance of ϵ_t formed at period t based on the information set (σ-field) of all information up to period t-1, note that by assumption (2) is a non-stochastic equation.

A further extension of the ARCH formulation, which imposes smoother behaviour on the conditional second moments, has been suggested by Bollerslev (1986). In Bollerslev's GARCH formulation, the conditional second moments are functions of their own lagged values as well as the squares and cross-products of lagged forecast errors. Bollerslev did not consider the GARCH-M extension although this is a fairly obvious one which was subsequently used in Bollerslev, Engle and Wooldridge(1988). Thus, for example, the GARCH-M (n, p) formulation of the above model would consist of (1) and

$$h_t^2 = A_0 + \sum_{i=1}^{n} A_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} B_i h_{t-1}^2 \qquad (3)$$

where the B_i and A_i are coefficients.

Stacking all of the parameters of the system into a single vector

$\mu = (\alpha, \delta, A_0, A_1, ..., A_n, B_1, ..., B_p)$

and applying Schweppe's (1965) prediction error decomposition form of the likelihood function, the log-likelihood for a sample of T observations (conditional on initial values) is proportional to

$$L(\mu) = \sum_{t=1}^{T} -\log | h_t^2(\mu) | - \sum_{t=1}^{T} \epsilon_t^2 / h_t^2 \qquad (4)$$

(where we have assumed normality of the forecast errors).

Although the analytic derivatives of (4) can be computed (see Engle, Lilien and Robins, 1987) variable-metric algorithms which employ numerical derivatives are simpler to use and easily allow changes in specification. Under the usual regularity conditions (Crowder, 1976), maximization of (4) will yield maximum likelihood estimates with the usual properties.

The difficulty with the GARCH equation (3) when it is not exact can be most easily appreciated by focusing on the information sets which are being used. Assume that h_0^2 and ϵ_0 are known and that n=p=1 then

$$h_1^2|\Omega_0 = A_0 + A_1\epsilon_0^2 + B_1h_0^2$$

so h_1^2 is properly conditioned on information at period 0. But when we use equation (3) to form an estimate of h_2^2 this is no longer the case, to begin with,

$$\epsilon_1 = y_1 - \alpha' x_1 + \delta(h_1^2|\Omega_0) \qquad (5)$$

So ϵ_1 is based on a mixture of information, most of which is dated at t=1 but the conditional variance is dated at t=0. In the actual use of the Garch equation (3) the lagged term in the conditional variance is also based on information dated at t=0.

$$h_2^2 = A_0 + A_1\epsilon_1^2 + B_1(h_1^2|\Omega_0) \qquad (6)$$

So the estimate of h_2^2 is based on Ω_0 and partly on y_1 and x_1. In order to fully utilize the information set Ω_1 we should update our estimate of h_1^2 when observations on y_1 and x_1 become available and also derive a new estimate of the error ϵ_1 based on Ω_1. These updated estimates should then be used in the GARCH equation.

The next two section considers how we might do this.

3. THE STATE SPACE FORM AND THE KALMAN FILTER

In this section a standard state space formulation is presented with the appropriate Kalman Filter equations for the univariate case,following Harvey(1987). Let

$$y_t = \delta' z_t + \alpha' x_t + \epsilon_t \tag{7}$$

be the measurement equation, where y_t is a measured variable z_t is the state vector of unobserved variables δ and α are parameters and ϵ_t ~ NID(0,Γ_t) and x_t is fixed. The state equation is then given as,

$$z_t = \Psi z_{t-1} + \beta' w_t + \omega_t \tag{8}$$

Where Ψ and β are parameters, w_t is fixed and ω~ NID(0,Q_t).

The only departure from the standard state space form given in many text books is in the introduction of x_t into the measurement equation and w_t into the state equations. This is not an important elaboration as long as both x_t and w_t are known at time t.

The appropriate Kalman Filter prediction equations are then given by defining $\hat{z}_t$ as the best estimate of z_t based on information up to t and P_t as the covariance matrix of the estimate $\hat{z}_t$, and stating;

$$\hat{z}_{t|t-1} = \Psi \hat{z}_{t-1} + \beta' w_t \tag{9}$$

and

$$P_{t|t-1} = \Psi P_{t-1} \Psi' + Q_t \tag{10}$$

Once the current observation on Y_t becomes available we can update these estimates using the following equations.

$$\hat{z}_t = \hat{z}_{t|t-1} + P_{t|t-1}\delta(y_t-\alpha' x_t-\delta'\hat{z}_{t|t-1})/(\delta' P_{t|t-1}\delta+\Gamma_t) \quad (11)$$

and

$$P_t = P_{t|t-1} - P_{t|t-1}\delta\delta' P_{t|t-1}/(\delta' P_{t|t-1}\delta+\Gamma_t) \quad (12)$$

Equations (9)-(12) then jointly represent the Kalman Filter equations.

If we then define the one step ahead prediction errors as,

$$v_t = y_t - \delta'\hat{z}_{t|t-1} + \beta' w_t$$

Then the concentrated log likelihood function is proportional to,

$$\log(l) = \Sigma \log(f_t) + N\log(\Sigma v_t^2/Nf_t)$$

where $f_t = \alpha' P_{t|t-1}\alpha + \Gamma_t$ and N=T-k where k is the number of periods needed to derive estimates of the state vector.

That is to say the likelihood function may be expressed as a function of the one step ahead prediction errors suitably weighted.

4. The GARCH-M MODEL IN STATE SPACE FORM

A comparison of equations (7) and (1) and (8) and (3) will quickly show that the GARCH-M model is already very close to a state space representation, all we need to do is to reinterpret some of the notation of the state space model. For simplicity I will only consider the GARCH(1,1) model but higher order GARCH models can be treated in an exactly analogous fashion. To begin with, in the state equation (8) we interpret w_t to be a 2x1 vector which is made up of a constant and ϵ^2_{t-1},this equation is then identical to the GARCH equation (3) where we identify z_t with h^2_t. The z_t term in the measurement equation is then seen as the variance term which is identical to the variance term in equation (1), we then only need to make explicit allowance for the structure of the variance of the measurement equation and this is done simply by equating Γ_t with z_t. The SGARCH-M state space model may then be written out as,

$$y_t = \delta' z_t + \alpha x_t + \epsilon_t \quad \epsilon_t \sim NID(0, \hat{z}_{t|t-1}) \tag{13}$$

and

$$z_t = \Psi z_{t-1} + A_0 + A_1 \epsilon^2_{t-1} + \omega_t \quad \omega_t \sim NID(0, Q_t) \tag{14}$$

This model would be identical to the equations (1) and (3) when Q_t=0 and P_0=0, that is when the GARCH equation is non stochastic with certain initial conditions. This can be seen by noting that if Q_t=0 and P_0=0, then the updating equations, (11) and (12), are none operational and $f_t = \Gamma_t = z_t$ and the likelihood function becomes equivalent to (4).

When this is not the case the Kalman Filter equations may be used to estimate the model for known values of the parameters of the model. When any of the parameters or variances are unknown this procedure may be used to define the likelihood function conditional on the unknown parameters and this may be used for maximum likelihood estimation of the stochastic GARCH-M model.

A further point of interest is that the SGARCH-M model also nests the standard ARCH and GARCH model when δ=0. In this case the updating equations (11) and (12) again become non operational and with P_0=0 the likelihood function is again equivalent to the standard GARCH likelihood function (4) (as $f_t=z_t$). So the problem of the dating of the information set only occurs when δ is non zero. Although if P_0 is non-zero the likelihood function is slightly different as the early observations receive less weight because of the uncertainty of the initial conditions.

The use of the standard Kalman Filter does however raise one potential problem with regard to the sign of the conditional variance z_t. Under the assumption that p_0 is non zero and Q_t is normally distributed it is always possible that z_t could be updated to a negative value. In practice this can be seen as extremely unlikely for sufficiently small values of P_0 and Q_t, but it is certainly an important conceptual problem. The solution is of course to assume that ω_t follows some distribution other that the normal one, the log-normal distribution would be a suitable choice, an extended Kalman Filter algorithm could then be defined to cope with this assumption. The standard Kalman Filter also presents certain practical difficulties as the use of diffuse priors in the form of a very large estimate of p_0 will often produce estimates of z_t which become negative in the early periods of the recursion, and so this attractive assumption is not fully operational. An extended Kalman Filter is however computationaly unatractive and would of course still be dependent on the specific distributional assumption made. One solution to the problem is to assume that P_0 follows a truncated normal distribution, thus the filter works in the usual way except when a negative value for z_t would result and then z_t would simply be set to some small (non zero value). This of course can not be justified on the grounds of maximum likelihood estimation but following the argument of Griliches(1983) or Van Praag(1983) it may be viewed as a Quasi- or pseudo-maximum likelihood estimator as long as $E(v_t^2)=f_t$. Thus under this assumption while the estimation will not be fully efficient it will be consistent. This is the estimation technique which will be used below in the example.

This generalization of the standard model has a number of advantages; It forms the expectation of the variance of the measurement equation conditional on information at t-1 during the prediction stage, it then updates it at the updating stage to use information at t. This means that the lagged term in (6) is conditional on information at t-1 (as it was updated after the initial estimate based on information at t-2). This is in contrast to the lagged term in the standard GARCH model which is not updated. The standard Kalman Filter smoothing algorithm can also be used to produce optimal estimates of the time varying variance conditional on the whole sample of data. Neither of these stages are possible in the standard procedure.

5. AN ILLUSTRATION

It has become almost traditional to illustrate ARCH and GARCH processes using the example of inflation, both Engle(1982) and Bollerslev(1986) showed that a simple AR(4) model of GNP inflation for the USA seemed to be a satisfactory model except that there was strong evidence of ARCH or GARCH processes in the residuals. This section will replicate these results for the UK using quarterly data for the GDP deflator over the period 1959Q1 to 1988Q4. The model is estimated in terms of the inflation rate defined as $\pi_t = \log(GDP_t - GDP_{t-1})$. Table 1 gives the estimation results for the three procedures; The OLS results show an absence of serial correlation with sensible parameter estimates but there is clear evidence of non-normality in the residuals as well as strong ARCH effects. The GARCH-M estimates clearly correct for the ARCH effects and even though δ is very small and poorly determined the GARCH correction is clearly important, the scaled residuals still exhibit considerable non normality however and the sum of A_1 and Ψ is obviously greater than unity. The SGARCH-M results find a significant variance for the GARCH equation (Q>0) and the results are rather more plausible in that the A_1 and Ψ now sum to less than unity. The normality test on the scaled residuals also suggests that the residuals are much closer to being normal. The 't' statistics are all rather high in the case of the SGARCH-M model (they were based on the inverse of the Hessian of the likelihood function) and may be misleading if the likelihood function is locally peaked at the maximum. So in order to confirm the significance of the variance of the GARCH equation the Likelihood ratio test for the restriction that Q=0 was also calculated, this gave LR=6.97, which is also highly significant.

Figures 1 and 2 show the estimated variance terms produced by the two procedures, the overall pattern is clearly quite similar with peaks in the variance following the bursts of inflation in 1973 and 1979. The detail is however different with the relative size of the peaks in the SGARCH-M model being less but rather smoother, this is due mainly to the different parameter estimates for the GARCH process which the two procedures yield. The estimate for the overall level of the variance is also rather different in the two models with the SGARCH-M variance being much smaller, due of course to the existence of the non-zero Q variance.

Table 1. RESULTS FOR THE ESTIMATION OF THE GDP INFLATION RATE

	OLS	GARCH-M	SGARCH-M
CONSTANT	0.0043(2.3)	0.0037(3.0)	0.0039(53)
π_{t-1}	0.22(2.4)	0.26(2.8)	0.32(48)
π_{t-2}	0.41(4.3)	0.27(3.4)	0.27(32)
π_{t-3}	0.20(2.2)	0.074(1.1)	0.14(47)
π_{t-4}	-0.05(0.5)	0.03(0.5)	0.011(37)
δ	-	6.2(0.13)	0.005(44)
A_0	-	0.00005(0.02)	0.00002(31)
A_1	-	0.46(11.6)	0.51(250)
Ψ	-	1.13(0.2)	0.18(33)
Q	-	-	0.000054(30)
LB(1)	0.0	0.24	0.31
LB(2)	0.0	0.37	0.67
LB(4)	1.75	2.21	2.56
LB(8)	7.72	3.4	7.19
BJ(2)	54.4	136.4	5.99
LB2(1)	2.9	0.21	0.25
LB2(2)	9.25	0.61	0.46
LB2(4)	13.11	0.90	1.29
LB2(8)	15.7	2.00	5.84

LB(n) is the Ljung-Box test of a random correlogram distributed $\chi^2(n)$, BJ is the Berra-Jarque test of normality in the scaled residuals distributed as $\chi^2(2)$ and LB2 is the Ljung-Box test carried out on the squared residuals.

SGARCH-M statistics are based on the smoothed estimate

FIGURE 1 VARIANCE TERM FOR GARCH-M MODEL

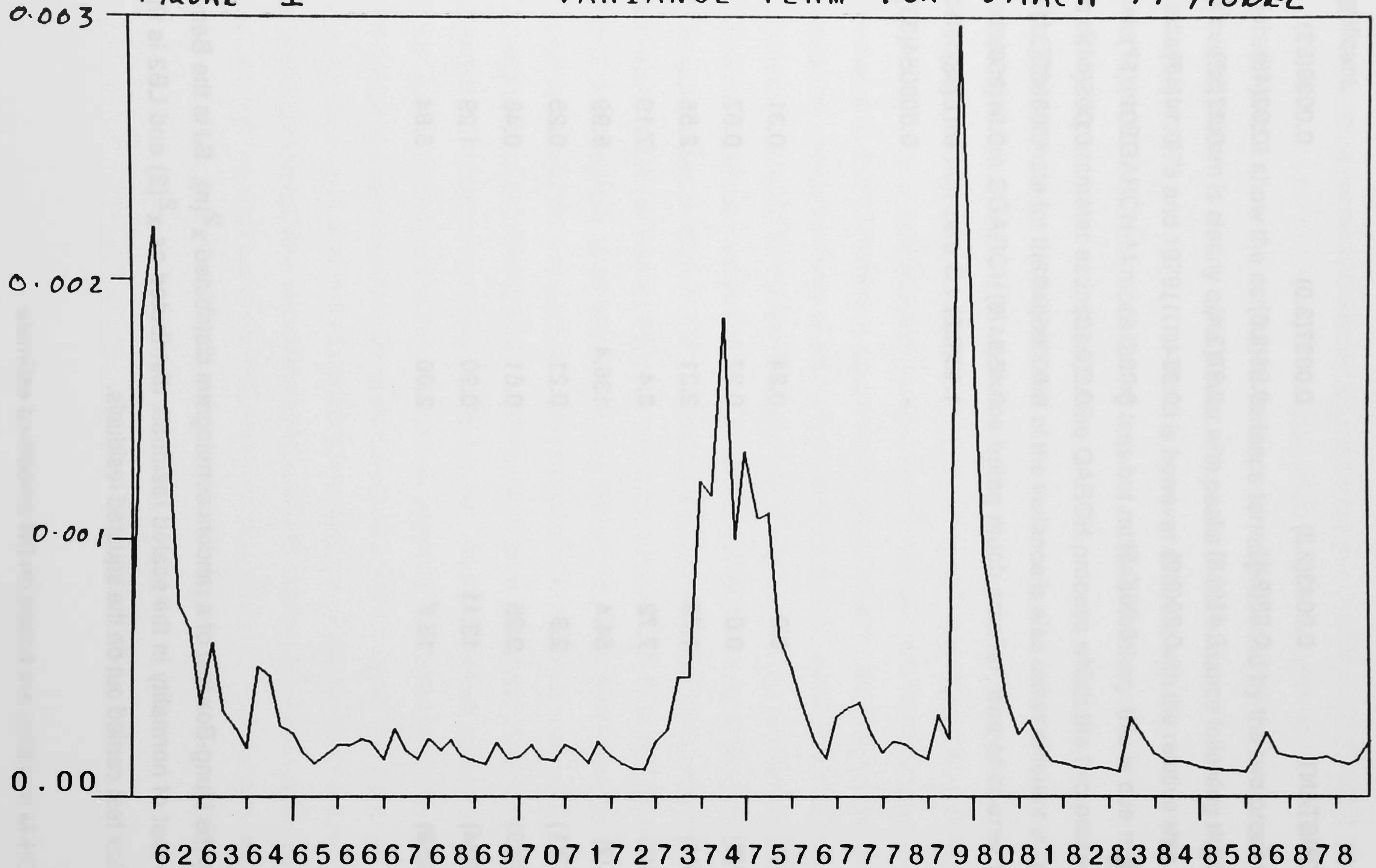
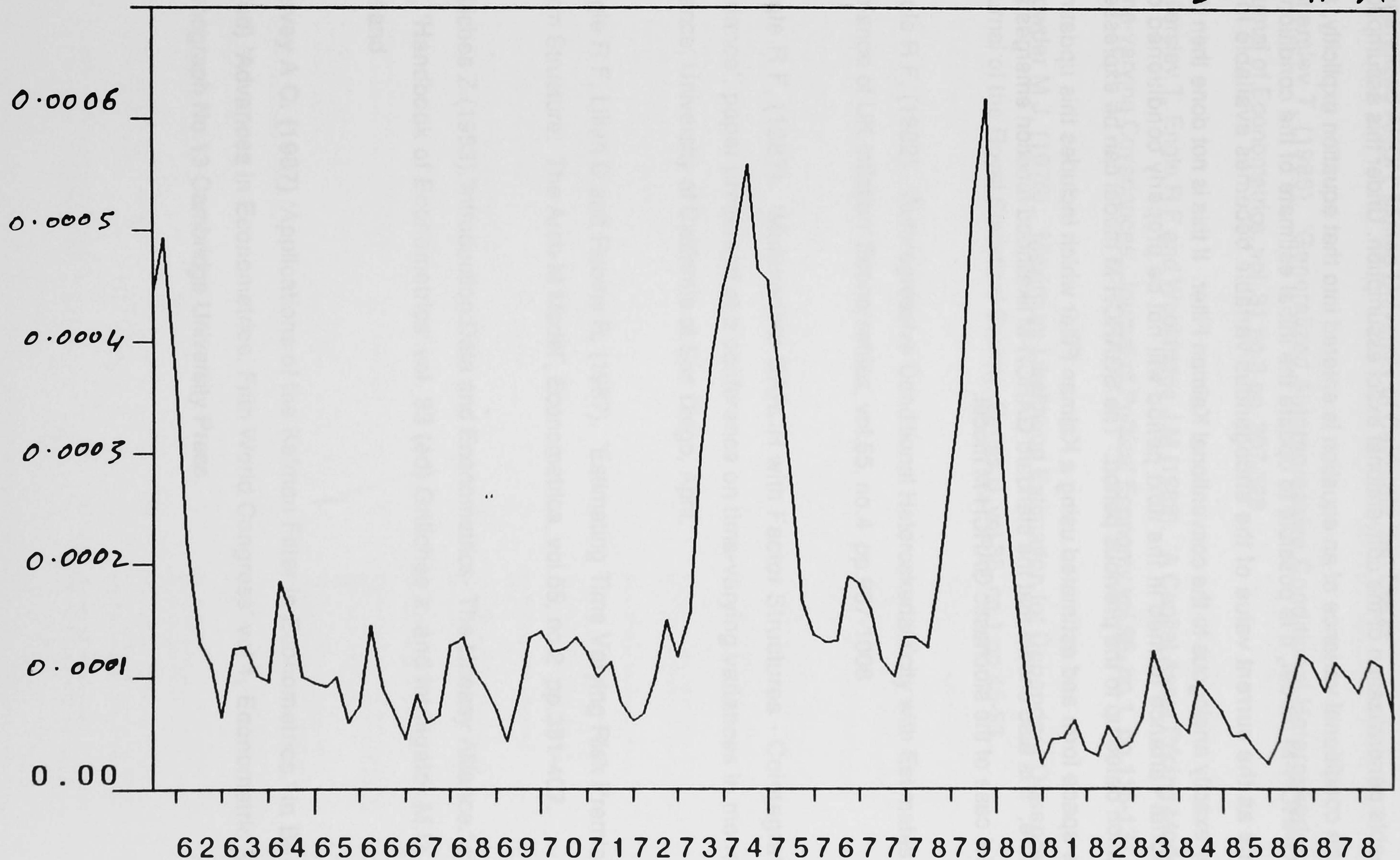


FIGURE 2 SMOOTHED TIME VARYING VARIANCE FOR SGARCH-M



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6. CONCLUSION

This note has argued that the assumption of a stochastic GARCH process is a reasonable generalization of the conventional exact assumption. Under this assumption when the conditional variance of an equation is entered into that equation explicitly, as in the GARCH-M model, it is possible to update the initial estimate of the conditional variance as the current value of the endogenous variable becomes available in a manner exactly analogous to the conventional Kalman Filter. If this is not done then the conditional variance formed in the next period will not be properly conditioned on information dated up to the previous period. The SGARCH-M model can be expressed in state space form and estimated using a Kalman Filter which includes this updating procedure, It is also shown that the standard GARCH-M likelihood function emerges as a special case of the stochastic GARCH-M model.

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