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No 38

A system approach to the relationship between consumptiion and wealth

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Abstract

In this study we extend the single-equation approach to the determination of consumption, due to Hendry and von Ungern-Sternberg (HUS), to a multi-equation system which links decisions on consumption and the components of wealth. The original HUS approach has proved to be a very durable one, being applied to a number of countries and macroeconomic models; we extend this to allow a rôle for rates of return as well as income, and show how this extension results in an error correction system. Our empirical analysis is based on Johansen's maximum likelihood extension of Engle and Granger's co-integration approach.

I Introduction

The framework, for explaining aggregation consumption, due to Hendry and von Ungern-Sternberg (1981), hereafter HUS, has been a very influential one in a number of countries. For the United Kingdom see, for example, Davis (1984) and see von Ungern-Sternberg (1986) for West Germany, Rossi and Schiantarelli (1982) for Italy, Steel (1987) for Belgium and Harnett (1988) for the US. There are two central characteristics of this framework: the functional form is non-linear (log-linear) and the ratio of net liquid assets to income is an important determinant of consumption expenditures. However, a number of model proprietors and some researchers have developed or modified the basic HUS framework. Patterson (1984, 1985) introduces assets other than net liquid assets into the consumption function, and both the LBS and HMT macroeconomic models include an interest rate term [for a survey of interest rate effects, arising from expenditure equations, in five leading models, see Easton and Patterson (1985)]. These studies suggest that the HUS framework is a durable one, indeed probably the dominant one in this area, providing the basis from which variations are considered.

Within the HUS framework both a consumption and a net wealth equation are derived in accordance with the specification of a target wealth-income ratio and a log-linearised accumulation constraint. However, it is the consumption equation rather than the wealth equation which has been estimated in the various studies referred to above, although in principle either could be estimated. In practice the constraint on deciding which equation to estimate has been the availability of sufficiently good data on wealth, indeed HUS originally used only a subset of wealth, that is net liquid assets, even though total wealth was theoretically required, because of the data constraint. Developments of balance sheet data by the UK CSO have, however, made available runs of wealth data of sufficient length to be useful for applied econometric analysis. Thus, nearly ten years on from the original HUS study we are in a position to consider a more explicit treatment of the various components of wealth as well as the determination of consumption.

The broadening of the database and the empirical success of the basic HUS model suggests that it would be fruitful to consider an extension to a system of equations comprising a disaggregated set of portfolio and consumption variables. To anticipate the system we derive leads to log-linear estimating equations and allows the wealth components to be a function of income and rates of return on the elements of the disaggregated portfolio. The implications for the consumption function follow as a matter of course from the system specification.

Estimation of a set of equations, the equilibrium and dynamic specification of each not being known in advance, leads to severe practical problems. Even conducting a 'general to specific' search for a single equation can lead to problems arising from several 'simplification' routes. To overcome these problems we adopt the co-integration approach originally due to Engle and Granger (1987) and developed by Johansen (1988). We first establish that, at least one co-integrating vector exists which defines the long- run or equilibrium; then in the second stage we concentrate on specifying the dynamic structure of the equations. Our empirical results are encouraging, though we would not claim any more at this stage than that viewing consumption and wealth decisions together rather than separately is likely to offer a fruitful line of research.

This study is organised as follows. Section II shows how to specify the disaggregated system of portfolio and consumption decisions; this generalises, and incidentally considerably simplifies the original derivation of the original HUS system—see HUS (1981, pp 240-43). Section III reports details of the empirical analysis; here we consider the joint determination of non-durable consumption and five components of wealth—net liquid assets, net housing wealth, equity in life assurance and pension funds, net other financial assets and consumer durables. Section IV contains some concluding remarks and an appendix outlines Johansen's (1988) maximum likelihood procedure which we use in Section III.

II Specification

(i) Asset and consumption targets

In the HUS approach consumers are assumed to specify a target (or dynamic equilibrium) wealth-income ratio which is constant. This is simply extended if there is more than one asset with different, but constant, asset-income ratios—see Patterson (1984). For example, suppose there are two assets W_{1t}, and W_{2t}; then letting a superscript e denote a target variable, the targets would be

$$w^e_{1t} = b_1 + y^*_t \text{ and } w^e_{2t} = b_2 + y^*_t \tag{1}$$

where lower case letters denote natural logarithms; Y^*_t is income defined below; assets and income are real variables obtained from the corresponding nominal values on deflation by a general price index, P_t. An obvious problem with this kind of specification is that it ignores the role rates of return on the two assets may have in determining the desired asset-income ratios. In particular, whilst, if income is correctly measured, the income effects of changes in the rates of return can in principle be captured through their effect on Y^*_t, the substitution effects will be missed entirely.

To remedy this particular defect consider the following specification which introduces rates of return into the asset functions,

$$w^e_{it} = b_i + y^*_t + \sum_{j=1}^{n} \beta_{ij} r_{jt} \qquad i = 1 \ldots n \tag{2}$$

where the r_{jt} are real rates of return; for example, if asset j is a building society deposit with nominal net of tax interest rate R_{jt}, then $r_{jt} = \dfrac{(R_{jt} - \pi_t)}{(1 + \pi_t)}$, where π_t is the rate of inflation (of P_t). Notice that it is *not* implicit in this framework that the total level of wealth (the portfolio size) is given. The flow of consumption and hence savings can in principle alter and therefore affect the total of net asset holdings. To derive the consumption function we first recognise that the accumulation of assets is governed by the following recursion, which holds for equilibrium and actual quantities, which effectively links portfolio and consumption decisions,

$$\frac{W_t}{P_t} = \sum_{i=1}^{n} (1 + r_{it}) \frac{W_{it-1}}{P_{t-1}} + \frac{(Y_t - C_t)}{P_t} \tag{3}$$

C_t is expenditure on non-durables and services, and thus consumers' durables are one of the n assets; Y_t is non-asset or labour income. The specifications in (2) are loglinear but the constraint (3) is linear; however, generalising the procedure followed in HUS the constraint is loglinearised as:

$$w_{1t} + \sum_{i=2}^{n} \mu_{it} w_{it-1} = w_{1t-1} + \sum_{i=2}^{n} \mu_{it} w_{it-1} + B_t (y^*_t - c_t) \tag{4}$$

where $w_{it} = Ln(W_{it}/P_t)$, $c_t = Ln(C_t/P_t)$, and

$$y^*_t = Ln\left[\left(\frac{Y_t}{P_t}\right) + \sum_{1=1}^{n} r_{it} \frac{W_{it-1}}{P_{t-1}}\right]; \text{ and}$$

$$\mu_{it} = \frac{W_{it-1}}{W_{1t-1}}$$

$$B_t = \frac{Y^*_t}{W_{1t-1}}$$

Note that Y^*_t is total (accrued) disposable income comprising labour income and net returns from the n assets.

Recognising that (2) must hold for target (and planned) quantities, we can solve for target (and planned) consumption on substituting (2) into the accumulation constraint, (4), to obtain:

$$c_t^e = y_t^* + \lambda_{yt}\, g_{yt} + \sum_{i=1}^{n} \lambda_i\, g_{it} \tag{5}$$

where

$$\lambda_{yt} = \frac{-(1+\mu_t)}{B_t} \quad ;$$

$$g_{yt} = \Delta y_t^* \quad ;$$

$$\lambda_{it} = \frac{-(\beta_{1i} + \sum_{h=2}^{n} \mu_{ht}\, \beta_{hi})}{B_t} \quad ;$$

$$g_{it} = \Delta r_{it} \quad ;$$

$$\mu_t = \sum_{h=2}^{n} \mu_{ht} \quad ; \text{ and for later reference we define}$$

$$k_t = \lambda_{yt}\, g_t + \sum_{i=1}^{n} \lambda_i\, g_{it} \quad .$$

Target consumption has a unit elasticity with respect to the level of income, so that (5) implies that the consumption-income ratio deviates from unity if either g_{yt} or any of the g_{it} are not equal to zero. This is just an implication of the fact that since asset stocks are functions of the levels of the determining variables, consumption, which is a flow, works on the differences of these variables.

(ii) A dynamic model

The equations (2) and (5) relate to the equilibrium determination of wealth and consumption, respectively. In practice, account needs to be taken of adjustment to equilibrium faced with shocks to income or the rates of return. By minimising a suitably extended version of the HUS loss function Patterson (1991) has shown that an error correction system can be derived. For example, in a system comprising consumption and two assets we have:

$$\begin{bmatrix} \Delta c_t \\ \Delta w_{1t} \\ \Delta w_{2t} \end{bmatrix} = \begin{bmatrix} (\pi_{12}-1) & \pi_{13} & \pi_{14} \\ \pi_{22} & (\pi_{23}-1) & \pi_{24} \\ \pi_{32} & \pi_{33} & (\pi_{34}-1) \end{bmatrix} \begin{bmatrix} e_{0t-1} \\ e_{1t-1} \\ e_{2t-1} \end{bmatrix} + \begin{bmatrix} \pi_{11} & \pi_{16} & \pi_{17} \\ \pi_{21} & \pi_{26} & \pi_{27} \\ \pi_{31} & \pi_{36} & \pi_{37} \end{bmatrix} \begin{bmatrix} \Delta y_t^* \\ \Delta r_{1t} \\ \Delta r_{2t} \end{bmatrix} + \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} + \begin{bmatrix} u_{0t} \\ u_{1t} \\ u_{2t} \end{bmatrix} \tag{6}$$

where $u_{0t} = c_t - c_t^p$ and $u_{it} = w_{it} - w_{it}^p$; and the γ_i are constants and a p superscript indicates a planned quantity. The e_{it} are the disequilibrium or error feedback terms; thus $e_{0t} = c_t - y_t^* - k_t$, and $e_{it} = w_{it} - y_t^* - \beta_{i1}\, r_{1t} - \beta_{i2}\, r_{2t}$. Notice that (6) is a system of equations; consumption potentially depends upon disequilibrium in consumption and both assets, and similarly all disequilibria or error correction terms could be present in the asset equations. In practice a number of the π_{ij} coefficients might be zero reflecting the separability of certain parts of the system. Our approach suggests that, for example, if one of the wealth components is net liquid assets then, at least potentially, disequilibria in consumption and the other assets could affect the adjustment path of that component. This contrasts with approaches which model liquid assets without reference to consumption or other decisions concerning portfolio allocation.

III Empirical analysis

In this section we consider the joint determination of real non-durable consumption and real wealth disaggregated into five components; net liquid assets (NLA), net housing wealth (NHW), equity in life assurance and pension funds (LAPF), the stock of consumer durables (SCD), and net other financial assets (OFA) which primarily comprises personal sector holdings of equity and unit trusts. In each case we have also derived an own rate of return on each asset, denoted by the prefix R (eg RNLA etc).

In defining income we found, initially, that real personal disposable income unadjusted for the inflation loss on net liquid assets was preferred to the adjusted measure [confirming earlier results in a single equation context reported by Patterson (1990)]. Even though further disaggregation of both consumption and wealth would be possible, the system we propose here introduces a number of practical problems arising from its complexity. We have to determine both the equi- librium—or long-run—specification of each equation as well as the dynamic adjustment process. The methodology we adopt is Johansen's (1988) development of the co-integration techniques due to Engle and Granger (1987); for a practical illustration of this method see Hall (1989). In this approach we separate modelling of the long run from the dynamic specification search, which provides a practical solution to a complex empirical problem.

Johansen's (1988) approach applied to our particular area is as follows (a more detailed exposition of the Johansen technique is given in an appendix). Let X_t denote the vector of variables at t comprising consumption and the five assets referred to above as well as other variables such as interest rates , such that

$$X_t = \sum_{i=1}^{k} \Pi_i X_{t-i} + \sigma_t \qquad (7)$$

where σ_t is a p dimensional random vector of residuals with zero mean and constant variance. Then define the matrix polynomial,

$$A(Z) = I - \sum_{i=1}^{k} \Pi_i Z^i \qquad (8)$$

$$\text{and} \quad \Pi \equiv A(Z)\,|_{z=1} = I - \sum_{i=1}^{k} \Pi_i \qquad (9)$$

with r, the rank of Π, <p. Π is simply related to the matrix of long-run coefficients which characterise the equilibrium relationships (2) and (5). Expressing Π as

$$\Pi = \alpha\beta' \qquad (10)$$

for suitable pxr matrices α and β, then if the linear combinations given by $\beta' X_t$ are stationary for X_t integrated of order 1 (denoted I(O) and I(1), respectively) the variables in X_t are said to form a co-integrating set. What we need to determine is how many co-integrating vectors exists for our set of variables and whether Π has positive elements along the diagonal. This latter condition is exactly the same as requiring a negative error correction coefficient in the conventional single equation error correction mech- anism. If these conditions are met we can then consider the second stage of modelling the dynamic adjustment process, building in the equilibrium defined in the first stage. It is thus analogous to the Engle and Granger (1987) two step procedure in a multivariate context. Johansen shows that a maximum likelihood (ML) technique can be applied in the first stage to determine whether co-integrating vectors exist for a particular data set.

Note that the Johansen framework is remarkably similar to that outlined in section II, in that each of the equilibrium terms in (6) implicitly corresponds to a co-integrating vector; thus, the first term on the right hand side of (6) corresponds (with the trivial difference of a change in sign) to the Π matrix of (10). The π coefficients are equivalent to the Johansen loading matrix α and the e_i are the co-integrating vectors made up of $\beta' X_t$. Π is, of course of deficient rank as r is always less than p, but the e_i terms in (6) are linked by the asset accumulation constraint (3) so they are not independent implying that the first term in (6) is also of deficient rank. Our requirement above that the Π matrix have positive diagonal terms is simply equivalent to requiring that the diagonal terms of the π matrix in (6) are negative given the implicit normalization of the e_i terms. So each row of the Π matrix represents the equilibrium terms combined together for each equation in the system. The spill-over effects from each equilibrium term to every variable are then contained within this matrix.

An initial step in the analysis is to determine the univariate properties of the data, which are summarised in Table 1.

The broad conclusion to be drawn from this table is that the ratios of wealth components and consumption to disposable income are consistently non-stationary, whilst the results for the rates of return on the various components of wealth are mixed; the return on liquid assets (RNLA), housing (RNHW) and consumers durables (RSCD) seem to be non-stationary while the other two are stationary.

We now consider the application of Johansen's ML procedure to the non-stationary variables in Table 1, that is the consumption-income ratio, the various wealth-income ratios and RNLA, RNHW and RSCD (whilst we could include the other interest rates, as the inclusion of an I(0) variable does not cause a problem within this procedure, so doing creates severe problems with degrees of freedom). We found that there were potentially seven co-integrating vectors in our data set confirming the view that the consumption and wealth income ratios and the rates of return are co-integrated (the likelihood ratio test that there were at most six co-integrating vectors was 33.9, 95% critical value=31.2). The Π matrix is reported in Table 2.

These results are reasonably encouraging for such a large system and intuitively fairly plausible. To help with the interpretation, note that the first row shows the long-run solution to the system for the consumption to income ratio; the positive coefficient on C-Y shows that the solution to this part of the model is stable. We would expect that consumption, income and the wealth components would all move in a similar fashion so we would not expect the wealth to income ratio effects to be very different from zero, and indeed they are all quite small. A similar pattern exists with respect to the long-run solution for the other variables, with the exception that NHW-Y and SCD-Y both have negative coefficients in their own equation (shown by the underlined diagonal terms) and so this system may not be determining a stable long-run solution for these variables.

One aspect which may account for the poor determination of the housing wealth variable is that we have not made any allowance for the financial liberalisation which has taken place in the housing market. In particular the

Table 1
The univariate properties of the data

Variable	I(0) DF	I(0) ADF	I(1) DF	I(1) ADF	ML
RNLA	-0.35	-0.27	-5.5	-3.8	0.008
RNHW	-3.9	-2.2	-4.7	-4.2	4.8
ROFA	-6.0	-3.5	-	-	15.5
RLAPF	-6.2	-3.5	-	-	15.8
RSCD	-5.8	-2.4	-5.1	-4.2	2.8
C-Y	-3.9	-0.8	-13.1	-3.4	1.1
NLA-Y	-2.3	-1.6	-8.9	-2.3	2.4
NHW-Y	-1.4	-1.4	-5.4	-3.6	1.5
OFA-Y	-2.6	-2.7	-5.5	-3.1	6.4
LAPF-Y	0.2	-0.03	-5.4	-2.4	0.8
SCD-Y	-2.3	-1.99	-9.8	-2.6	5.07
MINTY	1.3	-1.5	-4.8	-3.5	1.06

Notes: DF is the Dickey-Fuller test, 95% critical value=-2.9 [see MacKinnon(1990)].
ADF is the 4th order augmented Dickey-Fuller test (95% critical value=-2.9).
ML is the Johansen maximum likelihood procedure test for a single co-integrating vector involving one variable (95% critical value=8.03).

Table 2
The Π matrix for the 9 variable model

C-Y	NLA-Y	NHW-Y	OFA-Y	LAPF-Y	SCD-Y	RNLA	RNHW	RSCD
0.92	0.16	-0.02	-0.08	-0.02	0.02	4.9	0.8	-0.6
0.47	0.28	-0.02	-0.08	0.02	0.08	3.93	0.89	-0.0
0.67	0.212	-0.003	-0.08	0.046	0.11	8.001	0.165	-0.17
-5.04	0.443	0.118	0.37	-0.105	1.04	2.07	-1.26	-0.53
-0.823	0.03	0.23	0.10	0.05	0.08	10.8	0.62	1.38
0.85	0.09	0.023	-0.08	0.03	-0.05	5.46	0.866	-0.41
0.03	-0.007	0.002	0.001	0.001	-0.01	0.22	-0.003	0.007
-0.58	-0.07	0.04	-0.002	0.012	-0.281	3.51	0.99	0.19
0.61	-0.26	0.02	-0.02	0.41	-0.59	3.42	0.46	1.37

Table 3
The Π matrix for the 10 variable model

C-Y	NLA-Y	NHW-Y	OFA-Y	LAPF-Y	SCD-Y	RNLA	RNHW	RSCD	MINTY
1.07	0.14	0.05	-.08	0.06	0.03	5.5	0.5	-0.6	-2.1
0.51	0.23	0.017	-0.08	0.03	0.05	3.43	0.96	0.008	-0.5
0.83	0.18	0.065	-0.08	0.08	0.11	8.49	-0.03	-0.06	-1.95
-4.9	0.844	0.41	0.255	0.09	1.55	7.1	-4.2	-2.5	-10.0
-0.58	0.41	0.50	0.01	0.21	0.44	14.5	-1.92	0.17	-8.73
0.96	0.03	0.08	-0.08	0.06	-0.07	5.5	0.844	-0.25	-1.3
0.02	-0.01	0.0005	0.001	-0.002	-0.01	0.01	0.020	0.006	0.12
-0.61	-0.12	0.02	0.01	0.001	-0.32	3.05	1.25	0.47	0.59
0.34	-0.36	-0.06	-0.005	-0.01	-0.67	1.66	1.09	1.49	2.84
0.02	-0.009	-0.003	-0.0003	-0.005	-0.012	0.07	0.085	-0.03	0.22

relaxations which have taken place on the legal constraints on building societies and the entry of the clearing banks into the mortgage market. This omission may explain the lack of a long-run solution for the housing wealth to income ratio. One characteristic of financial liberalisation is the relaxation of previously established prudential ratios—for example, a relaxation of the loan to income ratio and, of particular importance when nominal interest rates are increasing, an implicit relaxation of the debt servicing costs to income ratio. We found that this latter, denoted here as MINTY—the ratio of mortgage interest, after tax relief, to disposable income, to be influential in an extension of our data set to provide a solution for the housing wealth variable.

Repeating the ML procedure including MINTY we found that the 10 variable model has a total of 8 co-integrating vectors (the likelihood ratio test that there are at most 7 co-integrating vectors is 32.03 compared with the 95% critical value of 31.2). The revised Π matrix is reported in Table 3.

Numerically the results are not greatly altered by the introduction of MINTY, the main qualitative change is, however, an important one: the diagonal term for the net housing wealth equation is now positive so that the full dynamic model should have a stable long-run solution for this variable. Of the consumption and wealth components the only negative diagonal term is in the stock of consumer durables equation, this is unfortunate but hardly surprising as there are clearly a number of institutional factors which have affected this variable which would be hard to include in such a large system (changes in consumer credit and higher purchase regulations are one obvious example).

The matrix in Table 3 is then our full system estimate of the Π matrix which represents the combinations of error correction terms in the generation of the equation system (6). We are now in a position to estimate dynamic equations for the consumption and wealth variables. We start our specification search by setting up a general dynamic error correction system, where the error correction term in each equation is given by the appropriate line from Table 3; and then carry out a conventional search over the dynamics of the system to produce our preferred dynamic equations. This procedure may be thought of as exactly analogous to the Engle and Granger two step procedure except that, as we are dealing with more than one co-integrating vector, we have derived the estimates of the total error correction term from the Π matrix of the Johansen procedure.

The preferred equations for each of the variables are presented in Table 4.

Table 4
The six preferred dynamic models

1 **Non-durable Consumption, C.**

$$\Delta(c-y) = 0.0004 - 0.38\,\Delta c_{t-1} - 0.54\,\Delta c_{t-2} + 0.36\,\Delta y_{t-1} + 0.40\,\Delta y_{t-3} - 0.21\,\Delta(scd-y)_{t-1} - 0.38\,\Pi_{t-1}$$

(0.2) (1.3) (3.2) (3.7) (4.2) (2.1) (5.1)

SEE=0.0115 DW=1.92 LM(1)=0.38 LM(2)=2.3 LM(4)=3.6 LM(8)=8.4 BP(1)= 0.03 BP(2)=1.07
BP(4)=1.56 BP(8)=6.7 BJ(2)=0.7 ARCH(1)=0.02 RESET(3)=2.8

2 **Net Liquid Assets, NLA**

$$\Delta(nla-y) = 0.038 + 0.45\,\Delta y_{t-1} + 0.27\,\Delta(nla-y)_{t-1} + 0.03\,\Delta(ofa-y)_{t-1} - 0.24\,\Pi_{t-1}$$

(2.85) (3.3) (1.91) (2.0) (3.1)

SEE=0.014 DW=2.06 LM(1)=0.74 LM(2)=2.9 LM(4)=6.0 LM(8)=6.2 BP(1)=0.2 BP(2)=2.5
BP(4)=3.2 BP(8)=3.9 BJ(2)=1.0 ARCH(1)=0.7 RESET(3)=6.9

3 Net Housing Wealth, NHW

$$\Delta(nhw - y) =$$

$$0.113 + 0.83\,\Delta c_{t-3} - 0.32\,\Delta(nla - y)_{t-1} + 0.36\,\Delta(nhw - y)_{t-2} - 0.63\,\Delta(scd - y)_{t-2} + 0.09\,D883 - 0.71\,\Pi_{t-1}$$

(5.6) (3.1) (2.1) (4.0) (3.7) (4.4) (5.9)

SEE=0.018 DW=2.02 LM(1)=0.6 LM(2)=0.9 LM(4)=6.2 LM(8)=6.5 BP(1)=0.04 BP(2)=0.05 BP(4)=2.2
BP(8)=2.71 BJ(2)=6.1 ARCH(1)=0.002 RESET(3)=1.01

4 Other Financial Assets, OFA

$$\Delta(ofa-y) = 0.425 - 0.358\,D874 + 0.241\Delta(ofa-y)_{t-1} - 0.17\Pi_{t-1}$$

(2.2) (3.6) (2.2) (2.2)

SEE=0.096 DW=2.03 LM(1)=0.5 LM(2)=0.7 LM(4)=2.3 LM(8)=17.6 BP(1)=0.08 BP(2)=0.11
BP(4)=2.1 BP(8)=14.2 BJ(2)=1.8 ARCH(1)=2.2 RESET(3)=4.1

5 Life Assurance and Pension Funds, LAPF

$$\Delta(lapf - y) = 0.52 - 1.35\,\Delta c_{t-2} - 0.26\,D874 - 0.37\,\Pi_{t-1}$$

(4.6) (2.0) (5.2) (4.4)

SEE=0.050 DW=2.12 LM(1)=0.55 LM(2)=4.4 LM(4)=7.1 LM(8)=15.1 BP(1)=0.5 BP(2)=3.9 BP(4)=6.5
BP(8)=11.5 BJ(2)=0.4 ARCH(1)=0.3 RESET(3)=6.8

6 The Stock of Consumer Durables, SCD

$$\Delta(scd - y) = 0.006 + 0.25\Delta y_{t-1} + 0.27\Delta y_{t-2} - 0.32\Pi_{t-1}$$

(2.9) (2.6) (2.8) (4.1)

SEE=0.013 DW=1.88 LM(1)=0.2 LM(2)=3.4 LM(4)=7.4 LM(8)=8.8 BP(1)=0.1 BP(2)=2.9 BP(4)=4.7
BP(8)=8.0 BJ(2)=0.02 ARCH(1)=1.3 RESET(3)=3.7

NOTES: Estimation period 1973Q2-1988Q3, D874,D883 are zero one dummies for 1987Q4 and 1988Q3. Π_{t-1} is the relevant row from the Π matrix specified in Table 3, SEE is the standard error of the regression, DW is the Durbin Watson Statistic, LM(i) tests serial correlation up to order i $[\chi^2(i)]$. BP is the Box-Pierce test for a random correlogram $[\chi^2(i)]$, BJ is the Bera-Jarque test for normality of residuals $[\chi^2(2)]$. ARCH is the test for first order ARCH effects $[\chi^2(1)]$. RESET is the reset test for functional form $[\chi^2(3)]$.

These results are broadly satisfactory, in every case the parameter on the Π matrix is significant and correctly signed (that is negative) and all the equations pass a wide variety of diagnostics.

IV Concluding remarks

We have outlined a system approach to the determination of consumption and portfolio decisions. In part this was explicit in the work of Hendry and von Ungern-Sternberg (1981); however, their chosen wealth variable—net liquid assets—omitted much of the wealth held by the personal sector and their equilibrium wealth (and consumption) income ratios were not affected by changes in the return on wealth. Our approach offers a solution to both of these omissions, and results in the consumption function being viewed as one of several inter-related equations.

Our empirical approach illustrated how progress could be made in estimating a system of equations. Even in a single equation context searching for both an equilibrium and a dynamic specification can be a difficult matter. In a system context this is almost inevitable, and we adopted the two stage co-integrating procedure familiar from the work of Engle and Granger (1987) as developed for a multi-variate context by Johansen (1988). In practice we found it necessary to augment our system variables to reflect the effects of financial liberalisation in the housing market. The resulting dynamic equations passed a battery of diagnostic tests. Given the efforts of the CSO to produce consistent disaggregated sectoral balance sheets, the focus of consumption modelling could usefully be extended to incorporate the determination of the components of wealth.

APPENDIX: THE JOHANSEN PROCEDURE

Johansen (1988) sets his analysis within the following framework. Begin by defining a general polynomial distributed lag model of a vector of variables X as

$$X_t = \pi_1 X_{t-1} + \ldots + \pi_k X_{t-k} + \varepsilon_t \qquad t = 1, \ldots, T \tag{A1}$$

where X_t is a vector of N variables of interest; π_i are NXN coefficient matrices, and ε_t is an independently identically distributed N dimensional vector with zero mean and variance matrix L. Within this framework the long run, or co-integrating matrix is given by

$$I - \pi_1 - \pi_2 \ldots - \pi_k = \pi \tag{A2}$$

Where I is the identity matrix.

π will therefore be an NXN matrix. The number, r, of distinct co-integrating vectors which exists between the variables of X, will be given by the rank of π. In general, if X consists of variables which must be differenced once in order to be stationary [integrated of order one or I(1)] then, at most, r must be equal to N-1, so that $r \leq N-1$ Now we define two matrices α, β both of which are Nxr such that

$$\pi = \alpha\beta'$$

and so the rows of β form the r distinct co-integrating vectors.

Johansen then demonstrates the following Theorem.

Theorem: The maximum likelihood estimate of the space spanned by β is the space spanned by the r canonical variates corresponding to the r largest squared canonical correlations between the residuals of X_{t-k} and ΔX_t corrected for the effect of the lagged differences of the X process. The likelihood ratio test statistic for the hypothesis that there are at most r co-integrating vectors is

$$-2\ln Q = -T\sum_{i=r+1}^{N} \ln(1 - \hat{\lambda}_i) \tag{A3}$$

where $\hat{\lambda}_{r+1} \ldots \hat{\lambda}_N$ are the N - r smallest squared canonical correlations. Johansen then goes on to demonstrate the properties of the maximum likelihood estimates and, more importantly, he shows that the likelihood ratio test has an asymptotic distribution which is a function of an N - r dimensional Brownian motion which is independent of any nuisance parameters. This means that a set of critical values can be tabulated which will be correct for all models. He demonstrates that the space spanned by β is consistently estimated by the space spanned by $\hat{\beta}$.

In order to implement this Theorem we begin by reparameterising (A1) into the following error correction model.

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \quad + \Gamma_{k-1} \Delta X_{t-k+1} + \Gamma_k X_{t-k} + \varepsilon_t \tag{A4}$$

where $\Gamma_i = -I + \pi_1 + \ldots \pi_i$; i=1 ... k

The equilibrium matrix π is now clearly identified as $-\Gamma_k$.

Johansen's suggested procedure begins by regressing ΔX_t on the lagged differences of ΔX_t and defining a set of residuals R_{ot}, then regressing X_{t-k} on the lagged differences and defining R_{kt}. The likelihood function, in terms of α, β and Ω is then proportional to

$$L(\alpha, \beta, \Omega) = |\Omega|^{\frac{-T}{2}} EXP\left[\frac{-1}{2}\sum_{t=1}^{T}(R_{ot} + \alpha\beta' R_{kt})' \Omega^{-1} (R_{ot} + \alpha\beta' R_{kt})\right] \qquad \text{(A5)}$$

If β were fixed we could maximise over α and Ω by a regression of R_{ot} on $-\beta' R_{kt}$ which gives

$$\hat{\alpha}(\beta) = -S_{ok}\,\beta\,(\beta' S_{kk}\,\beta)^{-1} \qquad \text{(A6)}$$

and

$$\hat{\Omega}(\beta) = S_{oo} - S_{ok}\,\beta\,(\beta' S_{kk}\,\beta)^{-1}\,\beta' S_{ko} \qquad \text{(A7)}$$

where

$$S_{ij} = T^{-1}\sum_{t=1}^{T} R_{it} R'_{jt} \qquad \text{i, j=O,k}$$

and so maximising the likelihood function may be reduced to minimising

$$|\, S_{oo} - S_{ok}\,\beta\,(\beta' S_{kk}\,\beta)^{-1}\beta' S_{ko}\,| \qquad \text{(A8)}$$

It may be shown that (A8) will be minimised when

$$\frac{|\,\beta' S_{kk}\,\beta - \beta' S_{ko} S_{oo}^{-1} S_{ok}\,\beta\,|}{|\,\beta' S_{kk}\,\beta\,|} \qquad \text{(A9)}$$

attains a minimum with respect to β.

We now define a diagonal matrix D which consists of the ordered eigenvalues

$\lambda_1 > \ldots > \lambda_N$ of $S_{ko} S^{-1}_{oo} S_{ok}$ with respect to S_{kk}. That is λ_i satisfies

$$|\,\lambda S_{kk} - S_{ko} S_{oo}^{-1} S_{ok}\,| = 0 \qquad \text{(A10)}$$

Define E to be the corresponding matrix of eigenvectors so that

$$S_{kk}\, E\, D = S_{ko}\, S_{00}^{-1}\, S_{ok}\, E \qquad \text{(A11)}$$

where we normalize E such that $E' S_{kk} E = I$

The maximum likelihood estimator of β is now given by the first r rows of E, that is, the first r eigenvectors of $S_{ko} S_{oo}^{-1} S_{ok}$ with respect to S_{kk}. These are the canonical variates and the corresponding eigenvalues are the squared canonical correlations of R_k with respect to R_o. These eigenvalues may then be used in the test proposed in (3) to test either for the existence of a co-integrating vector r = 1 or the number of co-integrating vectors $N > r > 1$.

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