

Bank of England

Discussion Papers

Technical Series

No 25

**An independent error feedback model
of UK company sector asset demands**

by

D G Barr

K Cuthbertson

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The object of this Technical Series of Discussion Papers is to give wider circulation to econometric work predominantly in connection with revising and updating the various Bank models, and to invite comment upon it; any comments should be sent to the authors at the address given below.

The authors are grateful for comments from David Miles. The views expressed are those of the authors and do not necessarily represent those of the Bank of England.

Issued by the Economics Division, Bank of England, London, EC2R 8AH to which requests for individual copies and applications for mailing list facilities should be addressed; envelopes should be marked for the attention of the Bulletin Group.

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ISBN 0 903313 75 8
ISSN 0263-6123

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Abstract

An interdependent error feedback system is used to model asset demands for the UK company sector. Tests of long-run and short-run symmetry, homogeneity, homotheticity and negativity are undertaken. The model is found to be consistent with the basic axioms of demand theory. The model performs adequately in statistical terms and parameter values are plausible.

I INTRODUCTION

There have been numerous systems models applied to asset decisions of the personal sector (Weale 1986, Green 1984) banks, (Courakis, 1975, 1980, Parkin et al 1970) and some to the non-bank private sector (Conrad 1980). Applied work, in our view, has largely neglected a systems approach to the asset decisions of the company sector. Such attempts as have been made have often yielded results that conflict with the chosen theoretical model or intuitive a priori views (Jackson 1984, Courakis 1988). In this paper we present a systems approach to the short-term asset decisions of the UK company sector. The theoretical structure is based on the Almost Ideal Demand System (AIDS) (Deaton and Muellbauer 1980) which provides our long-run asset demands. Dynamics are modelled by an interdependent error feedback model. Our dynamic AIDS model allows one to test the theoretical restrictions of homogeneity, symmetry and negativity which must hold if the behaviour of the representative agent is to conform to the basic axioms of rational choice (eg convexity, transitivity). We find that with a suitably flexible dynamic structure we obtain demand functions for company sector short-term assets that satisfy the theoretical restrictions implied by the AIDS model and are intuitively plausible. The rest of this paper is organised as follows. In Section II we outline the theoretical model and in Section III we consider the modelling of short-run dynamics in a systems framework and associated econometric problems. In Section IV we discuss data problems and in Section V we present our empirical results. We conclude with a brief summary.

II THE AIDS MODEL

The representative agent is assumed to distribute his wealth between alternative assets in order to minimise the cost of achieving a given level of utility.⁽¹⁾ The axioms of rational choice in demand theory (ie the existence of consistent preferences) are met providing we choose a cost function that is concave and homogeneous of degree one in prices.⁽²⁾ Of the several flexible functional forms available we select the PIGLOG (Price Independent Generalised Logarithmic) which, in common with others (eg indirect translog, Christensen et al 1975) is a second order approximation but has the advantage of desirable aggregation properties. Within the PIGLOG class we choose the AIDS cost function (Deaton & Muellbauer 1980).

The budget constraint is:

$$\sum_i p_{it}^r a_{it+1}^r = w_t^r \quad (1)$$

where

$$p_{it}^r = [(1 + r_{it})(1 - g_z)]^{-1}$$

$$a_{it+1}^r = \text{real asset holdings } (= a_{it+1}/z_{t+1})$$

$$w_t^r = \text{real wealth } (= w_t/z_t)$$

$$a_{it} = \text{nominal asset holdings of the } i^{\text{th}} \text{ asset}$$

$$r_{it} = \text{expected (proportionate) nominal return on asset } i, \text{ between } t \text{ and } t+1 \text{ (including any capital gains).}$$

$$g_z = \text{expected (proportionate) rate of goods price inflation between } t \text{ and } t+1.$$

$$p_{it}^r = \text{'real price' (ie approximately equal to the inverse of the expected real interest rate, } (r_{it} - g_z) \text{).}$$

$$z_t = \text{goods price index}$$

Solving the constrained cost minimisation problem leads to the AIDS share equations (see for example, Barr and Cuthbertson 1989, Weale 1986):

$$s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_{jt}^r + \beta_i \ln (w_t^r / P^{*\tau})_t \quad (2)$$

where

$$s_i = a_{it} / w_t \quad (2a)$$

$$\ln P_t^{*\tau} = \sum_i s_i \ln p_{it}^r \quad (2b)$$

Note that $\ln P_t^{*\tau}$ may be interpreted as a composite real interest rate. For assets that are used for transactions (ie acceptable as a means of exchange)

the cost of achieving a given level of utility will depend on the level of transactions, which may be represented by an additional term in the cost function. This results in an additional term in the level of real transactions in (2) (see Weale 1986).

The long-run AIDS share equations in vector notation are:

$$s_t^* = \Pi X_t = \Gamma \ln p_t^T + B RW_t + J X_{1t} + \alpha \quad (3)$$

$$s_t^* = k \times 1 \text{ vector of asset shares}$$

$$\ln p_t^T = k \times 1 \text{ vector of real asset prices}$$

$$RW_t = \log \text{ of real wealth } (= \ln(W^T/P^{*T})_t)$$

$$X_{1t} = m \times 1 \text{ matrix of 'm' additional independent variables (eg expenditure)}$$

$$\Gamma = (k \times k) \text{ matrix of 'price' coefficients } (\gamma_{ij})$$

$$B = (k \times 1) \text{ vector of real wealth coefficients } (\beta_i)$$

$$J = k \times m \text{ matrix of coefficients}$$

$$\alpha = k \times 1 \text{ vector of constants}$$

$$\Pi = [\Gamma : B : J : \alpha]$$

$$X_t = [\ln p_t^T, RW_t, X_{1t}, 1]$$

The theoretical restrictions of the AIDS model based on consumer theory are:

The adding up constraints

$$\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = 0, \quad \sum_i \beta_i = 0 \quad (3a)$$

Homogeneity:

$$\sum_j \gamma_{ij} = 0 \quad (3b)$$

Symmetry and negativity (of the Hicksian demand functions) are direct consequences of the axioms of rational choice. The former implies: (3)

$$\gamma_{ij} = \gamma_{ji} \quad (3c)$$

Negativity arises from the concavity of the cost function and implies that the matrix of coefficients k_{ij} :

$$k_{ij} = \gamma_{ij} + \beta_i \beta_j \ln(W^r/P^{r*}) - s_i \delta_{ij} + s_i s_j \quad (3d)$$

is negative semi-definite (δ_{ij} is the Kronecker delta).

Thus our systems approach implicitly imposes data admissability in the form of adding up constraints and the additional theoretical constraints of symmetry homogeneity and negativity. If symmetry and homogeneity hold then this reduces the number of parameters to be estimated and increases efficiency. In addition one might wish to judge the model on more intuitive notions, (eg that own price effects are negative, wealth and expenditure elasticities are 'reasonable' etc).

Our 'consumer demand' approach is flexible in that it allows transactions, hedging or perceptions of risk (ie variances and covariances of returns) to be represented in the parameters of the cost function. Should any of the latter change then this will be picked up empirically either by a failure of homogeneity or symmetry or by parameter instability.

The wealth and compensated own price and cross price elasticities are:

$$E_w = (\beta_i/s_i) + 1 \quad (4a)$$

$$E_{ij}(p) = (s_i)^{-1} k_{ij} \quad (4b)$$

Interest rate semi-elasticities are given by

$$E_{ij}(R) = E_{ij}(p)/4 \quad (4c)$$

III DYNAMIC ADJUSTMENT AND ECONOMETRIC ISSUES

In a system of asset demand equations if we include only own-lags, then we must implicitly accept that all assets adjust at the same rate (Smith 1975). To avoid this problem cross-lagged terms must also be included (Brainard and Tobin 1968). The latter can be rationalised by generalising the quadratic cost of adjustment function of Christofides (1976):

$$L^* = (s - s^*)_t' C_1 (s - s^*)_t + \Delta s_t' (C_2) \Delta s_t - (\Delta s_t) C_3 (\Delta s_t^*) \quad (5)$$

where C_i ($i = 1, 2, 3$) are conformable adjustment matrices. Minimising L^* with respect to s_t subject to the budget constraint we obtain generalised error feedback equations:

$$\Delta s_t = \Pi^* \Delta X_t + L (s - s^*)_{t-1} \quad (6)$$

where the disequilibria in $(k-1)$ asset shares at time $t-1$ influence the current period adjustment of any particular asset share. Since

$\sum_{i=1}^k (s_i - s_i^*)_{t-1} = 0$, only the $(k-1)$ independent disequilibrium shares are required in (Anderson and Blundell 1983) (6) (ie L is $(k \times (k-1))$). The adding up restrictions imply that the columns of Π^* and L sum to zero. In addition on intuitive grounds we might expect the diagonal elements of L to be negative. However the latter is not required for dynamic stability. As long as the eigenvalues of the appropriate adjustment matrix have modulus less than unity, then the system is dynamically stable.⁽⁴⁾

The above adjustment equation could be further generalised by including additional lagged own and cross-shares that is, terms in Δs_{t-j} and ΔX_{t-j} ; although the dynamics are then not consistent with the quadratic cost of adjustment function (5). Indeed one can also interpret (6) merely as a reasonably parsimonious method of incorporating dynamics while maintaining the adding up restrictions but without alluding to the cost function (5). We feel that quadratic costs are very unlikely to apply to adjustments in financial assets (eg costs are unlikely to rise smoothly with the size of the transaction undertaken) and we therefore interpret equation (6) merely as a convenient method of characterising sluggish adjustment (Hendry et al 1984). In principle it is possible to generalise (5) to yield a multi-period interdependent costs of adjustment function but this rapidly becomes intractable (Currie and Kennally 1985), and still retains the unrealistic quadratic form.

Drawing on (3) and (6), the short run equations may be represented by the following interdependent EFE:

$$\Delta s_t = C \Delta \ln p_t^r + K \Delta R W_t + F \Delta X_{1t} + L (s - s^*)_{t-1} \quad (7)$$

where C , K , F and L are suitably dimensioned matrices of short-run parameters. Substituting for s_{t-1}^* from (3) in (7) then gives our set of non-linear estimation equations. The advantage of (7) over an unrestricted ADL equation is that we can easily test and impose restrictions on the long-run parameters

(eg homogeneity, symmetry).⁽⁵⁾ In contrast Weale (1986) using an autoregressive distributed lag, ADL, system is only able to impose and test short-run symmetry and homogeneity and these properties do not hold in the long run, about which economic theory is thought to be more informative.⁽⁶⁾ We are also able to test the long-run parameters obtained to see if they form a co-integrating vector (Hall 1986, Hendry 1986, Granger 1986).

We use a general to specific modelling strategy with equation (7) as our initial maintained hypothesis without any restrictions on the parameter matrices (other than the adding-up restrictions to obtain data admissability). The theoretical structure imposed by adopting the system approach therefore limits the extent to which one can indulge in 'overfitting' and data-mining. The final parsimonious system of equations is subject to the usual test procedures (although understandably these are not as numerous as found in single equation studies).

The equations are estimated by 3SLS (Zellner and Theil 1962) treating current period prices, wealth and expenditure as endogenous. The instruments used are two lagged values (t-1, t-2) of all prices, wealth, expenditure and asset shares. Corrections for serial correlation in systems of equations are not possible with our current software (Berndt and Savin 1975) but because of our flexible lag response this was not found to be an acute practical problem.

Restrictions on parameters are tested using Wald tests and a quasi-likelihood ratio test statistic QLR.⁽⁷⁾ (QLR has degrees of freedom equal to the number of independent restrictions.) When using IV the likelihood ratio test (Gallant and Jorgenson 1979) is

$$QLR = T (Q_0 - Q_1) \quad (9a)$$

Where Q_0 (Q_1) is the value of the minimum distance criterion for the null (maintained) hypothesis. For a vector of parameters b , a stacked vector of residuals e , with variance-covariance matrix Σ and projection matrix of instruments P_z then:

$$Q = e' (\Sigma^{-1} \Theta P_z) e \quad (9b)$$

When using QLR, the same set of instruments must be used under both H_0 and H_1 and Σ must be held fixed at its value under the maintained hypothesis.

In testing parameter constancy we have used a systems analogue to the Salkever (1976) test which allows the use of a fixed instrument set and fixed variance covariance matrix. For testing parameter constancy over r additional periods we augment each equation with r (...0, 1, 0 ...) dummies. The coefficients and t-statistics on the dummies then yield estimates of the outside sample forecast errors and their statistical significance. A Wald test on all the dummies yields an asymptotically valid test of the parameter stability in the system as a whole. The dummies are included in the instrument set.

IV DATA USED

The asset categories modelled are 'short-term' assets of the company sector namely:

"M1" = transactions balances (= notes and coin + sterling sight deposits)
 TD = "time deposits" (= sterling time deposits + building society deposits +
 local authority temporary debt)
 PSL = public sector long-term debt
 FCD = foreign currency deposits

In order to apply the above model to the company sector we delineate the problem by assuming decisions concerning the portfolio of 'short-term' assets are weakly separable from other asset decisions (and real decisions). We also assume weak intertemporal separability.⁽⁸⁾ This allows our set of short-term assets to depend only on interest rates within this set of assets and the total wealth allocated to these assets. Formal non-parametric tests of separability (Varian 1983, Swofford et al 1986) are beyond the scope of the present paper. Evidence in (Mayer 1988 and Chowdhury et al 1987) provide some evidence that this asset split may be a reasonable working hypothesis.

The flow data is taken from the 'Flow of Funds' matrix in Financial Statistics. Revaluation indices are chosen to be consistent across sectors of the complete matrix. Benchmark stocks are then chosen such that all elements in the matrix satisfy the accounting identities (ie zero row-sums and column sums equal to the NAFA). The data on company sector assets used here therefore comes from a fully consistent complete stock-flow matrix. Data on the sight-time deposit split is only available from 1975(3).

The mean shares for our asset categories over the period 1977-1986 are M1 (23%), TD (55%), PSL (5%), FCD (17%). However, there has been a relatively

large secular fall in TD from 63% in 1977 to around 48% in 1986 and this mirrored by the rapid rise in FCD after 1979 from a share of 8% to 23% by 1983/4. Tax instruments were initially included in the asset set but proved problematic, probably because of the complexity of the 'true' rate of return (see Jackson 1984).

The rate of return on M1 is the negative of the inflation rate (of the price of total final expenditure, TFE) and for TD the own rate is taken to be the rate on "parallel money-market" assets, namely the 3 month rate on Local Authority bills. Various rates of return for PSL and FCD were tried all of which included the running yield and capital gains. The running yield plus a 3 year backward looking capital gain is used for the return on PSL and FCDs. The F-T actuaries price index for all government stock and the running yield are used for PSL while the return on FCD is the yield on 3 month dollar deposits in London plus capital gains due to changes in the dollar-sterling exchange rate. (A one quarter ahead and 1 year ahead capital gains variables were also tried but gave very un-satisfactory results). The expenditure variable is real TFE.

All data used are seasonally unadjusted but seasonal dummy coefficients are not reported. The regressions are run over the period 1976(4)-1986(4). Critical values of test statistics are given at a 5 percent significance level (unless stated otherwise).

V EMPIRICAL RESULTS

In reporting on a system of equations there is a danger in overwhelming the reader with a plethora of results and tests. We have therefore chosen to concentrate on the details of our preferred equations and their economic interpretation. We then discuss tests of parameter stability and other variants considered.

Unrestricted EFE

In the unrestricted EFE (equation 7) long-run and short-run own price effects are all negative. We can easily accept the hypothesis of long-run, homogeneity and the hypothesis of long-run symmetry plus homogeneity (Table 1). The model therefore satisfies two of the basic axioms of rational choice in demand theory. Short-run homogeneity (conditional on long-run

symmetry and homogeneity holding) and short-run symmetry are also accepted on a Wald test.⁽⁹⁾ Finally a Wald test that both long-run and short-run symmetry and homogeneity hold simultaneously is not rejected (table 1).

In the next stage of our simplification procedure we therefore impose long-run and short-run symmetry and homogeneity. We then sequentially exclude coefficients that have both low t-statistics and a relatively small numerical impact (Table 2). These exclusion restrictions are then tested as a group. Setting the long-run price coefficient $\gamma_{34}=0$, long-run expenditure effects in TD and PSL to zero ($J_{21}=J_{31}=0$), the corresponding short-run coefficients C_{34} , F_{21} , F_{31} to zero and K_2 , L_{23} , L_{24} , L_{32} , L_{34} , L_{43} to zero yields a Wald test statistic $W(12) = 8.1$ ($\chi^2_C = 21$). Of the remaining elements of the short-run price matrix C_{24} and C_{44} are then set to zero, $W(2) = 0.2$ ($\chi^2_C = 6.0$) yielding the preferred parsimonious equation of Table 2. The zero restrictions in Table 2 are also not rejected on a QLR test ($QLR(14) = 10.1$ $\chi^2_C = 23.7$).

Preferred Long-run Results (Table 2A)

The estimated parameters of the AIDS model (Table 2) are not very informative in terms of the usual economic concepts (eg elasticities) but the following general features are of interest.

The "own rate" price coefficients, γ_{ii} , are negative and greater in absolute value than the cross-rate effects, γ_{ij} 's ($i \neq j$). The γ_{ij} ($i \neq j$) for TD, PSL and FCD are positive indicating these assets are substitutes. An increase in inflation (for unchanged real prices on TD, PSL and FCD) leads to a fall in the share of M1 and a rise in the share of TD, PSL and FCD (Column 1, Table 2A). At any given level of wealth, a rise in domestic expenditure, y , causes a shift out of FCD into sight deposits (M1). The matrix of ' k_{ij} coefficients', evaluated at mean values of the variables, (see equation 3d) has all eigenvalues negative and is therefore negative semi-definite.

The semi-elasticities with respect to the annual rates of return, the elasticity with respect to wealth and expenditure (Table 3A) are shown together with the long-run impact on the absolute level of asset holdings (Table 3B).

The own rate semi-elasticities of TD, PSL and FCD are 2.3, 3.4 and 1.7 respectively, and are larger (in absolute value) than the cross rate semi-elasticities (this can be seen by looking across any row of interest rate effects in Table 3A).

The elasticity of M1 with respect to the annual inflation rate (g_2^a) is -5.6 while for TD, PSL and FCD the elasticities are 1.8, 1.2 and 1.3 respectively. If inflation increases there is a move away from M1 and into other assets. This, in part, reflects a move out of the non-interest bearing element M1 which now has a relatively higher negative return in terms of future purchasing power over goods.

It can also be seen from Table 3B (by looking down the columns R_2 , R_3 , R_4) that the effects of a change in any interest rate have a larger impact on the absolute value of the own asset, than on the 'other' assets. There is substantial switching between M1 and TD (Column 2, Table 3B) and between FCD and the capital certain domestic asset M1. (Column 4, Table 3B). These results are intuitively plausible although not strictly required by the theoretical model.

The long-run wealth and expenditure effects are shown in columns 5 and 6 of tables 3A and 3B. The wealth elasticity of TD (0.82) exceeds that for M1 (0.39) and FCD are a 'luxury good'. In the long run any additional wealth is held primarily in TD and FCD. At higher levels of domestic expenditure there is a switch from FCD to M1 with elasticities 2.2 and -1.8 respectively. The long-run wealth and expenditure effects are well determined statistically (Table 2) and yield plausible parameter values.

Considering the large number of parameters, even in this preferred equation the statistical significance of the individual parameter estimates particularly those on the price variables are encouraging. The R^2 for the preferred equations are reasonable given we are explaining the change in shares (Table 2C). The Box-Pierce statistics indicate the absence of serial correlation of up to order 4 (but there is a hint of 7th order negative serial correlation in equations 1 and 3).

Preferred Short-Run Equations

We do not analyse the short-run parameters in as greater detail as the long-run parameters. What is of key importance is the size and sign of the short-

run parameters relative to those for the long-run equations. All the short-run price effects (C_{ij} 's) are smaller than their long-run values (Table 2B). There is no short-run response of any asset to changes in the return to FCD (Column 4, Table 2B) and this may reflect a combination of the volatility of the uncovered return, risk aversion and transactions costs. The C_{ij} parameters are reasonably well determined statistically, given that we are modelling changes in shares and we have imposed symmetry and homogeneity restrictions on the C_{ij} matrix.

The point estimates of the short-run real wealth effects (Table 2B) are larger than their long-run values for TD and FCD. For example, a £100mn increase in current period wealth leads to an immediate increase of TD and FCD of £55mn and £52mn respectively: lightly larger than the long-run effects of £45mn for both assets. The point estimates of the short-run expenditure effects are smaller than their long-run values (final column, Tables 2A and 2B) and imply a switch of £99mn between FCD and M1 for a one percent increase in expenditure (the long-run effect is £138mn).

The matrix of adjustment parameters (Table 2C) has all diagonal elements negative, indicating that excess holdings of asset 'i' leads to a fall in the share of asset 'i' in the subsequent period. The eigenvalues of the (augmented) L matrix have 'real parts' 0.4, 0.53, 0.11 with zero imaginary parts, indicating a convergent response after a one quarter lag.

Further tests

Using the equations of table 2 we perform tests for a first difference and static model, for long-run homotheticity, weak separability, parameter stability and for a long-run co-integrating vector. We report each of these in turn and the results are summarised in Table 4.

The first difference model is a test that the L_{ij} coefficients of Table 2C are jointly zero and is decisively rejected [$W(4)=60.2$ ($\chi^2_5=9.5$)]. The static model suffers from severe first order serial correlation which is indicative of mis-specified dynamics and can be rejected on this count alone.⁽¹⁰⁾ Long-run homotheticity holds if all long-run wealth elasticities are unity: this is decisively rejected [$W(3)=49.1$, $\chi^2_3=7.8$].

We performed a (somewhat ad-hoc) test for separability by including a lagged levels and first difference term in the 'price' of bank loans. (The interest

rate used is the clearing banks base rate). Bank lending is perhaps the most likely asset one might expect not to be weakly separable from the short-term assets modelled in this paper (Sprenkle and Miller 1980, Spencer 1986). The additional variables in the 'price' of bank loans are insignificant ($W(6)=5.0$, $\chi^2_C=12.6$) as a group and no individual asymptotic t-statistic exceeds 1.4. Hence we accept that the assets modelled are weakly separable from bank loans.

Parameter stability is accepted over the period 1985(1) - 1985(4) ($W(12)=15.2$, $\chi^2_C=21$) but is marginally rejected for the period 1985(1)-1986(4) [$W(24) = 43.7$, $\chi^2_C = 36.4$]. The rejection is due to the statistically significant dummy variables for the TD equation in 1986(1)-1986(4). However, the long-run and short-run parameters are largely qualitatively unchanged over the two periods ending in 1984(4) and 1985(4) (the exception is the long-run own price effect on FCD which becomes statistically insignificantly different from zero).

The variables in the long-run equations appear to be I(1) (Table 5).⁽¹¹⁾ The long-run parameters of our preferred equations of Table 2 are used to construct the static long-run residuals which are then subject to the Dickey Fuller (DF) and Augmented Dickey-Fuller (ADF) tests for stationarity.

If $\hat{\Pi}$ are the long-run parameter estimates, X the matrix of long-run variables and s the vector of asset shares, the co-integrating residuals are $e=s-\hat{\Pi}X$. The test for stationarity in the residuals of equation i (i=1, 2, 3, 4) uses:

$$\Delta e_{it} = \theta_i e_{it-1} + \sum_{j=1}^4 \psi_j \Delta e_{it-j}$$

The residuals are stationary for $\theta_i < 0$ and $|t_{\theta}| > 4.0$, but critical values are only approximate (Engle and Yoo 1987). For the equations for TD and FCD the test statistics (Table 4) are somewhat below their critical values although the point-estimates of θ_i are always negative. But given that the tests have low power against highly dynamic stationary alternatives (Engle and Granger 1987) the results are indicative of stationary residuals and the existence of a co-integrating set of parameters (which may not be unique, Johansen 1988).

VI SUMMARY

We have estimated demand functions for short-term assets of the UK company sector within a coherent theoretical framework provided by the AIDS model. Dynamics are adequately modelled using a systems error feedback approach which allows tests of and restrictions to be imposed on long-run parameters across different equations. The estimated model satisfies the theoretical restrictions of homogeneity, symmetry and negativity and broadly characterises the data. The AIDS model, the error feedback approach and the allied concepts of co-integration have therefore provided a useful methodology for examining systems of asset demand equations.

Table 1: Wald Tests of Homogeneity and Symmetry

<u>Test</u>		<u>Wald Statistic</u>	<u>Critical Value</u>
1	LR homogeneity (without symmetry)	1.1	7.8
2	LR symmetry and homogeneity	2.5	12.5
<hr/>			
Conditional on LR symmetry and homogeneity; Test of:			
3	SR homogeneity	2.3	7.8
4	SR symmetry and homogeneity	10.2	12.5
<hr/>			
5	LR and SR symmetry and homogeneity	17.8	21.0

Table 2: Preferred Error Feedback Equation (LR and SR Symmetry and Homogeneity Imposed) ⁽¹⁾

A Long-run parameters

<u>PRICE MATRIX Γ</u>					<u>WEALTH</u>	<u>EXP</u>
	g_z	$\ln p_2^T$	$\ln p_3^T$	$\ln p_4^T$	RW	y
1. M1	-5.19(2.0)	4.04(2.4)	0.24(0.3)	0.91(3.8)	-0.14(3.4)	0.51(4.0)
2. TD	4.04	-4.93(4.2)	0.49(1.0)	0.40(1.4)	-0.10(3.6)	0*
3. PSL	0.24	0.49	-0.73(1.2)	0*	-0.055(3.7)	0*
4. FCD	0.91	0.40	0*	-1.31(3.9)	0.295(6.0)	-0.51(4.0)

B Short-run parameters

<u>PRICE MATRIX C</u>					<u>WEALTH</u>	<u>EXP</u>
	Δg_z	$\Delta \ln p_2^T$	$\Delta \ln p_3^T$	$\Delta \ln p_4^T$	ΔRW	Δy
1. M1	-3.48(1.5)	3.22(1.8)	0.26(0.4)	0*	-0.31(3.5)	0.37(3.1)
2. TD	3.22	-3.40(2.2)	0.18(0.5)	0*	0*	0*
3. PSL	0.26	0.18	-0.44(1.1)	0*	-0.04(1.5)	0*
4. FCD	0*	0*	0*	0*	0.35(4.2)	-0.37(3.1)

C Adjustment matrix and diagnostics

<u>ADJUSTMENT MATRIX, L</u>						
<u>Lagged Disequilibria L</u>				<u>Diagnostics</u>		
	2.	3.	4.	R^2	BP (4) ⁽²⁾	BP (8) ⁽²⁾
1. M1	1.1(5.5)	0.47(3.3)	0.89(4.5)	0.22	5.3	15.6
2. TD	-0.6(5.1)	0*	0*	0*	0.42	5.411
3. PSL	0*	-0.47(3.3)	0*	0.33	8.3	17.7
4. FCD	-0.5(2.6)	0*	-0.89(4.5)	0.32	6.5	14.1

(1) Asymptotic t-statistics in parenthesis. Imposed coefficients are starred.

(2) BP(k) is the Box-Pierce statistic for serial correlation of order 1 to k. Under the null of no serial correlation it is asymptotically distributed as central chi-squared with k degrees of freedom. Critical values at 5 percent significance level are $\chi_C^2(4)=9.5$, $\chi_C^2(8)=15.5$.

Table 3A

LONG-RUN ELASTICITIES ⁽¹⁾

	g_z^a	R_2	R_3	R_4	RW^2	EXP^3
1. M1	-5.6	-1.9	-0.27	-1.0	0.39	2.2
2. TD	1.8	2.3	-0.22	-0.22	0.82	0*
3. PSL	1.2	-2.5	3.4	0*	0.1	0*
4. FCD	1.3	-0.7	0	1.7	2.7	-1.8

1. The first 4 columns show the effect of a one percentage point change in g_z^a (the annual inflation rate), and the annual rate of return R_i ($i=2,3,4$) on the percentage change in asset holdings (ie $(\Delta a_i/a_i)*100$).

2. Wealth elasticity [ie $(\Delta a_i/a_i)(\Delta RW/RW)^{-1}$].

3. Expenditure elasticity [ie $(\Delta a_i/a_i)(\Delta y/y)^{-1}$]

Table 3B

LONG RUN IMPACT ON ASSET HOLDINGS (£mn) ⁽¹⁾

	g_z^a	R_2	R_3	R_4	RW^2	EXP^3	MF^5
1.	-350	-273	-16	-61	9	138	270
2.	273	333	-33	-27	45	0*	450
3.	16	-33	49	0*	1	0*	24
4.	61	-27	0	88	45	-138	278

1. The first 4 columns show the effect of a one percentage point change in g_z^a (the annual inflation rate) and the annual rate of return R_i ($i = 2,3,4$) on the holdings of the i^{th} asset Δa_i (£mn).

2. Impact on asset holdings (£mn) of £100mn increase in wealth.

3. The impact of a one percent change in expenditure on asset holdings, Δa_i (£mn).

4. The quarterly mean flow (£mn) into asset i over the period 1982-86.

Table 4: Further Tests

		<u>Wald Test</u>	<u>Critical</u>		
		<u>Statistic</u>	<u>Value</u>		
1.	1st difference model	60.2	9.5		
2.	LR homotheticity	49.1	7.8		
3.	Weak separability	5.0	12.6		
4.	Parameter Stability 85(1)-85(4)	15.2	21.0		
	85(1)-86(4)	43.7	36.4		
5.	Co-integration test on long-run parameters in:				
		<u>DF</u>	<u>ADF</u>		
		θ_i	t_θ	θ_i	t_θ
	M1	-0.31	-3.6	-0.28	-1.7
	TD	-0.18	-2.4	-0.22	-2.3
	PSL	-0.51	-3.8	-0.64	-2.5
	FCD	-0.12	-2.2	-0.15	-2.1

Table 5: Order of Integration of the Variables⁽¹⁾

	<u>I(0)</u>		<u>I(1)</u>	
	<u>DF</u>	<u>ADF</u>	<u>DF</u>	<u>ADF</u>
s ₁ (M1)	-2.0	-1.3	-7.5	-2.1
s ₂ (TD)	-1.6	-1.2	-6.6	-3.2
s ₃ (PSL)	-1.4	-0.3	-6.4	-3.8
s ₄ (FCD)	-1.1	-1.1	-5.9	-2.0
y (TFE)	+0.7	+0.3	-13.5	-4.0
RW	+0.8	+0.1	-4.9	-3.0
lnp ₁ ^T	-2.2	-2.1	-9.9	-3.4
lnp ₂ ^T	-2.5	-1.6	-10.8	-3.7
lnp ₃ ^T	-2.2	-2.2	-10.1	-3.4
lnp ₄ ^T	-2.0	-2.4	-9.8	-2.8

(1) The critical values at 5 percent significance level for DF and ADF are about 2.8. (Dickey and Fuller 1979).

FOOTNOTES

- (1) One can argue that ultimately utility depends only on current and future consumption. However, in the absence of fully contingent binding contracts, when saving takes place, agents must hold some asset stocks, and it is reasonable to assume that agents are not indifferent to the composition of their assets. Hence asset holdings represent purchasing power over future consumption goods. Any factors not in the wealth constraint that influence the composition of such assets, will implicitly be captured in the parameters of the utility function. For an alternative derivation see Taylor and Clements (1983).
- (2) The cost function is also usually assumed to be continuous in prices and that the first and second derivatives with respect to prices exist.
- (3) Note that 'adding up' and symmetry imply homogeneity. (Although homogeneity and 'adding up' do not imply symmetry.)
- (4) If the disequilibrium term for asset 1 is excluded then the estimated adjustment matrix is:

$$L = (\underline{l}_2, \underline{l}_3, \dots, \underline{l}_k) \quad k \times (k-1) \quad .$$

The dynamics of the full model may be written

$$s_t = (I_k + L^*)s_{t-1}$$

Where

s_t is $k \times 1$

$L^* = (i, \underline{l}_2, \underline{l}_3, \underline{l}_4)$ is $(k \times k)$

$i = (1, 0 \dots 0)$ is $(k \times 1)$

$I_k = k \times k$ identity matrix

One of the eigenvalues of $(I_k + L^*)$ is unity and stability requires that the other $(k-1)$ eigenvalues have negative real parts.

- (5) However one of the practical disadvantages for our particular model is the need to provide 'good' starting values for the parameters, to ensure convergence in the non-linear system.

- (6) An alternative is to use the procedure in Bewley (1979).
- (7) In heavily parameterised systems estimated with a relatively modest number of observations, there is likely to be over-rejection of null hypotheses if asymptotically valid test statistics are used (eg Deaton 1974, Bewley 1983). In general it appears that over-rejection becomes more severe as the number of equations in the system increases or as the sample size becomes smaller. Small sample adjustments are problematic in dynamic simultaneous systems but Pudney (1981) suggests an adjusted quasi-likelihood ratio test statistic:

$$QLR^* = QLR + nT \ln \{ (nT - p_1) / (nT - p_0) \}$$

where:

LR = likelihood ratio statistic

n = number of estimated independent equations

T = total number of observations

p_1 = number of parameters under H_1

p_0 = number of parameters under H_0 .

Clearly $QLR^* < QLR$. However we find that for acceptance of the null hypothesis under consideration we do not need to make such adjustments.

- (8) For AIDS models that relax this assumption see Weissenberger (1986) and Rossi (1987).
- (9) There is little economic rationale in testing short-run homogeneity and symmetry conditional on these restrictions not holding in the long-run. However for the record this null hypothesis is accepted on a Wald test $W(6) = 12.2$ ($\chi^2_C = 12.5$).
- (10) The Box-Pierce, BP(1), statistics for the residuals from the static regressions are 9.4, 12.4, 7.8, 21.8 respectively.
- (11) The Dickey-Fuller (DF) and ADF statistics (Table 5) clearly indicate that none of the series are $I(0)$ (Dickey and Fuller (1979)). In testing for $I(1)$ series some of the ADF tests are below their critical values but the DF statistics clearly indicate that the variables are $I(1)$. Given the small sample available we take these results to indicate $I(1)$ variables.

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