Bank of England

Discussion Papers

Technical Series

No 3

Composite monetary indicators for the United Kingdom; construction and empirical analysis

by

T C Mills

May 1983

↑r → ↑ weights on cash + nib and ↓ its rate of growth relative to other deposits (∵ substitute out).

If ↑r → ↓ activity + ↓ p in future then Divisia acts as a better leading indicator?

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The object of this Technical Series of Discussion Papers is to give wider circulation to econometric research work predominantly in connection with revising and updating the various Bank models, and to invite comment upon it; any comments should be sent to the author at the address given below. The views expressed are his, and not necessarily those of the Bank of England.

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Introduction

Monetary policy in the UK is widely regarded as an important instrument for the attainment of certain macroeconomic objectives and, as a consequence, various measures of money have been monitored over the last decade by HM Treasury and the Bank of England. Focussing attention on a range of monetary aggregates has been necessitated both by the lack of a clear theoretical definition of money and by the divergent historical behaviour of the financial assets used in constructing empirical definitions. This can clearly be seen from Charts 1 and 2, which plot, respectively, the time paths and growth rates of the asset groupings making up the broadest monetary aggregate currently available, PSL2(1).

Much less attention, however, has been paid to the way in which the individual assets are combined to form monetary aggregates. Aggregating assets by simple summation has been the traditional method, thus defining the quantity of money as the weighted sum of the total value of all assets, the assigned weights being either unity, if the asset is included in the definition, or zero if it is excluded.

With the recent publication of time series data on a wide set of financial assets, detailed examination of the joint questions of the appropriate definition of money and the correct method of aggregation has become possible. This paper presents the results of considerable research into these two questions.

Section 2 sets out and discusses the data base used in this research. One approach to the definition of money and the associated method of aggregation is to define money empirically as that weighted collection of assets which is the best predictor of a goal variable, say nominal income. Standard regression techniques may then be employed to determine the appropriate weighting scheme. Section 3 presents results using this methodology and also utilises information theoretic concepts to ascertain how much information on movements in nominal income is contained in the set of financial asset measurements and to assess how much of this information is lost by first restricting attention to subsets of these component assets and then summing these subsets to form the traditionally

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(1) Charts are contained in Appendix B.

defined monetary aggregates. Section 4 then provides alternative empirical definitions of money by constructing weighted aggregates using principal component analysis.

Such empirical definitions may be criticised as being essentially ad hoc and having no basis in economic theory. For 'money' to be an economic good measuring the economy's transactions services, it must be capable of being treated as a single quantity which can be selected without regard to the quantities of the assets over which it is defined. The level of transactions services provided by these assets is then a function of the asset quantities alone, this function being known as an aggregator function. The traditionally constructed aggregates assume that this aggregator function is a simple sum, thus implying that all assets included in the aggregate are perfect substitutes for each other but have zero substitutability with respect to all excluded assets. Section 5 develops this line of argument by assuming that all assets provide both liquidity (ie transactions) and store-of-value services to holders, although in different proportions. The broad sum aggregates thus implicitly view distant substitutes for money as perfect substitutes for currency and therefore tend to swamp the included transaction services with heavily weighted store of value services. Narrower aggregates, for example M1, regard assets close to transaction balances as providing no liquidity and hence tend to capture only part of the economy's transaction services. We therefore require aggregates that appropriately capture the contribution of all financial assets to the economy's flow of transaction services. The aggregator function that will accomplish this, known as an economic quantity aggregate, will rarely be available in practice. In principle, the function could be specified and estimated and attempts have been made to do so, but aggregates depending on estimated parameters are rarely thought satisfactory, particularly if they are to be regularly published by government agencies.

Alternatively, statistical quantity index numbers may be constructed. Such index numbers contain no unknown parameters but do contain prices. Thus to use index number theory to construct monetary quantity aggregates, price data are required for each component asset. Section 6 constructs such prices from interest rate data using an analogous theory to that of deriving the rental price or user cost of a durable good. Having available both quantity and price data for the component assets, a suitable index number may then be selected. This may be chosen from a class of index numbers having desirable properties and which are related to the aggregator functions introduced in Section 5. A particularly attractive index number in the present context is the Divisia, and Section 7 uses this index to construct a hierarchy of monetary quantity indices. Dual to a quantity index is a price index, and the corresponding hierarchy of monetary price indices are also constructed. These indices may be interpreted as the opportunity cost or "own price" of their dual quantity index.

The historical behaviour of these sets of indices and associated velocities are discussed in Section 8, comparing the time paths of the Divisia quantity indices and velocities with the evolution of the corresponding traditionally defined sum aggregates.

Time series representations of the sum aggregates and the Divisia indices and velocities are provided in Section 9. Univariate models of these series are obtained and their decomposition into trend, seasonal and irregular components is performed.

Section 10 estimates feedback measures between the Divisia monetary aggregates and nominal income and its price and output components. These measures are compared and contrasted with those obtained using the traditional sum aggregates. Demand functions for the hierarchy of Divisia monetary quantity aggregates are developed in Section 11, utilising the availability of the dual price indices to incorporate proper opportunity cost variables into the analysis.

The trend components obtained from the time series decompositions of the monetary aggregates are used in Section 12 to illustrate the implications of the quantity theory as a proposition concerning the long run behaviour of money and income. Section 13 finally draws together this battery of results to present overall conclusions regarding the appropriate definition of a monetary aggregate.

2

The Data Base

The components of "private sector liquidity" used in the construction of the PSL1 and PSL2 aggregates are available on a quarterly basis from 1963 and these series, plus the additional components required to construct the M3 aggregate, form the data base for the empirical work undertaken in this paper(1). The full set of monetary components, and the sum aggregates constructed from them, are listed as Table 1(2). These components and aggregates are used in the work reported in Section 3. Certain of the components have almost identical characteristics and in subsequent analysis these have been combined. The resulting set of assets, along with alternative aggregate groupings, are shown in Table 2. An interest rate can be identified with each of these assets and these are also shown in Table 2.

In the empirical work reported, various other macroeconomic time series are used. Gross domestic product at factor cost is employed as the proxy for nominal income and in the demand functions developed in Section 10, the price and output components of this series are also used. In the work of other sections, the retail price index is also employed.

All series are seasonally unadjusted and the data period ends in 1981 Q4, giving a total of 76 observations. During 1981, however, definitional changes were made to some of the monetary components and certain distortions to the data arose as a result of the civil servants' dispute(3). As a consequence, the data period for many of the empirical exercises was terminated at 1980 Q4, although all aggregates were constructed up to the end of 1981.

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(1) See Financial Statistics, August 1980 and subsequent issues.

(2) Tables are contained in Appendix A.

(3) The definitional changes are set out in the Bank of England Quarterly Bulletin, December 1981, pages 531-9.

3

Information Content and Aggregation Properties of the Monetary Components

Official targets for UK monetary growth have been announced since mid-1976, initially for M3, but, from December 1976, in terms of £M3. In the Government's medium-term financial strategy (MTFS) of 1980, future targets were set and it was stated that, although the growth of the money stock would be progressively reduced, the aggregate being targeted might be altered from time to time as circumstances change. Such an alteration occurred in March 1982 when a move was made towards monitoring a range of aggregates rather than simply £M3.

For monetary targeting to be successful, a chosen aggregate should satisfy two principal requirements. First, there should be a sufficiently well-established link between the aggregate and the goal variable the authorities wish to influence. This will permit a target growth path for the aggregate to be determined which will be consistent with a particular growth path of the goal variable. The second requirement is that the growth of this aggregate should be sufficiently sensitive to the actions of the authorities to allow the target growth path to be achieved quickly and smoothly.

This and the succeeding section provide a quantitative assessment of the alternative aggregates in terms of their ability to predict movements in nominal income, this variable being chosen because it is that considered in theoretical work on the role of monetary policy [Friedman (1970)] and has also been proposed as an appropriate goal variable in the UK by Brittan (1981). Almost identical qualitative, and very often quantitative, results were obtained for the price level, this being the goal variable stated in the MTFS.

In carrying out this assessment, the present section first concentrates attention on the behaviour of the individual monetary components underlying the constructed aggregates and on the role of aggregation itself. The information content of the monetary components in terms of their collective ability to predict movements in nominal income is calculated and an evaluation is then made of how much of this information is lost by first restricting attention to subsets of these components and then summing the components of these subsets to obtain the traditionally defined sum aggregates.

The information measure used is a variant of that introduced by Tinsley et al (1980) and is developed in Mills (1983a, b), the results of which are now summarised.

The information content of the vector of monetary components $\underline{c} = (c_1,...,c_k)$ with respect to a single goal variable y at a response lag d, denoted $I_d\ (y/\underline{c})$, is defined as:

$$I_d\ (y/\underline{c}) = -\ 1/2\ \ln\ [1 - R^2*(d)]$$

where $R^2*(d)$ is the Pierce (1979) multiple correlation coefficient measure obtained from the regression

$$y_t = \sum_{r=1}^{n} \alpha_r\, y_{t-r} + \sum_{i=1}^{k} \beta_i\, c_{i,t-d} + u_t \qquad t=1,\ 2,\ ...\ T \qquad (3.2)$$

Models of this form were estimated with nominal income as y and response lags running from d = 0 to 9.

Table 3(a) shows both the information content measures and R^2* statistics so obtained. Since $R^2* = 0$ corresponds to the restriction $\beta_1 = ... = \beta_k = 0$, the significance of the R^2* values can be examined by conventional testing procedures, the critical values being shown in parentheses. All R^2* statistics are significant, thus implying that the set of current monetary component measurements do contain significant information concerning future movements in nominal income.

Concentrating attention on a subset of the components, $\underline{c}^{(j)} = (c_1,\ ...,\ c_j)$, j < k, say, imposes the set of k - j restrictions

$\beta_{j+1} = ... = \beta_k = 0$ on (3.2). The information loss is then given by:

$$L(y/\underline{c}^{(j)} : \underline{c}) = 1/2\ \ln \left[\frac{1 - R^2*(j)}{1 - R^2*} \right] \qquad (3.3)$$

where $R^2*(j)$ is obtained from the restricted regression.

This information loss may be tested for statistical significance by noting that $2T.L(y/\underline{c}^{(j)} : \underline{c}) \sim \chi^2(k - j)$ on the null hypothesis $\beta_{j+1} = \ldots = \beta_k = 0$.

If the subset of components $\underline{c}^{(j)}$ are then summed to obtain the aggregate $M^{(j)} = \sum_{i=1}^{j} c_i$, the further set of j restrictions $\beta_1 = \beta_2 = \ldots = \beta_j$ are imposed on (3.2). Since these two sets of restrictions form a nest of hypotheses, they can be tested sequentially [see Mizon (1977) and Mills (1981a)].

Table 3(b) shows the consequences of restricting attention to subsets of $\underline{c}$. Restricting attention to anything less than the components of the broadest aggregate, PSL2, involves significant, and, for the narrower aggregates, almost complete, information loss. The consequences of constructing sum aggregates from these subsets are shown in Table 3(c). Because of the nesting of the implicit sets of hypotheses contained in sum aggregation, a significant information loss in Table 3(b) implies a significant information loss for sum aggregation and hence the incremental information loss defined in Table 3(c) need not be evaluated. For those that do require calculation, all incremental information losses are significant, thus rejecting the hypotheses of sum aggregation at the second stage of the sequential testing procedure(1).

These results therefore show that, if information content (or predictive power) with respect to a chosen goal variable is the relevant criterion for constructing a monetary aggregate, simple sum aggregation is an inappropriate procedure to adopt. The above analysis suggests the possibility of using information content to construct weighted sum aggregates based upon the estimated coefficients of (3.2). Such a procedure is, however, infeasible here because severe collinearity between the components of $\underline{c}$ prevents precise coefficient estimation, and a lack of degrees of freedom requires a set of response lags to be used rather than distributed lags on the c_i components.

As an alternative approach, the following models were considered (noting that £M3 and M3 form a separate nest to that of PSL1 and PSL2),

$$\alpha_1(B)y_t = \beta_1(B)\, M1_t + \gamma_1(B)(\pounds M3_t - M1_t) + \delta_1(B)(M3_t - \pounds M3_t) + u_{1t} \quad (3.4)$$

$$\alpha_2(B)y_t = \beta_2(B)\, M1_t + \gamma_2(B)(PSL1_t - M1_t) + \delta_2(B)(PSL2_t - PSL1_t) + u_{2t} \quad (3.5)$$

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(1) It is often argued [see eg Artis and Lewis (1981, ch 4)] that the choice of £M3 was dictated partly by the fact that a change in £M3 can be decomposed into a set of counterparts which reflect the policy operations of the authorities, and hence provides a link between fiscal and monetary policy. Mills (1983a) shows that the implicit aggregation of these counterparts into £M3 also involves a considerable and significant information loss.

where the $\alpha_i(B)$ etc are polynomials in the lag operator B.

In equation (3.4), for example, the hypotheses H_1: $\gamma_1(B) = \delta_1(B) = 0$, H_2: $\beta_1(B) = \gamma_1(B)$, $\delta_1(B) = 0$ and H_3: $\beta_1(B) = \gamma_1(B) = \delta_1(B)$ correspond, respectively, to M1, £M3 and M3 being the appropriate regressors while H_4: $\delta_1(B) = 0$ restricts the model to allow only M1 and (£M3-M1) to appear as regressors. These hypotheses may be tested to determine which, if any, of the "sum aggregation" hypotheses H_1, H_2 and H_3 are data acceptable or whether a "weighted aggregate", H_4 or the maintained model (3.4), is required. Similar sets of hypotheses can be constructed for equation (3.5).

On estimation of these equations, the test statistics shown in Table 4 were obtained. All the sum aggregation hypotheses are rejected at very small marginal significance levels, thus confirming the findings of Table 3, and the only hypothesis not strongly rejected (marginal significance level of approximately .04) is that only M1 and (£M3-M1) are required as regressors in equation (3.4), ie that foreign currency deposits (M3-£M3) do not help in predicting movements in nominal income.

Based on these estimated models, weighted aggregates may be constructed in the following manner [this may be regarded as an extension of the approach of Timberlake and Fortson (1967) and Laumas (1968, 1969)]. The static equilibrium solution of (3.4) is:

$$y = bM1 + c(£M3-M1) + d(M3-£M3) \tag{3.6}$$

where $b = \dfrac{\beta_1(1)}{\alpha_1(1)}$, $c = \dfrac{\gamma_1(1)}{\alpha_1(1)}$ and $d = \dfrac{\delta_1(1)}{\alpha_1(1)}$

We may either normalise by setting the 'moneyness' weight of M1 as unity to obtain:

$$\text{Weighted } (M3)_1 = M1 + \frac{c}{b}(£M3-M1) + \frac{d}{b}(M3-£M3) \tag{3.7}$$

or form either of the weighted averages:

$$\text{Weighted } (M3)_2 = \left(\frac{b}{b+c+d}\right) M1 + \left(\frac{c}{b+c+d}\right)(£M3-M1) + \left(\frac{d}{b+c+d}\right)(M3-£M3) \tag{3.8}$$

$$\text{Weighted } (M3)_3 = \left(\frac{b-c}{b}\right) M1 + \left(\frac{c-d}{b}\right) £M3 + \left(\frac{d}{b}\right) M3 \tag{3.9}$$

Again, similar constructions apply for the results obtained from estimating (3.5).

The static equilibrium solutions are:

$$y = -2250 + 1.46M1 + 0.67\ (£M3-M1) \tag{3.10}$$

and

$$y = -3081 + 1.60M1 + 0.62\ (PSL1-M1) - 0.09\ (PSL2-PSL1) \tag{3.11}$$

After deletion of the small negative coefficient in (3.11), the alternative weighted aggregates so constructed are shown in Table 5.

These weighted aggregates confirm that restricting attention to M1 alone is inappropriate, although it does take the largest weight in all forms. The results suggest that, in predicting future movements of nominal income, both M1 and a broader aggregate, either £M3 or PSL2, should be monitored, with a somewhat greater weight being given to the behaviour of M1.

4

Empirical Definitions of Money

The construction of weighted money aggregates based upon the ability of the components to predict movements in nominal income may be criticised for being an arbitrary, ad hoc procedure. The reduced forms estimated in the previous section may be structurally unstable [Mills (1980a) and Mills and Wood (1978) both emphasise the importance of the exchange rate regime in the interpretation of such models] and may yield biased estimates because of the omission of important additional explanatory variables, for example measures of world trade and prices. Thus the weights entering into the composite aggregates may be both biased and subject to periodic shifts, neither of which will instill great confidence in these aggregates.

An alternative statistical approach is to ask the question: if the variation in the individual monetary components is to be summarised as closely as possible by a linear combination of the components, what is the best linear function? Principal component analysis provides the solution to this question and in so doing furnishes a set of weights by which the components can be aggregated.

Because the components have wide differences in their orders of magnitude, growth rates of the assets listed in Table 2 were used as the data set. Thus principal component analysis was performed on the two sets of assets $\underline{M}_a = (\Delta \ln m_o, \Delta \ln m_1^{**}, \Delta \ln m_2)$ and $\underline{M}_b = (\Delta \ln m_o, \Delta \ln m_1^{*}, \Delta \ln m_4, \Delta \ln m_5, \Delta \ln m_6, \Delta \ln m_7, \Delta \ln m_8, \Delta \ln m_9, \Delta \ln m_{10}^{*})$ where $m_{10}^{*} = m_{10} + m_{11}$, ie analysis is carried out over the assets making up £M3 and PSL2. The first and second principal components, with the associated percentages of variation explained, are shown in Table 6, these being denoted P_a^1, P_a^2 P_b^1 and P_b^2 respectively.

The first principal component may be regarded as a measure of the liquidity services provided by the alternative assets. All factor loadings are positive and have a tendency to be smaller for the more distant substitutes for currency. As these first principal components explain only a modest proportion of the variance of the asset growth rates, the second principal component is of some importance. Interpretation of this component is generally rather difficult, but one possibility here is that it is a measure of the "store of wealth" characteristic of money, perhaps of the "rainy day nest egg" type, as many of the more illiquid assets enter with negative factor loadings.

Table 6 also constructs weighted aggregates of the growth rates, Q_a and Q_b, by scaling the factor loadings to obtain share weights. The asset m_o, transaction balances, is again weighted most heavily, although for the PSL aggregate the weights are spread over a wide spectrum of assets. Even so, the results again are consistent with the rejection of simple sum aggregation as an adequate means of summarising the asset data.

These principal component weighted aggregates were compared with the traditionally defined sum aggregates in their ability to predict movements in nominal income by estimating models of the form(1):

$$\alpha(B) \ln y_t = \theta(B) \ln M_t + u_t \tag{4.1}$$

Equation standard errors and R^2* statistics are shown in Table 7. The Q_a and Q_b aggregates are clearly inferior to all of the sum aggregates, of which £M3 and PSL1 are the best predictors of nominal income. Thus, on predictive criteria, the use of statistical data analysis to construct monetary aggregates is not validated. The small values of the R^2* statistics, however, show that, after the past history of nominal income is taken into account in predicting current nominal income, little of the remaining variation is explained by any of the money aggregates, sum or weighted, thus suggesting that predictive power is a poor criterion on which to base decisions concerning the appropriate definition of money. Indeed, it may be persuasively argued that money should not, and probably cannot, be defined empirically and that any definition must be developed from monetary theory itself [Mason (1976) provides a critique of empirical definitions of money].

(1) Weighted aggregates constructed from a predictive criterion are obviously excluded from this comparison as such tests would be biased in favour of them.

5

Economic Monetary Aggregates

The empirical definitions of monetary aggregates developed in the previous sections have paid scant consideration to crucial questions concerning the theoretical role of money in the economy. For such aggregates to be useful, they must coincide with a concept of money that is both economically meaningful and measurable. By this we mean that a monetary aggregate, M say, must be able to be treated in economic agents decision-making as the quantity of a single good, the desired level of which may be selected without regard to its composition. The allocation of M over its component elements can then be accomplished in a later second stage decision, conditionally upon the prechosen aggregate level of M. Varying the relative quantities of the components within M while holding the aggregate level constant must not affect the preference orderings over M and other goods. If M is not a good in this fundamental sense, preference orderings over M and other goods will appear to shift whenever the relative proportions of the components of M change.

Thus, if the concept of money is to have any economic meaning, an aggregate of financial assets must exist which is treated by the economy as if it were a single good, and that good can be termed 'money'. Following Barnett (1980, 1981 chapter 7), such an aggregate, known as an economic quantity aggregate, is a function of the asset quantities alone, the choice of these quantities being independent of the levels of any other variables. Without these "separability" conditions, any aggregate is inherently arbitrary and does not define an economic variable(1).

When such an economic monetary quantity aggregate exists, it can be shown to possess the properties of a known utility function for financial assets, and, if this is the case, such an aggregate is said to be consistent. The economic quantity aggregate cannot be known exactly without knowledge of this underlying utility function. All traditional monetary aggregates are constructed by simple summation of components. Thus, if these aggregates have any economic (as opposed to accounting) meaning, they must have been generated by a utility function for financial assets possessing the same

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(1) These separability conditions, derived from Green (1964), are analysed formally in Barnett (1980).

simple unweighted summation form used in constructing the aggregate. Such a utility function requires the assets over which it is defined to be perfect substitutes in identical ratios, ie the included financial assets must be indistinguishable.

The utility function of money that we are considering may be regarded as determining the level of transaction services provided by the set of financial assets. As we have noted, the broader sum aggregates implicitly view distant substitutes for money as perfect substitutes for currency, thus swamping the included transactions services with heavily weighted store of value or investment services. Narrow sum aggregates regard assets close to transaction balances as providing no liquidity and hence only capture part of the economy's transaction services.

Thus the appropriate economic monetary aggregate is that determined by the underlying utility function for financial assets, this function being generally known as the aggregator function. In practice, this aggregator function and hence the economic aggregate are not known. In principle, the function could be specified and estimated and attempts to do so have been made [see eg Chetty (1969)]. However, aggregates depending upon estimated parameters are unsatisfactory as they require both assumptions about the specified model and a choice of data and estimator to be made. The theory of statistical index numbers has been developed precisely to provide parameter-free aggregates, and it is to this theory that we now turn.

6

Statistical Index Numbers

An economic quantity aggregate depends only upon component quantities and unknown parameters, and does not involve prices. On the other hand, statistical index numbers do not depend upon any unknown parameters, but quantity index numbers can depend upon component prices as well as component quantities. The link between economic aggregates and statistical index numbers is provided in Diewert (1976), who also introduces the concepts of exact and superlative index numbers.

A quantity index between periods t-1 and t, $Q(\underline{\pi}_{t-1}, \underline{\pi}_t; \underline{m}_{t-1}, \underline{m}_t)$, is a function of the prices in periods t-1 and t, $\underline{\pi}_{t-1} > \underline{0}$ and $\underline{\pi}_t > \underline{0}$ and the corresponding quantities $\underline{m}_{t-1} > \underline{0}$ and $\underline{m}_t > \underline{0}$. Diewert (1976) defines such an index to be exact for a given aggregator function, f, if $Q(\underline{\pi}_{t-1}, \underline{\pi}_t; \underline{m}_{t-1}, \underline{m}_t) = f(\underline{m}_t)/f(\underline{m}_{t-1})$ whenever $\underline{m}_t > \underline{0}$ is the value of $\underline{m} > \underline{0}$ which maximises f(m) subject to $\underline{\pi}_t' \underline{m} < \underline{\pi}_t' \underline{m}_t$, ie the quantity index number is exact if it exactly equals the aggregator function whenever the data is consistent with maximising behaviour.

In continuous time, Hulten (1973) has proved that the Divisia index, defined by the differential $d\log Q = \Sigma w_i d\log m_i$, where $w_i = \pi_i m_i / \underline{\pi}' \underline{m}$, is always exact for any consistent aggregate. No always-exact index numbers are known in the discrete time case, but Diewert (1976) defines an index number to be 'superlative' if it is exact for some aggregator function which can provide a second order approximation to any linearly homogenous aggregator function.

Within the class of superlative index numbers (which also contains the Fisher Ideal), the Tornquist-Theil Divisia index has been found to be especially useful. It is defined as:

$$\log Q_t^T - \log Q_t^{T-1} = \sum_{i=1}^{k} s_{it} (\log m_{it} - \log m_{i,t-1}) \qquad (6.1)$$

where $s_{it} = (w_{i,t} + w_{i,t-1})/2$

and $w_{it} = \pi_{it} m_{it} / \sum_{j=1}^{k} \pi_{jt} m_{jt}$,

and provides a discrete time approximation to the optimal continuous time Divisia index. The growth rate of the index is a weighted average of the growth rates of the components, the weights being the share contributions of each component to the total value of all components.

This interpretation is particularly attractive in the application to the aggregation of financial assets, and Barnett (1980) has proposed the use of such an index to construct a hierarchy of monetary aggregates.

The theory of functional structure [see Blackorby et al (1977)] tells us that dual to an economic quantity aggregate there exists an economic price aggregate depending only upon component prices. We can therefore construct a dual statistical price index depending on both component prices and quantities that satisfies the accounting identity of equality between expenditure and the product of quantity and price.

As stated above, both price and quantity indices require component prices as well as component quantities and such financial asset prices must first be defined and calculated before the construction of monetary indices can become operational.

Barnett (1978) rigorously constructs such prices from interest rate data using an analogous theory to that of deriving the 'rental price' or 'user cost' of a durable good. This user cost of a financial asset is the price imputed to the service flow of that asset during any given period, and is the cost during that period of acquiring, using, and disposing of the asset. The user-cost formula derived by Barnett, and less formally obtained by Donovan (1978), is:

$$\pi_{it} = g_t (R_t - r_{it}) \qquad (6.2)$$

where $g_t = p_t (1 - \tau_t)/[1 + R_t (1 - \tau_t)]$.

R is the maximum available yield in the economy on any financial asset, r_i is the own rate of return on financial asset i, τ_t is the marginal tax rate and p_t is the general price level. Since g_t does not depend on asset i, the difference $R - r_i$ can be treated as the user cost of that asset and represents the foregone interest, and hence the opportunity cost, of holding asset i during that period. Unless transactions services are provided by the asset, no one would hold it and $R - r_i$ is the price paid in return for the receipt of those services.

Utilising the user costs given by (6.2) as the prices in (6.1) defines the weights s_{it} to be the user cost evaluated value shares. It is important to note that the user costs are not the weights, but are the prices used along with all of the quantities in computing these weights, each weight depending upon all prices and all quantities.

On constructing the quantity index given in (6.1), the dual price index P^T can be constructed as:

$$\log P_t^T - \log P_{t-1}^T = \sum_{i=1}^{k} s_{it} (\log \pi_{it} - \log \pi_{i,t-1}) \qquad (6.3)$$

However, as Diewert (1976) shows, Q^T and P^T do not satisfy the 'factor reversal test', ie $Q_t^T P_t^T \neq \underline{\pi}_t' \underline{m}_t$. If we require this test to hold, we may compute the monetary price index as:

$$P_{*t}^T = \frac{\underline{\pi}_t' \underline{m}_t}{Q_t^T} \qquad (6.4)$$

The Barnett (1980) proposals have led to a rapidly expanding body of research on the construction of Divisia monetary aggregates. A survey of the earlier literature is provided by Barnett, Offenbacher and Spindt (1981) and details of their construction for the US is given in Barnett and Spindt (1982). Divisia quantity aggregates have also been constructed for Canada by Cockerline and Murray (1981) and for the UK by Bailey et al (1982). Our intention in the remainder of this paper is to construct a hierarchy of Divisia monetary aggregates based on the monetary assets defined in Table 2, to investigate the historical behaviour of these quantity and price indices and to analyse their statistical properties and relationships with other macroeconomic variables.

7

The Construction of Divisia Monetary Aggregates for the UK

Divisia monetary quantity aggregates were constructed over the six asset groupings shown in Table 2, the interest rates also shown being used to construct the asset user costs. Dual monetary price or user cost indices were then calculated using equation (6.4). The time series behaviour of these quantity and price aggregates, along with the behaviour of the corresponding simple sum aggregates, are shown as Charts 3-14. Charts 3-8 plot the historical behaviour of the levels of the Divisia and simple sum aggregates, with annual growth rates being shown in Charts 9-14. The share weights of the Divisia indices [the s_i of equation (6.1)] are plotted in Charts 15-20(1). The user cost indices are plotted in Charts 21-26 and finally GNP velocities are shown as Charts 27-32.

(1) In constructing the share weights, note that, from using (6.2):

$$w_{it} = (R_t - r_{it})\, m_{it} / \sum_{j=1}^{k} (R_t - r_{jt})\, m_{jt}$$

and hence the share weights, and the monetary and quantity price indices, depend only upon asset quantities and interest rates.

All indices are normalised at 1 in 1963 Q1, and annual growth rates are calculated as:

$$\dot{Q}_t = (Q_t - Q_{t-4})/Q_{t-4}$$

Certain small share weights in the Divisia PSL1 and PSL2 aggregates have been combined to provide easier interpretation.

8

The Historical Behaviour of the Divisia Aggregates

The Divisia quantity indices measure the flow of monetary services produced by the stocks of assets that are components of the aggregates. In principle, the most informative Divisia index is the one that aggregates over as many financial assets as possible, since this aggregate will capture the appropriately weighted contribution of all assets to the monetary service flow of the economy. The behaviour of the highest level Divisia aggregate, that for PSL2, should therefore be of particular interest.

For the narrowest aggregate grouping, M1, the Divisia and sum aggregates are almost identical (see Charts 3 and 9), but, for all higher level groupings, the simple sum indices grow at higher rates than do the corresponding Divisia indices. This is because the simple sum indices give more weight to the contributions of distant substitutes for money in the aggregation than do the Divisia indices, and those distant substitutes have been growing at faster rates than more money-like components, such as currency and demand deposits(1). The size of this "aggregation bias" in the simple-sum index therefore increases as the level of aggregation increases; the maximum divergence of the simple-sum from the Divisia aggregate being 26 per cent for both £M3 and PSL2 by the end of 1981.

Concentration is therefore focussed on the historical behaviour of these two aggregates, the evolution of the M2, PSL1 and M3 aggregates being reasonably similar. The growth paths of the Divisia and simple-sum aggregates diverge quite considerably (Charts 11 and 14). In the early years of the data period (pre-1970), growth rates are low, generally between zero and ten per cent, with the Divisia growth rates being consistently below their simple-sum counterparts. Between 1970 Q2 and 1972 Q2, both forms of aggregates increase substantially and at almost identical rates. After the latter date, however, the Divisia growth rates begin to fall, whereas the simple-sum growth rates continue to increase for a further six to eight quarters before peaking. The fall in the growth rates of these aggregates is then rapid and between 1975 Q1 and 1977 Q4 the two sets of growth rates are again similar, although, in contrast to the pre-1970 period, the Divisia growth rates are slightly above the simple-sum growth rates. Both sets of rates accelerate in 1978 Q1, peaking in 1978 Q4, from which date the Divisia growth rates fall

(1) Although the Divisia and simple-sum M1 aggregates have been almost identical for most of the period, divergences are now appearing as the interest bearing part of M1 becomes proportionately more important.

continuously until 1980 Q2, whereas the simple-sum aggregates return to their previous growth paths after a brief downturn. Since 1980 Q2, the Divisia aggregates have also returned to upward growth paths, although remaining consistently below those of the simple-sum aggregates.

Interpretation of these divergences may be helped by examining the time paths of the share weights of the Divisia aggregates (Charts 17 and 20). The weight given to transaction balances in the £M3 aggregate falls from approximately 0.75 in the 1960s to just over 0.50 in 1977, with two brief upward movements in 1973 and 1976. It then rises to over 0.80 in early 1980 before falling back to around 0.60 by the end of 1981. The weights assigned to retail deposits tend to mirror those just discussed as wholesale deposits have only a small weight throughout the period, a consequence of both low user cost and quantity.

For ease of presentation, groups of weights in the PSL2 aggregate have been combined. The movements in the share weights for transactions balances and retail deposits follow the pattern described for £M3, but the size of the weights are rather lower. This is, of course, because of the introduction of additional assets. 'Other money market instruments' have low weight throughout but the behaviour of the other two assets, building society deposits and 'other national savings', is of considerable interest. The latter asset has declined in importance over the sample period (from a weight of approximately 0.15 in 1965 to 0.05 in 1981) because almost constant asset quantities have not been compensated by sufficiently higher user costs. Building society deposits, however, have increased in weight from 0.01 in 1963 to 0.20 in 1981, thus reflecting the increased importance of this asset in providing liquidity services.

The behaviour of these share weights provides an explanation for the divergent behaviour of the Divisia and sum aggregates in the 1972-1974 and 1979-1981 periods. The general increase in interest rates in 1973 and 1974 decreased the relative user costs and hence the monetary services provided by assets other than transaction balances, this being reflected by the increased weight given to this asset in these years. But, as this asset grew at a slower rate than other assets, the Divisia aggregates show a slower growth rate increase than their simple-sum counterparts. A similar explanation holds for the later period, when the sharp increase in interest rates pushed up the weight given to transaction balances considerably.

Again, this asset grew at a slower rate than other assets, forcing the growth rate of the Divisia aggregates below that of the simple-sum aggregates, and thus giving a different indication of the 'tightness' of monetary conditions during this period. In general, the cyclical behaviour of a high level Divisia aggregate will be more like that of a narrow simple-sum aggregate than the corresponding broad simple-sum aggregate. This is because, as interest rates rise, funds will be shifted to less liquid assets. More weight will therefore be given to the more liquid, low yielding assets, whose growth rates have been reduced.

The time paths of the velocities of Divisia £M3 and PSL2 and their simple-sum counterparts are shown in Charts 29 and 32. The Divisia velocities are always higher than the simple-sum velocities and the large falls in the latter between 1972 and 1974 are less marked in the Divisia velocities. This is a consequence of the lower growth rate of the Divisia aggregates, but is also to be expected from substitution out of the lowest yielding and most highly liquid assets in response to rising interest rates and inflationary expectations. Indeed, for the interest rate elasticity of the demand for money to be correctly (negatively) signed, velocity should be positively correlated with interest rates and a rising velocity in a period of rising interest rates is therefore theoretically consistent. If the Divisia monetary price indices are regarded as measures of the opportunity cost of holding monetary assets, then Charts 23 and 26 show that the general increase over time of the £M3 and PSL2 price indices is also consistent, on these grounds, with rising velocity.

Detailed statistical and econometric analysis of these Divisia indices is performed in subsequent sections, but we end this by discussing some advantages and disadvantages of such indices specific to their use as monetary indicators.

An important feature of Divisia monetary aggregates, not shared by the composite aggregates developed in earlier sections, is that the share weights vary over time. If the general level of interest rates rises, transactions balances will tend to receive greater weight. If interest were paid on current accounts, however, this asset would receive less weight. This is not because its intrinsic 'moneyness' has altered, but reflects the fact that wealth holders will increase their holdings of such an asset until its marginal liquidity return is equal to that on all other assets. Higher interest rates will therefore encourage holders to economise on non-interest-bearing accounts.

The index can also provide a framework for dealing with the effects of some financial innovations in something other than a purely ad hoc manner. For example, certificates of deposit only appeared towards the end of 1968, but can be readily incorporated into the index. Initially, such new assets will enter with small weights, due both to their small quantity and usually relatively high rate of return (ie low user cost), but, as their importance increases, so will their weight in the index. This property could be useful if the clearing banks introduce interest-bearing current accounts on a major scale, although it is debatable whether simply an interest rate differential will be an adequate measure of the relative moneyness of this form of current account vis-a-vis the conventional, non-interest-bearing account. The index may also be relatively robust to changes in methods of monetary control, for example picking up the effects of 'corset' disintermediation out of controlled assets into close substitutes.

There are some operational difficulties with the use of Divisia aggregates. Use of such an index would make any counterpart analysis impossible, but, on the other hand, a Divisia aggregate is likely to have a higher interest elasticity than a simple-sum aggregate and thus be potentially more controllable(1). The Divisia index also implicitly assumes that holders respond almost immediately to the differential rates of return on various assets. In reality, there are lags in behaviour and adjustment takes time. Moreover, there are transaction costs involved in changing the structure of portfolios so that shifts are only likely to occur if the change in differentials is likely to persist or is large enough to make arbitrage profitable. Nonetheless, the advantages in terms of economic and statistical theory are sufficient to make the Divisia indices attractive aggregates for further study.

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(1) See Artis and Lewis (1981) for detailed discussion of methods of monetary control in the UK.

9

Univariate Time Series Representations of the Indices

Following the methodology of Box and Jenkins (1976), autoregressive-integrated moving average (ARIMA) models were developed for the simple-sum and Divisia quantity indices, the user cost indices and the simple-sum and Divisia velocities. As the models for the alternative quantity indices were of similar form, attention is again concentrated on the M1, £M3 and PSL2 aggregates. The ARIMA models developed for these aggregates are shown in Table 8.

As noted in the previous section, the Divisia and simple-sum aggregates for M1 are almost identical and the logarithms of both series can be modelled by ARIMA $(0,1,2)(0,1,1)_4$ processes with similar parameters. The simple-sum £M3 and PSL2 aggregates require ARIMA $(0,1,3)(0,1,1)_4$ processes to adequately model their logarithms, while their Divisia counterparts can again be modelled by ARIMA $(0,1,2)(0,1,1)_4$ processes.

For the user cost indices, the logarithms of all series, except M1, can be modelled by ARIMA (0,1,0) models, ie random walks with no seasonal patterns. The M1 user cost series, however, requires a moving average term. All velocities, whether simple-sum or Divisia, can be adequately modelled by ARIMA $(0,1,0)(0,1,1)_4$ models, these being shown in Table 9.

Given an ARIMA representation, each of the above time series can be additively decomposed into trend, seasonal and noise components using the technique of signal extraction. Suppose that, in general, an observed time series x_t can be decomposed as:

$$x_t = T_t + S_t + N_t \tag{9.1}$$

where T_t, S_t and N_t are unobservable trend, seasonal and noise components. Assume that each of the components follows an ARIMA model:

$$\phi_T(B)T_t = \eta_T(B)b_t \tag{9.2a}$$

$$\phi_S(B)S_t = \eta_S(B)c_t \tag{9.2b}$$

$$\phi_N(B)N_t = \eta_N(B)d_t \tag{9.2c}$$

Each of the pairs of polynomials $(\phi_T(B), \eta_T(B))$, $(\phi_S(B), \eta_S(B))$, $(\phi_N(B), \eta_N(B))$ are assumed to have roots lying on or outside the unit circle and to have no common roots and b_t, c_t and d_t are assumed to be orthogonal white noise sequences with finite variances σ_b^2, σ_c^2 and σ_d^2. It can then readily be shown that x_t has the form:

$$\psi(B)x_t = \theta(B)a_t \tag{9.3}$$

where $\psi(B)$ is the highest common factor of $\phi_S(B)$, $\phi_T(B)$ and $\phi_N(B)$, and $\theta(B)$ and σ_a^2 can be obtained from:

$$\frac{\theta(B)\theta(F)\sigma_a^2}{\psi(B)\psi(F)} = \frac{\eta_T(B)\eta_T(F)\sigma_b^2}{\phi_T(B)\phi_T(F)} + \frac{\eta_S(B)\eta_S(F)\sigma_c^2}{\phi_S(B)\phi_S(F)} + \frac{\eta_N(B)\eta_N(F)\sigma_d^2}{\phi_N(B)\phi_N(F)} \tag{9.4}$$

where $F = B^{-1}$ [see, eg Hillmer and Tiao (1982)]. When the stochastic structures (9.2) of T_t, S_t and N_t are known, the minimum mean square error (MSE) estimate of T_t, for example, is given by:

$$\hat{T}_t = \nu_T(B)x_t = \sum_{-\infty}^{\infty}\nu_{Tj}x_{t-j} \tag{9.5}$$

where the filter $\nu_T(B)$ is defined as:

$$\nu_T(B) = \frac{\sigma_b^2\psi(B)\,\psi(F)\,\eta_T(B)\,\eta_T(F)}{\sigma_a^2\theta(B)\theta(F)\phi_T(B)\phi_T(F)}$$

Similarly, for the seasonal component, we have

$$\hat{S}_t = \nu_S(B)x_t = \sum_{-\infty}^{\infty}\nu_{Sj}x_{t-j} \tag{9.6}$$

where the filter $\nu_S(B)$ is defined as:

$$\nu_S(B) = \frac{\sigma_c^2\psi(B)\,\psi(F)\,\eta_S(B)\,\eta_S(F)}{\sigma_a^2\theta(B)\,\theta(F)\,\phi_S(B)\,\phi_S(F)}$$

Consequently, the noise component is estimated as:

$$\hat{N}_t = x_t - \hat{T}_t - \hat{S}_t \tag{9.7}$$

Because in practice the component series T_t, S_t and N_t are unobservable it is usually unrealistic to assume that their models (9.2) are known. An accurate estimate of the model (9.3) can be obtained, however, from the observable x_t series and, based upon this, estimates of the components can be determined. In general, the information in the known model for x_t is not sufficient to uniquely determine T_t and S_t but by maximising the innovation variance σ_d^2 of the noise component N_t a 'canonical decomposition' of x_t is obtained which uniquely identifies the trend and seasonal components [see Tiao and Hillmer (1978) for discussion of admissible and canonical decompositions of a time series].

A convenient algorithm for providing such a canonical decomposition has been developed by Burman (1980). By writing equation (9.3) as:

$$x_t = \frac{\theta(B)}{(1-B)^d(1-B^s)^D\ \psi_1(B)\ \psi_2(B^s)}\ a_t \tag{9.8}$$

and noting that $(1-B^s)^D$ and $\psi_2(B^s)$ can, in principle, be factored into seasonal and non-seasonal parts [for example $(1-B^s)^D = (1-B)^D(1+B+...+B^{s-1})^D$], the model for x_t can be expressed as:

$$x_t = \frac{\theta(B)}{\psi_T(B)\,\psi_S(B)}\ a_t \tag{9.9}$$

By performing a partial fractions expansion of (9.9), Burman's algorithm, known as Minimum Signal Extraction (MSX), estimates T_t and S_t through a partitioning of the spectrum of x_t. Although MSE estimates of the components require doubly infinite x series and filters, the expected values of the outside sample observations can be obtained by extending the observed series with backcasts and forecasts. Only a limited number of these predictions are, in fact, required in practice because of the particular algorithm employed.

This algorithm, primarily designed for seasonal adjustment, also has a procedure for modifying extreme residuals after preliminary estimates of the components have been made and has a further refinement to deal with bias in multiplicative models, ie models in which $x_t = \log (X_t)$.

The use of such 'unobserved' components in modelling macroeconomic relationships has been documented in Nerlove et al (1979) and further discussion and applications of the MSX methodology may be found in Mills (1981b, 1982a, 1982b).

Using the ARIMA models estimated above, the trend, seasonal and noise components extracted from the Divisia M1, £M3 and PSL2 quantity indices are shown in Charts 33-35, the simple-sum counterparts providing similar decompositions.

For all indices, the trend components are much smoother than the observed series, and, although capturing the basic underlying movements of the indices, they are by no means deterministic. Because each seasonal parameter is relatively large in magnitude, all seasonal components are slowly changing, but the numerical size of these components are rather small, being at most 2.5% of the level of the series. Both the M1 and £M3 indices have large (greater than two standard deviations) noise components at 1969 II, while both £M3 and PSL2 have outliers at 1976 III. This latter index has a pair of outliers at 1977 IV and 1978 I, while the M1 index has outliers at 1971 I and 1973 II, this being the only noise observation to exceed three standard deviations in magnitude.

As the user cost indices for all Divisia aggregates wider than M1 follow non-seasonal random walks, the best estimate of the trend component for these series is the currently observed value of the series itself. As stated above, the M1 user cost series follows an ARIMA (0,1,1) process. Such a process can be decomposed into a random walk trend plus a white noise error if the first order autocorrelation of the first differences of the observed series lies between -.5 and 0. For this series, however, the autocorrelation is positive and hence the parameters of the unobserved components are unidentified. Following Nelson and Plosser (1982), if it is assumed that the trend and noise components are independent and the noise component is stationary, the observation that autocorrelations in the first differences of the M1 user cost are positive at lag one and zero elsewhere is sufficient to imply that the variation in actual M1 user cost changes is dominated by changes in the trend component rather than the noise component, even though the standard deviation of the innovations in the trend component is over twice as large as the standard deviation of the innovations in the noise component.

Each velocity series, being adequately modelled by ARIMA $(0,1,0)(0,1,1)_4$ processes, may also be decomposed into trend and seasonal components that follow random walks and a white noise component. In this case, however, since $\phi_1 = 0$ the standard deviation of the innovations of the trend component will theoretically be infinitely larger than the standard deviation of the innovations of the noise component. Charts 36-38 give the MSX decomposition of the Divisia velocities, showing trend velocities to be essentially random walks, with the seasonal and noise components being of very small relative magnitude. Such a decomposition is consistent with the US findings of Gould and Nelson (1974) and contradicts the more recent assertions of Friedman and Schwartz (1982).

Having obtained univariate representations of the monetary quantity indices and associated time series, their relationships with other macroeconomic variables are investigated in the following three sections.

10

Measures of Feedback between Money and Income

Sims' (1972) influential work on the causal relationship between money and income has spawned a voluminous literature of attempts to ascertain the direction of causality between monetary and real variables over a variety of countries and historical episodes. Williams et al (1976), Mills (1980a, b) and Holly and Longbottom (1982) have investigated such relationships for the UK using post-war quarterly data, with no real consensus of opinion emerging from the battery of tests performed on different data series.

These studies have been restricted to the analysis of simple-sum monetary aggregates and the present section extends this research to incorporate the newly constructed Divisia aggregates. The empirical analysis also takes advantage of some recent developments in the modelling of dependencies between time series. Geweke et al (1983) and Nelson and Schwert (1982) have presented simulation evidence suggesting that reduced form (multivariate autoregressive) models provide more powerful tests of predictive relationships between time series than do the more common two-sided regression or cross-correlation of univariate ARIMA residuals approaches. Using such a representation, Geweke (1982) develops a simple measure of dependence between two vectors of time series which has a useful decomposition into feedback and contemporaneous components, and which is also a transformation of Pierce's (1979) R^2 measure introduced in Section 3. The results presented in this section utilise the bivariate form of Geweke's (1982) more general multiple time series framework.

Thus, consider a bivariate time series x_t and m_t with the following invertible moving average representation:

$$\begin{bmatrix} x_t \\ m_t \end{bmatrix} = \begin{bmatrix} \theta_{11}(B) & \theta_{12}(B) \\ \theta_{21}(B) & \theta_{22}(B) \end{bmatrix} \begin{bmatrix} a_t \\ b_t \end{bmatrix} \qquad (10.1)$$

where $(a_t b_t)'$ is a bivariate white noise series with mean $\underline{0}$ and variance-covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} \qquad (10.2)$$

Using (10.1) and (10.2), the following linear projections (or canonical representations) may be defined:

$$x_t = \sum_{j=1}^{\infty} \alpha_{1j} x_{t-j} + u_{1t} \tag{10.3}$$

where u_{1t} is white noise with mean 0 and variance σ_1^2

$$x_t = \sum_{j=1}^{\infty} \alpha_{2j} x_{t-j} + \sum_{j=1}^{\infty} \beta_{2j} m_{t-j} + u_{2t} \tag{10.4}$$

where u_{2t} is white noise with mean 0 and variance σ_2^2 $(= \sigma_a^2)$

$$x_t = \sum_{j=1}^{\infty} \alpha_{3j} x_{t-j} + \sum_{j=0}^{\infty} \beta_{3j} m_{t-j} + u_{3t} \tag{10.5}$$

where u_{3t} is white noise with mean 0 and variance σ_3^2

$$m_t = \sum_{j=1}^{\infty} \beta_{4j} m_{t-j} + u_{4t} \tag{10.6}$$

where u_{4t} is white noise with mean 0 and variance σ_4^2

$$m_t = \sum_{j=1}^{\infty} \alpha_{5j} x_{t-j} + \sum_{j=1}^{\infty} \beta_{5j} m_{t-j} + u_{5t} \tag{10.7}$$

where u_{5t} is white noise with mean 0 and variance σ_5^2 $(= \sigma_b^2)$

Based on these linear projections, Geweke defines the following measures:

$$F_{m\to x} = \ln\ (\sigma_1^2/\sigma_2^2)$$

$$F_{x\to m} = \ln\ (\sigma_4^2/\sigma_5^2)$$

$$F_{x.m} = \ln\ (\sigma_2^2/\sigma_3^2)$$

$$F_{x,m} = F_{m\to x} + F_{x\to m} + F_{x.m} = \ln\ (\sigma_1^2\ \sigma_4^2/\sigma_3^2\ \sigma_5^2)$$

$F_{m\to x}$ and $F_{x\to m}$ are measures of linear feedback from m to x and x to m respectively, and $F_{x.m}$ is a measure of instantaneous linear feedback. $F_{x,m}$ is, therefore, a measure of the total linear dependence between m and x. Furthermore, for each F measure, eg $F_{x,m}$, we may define the associated statistic:

$$R^2_{x,m} = 1 - \exp\ (-F_{x,m}) \qquad (10.8)$$

which is the Pierce (1979) R^2 measure introduced in Section 3.

In order to calculate such feedback measures, the linear projections (10.3) to (10.7) must first be estimated. This requires the lags appearing in these canonical forms to be truncated at finite lengths. If we suppose that each lag length has been so truncated, then (10.3) to (10.7) may be estimated by ordinary least squares. Setting the lag lengths at order p, say, then, on the assumption that the regression disturbances are independently and identically distributed, if $F_{m\to x} = 0$

$$T\hat{F}_{m\to x} \overset{a}{\sim} \chi^2(p)$$

where $\hat{F}_{m\to x} = \ln\ (\hat{\sigma}_1^2/\hat{\sigma}_2^2)$, $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ being the residual variances from the regresssions (10.3) and (10.4) estimated using T observations. Similarly, if $F_{x\to m} = 0$:

$$T\hat{F}_{x\to m} \overset{a}{\sim} \chi^2(p)$$

and if $F_{x.m} = 0$

$$\hat{TF}_{x,m} \overset{a}{\sim} \chi^2(1)$$

Since these are tests of nested hypotheses, $\hat{F}_{m\to x}$, $\hat{F}_{x\to m}$ and $\hat{F}_{x.m}$ are asymptotically independent. All three restrictions can be tested at once since :

$$\hat{TF}_{x,m} \overset{a}{\sim} \chi^2(2p + 1)$$

Regressions of the form (10.3) to (10.7) were estimated for (x , m) systems in which x was, in turn, nominal income and its price and output components and m was, in turn, the set of Divisia monetary indices and their simple-sum counterparts. Rather than set p in each of these systems arbitrarily, it was chosen by using the Minimum Final Prediction Error (MFPE) criterion [Caines et al (1981)]:

$$\text{MFPE}(p) = \left[\frac{T + 1 + 2p}{T - 1 - 2p}\right] \hat{\sigma}_3(p)\ \hat{\sigma}_5(p) \qquad (10.9)$$

choosing that value of p which minimises (10.9); $\hat{\sigma}_3(p)$ and $\hat{\sigma}_5(p)$ being the residual variances from the order p regressions of (10.5) and (10.7).

In accordance with the discussion of Sections 8 and 9, only the results for the M1, £M3 and PSL2 aggregates are reported, the estimation period being from 1963 Q1 to 1980 Q4. Table 10 reports R^2 statistics calculated from (10.8) and associated marginal significance levels for the corresponding test of F=0 for all (x,m) combinations.

From the $R^2_{m.x}$ statistics and associated significance levels, no strong evidence of contemporaneous feedback is found. As might be expected from preceding discussions, the behaviour of the Divisia and simple-sum M1 aggregates are very similar. There is a strong relationship between the indices and income, feedback running in both directions. With regard to the price and output components of income, feedback is from these components to M1, with no evidence of an effect in the reverse direction, thus lending support to the conventional view that narrow monetary aggregates are predominantly demand determined.

The broader Divisia indices are more strongly related to income than their simple-sum counterparts, for both aggregates feedback predominantly running from income to money. A similar feedback pattern is found with respect to output, although the simple-sum indices are now more closely related to the real variable.

The broader simple-sum indices are strongly related to prices with feedback effects running in both directions. The Divisia indices are less closely related to prices, with certainly little evidence supporting a feedback relation running from money to prices.

Thus, while the Divisia monetary quantity indices are closely related to income, there is no evidence to support the view that there is feedback from these indices to prices. Simple-sum indices, while less closely related to income, are closely related to prices. In particular, there is a very strong relationship between simple-sum £M3 and prices with feedback running in both directions, thus providing support for the findings of Holly and Longbottom (1982).

There are, however, a number of statistical and economic caveats that need to be discussed when assessing these results. As Tiao and Wei (1976) and Wei (1982) show, systematic sampling and temporal aggregation have important consequences in time series analysis, the former substantially weakening feedback relationships and the latter turning a one-sided causal model into a complete two-sided feedback system. Both problems are present here with the use of quarterly data, monetary quantity indices being constructed using systematic sampling while income series, being flow variables, require temporal aggregation for their construction. Unfortunately, income data are not available on a more frequent basis, and monthly financial asset series are only available from 1975.

A second statistical matter concerns the exclusion of other variables from the systems under consideration, for example interest rates and world prices. If such variables were to be included in the analysis, the feedback measures and their R^2 conversions could be calculated after conditioning on these additional influences, thus measuring the strength of the relationship between x and m after eliminating their common dependency on these variables. In their absence, the present measures may not reflect the true strength of the relationship, although they may not necessarily over-estimate it.

Turning now to economic caveats, Mills and Wood (1978) emphasise the importance of the exchange rate regime in interpreting feedback relationships between money and income. Only in a period when the exchange rate is freely floating can the UK monetary authorities gain full control over its monetary conditions and hence only in such a period will a feedback from money to real conditions be consistently observed. The exchange rate was fixed before 1972, and has not always floated freely since then, hence the estimation period employed here may be interpreted as a collection of subperiods, heterogeous with respect to feedbacks between money and income. In such a situation, findings of feedbacks in both directions, or indeed of independence, may be unsurprising.

Finally, in many theoretical macroeconomic models, money does not affect output in the long run, although in the short run variations in money can have substantial effects, but, in contrast, the price level always responds eventually to persistent movements in the money supply but may not be much affected by money in the short run. The notions of 'long run' and 'short run' are difficult to make analytically precise when empirical models are being considered and it may be the case that time domain analysis of the observed money and income series, as performed here, will fail to capture the essential predictive differences between the 'short' and 'long' run. Lucas (1980) and Geweke (1982) have proposed related methods for empirically modelling long run quantity theory propositions and these are considered in Section 12. The next section discusses more conventional empirical modelling for, given the strong general evidence of one way feedback from observed output and prices to observed money, demand functions for the monetary quantity indices are specified and estimated.

11

Demand Functions for the Divisia Monetary Aggregates

The UK demand for money function has, in the last decade, become one of the most heavily researched areas in applied macroeconomics, a concise survey of the literature being found in Mills (1980b, chapter 2). In response to the breakdown and subsequent instability of conventional demand for broad money functions in the mid 1970s, attention has since focussed on the issue of dynamic specification, resulting in the studies of Hendry and Mizon (1978), Hendry (1980) and Mills (1980, chapter 4).

The construction of Divisia monetary indices enables an important aspect of conventional demand theory missing in the estimation of simple-sum money demand functions to be considered. The availability of a dual set of Divisia monetary price indices allows a true opportunity cost variable to be incorporated as a regressor, rather than the more or less arbitrary proxy by an interest rate, and also enables a competing price variable to be included.

Because the dynamic specification of demand functions for the entire set of Divisia monetary quantity indices would require a considerable research effort, "error correction" models were specified and estimated for each index. Such models have been found to be particulary useful in the present context, for they embody sensible equilibrium behaviour and rich short run dynamics. Both Hendry (1980) and Mills and Wood (1982) have successfully employed models of this type to investigate the demand for money in different historical periods and policy regimes, although Granger and Weiss (1982), in developing the relationship between these models and conventional time series analysis, have emphasised a number of difficulties inherent in such dynamic specifications. The use of this particular model should be seen as a first attempt at dynamically modelling the Divisia money demand functions, obvious deficiencies in these models being regarded as pointers to the direction any respecification should take.

Thus the following error correction model was specified:

$$\Delta_1 (m-p)_t = \beta_0 + \beta_1 \Delta_1 q_t + \beta_2 \Delta_1 p_t + \beta_3 x_t + \beta_4 z_t - \beta_5 (m-p-q)_{t-1} + u_t \qquad (11.1)$$

where all variables are in logarithms, m is the particular Divisia quantity index, x is the dual Divisia user cost index, z is the price of competing assets, and q and p are output and the price level respectively. In constant growth equilibrium, after all short run dynamics have worked themselves out, (11.1) reproduces:

$$M = A\ (P.Q)\ X^{\gamma_1} Z^{\gamma_2} \tag{11.2}$$

where upper case letters denote actual values of the variables, m = p = q - g, $A = \exp\ (\beta_0 + g\ (\beta_1 + \beta_2))$, $\gamma_1 = \beta_3/\beta_5$ and $\gamma_2 = \beta_4/\beta_5$

The parameters γ_1 and γ_2 are the long run own and competing price elasticities of money demand, the long run price and output elasticities being constrained to unity. Noting that P.Q = Y, (11.2) can be written as the 'inverse velocity' function:

$$\frac{M}{Y} = AX^{\gamma_1} Z^{\gamma_2} \tag{11.3}$$

Equation (11.1) was estimated for each Divisia quantity index over the sample period 1963 Q1 to 1980 Q4 with the dual user cost for Divisia PSL2 entering as the competing price variable z. (This means that there was no z variable for the PSL2 demand function.) Serial correlation in the residuals was modelled by autoregressive schemes, although its presence may be taken as an indication of the dynamic misspecification inherent in the error correction model. The estimates of the demand functins for the Divisia M2, £M3, M3 and PSL1 indices were almost identical and hence only the results for Divisia £M3, along with those for M1 and PSL2, are presented in Table 11.

All the parameter estimates in the Divisia M1 demand function are significant and correctly signed, the equilibrium solution being:

$$\frac{M}{Y} = AX^{-.21} Z^{.18}, \quad A = \exp\ (-4.12 - .29g)$$

Own and competing price elasticities are of the same inelastic order of magnitude, but of course opposite signed, while, in the absence of any asset price changes, the money-income ratio would decline in steady state growth, this being consistent with the theory of transaction balance demand. Mean response lags for all variables are under two quarters and, in comparison with the univariate model for Divisia M1, the residual standard error is some 14% lower.

This minimal requirement for a multivariate model is not met for the broader indices. While the parameter estimates are correctly signed for Divisia PSL2, the own price and error correction coefficients are insignificant and mean response lags are between 10 and 15 quarters. Moreover, the residuals require a fourth order scheme for adequate modelling, this being strong evidence of the need for dynamic respecification. The positive, albeit insignificant, error correction coefficient in the Divisia £M3 function implies that the equation is dynamically unstable, suggesting that a model in differences alone may be necessary.

These results, although exploratory, are interesting in a number of respects. Error correction models are often too simplistic to capture the dynamic relationships existing between economic time series, thus lending support to the findings of Granger and Weiss (1982). Nevertheless, such a formulation does adequately model the demand for Divisia M1, although, given the close relationship between the Divisia and simple-sum indices for this aggregate, and the known success of error correction models in modelling this latter index [Hendry (1980)], this should not be surprising. The success of the user cost variables is noteworthy, and represent a useful addition to the menu of explanatory variables for modelling money demand.

As with the conventional simple-sum aggregates, specification problems arise when modelling broader quantity indices. Further dynamic respecification is obviously required and it is interesting to note that the dynamic instability of Divisia £M3 is a problem that has consistently beleaguered the modelling of simple-sum £M3.

12

Long Run Feedback between Money and Income

As has been discussed in Section 10, the quantity theoretic implications of a given change in money inducing a proportionate change in the price level and a zero change in output are essentially statements about the characteristics of the "long-run average" behaviour of an economy. This is the position taken by Lucas (1980) and Geweke (1982). The latter has proposed testing these long run implications by decomposing the feedback measures introduced in Section 10 by frequency, while the former has examined these questions by comparing the behaviour of two sided moving averages of the variables in question. As noted by Lucas, and developed by Mills (1982b), these methods are consistent with the interpretation of long-run average behaviour as the relationships existing between the trend components of these time series.

Consequently, this section constructs corresponding measures of feedback to those presented in Section 10 for the trend components extracted from the alternative monetary quantity indices and nominal income and its components. The decomposition of the quantity indices has been discussed in Section 9 and, using Burman's (1980) MSX methodology, the trend components of nominal income, output and the price level were extracted using the ARIMA models for these series shown in Table 12. Both the logarithms of income and output follow ARIMA $(0,1,1)(0,1,1)_4$ processes and hence their trend and seasonal components are random walks. The logarithm of price has an autoregressive component, imparting a greater smoothness to its trend.

To reduce computational costs, the model orders were set at the maximum found in the previous analysis and the feedback measures so calculated are reported as Table 13.

There is strong unidirectional feedback from trend income to the M1 trend components, with no evidence of feedback in the other direction. For the broad aggregates, however, the direction of feedback is reversed, running from trend money to trend income.

With regard to trend output, there is bidirectional feedback for all aggregates, the relationship being stronger for the Divisia indices. Trend prices and the trend components of the narrow money aggregates have feedbacks operating in both directions, with the Divisia indices again having larger feedback measures. The relationship between trend prices and the broad aggregate trends is one of unidirectional feedback running from money to prices alone, the Divisia indices outperforming the simple-sum aggregates yet again. In general, the trend broad aggregates are more closely related to trend income and its components than are narrow aggregates with the Divisia indices having a closer relationship than their simple-sum counterparts.

The relationships between the trend components are uniformally stronger than those between the observed series, with the pattern of feedback being more consistent. These results lend support to the views of Lucas (1980) and Geweke (1982) concerning the interpretation of the quantity theory as a set of long run propositions about the interaction between money and income, but further research is needed to test the specific quantity theoretic implications, although a start has been made in Mills (1982a, b).

13

Concluding Remarks

The results presented in the earlier sections of this paper show that, on an information criterion, the practice of simple-sum aggregation over monetary assets is severely defective. If weighted monetary aggregates are important, as these results and economic aggregation theory clearly suggest, then the use of statistical monetary quantity index numbers employing user costs as prices provides an appropriate method for their construction. Within the class of 'superlative' index numbers, the Divisia has a particularly clear interpretation in the present context, being the weighted average of the growth rates of the monetary assets, the weights being the share contributions of each asset to the total value of transaction services provided by all assets.

A hierarchy of Divisia monetary quantity and dual price indices have therefore been constructed and, although there are a number of (primarily institutional) difficulties with these indices, their historical behaviour is both sensible and readily explainable in terms of movements in financial variables.

On subsequent statistical analysis, the traditional simple-sum aggregates are found to be somewhat more closely related to income and prices than their Divisia counterparts, although both sets of results do not provide feedback patterns consistent with any economic theory. Interpreting the quantity theory as a set of propositions concerning the long run average behaviour of an economy, however, suggests modelling the relationships between the trend components extracted from the observed series and such modelling indeed provides a set of feedback patterns consistent with this interpretation. For such models, the Divisia indices are now more closely related to income and prices and the overall set of results point to a broad Divisia quantity index as being the most appropriate monetary aggregate, the choice being either Divisia £M3 or PSL2. Such a finding is consistent with that for the US discussed in Barnett (1982), who favours US Divisia L, and is also consistent with, his conditions for the optimal level of monetary aggregation.

Appendix A - Tables

TABLE 1

Monetary Components

C_1	Notes and coin in circulation with the public
C_2	Non-interest-bearing UK private sector sterling sight deposits
C_3	Interest-bearing UK private sector sterling sight deposits
C_4	UK private sector sterling time deposits excluding deposits of over 2 years
C_5	UK private sector sterling time deposits (over 2 years)
C_6	UK public sector sterling deposits
C_7	UK residents' deposits in other currencies
C_8	Treasury bills
C_9	Bank bills
C_{10}	Deposits with local authorities
C_{11}	Deposits with finance houses
C_{12}	Shares and deposits with building societies
C_{13}	Deposits with Trustees Savings Banks
C_{14}	National savings securities
C_{15}	Deposits with the National Savings Bank
C_{16}	Certificates of tax deposits

Monetary Sum Aggregates

M1 $= C_1 + C_2 + C_3$

£M3 $= M1 + C_4 + C_5 + C_6$

M3 $= £M3 + C_7$

PSL1* $= M1 + C_4 + C_8 + C_{10} + C_{11}$

PSL2* $= PSL1 + C_{12} + C_{13} + C_{14} + C_{15} + C_{16}$

* Adjusted for double-counting

For exact definitions of monetary components, see Financial Statistics explanatory handbook.

TABLE 2

Monetary Assets

$m_0 = C_1 + C_2$ (Transaction balances)

$m_1 = C_3$

$m^*_1 = C_3 + C_4$ (Retail deposits)

$m^{**}_1 = C_3 + C_4 + C_6$

$m_2 = C_5$ (Wholesale deposits)

$m_3 = C_7$

$m_4 = C_8$

$m_5 = C_9$

$m_6 = C_{10}$

$m_7 = C_{11}$

$m_8 = C_{12}$

$m_9 = C_{13}$

$m_{10} = C_{14} + C_{15}$

$m_{11} = C_{16}$

$m^*_4 = C_8 + C_9 + C_{10} + C_{11}$ (Other money market instruments)

$m^*_9 = C_{13} + C_{14} + C_{15} + C_{16}$ (Savings deposits and securities)

Alternative Aggregate Groupings

M1 = (m_0, m_1)

M2 = (m_0, m^*_1)

£M3 = (m_0, m^{**}_1, m_2)

M3 = $(m_0, m^{**}_1, m_2, m_3)$

PSL1 = $(m_0, m^*_1, m_4, m_5, m_6, m_7)$

PSL2 = $(m_0, m^*_1, m_4, m_5, m_6, m_8, m_9, m_{10}, m_{11})$

TABLE 2 (continued)

Rates of Return

r_0	= 0
r_1	= LCB 7-day ordinary deposit account (= r^*_1 = r^{**}_1)
r_2	= Sterling certificates of deposit (3 months)
r_3	= Eurodollar (3 months)
r_4	= Treasury bill yield (91 days)
r_5	= Prime bank bill (3 months)
r_6	= Local authority deposit yield (3 months)
r_7	= Finance house deposit (3 months)
r_8	= Building societies ordinary share deposit account (gross)
r_9	= Trustee Savings Bank deposit rate
r_{10}	= National Savings Bank deposit rate
r_{11}	= Certificates of tax deposit (gross)
R	= max (r_0, ... r_{11}, r_G) + 0.10 points, where
r_G	= Long term (20 year) government stock

TABLE 3

(a) Information content of $\underline{c}$

Lag	0	1	2	3	4	5	6	7	8	9
$I(y/\underline{c})$	.26	.38	.35	.42	.55	.46	.67	.43	.57	.48
R^{*2}	.41 (.32)	.53 (.32)	.50 (.32)	.57 (.32)	.67 (.33)	.60 (.33)	.74 (.33)	.58 (.34)	.68 (.34)	.62 (.35)

(b) Relative information loss of $\underline{c}^{(j)}$: $L(y/\underline{c}^{(j)} : \underline{c})/I(y/\underline{c})$

Lag	0	1	2	3	4	5	6	7	8	9
$\underline{m}^{(1)}$	.89 (.70)	.93 (.50)	.98 (.54)	.92 (.44)	.91 (.34)	.99 (.41)	.93 (.28)	.97 (.43)	.97 (.34)	.96 (.41)
$\underline{m}^{(2)}$	.81 (.63)	.85 (.44)	.92 (.49)	.84 (.39)	.88 (.31)	.95 (.35)	.93 (.25)	.77 (.39)	.82 (.30)	.91 (.41)
$\underline{m}^{(3)}$	.54 (.47)	.59 (.36)	.78 (.39)	.68 (.32)	.63 (.25)	.65 (.30)	.68 (.20)	.62 (.32)	.49 (.25)	.58 (.34)
$\underline{m}^{(4)}$	.49 (.47)	.58 (.34)	.74 (.36)	.63 (.29)	.62 (.23)	.49 (.28)	.65 (.19)	.61 (.29)	.49 (.23)	.48 (.31)
$\underline{m}^{(5)}$	.51 (.44)	.26* (.30)	.55 (.33)	.59 (.27)	.50 (.21)	.53 (.25)	.56 (.17)	.52 (.27)	.61 (.21)	.59 (.28)
$\underline{m}^{(6)}$	.05* (.22)	.03* (.15)	.06* (.17)	.10* (.13)	.04* (.10)	.22 (.12)	.26 (.09)	.13* (.14)	.23 (.11)	.28 (.15)

(c) Incremental information loss of $M^{(j)}$: $L(y/M^{(j)} : \underline{c}) - L(y/\underline{c}^{(j)} : \underline{c})/I(y/\underline{c})$

Lag	0	1	2	3	4	5	6	7	8	9
$M^{(2)}$	nc	nc	nc	nc	nc	nc	nc	nc	nc	nc
$M^{(3)}$	nc	nc	nc	nc	nc	nc	nc	nc	nc	nc
$M^{(4)}$	nc	nc	nc	nc	nc	nc	nc	nc	nc	nc
$M^{(5)}$	nc	.49 (.27)	nc	nc	nc	nc	nc	nc	nc	nc
$M^{(6)}$	.74 (.59)	.78 (.42)	.74 (.47)	.69 (.38)	.74 (.30)	nc	nc	.47 (.37)	nc	nc

nc = not computed. Figures in parentheses are .05 significance values of the statistic based on the appropriate transformation to the 2 distribution.

* denotes an insignificant information loss.

TABLE 4

Hypothesis/Regressors	LR Test Statistic
Maintained (3.4)/M1; £M3-M1, M3-£M3	
H1/M1	88.4
H2/£M3	39.9
H3/M3	38.0
H4/M1; £M3-M1	14.9
Maintained (3.5)/M1; PSL1-M1; PSL2-PSL1	
H1/M1	72.9
H2/PSL1	40.6
H3/PSL2	53.8
H4/M1; PSL1-M1	29.0

H1, H2, H3 test statistic $\sim\chi^2(14)$; H4 test statistic $\sim\chi^2(17)$

TABLE 5

Weighted $(£M3)_1$ = M1 + .46 (£M3-M1)

Weighted $(£M3)_2$ = .69M1 + .31 (£M3-M1)

Weighted $(£M3)_3$ = .54M1 + .46 £M3

Weighted $(PSL1)_1$ = M1 + .39 (PSL1-M1)

Weighted $(PSL1)_2$ = .72M1 + .28 (PSL1-M1)

Weighted $(PSL1)_3$ = .62M1 + .39 PSL2

TABLE 6

$P_a^1 = \underline{M}'_a\ \underline{p}_{a1}$, where $\underline{p}'_{a1}$ = (.81, .11, .58)

$P_a^2 = \underline{M}'_a\ \underline{p}_{a2}$, where $\underline{p}'_{a2}$ = (.65,-.56,-.52)

$P_b^1 = \underline{M}'_b\ \underline{p}_{b1}$, where $\underline{p}'_{b1}$ = (.57, .57, .38, .07, .25, .18, .20, .01, .28)

$P_a^2 = \underline{M}'_b\ \underline{p}_{b2}$, where $\underline{p}'_{b1}$ = (-.06, .61,-.24, -.50, .24, -.11, -.40, -.29, -.01)

Percentage of variation explained by:

$P_a^1 = 39$ $\quad P_b^1 = 24$

$P_a^2 = 35$ $\quad P_b^2 = 17$

Weighted aggregates

$$\ln Q_a = .54 \ln m_0 + .07 \ln m_1{}^{**} + .39 \ln m_2$$

$$\ln Q_b = .23 \ln m_0 + .23 \ln m_1{}^{*} + .15 \ln m_4 + .03 \ln m_5 + .10 \ln m_6 + .07 \ln m_7$$

$$+ .08 \ln m_8 + .00 \ln m_9 + .11 \ln m_{10}{}^{*}$$

TABLE 7

Aggregate	Equation standard error	R^2_*
Sum M1	.024	.05
Sum £M3	.021	.09
Sum M3	.022	.06
Sum PSL1	.021	.09
Sum PSL2	.024	.03
Q_a	.026	.02
Q_b	.026	.02

TABLE 8

ARIMA Models for Divisia Quantity Indices

Model: $\Delta_1\Delta_4 \ln x_t = (1-\theta_1 B-\theta_2 B^2)(1-\theta_4 B^4)\ a_t$

	x	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_4$	$\hat{\sigma}_a$	Q(9)
Divisia	M1	-.280 (.105)	-.492 (.109)	.856 (.078)	.0208	13.7
Divisia	£M3	-.271 (.115)	-.248 (.115)	.836 (.075)	.0153	5.2
Divisia	PSL2	-.191 (.114)	-.296 (.114)	.794 (.079)	.0131	5.8

ARIMA Models for Simple-Sum Quantity Indices

Model: $\Delta_1\Delta_4 \ln x_t = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)(1-\theta_4 B^4)\ a_t$

	x	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\sigma}_a$	Q(9)
Sum	M1	-.134 (.125)	-.443 (.206)	.089 (.119)	.886 (.119)	.0208	13.7
Sum	£M3	-.406 (.117)	-.337 (.155)	-.251 (.129)	.842 (.089)	.0160	8.8
Sum	PSL2	-.389 (.123)	-.483 (.170)	-.258 (.129)	.851 (.083)	.0108	10.9

TABLE 9

ARIMA Models for Income Velocities

Model: $\Delta_1\Delta_4 \ln v_t = (1-\theta_4 B^4)\, a_t$

	v	$\hat{\theta}_4$	$\hat{\sigma}_a$	Q(11)
Divisia	M1	.799 (.084)	.0421	19.5
Divisia	£M3	.724 (.095)	.0339	13.1
Divisia	PSL2	.708 (.098)	.0320	13.5
Sum	M1	.797 (.084)	.0408	17.3
Sum	£M3	.705 (.098)	.0317	11.8
Sum	PSL2	.687 (.098)	.0264	11.3

TABLE 10

(a) Measures of Linear Dependence between Money and Income

(x,m)	p	$R^2_{x \to m}$	$R^2_{m \to x}$	$R^2_{m.x}$	$R^2_{m,x}$
Income Sum M1	2	.151 (.006)	.099 (.040)	.003 (.671)	.238 (.005)
Income Sum £M3	1	.049 (.219)	.004 (.896)	.009 (.221)	.061 (.554)
Income Sum PSL2	1	.144 (.046)	.014 (.929)	0 (.999)	.155 (.312)
Income Divisia M1	2	.142 (.009)	.078 (.081)	.028 (.693)	.211 (.011)
Income Divisia £M3	2	.098 (.041)	.057 (.177)	.008 (.480)	.156 (.060)
Income Divisia PSL2	2	.128 (.015)	.073 (.094)	.005 (.607)	.196 (.019)

(b) Measures of Linear Dependence between Money and Output

(x,m)	p	$R^2_{x \to m}$	$R^2_{m \to x}$	$R^2_{m.x}$	$R^2_{m,x}$
Output Sum M1	3	.115 (.055)	.028 (.625)	.005 (.607)	.143 (.214)
Output Sum £M3	3	.139 (.024)	.002 (.986)	0 (.999)	.141 (.224)
Output Sum PSL2	4	.123 (.086)	.101 (.165)	.018 (.279)	.226 (.068)
Output Divisia M1	2	.128 (.037)	.032 (.563)	.006 (.571)	.161 (.149)
Output Divisia £M3	3	.116 (.052)	.032 (.563)	.001 (.838)	.145 (.207)
Output Divisia PSL2	3	.113 (.059)	.038 (.489)	.003 (.707)	.149 (.194)

TABLE 10 (continued)

(c) Measures of Linear Dependence between Money and Prices

(x,m)	p	$R^2_{x\ m}$	$R^2_{m\ x}$	$R^2_{m.x}$	$R^2_{m,x}$
Prices	2	.089	.008	.015	.111
Sum M1		(.053)	(.774)	(.348)	(.206)
Prices	4	.168	.144	.044	.319
Sum £M3		(.022)	(.046)	(.094)	(.004)
Prices	3	.095	.100	.043	.221
Sum PSL2		(.099)	(.088)	(.097)	(.029)
Prices	3	.102	.025	.027	.148
Divisia M1		(.083)	(.664)	(.432)	(.198)
Prices	1	.025	.011	.011	.047
Divisia £M3		(.219)	(.428)	(.425)	(.408)
Prices	1	.039	.016	.008	.061
Divisia PSL2		(.124)	(.342)	(.486)	(.267)

Figures in parentheses are marginal significance levels of the $T\hat{F}$ statistics; $T = 71-P$.

TABLE 11

Demand Functions for Divisia Quantity Indices

Model: $\Delta_1 (m-p)_t = \beta_0 + \beta_1\Delta_1 q_t + \beta_2\Delta_1 p_t + \beta_3 x_t + \beta_4 z_t - \beta_5 (m-p-q)_{t-1} + u_t$

m	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\bar{R}^2$	$\hat{\sigma}$	r*
Divisia M1	-4.12	.48	- .77	- .10	.08	- .46	.74	.0179	2
	(.77)	(.05)	(.19)	(.02)	(.03)	(.09)			
Divisia £M3	.64	.34	-1.14	.03	- .09	.07	.65	.0179	0
	(.46)	(.05)	(.19)	(.01)	(.06)	(.05)			
Divisia PSL2	- .62	.27	-1.00	- .01	-	- .07	.66	.0148	4
	(.49)	(.05)	(.16)	(.01)		(.06)			

* r is the order of the autoregressive scheme modelling u_t.

TABLE 12

ARIMA Models for Nominal Income and its Components

$$\Delta_1\Delta_4 y_t = \left(1 - \underset{(.12)}{.16}B\right)\left(1 - \underset{(.10)}{.65}B^4\right)a_t \quad , \hat{\sigma}_a = .0231, \ Q(10) = 10.4$$

$$\Delta_1\Delta_4 q_t = \left(1 - \underset{(.11)}{.41}B\right)\left(1 - \underset{(.09)}{.73}B^4\right)a_t \quad , \hat{\sigma}_a = .0196, \ Q(10) = 5.9$$

$$\Delta_1\Delta_4 p_t = \frac{\left(1 - \underset{(.10)}{.65}B^4\right)}{\left(1 - \underset{(.11)}{.52}B\right)}a_t \quad , \hat{\sigma}_a = .0231, \ Q(10) = 10.4$$

TABLE 13

(a) Measures of Linear Dependence between Trend Money and Trend Income

(x,m)	p	$R^2_{x\to m}$	$R^2_{m\to x}$	$R^2_{m.x}$	$R^2_{m,x}$
Income Sum M1	4	.215 (.003)	.031 (.719)	0 (.999)	.240 (.033)
Income Sum £M3	4	.134 (.048)	.255 (.001)	.033 (.150)	.376 (.001)
Income Sum PSL2	4	.029 (.741)	.244 (.001)	.025 (.211)	.284 (.008)
Income Divisia M1	4	.220 (.003)	.020 (.846)	.003 (.664)	.238 (.034)
Income Divisia £M3	4	.093 (.178)	.153 (.024)	.013 (.788)	.233 (.040)
Income Divisia PSL2	4	.064 (.367)	.171 (.015)	.001 (.770)	.226 (.048)

(b) Measures of Linear Dependence between Trend Money and Trend Output

(x,m)	p	$R^2_{x\to m}$	$R^2_{m\to x}$	$R^2_{m.x}$	$R^2_{m,x}$
Output Sum M1	4	.178 (.008)	.186 (.008)	.005 (.633)	.334 (.002)
Output Sum £M3	4	.149 (.030)	.115 (.088)	.017 (.307)	.259 (.019)
Output Sum PSL2	4	.148 (.031)	.118 (.079)	0 (.999)	.249 (.024)
Output Divisia M1	4	.177 (.011)	.234 (.002)	0 (.999)	.370 (.001)
Output Divisia £M3	4	.185 (.086)	.145 (.035)	0 (.999)	.303 (.004)
Output Divisia PSL2	4	.239 (.001)	.141 (.039)	0 (.999)	.366 (.001)

TABLE 13 (continued)

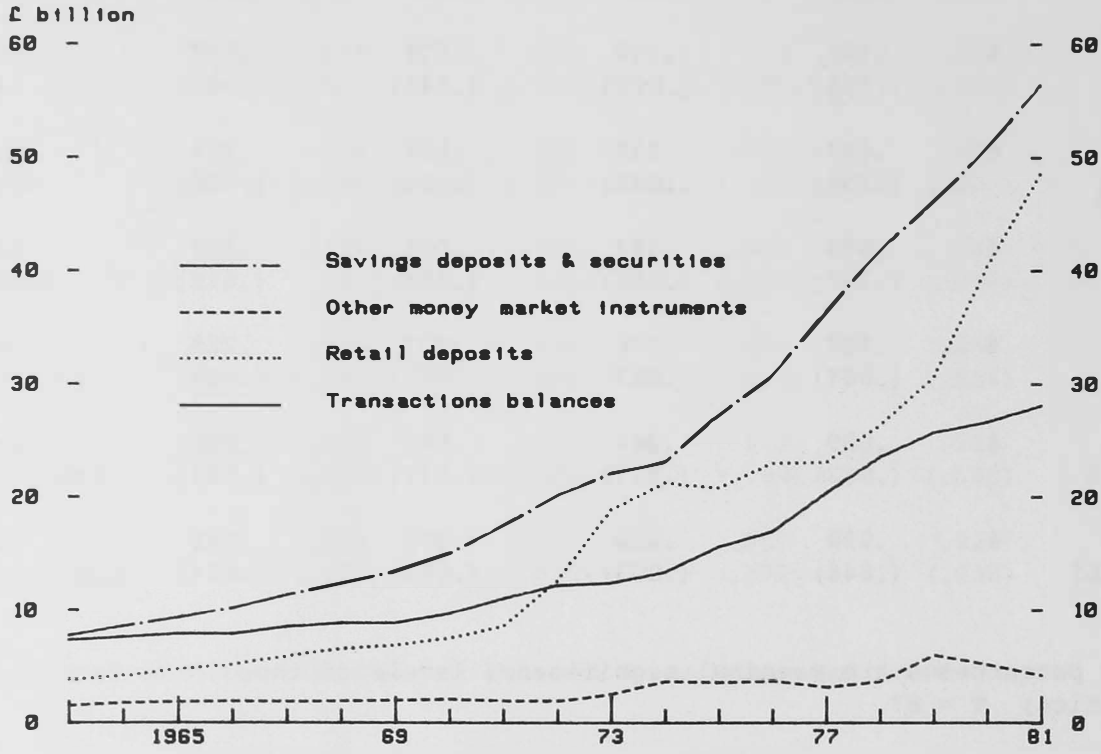
(c) Measures of Linear Dependence between Trend Money and Trend Prices

(x,m)	p	$R^2_{x \to m}$	$R^2_{m \to x}$	$R^2_{m.x}$	$R^2_{m,x}$
Prices Sum M1	4	.105 (.123)	.110 (.099)	.034 (.145)	.230 (.043)
Prices Sum £M3	4	.077 (.256)	.232 (.002)	.004 (.621)	.294 (.006)
Prices Sum PSL2	4	.078 (.247)	.197 (.006)	.001 (.806)	.260 (.019)
Prices Divisia M1	4	.127 (.061)	.116 (.087)	.001 (.825)	.229 (.044)
Prices Divisia £M3	4	.009 (.962)	.241 (.001)	.003 (.671)	.250 (.024)
Prices Divisia PSL2	4	.020 (.846)	.224 (.003)	.010 (.684)	.248 (.024)

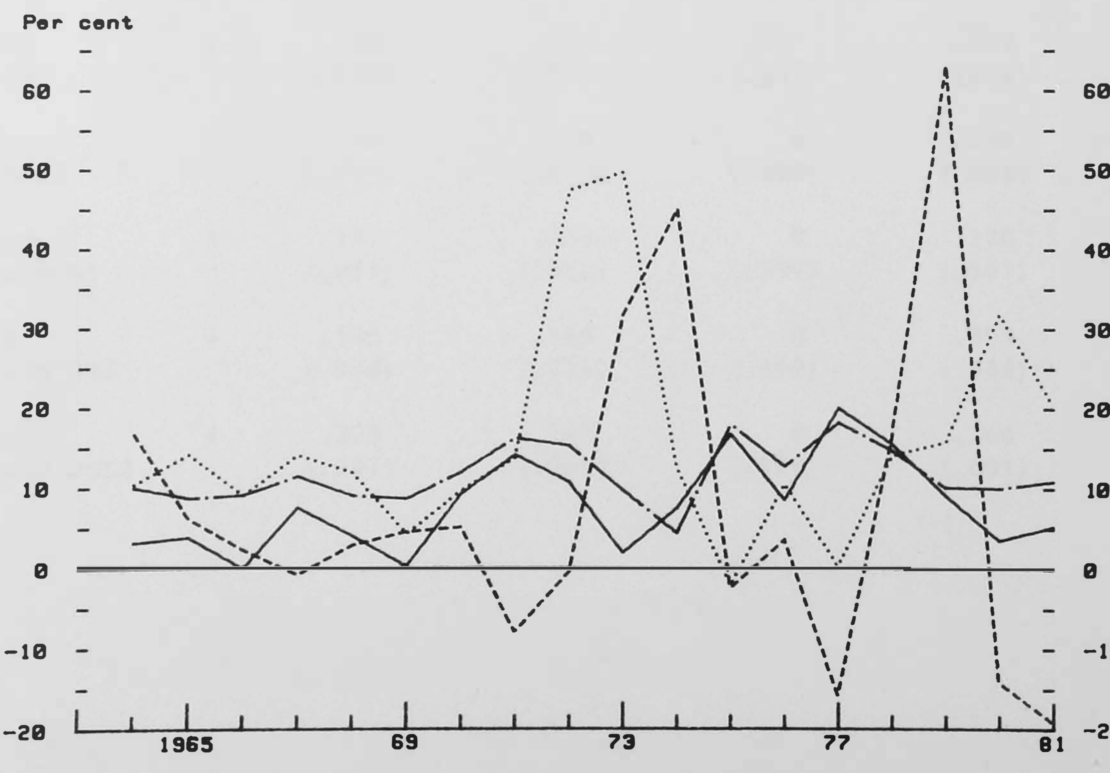
Figures in parentheses are marginal significance levels of the $T\hat{F}$ statistics; T = 67.

Appendix B - Charts

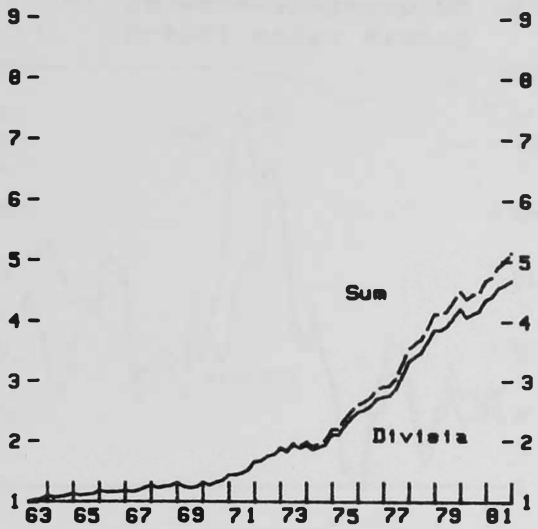
1. Components of private sector liquidity (levels)



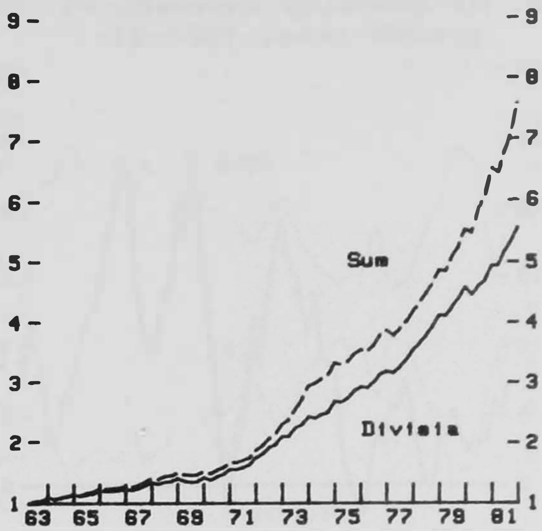
2. Components of private sector liquidity (growth rates)



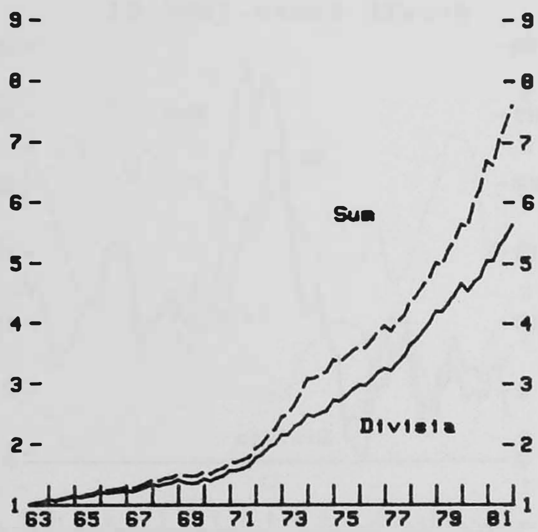
3. M1 quantity indexes 1963-81



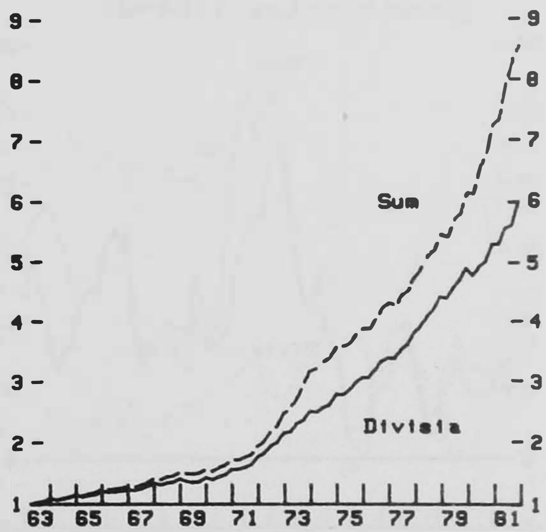
4. M2 quantity indexes 1963-81



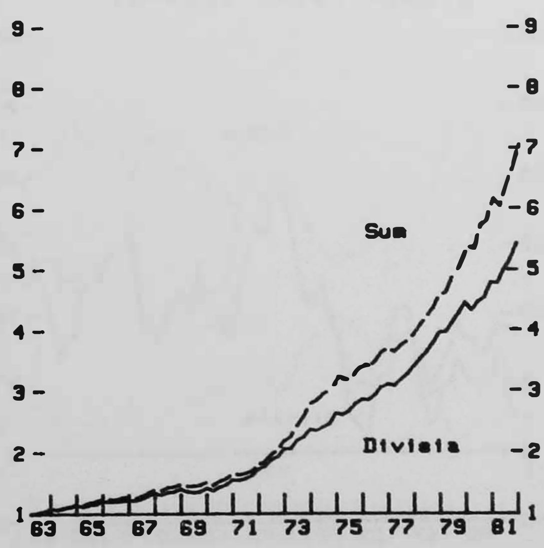
5. £M3 quantity indexes 1963-81



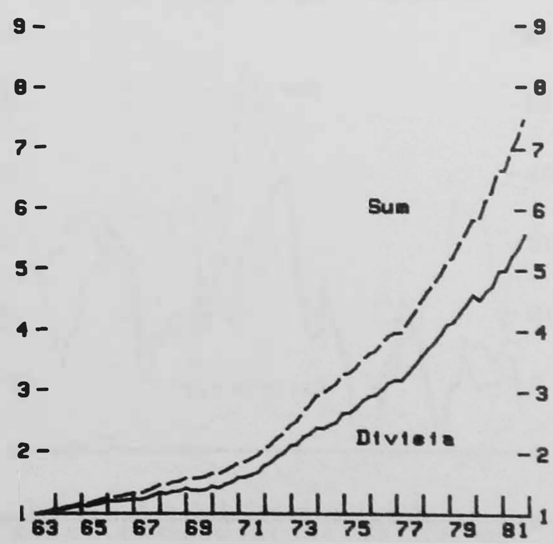
6. M3 quantity indexes 1963-81



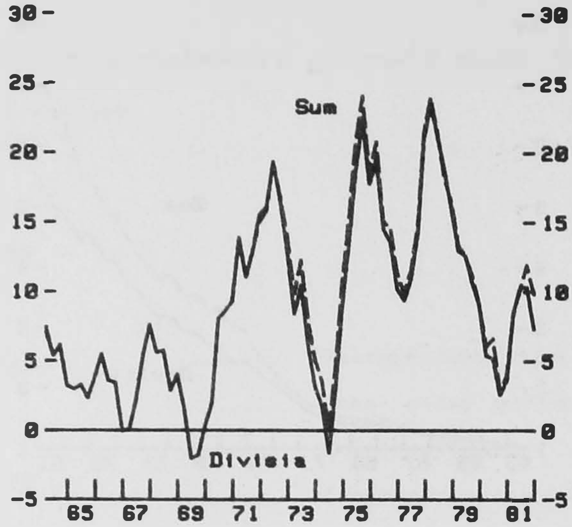
7. PSL1 quantity indexes 1963-81



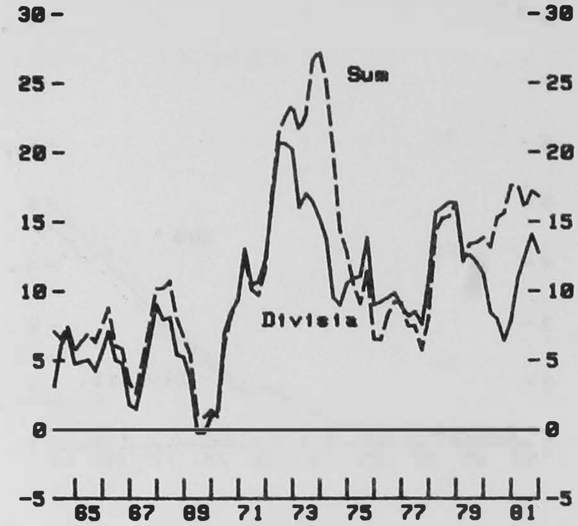
8. PSL2 quantity indexes 1963-81



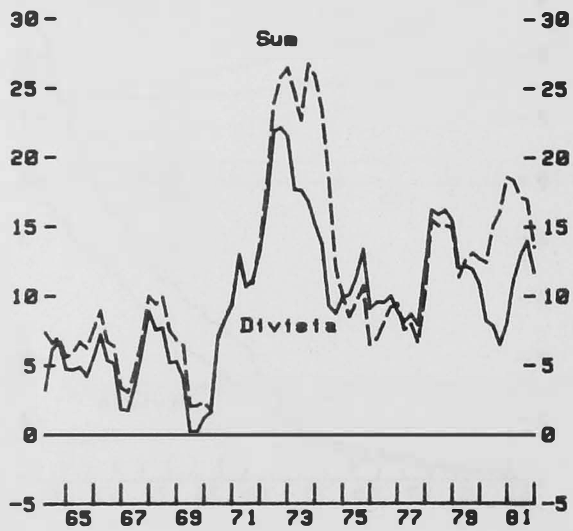
9. M1 quantity indexes growth rates 1964-81



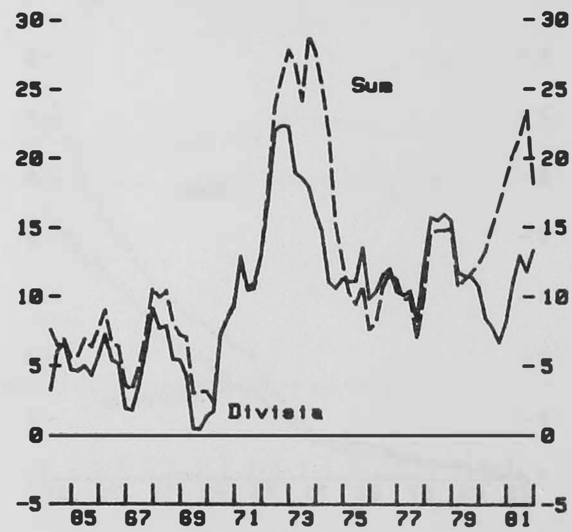
10. M2 quantity indexes growth rates 1964-81



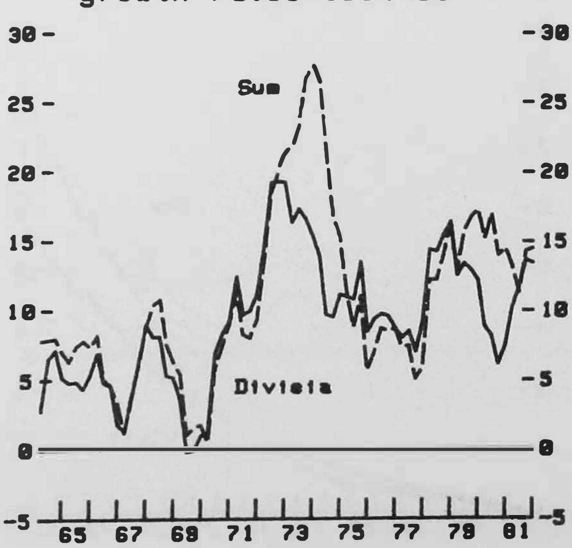
11. £M3 quantity indexes growth rates 1964-81



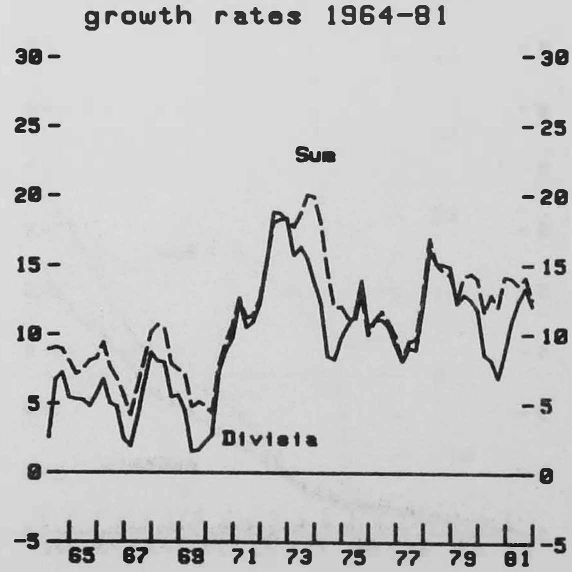
12. M3 quantity indexes growth rates 1964-81



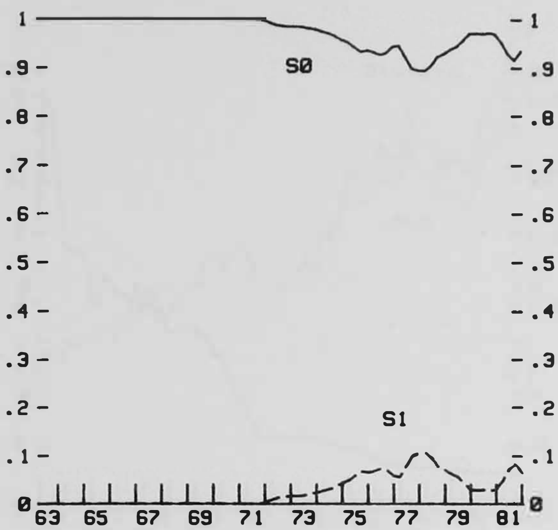
13. PSL1 quantity indexes growth rates 1964-81



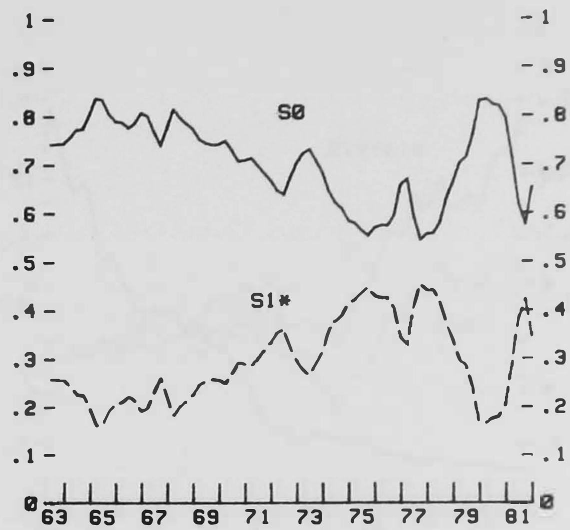
14. PSL2 quantity indexes growth rates 1964-81



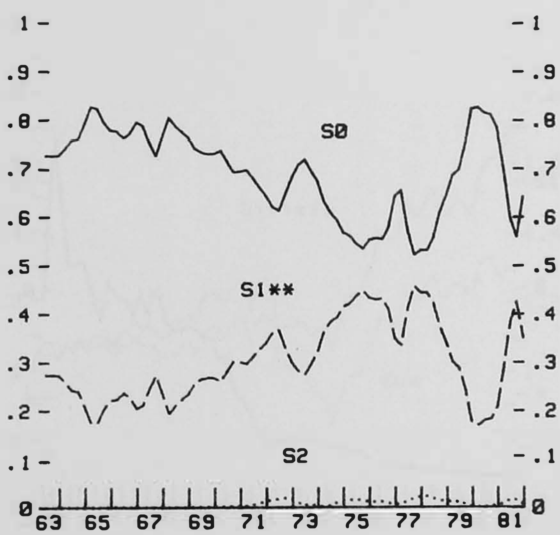
15. M1 share weights



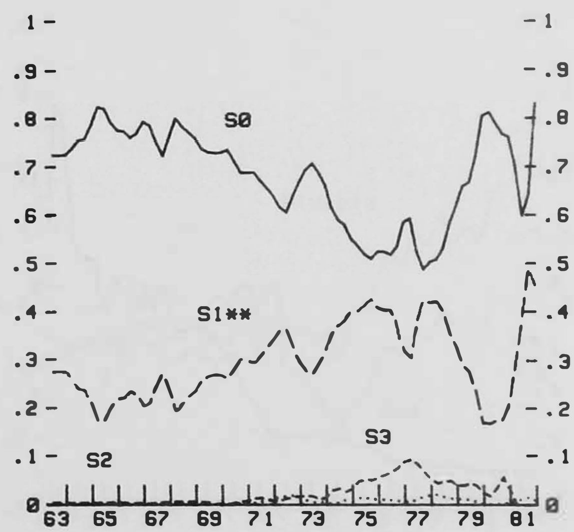
16. M2 share weights



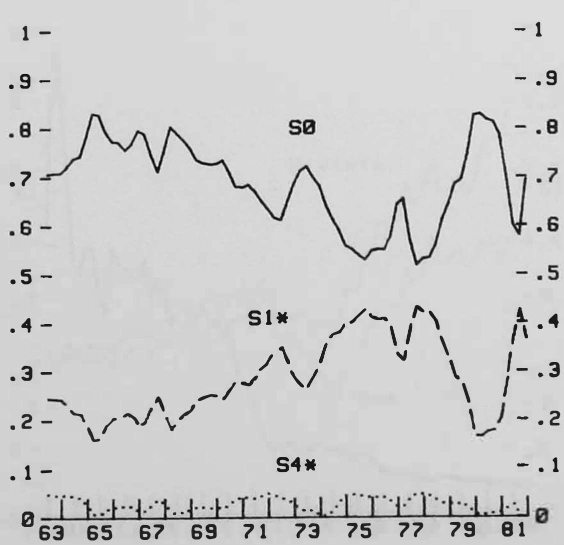
17. £M3 share weights



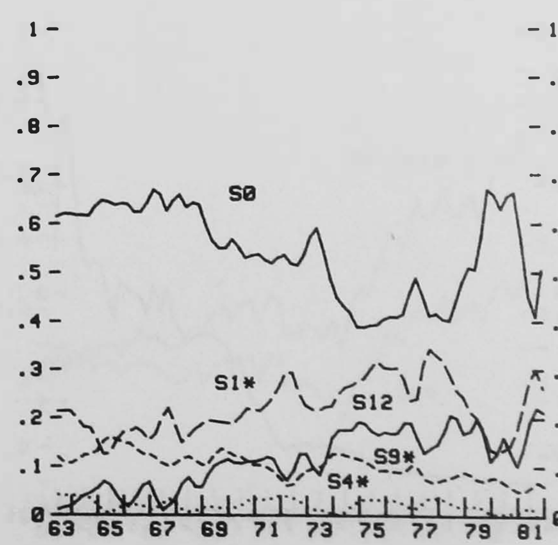
18. M3 share weights



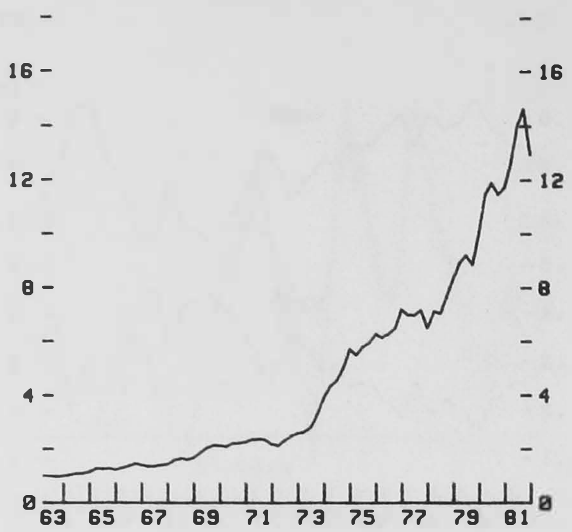
19. PSL1 share weights



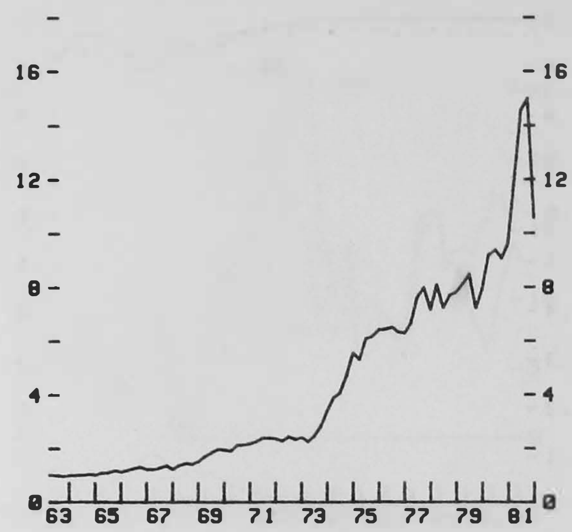
20. PSL2 share weights



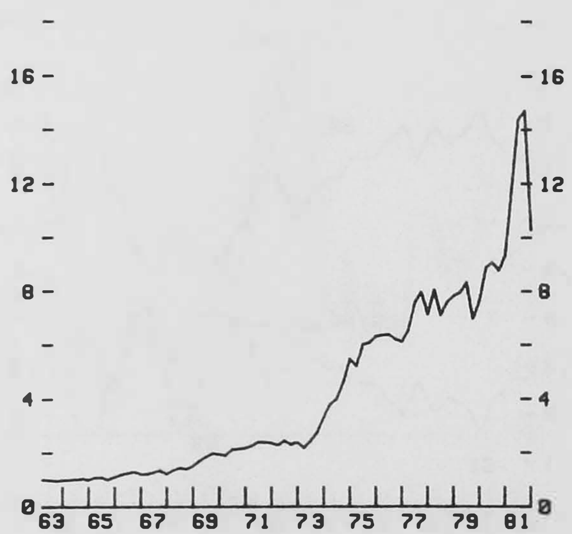
21. M1 user cost index 1963-81



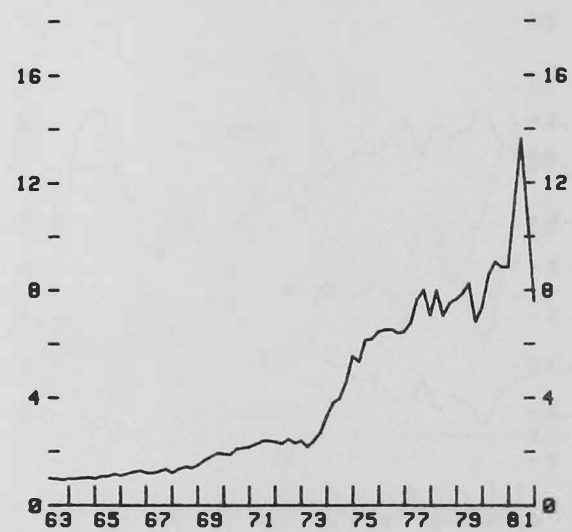
22. M2 user cost index 1963-81



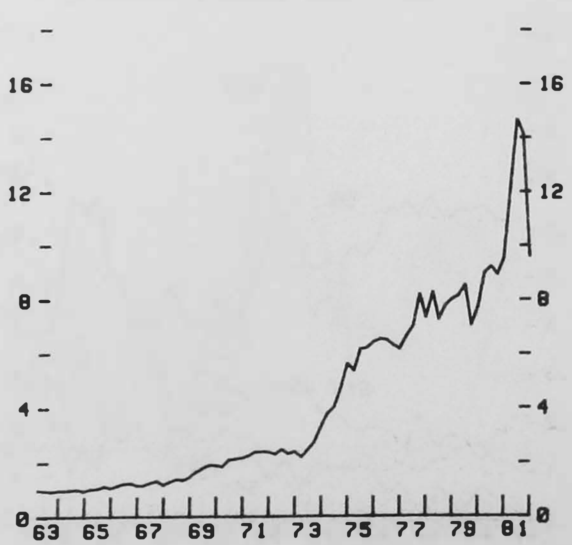
23. £M3 user cost index 1963-81



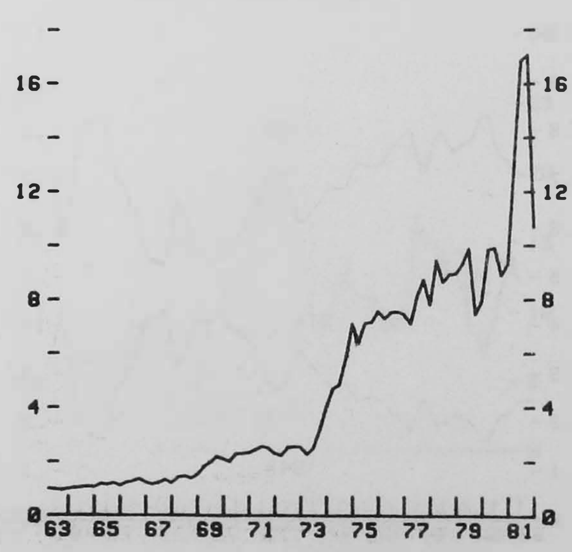
24. M3 user cost index 1963-81



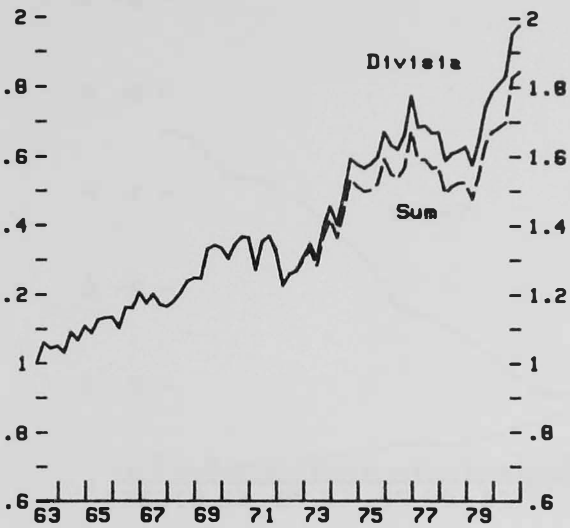
25. PSL1 user cost index 1963-81



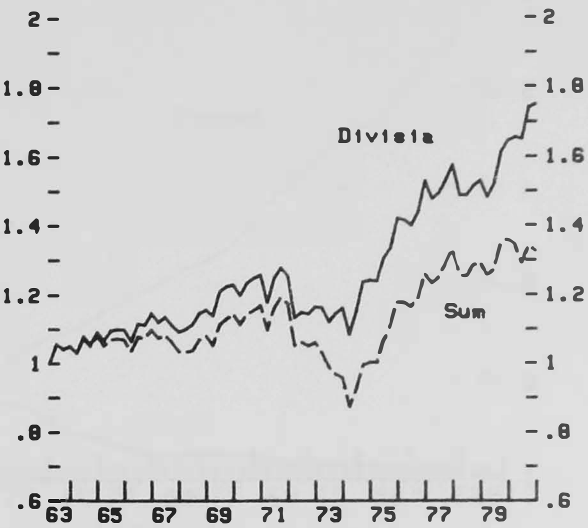
26. PSL2 user cost index 1963-81



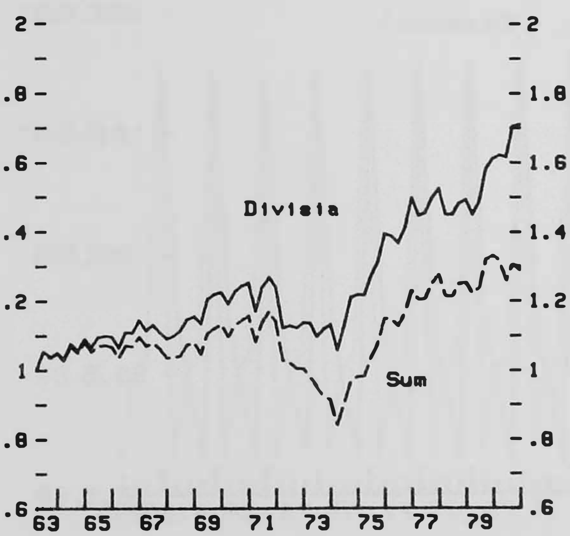
27. M1 velocities 1963-80



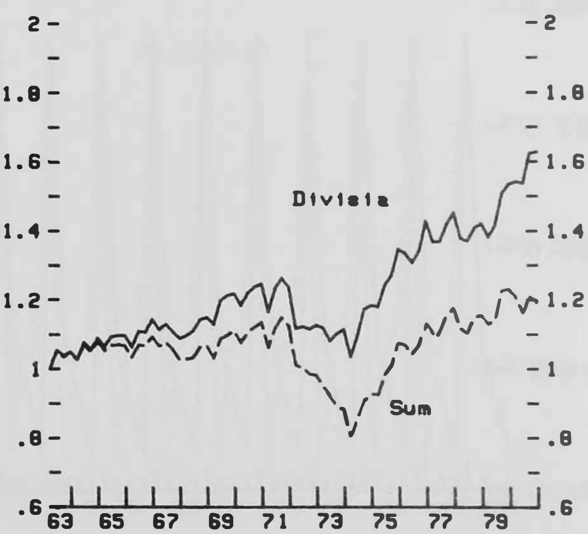
28. M2 velocities 1963-80



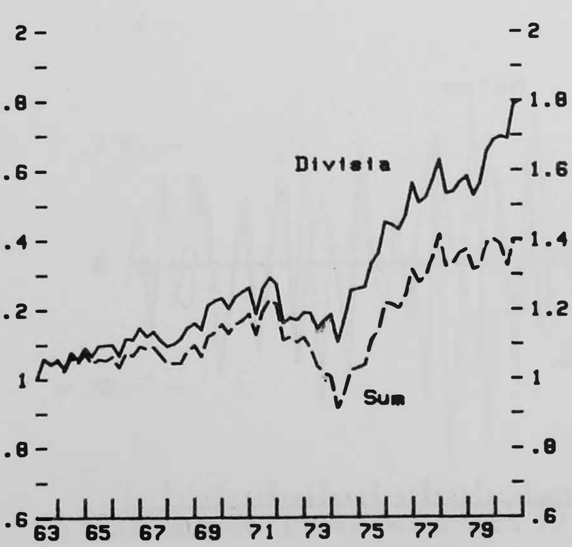
29. £M3 velocities 1963-80



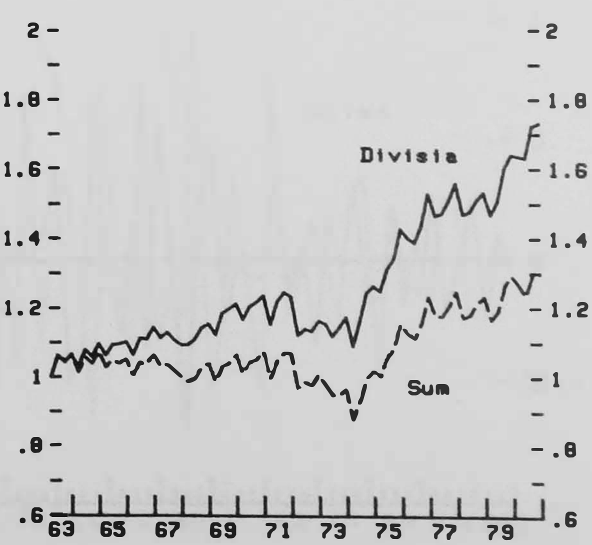
30. M3 velocities 1963-80



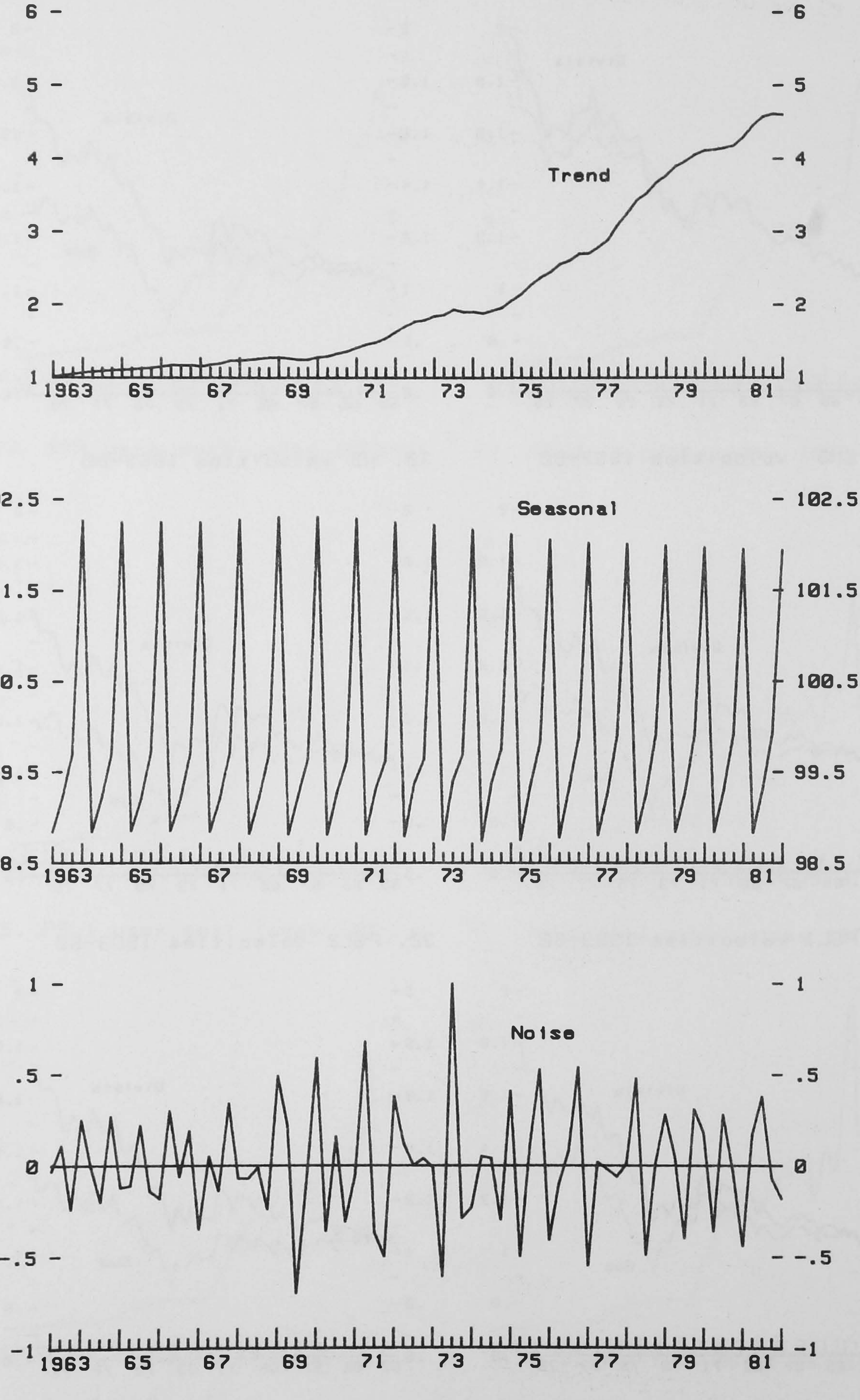
31. PSL1 velocities 1963-80



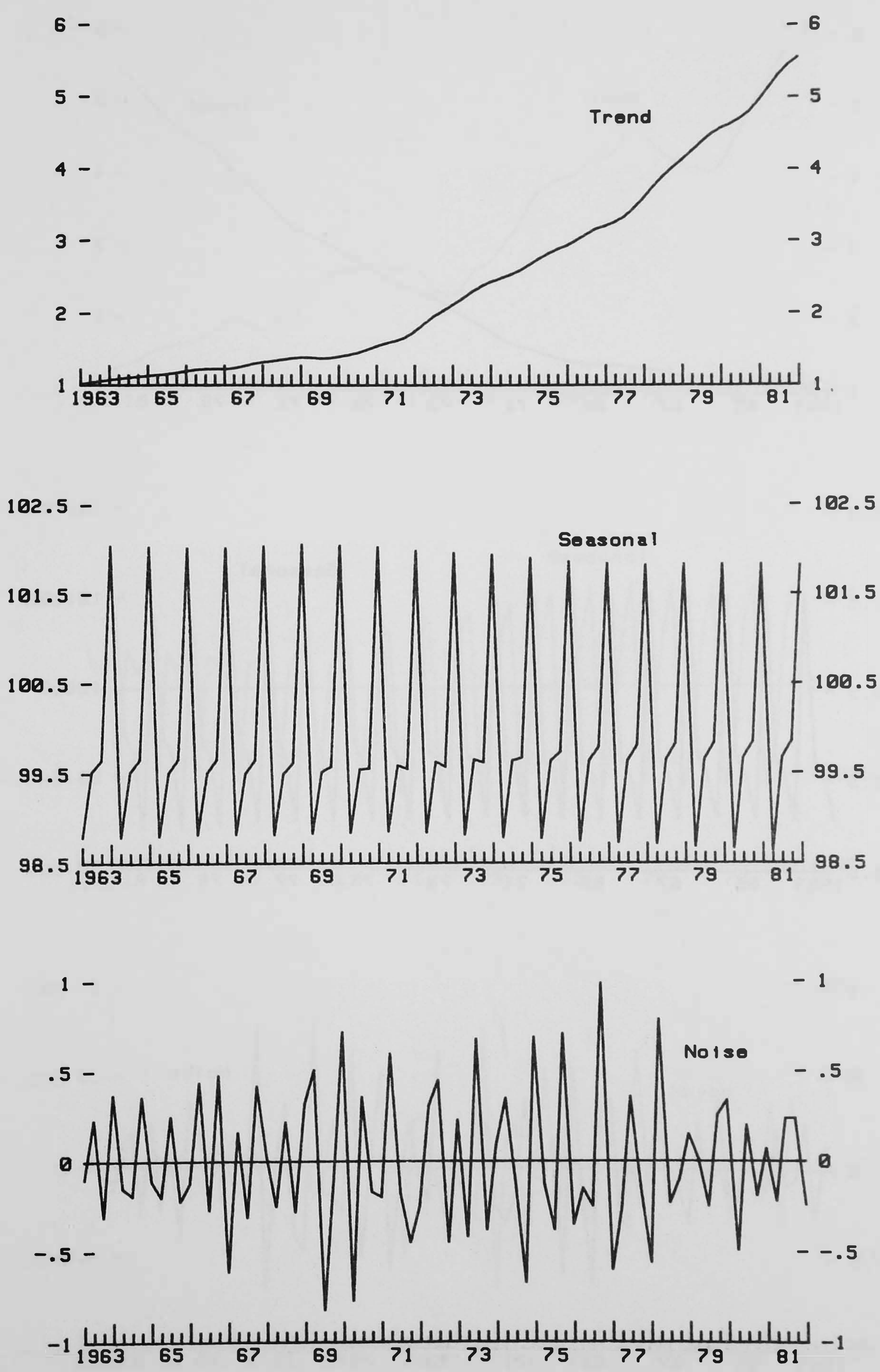
32. PSL2 velocities 1963-80



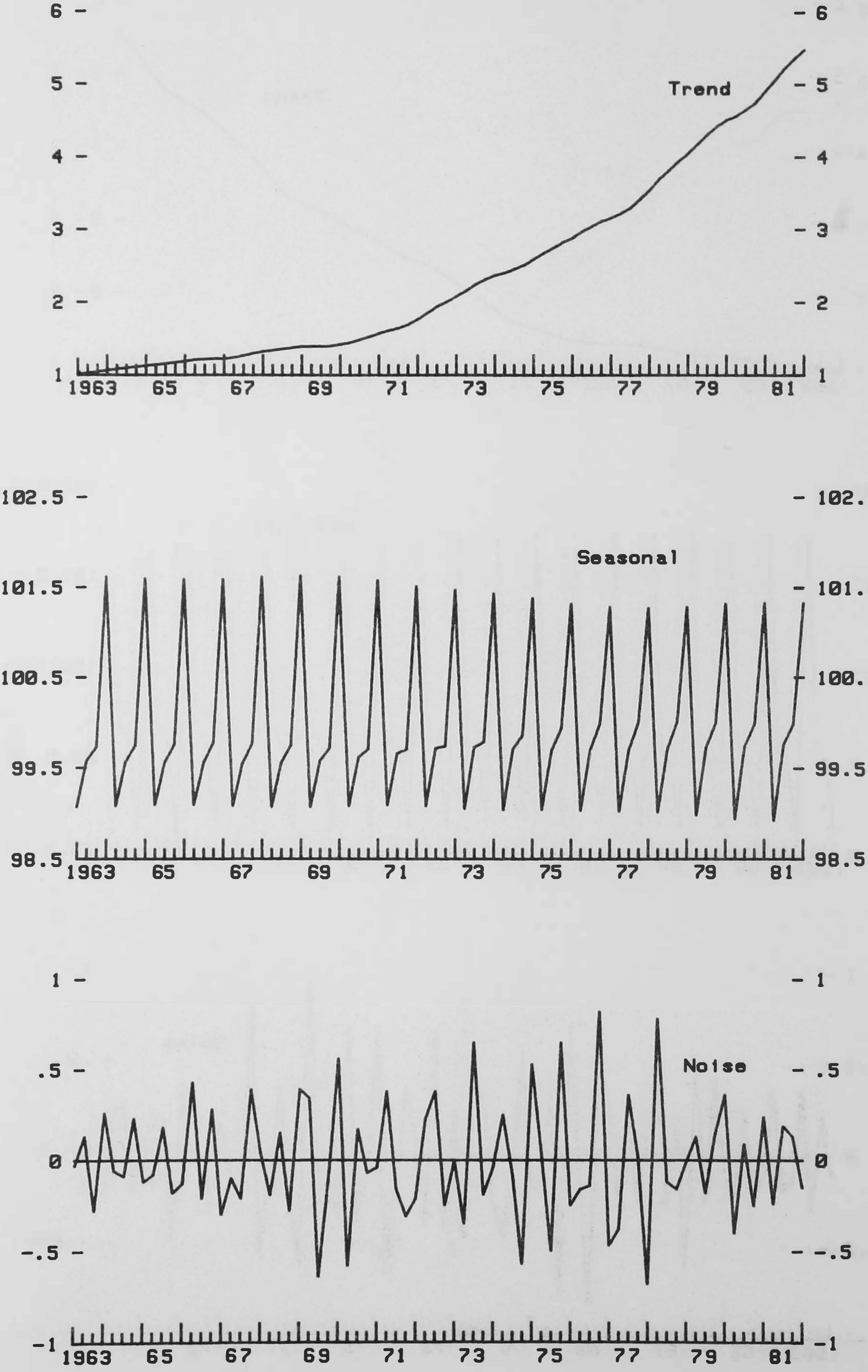
33. Decomposition of divisia M1



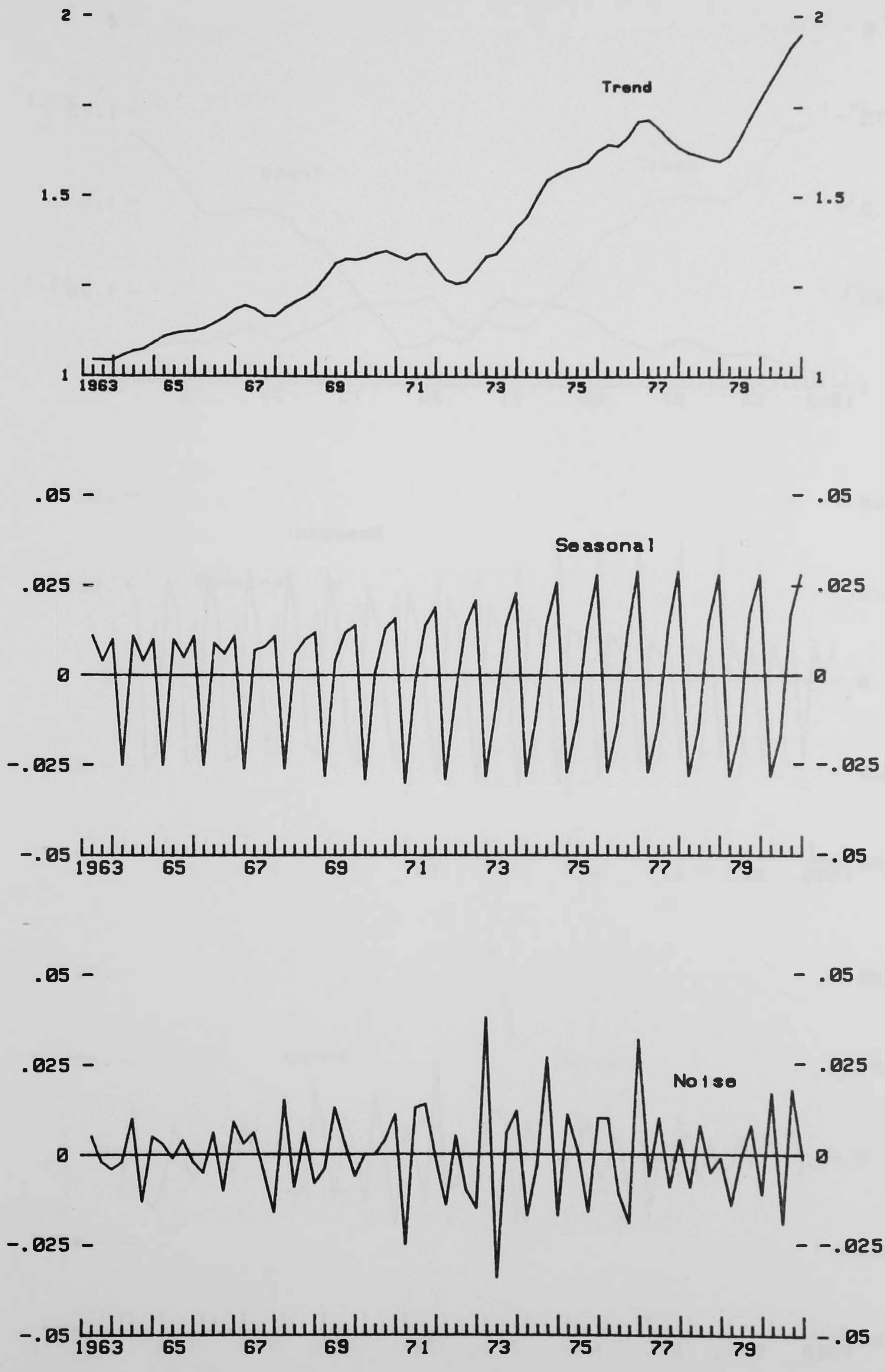
34. Decomposition of divisia £M3



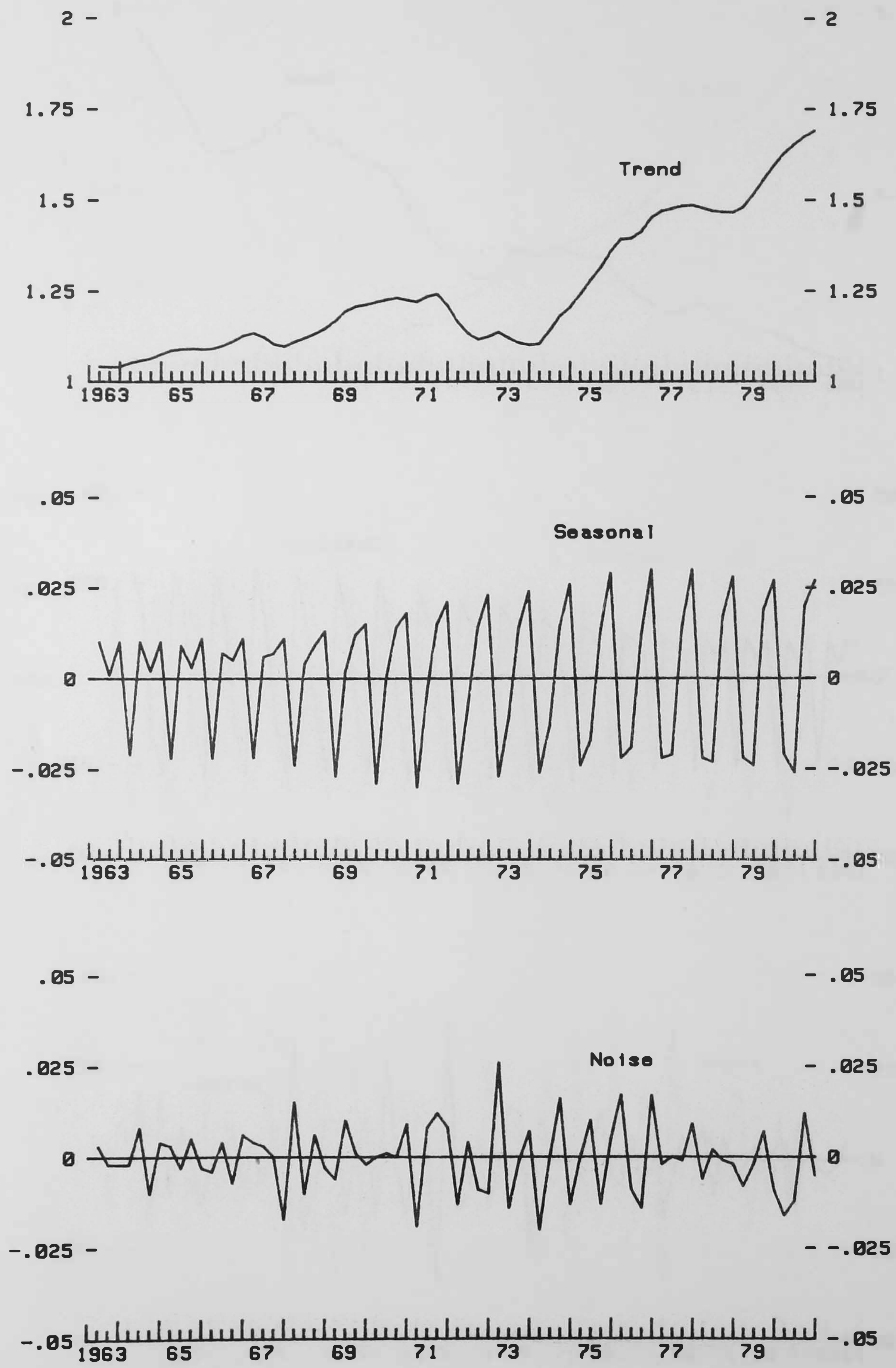
35. Decomposition of divisia PSL2



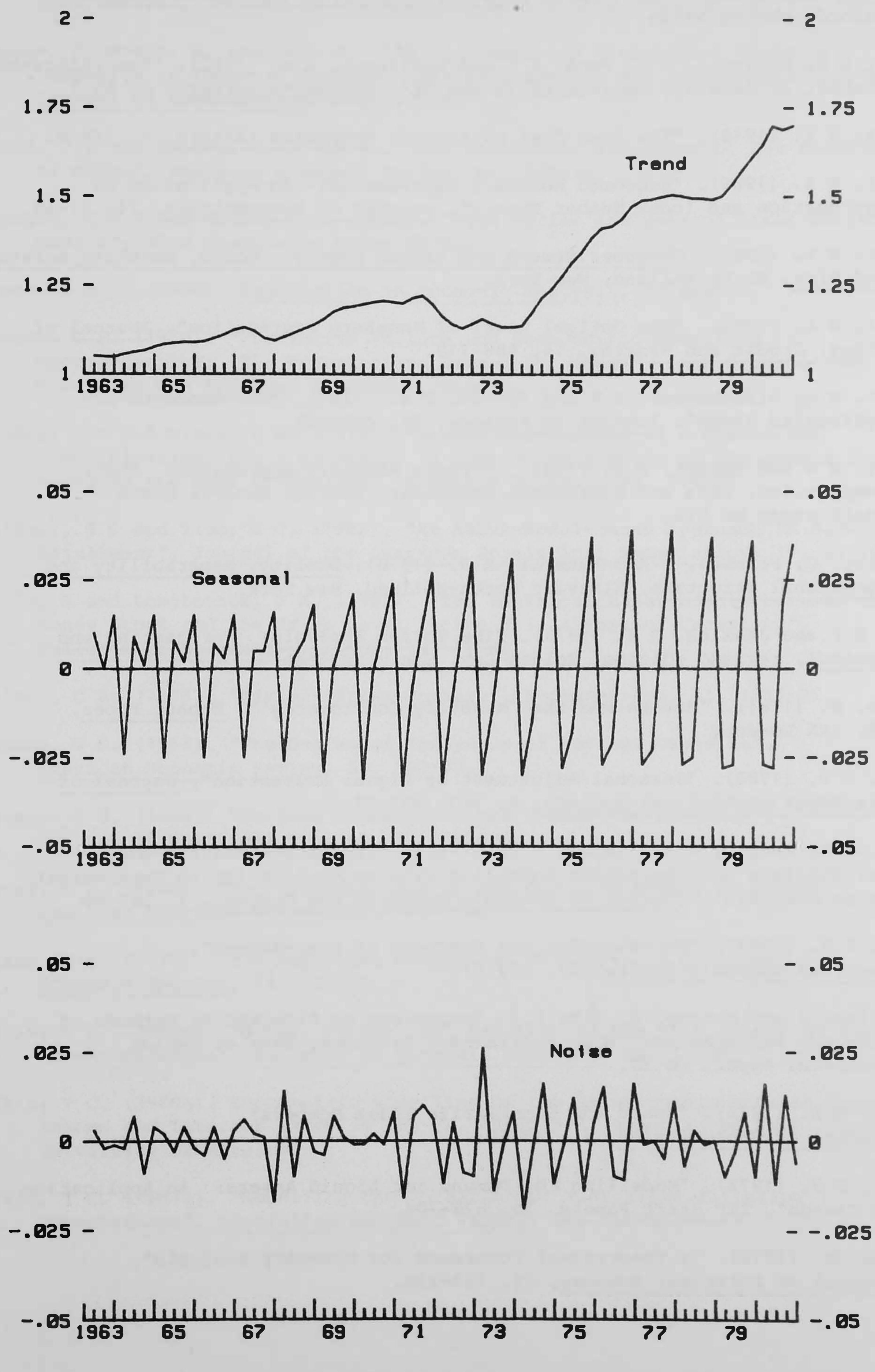
36. Decomposition of divisia M1 velocity



37. Decomposition of divisia £M3 velocity



38. Decomposition of divisia PSL2 velocity



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Bank of England Discussion Papers

	Title	Author
1-5, 8 12-14, 16 & 17	*A list of these papers can be found in the December 1981* Bulletin, *or can be obtained from the Bank. These papers are now out of print, but photocopies can be obtained from University Microfilms International (see below).*	
6	'Real' national saving and its sectoral composition	C T Taylor A R Threadgold
7	The direction of causality between the exchange rate, prices and money	C A Enoch
9	The sterling/dollar rate in the floating rate period: the role of money, prices and intervention	I D Saville
10	Bank lending and the money supply	B J Moore A R Threadgold
11	Building societies: an econometric model	M J Pratt
15	Influences on the profitability of twenty-two industrial sectors	N P Williams
18	Two studies of commodity price behaviour: Interrelationships between commodity prices Short-run pricing behaviour in commodity markets	 Mrs J L Hedges C A Enoch
19	Unobserved components, signal extraction and relationships between macro-economic time series	T C Mills
20	A portfolio model of domestic and external financial markets	C B Briault Dr S K Howson
21	Deriving and testing rate of growth and higher order growth effects in dynamic economic models	K D Patterson J Ryding

Technical Series

	Title	Author
1	The consumption function in macroeconomic models: a comparative study*	E P Davis
2	Growth coefficients in error correction and autoregressive distributed lag models	K D Patterson
3	Composite monetary indicators for the United Kingdom; construction and empirical analysis	T C Mills

Papers presented to the Panel of Academic Consultants[(a)]

	Title	Author
8	International monetary arrangements: the limits to planning*	P M Oppenheimer
9	Institutions in the financial markets: questions, and some tentative answers*	M V Posner
10	The arguments for and against protectionism*	M Fg Scott The Hon W A H Godley
14	The usefulness of macroeconomic models*	Prof W H Buiter T F Cripps Prof Angus Deaton Prof A P L Minford M V Posner
15	Factors underlying the recent recession	G D N Worswick Dr A Budd
17	Why do forecasts differ?	Prof M J Artis
19	Bank lending, monetary control and funding policy*	Prof A D Bain

* These papers are no longer available from the Bank, but photocopies can be obtained from University Microfilms International, at 30-32 Mortimer Street, London W1N 7RA.

(a) Other papers in this series were not distributed.