Bank of England

Discussion Papers

Technical Series

No 39

Exchange rate equations

by

Helen Allen

Brian Henry

and

Bahram Pesaran

March 1991

No 39

Exchange rate equations

by

Helen Allen

Brian Henry

and

Bahram Pesaran

March 1991

The object of this Technical Series of Discussion Papers is to give wider circulation to econometric research work predominantly in connection with revising and updating the various Bank models, and to invite comment upon it; any comments should be sent to the authors at the address given below.

The authors would like to thank John Dorrington, Paul Fisher, John Flemming, Stephan Hall, Ian Michael and Mike Wickens for comments on an earlier draft. Final responsibility for the contents rests solely with the authors.

The authors would like to thank the Bulletin Group for editorial assistance in producing this paper.

Issued by the Economics Division, Bank of England, London, EC2R 8AH to which requests for individual copies and applications for mailing list facilities should be addressed; envelopes should be marked for the attention of the Bulletin Group. *(Telephone: 071-601-4030)*

ISBN 0 903315 75 0

ISSN 0263-6123

Exchange rate equations

1 Introduction

This paper presents estimates of single equation exchange rate models, estimated on quarterly UK data. Such single equation models may be distinguished from implicit models of the exchange rate, which are obtained, for example, by solving the entire financial sector for market clearing prices, including the price of foreign exchange. An example of such an implicit model is the Financial Sector model described in Keating (1985). The disadvantage in determining the exchange rate in this way, by inverting a large set of financial asset equations is, however, clear. In such models, the determinants of the exchange rate are implicit, and indeed may not be *a priori* acceptable since misspecifications which arise in estimating the financial sector will in part determine the properties of the implied exchange rate. Hence there are reasonable grounds for adopting an alternative—single equation—approach as we do here. In the alternative the single equation may still be interpreted as arising from general asset market behaviour, where the exchange rate equation is interpreted as the reduced form of the system of asset demands and supplies. The overriding advantage of this alternative, however, is that its properties are clear, and their coherence with the evidence can be tested directly.

Using the single equation approach as we do in this paper enables us to search for the determinants of the equilibrium level of the exchange rate. Like much empirical work directed at the identification of a dynamic behavioural equation, the present work employs a two-stage estimation procedure, and considerable emphasis will be placed on the first stage co-integration part of this. By attempting to identify the minimal set of determinants of the level of the exchange rate, the alternative variables which have figured in the discussions of exchange rate determination can be evaluated, and a parsimonious specification chosen. The results can then be introduced into the second stage-dynamic-equation following the precepts of Granger and Engle. However, there is widespread concern about the (possibly) misleading properties of two-stage estimation where these are based on OLS. The first stage estimates may be badly biased for example [Banerjee et al (1986)]. To overcome some of these problems we have used two additional pieces of supporting estimation: The Johansen maximum likelihood technique for determining the set of co-integrating vectors, and a three step estimation procedure recently proposed by Engle and Yoo (1990). Further details of these are given in Section 3 below.

The plan of the paper is first to briefly describe underlying theoretical issues, and to comment on the choice of models which are selected for further investigation. This is the subject of the next section. The third section then provides empirical results and section four concludes.

2 Single equation exchange rate models

A number of alternative exchange rate models can be derived from the augmented uncovered interest parity condition (*UIP*),

$$\varepsilon_t = \varepsilon^e_{t+1} + r_t - r^*_t + \delta$$

Where ε_t is the log of the exchange rate, r the interest rate (r^* denotes the overseas rate) and δ may be interpreted as indicating the degree of capital mobility so $\delta = 0$ implies perfect capital mobility. The equation may be expressed in real terms, with the real exchange rate as the dependent variable depending on the real interest differential, where the real exchange rate is defined as $\varepsilon+p-p^*$, where p is the log of the price level (and p^* the overseas equivalent).

Hence we may write

$$\rho_t = \rho^e_{t+1} + (i - i^*)_t + \gamma X_t \tag{1}$$

where ρ is the real exchange rate, and i the real interest rate. The variable X_t is included as a (set of) determinant(s) of the risk premium. [See Fisher et al (1990).]

The model has been extended to allow for a 'risk' premium. Among the variables which may affect the risk premium are the current balance, or other functions of the external account, overall fiscal measures (eg the real PSBR), and other variables such as oil prices. Note that the current balance can also be entered into an exchange rate equation by solving the balance of payments equation for the exchange rate, though the formulation above—interpreting external balance terms as determinants of the risk premium—is adopted here.

Before implementing the equation suggested by (1), there are two things which need further clarification; the role of expectations in the model, and second the actual form of equation we estimate.

On the first point the equations presented here will not use the expected exchange rate as a determinant of the current exchange rate. This reflects a conscious decision to develop an exchange rate model dependent upon current and lagged variables only, since it is planned to use it in a full macro-model which is not based on model consistent expectations. Such lagged exchange rate equations may be interpreted as unrestricted versions of equation (1), however, where the expected future value of the exchange rate has been substituted out using lagged regressors so that, on this assumption, the underlying interpretation of the equation as dependent on relative rates of return still holds.

On the second point the next section will report estimates of equations for the level of the real exchange rate, (ρ) of the form (in logs)

$$\rho \equiv \left(\varepsilon - p^* + p\right) = \Gamma' \tilde{Z} + \mu \tag{2}$$

where $\tilde{Z}$ is a vector of variables, including the real interest differential, a measure of the external balance in real terms, etc, and μ is an error term. Within this general definition, the alternatives reported in the next section, and their interpretation, are as follows.

(i) If $\tilde{Z}$ includes the real interest differential then the real exchange rate equation can be written,

$$\rho = \gamma_0 + \gamma_1 (i - i^*) + \gamma_2 Z + \mu \tag{3}$$

where i_t, for example, is the real interest rate given by $r_t - (p^e_{t+1} - p_t)$, and proxied here by $r_t - \Delta p_t$, and where Z is a subset of $\tilde{Z}$. The interpretation of the role of relative asset returns in the equation is not the same as implied in the UIP conditions in which the rate of exchange rate *depreciation* is equal to the interest differential. Equation (3) suggests instead that the *level* of the real exchange rate depends upon the *level* of real interest differentials. In other words the underlying asset demand is based on a stock-adjustment model, whereby a change in relative interest rates causes the exchange rate to move to a new equilibrium level.

(ii) An alternative formulation which is more consistent with the UIP condition includes the integral of interest rate differentials as a determinant of the level of the exchange rate, ie

$$\rho = \gamma'_0 + \gamma'_1 \sum_k (i - i^*)_{t-k} + \gamma'_2 Z + \mu \qquad (4)$$

Thus if the arbitrage condition is written as $\Delta\rho = \gamma_1 (i - i^*)_{t-1}$, on integrating this gives

$$\rho_t = \gamma_1 \sum_k^{\infty} (i - i^*)_{t-k-1} + \text{constant}$$

This simple point serves to motivate the use of the cumulation rather than the level of the interest rate differential in an equation for the level of the exchange rate. But, there are additional problems with this approach. One in particular is its implication for the error term. Thus, on integrating the real UIP term, augmented by a risk term φ, ie

$$\Delta\rho = (i - i^*)_{t-1} + \varphi_t + \mu_t \qquad (5)$$

gives

$$\rho = \sum_k (i - i^*)_{t-k-1} + \sum_k \varphi_{t-k} + \sum_k \mu_{t-k} \qquad (6)$$

The presence of the cumulative error term $\sum \mu_{t-k}$ implies that ρ and the cumulative interest differential cannot be co-integrated, as $\sum \mu_{t-k}$ is I(1).(1) What this serves to illustrate is that the augmented *UIP* [equation (5)] may not be a valid representation of the way the data actually behave. Thus, if equations like (6) appear to have I(0) error processes when estimated, this suggests that the assumption made in (5) that μ_t is a stationary white noise is not true.

(iii) Z_t includes the current deficit as a percentage of GDP or the trade deficit in real terms. Since we are concerned here with a model of the effective exchange rate, it is appropriate to use such a current deficit term rather than, say, the UK deficit relative to an overseas one. The use of relative deficits may be relevant when modelling bilateral exchange rates, but are clearly not sensible in equations for the effective rate. In the present context, as we previously noted, the external deficit can be interpreted as a determinant of the risk premium. (Although it could be argued that the UK deficit relative to a weighted average of all other countries deficits—with weights equal to those in the calculation of the EER—could have been used.) Another alternative uses the cumulative deficit [see eg Hooper and Morton (1982) and Meese and Rogoff (1983)]. Excluding revaluations, this is equivalent to the use of real net overseas assets (ie expressed as a percentage of GDP), as used by Barrell et al (1988). In practice, however,

(1) We are grateful to Mike Wickens for pointing this out.

the presence of the large balance of payments balancing item and the fact that revaluations have historically been the principal element of change in identified net assets render this equivalence somewhat tenuous.

(iv) Instead of the cumulative current deficit, a similar term in the PSBR may be used. This variant can have one of two interpretations. One of these interprets the sum of fiscal deficits as an influence upon the risk premium, this time originating from concern about the 'stock' of the government's credibility. The PSBR is likely to be perceived as more directly under the authorities' control than is the current balance, so may affect confidence in the foreign exchange market more directly. The other interpretation is that the fiscal deficit proxies the external deficit used in (iii) above, and since data on the fiscal deficit is more reliable, there may be a (admittedly fairly weak) statistical argument for preferring this to direct measures of the external account.

Portfolio considerations may be used to justify the presence of either the stock of government debt (cumulative PSBR) or the net stock of overseas assets (the cumulative current account). Although we have already observed that there are problems in giving this stock interpretation to the cumulative current account deficit, nonetheless the general point is that it, and the cumulative PSBR, can be thought of as proxying asset supplies. In turn this points to a portfolio-type interpretation of the real exchange rate equation where this depends upon cumulative current or fiscal deficits, or both.

Having discussed the alternative variables which can be used in exchange rate equations, the next section describes estimation results for these alternatives.

3 Estimation results

The equations reported next were estimated on quarterly data, from 1979:1 to 1989:3, this sample being taken as a period with relatively free capital movements.

Exchange Rate Equations

We start with preliminary time-series tests. First orthodox tests for the order of integration of the variables which are to be used in the exchange rate model are provided. Table 1 below gives the details.

Table 1: Orders of integration

Variable	Level DF	Level ADF	Difference DF	Difference ADF	Order of Integration
LEERR	-2.67	-1.99	-5.42	-3.47	I(1)
PSBRR	-9.17	-2.53			I(0)
FINTRS	-3.56	-2.77	-6.20	-4.75	I(1)
BALR	-2.98	-2.26	-7.71	-2.78	I(1)

Critical value for DF/ADF is -3.5(5%) where the test uses a time trend.

L refers to a logarithm.

Where: *EERR* is the real effective exchange rate.

PSBRR is the PSBR as a percentage of nominal GDP.

FINTRS is the short term interest rate relative to the US, deflated by consumer prices.

BALR is the current balance as a percentage of nominal GDP.

First, some further comments about the results shown in Table 1 are in order. The order of integration of the real exchange rate is clearly I(1). The proportion of the PSBR to GDP ratio does not have clear integration properties, with the DF and ADF pointing to different conclusions. A DF regression for the variable suggests that the DF may be the more meaningful of the summary tests, however, as there is no evidence of serial correlation in the regression. Hence we are probably justified in treating the variable as I(0) in spite of the finding that the ADF is -2.53. In turn this implies that the cumulation of *PSBRR* is an I(1) variable and this is important for our later attempts at explaining the level of the exchange rate. A number of additional tests were undertaken in the case of the interest rate differential since its univariate properties were also not clear. According to theory, the interest differential should be I(0), since we might expect that arbitrage should eliminate excess returns. The DF statistic is compatible with that expectation, though the ADF is not. Moreover, inspection of the spectral density function for the variable shows that the variations in the series is dominated by the very low frequencies, indicating non-stationarity. What the balance of this evidence suggests therefore is that the interest rate differential—for this sample—is an I(1) series, and hence the cumulative differential (such as would appear in equation (4) above) is I(2). There are, inevitably, important issues raised by the finding, which is in any event not a clear cut one. At present we proceed with the view that *FINTRS* is I(1). Later in Section (2) we report on the transformation of the series which renders it an I(0) series. The consequences of each of these alternatives for the behaviour of the exchange rate is explored in subsequent—co-integrating—exercises. Continuing for the moment with the results from Table 1, the final variable *BALR* is I(1), although the ADF for the difference of the variable raises doubts about this. Again a DF regression points to the DF being the more relevant statistic for this variable, hence we conclude that *BALR* can be treated as an I(1) variable.

Next, we consider the evidence for co-integration, based on the proposition that the interest rate variable (*FINTRS*) is I(1). This is the subject matter of the next section. In it we will present evidence on co-integration based on a number of approaches; the Engle-Granger two step method, the Engle-Yoo 3-step, and maximum likelihood.

(1) Treating *FINTRS* as I(1)

Based on the findings of Table 1—in particular that the interest differential is I(1)—we proceed to search for a levels equation for the real exchange rate. This part of the exercise follows the Engle-Granger 2-step estimation procedure initially. But, as there are serious doubts about this form of estimation, especially with the biasses which can arise at the first stage, we augment this by re-estimating the equation using the 2-stage procedure described by Engle and Yoo [1990] and by the exact maximum-likelihood procedure developed by Johansen (1988).

(a) Granger-Engle estimates

The real exchange rate equation estimated by OLS is shown next. This uses the cumulative interest differential (CFINTRS) and the cumulative *PSBRR* and *BALR* (*CPSBRR* and *CBALR* respectively). The use of the cumulative interest rate differential is dictated by the discussion around equation (4) above; that it can be viewed as an integration of the (lagged) UIP conditions to give an equation between the level of the exchange rate and the cumulative sum of the interest rate differential. According to our tests of integration, this cumulative variable is I(2). Hence to search for a co-integrating equation, a further I(2) variable is necessary, and we use *CBALR* which is also I(2). However, both the cumulative fiscal and external deficit are used in the equation to measure the effect of stocks of assets of the government and overseas sector respectively, though only the latter is I(2) (See Table 1).

$$LEERR_t = 4.79 + .2632\, CFINTRS_t + 0.127\,(10^{-3}) CBALR_t - 0.105\,(10^{-2}) CPSBRR_t \qquad (7)$$

$R^2 = 0.628$ CRDW = 0.877 DF = -3.77 ADF(1) = -4.67
Sample 1980:Q1–1989:Q3

According to the criteria usually applied in these equations, the present example is fairly satisfactory. The Durbin-Watson and Dickey-Fuller tests point to there being a stationary error process in the equation. The parameters have acceptable signs; in particular a worsening current deficit to GDP ratio lowers the exchange rate, as does an increase in the fiscal deficit. In the long run, according to the estimated equation, the higher the level of domestic interest rates, given overseas rates, the higher the exchange rate.

The next step incorporates the lagged residuals (*RES*) from the levels equation above, as an ECM term in a dynamic equation for the exchange rate. The results for the dynamic equation are shown next.

$$\underset{}{\Delta LEERR_t} = \underset{(2.52)}{0.304\Delta LEERR_{t-1}} - \underset{(2.459)}{0.595\Delta FINTRS_t} + \underset{(1.356)}{0.378\Delta FINTRS_{t-1}} - \underset{(2.119)}{0.002PSBRR_t} - \underset{(4.66)}{0.578RES_{t-1}} \quad (8)$$

$R^2 = 0.599$ $\sigma = 0.026$ DW = 2.09 LM(4) = 5.56
RESET(1) = .83 HETEROSC = .18 NORM(2) = .11

Again the properties of this dynamic equation are plausible, and its overall statistical features are good. It implies that increases in interest rates (given overseas rates) are first associated with a *decline* in the exchange rate, though in the long run the effect is to increase the exchange rate. This kind of behaviour is broadly consistent with overshooting, with an immediate overshooting following the increase in interest rates (and this happens too quickly to be identified in quarterly observations) thereafter the rate declines to its new equilibrium but at a level higher than that existing before the increase in interest rates.

A further test of this model investigates its forecasting performance. To do this the model is completely re-estimated over a shorter sample period, ie both the levels and difference equation is re-estimated, this time over the period 1980:Q2–1988:Q3. The model is then used to produce the four remaining data points. Its one-step predictive performance is good, a Chow test F(4, 29) is 0.568 which is acceptable. The model tends to underpredict, but it outperforms a random walk model over the forecast period.

(b) Engle 3-stage estimation

Because, as we have already noted, there is concern that the Engle-Granger two step estimation procedure can lead to biasses, equation (8) is re-estimated by applying the recent suggestion of Engle and Yoo (1990) for estimating ECM equations. By following this extension the parameter estimates at the first stage-levels—regressions may be corrected so that they are asymptotically equivalent to *FIML* and valid standard errors for the parameters can be calculated. Hence this extended procedure enables us to conduct 't' tests on the variables at the first (levels) stage of the two-stage estimation exercise, which the Engle-Granger procedure does not. The primary disadvantage of the three stage procedure however, is that it assumes there is a unique co-integrating vector and moreover it assumes the weak exogeneity of the conditioning variables in the dynamic model. [See Hall and Taylor (1989).] Although these are assumptions which are implicitly made when applying the Engle-Granger two step technique, nonetheless they need to be tested, and the three stage extension does not do this. [Section (c) below rectifies this somewhat as the uniqueness or otherwise of the co-integrating vector is tested via the application of Johansen's maximum likelihood technique.] We provide an illustration of equation (8) estimated by the three-step procedure below, where the third step regresses the levels variables times the error correction parameter, on the errors from the second-stage dynamic model. The adjusted coefficients of the long-term relationship (7) along with their t-ratios obtained by performing this third estimation step to (7) and (8) are reported next.

Variable	Original coef	adjusted coef	t-ratio
constant	4.79	4.80	152.7
CFINTRS	.263	.229	2.04
CBALR	.000127	.00116	-.49
CPSBRR	-.00105	-.000850	-1.28

We should treat the above results with caution since it will be established in the next section that a unique co-integrating vector does not exist. But according to the present results the adjusted coefficient of *CBALR* has the wrong sign although admittedly is not significantly different from zero.

(c) Maximum likelihood estimates

The final application which are based on the results in Table 1—which suggest that the cumulative *CFINTRS* and *CBALR* are I(2) variables—applies the Johansen procedure. This not only gives ML estimates of the co-integrating vectors, but also gives a test procedure for determining the number of distinct co-integrating vectors which exist within the set of variables (the uniqueness question).

In applying the Johansen procedure however, only I(1) variables may be used. As Table 1 shows, two of the relevant variables are I(2). To proceed with the ML procedure therefore we use the ploy of forming an I(1) combination of the two I(2) variables, simply by regressing one on the other. This gives:

$$CFINTRS_t = -0.027 - 0.00108 CBALR_t + z_t \qquad (9)$$

The residual from this regression z should be I(1). The DF/ADF statistic for the level and difference of *z* are -1.99 and -3.46 respectively, suggesting that *z* is I(1) as required. The next step is then to derive tests for the number of independent co-integrating vectors in the set of variables—*z* (from (9) above), *LEERR* and *CPSBRR*. The results of this are shown next.

Table 2: Number of distinct co-integrating vectors

Null	LR[a]	LR[b]
$r = 0$	29.4 (22.0)	56.0 (34.9)
$r \leq 1$	21.3 (15.7)	26.7 (19.9)
$r \leq 2$	5.37 (9.2)	5.37 (9.2)

(a) Is an LR test based on the maximum eigenvalue.
(b) Is an LR test based on the trace of the matrix.

Figures in brackets are the 95% cut-off values.

According to these tests, there are two distinct co-integrating vectors among this set of variables. The two vectors associated with the two largest eigenvalues are:

LEERR	*CPSBRR*	z	c
1.56	0.0131	1.69	-8.59
3.90	0.005	-1.52	-18.51

Of these vectors the first may be interpreted as a a policy reaction function if it is normalised on *CFINTRS* (an element of z). This implies a negative relationship between interest rates and the exchange rate, "leaning into the wind", and a negative relation between interest rates and the current deficit which is consistent with tightening monetary policy when demand expansion worsens the external account. The relationship with the fiscal deficit is difficult to justify, however, as a positive relationship is normally expected, unless monetary policy accommodates fiscal expansions in which case a negative correlation could occur. The second vector may be normalised on the exchange rate, yielding an equation not very different from the OLS equation (7).

At this point, given the existence of two distinct co-integrating vectors, both may be used in a dynamic equation. Providing the conditioning variables are I(0), then the usual 't' statistic can be used to discriminate between them, leading to the elimination of one if it proves to be insignificant. But, as the following result shows, both vectors appear to be significant when entered into a dynamic equation for the exchange rate. (*RES*1 is the residual from the first co-integrating equation above, *RES*2 the residual from the second.)

$$\Delta LEERR_t = \underset{(1.93)}{0.25}\,\Delta LEERR_{t-1} - \underset{(2.82)}{0.72}\Delta^2 CFINTRS_t + \underset{(1.46)}{0.4}\Delta^2 CFINTRS_{t-1} - \underset{(3.30)}{0.005}PSBRR_t$$

$$- \underset{(1.76)}{0.002}PSBRR_{t-1} - \underset{(3.36)}{0.096}RES1_{t-1} - \underset{(4.12)}{0.42}RES2_{t-1}$$

$R^2 = 0.62$, $\sigma = 0.026$, LM(4) = 3.65, RESET(1) = 0.099

This result is interesting because more than one levels equation turns out to be significant and both levels equations can be given an economic interpretation. As a practical matter if the first of these were true, but not the second, then the obvious thing to do would be to incorporate only that residual from the levels equation which could be given an economic justification. The residual from the other equation could be excluded on *a priori* grounds. But this is not the case in the example above. The residual from both equations are significant, and their joint presence may be rationalised as representing the asset behaviour underlying the foreign exchange market itself (the second equation in the pair of co-integrating equations is consistent with this view), and a policy reaction function (the first equation). The level of the exchange rate is then determined by the joint presence of both of these equations. Although such an interpretation is possible, it may not be satisfactory. This is especially so when an equation such as (9) is used in a complete macro model. Usually when using a macro model for forecasting, these forecasts are conditioned upon an exogenous policy judgement. The added complexity which an equation like (9) introduces is that the policy rule embedded within it may not be consistent with the policy judgement on which the forecast is conditioned. This is not an issue which can be resolved easily, and it is not proposed to try to do so here. So, although the results in (9) are of interest, we next report on an levels equation with a simpler interpretation to those given in Table 2. This alternative is based on the transformation of the interest differential (*FINTRS*) to an I(0) variable, and the nature of this transformation and the implications of it for the model of the exchange rate are the subjects of the next section.

(2) Treating *FINTRS* as I(0)

Although as it stands *FINTRS* is 1(1), inspection of this variable reveals that there has been an increase in the level of the series since 1985:Q1. It can be argued that the change in the underlying time series behaviour is due to eg a regime change, and behaviour before and after that can be represented by a stationary series. It is recognised, of course, that this does not explain why this apparent change occurred. But the interest differential is likely to be I(0) according to *a priori* reasoning and, apart from the jump in the series in 1985:Q1, its time series properties are consistent with this, as we now show. To proceed with this approach we fit a regression of *FINTRS*, a constant term, and a dummy variable (*DUM*851). The dummy variable takes the value of 1 from 1985:Q1 onwards and is zero otherwise. The residuals of this regression are labelled *NEWFINTRS*, ie:

$$NEWFINTRS = FINTRS + 0.0202 - 0.0416\,DUM851$$

Carrying out the usual tests on *NEWFINTRS* reveals that this variable is indeed I(0). The cumulated value of this variable (*CNEWFINTRS*) is, therefore, I(1) and we have used this together with *LEER* and cumulated *PSBRR* to identify a co-integrating relationship between these variables, all of which are now I(1). When the Johansen maximum likelihood technique is used, it turns out there exists a unique co-integrating vector, and hence the application of the Engle and Yoo (1990) 3-step method is justified in this case.

The details of the estimation of an error-correction model are then as follows. Firstly, to decide on the number of independent co-integrating vectors tests for the number of distinct co-integration vectors were conducted, and the results are shown in Table 3:

Table 3: Further tests for co-integrating vectors

Null	LR^a	LR^b
$r = 0$	32.0 (22.0)	47.99 (34.9)
$r \leq 1$	11.95 (15.76)	15.97 (19.96)
$r \leq 2$	4.03 (9.24)	4.03 (9.24)

(a) Is an LR test based on the maximum eigenvalue.
(b) Is an LR test based on the trace of the matrix.

Figures in brackets are the 95% cut-off values.

According to these, there is one co-integrating vector, and the unique co-integrating vector obtained by the Johansen method is,

$$LEERR = 4.951 + 0.416\,CNEWFINTRS - 0.00309\,CPSBRR \qquad (13)$$

This equation is simple, and has acceptable (ie interpretable) signs on the coefficients. Consequently the residuals from (13) are used in the estimation of the dynamic equation for the exchange rate. A simplified dynamic model was obtained, and this is shown next.

$$\begin{aligned} DLEER_t = & -.529NEWFINTRS_t + 0.977NEWFINTRS_{t-1} - 0.00631PSBRR_t \\ & \quad (1.80) \qquad\qquad (3.18) \qquad\qquad (4.21) \\ & -.00412PSBRR_{t-1} - .410RES_{t-1} \\ & \quad (3.30) \qquad\qquad (4.71) \end{aligned} \qquad (14)$$

$R^2 = .56$ $\sigma = 0.027$ DW = 1.88 LM(4) = 3.41 RESET(1) = .49
HETEROSC(1) = .39 NORM(2) = 0.85

The resulting equation has plausible economic properties and its statistical properties are good, so the results of this version of the model are encouraging. According to (14) the interest differential shows a negative followed by a positive dynamic response in the exchange rate, with a positive relationship holding between the variables in the long run. This is broadly consistent with overshooting in the real exchange rate. Furthermore, the equation includes well established effects from the fiscal balance—both the level and its cumulative sum—upon the exchange rate.

As a final exercise on this model, we have also conducted an illustrative exercise using the Engle-Yoo three step procedure. Since there is a unique co-integrating vector the use of 3-step estimation is justified. When it is used, the final results are broadly similar to the ones obtained using the Johansen method as shown by equations (13) and (14). To implement the Engle-Yoo three step estimation we first estimate an OLS levels equation. This estimation is reported in (15) below:

$$LEERR = 4.7791 + 0.0938\,CNEWFINTRS - 0.00163\,CPSBRR \quad \textbf{(15)}$$

$R^2 = .43$ CRDW = .58 DF = -2.78 ADF(4) = -1.98

Although using the DF/ADF statistic the residuals from (15) do not appear to be stationary, inspection of the correlogram of these residuals suggests on the contrary that they could be. So the results are ambiguous. But proceeding to use the residuals from this equation as if they were stationary, the dynamic equation which results is shown by equation (16)

$$\begin{aligned} DLEERR_t = {} & \underset{(1.60)}{.242DLEERR_{t-1}} - \underset{(1.22)}{.428NEWFINTRS_t} + \underset{(2.05)}{.777NEWFINTRS_{t-1}} - \underset{(1.95)}{0.00255PSBRR_t} \\ & - \underset{(.77)}{.001PSBRR_{t-1}} - \underset{(3.04)}{.384RES_{t-1}} \end{aligned} \quad \textbf{(16)}$$

$R^2 = .43$ $\sigma = .032$ DW = 1.89 LM(4) = 7.67 RESET(4) = 2.53
HETEROSC(1) = 0.003 NORM(2) = .73

Thus, to complete the procedure, we adjust the coefficients of (15) as recommended in the Engle-Yoo procedure, and these adjusted coefficients are reported below,

variable	original coeff	adjusted coeff	t-ratio
constant	4.779	4.826	97.93
CNEWFINTRS	0.0938	0.3130	.73
CPSBRR	-0.00163	-0.00199	3.31

Comparing the adjusted coefficients in the results above with those obtained by the ML method in (13) shows that the results are broadly similar. Of course, the 3-step estimator is asymptotically equivalent to the ML method, but generally the ML results are preferred if they are available. It should be noted that where a direct test on the residuals of the adjusted equations was undertaken it produced a result similar to the OLS result in (15), namely that both the DF/ADF statistics and the spectral density did not appear consistent with stationarity. However, the correlogram of the residuals did. In this sense, the use of the 3-step procedure has not improved upon the results from the two-stage one. However, the 3-step procedure gives results which are broadly in line with the Johansen method, which is as expected given the uniqueness of the co-integrating vector.

3 Conclusions

In this paper we have applied alternative estimation procedures to an eclectic model of the real exchange rate. Although we have used the Granger-Engle two stage procedure for part of this work, we acknowledge that there are well recognised difficulties with this. So we describe alternative results which use the 3-stage procedure introduced by Engle and Yoo and results using the Johansen procedure. When treating the interest differential variable as I(1), which a purely time series interpretation suggests, the eventual result for the ECM model of the exchange rate was an equation risk *two* error correction terms. Each error correction term has a reasonable economic interpretation. There are, however, problems with an equation of this sort, not least the problems in using it in a full blown macro model. Hence the equation we prefer is based on a single levels or ECM component, which we are able to derive after transforming the interest differential variable to an I(0) process, after effectively dummying out one observation. The paper provides empirical results for this using both the 3-stage and Johansen maximum likelihood procedures, and the results for either of these are acceptable on economic and statistical grounds.

Appendix: data definitions

Full data set 1979:1–1989:3.

The prefix L in the text denotes a logarithm. Codes refer to CSO codes and/or Economics Division model codes.

BALR	: current balance as percentage of nominal GDP. (*BAL / GDPN*)*100 where *BAL*=current balance; *GDPN*=nominal GDP at market prices, average measure.
PSBRR	: PSBR as percentage of nominal GDP. (PSBR/*GDPN*) *100
EERR	: real effective exchange rate. (*EER*/100)*[*PPOX* / (*WPP*/100)] where PPOX=producer price index, WPP=world producer price index.
BOP	: ratio of non oil exports to non-oil imports. [*XG£*- (*PXGO***XGO*)] / [*MG£*-(*PMGO***MGO*)] where XG£=total visible exports: PXGO*XGO= total oil exports. Similarly for imports.
CPSBRR	: cumulation of *PSBRR* from 1979:1.
INTRL	: long interest differential, relative to US, consumer price deflated.
CFINTRS	: short interest differential cumulated from 1979:1. Relative to the US, consumer price deflated.
DUM851	: dummy variable = 1 1985:1 to 1989:3 = 0 elsewhere

References

Banerjee A, Dolado, J J, Hendry D F, and Smith G W (1986), 'Explaining equilibrium relationships in econometrics through state space models; some Monte Carlo evidence'. *Oxford Bulletin of Economics and Statistics*, 43(3) p 253-78.

Barrell R, A Gurney, B Pesaran and S Wren-Lewis (1988), 'Three forward-looking exchange rate equations and their properties' paper presented to the ESRC Macroeconomic Modelling Bureau. November.

Engle R F, and B S Yoo (1990), 'Co-integrated Economic Time Series: A Survey with New Results', University of California, San Diago, discussion paper.

Fisher P G, S K Tanna, D S Turner, K F Wallis and J D Whitley (1990), "Econometric Evaluation of the Exchange Rate in Models of the UK Economy", ESRC Macro Economic Modelling Bureau, Working Paper.

Hall S G (1987), 'A forward-looking model of the exchange rate' *Journal of Applied Econometrics,* vol 2, no. 1.

Hall and Taylor (1989), 'Co-integration'. Bank of England mimeo.

Hooper P, and J E Morton (1982), 'Fluctuations in the dollar: A model of nominal and real exchange rate determination' *Journal of International Money and Finance.* 1 April p 39-56.

Johansen S (1988), 'The statistical analysis of co-integrating vectors'. *Journal of Economic Dynamics and Control.* 12(2-3) p 231-54.

Keating G (1985), 'Estimated and simulation of the London Business School finanial model with rationally-expected asset prices'. London Business School, Centre for Economic Forcasting Discussiion Paper no 148.

Meese R A and K Rogoff (1983), 'Empirical exchange rate models of the seventies: do they fit out of sample?' *Journal of International Economics 14*, February, p 3-24.

Bank of England Discussion Papers

	Title	Author
1-5, 8, 11-14, 16-17, 19-22, 31, 44	*These papers are now out of print, but photocopies can be obtained from University Microfilms International* (a)	
6	'Real' national saving and its sectoral composition	C T Taylor A R Threadgold
7	The direction of causality between the exchange rate, prices and money	C A Enoch
9	The sterling/dollar rate in the floating rate period: the role of money, prices and intervention	I D Saville
10	Bank lending and the money supply	B J Moore A R Threadgold
15	Influences on the profitability of twenty-two industrial sectors	N P Williams
18	Two studies of commodity price behaviour: Interrelationships between commodity prices Short-run pricing behaviour in commodity markets	 Mrs J L Hedges C A Enoch
23	A model of the building society sector	J B Wilcox
24	The importance of interest rates in five macroeconomic models	W W Easton
25	The effects of stamp duty on equity transactions and prices in the UK Stock Exchange	Mrs P D Jackson A T O'Donnell
26	An empirical model of company short-term financial decisions: evidence from company accounts data	Mrs G Chowdhury C J Green D K Miles
27	Employment creation in the US and UK: an econometric comparison	I M Michael R A Urwin
28	An empirical model of companies' debt and dividend decisions: evidence from company accounts data	Ms G Chowdhury D K Miles
29	Expectations, risk and uncertainty in the foreign exchange market: some results based on survey data	M P Taylor
30	A model of UK non-oil ICCS' direct investment	E J Pentecost
32	The demographics of housing demand; household formations and the growth of owner-occupation	M J Dicks
33	Measuring the risk of financial institutions' portfolios: some suggestions for alternative techniques using stock prices	S G F Hall D K Miles
34	An error correction model of US consumption expenditure	I R Harnett
35	Industrial structure and dynamics of financial markets; the primary eurobond market	E P Davis
36	Recent developments in the pattern of UK interest rates	D K Miles
37	Structural changes in world capital markets and eurocommercial paper	J G S Jeanneau
38	Stockbuilding and liquidity: some empirical evidence for the manufacturing sector	T S Callen S G B Henry
39	The relationship between employment and unemployment	M J Dicks N Hatch
40	Charts and fundamentals in the foreign exchange market	Mrs H L Allen M P Taylor
41	The long-run determination of the UK monetary aggregates	S G Hall S G B Henry J B Wilcox
42	Manufacturing stocks; expectations, risk and co-integration	T S Callen S G Hall S G B Henry
44	Corporate governance and the market for companies: aspects of the shareholders' role	J Charkham
43	Instability in the euromarkets and the economic theory of financial crises	E P Davis
45	Stock-flow consistent income for industrial and commercial companies: the UK experience	K D Patterson
46	The money transmission mechanism	D K Miles J B Wilcox
47	Monetary aggregates in a changing environment: a statistical discussion paper	R D Clews Ms J E C Healey Glenn Hoggarth C R Mann
48	A model of manufacturing sector investment and employment decisions	J W Lomax
49	A simple model of the housing market	M J Dicks
50	An industrial approach to financial instability	E P Davis
51	International financial centres—an industrial analysis	E P Davis
52	A model of ICCs' dividend payments	J W Lomax
53	The determination of average earnings in Great Britain	M A S Joyce

Technical Series

	Title	Author
1-11, 14, 20, 23	*These papers are now out of print, but photocopies can be obtained from University Microfilms International* (a)	
12	The development of expectations generating schemes which are asymptotically rational	K D Patterson
13	The arch model as applied to the study of international asset market volatility	R R Dickens
15	International comparison of asset market volatility: a further application of the ARCH model	R R Dickens
16	A three sector model of earnings behaviour	D J Mackie
17	Integrated balance sheet and flow accounts for insurance companies and pension funds	Raymond Crossley
18	Optimal control of stochastic non-linear models	S G Hall I R Harnett M J Stephenson
19	A multivariate GARCH in mean estimation of the capital asset pricing model	S G Hall D K Miles M P Taylor
21	Modelling of the flow of funds	D G Barr K Cuthbertson
22	Econometric modelling of the financial decisions of the UK personal sector: preliminary results	D G Barr K Cuthbertson
24	Modelling money market interest rates	J S Flemming D G Barr
25	An independant error feedback model of UK company sector asset demands	D G Barr K Cuthbertson
26	A disequilibrium model of building society mortgage lending	S G Hall R A Urwin
27	Balancing the national accounts: an asymptotically maximum likelihood approach using trends	G P Dunn D M Egginton
28	Testing a discrete switching disequilibrium model of the UK labour market	S G Hall S G B Henry M Pemberton
29	The Bank of England Model 1989: recent developments and simulation properties	F J Breedon A J Murfin S H Wright
30	A data-based simulation model of the financial asset decisions of UK, 'other' financial intermediaries	D G Barr K Cuthbertson
31	The demand for financial assets held in the UK by the overseas sector: an application of two-staged budgeting	D G Barr K Cuthbertson
32	A note on the estimation of GARCH-M models using the Kalman Filter	S G Hall
33	Modelling the sterling effective exchange rate using expectations and learning	S G Hall
34	Modelling short-term asset holdings of UK banks	D G Barr K Cuthbertson
35	A Monte Carlo study of alternative approaches to balancing the national accounts	D M Egginton
36	Sterling's relationship with the dollar and the deutschemark: 1976–89	A G Haldane S G Hall
37	Using and assessing CBI data at the Bank of England	Bahram Pesaran C B Wright
38	A system approach to consumption and wealth	S G Hall K D Patterson
39	Exchange rate equations	Helen Allen Brian Henry Bahram Pesaran

(a) These papers are no longer available from the Bank, but photocopies can be obtained from University Microfilms International, at White Swan House, Godstone, Surrey RH9 8LW.