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Bank of England

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Growth coefficients in error correction and autoregressive distributed lag models

by

K D Patterson

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Introduction [1]

There has been considerable interest lately in the use of error correction models in applied econometrics - see, for example, the articles by Davidson et al (1978), Hendry et al (1982), Pagan and Volcker (1981), and Salmon (1982). An error correction model, ECM, is one in which some order of difference of the dependent variable is taken to be a function of the discrepancy between the target value and some actual value of dependent variable. Salmon (1982), for example, considers whether dynamics of particular ECMs are adequate to ensure that the difference between the target value and the steady state value of the dependent variable is zero. Along what was hitherto considered an unrelated line of research some work has recently been carried out on the long-run properties of autoregressive distributed lag models, ADM, see Hendry and Mizon (1978), Currie (1981 and 1982), Patterson and Ryding (1982); for example, Patterson and Ryding and Currie (1981) consider the implications of setting certain of the growth coefficients, in an ADM, to zero.

One of the main purposes of this paper is to demonstrate the intimate connections between ADMs and ECMs, and highlight the central role of growth coefficients in both types of model. A by-product of this analysis is the clarification of some concepts in Salmon (1982). The analysis of ECMs is also extended to look at the steady state integral error as well as the steady state flow error, since to concentrate on the latter is often to give only a partial analysis of the importance of the dynamic characteristics of particular ECMs (as would concentrating on flow equilibrium without reference to the stock position). An important conclusion of the analysis is that the concept of growth coefficients serves to unify a number of recent articles and provide a useful heurestic and analytical aid to the determination of an appropriate dynamic (empirical) specification.

The structure of this paper is as follows: the next section gives a brief introduction to the derivation and use of growth coefficients;

[1] I would like to thank Mark Salmon and John Flemming for helpful discussions, and I am indebted to Gareth Evans for his insights which enabled the proofs to be considerably simplified.

the third section defines an error correction model and establishes some initial properties; the fourth section considers the particular characteristics of ECMs expressed in terms of the growth coefficients, and hence derives necessary and sufficient conditions for zero steady state error and derives easily calculated expressions for the size and magnitude of the steady state error; the fifth section considers the steady state integral error; the sixth section looks briefly at the implications of the earlier analysis for the shape of the implied lag distributions; and the last section contains concluding remarks and summarises the main results. An appendix extends the analysis to continuous time models, and contains proofs of some of the results in the text.

2
Growth coefficients

1 Hendry and Mizon (1978) and Davidson et al (1978) established the practice of deriving the dynamic steady state associated with a particular estimated equation. For example, in their estimation of the consumption function, Davidson et al obtained the following dynamic steady state with constant growth rates for real income π_x and prices π_p, (a superscript ss indicates the steady state):

$$Y_t^{ss} = \exp\,(-5.3\ \pi_x - 1.3\ \pi_p)\ X_t$$

where Y and X are real consumption and real income respectively (for detailed data definitions, see the original article); or letting lower case letters denote logarithms:

$$y_t^{ss} = x_t - 5.3\ \pi_x - 1.3\ \pi_p$$

Notice that $y_t^{ss} = x_t$ if both π_x and π_p are zero, otherwise non-zero growth rates for income and prices causes variations in the consumption income ratio. The coefficients on π_x and π_p are the (first order) growth coefficients.

2 In a subsequent article, Currie (1981) drew attention to the existence of significant (in the numerical sense) growth coefficients in a number of extant studies of the demand for money and wage determination, and suggested that insignificant (in the statistical sense) growth coefficients could be set equal to zero (see Currie 1981, pages 706-7 and 714-5). Following and extending this work, Patterson and Ryding (1982) provided: an efficient statistical framework for testing hypotheses on the growth coefficients; a theoretical and empirical analysis of setting growth coefficients to zero in some commonly occurring models, and demonstrated that the effects of so doing are unlikely to be benign on the short-run dynamic and steady state characteristics of estimated equations; and, most importantly in the present context, showed that the lag generating function $W(L) = \sum_{i=0}^{\infty} W_i L^i$ could be used to derive the growth coefficients to any desired order.

That is in the autoregressive distributed lag model:

$$y_t = \frac{\beta(L)}{\gamma(L)} x_t = W(L)\, x_t \tag{1}$$

the growth coefficients defined by:

$$\lambda_k \equiv \frac{\partial y_t^{ss}}{\partial\, [(1-L)^k x_t]} \qquad k = 0,\ 1\ \ldots \tag{2}$$

where y_t^{ss} is the steady state value of y_t, are obtained from:

$$\lambda_k = \frac{\partial^k W(L)}{\partial[(1-L)]^k}\Big|_{L=1} \Big/ k! = (-1)^k \frac{\partial^k W(L)}{\partial L^k}\Big|_{L=1} \Big/ k! \tag{3}$$

Further details and proofs are in Patterson and Ryding (1982); and the dynamic (of any order) non-stochastic steady state is written as:

$$y_t^{ss} = \sum_{k=0}^{n} \lambda_k \Delta^k x_t \quad \text{where n is such that } \Delta^{n+1} x_t = 0 \tag{4a}$$

3 With x_t exogenous, equation (4a) can be interpreted as the steady state y_t with growth in x_t of order n. The λ_k, which usually have the interpretation of semi-elasticities, are known as the (order of) growth coefficients or multipliers (static, dynamic, etc).

Growth coefficients can also be defined in a similar way for continuous time models. Thus, let x(t) and y(t) be continuous functions of time, t, then equation 4(a) is replaced by:

$$y^{ss}(t) = \sum_{k=0}^{n} \lambda_k \frac{d^k x(s)}{ds^k}\Big|_{s=t} \tag{4b}$$

where $\frac{d^{n+j} x(s)}{ds^{n+j}}\Big|_{s=t} \simeq 0$ for $j > 0$; and if $\simeq$ can be replaced by =, as would be the case if x(t) is an n-th degree polynomial in t, the steady state is exact (global) rather than approximate (local). This definition shows that whilst most examples have taken n=1, corresponding to constant growth, or, less frequently, n=2, the steady state can be defined for other, and perhaps more realistic, time paths by specifying n such that an acceptable degree of approximation is obtained. The remainder of the text considers discrete time models, whilst the appendix considers continuous time models in greater detail.

3
Error correction mechanisms

4 In a subsequent article, Salmon (1982) generalised the concept of an error correction model, ECM, due to Davidson et al (1978), and see also Davidson and Hendry (1981), Hendry and Von Ungern-Sternberg (1980) (HUS) and Hendry et al (1982). If y_t is the dependent variable, let y^*_t denote the target value of y_t, then an ECM is defined by:

$$y_t = A(L)\ (y^*_{t-j} - y_{t-i}) \tag{5}$$

with $0 \leqslant j \leqslant i$, and where $A(L) = a(L)/b(L)$ is a rational polynomial in the lag operator L, a(L) is of order m, and b(L) of order n. If s unit roots can be extracted from b(L), without cancellation of the same in a(L), then equation (5) will be described as an ECM of order, or type, s [see Salmon (1982), page 618]. The target error is defined by $e \equiv y^*_{t-j} - y_{t-i}$, and the steady state error by $e_{ss} \equiv y^*_t - y^{ss}_t$, where y^{ss}_t is the steady state value of y_t.

5 Actually equation (5) differs from the ECM of Salmon (1982) in the subscripting of the components of the target error; this is important for we show later e_{ss} is not invariant to certain choices of j and i. The most common choices for j and i are j = 0 and i = 1 or s, or j = i = s; for expositional purposes, we take the latter case and write the ECM in its ADM form. The ECM of order s is given by:[1]

$$(1-L)^s\ y_t = \frac{a(L)}{(1+B^*(L))}\ (y^*_{t-s} - y_{t-s}) \tag{6}$$

where $b(L) = (1-L)^s(1+B^*(L))$ implicitly defines $B^*(L)$;

hence:

$$y_t = W(L)\ y^*_t \tag{7}$$

$$\text{with } W(L) = \frac{a(L)L^s}{(1-L)^s(1+B^*(L)) + a(L)L^s} \tag{8}$$

[1] For expositional purposes, the models herein are assumed to be nonstochastic, as in Salmon (1982), pages 615-622; stochastic considerations are taken up at length in an earlier version of this paper - see Patterson (1982).

Setting L=1 in equation (8) gives the static multiplier, or zero order growth coefficient, on y*, which is equal to unity, ie:

$W(1) = 1$

Additionally, if y^*_t is a multiple of an observable exogenous variable x_t, say[1] $y^*_t = \theta x_t$, then the static multiplier on x_t is equal to the target coefficient, ie $\lambda_0 = \theta$.

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[1] In fact, both θ and x_t could be vectors; if y^*_t is known, then set $\theta = 1$ and $x_t \equiv y^*_t$.

4

Steady state error, growth coefficients and ECMs

6 In the ECM interpretation $W(L) = \beta(L)/\gamma(L)$ with $\beta(L) = a(L)L^s$ and $\gamma(L) = [(1-L)^s (1+B^*(L)) + a(L)L^s]$, polynomials of orders p and q respectively; hence an ADM and in particular equation (7) is a linear difference equation of order q. The solution of a linear difference equation consists of two parts: the particular integral, which can be viewed as the equilibrium, or steady state, value of y_t, not necessarily constant; and the complementary function which describes the transient behaviour of y_t. The meaning of solution here is the representation of y_t in terms of constant parameters and some function of time. As equation (4a) describes the dependence, in steady state, of y_t on the orders of growth of x_t, including the level of x_t, it must be an alternative way of viewing the particular integral part of the difference equation solution. This connection can be illustrated simply with the cases of x_t growing with (a)t and (b)t^2.

7 In the first case, take $x_t = Kt$, ie constant growth.[1] Then $\Delta x_t = K$, and $\Delta^j x_t = 0$ for $j > 1$; substituting these into equation (4a) gives:

$$y_t^{ss} = K(\lambda_0 t + \lambda_1) \tag{10}$$

In the second case, $x_t = Kt^2$, and $\Delta x_t = 2Kt - K$, $\Delta^2 x_t = 2K$, $\Delta^j x_t = 0$ for $j > 2$; substituting these into equation (4a) gives:

$$y_t^{ss} = K\,[(2\lambda_2 - \lambda_1) + 2\lambda_1 t + \lambda_0 t^2] \tag{11}$$

8 Equations (10) and (11) are the particular integrals of their respective solutions written explicitly as functions of the order of growth coefficients and polynomial functions of time. If $x_t = t^n$, that is, we set K=1 for convenience, and n is arbitrary, then some features of the steady state are easy to determine: the coefficient on t^n will

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[1] Whilst the text deals with simply polynomial functions of time, the appendix shows that more general functions of time do not introduce any further complications; ie in general,

$$y_t^{ss} = \sum_{k=0}^{n} \lambda_k \left.\frac{d^k x(s)}{ds^k}\right|_{s=t}$$

always be λ_0, the coefficient on t^{n-1} will be a multiple of λ_1, the coefficient on t^{n-2} will be a simple linear function of λ_2 and λ_1, and the coefficients on t^{n-j} will be a simple linear function of λ_j, λ_{j-1}, ..., λ_1; see the appendix for further details.

9 The steady state error e_{ss} is defined as $y^*_t - y^{ss}_t$, but $y^*_t = \theta x_t = \lambda_0 x_t$ from the argument following equation (9), hence it is now easy to determine the characteristics of the e_{ss}. For illustrative purposes, take the three growth patterns considered in Salmon (1982):

Steady state error in terms of the growth coefficients

	Static	Constant growth	Dynamic growth
	$x_t = K$	$x_t = Kt$	$x_t = Kt^2$
y^*_t	$\lambda_0 K$	$\lambda_0 Kt$	$\lambda_0 Kt^2$
y^{ss}_t	$\lambda_0 K$	$(\lambda_0 t + \lambda_1)K$	$[(2\lambda_2 - \lambda_1) + 2\lambda_1 t + \lambda_0 t^2]K$
e_{ss}	0	$-\lambda_1 K$	$-[(2\lambda_2 - \lambda_1) + 2\lambda_1 t]K$

10 Inspection of this table shows that, in the static case, $e_{ss} = 0$; but for constant growth e_{ss} is the negative of the rate of growth effect, ie the (rate of) growth coefficient times the rate of growth[1] and therefore given $K \neq 0$, e_{ss} will be zero if, and only if, $\lambda_1 = 0$; for the case of dynamic growth, e_{ss} increases or decreases without limit[2] as $t \to \infty$. In the latter case, for the e_{ss} to tend to zero, it is now necessary and sufficient to have $\lambda_1 = \lambda_2 = 0$; if $\lambda_1 = 0$ but $\lambda_2 \neq 0$, then e_{ss} tends to the finite constant $-2\lambda_2 K$.

[1] Strictly for K to be the rate of growth, x_t should be the logarithm of the variable of interest.

[2] Notice that for both constant and dynamic growth $y^*_t / y^{ss}_t \to 1$ as $t \to \infty$, even though $e_{ss} \to +/- \infty$.

11 These results are generalised in the following proposition:

Proposition 1: Define the order of growth by n in $x_t = Kt^n$, then necessary and sufficient conditions to ensure that the steady state error tends to a constant equal to $-n!\lambda_n K$, as $t \to \infty$ are $\lambda_k = 0$, $k = 1 \ldots n-1$; and if that constant is to be zero, λ_n must also be zero. For $n > 1$, if any of the λ_k, $k = 1, \ldots, n-1$ are non-zero, then the steady state error will tend to +/- infinity as $t \to \infty$.

12 A fundamental characteristic of ECMs can now be stated:

Proposition 2: In an ECM of order, or type, s with target error given by $y^*_{t-s} - y_{t-s}$ all the growth coefficients:

λ_1 to λ_{s-1} are zero, for $s > 1$.

Corollary 2.1 An ECM of order s will have a zero steady state error, defined by $y^*_t - y^{ss}_t$, for $y^*_t = \theta x_t$, where $\theta = \lambda_0$, and x_t subject to growth of order up to and including s-1.

13 This proposition and its corollary demonstrate that an ECM of order s serves to achieve a zero e_{ss} against growth in x_t to order s-1, precisely because in that ECM the growth coefficients λ_1 to λ_{s-1} are identically zero; that is, an ECM is a parameterisation which prevents growth, of a certain order, in the target variable from affecting the steady state of the dependent variable. To see how this works, consider the simple consumption function in Hendry and Von Ungern-Sternberg (1981); the basic model is:

$$\Delta y_t = \mu_1 \Delta x_t + \mu_2 (x_{t-1} - y_{t-1}) \qquad (12)$$

with steady state for constant growth in x_t

$$y^{ss}_t = x_t + \lambda_1 \Delta x \qquad (13)$$

where $\lambda_1 = (\mu_1 - 1)/\mu_2$.

14 Now equation (12) is a first order ECM:

$$\Delta y_t = (k_p + (k_i - k_p) L)(y^*_t - y_t) \qquad (14)$$

with $y^*_t = x_t$, and $\mu_1 = \frac{k_p}{1+k_p}$, $\mu_2 = \frac{k_i}{1+k_p}$;

where k_p and k_i are the proportional and integral feedback gains, respectively - see Salmon (1982, page 622). As equation (12) is a first order ECM, it will not be able to deliver zero e_{ss} against constant growth in x_t, and equation (13) makes it clear the non-zero e_{ss} is due to the effect of the growth in x_t on the steady state y_t; hence, constraining λ_1 to zero will ensure zero e_{ss}, and proposition 2 shows that this will be achieved with an ECM of one higher order.

15 A further corollary serves to demonstrate how an AD (p,q) model can be interpreted as an ECM.

Corollary 2.2: An AD (p,q) model with $p \geqslant q$, $\beta_j = 0$ for $j = 0, ..., h-1$, and $\lambda_k = 0$, $k = 1, ..., f$ can be interpreted as an ECM of order $f + 1 \equiv s$ with target error $y^*_{t-h} - y_{t-h}$ and $n = p$, $m \equiv q - h$.

Thus, if an AD (p,q) model is estimated,[1] perhaps without being motivated by the ECM rationale, and satisfies the conditions of the corollary, it can be interpreted, ie reparameterised, as an ECM. For example, if $\lambda_1 = 0$ and $h = 2$, then an AD (p,q) model with $\beta_0 = \beta_1 = 0$, $q \geqslant h$ and $p \geqslant q$ can be interpreted as a second order ECM.

16 Salmon (1982, page 619) describes the following partial adjustment model as being of type 1:

$$\Delta y_t = \gamma (y^*_t - y_{t-1})$$

Notice that in this model there is a one period lag in the definition of the target error; that this is a crucial difference is shown in the following proposition:

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[1] Once outside the deterministic framework, we have also to assume that the stochastic properties of the ADM are as required by ECM considerations; this point is considered at greater length in Patterson (1982).

Proposition 3: In an ECM of type s with the target error given by $y^*_{t-j} - y_{t-i}$, $0 \leqslant j \leqslant i$, and $\delta \equiv i-j$,

then $\lambda_k = {}_{(\delta+k-1)}C_{(\delta-1)} \lambda_0$ k=1, ..., s-1; s > 1; $\delta > 0$

where ${}_gC_h$ is the binomial coefficient for choosing h from g and ${}_gC_h \equiv 0$ for h < 0.

17 Actually, proposition 2 is the special case of proposition 3 for $\delta = 0$ but was separated out because of its importance in Salmon (1982). Another special case is j=0 and $0 < i \leqslant s$, then we have:

$$\lambda_k = {}_{(i+k-1)}C_{(i-1)} \lambda_0 \qquad k=1, ..., s-1.$$

For example, in the second order ECM, if the target error is $y^*_t - y_{t-1}$, then $\lambda_1 = \lambda_0$; and if the target error is $y^*_t - y_{t-2}$, then $\lambda_1 = 2\lambda_0$. Hence, in neither of these cases will the second order ECM deliver zero e_{ss} against the target growing at a constant rate. We may formalise this result as:

Corollary 3.1: An ECM with target error $y^*_{t-j} - y_{t-i}$

with $\delta \equiv i-j$, will not, under the same conditions as corollary 2.1, have a zero steady state error unless $\delta = 0$.

18 As the following corollary demonstrates the subscripting in the target error is an incidental matter only if the steady state error is redefined:

Corollary 3.2: If in an ECM of order s the target error is defined by $y^*_{t-j} - y_{t-i}$ and $\delta > 0$, then if the steady state error is defined as $y^*_t - y^{ss}_{t-\delta}$ such a model will have zero e_{ss} for growth in x_t up to and including order s-1.

19 Corollary 3.2 points out that the reconciliation takes place through an unusual and variable definition of the steady state error. This corollary could be proved in the same manner as corollary 2.1; an alternative method is to use proposition 3 and the representation of the steady state in terms of the growth coefficients. For example, for s=3, j=0, i=3, we have:

$\lambda_1 = 3\lambda_0$, $\lambda_2 = 6\lambda_0$, and if $x_t = Kt^2$ then:

$$y^*_t - y^{ss}_{t-3} = \lambda Kt^2 - [(2\lambda_2 - \lambda_1)K + 2\lambda_1 K(t-3) + \lambda_0 K(t-3)^2]$$

$$= 0 \text{ on substituting for } \lambda_1 \text{ and } \lambda_2.$$

20 An interesting implication of proposition 2 is that, since it applies for $s > 1$, it does not rule out the possibility that for s=1 and target error $y^*_t - y_{t-1}$, ie a generalised partial adjustment model, we could obtain $\lambda_1 = 0$, and hence some [1] first order ECMs will have a zero e_{ss} even though the target is growing at a constant rate. Now such cases must have an AD(p,q) representation with $p \geqslant 2$ and this implies that they could, according to corollary 2.2 be reformulated as a second order ECM with target error $y^*_t - y_t$. Thus, the transition from the AD form to the ECM (rather than the other way) will depend upon the definition of the target error.

Sign of the steady state error

21 A virtue of analysing ECMs and ADMs through the growth coefficients is that the latter summarises a great deal of information which would otherwise be tedious to obtain. For example, both the sign and magnitude of the steady state error are functions of the growth coefficients. Consider, for example, the case of dynamic growth in x_t (or equivalently y^*_t) then the limiting behaviour of the difference $y^*_t - y^{ss}_t$ is determined by the term $-2\lambda_1 Kt$; for $K > 0$ this will be positive if $\lambda_1 < 0$, that is the target will be undershot (overshot) if the (rate of) growth coefficient is negative (positive). This suggests that the table in Salmon (1982, page 618) should be modified to reflect the possibility of both divergent under and overshooting (ie $e_{ss} \to +\infty$ and $e_{ss} \to -\infty$).

[1] For example, consider the first order ECM

$$\Delta y_t = \frac{a_0}{(1+b^*_1 L)} (\theta x_t - y_{t-1})$$

and obtain its AD form,

$$y_t = \beta_0 x_t + \gamma_1 y_{t-1} + \gamma_2 y_{t-2}$$

then setting $\gamma_1 = -2\gamma_2$ ensures $\lambda_1 = 0$; Note: $a_0 = 1 - \gamma_1 - \gamma_2$ and $b^*_1 = \gamma_2$

22 For example, in the simple partial adjustment model, which would be estimated as an AD(1,0) model:

$$y_t = \beta_0 x_t + \gamma_1 y_{t-1}$$

with $\lambda_0 = \beta_0/(1-\gamma_1)$, then $\lambda_1 = -\gamma\beta/(1-\gamma_1)^2$ is negative for the usual case of β_0, $\gamma_1 > 0$, hence $e_{ss} \rightarrow + \infty$.

23 In the following generalisation of the partial adjustment model:

$$(1-L) y_t = \frac{a_0}{1+b^*_1 L} (y^*_t - Y_{t-1})$$

which is the AD(2,0) model

$$y_t = \beta_0 x_t + \gamma_1 y_{t-1} + \gamma_2 y_{t-2}$$

with $a_0 = 1- \gamma_1 - \gamma_2$, $b^*_1 = \gamma_2$, $y^*_t = \theta x_t$, $\theta = \lambda_0 = \beta_0 /(1-\gamma_1-\gamma_2)$ and

$\gamma_1 = - \beta_0(\gamma_1 + 2\gamma_2)\ (1-\gamma_1 -\gamma_2)^{-2}$; then for β_0, $(1-\gamma_1-\gamma_2) > 0$

the sign of λ_1 will depend on $\gamma_1 + 2\gamma_2$, ie $\lambda_1 \gtrless 0$ as $\gamma_1 \gtrless -2\gamma_2$.

If $\lambda_1 > 0$ then, with $x_t = Kt^2$, $e_{ss} \rightarrow - \infty$ indicating divergent overshooting of the target $y^*_t = \theta x_t$.

24 If the static multiplier is positive, we know from proposition 3 that overshooting, in for example cases of constant or dynamic growth of the target variable, will be an <u>inherent</u> feature of ECMs with target error defined by $y^*_{t-j} - y_{t-i}$, $i > j$ and $s > 1$, since the growth coefficients λ_k, $k=1, ..., s-1$ will all be positive.

5

Steady state integral error

25 The discussion so far has been concerned with establishing necessary and sufficient conditions for the steady state error $y^*_t - y^{ss}_t$ to be zero; however, this is quite often only a partial characterisation of equilibrium. For example, if y_t is a flow, and y^*_t is a function of a variable(s) of the same dimension, then there is an implicit stock which is changing by the extent of the disequilibria, as measured by the (current) target error, $y^*_t - y_t$; a measure of the stock disequilibrium is then naturally given by the cumulative sum of target errors, that is the integral error:

$$e_{iT} \equiv \sum_{t=1}^{T} (y^*_t - y_t)$$

where flow and stock equilibrium are assumed for $t < 1$, and the steady state integral error, e_{ssi}, is obtained as $T \rightarrow \infty$. (Even if y_t is not a flow, the integral error may often be given economic meaning.[1]) This suggests that full equilibrium may require both the steady state error and the steady state integral error to be zero. How this can be achieved is shown in the following proposition.

Proposition 4: If the target variable is growing at order n, that is,

$$y^*_t = \theta x_t = \theta K t^n,$$

the steady state integral error will be infinite if any of λ_i, i=1...n are non-zero; the steady state integral error will be finite if λ_i=0, i=1, ..., n, and in this case is given by:

$$e_{ssi} = n!\, \lambda_{n+1} K$$

26 There are two obvious corollaries:

[1] For the cumulative sum to make economic sense y_t should be in 'real' rather than 'nominal' terms.

Corollary 4.1: If growth in x_t is at order[1] n, and $\lambda_i = 0$, $i=1, \ldots, n$ then to ensure $e_{ssi} = 0$ it is necessary and sufficient to have $\lambda_{n+1} = 0$.

Corollary 4.2: If growth in x_t is at order n, then in an error correction model of order, or type, n+2 with the target error defined by $y^*_{t-s} - y_{t-s}$, both e_{ss} and e_{ssi} will be zero; if the order of the ECM is n+1, then $e_{ssi} = -n!\lambda_{n+1}K$.

Proposition 4 is proved in the appendix; corollary 4.1 follows from proposition 4, and corollary 4.2 follows from propositions 4 and 2 and corollary 2.1; it can be justified intuitively by noting that the integral of the target will involve one higher order of growth than growth in y_t.

27 Example one

Let y_t be the real wage, y^*_t the target (or desired) real wage, then the cumulative target error

$$\sum_{t=1}^{T} (y^*_t - y_t)$$

could be interpreted as the 'catch-up' due to past disequilibria. If the target real wage is growing at a constant rate, a second order ECM would ensure that in steady state the actual and the desired real wage were equal and it does this by setting to zero the rate of growth effect (ie the long-run homogeneity of the real wage to the desired real wage is not disturbed by the latter growing at a constant rate); the extent of outstanding 'catch-up' (or past disequilibria) would be given by $-\lambda_2 K$, and to remove this in steady state it would be necessary to formulate the behavioural equation as a third order ECM. Notice that, whilst in this example the real wage is not a 'flow' variable, there is nevertheless an appropriate stock concept which makes the integral error relevant to the modelling process.

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[1] Alternatively, let x(t) denote a, not necessarily polynomial, function of time then n is defined by:

$$\left.\frac{d^{n+j}x(s)}{ds^{n+j}}\right|_{s=t} \simeq 0, \text{ for } j > 0.$$

Example two

Consider again the simple consumption function referred to earlier as equations (12) to (14). The (rate of) growth coefficient [1] is $\lambda_1 = (\mu_1 - 1)/\mu_2$, which on substitution for μ_1 and μ_2, or direct derivation from equation (14), gives:

$$\lambda_1 = -1/k_i$$

that is, apart from sign, the growth coefficient is the inverse of the integral feedback coefficient; and hence the steady state integral error which results when y* moves from one static equilibrium to another is K/k_i.

Note that $\lambda_1 \to 0$ as $k_i \to \infty$; that is, the steady state integral error tends to zero as the integral feedback gain - the weight given to past disequilibria - becomes more important in the PI control rule. Increasing the order of the ECM by one is necessary to ensure $\lambda_1 = 0$, and hence that $e_{ssi} = 0$ for changes in static equilibria. A second order ECM also ensures that in steady state, with income growing at a constant rate, consumption equals income - a zero steady state error - and it does this by constraining the growth of income to have no effect on consumption.

If the variables in equation (12) are in levels rather than logarithms, and y_t refers to total consumption, then the integral error is cumulated savings, ie:

$$\sum_{t=1}^{T} (y^*_t - y_t) = \sum_{t=1}^{T} s_t = A_{T-1} + s_T$$

where $s_t = y^*_t - y_t = x_t - y_t$, and A_t is the asset stock. Note that saving is a disequilibrium phenomenon in an ECM which ensures that in steady state consumption equals income. The interpretation of a zero e_{ssi} is that not only should consumption equal income in the steady state, but that past [that is, since the previous flow (e_{ss}) and stock (e_{ssi}) equilibrium] savings and dissavings should net to zero - that is, $A_T \to 0$ as $T \to \infty$. In the ECM of equation (12) savings arise out of a non-zero steady state error, through being positively related to changes in income (we expect $\lambda_1 < 0$); indeed, with a constant (non-zero) change in income cumulated savings in equation (12) increase without limit, that is, $A_T \to \infty$ as $T \to \infty$, whereas, in

[1] In this model λ_1 is also the negative of the 'mean' lag.

a second order ECM, $A_T \rightarrow -\lambda_2 K$ as $T \rightarrow \infty$, and in a third order ECM $A_T \rightarrow 0$ as $T \rightarrow \infty$. Whether or not the implications for the steady state integral error of a particular ECM against different growth paths of the target variable are desirable, it is clearly important to be aware of the inherent properties of ECMs. [For example, the illustration in Salmon (1982, Figure 1 (c)) of a second order ECM against a change in static equilibria clearly does not have the required property of zero steady state integral error.]

6

Lag distributions

28 In a previous paper, Patterson and Ryding (1982) explored the implications, both analytically and with some empirical examples, of setting λ_1 and λ_2 equal to zero on the shape of the lag distribution in various autoregressive distributed lag models; they found considerable evidence that such a constraint was likely to substantially alter both the short-run dynamic and steady state characteristics of estimated equations, and recommended caution in applying the suggestion in Currie (1981, pages 704 and 706-7) that insignificant growth coefficients should be set equal to zero in the context of the modelling strategy associated particularly with Hendry and Mizon (1978).

29 Setting λ_1 to zero implies a zero 'mean' lag, and in general simultaneously setting λ_1 to λ_k equal to zero implies that the k-th order 'moment about the mean' is zero; in these cases, the implied lag distribution must be non-monotonic,[1] hence a 'smooth' lag distribution is necessarily precluded. Sometimes such non-monotonicity is viewed with consternation by researchers; however, if the modelling rationale is that of an error correction mechanism, non-monotonicity is to be expected. Indeed, the condition that the model specification be chosen to ensure a zero steady state integral error is a strong rationale for non-smooth distributions, and leads one to expect that initial undershooting, say, of the target must be matched by subsequent overshooting - hence, there must be oscillations in the implied lag distribution; and all ECMs, with target error defined by $y^*_{t-j} - y_{t-j}$, of order $\geqslant 2$ will have λ_1 equal to zero.

[1] Patterson and Ryding (1982) show that 'approximate' non-monotonicity of the lag distribution is possible in some models with $\lambda_1 = 0$, but that the part of the parameter space generating such models was small.

7
Conclusions

30 The purpose of this paper has been to make explicit the connections between autoregressive distributed lag models, error correction models (or mechanisms) and growth coefficients; in so doing, this has demonstrated the links among a series of papers by Hendry, starting with Hendry and Mizon (1978), and papers by Currie (1981), Salmon (1982) and Patterson and Ryding (1982). The main conclusions to emerge are:

(i) The equilibrium or steady state values of the difference equation, implied by an autoregressive distributed lag model, ADM, can be written in terms of the growth coefficients, and these are easily derived from the lag polynomials $\beta(L)$ and $\gamma(L)$ which define the ADM. Bearing in mind the particular interpretation of $\beta(L)$ and $\gamma(L)$ for an error correction model, ECM, then its steady state can also be written in terms of the growth coefficients.

(ii) By using the growth coefficients, λ_i, in the steady state representation, it is easy to derive necessary and sufficient conditions, in terms of these coefficients, for the steady state error to be zero. An ECM of order s with target error $y^*_{t-s} - y_{t-s}$ was shown to satisfy the necessary and sufficient conditions which ensure a zero steady state error growth in the target variable of order s-1; and, heuristically, ECMs can hence be seen to ensure zero steady state error by effectively deleting the growth coefficients up to and including the order of growth in the target variable.

(iii) The growth coefficients give both the sign and magnitude of the steady state error given specific values of t and the growth path of the target variable. Hence, they provide an explicit means to evaluating the importance of a particular steady state error [1]; and the analysis

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[1] It is also important in assessing the properties of a particular dynamic specification not to rely entirely on the steady state properties, for two alternative specifications may have the same long-run properties but be very different in their short-run implications; some simple numerical examples are reported in Patterson (1982) which demonstrate the importance of looking not only at the difference between y^*_t and y^{ss}_t, but also at the approach of y_t to y^{ss}_t given an initial displacement from equilibrium; and whilst the steady state properties of the ECMs, alike apart from target errors given by $y^*_{t-j}, - y_{t-j}$ and $y^*_{t-i}, - y_{t-i}$ respectively, with i > j, are identical their small sample properties could be very different.

of section 4 shows that both (divergent) under and overshooting of the target are possible [compare Salmon (1982) page 618].

(iv) The definition of the target error in the ECM has important implications for the steady state error. For example, in an ECM of order s, s > 1, the target error $y^*_t - y_{t-s}$ implies that the growth coefficients $\lambda_1 \ldots \lambda_{s-1}$ are constant multiples of λ_0; with such a definition, in order for the steady state error to be zero, for growth in the target variable up to and including s-1, the latter has to be defined as $y^*_t - y^{ss}_{t-s}$. This clarifies a confusion which might otherwise arise in interpreting the results of Salmon (1982).

(v) A first order ECM with target error $y^*_t - y_{t-1}$ will track a target growing at a constant rate if (and only if) λ_1 (the 'dynamic' multiplier) is zero; this emphasises the importance of specifying the necessary and sufficient conditions for zero steady state error in terms of the growth coefficients.

(vi) As a necessary and sufficient condition to track a target variable which is growing (declining) is that the mean lag, and <u>some</u> higher order moments if the order of growth is greater than one, in the AD form of the ECM, be zero the implied lag distribution <u>must</u> be non-monotonic - see Patterson and Ryding (1982) and Currie (1981). Indeed, the reservations often expressed over 'irregular' shaped lag distributions are misplaced if the modelling rationale is to ensure zero steady state flow/integral error.

(vii) The choice of dynamic specification, in the class of error correction models, has implications for the steady state integral error as well as the steady state flow error. It was shown that the steady state integral error[1] if finite is equal to $-n!K\lambda_{n+1}$, where n is the order of growth of the target variable; and hence a zero steady state integral error was implied by an ECM of one higher order than that necessary to ensure a zero steady state (flow) error.

(iv) Time paths more realistic than constant or dynamic growth can be used to define a steady state by determining an acceptable degree of approximation from a Taylor series expansion.

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[1] This result generalises to $-\lambda_{n+1}\left.\frac{d^n x(s)}{ds^n}\right|_{s=t}$.

Appendix

Proofs, of the propositions and corollaries, not given in the text are collected in this appendix, and the opportunity is taken to extend the analysis to continuous time.

Proposition 1

Write the infinite distributed lag model in continuous time as,

$$y(t) = \int_{j=0}^{\infty} w(j)\, x(t-j)\, dj$$

Now expand x(t-j) about x(t) in the Taylor series,

$$y(t) = \int_{j=0}^{\infty} w(j) \left[\sum_{k=0}^{\infty} (-1)^k \frac{j^k}{k!} \left.\frac{d^k x(s)}{ds^k}\right|_{s=t} \right] dj$$

$$= \sum_{k=0}^{\infty} \left[(-1)^k \int_{j=0}^{\infty} w(j) \frac{j^k}{k!} dj \right] \left.\frac{d^k x(s)}{ds^k}\right|_{s=t}$$

$$= \sum_{k=0}^{\infty} \lambda_k \left.\frac{d^k x(s)}{ds^k}\right|_{s=t}$$

where $\lambda_k = (-1)^k \int_{j=0}^{\infty} w(j) \frac{j^k}{k!} dj$, are the growth coefficients; in the discrete time case $\lambda_k = (-1)^k \sum_{j=k}^{\infty} w_j \, {}_jC_k$, see Patterson and Ryding (1982) and the proof of propositions 2 and 3.

If x(t) is an n-th degree polynomial in t, then $\frac{d^{n+j} x(t)}{dt^{n+j}} = 0$ for j > 0 and dynamic equilibrium of order n is defined by:

$$y^{ss}(t) = \sum_{k=0}^{n} \lambda_k \frac{d^k x(t)}{dt^k}$$

Even if x(t) is not a polynomial a local dynamic equilibrium can be defined by $\left.\frac{d^{n+j} x(s)}{ds^{n+j}}\right|_{s=t} \simeq = 0$ for j > 0, and hence the analysis is not restricted to the simple time paths which have so far been considered in the text or elsewhere.

The steady state error is given by:

$$y^*(t) - y^{ss}(t) = \lambda_0 x(t) - \sum_{k=0}^{n} \lambda_k \left.\frac{d^k x(s)}{ds^k}\right|_{s=t}$$

and hence for this to be zero we must have $\lambda_k = 0$, for k = 1, ..., n. If $\lambda_k = 0$, k = 0, ..., n-1 the steady state error is finite (provided λ_n is finite) and equal to $-\lambda_n \left.\frac{d^n x(s)}{ds^n}\right|_{s=t}$; and if $x(t) = Kt^n$, the steady state error, if finite, is $-\lambda_n\, n!K$.

Propositions 2 and 3.

If the target error is an ECM of order s is defined as:

$y^*_{t-j} - y_{t-i}$, $0 \leqslant j \leqslant i$, and $\delta \equiv i-j$

then W(L) is given by:

$$W(L) = \frac{a(L)L^j \theta}{((1-L)^s(1+B^*(L) + a(L)L^i)} \qquad A(1)$$

The growth coefficients are defined by,

$$\lambda_k \equiv \left.\frac{\partial^k W(L)}{\partial[(1-L)]^k}\right|_{L=1} \Big/ k! \equiv (-1)^k \left.\frac{\partial^k W(L)}{\partial L^k}\right|_{L=1} \Big/ k! \qquad A(2)$$

For more details see Patterson and Ryding (1982).

The proof requires evaluation of A(2) for W(L) given by A(1) with s > 1 and k = 1, ..., s - 1. However, direct evaluation is not necessary given that if k, the order of the derative, is less than s, the model order, then terms in powers of (1-L) will drop out when evaluated at L=1. Ignoring the term $(1-L)^s$ (1+B*(L)) in A(1) leaves evaluation of,

$$\left.\frac{\partial j^k}{\partial L^k} \frac{a(L)L^j}{a(L)L^i}\theta\right|_{L=1} = \lambda_0 \left.\frac{\partial^k}{\partial L^k}(L^{-\delta})\right|_{L=1}, \text{ for } k=1, ..., s-1 \qquad A(3)$$

With this simplification and taking the indicated partial derivative, we obtain:

$$\lambda_k = {}_{(\delta+ k-1)}C_{(\delta-1)}, \; k = 1, \dots s-1; \quad s > 1; \; \delta \geqslant 0 \qquad A(4)$$

where ${}_gC_h$ is the binomial coefficient for choosing h from g and

${}_gC_h \equiv 0$ for $h < 0$.

Proposition 2 follows on setting $\delta = 0$. To illustrate proposition 3 if j=0 and i=1: $\lambda_1 = \lambda_2 = \dots = \lambda_{s-1} = \lambda_0$ (nb: ${}_jC_0 \equiv 1$); and if j=0 and i=2: $\lambda_1 = 2\lambda_0, \lambda_2 = 3\lambda_0, \dots, \lambda_{s-1} = s\lambda_0$.

<u>Proof of corollary 2.1</u> Proposition 2 established that in an ECM of order s, with target defined by $y^*_{t-s} - y_{t-s}$, $\lambda_1, \dots \lambda_{s-1}$ are all zero, but reference to proposition 1 shows that this is necessary and sufficient to ensure a steady state error for growth in x_t up to and including s-1.

<u>Proof of corollary 2.2</u> Consider the AD (p,q) model with $\gamma(L)$ and $\beta(L)$ polynomials in L of orders p and q respectively; then for an ECM interpretation with target error $y^*_{t-h} - y_{t-h}$,

$$\beta(L) = a(L)L^h \qquad A(5)$$

$$\gamma(L) = 1+B(L) + a(L)L^h, \text{ and } 1+B(L) = (1-L)^s(1+B^*(L)) \qquad A(6)$$

Notice that the definition of the target error implies $\beta_0 = \dots \beta_{h-1} = 0$

From A(5) the order of a(L) is $q-h \equiv m$; also note that $\beta(L)$ and a(L) have the same number of coefficients, and from A(5) and A(6) we must have $p \geqslant q$.

Now, if in the AD(p,q) model $\lambda_1 = \dots = \lambda_f = 0$, then such a model has the essential characteristics of an ECM of order $s \equiv f+1$, see corollary 2.1, and hence the parameter space is of dimension (m+1) + p-f. In order to map this AD(p,q) model into the class of ECMs we have to determine the order of $B^*(L)$, and hence B(L). In an ECM of order s there are (m+1) + (n-s) + 1 coefficients, those in a(L), B*(L), and the target coefficient θ, respectively. The difference between the number of coefficients in the AD model and in a(L) and θ is the order[1] of B*(L): that is,

$$(m+1) + (p-f) - [(m+1) + 1] = p-f-1 = p-s$$

and hence B(L) is of order p-s+s = p (which defines n, the order of B(L)).

Proof of corollary 3.1 In order to ensure a zero steady state error for $x_t = Kt^n$, where n = s-1, then from proposition 1 we must have $\lambda_1, \ldots, \lambda_{s-1}$ all equal to zero; inspection of A(4) reveals that this only occurs for $\delta = 0$, that is i = j implying a target error $y^*_{t-j} - y_{t-j}$.

Proposition 4

The integral error at T is defined as $e_{iT} = \int_{t=0}^{T} (y^*(t) - y(t))\, dt$,

with the steady state integral error, e_{ssi} defined as the limit of e_{iT} as $T \to \infty$. We assume that t=0 represents the 'starting date' prior to which there were flow and stock (ie integral) equilibria; without loss of generality, x(t) is assumed to be zero for t < 0, and for $t \geqslant 0$ to be such that

$$\left.\frac{d^{n+j}x(s)}{ds^{n+j}}\right|_{s=t} \simeq 0, \text{ for } j > 0.$$

---

[1] Strictly p-s and p are the maximum orders of B*(L) and B(L), respectively; however, these polynomials will only be of lesser order if further constraints are satisfied by γ(L). For example, consider the AD(3,3) model which satisfies $\lambda_1 = 0$:

$$y_t = \beta_2 x_{t-2} + \beta_3 x_{t-3} + \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \gamma_3 y_{t-3}$$

then applying corollary 2.2 the target error is $y^*_{t-2} - y_{t-2}$, m = 3-2 = 1, and B*(L) is of order p-s = 1. Solving for a_0, a_1, θ, and b_1 reveals that if and only if $\gamma_1 = 2$ does $b_1 = 0$, and then B*(L) is of order 0.

$$\int_{t=0}^{T} (y^*(t) - y(t))\, dt = \int_{t=0}^{T} (x(t) \int_{j=0}^{\infty} w(j)\, dj - \int_{j=0}^{t} w(j)\, x(t-j)\, dj)\, dt$$

$$= \int_{t=0}^{T} (x(t) \int_{j=0}^{\infty} w(j)dj - \int_{j=0}^{t} w(j) \sum_{k=0}^{\infty} (-1)^k \frac{j^k}{k!} \frac{d^k x(s)}{ds^k}\Big|_{s=t} dj)\, dt$$

$$= \int_{0}^{T} (x(t) \; (\int_{j=0}^{\infty} w(j)\, dj - \int_{j=0}^{t} w(j)dj) - \int_{j=0}^{t} w(j) \sum_{k=1}^{\infty} (-1)^k \frac{j^k}{k!} \frac{d^k x(s)}{ds^k}\Big|_{s=t} dj)\, dt$$

$$= \int_{j=0}^{T} w(j) \int_{t=0}^{j} x(t)\, dj\, dt - \int_{j=0}^{T} w(j) \sum_{k=1}^{n} (-1)^k \frac{j^k}{k!} \int_{t=j}^{T} \frac{d^k x(s)}{ds^k}\Big|_{s=t} dj\, dt$$

$$+ x(T) \int_{j=T+}^{\infty} w(j)dj$$

$$= \sum_{k=0}^{n-1} \frac{d^k x(s)}{ds^k}\Big|_{s=T} \int_{j=0}^{T} w(j)\, (-1)^{k+1} \frac{j^{k+1}}{(k+1)!} dj + \int_{j=0}^{T} w(j) \; (\sum_{k=0}^{n} (-1)^k \frac{j^k}{k!} \frac{d^{k-1} x(s)}{ds^{k-1}}\Big|_{s=t})$$

$$+ x(T) \int_{j=T+}^{\infty} w(j)dj \qquad \text{A(7)}$$

(Note: $\frac{d^{-1} x(s)}{ds^{-1}}\Big|_{s=j} \equiv X(j)$ is the integral of x(s) evaluated at s=j)

Consider the first set of terms in A(7): as $T \to \infty$ $\int_{j=0}^{T} w(j)(-1)^{k+1} \frac{j^{k+1}}{(k+1)!} dj \to \lambda_{k+1}$, and as each of the derivatives will, by hypothesis, involve T each term in the sum will $\to \infty$; hence,[1] to ensure a finite e_{ssi} it is necessary to have $\lambda_{k+1} = 0$, k = 0, ..., n-1. We derive proposition 4 and corollaries 4.1 and 4.2 from A(7) on noting that:

$$\int_{j=0}^{T} w(j) \; [\sum_{k=0}^{n} (-1)^k \frac{j^k}{k!} \frac{d^{k-1} x(s)}{ds^{k-1}}\Big|_{s=j} + (-1)^{n+1} \frac{j^{n+1}}{(n+1)!} \frac{d^n x(s)}{ds^n}]\, dj = 0$$

as the term in square brackets is the expansion of X(t) about X(j) for t=0. Thus, if $\lambda_{k+1} = 0$, k = 0, ..., n-1 then $e_{ssi} = -\lambda_{n+1} \frac{d^n x(t)}{dt^n}$.

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(1) We assume $\int_{j=T+}^{\infty} w(j)\, dj$ dominates x(T) in convergence, such that the product of the two terms → 0 and T.

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