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Inventories, technological change, financial effects and risk

by

Keith Cuthbertson
and
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Abstract

We examine the determinants of inventory holding by the UK manufacturing sector over the period 1968–89. We find that inventory levels respond positively to output and its varience and negatively to the level of capital gearing. Technological innovation in the form of computerised inventory control leads to a fall in the stock-output ratio. Empirical results for inventories are examined in a co-integration and a general-to-specific error correction modelling framework with conditional variances estimated using the GARCH procedure.

I Introduction

Movements in inventories have had a major impact on cyclical changes in GDP in the United Kingdom in the post-war period, particularly in the downturns of 1974–5 and 1980–2. However, as Wallis et al (1987) note models of inventory behaviour (on UK data) are plagued by parameter instability. In this paper we develop a model of inventory holding which embodies most of the key ideas of earlier theoretical work. Previous theories include the production level smoothing and production cost smoothing models, the accelerator principle and the precautionary model. We are also able to modify the above models to incorporate the effects of financial factors as well as technological change. In the empirical implementation of the model we use the general-to-specific methodology to directly estimate an error correction model (ECM). However, as the latter *implicitly* assumes the presence of a co-integrating relationship we also estimate the long-run equilibrium solution of the model using techniques from the unit root and co-integration literature. Finally to model the time-varying variance of output we use the ARCH and GARCH framework (Engle, Lilien and Robins 1987, Bollerslev 1986). Our primary aim in this paper is, therefore, to take up the challenge noted in Wallis et al (1987) and develop a model of inventory behaviour that is consistent with existing theories and has stable parameters.

To anticipate our empirical results, we find that the level of manufacturing inventories has a unit elasticity with respect to output, is positively related to the conditional variance of output and negatively related to the overall gearing position of the firm. In addition, the introduction of computer technology has led to a fall in the stock-output ratio.

The rest of the paper is organised as follows. In Section II we present our theoretical model and in Section III we discuss our estimation strategies. Empirical results are to be found in Section IV and we end with a brief summary.

II Theoretical considerations[1]

Perhaps the basic model of inventory holding is that of 'production level smoothing'. In the face of fluctuating sales, and convex production costs, inventories are used as a 'buffer' and allow the firm to smooth production levels and hence minimise costs (see for example Blinder 1988). This simple model predicts that the variance of sales exceeds the variance of output: a prediction that is at odds with the empirical evidence (eg Blanchard 1983, West 1986).

Kahn (1987) develops a model of the precautionary demand for inventories. As the firm holds more inventories it reduces the probability of a stockout for any level of the variance of sales. The stock of inventories depends on the *conditional* variance of sales.[2]

If the production level smoothing model is combined with an accelerator relationship, (ie inventories proportional to expected sales) then it is possible for the model to predict output variation that is in excess of sales variation. However, the latter prediction is also possible in a 'production *cost* smoothing' model of inventories (Eichenbaum 1984, 1989). Here inventories are used to shift production to periods in which production costs are expected to be relatively low. It therefore introduces an intertemporal element to modelling inventory behaviour.

We present a model of inventory behaviour that embraces the above conceptual framework and also explicitly introduces technological change and financial effects. Since we require a closed form solution for inventories we shall use fairly simple functional forms (this also aids exposition and interpretation). The optimisation problem facing the firm is assumed to be intertemporal but we also demonstrate the conditions under which the reduced form solution may be interpreted as a purely backward-looking equation.

We begin with the different cost elements facing the firm. The generalised perceived cost of a stockout may be written $C^A(S_t, Y_t^s, h_t, RC_t)$ where S_t = level of inventories, Y_t^s = expected sales, h_t = variance of expected sales, RC_t = technological changes in inventory control procedures. A higher level of inventories reduces the probability (and hence the expected cost) of a stockout for any given level of expected sales hence, $C_1^A < 0$ and $C_2^A > 0$ (where $C_2^A = \dfrac{\partial C^A}{\partial S_t}$ etc). An increase in the variance of output increases the probability of a stockout, hence $C_3^A > 0$. The introduction of computerised methods of inventory control improves information flows about the different stages of the production process and the state of demand.[3] Thus for any given expected level or variance of output, technological advances lower the desired level of inventories. Thus linearising, we assume a desired precautionary level of inventories, $S_t^* = cY_t^s + dh_t - gRC_t$ with quadratic costs of being out of equilibrium:

$$C^A = \left(\frac{b}{2}\right)(S_t - cY_t^s - d\,h_t + gRC_t)^2 \tag{1}$$

Physical storage costs C^B (labour, machines, space, co-ordination and monitoring) we assume are quadratic in S_t:

$$C^B = e_1 S_t + \left(\frac{e_2}{2}\right) S_t^{\,2} \tag{2}$$

As is usual in the literature we assume production costs C^c are quadratic in output, Y_t^o:

$$C^c = \upsilon_t\, Y_t^o + \left(\frac{a}{2}\right)(Y_t^o)^2 \tag{3}$$

If $a>0$ this embodies the production *level* smoothing motive for inventory holding (Blinder 1988) whereas the time dependent element of marginal cost term υ_t embodies the production *cost* smoothing role of inventories (eg Eichenbaum 1984, 1989). Marginal cost is partly determined by the production level smoothing component, aY_t^o and partly by the term υ_t.

The term υ_t is usually just referred to as a 'stochastic shock' but clearly it might refer to changing relative factor prices[4] or the financial position of the firm. In our empirical implementation of the model we consider the additional major influence on expected marginal production costs to be the overall financial position of the firm. The perceived costs of an additional unit of output will be higher the more vulnerable is the firm's overall financial position.[5] A permanent increase in the level of output will usually involve the firm having recourse to additional debt or equity finance. To the extent that the former is perceived as increasing the risk of bankruptcy (with consequent interference from creditors or adverse movements in the share price), the managers of the firm will take account of such financing requirements in their assessment of marginal cost.[6] Since it is known that firms make use of financial ratios in corporate decision making we use a measure of gearing to pick-up these financial effects.

The total costs facing the firm are therefore[7]

$$C_t = \left(\frac{b}{2}\right)(S_t - c\,Y_t^s - dh_t + g\,RC_t)^2 + (e_1 s_t + \left(\frac{e_2}{2}\right) S_t^{\,2}) + \upsilon_t Y_t^o + \left(\frac{a}{2}\right)(Y_t^o)^2 \tag{4}$$

The inventory constraint is:

$$Y^{o} = Y^{s} + \Delta S_{t} \tag{5}$$

We assume the firm maximises the expected discounted present value of real profits

$$E_{t}(\Pi) = E_{t}\left[\sum_{t}^{\infty} D^{t}(Y_{t}^{s} - C_{t})\right] \tag{6}$$

where D is the discount factor ($0 < D < 1$) and E is the expectations operator conditional on information at time t. Since we take S_t as end of period inventories it seems reasonable to assume current period output, sales etc are known.

Substituting (4) and (5) in (6) and setting $\dfrac{\partial E_t(\Pi)}{\partial S_t} = 0$ results in the following first order condition, FOC, (L is the lag operator)

$$E_{t}[L^{-1} + \theta + (D^{-1})L]S_{t} = E_{t}[-Y_{t+1}^{s} + (a - bc)(aD)^{-1}Y_{t}^{s} + (aD)^{-1}\upsilon_{t} - a^{-1}\upsilon_{t-1} + (aD)^{-1}e_{1t} - bd(aD)^{-1}h_{t}] \tag{7}$$

where

$$\theta = -\left[\left(\frac{1+D}{D}\right) + \frac{b + e_{2}}{aD}\right] \tag{8}$$

Following Sargent (1979) the LHS of (7) may be factorised $(1 - \lambda L)(1 - (\lambda D)^{-1}L)$ *where* $0 < \lambda < 1$ for $a, b, e_2 > 0$. The closed form solution (see appendix I) is:

$$S_{t} = \lambda S_{t-1} + q\left[E_{t}\sum_{0}^{\infty}(\lambda D)^{i}Y_{t+1}^{s}\right] - Y_{t}^{s} - (\lambda a^{-1})E_{t}\left[\sum_{o}^{\infty}(e_{1} + (\lambda D)^{i}bgRC_{t+1}\right] - \lambda a^{-1}\upsilon_{t} + (1 - \lambda)a^{-1}E_{t}\sum_{0}^{\infty}(\lambda D)^{i}\upsilon_{t+i} + \Psi\sum_{o}^{\infty}(\lambda D)^{i}bdh_{t+i} \tag{9}$$

Given this closed form solution the following predictions emerge:

(i) An increase in current sales (given unchanged expected future sales) reduces current inventory levels (ie production level smoothing).

(ii) An increase in current and all expected future sales increases current period inventories (ie accelerator principle).

(iii) An exogenous increase in computerised inventory control procedures (RC_t) reduces current inventory levels.

(iv) An increase in *current* marginal costs, y_t reduces inventories given constant expected future costs (ie production cost smoothing).

(v) An increase in current and perceived marginal (production) costs in all future periods, reduces inventories.

There are a number of shortcomings of the above model. The model predicts that S_t depends only on S_{t-1} but Callen et al (1990) for the United Kingdom and Blanchard (1983) for the United States (automobile industry) demonstrate that an AR(2) model for S_t is empirically valid. If we add a term $[(\frac{f}{2})(\Delta Y_t^o)^2]$ to the cost function C^c the Euler equation contains an additional linear term in S_{t-2} and the latter enters the closed form solution (9). In this case the forward convolution is more complex (Sargent 1979) but is of the general form (9).

The maximisation problem in (6) contains only the first moment of profit and this sits uneasily with the assumption that the level of inventories depends on the uncertainty (variance) of sales, h_t. One tractable way to mitigate this problem is to assume different discount factors for the different cost elements. For example, if 'shocks' to marginal product costs υ_{t+j} are more difficult to forecast than the level of output (in the cost function C^c) then different discount factors apply to Y^s_{t+j} and υ_{t+j} in the closed form solution (9).

As expectations about future variables become more uncertain D $\rightarrow$ 0 the model yields a static equilibrium solution of the form $S_t = S(Y_t, h_t, RC_t, \upsilon_t)$ with $S_1, S_2 > 0$ and $S_3, S_4 < 0$. Of course this is also the form of the long-run 'static' equilibrium solution obtained in the forward model when $0 < D < 1$ and we set all future variables equal to some constant value. (eg if all forcing variables are random walks the optimal forecast is the current period value, for all t+j.)

The forward model reduces to an autoregressive distributed lag (ADL) model if we assume a VAR for the forcing variables $Z_t = \{Y_t^s, RC_t, \upsilon_t\}$ and a GARCH process for the conditional variance of output h_t. Representing the VAR process for Z_t in companion form:

$$Z_{t+1} = \Phi Z_t + w_t \tag{10a}$$

then the optimal predictor is:

$$E_t(Z_{t+j}) = \Phi^j Z_t \qquad \text{(for all j > 0)} \tag{10b}$$

If we assume the residual w_{it} from the VAR for Y_t^s follows a simple GARCH process

$$h_{t+1} = \alpha_0 + \alpha_1 h_t + \alpha_2 w_{it}^{\ 2} \tag{11}$$

then

$$E_t(h_{t+j}) = g(h_t, w_{it}^{\ 2}) \qquad \text{(for all j > 0)} \tag{12}$$

Substituting (10) and (11) in (9) then gives a solution for inventories of the form

$$S_t = \lambda S_{t-1} + \theta_1(L) Y_t + \theta_2(L) RC_t + \theta_3(L) \upsilon_t + \theta_4(L) h_t \tag{13}$$

where θ_i(L) are polynominals in the lag operator. Equation (13) is an ADL model where the θ_i depend on the parameters of the cost function (4) and the 'expectations parameters' from (10) and (11). If the latter are stable and the forward model is the 'correct model' then the ADL model will perform adequately.

Note also that the ADL model can be reparameterised into an (unrestricted) ECM.

$$\Delta S_t = \delta_1(L) \Delta S_t + \delta_2(L) \Delta S_t^* + \delta_3(L)(S - S^*)_t \tag{14}$$

It is also worth noting that in the general framework adopted we may obtain an ADL or ECM model of the form (14) if we assume (i) maximisation of (6) over a *single* period to yield the desired long-run inventory level $S_t^* = S(Y_t, h_t, RC_t, \upsilon_t)$ and (ii) a general distributed lag adjustment of S_t to S_t^*.

Recent papers that deal explicitly with a forward looking model of inventories include Eichenbaum (1983, 1984, 1989) Blanchard (1983) and Callen et al (1990). Eichenbaum (1989) uses the orthogonality restrictions from a Hansen 'methods of moments' estimator in an attempt to discriminate between the production level smoothing and production cost smoothing models. The strengths of Blanchard's (1983) model are that it examines a model based on quadratic production cost and the accelerator relationship in an intertemporal RE framework using industry level data. He is able to recover the adjustment cost parameters and test the implicit RE cross equation restrictions: the latter are found to hold. However, Blanchard (p373) conjectures that his deep parameters (ie adjustment costs and inventory sales coefficient) are *"very likely to depend at least on the second moment of the distribution of sales"*.

Callen et al (1990) do incorporate the conditional variance of output into the inventory equation but do so by assuming a two stage decision procedure and a simpler cost structure to that in equation (4) above.[8] They *impose* but do not test the implicit RE cross equation restrictions.

In principle we could estimate (9) using the errors in variables method (EVM) under the restrictive assumption that quadratic costs are appropriate, the discount factor is constant and the same for all forcing variables and that the marginal process for the forcing variables have constant parameters (Hendry 1988). This would take us beyond the focus of the present paper, however.[9] We therefore seek only to investigate the long-run cointegrating parameters and to establish a dynamic equation with stable parameters: the latter enables us to avoid imposing a strict quadratic costs of adjustment framework in estimation. The 'price' we pay is not being able to retrieve the cost of adjustment parameters but we do not consider these as of much interest relative to the 'deep parameters' from the long-run cointegrating relationships. However we do assess the *speed of adjustment* of inventories but do not partition this between the various cost parameters. Thus, although we do not explicitly estimate the RE version of the model, our estimated equation is not necessarily inconsistent with a forward looking model. It has the added advantage of incorporating variables in the conditional variance of output, technology and financial factors.[10] We therefore estimate the ECM model (14) and also test for the cointegrating properties of the long-run solution (13), recognising that our results are consistent with a number of theories of inventory behaviour.

III Estimation issues

The Engle-Granger Representation Theorem (Engle and Granger 1987) demonstrates that the presence of a cointegrating vector implies an error correction model and vice-versa. There are a number of strategies one can adopt for obtaining a dynamic model with 'sensible' long-run properties. In a single equation context we have the general-to-specific methodology of Hendry (Hendry 1983, Hendry and Ericsson 1990). The drawback of this approach is the possible absence of a co-integrating vector implicit in the set of variables in the unrestricted ADL model and hence in the final parsimonious ECM. However, the approach may embody less small sample bias than attempting to estimate long-run parameters in an OLS (or maximum likelihood) regression on the non-stationary variables (Banerjee et al 1986). The algebraic long-run solution from the dynamic ECM can of course be tested for the presence of a co-integrating vector. The latter does not, however, deal with the problem of multiple co-integrating vectors. For this we require the Johansen (1988) procedure but note that with multiple co-integrating vectors the researcher must choose (identify) a 'sensible' co-integrating vector based on the economic theory under investigation.

A frequently used alternative to the general-to-specific methodology is to employ the Engle-Granger two-step procedure. Here one first establishes a co-integrating vector (usually by OLS) and the 'residuals' are then used as the error correction term in a dynamic model. This approach gets us directly to a co-integrating vector but with the attendant risk of small sample bias (Banerjee et al 1986). Also the t-statistics in the OLS co-integrating parameters of the first stage cannot be used for hypothesis testing. However Engle and Yoo (1989) propose a three-stage procedure which they suggest mitigates these problems and provides valid t-statistics as well as correcting for small sample bias in the co-integrating parameters.

Given that many unit-root tests have low power against highly dynamic stationary alternatives (Perron 1989, Johansen and Juselius 1990) and clear cut results often do not emerge in 'moderate' sample sizes we prefer to examine both the direct general-to-specific approach and the co-integration (three-step and Johansen) procedures.[11] Both would yield similar inferences in an infinite sample given the Engle-Granger representation theorem but in small samples results are likely to differ.

A General to specific approach

The level of sales is assumed to be generated by a set of variables Q_t

$$Y_t = \alpha_2' Q_t + v_t \tag{15}$$

The disturbance term v_t in (15) is assumed $N(0, h_{22t})$ where h_{22t} is the time varying *conditional* variance of sales. We can re-parameterise the general ADL model into an unrestricted error-correction form:

$$\Delta S_t = \beta_0 + \beta_1(L)\Delta S_t + \beta_2'(L)\Delta X_t + \beta_3(L)\Delta h_{22t} + \gamma[S_{t-1} - \theta_1' X_{t-1} - \theta_2 h_{22t-1}] + \varepsilon_t \tag{16}$$

where S_t = logarithm of (the stock of) inventories, X_t = vector of determining variables and $\varepsilon_t \sim N(0, h_{11})$. The advantage of this (non-linear) reparameterisation is that one obtains direct estimates of the long-run parameters θ_i on the I(1) variables X_t and h_{22t} and their asymptotic standard errors. The model can then easily be estimated recursively to examine parameter stability and the statistical significance of the long-run and short-run parameters.

If we denote $w_t = (\varepsilon_t, v_t)'$ then the generalised GARCH (n, p) model is

$$\text{vech}(H_t) = A_0 + \sum_{i=1}^{n} A_i \ \text{vech}(w_{t-1} w_{t-1}') + \sum_{i=1}^{p} B_i \ \text{vech}(H_{t-1}) \tag{17}$$

where A_o, A_i, B_i are (2x2) matrices and 'vech(.)' denotes the column-staking operator of the lower portion of a symmetric (2x2) matrix.

The log-likelihood function (conditional on initial values) is proportionate to

$$\text{Log}(L) = -\sum_{1}^{T} \log |H_t| - \sum_{1}^{T} w_t' H_t^{-1} w_t \tag{18}$$

where we have assumed normality of the forecast errors.

Although the analytic derivates of (16) can be computed (see Engle et al 1987) variable-metric algorithms which employ numerical derivatives are simpler to use and easily allow changes in specification: the latter approach is applied in this paper. Under the usual regularity conditions, maximisation of (18) will yield maximum likelihood estimates with the usual properties.

As they stand, the above formulations are very general and contain a large number of parameters to be estimated, which may be problematic given the non-linearities of the system. Since inversion of any GARCH model of non-zero order implies an infinitely long memory with respect to past surprises, we limit the estimated GARCH model to first order—ie GARCH (1, 1). At this point a number of studies assume that the conditional variances h_{11t}, h_{22t} and covariance h_{12t} have the same weight in the GARCH process and hence shocks to the variance and covariance decay at the same exponential rate. The latter seems unduly restrictive given we are dealing with stocks and output. The alternative of assuming different rates of decay in each case proved impossible to implement (ie non-convergence). We therefore assume that only the conditional variance of output h_{22t} is time varying (we test for a constant h_{11} below). Note that if the conditional variance of output in the GARCH (1, 1) model is an integrated process (ie $a_{22} + b_{22} = 1$), it is known as IGARCH and h_{22t} is an I(1) variable.

B Co-integration approach

Here we are interested in direct estimation of the co-integrating vector

$$S_t = \theta_1' X_t + \theta_2 h_{22t} + \eta_t \qquad (19)$$

where we have assumed S_t, X_t and h_{22t} are all I(1) variables. OLS on (19) yields superconsistent parameter estimates of θ_i if the variables are cointegrated. We use the predictions from our univariate IGARCH (1, 1) model to obtain the conditional variance of output h_{22t}. The cointegration parameter estimates from (19) may be biased (Banerjee et al 1986). Hence we utilise the Engle-Granger three-step procedure which gives 'corrected' long-run parameter estimates of the θ_i and 'correct' standard errors (and also provides estimates of the short-run parameters). The 'corrected' third-stage long-run parameters θ_1^* and θ_2^* can then be used to generate the 'corrected' residuals η_t^* on which we perform the usual unit root tests for co-integration.

IV Empirical results

Data

The data is quarterly, constant price, seasonally adjusted from 1968(1) to 1989(4). The inventory data S_t is for the manufacturing sector. We do not have data on sales and we therefore use manufacturing output Y_t (if ΔS_t is stationary then our long-run but not our short-run results should be unaffected). The financial variable is capital gearing G_t that is, total debt at market value as a proportion of the value of the capital stock at historic cost. The gearing variable is only available for all industrial and commercial companies.[12]

The variable we use to pick-up changes in inventory control technology is the real value of the *stock* of computers in the United Kingdom, RC_t, (ie not just for the manufacturing sector). Nominal data on computer sales is deflated using a price index for durable goods[13] and the real series is cumulated using a depreciation rate of 0.125 per quarter. This we feel is the most important technological development in this area (it is obviously related to the adoption of 'Just-in-Time' methods of production). As the manufacturing sector has been a leading sector in the adoption of computer based technology our economy-wide data series should reflect this (see *British Business* CSO, various issues in 1980s). An alternative variable tried was the number of Japanese manufacturing companies setting up in the UK since it is known that these companies operate sophisticated inventory control methods. This variable also performed adequately (it is highly correlated with RC_t) but on a priori grounds we prefer the wider coverage of the computer variable.[14]

Table 1A
Dickey-Fuller and augmented Dickey-Fuller tests for unit roots[1]

	Dickey Fuller (DF)			Augmented Dickey Fuller (ADF)		
	$(t\,\gamma)$	$\hat{\alpha}$	$(t\,\alpha)$	$\hat{\alpha}$	$(t\,\alpha)$	$10^4\,(t\,\gamma)$
S_t	2.4	-0.03	(-1.5)	-0.03	$(-1.7)^4$	-1.6
Y_t	-0.4	-0.08	(-1.9)	-0.10	$(-2.3)^2$	-0.3
RC_t	-1.2	0.01	(0.7)	-0.02	$(-1.1)^4$	-1.7
h_{22t}	-0.6	-0.10	(-2.0)	-0.10	$(1.9)^4$	-0.5
G_t	-1.5	-0.10	(-2.2)	-0.12	$(-2.1)^4$	-1.5
ΔS_t	-1.5	-0.5	(-4.8)	-0.6	$(-4.8)^3$	-2.0
ΔY_t	-0.1	-0.9	(-4.0)	-0.8	$(-4.0)^3$	-0.1
ΔRC_t	1.8	-0.4	(-4.6)	-0.4	$(-3.2)^3$	1.5
Δh_{22t}	-0.7	-1.1	(-10.0)	-0.8	$(-3.8)^4$	-0.5
ΔG_t	-0.1	-1.0	(-8.8)	-1.3	$(-5.2)^4$	-0.2

Critical values of test statistics

5%	2.8	-2.89		-2.89		2.8
10%	2.3	-2.58		-2.58		2.3

(1)The Dickey-Fuller (DF) and Augmented Dickey Fuller (ADF) tests consist of regressions of the form

$$\Delta X_t = \alpha_0 + \alpha x_{t-1} + \sum_{j=1}^{m} \beta_j\, \Delta x_{t-j} + \gamma t + \varepsilon_t$$

where $\beta_k = 0$ (j=1-m) for the DF test and 't' is a deterministic time trend. The coefficient α and its associated t-statistic $(t\,\alpha)$ are reported above. For the ADF test the superscript denotes the minimum value of m to achieve white noise errors ε_t. If the absolute value of any test statistic exceeds its (absolute) critical value then we reject the null hypothesis of a unit root. Critical values are taken from Table 1 in MacKinnon (1990). The data period is 1968(2)–89(4).

Table 1B
Phillips-Perron tests for unit roots[1]

	$Z(t\,\tilde{\alpha})$	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(t_{\alpha^*})$	$Z(\Phi_1)$
S_t	-2.1	4.0	3.1	-2.7	4.2
Y_t	-1.6	1.3	1.4	-1.3	1.7
RC_t	0.1	3.0	13.0	-1.2	19.5
h_{22t}	-2.0	2.3	1.5	-2.1	2.2
G_t	-1.3	1.0	0.9	-1.4	1.3
ΔS_t	-5.4	14.5	9.7	-5.2	13.5
ΔY_t	-8.9	26.7	40.0	-8.9	40.4
ΔRC_t	-4.6	10.4	7.0	-4.1	-8.5
Δh_{22t}	-11.1	62.0	41.3	-11.1	62.0
ΔG_t	-9.2	42.8	28.5	-9.3	43.0

Critical values of test statistics

5%	-3.45	6.49	4.88	-2.89	4.71
10%	-3.15	5.47	4.16	-2.58	3.86

(1) The Phillips-Perron (1988) tests are based on the following alternative models for any variable y_t:

$$y_t = \tilde{\mu} + \beta\left(t - \frac{n}{2}\right) + \tilde{\alpha} y_{t-1} + \tilde{u}_t$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u_t^*$$

The test statistics correspond to the following null hypotheses:

H_0: $\tilde{\alpha} = 1$	$Z(t\,\tilde{\alpha})$	H_0: $\alpha^* = 1$	$Z(t\,\alpha^*)$
H_0: $\tilde{a} = 1, \beta = 0$	$Z(\Phi_3)$	H_0: $\alpha^* = 1,\ \mu^* = 0$	$Z(\Phi_1)$
H_0: $\tilde{\alpha} = 1, \beta = 0, \tilde{\mu} = 0$	$Z(\Phi_2)$		

If the absolute value of any test statistic exceeds its (absolute) critical value then we reject the null. Critical values are taken from Fuller (1976) and Dickey and Fuller (1981). If we do not reject H_0: $\tilde{\alpha}=1, \beta = 0$ we use the $Z(t_{\alpha^*})$ and $Z(\Phi_1)$ statistics. The truncation lag parameter is set at 4. The data period used is 1968(2)–1989(4).

Inference in both the general-to-specific and three-step approaches is conditional on the order of integration of the variables, and appropriate tests are shown in Tables 1A and 1B. In Table 1A the results using the ADF tests (including a deterministic time trend, MacKinnon 1990) are applicable since additional lagged difference terms are significant. The results in Table 1B for $Z\ (\Phi_3)$ for the levels of the variables indicate that we cannot reject the null of a unit root and a zero deterministic time trend (Table 1B). The $Z\,(t\alpha^*)$ statistic also does not reject the null of a unit root given a zero deterministic trend. The results in the bottom half of Table 1B indicate rejection of the null of a unit root in the first differenced variables (using either $Z\,(t\tilde{\alpha})$ to $Z\,(t\alpha^*)$). Both sets of result in table 1A and 1B indicate that we can reject the hypothesis that the level of the variables are I(0). However, we can reject non-stationarity in the first differences. The variables are therefore I(1) with the possibility of a deterministic trend in S_t, (see $t\,\gamma$, Table 1A).

Results using the general to specific approach

Our model for inventories is

$$\Delta S_t = \beta_0 + \beta_1\ (L)\ \Delta S_{t-1} + \beta_2'\ (L)\ \Delta X_t + \beta_3 \Delta\ (L)\ h_{22t} + \gamma [S_{t-1} - \theta_1' X_{t-1} - \theta_2 h_{22t-1}] + \varepsilon_t \quad (20)$$

$$\Delta Y_t = \alpha_{21} + \sum_{i=1}^{4} \alpha_{2i} \Delta Y_{t-i} + v_t \quad (21)$$

where

$$\varepsilon_t \sim N(0, h_{11}) \qquad v_t \sim N(0, h_{22t})$$

$$w_t = (\varepsilon_t, v_t)' \qquad E(ww') = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22t} \end{bmatrix}$$

$$X_t = [Y_t, G_t, RC_t]$$

and S_t = manufacturing stocks, Y_t = manufacturing output, G_t = capital gearing, RC_t = stock of computers, h_{22t} = conditional variance of output (based on the fourth order autoregressive model, equation 21).

Our preferred specification of (20) is:[15]

$$\Delta S_t = \underset{(3.6)}{0.16} + \underset{(4.5)}{0.43}\, \Delta S_{t-1} + \underset{(3.8)}{0.16}\, \Delta Y_t - \underset{(3.9)}{0.10}\, [(S - Y)_{t-1} + \underset{(2.0)}{0.317}\, (10^{-1}) RC_{t-1}$$

$$+ \underset{(2.9)}{0.144}\, G_{t-1} - \underset{(1.9)}{0.548}\, (10^{-7})\, h_{22t-1}] \quad (22)$$

1968(1)–1987(4) , SEE = 0.7(%), LM(4) = 8.3, HF(8) = 10.7, ARCH(1) = 0.1, RAM(3) = 2.1, BJ(2) = 2.9, (.) = t-statistic.

The IGARCH process for the conditional variance of output is

$$h_{22t} = \underset{(17.5)}{(0.85)^2}\, h_{22t-1} + [1 - (0.85)^2]\, v^2_{t-1} \quad (23)$$

The likelihood ratio test for imposing the IGARCH restriction $a_{22}+b_{22} = 1$ is LR(1) = 1.7 and is easily acceptable at a 5 per cent significance level ($\chi^2_c = 3.8$).

The standard error of the inventory equation is reasonable at 0.7 percent and there is no evidence of serial correlation (LM(4) = 8.3, χ^2_c = 9.5), functional form misspecification (RAM(3) = 2.1, χ^2_c = 7.8) and the residuals do not exhibit an ARCH process ARCH(1) = 0.1, χ^2_c = 3.8). The Jarque-Bera statistic BJ(2) = 2.9 (χ^2_c = 6.0) does not reject the null of a normally distributed error term. The Hendry forecast test for (relative) numerical parameter constancy HF(8) = 10.7 is below its critical value (χ^2_c = 15.5). (These tests use the residuals from the inventory equation.)

The unit elasticity on output is tested by including the term $(S - \beta Y)_{t-i}$ in (22) and $\hat{\beta} = 1.2$ with asymptotic standard error 0.28: hence the long-run unit elasticity restriction is easily accepted. The model therefore solves to yield a long-run stock-output ratio that is independent of output, is negatively related to the stock of computers RC_t (which we interprete as technological developments in stock control procedures) and to gearing, G_t, and is positively related to the conditional variance of output, h_{22t}.

Table 2.1
ECM model: LR parameters: inventory equation[1]

	ECM[2]	Y_t	RC_t	G_t	$10^7.h_{22t}$
1968(1)-87(4)	-0.10(3.9)	1	-0.032(2.0)	-0.14(2.9)	0.55(1.9)
1968(1)-88(4)	-0.09(4.2)	1	-0.040(2.9)	-0.16(3.3)	0.55(1.8)
1968(1)-89(4)	-0.09(4.2)	1	-0.048(3.9)	-0.19(4.2)	0.56(1.8)

(1) Unit output elasticity imposed. t-statistics in parentheses.
(2) The coefficient on the error correction adjustment term.

The behaviour of the long-run parameters and the ECM term of the inventory equation (20) over successive 4 quarter periods are given in Table 2.1 and exhibit considerable stability in terms of point estimates. (The parameters of the GARCH process remained virtually unchanged and are not reported.)

When we take the algebraic long-run solution from the ECM equation for inventories as a potential cointegrating vector and test the residuals for stationarity we obtain DF = -2.3, ADF = -2.3 (the ADF has no additional lagged difference terms that are significant). These results do not reject the null of a unit root in the residuals and suggest the absence of a co-integrating vector. This is investigated further below.

Co-integration and three-step estimates

OLS on the I(1) variables of the model (including the conditional variance h_{22t} obtained from the estimation of the output equation and IGARCH process only) over the same data period as the ECM equation (22) yields

$$\underset{}{S_t} = \underset{(7.9)}{5.9} + \underset{(6.7)}{0.53}\, Y_t - \underset{(1.0)}{0.72}\,(10^{-2})\, RC_t - \underset{(4.6)}{0.12}\, G_t + \underset{(4.5)}{0.61}\,(10^{-7})\, h_{22t} \tag{24}$$

OLS 1968(1) - 1987(4), $R^2 = 0.54$.

Compared with the long-run results in (22) we note a much smaller output elasticity of 0.53 and a smaller coefficient on the real computer variable RC_t which also has a low t-statistic (but note that this is not distributed as a Student's t-distribution). The coefficients on G_t and h_{22t} are similar to those in the ECM model.[16] However the low value for R^2=0.54 indicates the possibility of substantial small sample bias in some or all of the coefficients of the OLS co-integration regression (24). We therefore apply the Engle-Yoo (1987), three-step procedure incorporating the same dynamics as in the ECM model.

The 'corrected' co-integrating parameters with corrected t-statistics in parentheses are:

$$S_t = \underset{(4.8)}{0.90}\, Y_t - \underset{(2.7)}{0.045}\, RC_t - \underset{(3.1)}{0.18}\, G_t + \underset{(1.8)}{0.57}\,(10^{-7})\, h_{22t-1} \tag{25}$$

The output elasticity is much closer to unity (the latter restriction is easily accepted on a t-test) and all other parameter estimates are statistically significant and similar in magnitude to those in the ECM equation (22). (The short-run parameters on ΔS_{t-1} [=0.58(t=7.4)] and ΔY_t [=0.17(t=4.0)] in the three-step procedure are very similar to those in the ECM model as one might expect.)

Table 2.2
Three-step estimates: LR parameters: inventory equation

	ECM[1]	Y_t	RC_t	G_t	$10^7 h_{22t}$
1968(1)-87(4)	-0.08(3.0)	0.90(4.8)	-0.045(2.7)	-0.18(3.1)	0.57(1.8)
1968(1)-88(4)	-0.09(3.3)	0.89(5.1)	-0.047(3.2)	-0.19(3.4)	0.55(1.8)
1968(1)-89(4)	-0.09(3.4)	0.87(5.3)	-0.050(3.9)	-0.20(3.9)	0.54(1.8)

(1) Unit output elasticity imposed. t-statistics in parentheses.

A comparison of the three-step estimates over sucessive 4 quarter periods is made in Table 2.2. The residuals e_t from the co-integrating vector from the three-step procedure only exhibit serial correlation of order one and therefore the DF statistic provides the appropriate unit root test. The coefficients on e_{t-1} and the t-statistics over the three periods ending 87(4), 88(4) and 89(4) are -0.18 (-2.9), -0.19 (-3.1) and -0.19 (-3.2). The point estimates are well away from zero but the t-statistics do not reject the hypothesis of a unit root (the critical values at 5 and 10 percent significance levels are -5.7 and -4.2, MacKinnon 1990). The weak power of these tests against highly dynamic stationary alternatives has been well documented (Johansen and Juselius 1990, Engle and Yoo 1987) but the results here certainly cast some doubt on the presence of a co-integrating vector.

We therefore tested for the presence of a set of co-integrating vectors using the Johansen (1988) procedure.[17] The results in Table 3 (for VAR lag length = 2) clearly indicate the presence of a unique co-integrating vector (LR(r = 0) = 90.4, χ^2_c= 68.5, and LR ($r\leq 1$) = 35.9, χ^2_c = 47.2). The eigenvector corresponding to the largest eigenvalue yields parameter estimates of the correct sign which are closer to those found in the OLS co-integrating regression (24) than in the 3-step equation (25). The Johansen procedure confirms the presence of a unique cointegrating vector[18] but the point estimates from the co-integrating vector (especially that on output) are at variance with those from the ECM equation (22).

On balance we take the results from the ECM equation (which has long-run parameters similar to those in the 3-step procedure) as our preferred equation.[19] The performance of

Table 3
Johansen maximum likelihood estimation: 1968:1–1987:4

(a) Test for cointegration: VAR lag length = 2

Number of cointegrating vectors, r	Likelihood Ratio statistics, LR	Critical Value 5% significance level
$r\leq 4$	0.71	3.76
$r\leq 3$	5.65	15.41
$r\leq 2$	19.18	29.68
$r\leq 1$	35.88	47.21
$r\leq 0$	90.37	68.52

(b) Eigenvectors and eigenvalues

	Eigenvalue	Eigenvector (normalised on S_t) Y_t	RC_t	G_t	h_{22t}
1	0.503	0.352	-0.020	-0.135	0.484 (10^{-8})
2	0.193	1.720	-0.151^{-2}	0.201	0.158 (10^{-6})
3	0.159	1.491	-0.083	-0.256	0.139 (10^{-6})
4	0.009	2.835	0.161	-0.374	0.869 (10^{-7})

this model in terms of sign, size and stability of parameters and in terms of its statistical performance is encouraging.

Further economic implications

The cumulative response functions for the percentage change in inventories [using the ECM equation (22)] consequent on a one percent change in output Y_t, the stock of computers RC_t and gearing G_t are given in Table 4. All step responses are smooth and monotonic (quarter by quarter). Over half the output effect has taken place after 4 quarters while half the total effect due to a change in the stock of computers and the impact of gearing takes about 5–6 quarters. The response of inventories to a rise in the conditional variance of output also has about a five quarter median lag. These effects do not seem unreasonable. (However, Blinder and Maccini (1990) are sceptical of long lags based on arguments of quadratic costs but conjecture that 'return point' (S, s) type models are not inconsistent with such lag lengths.)[20]

Table 4
Step response function: one percent shock to independent variables[1]

Effect after x quarters	Y_t	RC_t	G_t	$10^7.h_{22t}$
4	0.53	-0.01	-0.05	0.19
8	0.79	-0.02	-0.10	0.39
12	0.91	-0.03	-0.13	0.48
long run	1.0	-0.03	-0.14	0.55

Notes

(1) Y_t, RC_t and G_t are in logarithms as is the dependent variable (inventories). Therefore, response coefficients for these variables are elasticities. h_{22t} is not a logarithmic variable and hence only the time profile is readily interpretable.

Table 5
Contribution of the Independent variables to the annual percentage change in inventories

	Change in Inventories	Contributions of: Output Y_t	Technology RC_t	Gearing G_t	Variance h_{22t}
1978	1.49	0.74	-0.29	1.68	-1.45
1979	1.00	0.72	-0.26	1.78	-1.29
1980	-2.42	-2.58	-0.27	0.63	-0.56
1981	-7.54	-7.05	-0.32	-0.06	0.98
1982	-1.71	-3.33	-0.37	0.14	0.68
1983	-1.99	-0.62	-0.36	-1.44	-0.53
1984	1.78	2.93	-0.47	-0.45	-1.00
1985	0.84	3.29	-0.54	-0.83	-0.71
1986	-1.31	1.68	-0.52	-0.33	-0.38
1987	-0.89	3.17	-0.47	0.14	-0.08
1988	0.84	5.34	-0.49	-0.73	0.09
1989	1.96	6.03	-0.49	-2.75	0.02

For any variable X_t, the ECM inventory equation may be written $\theta_1(L)S_t = \theta_2(L)X_t$. We can then solve for $S_t=[\theta_1(L)]^{-1}\ \theta_2(L)X_t$ and for the contribution of X_t to the *annual* change in S_t. It is immediately clear (Table 5) that computer sales have contributed a downward trend in inventories which accelerates through the 1980s. Gearing G_t, and the conditional variance h_{22t} have contributed to cyclical changes in inventories with both having a predominantly negative impact in the 1980s. However the main cyclical impact on inventories in the 1980–82 recession and the subsequent recovery is the change in manufacturing output.

Turning now to the long-run inventory-output *ratio* our model is consistent with the view that this has fallen throughout most of the 1980s because of the increased use of computerised inventory control, a rise in gearing and a fall in the conditional variance of output. Studies of US inventory behaviour do not incorporate the latter variables even though they are not ruled out on theoretical grounds (the financial variable used in US studies is usually an interest rate which is often found to be statistically insignificant: Blinder and Maccini 1990). Callen et al (1990) find the conditional variance of output important in determining inventory holdings in the United Kingdom but do not incorporate our computer technology variable and contrary to much previous work do not find the level of output to be statistically significant.

V Conclusions

We have presented a theory of inventory holding which is consistent with the production level and production cost smoothing models. In addition we have incorporated a precautionary demand for inventories based on the conditional variance of output and financial effects on inventories which work via a gearing variable. The model can be viewed as a purely backward looking model (eg static optimisation over a single period plus lagged adjustment) or as the reduced form of a forward looking model (based on multiperiod optimisation). The estimated model is also broadly consistent with the (S, s) inventory model, particularly the use of the conditional variance of output. However, as (S, s) models often do not yield closed form solutions this correspondence is not exact. Because of the problems in discriminating between alternative models using aggregate data we concentrate on obtaining a valid co-integrating vector (which must exist under all the theory models considered) and a dynamic model with stable parameters.

We apply the general to specific/ECM modelling approach, co-integration and GARCH modelling procedures. We find a unit elasticity of inventories with respect to output, a positive response of inventories to an increase in the conditional variance of output and a negative response to increased 'financial stress' as measured by capital gearing. The steady decline in the inventory-output relationship in the 1980s we attribute (in part) to the increased use of computer based methods of inventory control.

Further research could profitably examine the model using industry level data (where one might obtain separate series for sales and output) and in providing valid encompassing tests against the RE version of the model.

Footnotes

(1) The set of inventory models considered in this section is not exhaustive. For a comprehensive account see Blinder 1990 and Blinder and Maccini (1990).

(2) In Kahn's (1987) basic model the firm has linear production costs (to abstract from the production level smoothing motive) and serially correlated exogenous demand (his alternative model also incorporates backlogs\excess demand). The firm maximises the DPV of profits where sales are the minimum of demand or 'production plus last periods inventory'. The non-negativity constraint requires a dynamic programming solution which in this version of the model results in $S_t = k\text{-}min[k, u_t]$ where u_t = demand shock and k depends on the conditional variance of the forecast error of sales, the discount factor and the price-cost mark-up. In a variant of the model Kahn also introduces the idea that a proportion of any backloged orders become a component of demand in the next period: again the variance of production can exceed the variance of sales. Kahn notes (p668) that his model *"suggests some ways in which the linear quadratic framework employed in much empirical research ... might be modified to make a better approximation to more rigerous models"*. Since Kahn does not undertake empirical work we hope our model throws some light on this issue.

(3) Dudley and Lasserre (1989) stress the improvement in information about the state of demand resulting from the reduction in the unit cost of telecommunications.

(4) Ramey (1989) explicitly considers inventories as a factor of production and hence relative factor prices influence inventory holdings. We take the inventory and employment decisions as weakly separable. West (1990) finds that the importance of cost-shocks relative to demand-shocks depends on the value of the target inventory-sales ratio.

(5) In principle v_t measures the product of the increased probability of bankruptcy and the unit cost of bankruptcy. With asymmetric information this probability may differ between managers, shareholders and creditors but we do not deal with this aspect in our model.

(6) We do not model the fixed investment and employment decisions of the firm. However, given that fixed capital and to a lesser extent additional employment is (in the aggregate) financed in part from external borrowing, we would expect a high level of output in the long run to be accompanied by an increase in employment and the capital stock and a higher level of gearing.

(7) Callen et al (1990) utilise a similar cost and profit function to (4) and (6). Agents are assumed to minimise the *one period* objective function

$$\pi = P_t Y_t^s - C(Y_t^0) - K(h_t, S_t, Y_t^s) - r_t B_t - H_t . S_t \qquad \text{(i)}$$

where $C(Y_t^o)$ are general costs of production, $K(h_t, S_t, Y_t^s)$ is a general functional form for the cost of a stockout, H_t is the storage costs per unit of inventories (which depends on the real interest rate and dummy variables to reflect changes in the tax regime). B_t is net borrowing and r_t the interest cost of net borrowing. In our model $C(Y_t^o)$ and $K(h_t, S_t, Y_t^s)$ are quadratic and we replace $H_t S_t$ by a quadratic in S_t to model storeage costs. Callen et al introduce a net borrowing constraint

$$B_t = (1+r)\, B_{t-1} + C(Y_t^o) - PY_t^s + H_t S_t \qquad \text{(ii)}$$

into the model together with the inventory identity

$$\Delta S_t = Y_t^o - Y_t^s \qquad \text{(iii)}$$

In our approach a financial effect appears via the term υ_t, a shift in expected costs. The latter simplifies what is a complex decision process but enables financial effects to be included in a tractable and reasonably realistic manner.

Eichenbaun (1984) also has a different cost function to (4). There is no output costs of adjustment term but he has quadratic adjustment costs in inventories *and* employment of the form $\Delta x_t H_1 \Delta x_t'$ and $(\Delta^2 x_t) H_2 (\Delta^2 x_t')$ where $x_t = \{L_t, S_t\}$, where L = employment and H_1 and H_2 are symmetric positive semi-definate (2x2) matrices. The closed form solution then depends on lagged values of L_t as well as lagged values of S_t. It is a form of interdependent adjustment. Eichenbaun also incorporates endogenous wages and prices but does not consider the conditional variance of output or financial factors in his model. The interaction between employment and inventory decisions has been extensively analysed in a series of papers by *inter alia* Maccini, Rossana and Haltiwanger and these are documented in Blinder and Maccini (1990).

(8) Callen et al (1990) estimate a dynamic forward looking model under RE using the errors in variables procedure after estimating a co-integration equation for S_t^*. They are able to make the problem tractable by assuming an independent *two-stage* decision process. First the agent decides his long-run desired inventory level $S_t^* = X_t \hat{\pi}$ (as described in footnote 4) where $\hat{\pi}$ is a co-integrating vector. The agent then chooses the actual inventory level S_t by minimising

$$C = \sum D^t \left[\left(\frac{a}{2}\right)(S_t - S_t^*)^2 + \left(\frac{b}{2}\right)(\Delta S_t)^2 + \left(\frac{c}{2}\right)(\Delta^2 S_t) \right] \quad \text{(i)}$$

The closed form solution is of the form

$$S_t = \lambda_1 S_{t-1} + \lambda_2 S_{t-2} + \sum \gamma_i S_{t+1}^* \quad \text{(ii)}$$

where the γ_i are a complex convolution of (λ_1, λ_2).

(9) There are a number of difficult problems in estimating the RE model some of which have arisen as a consequence of the co-integration literature. First and most obviously there is often a near unit root in the lagged dependent variables which may imply a very 'long' forward memory. Given the latter, the desirability of using either a fixed discount factor given 'risk' in the real world or of arbitrary truncation of the forward terms is bought into question. The co-integration properties of the expectations generation (marginal model) has to be addressed given the presence of non-stationary series. Next, following Hendry (1988) the marginal model has to have stable parameters if the ECM model also has stable parameters, otherwise the forward model is ruled out. Favero and Hendry (1990, p 31) argue that 'the force of the Lucas critique is not strong even for marked changes in input processes and that 'conventional' model mis-specifications remain of far greater practical concern'. In addition Milne (1990) argues that the Euler equation derived from the first order conditions of the intertemporal RE version of the production smoothing model are violated by the data for the UK manufacturing sector. Finally in testing RE cross-equation restrictions the information set for the VAR of the marginal model often excludes the lagged dependent variables and if these Granger-cause the variables in the conditional model, estimates will be biased.

(10) As Blinder and Maccini (1990) point out it is likely that different categories of inventories (eg finished goods, work-in-progress) require different models. For the work-in-progress they suggest using the (*S*, *s*) inventory model (Caplin 1985, Blinder 1981). As Blinder and Maccini (1990) note, 'the estimation of (*S*, *s*) models has barely begun'. This is another reason we do not wish to restrict our empirical model to the strict quadratic cost framework. It may be that our conditional variance of sales picks-up 'uncertainty' found in (*S*, *s*) type models and in addition that the time varying nature of the thresholds (*S*, *s*) are influenced by the financial position of firms. However, further development of (*S*, *s*) models is required before these conjectures can be examined further (see also, Granger and Lee 1989 who interpret a non-symmetric ECM for production depending on inventories and sales as being non-supportive of an (*S*, *s*) model).

(11) To examine in detail all the equations in the VAR system of the Johansen procedure (Johansen and Juselius 1990, Hendry and Mizon 1989) requires that we produce economically meaningful and statistically acceptable equations for *all* the variables in

our system (and we have to make the implicit assumption that all excluded variables would not materially effect our conclusions). The scope of this task is such that it often involves the use of somewhat arbitrary dummy variables in the specification of the auxiliary equations. We do not tackle these wider issues in our paper.

(12) We tried a number of 'gearing variables'. Income gearing (ie net interest payments as a proportion of non interest income net of stock appreciation and tax payments) did not yield satisfactory results. Two measures of capital gearing were tried both of which are highly correlated. The first is the ratio of the market value of debenture and loan stock plus lending from banks and other financial institutions minor holdings of liquid assets plus issues of commercial bills minus holdings of liquid assets to the value of trading assets at replacement cost. The second measure has the same numerator but the denominator is trading assets at historic cost. It makes no qualitative difference to our results which of these variables is used. However we believe the latter is more often used by managers and also more closely reflects creditors views about the probability and timing of any bankruptcy proceedings. Under certain restrictive assumptions the Modigliani-Miller theorem suggests that the capital structure of the firm is unimportant. Our empirical results clearly reject this hypothesis.

(13) Using an alternative price series (eg electrical goods only) did not materially affect the results. For the United Kingdom we are not aware of a more sophisticated price series for computers (but for the United States see Berndt and Griliches 1990).

(14) Dudley and Lasserre (1989) incorporate the impact of technological advances into the inventory decision and test for the importance of the 'number of minutes of international telephone calls', on inventory holdings. The hypothesis is that additional information on sales reduces the stock-output ratio. Our stock of computers variable is slightly different and provides information flows primarily *within* the organisation and its immediate suppliers.

(15) Using instrumental variables (instruments are 4 lagged values of S_t, RC_t, G_t, Y_t, world trade and oil prices) for ΔY_t does not materially alter the results obtained.

(16) In Callen et al (1990) their theoretical model predicts $\frac{\delta S}{\delta r} < 0$ (where r = interest rate) and $\frac{\delta S}{\delta Y^s} > 0$: these results are contradicted in their preferred empirical model (see their equation on p766) where $\frac{\delta S}{\delta r} > 0$ and $\frac{\delta S}{\delta Y^s} < 0$ (although the latter effect appears to statistically insignificant).

(17) As there may be deterministic trends in the inventory series we use an 'unrestricted constant' in the Johansen procedure (Johansen and Juselius 1990).

(18) The results for VAR lag length of 4 also did not reject a unique co-integrating vector.

(19) We do so in part because of the similar coefficients in the ECM equation (22) and the three-step equation (25). The latter are both 'single equation' techniques which incorporate dynamics when estimating the long-run cointegrating parameters. The Johansen (1988) VAR procedure is equivalent to FIML estimation and we conjecture that mis-specification in the (arbitrary) equations fo the VAR other than that for inventories may contaminate the long-run estimates of the inventory equation. Theoretical work on the latter problem has not yet appeared in the literature.

(20) Eichenbaum (1989) uses monthly inventory and sales data on specific industries to test a forward looking model. He finds short median lags and 95 per cent of adjustment takes place within 4 months. However, Eichenbaum uses either detrended data or first-differences to estimate the model. All the long-run (co-integration) information in the data is therefore lost.

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Appendix I: Solution of the model

The objective function is

$$E_t(\Pi) = E_t\left[\sum_t^{\infty} D^t (Y_t^s - C_t)\right] \tag{A1}$$

Where

$$C_t = (\frac{b}{2})(s_t - cY_t^s - dh_t + gRC_t)^2 + (e_1 S_t + e_2 S_t^2) + \upsilon_t Y_t^0 + (\frac{a}{2})(Y_t^0)^2 \tag{A2}$$

with the inventory constraint

$$Y^0 = Y_t^s + \Delta S_t \tag{A3}$$

Substituting (A3) in (A2) and (A4) in (A1) and differentiating we obtain the following FOC (and the usual transversality condition):

$$E_t[(a(1+D) + s_2 + b)S_t - aDS_{t+1} - aS_{t-1} + (a - bc)Y_t^s - aDY_{t+1}^s + \upsilon_t - D\upsilon_{t+1}$$

$$+ e_1 - bdh_t + bgRC_t] = 0$$

Re-arranging and using the lag operator L we have

$$E_t[L^{-1} - (1+D)D^{-1} + (e_2 + b)aD)^{-1}L]S_t = E_t[-Y_{t+1}^s + (a - bc)(aD)^{-1}Y_t^s$$

$$+ (aD)^{-1}\upsilon_t - a^{-1}\upsilon_t + (aD)^{-1}(e_1 - bdh_t + bgRC_t)] \tag{A4}$$

Following Sargent (1979) the LHS may be factorised as

$(1-\lambda L)(1-(\lambda D)^{-1}L)$

where λ and $(\lambda D)^{-1}$ are the roots of $(1 - \varphi X + D^{-1}X^2) = 0$ and

$$\varphi = \left[\frac{(1+D)}{D} + \frac{(e_2 + b)}{aD}\right]$$

If a, b and e_2 are positive then $0 < \lambda < 1$. Expanding the unstable root forward, the closed form solution is

$$S_t = \lambda S_{t-1} + qE_t \sum_0^{\infty}(\lambda D)^i Y_{t+i}^s - Y_t^s - (a^{-1}\lambda)E_t \sum_0^{\infty}(\lambda D)^i (e_1 + bgRC)_{t+i}$$

$$- a^{-1}\lambda\upsilon_t + a^{-1}(1-\lambda)E_t \sum_0^{\infty}(\lambda D)^i \upsilon_{t+1} + \Psi \sum(\lambda D)^i bdh_{t+i} \tag{A5}$$

where $q = 1 - \lambda(1 - bc/a)$, $\Psi = \lambda a^{-1}$. The coefficient $q > 0$ if $(1-bc/a) < 1$ the latter inequality holds since b, c and 'a' are all positive.

Long-run solution

The long-run static equilibrium solution (for given constant values of the forcing variables in all future periods) is:

$$S_t = [(1-\lambda)(1-\lambda D)]^{-1} [(1-\lambda z) Y^s_t - \lambda a^{-1} e_{-1} - \lambda (1-D) a^{-1} \upsilon_t + \lambda bda^{-1} h_t - \lambda bga^{-1} RC_t] \tag{A6}$$

where z = (1 - bc/a) < 1

Hence $\frac{\delta S}{\delta Y^s_t}$, $\frac{\delta S}{\delta h_t} > 0$, $\frac{\delta S}{\delta RC_t}$, $\frac{\delta S}{\delta \upsilon_t} < 0$

Thus if the forward model is the true model or if the objective function is for the current period only, the coefficients in the long-run solution have the static equilibrium partial derivatives given above. If the forcing variables are I(1) then we expect both the RE model and backward looking model to yield a cointegrating vector in the above variables.

Costs in adjusting output

If the cost function (A2) has an additional term $(f/2)(\Delta Y^0_t)^2$ then substituting the inventory constraint this becomes $(f/2)(Y^s_t + \Delta S_t)^2$. Therefore only terms in S and Y^s are affected in the optimisation problem. The FOC after tedious algebra yields

$$(\varphi_1 L^{-2} + \varphi_2 L^{-1} + \varphi_3 + \varphi_4 L + \varphi_5 L^2) S_t = \theta_1 Y^s_{t-1} - \theta_2 Y^s_t + \theta_3 Y_{t+1}{}^{s+} \theta_4 Y^s_{t+2} \tag{A7}$$

where

$\varphi_1 = D^2 f$

$\varphi_2 = -D(a + 2f + 2Df)$

$\varphi_3 = (b + e_2 + a(1+D) + 4fD + D^2 f)$

$\varphi_4 = -(a + 2f(1 + D))$

$\varphi_5 = f$

$\theta_1 = f_1$

$\theta_2 = -(a - bc + f(1 + 2D))$

$\theta_3 = D(a + f(2 + D))$

$\theta_4 = - D^2 f$

The LHS contains terms in S_{t+2}, S_{t+1}, S_t, S_{t-1}, S_{t-2} and may be factorised with roots λ_1, λ_2. For λ_1, $\lambda_2 < 1$ the stable roots give rise to terms in S_t, S_{t-1}, S_{t-2} while the unstable roots may be used in the forward expansion.

Appendix II: Data definitions

S_t = logarithm of inventory level (manufacturing sector): £m 1985 prices. Source, *Economic Trends*

Y_t = logarithm of output (manufacturing sector): £m 1985 prices. Source, *Economic Trends.*

G_t = logarithm of capital gearing. Capital gearing is the ratio of debt at market value to the value of the capital stock at historic cost. Debt consists predominantly of lending from banks (in sterling and foreign currency, including lending in the form of sterling and foreign currency bills) plus the Issue Departments holding of commercial bills net of holdings of liquid assets. The market valuation of debenture and loan stock is also included in debt although it is small relative to the aforementioned components. The denominator 'trading assets' is measured at historic cost. [*Source, Bank of England, Company Sector Database.*]

RC_t = logarithm of the stock of computers. The flow variable is taken from *British Business,* No 70, CSO. The series is deflated using the consumer durables price index. A depreciation rate of 0.125 per quarter is applied when cumulating the stock series.

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