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No 42

Manufacturing stocks;
expectations, risk and cointegration

by

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The object of this series is to give a wider circulation to research being undertaken in the Bank and to invite comment upon it; and any comments should be sent to the authors at the address below.

The authors are with the Economics Division of the Bank of England. Thanks for comments on an earlier draft are extended to J S Flemming. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Bank of England.

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1 Introduction

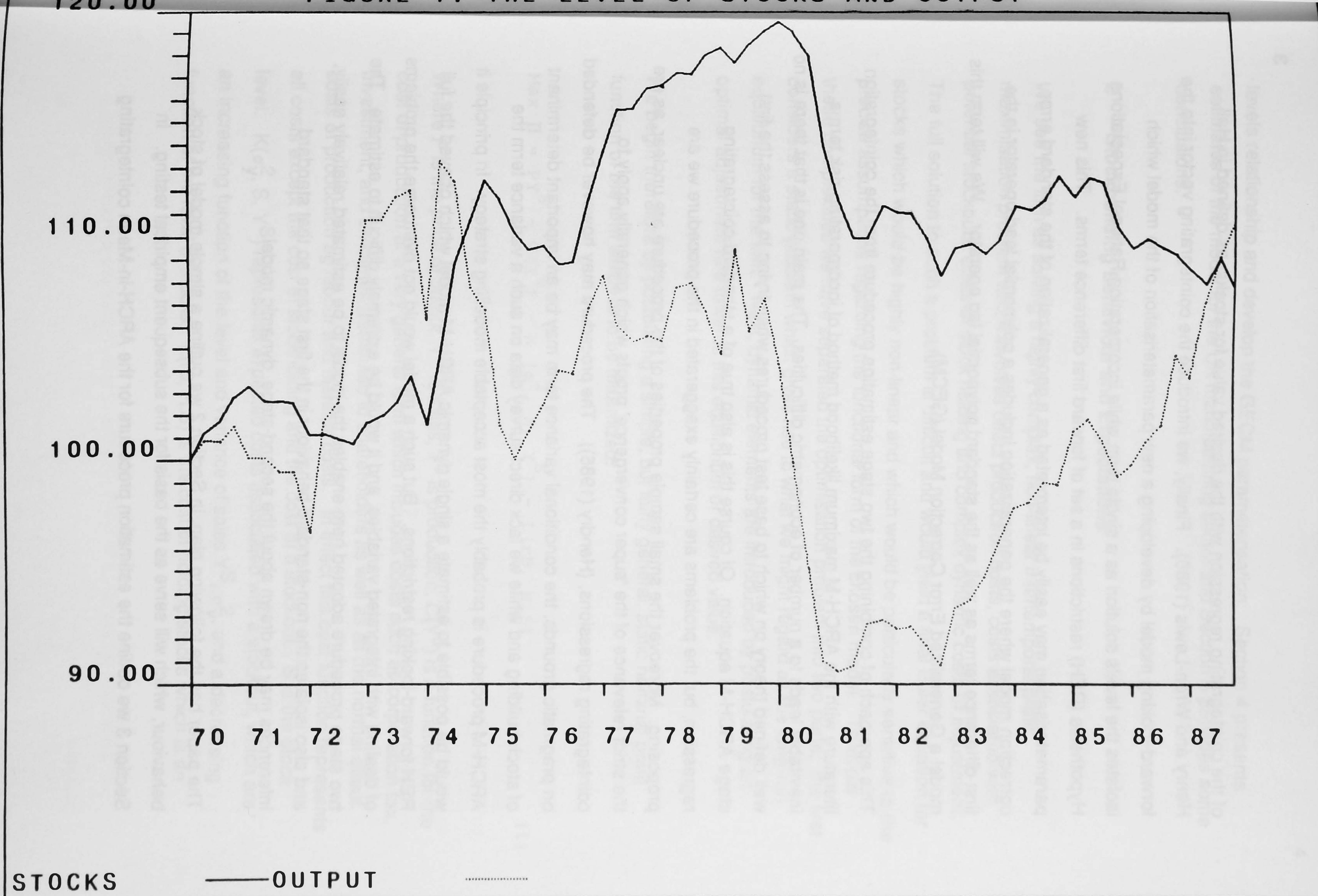
The modelling of firms inventory behaviour has been plagued by structural instability and despite a great deal of research effort over recent years little headway has yet been made in producing a structurally stable model of stock levels. Wallis et al (1987) surveyed the main UK models of inventory behaviour and concluded that 'the tests of predictive failure are particularly powerful when conducted over periods in which data characteristics change, and over half the equations we consider are rejected on the basis of their predictive performance in the early 1980s. This is a surprising result, since the poor performance of their predecessors over this period was a prime motivation for the research that has led to the current specifications. The similarity in turning points in the forecast errors, both for the aggregate stockbuilding equations and for the different categories of stocks, suggests the omission of some factor(s) common to all specifications.' (Wallis et al 1987 p144.) Their analysis was conducted using within-sample stability tests, and more recent data would undoubtedly find an even higher rejection rate. Accordingly, this paper seeks to identify important omitted components or misspecification in the existing models of inventory behaviour in response to the suggestion in the last sentence of the Wallis et al quotation.

We take as our point of departure a number of recent papers on company sector behaviour which have focussed on intertemporal optimisation under rational expectations. The seminal work of Sargent (1978) on the demand for labour has been followed by a number of studies which seek to treat the firms expectations of future variables in an explicit way. This work includes Nickell (1984) and Henry and Wren-Lewis (1984) on the labour market and Hall, Henry and Wren-Lewis (1986) on the determination of stock levels. One common finding amongst these, and many other papers is that of a root in the dynamic process under consideration which is close to unity. This means that the long-run solution for the model is poorly determined and this finding is a cause for some concern. Recently, with the growth of the literature on cointegration (see Engle and Granger (1987), Hall (1986)), this finding takes on an even more serious aspect, since a symptom of non-cointegration is the lack of a well-defined long-run solution. So the finding of a near unit root may be indicative that earlier researchers were in fact working with sets of variables which failed to cointegrate, which would inevitably lead to 'spurious regression' problems in the sense of Granger and Newbold (1974).

A recent paper by Callen and Henry (1989) investigated the cointegration properties of variables most often used in modelling stockbuilding and found that indeed they did not cointegrate and that it was necessary to add a range of financial variables to the model before cointegration was achieved. Two modelling issues remained unresolved however; first, how should a cointegrating vector, once found, be introduced into the forward-looking model? Second, theory suggests that the conditional variance of output is a major determinant of the stock level; the more uncertain we are about the level of output the more stocks need to be held, and so there is a need to find a way to identify a cointegrating vector which includes this uncertainty term. A further issue arises from the recent performance of existing empirical models which is illustrated in figure 1. The figure shows the behaviour of both the level of inventories and output for the manufacturing sector. While the period pre-1980 shows a strong positive correlation between output and the stock level this relationship seems to have broken down subsequently. Since the early 1980s output has recovered and has risen steadily but there has been no discernible rise in stock levels. This steady fall in the stock output ratio has puzzled many commentators and proved a cause of structural instability in most empirical models. Indeed many of the models surveyed in Wallis et al (1987) actually imposed a constant long-run stock output ratio, and so have great difficulty explaining the recent data.

In this paper we attempt to find solutions to these problems, by estimating cointegrating vectors which include terms in the conditional variance of output using a generalisation of the Auto Regressive Conditional Heteroscedasticity in-Mean (ARCH-M) process proposed by Engle, Lilien and Robins (1987). We then identify the implied equilibrium

FIGURE 1: THE LEVEL OF STOCKS AND OUTPUT



of the cointegrating regression with the desired value for stocks, as defined in Hall, Henry and Wren-Lewis (1986). Finally, we introduce the cointegrating vector into the forward-looking model by developing a new parameterisation of the model which isolates the levels solution as a single term while incorporating Rational Expectations Hypothesis (REH) restrictions in a set of forward first difference terms. This new parameterisation may easily be interpreted as a generalization of the standard error correction model where the generalization includes a polynomial lead operator in the first difference terms as well as the standard polynomial lag operator. We will term this model a Generalized Error Correction Model (GECM).

This approach of combining the two stage estimation procedure from the cointegration literature with the ARCH-M maximum likelihood method of incorporating risk terms inevitably leads to a number of econometric difficulties. The main one is that there is no well defined theory on which to base test procedures when trying to assess the first stage ARCH-M equation. Of course this is also true of a standard cointegrating regression, but the problems are certainly exaggerated in the procedure we are proposing. Moreover the small sample properties of the procedure are unclear, as are the strict relevance of the 'super convergence' proofs which generally apply to cointegrating regressions, (Hendry (1986)). The procedure may however be defended on pragmatic grounds, the conditional variance term may be an important determinant of stock building and while we lack direct survey data on such a variance term the ARCH-M procedure is probably the most acceptable modelling strategy. In principle it would be possible to estimate a single dynamic ARCH-M model which obeyed the full REH forward-looking restrictions. But such a model would not overcome the problems of dealing with integrated variables, and it would be extremely difficult to estimate. The two stage procedure adopted here enables the model to be estimated relatively easily, and also isolates the non-standard behaviour in the first stage so that standard inferences may be drawn about the second-stage, dynamic model.

The paper has the following plan, in Section 2 we outline a simple model of stock behaviour, which will serve as the basis for the subsequent empirical testing. In Section 3 we outline the estimation procedure for the ARCH-in-Mean cointegrating levels relationship and develop the GECM parameterisation. Section 4 presents estimates of the model for the UK manufacturing sector and Section 5 provides some conclusions.

2 Stockbuilding Behaviour and Expectations

We see the firm as solving a highly complex, non-linear intertemporal profit maximisation problem which involves direct production costs, costs associated with potential stockouts, costs of stock holding and complex dynamic costs of adjustment. The full solution to such a problem is difficult and would yield a final Euler equation for stocks which would be highly non-linear and which would be particularly sensitive to the precise assumptions regarding the functional form of the chosen model. To avoid these complexities we have separated the optimisation problem into two parts; the first considers a static problem which gives rise to what we might regard as the long-run equilibrium for the firm. Next, dynamic elements are introduced by postulating an optimal adjustment rule towards this equilibrium or desired value.

We begin therefore by postulating a simple, but fairly general, static nominal profit function of the firm which is to be maximized:

$$\text{Max } \Pi = PY^S - C(Y^0) - K(\sigma_Y^2, S, Y^S) - L(rB, H.S) \qquad (1)$$

Where P is the price of output, Y^S is sales, Y^0 is production, $C(Y^0)$ is a function for the cost of production of a very general form which includes costs such as corporation tax, investment, and dividends distributed to shareholders as well as all the normal direct costs of production eg labour and material costs. In effect this cost function represents all costs except the cost of stockholding and the cost of a stockout. S is the stock level. $K(\sigma_y^2, S, Y^S)$ represents expected costs associated with a stockout, which are an increasing function of the level and variance of sales Y^S, σ_y^2, and a decreasing function of the level of stocks. L(rB, H.S) is the cost of holding stocks which is an increasing function of the cost of net borrowing rB and the direct cost of holding stocks H.S, where H is the cost of holding a unit of stocks.

Now we have two identities which link the variables of the system, the first is the net borrowing identity which links the change in borrowing with the change in direct net revenue:

$$\Delta B_t = C(Y_t^0) - P Y_t^S + rB_{t-1} \tag{2}$$

$$\Rightarrow B_t = B_{t-1}(1 + r_t) + C(Y_t^0) - P Y_t^S$$

The second is the stock building identity which defines stockbuilding as the difference between production and sales:

$$\Delta S_t = Y_t^0 - Y_t^S \tag{3}$$

$$\Rightarrow S_t = S_{t-1} + Y_t^0 - Y_t^S$$

The consequence of these two identities is that, in this simple model, the firm has effectively only one choice variable when its sales are set exogenously by demand. In effect if the firm chooses the level of either production, stocks or net borrowing the other two are determined by identity. As we are focussing here on stocks the maximization is done with respect to this variable. This choice is, however, arbitrary and unimportant.

Thus re-write (3) as

$$Y^0 = S_t - S_{t-1} + Y^S \tag{4}$$

Then substitute (3) and (2) into (1) to give the unconstrained profit maximization problem

$$\text{Max } \Pi = PY_t^S - C(S_t - S_{t-1} + Y_t^S) - K(\sigma_Y^2, S_t, Y_t^S)$$

$$- L(r_t(B_{t-1}(1+r)_t + C(S_t - S_{t-1} + Y_t^S) - PY_t^S), H_t . S_t) \qquad (5)$$

This function may then be maximized with respect to S_t by satisfying the following first order condition.

$$\frac{\partial C}{\partial S_t} = - C'(S_t - S_{t-1} + Y_t^S) - K'(\sigma_Y^2, S_t, Y_t^S)$$

$$- L_1 C'(S_t - S_{t-1} + Y_t^S) - L_2 H_t = 0 \qquad (6)$$

In static equilibrium (where $S_t = S_{t-1}$) this yields a general solution for S_t of the form

$$S_t = f(\underset{+}{Y^S}, \underset{+}{\sigma_Y^2}, \underset{-}{B_{t-1}}, \underset{-}{r}, \underset{-}{H}) \qquad (7)$$

Where the signs under each variable suggest the signs of its comparative static effect.

Note that when the firm is unconstrained in its net borrowing position there is a negative relationship between stocks, B_{-1}, and r the rate of interest on net borrowing. A number of recent studies (eg Ireland and Wren-Lewis (1988) or Melliss (1986)) have postulated a positive effect on the grounds that the firm is constrained in its net borrowing position. This proposition may also be illustrated in our simple model. If net borrowing is constrained to be $B < B^*$ where B^* is the solution to (2) given S^*, then the constrained stock level equation may be derived from (2) and (4) as

$$S_t = S_{t-1} - Y_t^S + C^{-1}(B_t^* - B_{t-1}(1+r_t) + PY_t^S) \qquad (8)$$

where $C^{-1\prime} > 0$

so that $\frac{\partial S}{\partial B} > 0$

That is to say when the firm is constrained in its net borrowing an increase in borrowing is reflected in an increase in its ability to finance and hold stocks. In general therefore if the firm is unconstrained in its borrowing the equilibrium is defined by (7) while if it is constrained it is defined by (8). In a statistical sense (8) is nested within (7) so estimation will be based on (7) allowing the parameter estimates to constitute a test between the constrained and the unconstrained model. However, the approach taken here is that the theory is of a representative firm, and thus motivates the form of equations used on the later empirical section which concentrates on the behaviour of the entire manufacturing sector. But these aggregate equations are not derived by explicit aggregation so do not, eg, identify the proportions of individual firms which may face borrowing constraints.

It is convenient at this stage in the development of the model to mention two further problems: the derivation of equations for real stocks and the problem of sales as a determinant of stocks. On the first point, the model derived in equation (1)-(7) is explicitly nominal as it is based on optimising a constrained nominal profits function. However, it can be assumed that equation (7), which defines the static equilibrium for nominal stock levels is homogenous of degree one in prices. The function may thus be redefined to express the equilibrium or target value for real stocks. What is excluded by this assumption is the proposition that the price level (or price inflation) may affect real stocks. It may be argued that the level of prices or changes in prices can affect stock holding, due to price misspecifications by firms and/or non-neutralities in the corporation tax system. For simplicity these possibilities are not explored here. One exception in this however arises with the cost of stockholding variable we use, which allows for differential tax treatment on stock appreciation. (See Section 4 and the data definitions.) The second point is an important empirical issue, but which can conveniently be confronted in this section. It is that the theory developed so far distinguishes between sales and output. In the empirical section (section 4), stockbuilding equations are estimated which depend upon expected output (and its conditional variance). The reasons for using output are pragmatic ones. The studies alluded to in the Introduction typically use output as the scale variable. Also no data on manufacturing sales are available, so a proxy for the variable has to be used. The practice we adopt is to proxy sales by output. One implication is that the use of output induces measurement error into the structural equation which is based on sales. We accept this state of affairs, essentially as there is no alternative which avoids introducing measurement errors of some kind in the stocks equation. Sales data might be recovered from the identity ($\Delta S = Y - Y^S$ where Y is output, Y^S is sales) above by 'stock adjusting' output. This is implied in the CSO's derivation of 'stock-adjusted' production series (see CSO 1976 'The measurement of industrial production'). But the published data on production and output refer to net output of the sector, meant to approximate value added, by subtracting factor inputs. It is clear that sales data cannot be reconstructed from net output, since information on sectoral inputs is not available. Nevertheless, some model builders use a sales proxy, by stock adjusting net output (see eg H M Treasury (1982), p 155), although this also clearly induces measurement error into their stocks equation. For this reason we have elected not to do this, and in all the empirical work reported here, sales are replaced by an index of manufacturing production.

Having determined the static equilibrium we turn now to introducing dynamics into the model. We define the appropriate solution to (7) or (8) ,in real terms, to be S^*, the desired stock level, and we postulate a dynamic cost minimization problem which imposes costs of deviating from S^* and also costs of adjustment in terms of both the first and second derivative.

Thus

$$\min_{S_t} E\left[\sum_{t=1}^{\infty} \{a\,(S_t - S_t^*)^2 + b\,(S_t - S_{t-1})^2 + c\,(\Delta S_t - \Delta S_{t-1})^2\} \mid \Omega_t\right] \tag{9}$$

That is we minimize the undiscounted expectation of the future stream of costs conditional on the current information set Ω_t, setting the discount factor to unity for simplicity.

Invoking the certainty equivalence theorem and the rational expectations hypothesis, so that expectations may be replaced by future realizations, the appropriate Euler condition for this problem may then be stated as

$$- a S_t^* + (a + 2b + 6c) S_t - (b + 4c) S_{t+1} + c S_{t+2}$$

$$- (b + 4c) S_{t-1} + c S_{t-2} = 0 \qquad (10)$$

or

$$B(L) S_t = a S_t^*$$

where

$$B(L) = (cL^{-2} - (b + 4c) L^{-1} + (a + 2b + 6c) - (b + 4c) L + cL^2) \qquad (11)$$

and a suitable transversality condition is

$$\lim_{T \to \infty} S_T = \bar{S} \qquad (12)$$

A solution for (10) may be defined by factorising

$$B(L) = (1 - \lambda_1 L - \lambda_2 L^2)(1 - \lambda_1 L^{-1} - \lambda_2 L^{-2})$$

where

$$c = -\lambda_2$$

$$(b + 4c) = \lambda_1 - \lambda_2 \lambda_1 \qquad (12a)$$

$$(a + 2b + 6c) = (1 + \lambda_1^2 + \lambda_2^2)$$

and so we may write (10) as

$$S_t (1 - \lambda_1 L - \lambda_2 L^2) = \frac{a S_t^*}{1 - \lambda_1 L^{-1} - \lambda_2 L^{-2}} \qquad (13)$$

and further rearranging (13) yields

$$S_t = \lambda_1 S_{t-1} + \lambda_2 S_{t-2} + a \sum_{i=0}^{\infty} \gamma_i S_{t+i}^* \qquad (14)$$

The γ_i are implied by a set of non linear restrictions on λ_1 and λ_2, so the model has essentially only two free parameters (λ_1, and λ_2) and the determinants of S^*. The restrictions on the γ_i may be obtained by applying the method of undetermined coefficients (Muth (1961), Blanchard (1979)).

3 Estimation Methods

Introducing the Variance

Procedures for estimating a model such as (14) under rational expectations are well known when S^* is a function of observed variables (see Pesaran (1987) for a survey). In our case, however, one of the main determinants of S^* is the expected conditional variance of Y which is not directly observed. The conventional assumption is either that σ_y^2 is constant or that it varies solely with the level of output. This assumption is far from satisfactory, σ_y^2 is unlikely to be constant given the recession in manufacturing output in the early 1980 which was largely unexpected. It is also unlikely to be positively related to output, indeed the fall in output during the recession might well be expected to increase uncertainty rather than reduce it. So a more sophisticated measure of uncertainty would seem to be necessary. One approach would be to find some direct measure of expectations, based perhaps on survey data (eg Pesaran (1985) or Wren-Lewis (1986)). Such data however is difficult to obtain when we are dealing with higher moments of the probability distribution such as the variance. The recent work by Pagan and Ullah (1988) on constructed measures of uncertainty strongly suggests that the ARCH-M model offers the most direct method for obtaining consistent estimates of both parameters and their estimated variances. The technique we use to incorporate the GARCH-M model follows the work of Engle, Lilien and Robins (1987) and Bollerslev (1986).

Engle Lilien and Robins (1987), suggest an extension of Engle's (1982) Auto-Regressive Conditional Heteroscedasticity (ARCH) model to allow the conditional first moment of a time series to become a function of the conditional second moment which itself follows an ARCH process. Thus, suppose X_t is a vector of variables affecting S_t

$$S_t = \alpha' X_t + \delta h_t + \epsilon_t \tag{15}$$

where

$$h_t = E(\epsilon_t^2 \mid \Omega_{t-1}) = \gamma_0 + \sum_{i=1}^{N} \gamma_i \epsilon_{t-i}^2 + \phi. \tag{16}$$

The likelihood function for this model is then proportional to

$$\log(L) = \sum_{t=1}^{T} (-\log(h_t) - \epsilon_t^2/h_t)$$

and so the model (15), (16) may be estimated jointly by maximum likelihood procedures.

Two further extensions are made to the model given by (15), (16). The first follows Bollerslev (1986) in generalizing the ARCH process to include lagged terms in the conditional variance in the ARCH equation (16) which becomes

$$h_t = \sum_{i=1}^{P} \beta h_{t-i} + \sum_{i=1}^{N} \gamma_i \epsilon_{t-i}^2 + \phi \tag{18}$$

and the model is then termed a Generalized ARCH in mean process or GARCH-M.

The second generalizes the system to be a two equation system (following Bollerslev, Engle and Wooldridge (1985)) by including an equation for output (Y). This allows us to enter the conditional variance of the output equation into the stocks equation as an uncertainty term. So the model has the general form

$$S_t = \alpha_1' X_t + \delta\, h_{22t} + \epsilon_t \tag{19}$$

$$Y_t = \alpha_2' Z_t + \omega_t \tag{20}$$

Where Z is a set of variables determining Y. [In the application below we use lagged Y's.]

The conditional covariance matrix H is,

$$H_t = \begin{bmatrix} h_{11t} & h_{12t} \\ h_{12t} & h_{22t} \end{bmatrix} = \beta(L) \begin{bmatrix} h_{11t} & h_{12t} \\ h_{12t} & h_{22t} \end{bmatrix} + \gamma(L) \begin{bmatrix} \epsilon_t \epsilon_t & \epsilon_t \omega_t \\ \epsilon_t \omega_t & \omega_t \omega_t \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{bmatrix} \tag{21}$$

and the likelihood function for this system (conditional on the initial conditions) is proportional to

$$\log(L) = \sum_{t=1}^{T} \left(-\log(|H_t|) - w_t' H_t^{-1} w_t\right) \tag{22}$$

where $w_t = (\epsilon_t, \omega_t)$

Maximizing this function with respect to the parameters α_1, α_2, δ, $\beta(L)$, $\gamma(L)$, and the matrix ϕ then produces a simultaneous estimate of the complete model.

The Generalized Error Correction Model

The preceding discussion considers the way we might estimate an equation for S* which includes a term in the conditional variance of output. The question remains as to how this term can be incorporated in the dynamic model of stock behaviour, equation

(14). The association between the equation which determines S* and a cointegrating relationship in the sense of Engle and Granger (1987) should by this point be clear. By following the Engle and Granger two step estimation procedure the residuals from the S* equation can be used in a dynamic model in first differences to estimate a standard error correction model. Given a well determined equation for all the expected variables in equation (14) the expectations for the current information set may be substituted out and the model reparameterised in the form of a standard ECM incorporating the full non-linear restrictions imposed by the REH assumption and the model following the analysis of Campbell and Shiller (1988, 1987). In fact we do not have a good idea of the expectation formation mechanism and so we are extremely reluctant to put the model into the 'decision rule' form as this would mix the forward-backward restriction of (14) with the less plausible models of expectation formation effectively losing those restrictions. The aim therefore is to find another parameterisation of (14) which will not lose these restrictions but which also allows the explicit use of the cointegrating regression for S*.

The alternative begins by reparameterising (14) as:

$$S_t = \lambda_1 S_{t-1} + \lambda_2 S_{t-2} + a \sum_{i=1}^{\infty} d_i \Delta S^*_{t+i} + a e S^*_t \quad (24)$$

where $e = \dfrac{1}{1 - \lambda_1 - \lambda_2}$

and $d_i = e - \gamma_0 \ldots - \gamma_{i-1}$

Then by further reparameterising we get

$$\Delta S_t = (\lambda_1 - 1 + \lambda_2 + ae) S_{t-1} - \lambda_2 \Delta S_{t-1} + a \sum_{i=1}^{\infty} d_i \Delta S^*_{t+i}$$

$$+ ae \Delta S^*_t + ae (S^*_{t-1} - S_{t-1}) \quad (25)$$

Now the first term in (25) is

$$(\lambda_1 - 1 + \lambda_2 + ae) S_{t-1} = 0$$

as

$$a = (1 - \lambda_1 - \lambda_2)^2$$

and

$$e = 1/(1 - \lambda_1 - \lambda_2).$$

So we arrive at a parameterisation which is similar to the standard ECM model except that it has lead terms in the differences of the variables as well as lags.

$$\Delta S_t = - \lambda_2 \Delta S_{t-1} + a \sum_{i=1}^{\infty} d_i \Delta S^*_{t+i} + ae\Delta S^*_t + ae (S^*_{t-1} - S_{t-1}) \quad (26)$$

The final term is the error correction one and we may use the residuals from the cointegrating regression here in exactly the same way as in the Engle-Granger procedure. The future terms in ΔS^* may be constructed using the fitted values from the cointegrating regression, although as these will be subject to an REH error these variables are instrumented in estimation.

Hence to estimate the model we first investigate cointegrating equations for the level of stocks and then to build the residual and forecast from this equation into our GECM model (26) which incorporates the full set of non linear REH restrictions.

4 Empirical Results

To implement the modelling strategy outlined in the previous section, the first stage is to consider an equation for the long-run stock level based on (7) using the GARCH-M procedure. The model is specified in the natural level of the variables (rather than logs), where Y is manufacturing output, S is manufacturing stocks, B is net liquidity, r is the interest rate on lending to the financial sector and H is based on the Treasury's measure of the cost of stockholding (Kelly (1984), Melliss (1986)). We also introduce a term in retained earnings which would affect the financing decision of the firm but which, for simplicity was omitted from the analytical exposition, and this variable is referred to as RE. [The full definitions of the data used is given in detail in the data appendix.]

The model is estimated by a two stage modelling procedure where the first stage should determine the long-run or desired stock level. If this is treated as a cointegration exercise it is necessary to establish that the main variables are in fact non-stationary and that they potentially form a cointegrating set. This is investigated in the following table which reports the Dickey-Fuller (DF) and Augmented Dickey Fuller (ADF) statistics for both the levels and first differences of the variables used.

	Level		Difference	
	DF	ADF	DF	ADF
S	-1.7	-1.46	-5.6	-3.5
H	-1.99	-1.2	-8.4	-4.4
r	-2.23	-2.16	-7.8	-4.7
Y	-2.06	-2.09	-8.9	-4.2
RE	-0.18	0.77	-12.8	-4.4
B	0.42	-0.6	-6.8	-3.65

The critical value for the DF and ADF statistic which would reject the hypothesis of non stationarity is 2.8. This hypothesis cannot be rejected for the level of any of the variables but in all cases it can be rejected for the first difference of the variable. Thus it may be concluded that all the variables need to be differed once to induce stationarity, that is they are all integrated of degree one (I(1)). As the conditions for cointegrating are met we can now proceed to estimate the long-run relationship.

The exposition of the GARCH model in (21) is for a very general form of the model. It is common practice, however, to impose a number of simplifying assumptions on the structure of the GARCH process. In the present case, these are to assume that both B(L) and γ(L) are first order polynomials and that they are scalars rather than matrices.

The model for output is postulated to be a simple fourth order AR model, though the estimated parameters of this equation are not quoted as they are of no direct interest from the point of view of this paper.

The estimated equation for the desired stock level is then

(t statistics in parenthesis)

$$S_t = \underset{(7.8)}{20374} - \underset{(0.3)}{0.08}\, Y_t + \underset{(5.6)}{0.007}\, \sigma^2_{Yt} - \underset{(6.7)}{0.22}\, B_t + \underset{(8.5)}{10.7} r_t - \underset{(8.8)}{64.2}\, H_t + \underset{(6.9)}{0.5}\, RE_t$$

Garch Parameters

β	0.8	(31.4)
γ	0.12	(5.7)
ϕ_{11}	147.9	(4.7)[1]
ϕ_{12}	10.4	(0.5)
ϕ_{22}	122.9	(4.2)

Stock equation diagnostics

SE = 718, BJ(2) = 3.6, ADF = -3.7

Correlogram: 0.65 0.52 0.44 0.29 0.18 0.20 0.19 0.08

Where,

SE = standard error

BJ = Berra Jarque Test for normality distributed as $\chi^2(2)$

ADF = Augmented Dickey-Fuller Test

This equation can be interpreted as a cointegrating regression in the sense of Engle and Granger (1987), although this raises a number of difficulties when drawing inferences from the model. It is now well known that the assymptotic distribution of the parameters of a cointegrating regression is non standard and that conventional t tests are not appropriate. This point certainly applies to the model above as the elaboration provided by the GARCH-M structure would not cure this problem. The reported ADF statistic, however, strongly suggests that the model cointegrates according to the usual criteria for testing for cointegration. But existing Monte-Carlo studies have not been carried out on the basis of a GARCH-M model and so we have little firm guidance as to the correct critical values to use here. Engle (1987) stresses the difficulty and uncertainty of formal testing in such a model.

Despite these qualifications the equation seems to perform well, all coefficients are of an appropriate sign and magnitude and the parameter on the variance term is significantly different from zero (bearing in mind the qualifications made above) suggesting that the variance effect is important. The GARCH parameters suggest that time variation in the covariance matrix is important, B, γ are both significant, with an estimated fairly long memory. β and γ sum to 0.92 which is close to unity suggesting that the model is integrated in variance (Engle 1987).

This first stage regression may now be implanted in a dynamic GECM model as described by equation (26). Only two parameters need to be estimated for this equation, and in the present case the following estimates were obtained. (Quarterly seasonal dummies were also included in the model.)

λ_1 = 1.22 (9.01)

λ_2= -0.38 (3.2)

SE = 262.1 DW = 2.3 BP(1) = 1.9 BP(4) = 3.5 BP(6) = 7.8 BP(12) = 18.7

BP2(1) = 3.4 BP2(4) = 6.8 BP2(6) = 7.7 BP2(12) = 9.6

DW = Durbin Watson statistic.

BP = Box Pierce test for a random correlogram.

BP2 = Box Pierce test for a random correlogram for the squared residuals.

To give some idea of the validity of the REH restriction another version of the model was estimated which did not incorporate the restriction. This had a poorly determined forward convolution and the standard error of the unrestricted model actually rose to

335.6. A quasi likelihood ratio test of the restriction was $x^2(7) = 4.8$ (see Gallant and Jorgenson (1979)). So the REH restrictions would seem to be an absolute improvement.

In their survey of UK models of inventory behaviour Wallis et al (1987) found that structural instability plagued all the models examined. In the general spirit of encompassing, if the present model is correct in claiming that the level of output is almost completely irrelevant to stock behaviour and that the true model should include only the variance of output, then we would expect the earlier models to fail to capture the movement in stock levels over the eighties as stocks and output have diverged. The structural instability of existing models has already been noted, but there is a need to present some evidence of the structural stability of our model over this period before deciding whether it is an improvement. But most of the conventional stability tests are not properly defined for models estimated using instrumental variable procedures. So by way of investigating the parameter stability of the present model the dynamic model was estimated recursively over the last 24 quarters of the data, from 1980Q2-1986Q2. The resulting time series of the parameter estimate for λ_1 and λ_2 are shown in figures 2A and 2B, and by most standards these parameters are remarkably stable. A more formal test is provided by a version of the Salkever (1976) procedure which uses additional dummy variables, and tests for the joint significance of the additional dummy variable using a quasi likelihood ratio test (Gallant and Jorgenson 1979). This test for structural stability over the last 8 quarters of the data set gave a test statistic of 8.71 and as the test is distributed as a $x^2(8)$ with a 5% critical value of 15.5, this is well within the region for accepting the hypothesis of structural stability.

A further question is the size of the root of the dynamic equation. This matter was raised in the introduction where it was noted that many models had a near unit root. In the present case the model has a root of 0.84 which is well away from unity. Furthermore it is obvious from the recursive parameter estimates that this is not a chance calculation due to a specific data period but that the root is well away from unity regardless of the estimation period. This is further informal evidence that the levels equation does appear to constitute a true cointegrating vector.

FIGURE 2A:RECURSIVE 2SLS

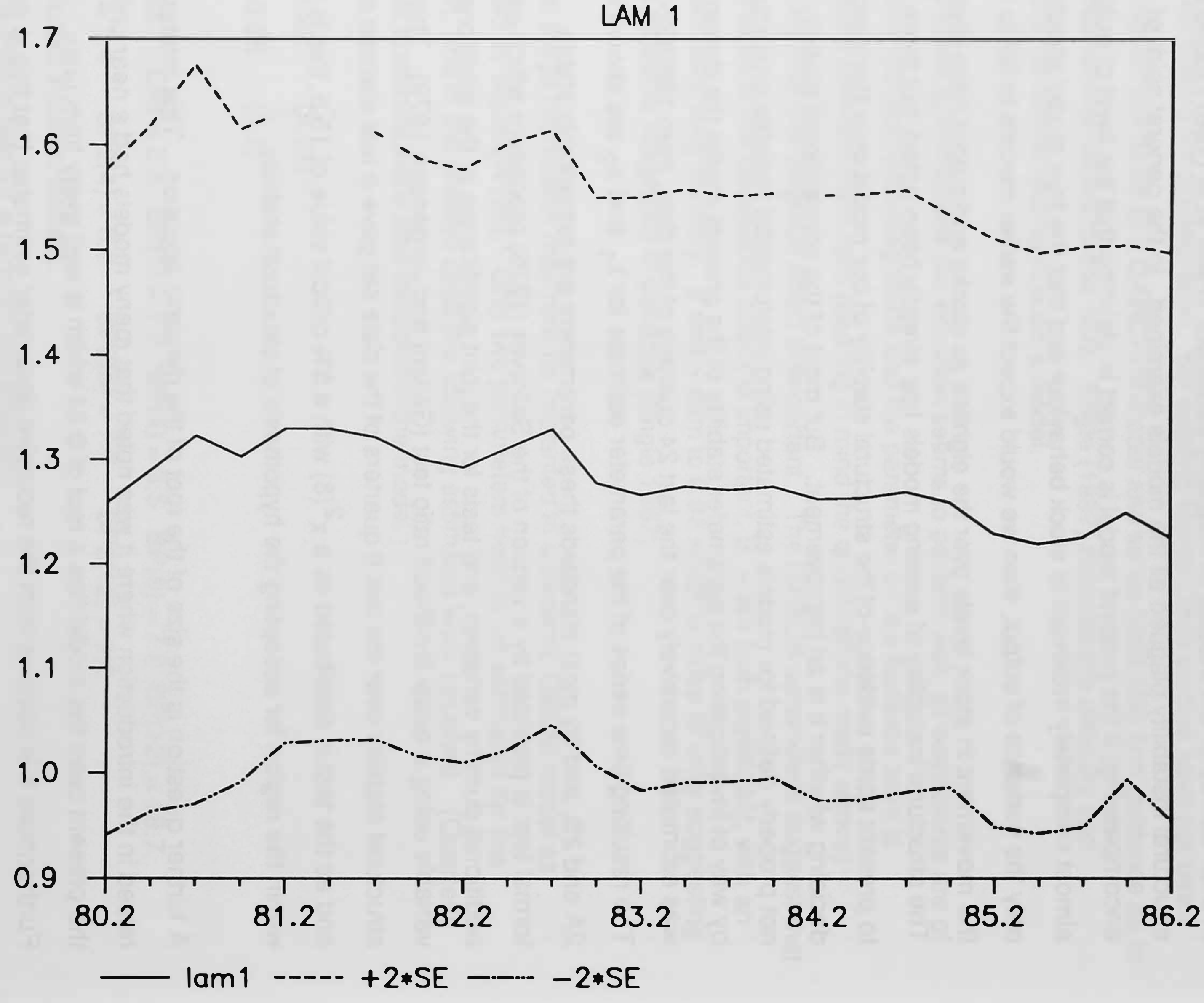


FIGURE 2B:RECURSIVE 2SLS

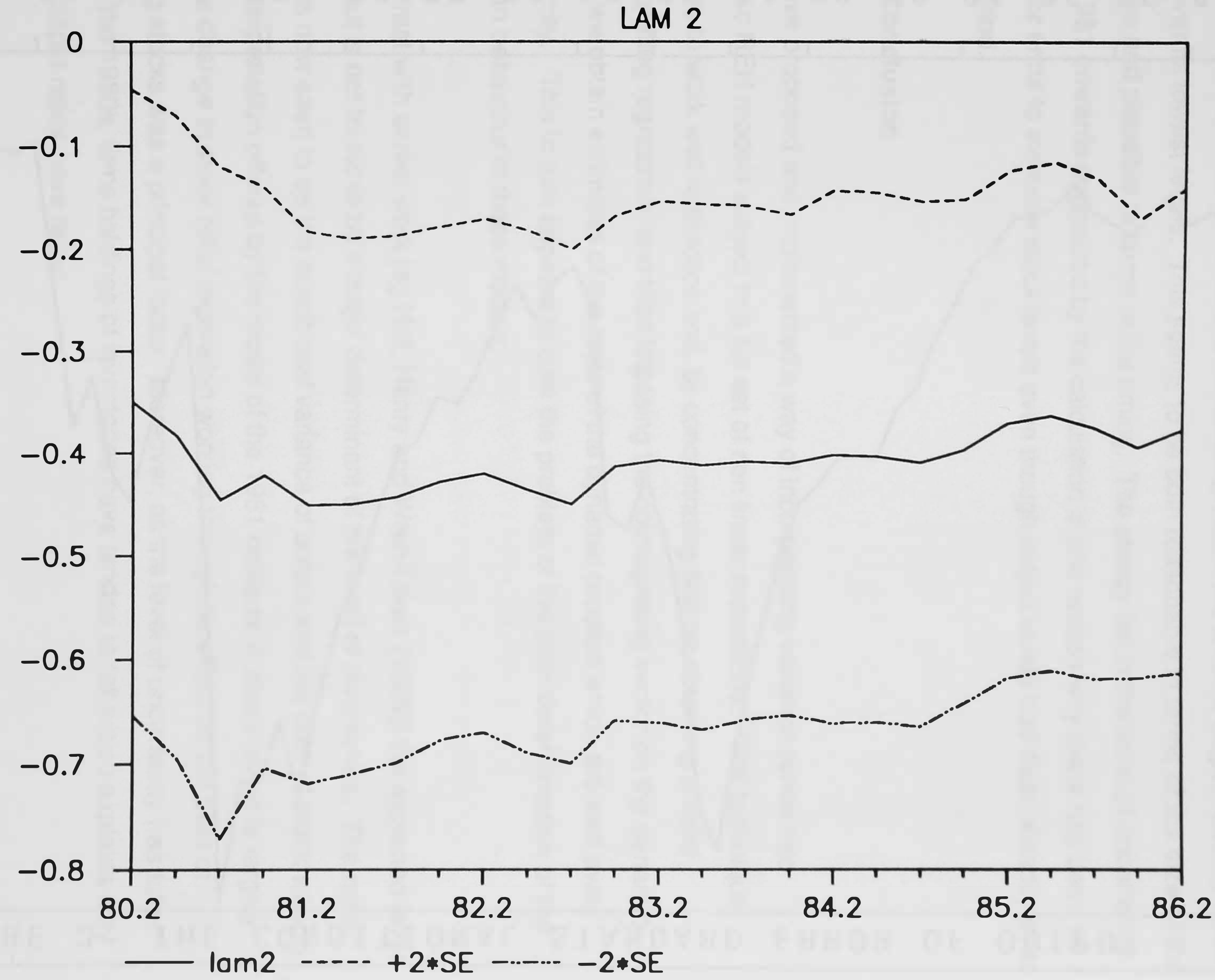
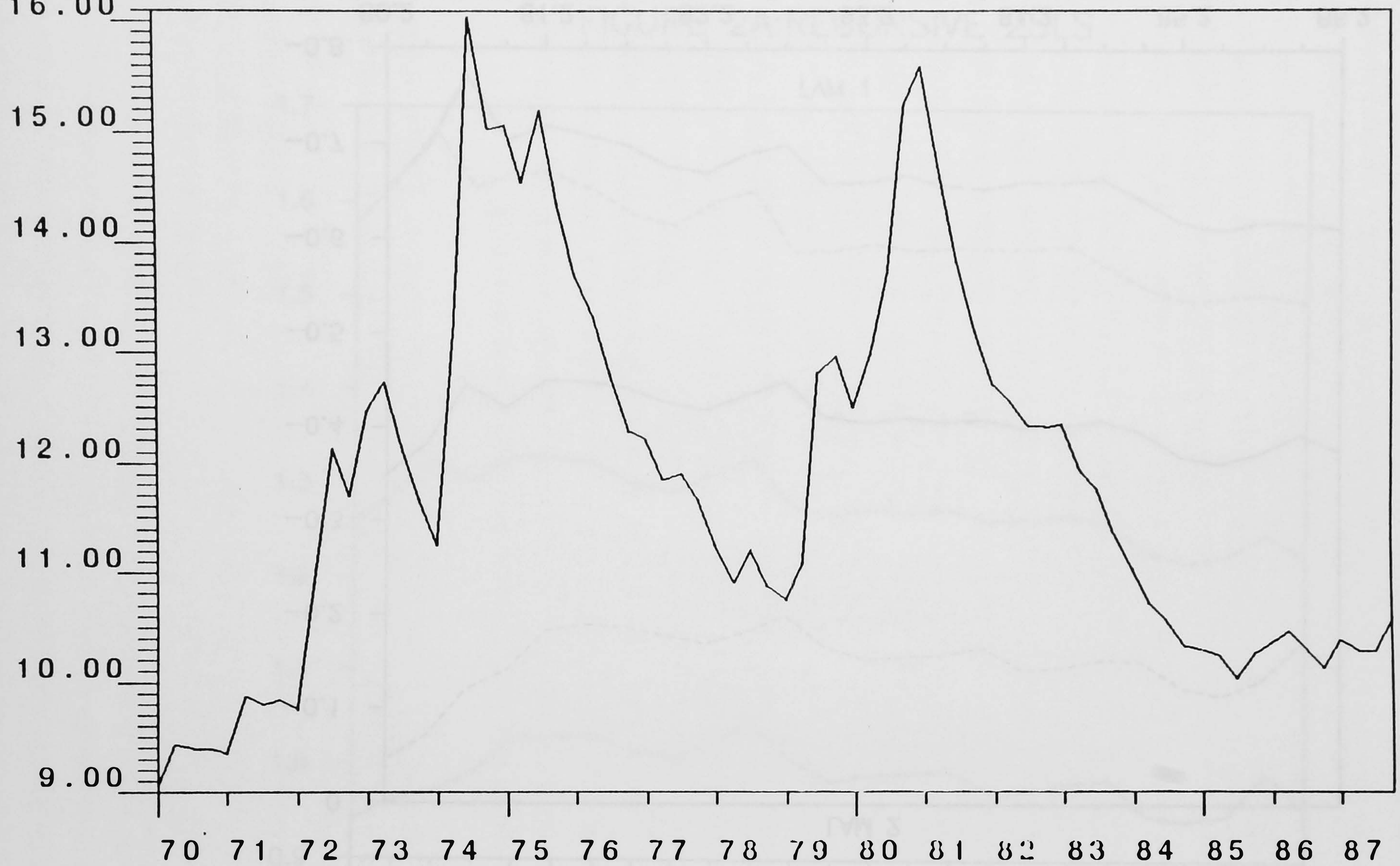


FIGURE 3: THE CONDITIONAL STANDARD ERROR OF OUTPUT



Finally figure 3 provides information on the estimated conditional standard error of output used in the stocks equation. This shows two periods of sharp increases in uncertainty, from 1974-75 following the oil price rise, and in 1980-81 following the recession. In both cases the level of uncertainty about the level of output rose by about 50% over its lowest levels. This seems to be both reasonable in terms of the size of the increase and plausible in terms of the timing. The steady fall in the level of uncertainty from 1981 onwards suggested by the calculation is one reason why there has been no need for firms to increase stock levels even though output levels had risen steadily over this period.

5 Conclusion

We have proposed and implemented a way of incorporating variance terms into dynamic REH models subject to a full set of non linear restrictions. The technique appears to work well in practice and, by concentrating first on obtaining a valid cointegrating regression, and then imposing the cointegrating vector on the dynamic model we obtain estimates of the roots of the dynamic process which are well away from unity. This in turn appears to cure the problem of the poor determination of the long-run behaviour in these models.

In contrast with earlier work [eg Hall, Henry and Wren-Lewis (1986)] the expected level of output is not found to be a major determinant of the level of inventories. The main effect is now seen to be the conditional variance of output and the cost of stockholding. The interpretation offered by the model of the 1981 collapse in stock levels is largely that the change in stock relief legislation working through its effect on the cost of holding stocks was a principal factor. Moreover, as the level of uncertainty has fallen during the 1980s, firms holdings of inventories have tended to fall,which explains why stock output ratios have fallen.

Notes

1 The Garch equation must produce an estimate of the covariance matrix which is positive semi definite, in a general formulation this is not necessarily the case. The restricted terms B and γ obey the restriction as long as they are positive, the constants may not obey this. As a result we actually use the following parameterisation as our constant matrix,

$$\begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{bmatrix}' \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{bmatrix}$$

Data Definitions

r - 3-month inter-bank rate [Financial Statistics Table 13.15.Code:AMIJ].

H - defined as,

H = (PS/PEF)x[((1-T/100))((BR/100+0.02) - Q)(1+((1-ZTWO)*(T/100))/(1-(T/100)))+(1-ZONE)(T/100)*Q/(1-(T/100))].

where PS - stock price deflator. Defined as BV/S, where BV is the book value of stocks (Blue Book, Table 14.1).

PEF - price deflator for total final expenditure defined as DIAB/DIAU where DIAB is total final expenditure at current market prices (Economic Trends, Table 7) and DIAU is total final expenditure at constant market prices [Economic Trends, Table 3].

BR - clearing banks' base rate (Financial Statistics, Table 13.15 Code:AMIH).

Q - (PS(+2)-PS(-2))/PS(-2).

T - Corporate tax rate.

ZONE - dummy variable to capture tax relief on nominal stock appreciation.

ZTWO- dummy variable to capture tax relief on physical increase in stocks.

S - stock level: manufacturer's work in progress and finished goods, 1980 prices [Blue Book, Table 14.3].

Y - manufacturing production [Economic Trends, Table 16 Code:DVIS].

RE - Industrial and commercial companies undistributed income adjusted for net unremitted profits [calculated from CSO printout reference DB14].

B - Real net liquidity. Calculated as gross liquid assets (AIEL) minus bank borrowing (AIEM) by industrial and commercial companies [Financial Statistics Table 8.4] deflated by the GDP deflator (Economic Trends, Table 2 Code:DJCM)].

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