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Measuring the risk of financial institutions' portfolios: some suggestions for alternative techniques using stock prices

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CONTENTS

MEASURING THE RISK OF FINANCIAL INSTITUTIONS' PORTFOLIOS: SOME SUGGESTIONS FOR ALTERNATIVE TECHNIQUES USING STOCK PRICES

This paper uses a model of share prices with time varying risk premia to analyse market perceptions of volatility. Measuring market perceptions of risk may help assess the volatility of financial intermediaries' capital and the chances of insolvency and could thereby be useful in regulation of financial markets.

Introduction

Financial intermediaries in many countries are subject to unique forms of regulation over the structure of their balance sheets. In the UK regulation of banks has involved the supervisor, the Bank of England, issuing guidelines on the adequacy of capital funds and on the risks of large and connected exposures to particular borrowers. The Financial Services Act will result in new forms of regulation for a wide range of non-bank financial intermediaries. Analysis of the risks of portfolios held by market makers, by securities dealers and other intermediaries who take open positions will form a key part of the job of the new teams of regulators. Draft guidelines on minimum levels of capital, or maximum leverage, are already being produced by these new regulatory bodies.

Non-financial corporations are not subject to the type of regulation which applies to many financial institutions. This raises the question as to what is special about financial firms. There is a large and rapidly growing literature on this topic. (See Marquand (1987), Chant (1987) and Goodhart (1987)) for good reviews of the issues involved.) What is often argued is that information problems and externalities, whilst not unique to the financial sector, are sufficiently more serious there to warrant special supervision.

Many of the information problems arise from the difficulty which individual customers of a financial institution may have in evaluating the risk involved in a transaction. This can give rise to incentives for financial intermediaries to take more risks than customers would choose. It has long been argued that such information problems also create an environment in which bank runs may occur (but for a counter view see Kaufman (1987)).

The externalities argument starts from the observation that financial institutions are heavily interlinked. The failure of one institution will have repercussions for other institutions. Whilst this is no less true for non-financial institutions a common argument is that such inter-dependencies are particularly important in financial markets and that the default of one institution - even if it did not directly affect a large part of the financial system - might, by undermining confidence, come to trigger a system-wide problem. This contagious undermining of confidence is really dependent upon the existence of information problems and so the externalities and information problems are rather hard to isolate.

There is much academic debate about both the scale and importance of these, supposedly unusual, features of financial markets and about the appropriate response to them. The significance of forms of market failure has, however, appeared sufficient for governments over much of the world to construct special regulatory systems for financial intermediaries. What these systems have in common is an analysis of the riskiness of financial institutions. In particular there has been great attention paid to the adequacy of financial institutions' own capital funds in reducing default probabilities to acceptably low levels. The recent proposals on the convergence of bank regulation across the major economies, for example, represent a detailed scheme for relating adequate capital to the riskiness of institutions' asset portfolios (see Bank for International Settlements (1987) and Bank of England (1988)).

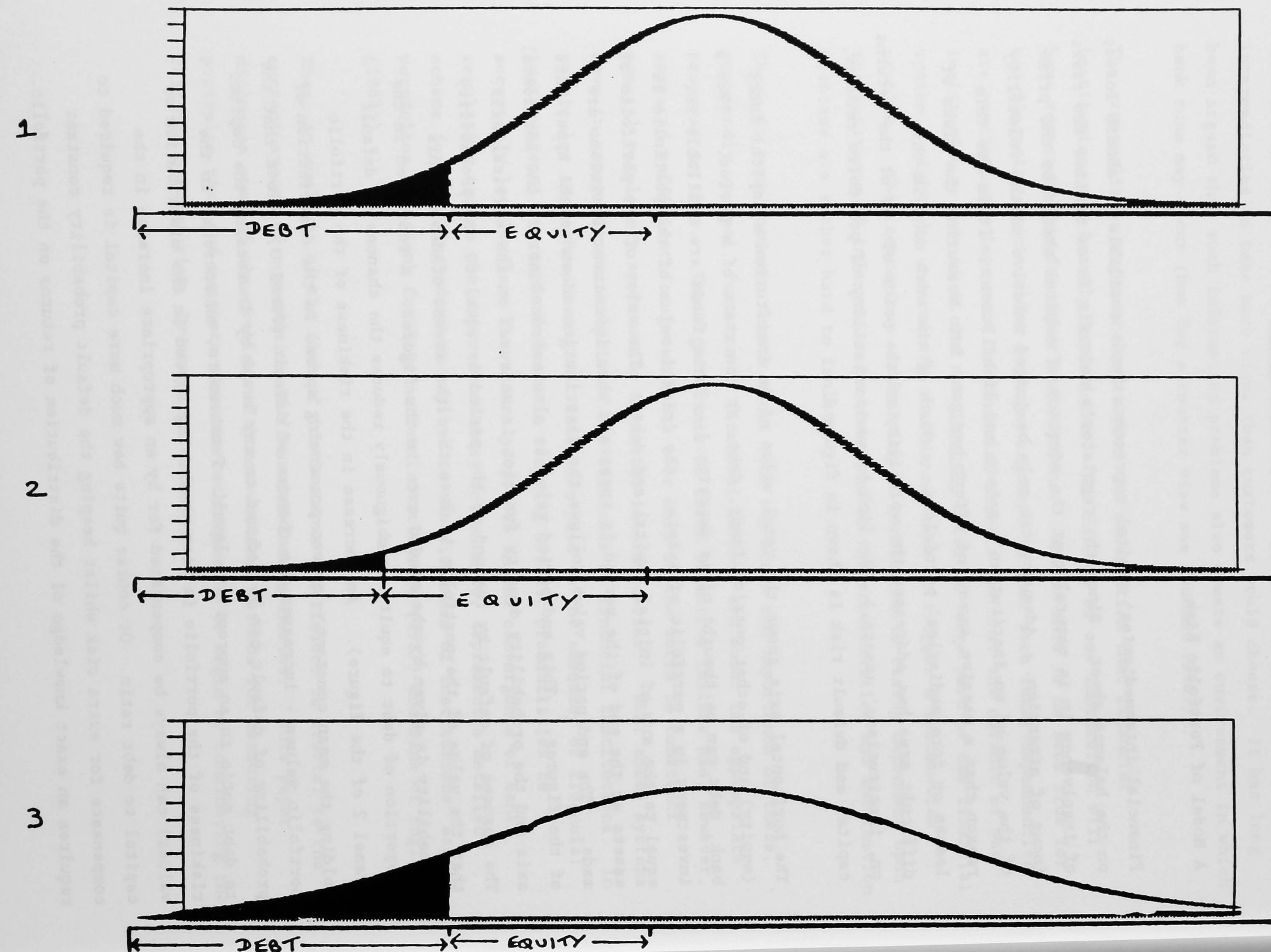
Thus, an analysis of the risk of institutions' portfolios is crucial to the current system of supervision of financial intermediaries. This paper discusses some of the problems with conventional methods for measuring portfolio risk and suggests some new techniques. We begin by considering the relation between risk, leverage and defaults.

A Model of Portfolio Risk

Financial intermediaries' capital represents funds available to absorb losses on the balance sheet. Since the regulator's main aim is to monitor the risks of insolvency it is natural that the adequacy of capital should be the prime focus of attention. Adequacy can only be judged relative to the volatility of the value of an institution's assets and liabilities. It is for this reason that a massive amount of effort has gone into measuring the risks of losses on intermediaries' portfolios. (Much of the work done in this difficult area has addressed the volatility of the price of, or of the returns on, institutions' assets.) The link between volatility of portfolio returns, capital and default risk is shown in figure 1.

The horizontal axis shows the total value of an institution's capital funds (equity) and of other outside funds, debt, at the start of a period. [For a bank, debt can be thought of as deposits.] These funds are available for investment in a portfolio of assets; the initial value of the portfolio is equal to the sum of initial capital and debt. The value of the portfolio of assets at the end of the period is uncertain when investment decisions are made. The end period value follows the distribution shown in the upper part of the figures. This end period value is also measured across the horizontal axis and the probability density function is measured on the vertical axis. The chances of default by the end of the period is equal to the probability that the value of the portfolio is less than the amount of debt. This probability is given by the shaded area in the figures. A reduction in the proportion of debt to equity unambiguously reduces the chances of default (Panel 2 of the figure). An increase in the riskiness of the portfolio holding the mean constant - a mean-preserving spread in the distribution of portfolio values - increases the chances of default (Panel 3). Note that the probability of default can be reduced to any level by increasing the capital to debt ratio to an appropriate level. Furthermore, an increase in the riskiness of the portfolio (a mean-preserving spread in the distribution of values) can always be compensated for by an appropriate increase in the capital to debt ratio. Of course quite how much more capital is required to compensate for extra risk whilst keeping the default probability constant requires an exact knowledge of the distribution of returns on the portfolio.

FIGURE 1



requires an exact knowledge of the distribution of returns on the portfolio. Estimating the risk of the value of assets and liabilities is, however, difficult. There are three particularly hard problems to face in analysing the risk on portfolios:

(1) The number of different classes of assets and liabilites is large and is changing.

(2) Many balance sheet items do not have market prices so analysis of the volatility of the value of such items is especially hard.

(3) The variability of asset returns is likely to change, often dramatically, over time.

The first problem is particularly tough. Suppose we can aggregate assets and liabilities held by an institution into J classes. Let the prices of the assets and liabilities be denoted by P_j (j = 1,2...J). The value of capital can be written as:

$$K = \sum_{j=1}^{J} P_j X_j$$

where X_j is the amount of the jth asset or liability held; X_j is negative for liabilites and positive for assets. The variance of the value of the portfolio - the variance of the capital - is then:

$$\sum_{j=1}^{J} \sigma^2_{p_j} X_j^2 + \sum_{\substack{j=1 \\ j \neq k}}^{J} \sum_{k=1}^{J} \sigma_{p_j p_k} X_j X_k$$

Where $\sigma^2{}_j$ is the variance of the jth asset, or liability, price

$\sigma_{p_j p_k}$ is the covariance of prices between the jth and kth item on the balance sheet.

If there are as few as 10 classes of asset and liability there are 55 variances and covariances to calculate (10 variances; 45 covariances). The number of variances and covariances goes up roughly in line with the square of J; the number is $\frac{J(J+1)}{2}$.

Because of the scale of the problem the empirical analysis of the variance of the value of capital, which under certain assumptions is a highly informative measure of volatility,[1] has usually proceeded by focussing on the individual variances of a small class of broadly defined balance sheet items. Covariances are often ignored. Furthermore, analysis of the variability is sometimes only carried out on the asset side of the balance sheet. A ranking of the variability of asset prices is then produced from which a set of weights is defined to calculate minimum acceptable levels of capital.

In the UK much valuable work both within the Bank of Englnad (on the volatility of banks' and securities dealers' portfolios) and outside the Bank, by the T.S.A and S.I.B, has been done along these lines. Charts showing the variances of various asset prices have been produced to inform the capital adequacy debates. One feature of these charts is the frequent dependency of the measures of volatility on the time period used for the calculation. This presents a particularly tough problem for the regulator. One (perhaps ultra) risk-averse response is to use as the measure of volatility the maximum of the variances calculated over the periods analysed.

There may be a way around the three problems associated with measuring volatility noted above. The idea is to use the market's valuation of an institution's balance sheet to infer its expectations of the volatility of the value of the underlying portfolio of assets and liabilties. Of course one cannot get something for nothing and the crucial assumption upon which this approach rests is that the market is efficient at evaluating companies. In the light of the stock market crash of last October one has to be sceptical about the validity of such an assumption. Provided, however, one does make the market efficiency assumption we can proceed by making a further, but far less controversial, assumption that the valuation of a company depends in part

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1 If asset prices are normally distributed about their expected values the variance is a complete measue of risk.

upon the perceived risk associated with the company's balance sheet and upon an evaluation of the price of risk - ie the compensation that the market requires to accept an increase in risk. The rest of this paper sketches out how one might use stock market valuations of an institution over time to derive estimates of perceptions of the changing risk of the balance sheet. Preliminary results on the risks of the portfolios of the big four UK banks are presented.

Begin by assuming that the market values an institution efficiently. If we denote an institution's share price at time t as Q_t this implies:

$$Q_t = \sum_{j=1}^{J} P_{jt} X_{jt} / N \qquad (1)$$

Where N is the number of shares (assumed constant) and where $\Sigma P_j X_j$ is the market value of the assets and liabilities, as above. Since for many institutions there will be items on the balance sheet which do not have market prices there will not usually be a means of testing the validity of (1). Assuming the validity of equation (1) will, however, allow us to implicitly value balance sheet items for which secondary market prices do not exist.

Basically the idea behind the proposed technique is that if equation (1) holds and if one can model how the market assesses the expected value of a financial intermediary one can use the variability in actual market valuations around their expected values to estimate the market's perceptions of the volatility of the institution's underlying portfolio. To implement the technique requires some theory about how share prices are determined. In the next part of this paper a theory is outlined and a procedure for estimating the volatility of an institution's capital is described. The technique is then used to assess the risk of the big four UK banks.

The most widely used theory of how equilibrium prices of assets evolve is the capital asset pricing model (CAPM) originally developed almost thirty years ago. (See Sharpe (1964), Lintner (1965) and Markowitz (1952).) According to this theory the return on an asset, in our case a share of the financial institution, depends on the safe rate of interest and the expected or perceived risk of the asset. In periods when no dividends are paid the return on a share in an institution is just the capital gain. Thus, we can write:

$$E\,\frac{(Q_t - Q_{t-1})}{Q_{t-1}} = R_{s_t} + (rp)_t \qquad (2)$$

where

R_s is the safe rate of return over one period (eg a Treasury Bill yield) measured at the beginning of the period

$(rp)_t$ is the risk premium at time t.

E is the expectations operator. Expectations are formed at the end of t-1 when it is assumed R_{st} is known. We will inititally draw no distinction between true mathematical expectations and the market's expectations. Thus, implicit in our definition of market efficiency is the assumption of rational expectations.

The risk premium can be expressed as the price of risk multiplied by the perceived amount of risk for which compensation is required. The price of risk, denoted λ_t, is dependent on the preferences of shareholders. The amount of risk needs to be carefully defined. In stock market equilibrium only certain types or risk are costly. By "costly risk" is meant the risk in the return on the institution's shares which the shareholders will require compensation for. This is often referred to as the non-diversifiable risk. The CAPM has the implication that this measure of risk, which is not the same as risk from a regulatory perspective, is that part of the conditional variability in the institution's share price which is correlated with the return on an efficiently diversified market portfolio. (We return to this point below.) Denoting the expected non-diversifiable risk as $E(ND)_t$ we therefore have

$$(rp)_t = \lambda_t E(ND)_t$$

If, on the whole, the market gets it right the actual return on the institution's shares between t-1 and t will equal the expected return, given by equation (2), plus some random mistake which is on average zero. Denoting this mistake e_t we now have:

$$\frac{Q_t - Q_{t-1}}{Q} = R_{s_t} + \lambda_t E(ND)_t + e_t \qquad (3)$$

The value of capital at time t is, by equation (1), Q_tN. So at the end of period t-1, when Q_{t-1} can be observed, the expected value of capital at time t can be derived by re-arranging (2) to give:

$$E(Q_tN) = Q_{t-1}N\,(1 + R_{s_t} + \lambda_t E(ND_t)) \qquad (4)$$

The variability of Q_tN about its expected, or average, value depends on e_t since by (3) we have:

$$Q_tN = Q_{t-1}N\,(1 + R_{s_t} + \lambda_t E(ND_t) + e_t) \qquad (5)$$

thus

$$Q_tN - E(Q_tN) = Q_{t-1}N\,e_t \qquad (6)$$

(6) is the one step ahead forecast error made by the market on the assumption that the CAPM is used to predict prices. By assumption the CAPM is the true model of share prices so (6) are rational expectation forecast errors.

Now we can immediately write down the conditional variance, as measured at t-1, of the value of capital at time t (ie $E((Q_tN - E(Q_tN))^2)$. This is:

$(Q_{t-1}N)^2\sigma e_t^2$ where σe_t^2 is the variance of e at time t

This is a variability measure relevant to a regulator. It is the variability in the market value of the capital funds around the market's expected value. <u>If</u> the assumption of market efficiency is valid this simple measure will be equal to the sums of variances and covariances of individual balance sheet items given on page 2. This is because the conditional variance of the financial institution's portfolio can be expressed in terms of the conditional variability and covariability of individual assets and liabilities or, more directly, in terms of the conditional variance of the value of the total portfolio. Assuming stock market efficiency we can estimate the conditional variance of the value of the total portfolio by estimating the differences between the expected value of the market valuation and the actual market value over time. Strong market efficiency implies that actual stock market valuations equal the value of the underlying portfolio. The conditional variance of market valuations around their expected values (ie $(Q_{t-1}N)^2\sigma^2 e_t$) is therefore equal to the conditional variance of the underlying portfolio. This is the beauty of the technique. If a measure of σ^2_{et} can be derived, and if e_t is approximately normally distributed, a natural measure of the adequacy of capital then suggests itself:

$$Q_{t-1}N / \sqrt{(Q_{t-1}N)^2 \sigma^2_{e_t}} = 1/\sigma_{e_t}$$

This expression [2] shows the number of standard deviations the value of capital represents at time t-1 when capital is exhausted the institution is insolvent. So if $1/\sigma_{e_t}$ were around 3 the probability of bankruptcy by end t would be around one in one thousand.

We now outline a method for estimating σ_{e_t}. Under the CAPM one can re-write equation (2) as:

$$E\frac{(Q_t - Q_{t-1})}{Q_{t-1}} = R_{s_t} + \beta_t \, E(RM_t - R_{s_t}) \qquad (7)$$

Where RM_t is the return on an efficient market portfolio and where β_t is the expected conditional covariance between the returns on the institution's share and that of the market portfolio, divided by the expected conditional variance of the return on the market portfolio ($\sigma^2 Rm_t$). β_t is the 'Beta' as usually

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2 Strictly speaking the expression should also take account of the expected return over the period. This is not, in practice, significant over short periods since <u>expected</u> returns over short periods are small.

defined in the theory of finance. [One could think of the return on the market portfolio (RM_t) as being, for example, the return from holding a value weighted market index like the F.T 500.] Now the CAPM also predicts that the expected return on the market portfolio depends on its risk. This risk is its variance since, by definition, no variability on the return from the efficient market portfolio is diversifiable. So the costly, or non-diversifiable, risk of the market portfolio just is its variance.
Thus:

$$E(RM_t) = R_{s_t} + \lambda_t E(\sigma^2 Rm_t) \qquad (8)$$

$$\text{where } \lambda_t = \frac{E(RM_t - R_{s_t})}{E(\sigma^2 Rm_t)}$$

$\sigma^2 Rm_t$ is the variance of the returns on the market portfolio. λ_t is the price of risk, the required excess in the return on the market portfolio over the risk free rate divided by the amount of risk.

Now if expectations are on average correct but subject to a random error, v_t, we can write (8) as:

$$RM_t = R_{s_t} + \lambda_t E(\sigma^2 Rm_t) + v_t \qquad (9)$$

$$\text{and so} \quad E(\sigma^2 Rm_t) = E(v_t^2)$$

Adding the random noise term e_t to (7) and recalling the definition of β_t gives:

$$\frac{Q_t - Q_{t-1}}{Q_{t-1}} = R_{s_t} + \frac{E(\mathrm{cov}(\frac{Q_t - Q_{t-1}}{Q_{t-1}}, RM_t))\, E(RM_t - R_{s_t})}{E(v_t^2)} + e_t$$

From (9) $E(RM_t - Rs) = \lambda_t E(v_t^2)$ and we also know that
$\qquad\qquad t$

$$E\left(Cov\left(\frac{Q_t - Q_{t-1}}{Q_{t-1}}, RM_t\right)\right) = E(v_t e_t)$$

Putting all this together we have the following two equation system:

$$RM_t = R_{s_t} + \lambda_t E(v_t^2) + v_t \qquad (10)$$

$$\frac{Q_t - Q_{t-1}}{Q_{t-1}} = R_{s_t} + \lambda_t E(v_t e_t) + e_t \qquad (11)$$

To estimate the unknown parameters we need to specify a process whereby expectations of variances and covariances are formed. Engle et al (1987) outline just such a process and describe how the unknown parameters of the system can be estimated from observable data. Their technique draws upon the idea that the variability in the return on an asset tends to follow patterns. Mandlebrot noted as far back as 1963 that in looking at the change in asset prices "large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes......". If this is so an ARCH (Auto-Regressive Conditional Heteroscedasticity) process can be fitted to the variances and covariances of equations (10) and (11). The simple ARCH model suggests equations of the form:

$$E(v_t^2) = \alpha_o + \alpha_1(v_{t-1}^2) + \alpha_2(v_{t-2}^2) + \ldots\ldots\alpha_n(v_{t-n}^2) \qquad (12)$$

$$E(v_t e_t) = \Psi_o + \Psi_1(v_{t-1}e_{t-1}) + \Psi_2(v_{t-2}e_{t-2}) + \ldots\ldots \Psi_n(v_{t-n}e_{t-n}) \qquad (13)$$

Where α_o, α_1, ... α_n; Ψ_o, Ψ_1, ... Ψ_n are parameters. A special case of (12) and (13) is where α_1, α_2.... $\alpha_n = 0$, Ψ_1; Ψ_2....$\Psi_n = 0$ when the variances and covariances would be constant over time. The procedure outlined in Engle et al shows how to simultaneously estimate all the unknown parameters of the system by maximum likelihood methods. (To use their procedure it is much simpler if we could assume the price of risk, λ_t, is constant, ie $\lambda_t = \lambda$ for all t. In our preliminary estimation work we have made this assumption.)[3]

A third ARCH equation for $E(e_t^2) = E(\sigma_{et}^2)$ is also estimated to give the market's perception of the total expected risk, both diversifiable and non-diversifiable, of the institution's portfolio in period t, as perceived at t-1. This equation is:

$$E(e_t^2) = \delta_0 + \delta_1(e_{t-1}^2) + \delta_2(e_{t-2}^2) + \ldots \delta_n(e_{t-n}^2) \qquad (14)$$

$E(e^2{}_t)$ is the key variable for determining the risk of the financial institution's portfolio. Notice that $E(e^2{}_t)$ does not enter into the equation for the return on the institution's share; it is only the non-diversifiable element of the total share price volatility which is relevant in defining equilibrium returns. For the supervisor of the financial institutions it is, however, the risk of default which is crucial. This depends on the _total risk_ of the institution's portfolio regardless of whether, from the shareholders' point of view, much of this risk may be diversifiable; shareholders and regulators of financial institutions do not have common interests. Both the market and the regulator are, however, concerned about the volatility in the value of the financial institutions' portfolios. Indeed our technique depends crucially upon this being so. What is important to note is that the relevant type of volatility is not the same for regulators and shareholders.

What the estimation technique rests upon is the simple idea that if the random or unexpected element in the return on an asset rises in one period this increases the perceived risky or random element in the near future. It takes

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3 The assumption of a constant price of risk is restrictive. We believe, however, that over the relatively short period used for estimation the variability in the price of risk is likely to be small relative to the changes both in the perceived risk of holding shares and in the safe rate of interest. The price of risk is a 'deep parameter' in the sense that it depends upon the preferences of individual shareholders; a usual assumption in economics is that parameters of utility functions are unchanging. Changes in the distribution of wealth allied with variability in attitudes to risk across individuals would, of course, be sufficient to cause variability in the market price of risk.

a period of less volatile returns to bring people's perceptions of underlying volatility back down. This is an intuitively plausible idea and probably accords well with post-crash events since it seems almost certain that people's ideas of the riskiness of stocks has increased dramatically compared with perceptions in the summer of last year.

What the technique outlined does allow us to do is to calculate $E(\sigma^2_{et})$, ie to derive a measure of the market's perception of the variance of the value of capital and to model how it changes over time. This itself is potentially highly informative, especially if changes in σ^2_{et} can be linked to changes in the structure of the balance sheet. Indeed the ARCH model allows a natural way to test hypotheses about precisely how changes in balance sheet structure, or in regulation or in macroeconomic policy, influence the perceived volatility of the value of financial intermediaries' capital. We now describe preliminary results on the volatility of the value of the four big UK banks.

Results

Monthly data on the share prices of Barclays, Midland, National Westminster and Lloyds over the period 75.6-87.9 were used to define one month rates of return. Adjustments were made for ex dividend days using reported per-share dividend payments. We used the (value weighted) FT 500 index to construct a proxy for the return on the market index (RM_t). The one month Treasury bill rate was used for the safe rate (R_{st}). The most general version of the five equation system which we estimate can be written:

$$RM_t = a_0 + a_1 R_{st} + a_2 E(v_t^2) + v_t \tag{15}$$

$$\frac{Q_t - Q_{t-1}}{Q_{t-1}} = b_0 + b_1 R_{st} + b_2 E(v_t e_t) + e_t \tag{16}$$

$$E(v^2{}_t) = \alpha_0 + \alpha_1(v^2_{t-1}) + \alpha_2(v^2_{t-2}) + \ldots \alpha_n (v^2_{t-n}) \tag{17}$$

$$E(v_t e_t) = \Psi_0 + \Psi_1 (v_{t-1} e_{t-1}) + \ldots \Psi_n (v_{t-n} e_{t-n}) \tag{18}$$

$$E(e^2_t) = \delta_0 + \delta_1 (e^2_{t-1}) + \ldots \delta_n (e^2_{t-n}) \tag{19}$$

where a_i, b_i, α_i, Ψ_i, δ_i i=0,1,2, ... are coefficients to be estimated

Comparing (15) and (16) with (10) and (11), and assuming λ_t is constant, gives the following restrictions implied by the CAPM:

(a) $a_0 = b_0 = 0$

(b) $a_1 = b_1 = 1$

(c) $a_2 = b_2$ (> 0)

Further restrictions are:

(d) $E(e_t^2)$ should not help explain $\frac{Q_t - Q_{t-1}}{Q_{t-1}}$

(e) expected variances (or covariances) only, and no other expected moments, are sufficient measures of risk for which compensation is required.

(f) the errors e_t, v_t are normally distributed (an assumption needed to justify both the CAPM and the estimation technique) and serially uncorrelated.

We tested each of these assumption by relaxing, in turn, the restrictions implicit in equations (10) and (11) ie we take as our null the case where $a_0 = b_0 = 0$; $a_1 = b_1 = 1$ and $a_2 = b_2$ and test the restrictions (a) - (e) in turn. We also test for nonnormality and serial independence of errors - condition (f). For all specifications we initially use a much restricted version of the general forms of the ARCH processes (17) - (19). We restrict the order of the highest lag in the ARCH equations to be 8 and impose a linearly declining pattern on the coefficients. Thus, our estimated version of (17) is

$$v_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \left[\sum_{i=1}^{8} c_i (v_{t-i}^2)\right] + \text{error} \qquad (20)$$

where $c_i = \frac{9 - i}{36}$

i:	1	2	3	4	5	6	7	8
c_i:	.22	.19	.17	.14	.11	.08	.06	.03

This restricted version of the ARCH process is similar to that used by Engle et al (op cit) who argue that a parsimonious specification generally proves to be statistically acceptable and is more likely to be economically sensible. Unrestricted versions, they found, often threw up some negative parameters and in their application the likelihood of the unrestricted versions was ill-behaved.

The test of the significance of the ARCH element of the conditional variances/covariances is simply a test of the significance of $\hat{\alpha}_1$ in (20). In our preliminary regressions we have also restricted the coefficients on the weighted averages of past variances and covariances for the three ARCH equations for a particular bank to be equal. The common parameter we denote $\hat{\alpha}_1$; it is the weight given to past forecast errors in estimating the future variability of asset returns.

Thus, for each bank we estimate 4 parameters:

$\hat{\lambda}$ the price of risk

$\hat{\alpha}_1$ the weight given to past variances (covariances) in estimating future variances (covariances).

Two constants for the ARCH processes denoted $\hat{\alpha}_0$, for the market variances (v_t^2), and $\hat{\delta}_0$, for the bank specific variances (e_t^2).

Table 1 shows the results from the restricted equations. Table 2 shows the tests of the restrictions.

TABLE 1:

t statistics in parenthesis

	$\hat{\lambda}$	$\hat{\alpha}_1$	$\hat{\alpha}_0$	$\hat{\delta}_0$	Log Likelihood
Midland	2.225	1.748	$2.5(10^{-7})$	$9(10^{-10})$	-1.969
	(3.1)	(33.3)	(0)	(0)	
Barclays	2.174	1.096	.254	.100	24.564
	(2.8)	(20.5)	(8.8)	(12.9)	
NatWest	4.219	.693	.527	.139	5.220
	(4.5)	(16.7)	(17.1)	(16.1)	
Lloyds	1.723	.810	.520	.153	-48.477
	(1.9)	(15.1)	(12.0)	(11.7)	

Estimation period: 1976.2 - 1987.8

Number of observations: 139

TABLE 2

Tests of Restrictions of the CAPM Models

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
Midland	23.67	4.46	4.55	26.81	.82	1.21	7.82
Barclays	.66	10.80	5.90	.03	.80	1.39	8.50
NatWest	18.40	14.32	10.40	9.66	15.68	1.41	8.43
Lloyds	12.19	8.69	1.12	4.65	3.07	1.50	8.85

L_1 = likelihood ratio test of no constants in CAPM equations.

L_2 = likelihood ratio test that coefficients on safe rate = 1.

L_3 = likelihood ratio test of equality of price of risk coefficients in market return and individual bank return.

L_4 = likelihood ratio test that $E(e_t^2)$ is irrelevant in predicting return on bank share - a test that the expected covariance, and not the expected variance, of the bank return is relevant.

L_5 = likelihood ratio test that the expected conditional variance of the market return, and not other moments, is relevant in predicting the market return. This is a test that y=1 where $E(v_t^2)^y$ is entered as an explanatory variable for RM_t.

L_6 = test of normality of errors on bank return (e_t); this is the Bera and Jarque normality test.

L_7 = Box Pierce test for up to 8th order Serial Correlation of bank return errors.

L_1, L_2, L_6 are distributed χ_2^2 under null. 95% critical value = 6.00
97.5% critical value = 7.38

L_3, L_4, L_5 are distributed χ_1^2 under null. 95% critical value = 3.84
97.5% critical value = 5.02

L_7 is distributed χ_8^2 under null. 95% critical value = 15.51
97.5% critical value = 17.53

The price of risk is consistently estimated as being significantly positive; the hypothesis of time varying variances and covariances is strongly confirmed (all estimates of $\hat{\alpha}_1$ are highly significant); errors on individual bank returns do not appear to deviate from being normally distributed and serial correlation does not seem significant. Tests of only up to 8th order serial correlation are shown in Table 2. Tests of up to 1st and 4th order serial correlation showed no sign of misspecificaton. Tests of the other strong CAPM restrictions give mixed results. In general the hypotheses that it is only the variances of market returns that matter is not rejected. There are signs that conditional variances do, however, add information in predicting returns on individual assets; conditional covariances with the market return would appear not to be a sufficient statistic for risk. In three out of four cases the restriction of no constants in the equations is, rejected. This may reflect mismeasurement of the safe rate of return. In one case, NatWest, the hypothesis of a common price of risk for the market and bank returns - a crucial restriction - is clearly rejected; for the other banks the restriction cannot be clearly rejected.

Charts 1-4 show the time-varying β's derived from the estimated models. The β's here are:

$$\frac{E\left(\mathrm{cov}\left(\dfrac{Q_t - Q_{t-1}}{Q_{t-1}}, RM_t\right)\right)}{E\left(\sigma^2 R_{mt}\right)} = \frac{E(e_t v_t)}{E(v_t^2)} \tag{21}$$

The numerator and denominator of (21) are derived from the ARCH models - our estimated versions of equations (17) and (18).

The charts show considerable variation in the β's for each bank. Average values are around unity; at times some of the estimated expected covariances are negative. Given the changing structure of bank portfolios of assets and liabilities over time significant variation is to be expected, though the volatility in the estimated β's over very short periods looks excessive. We now consider a different specification of the model which avoids this extreme volatility.

A GARCH Specification

There are two potentially serious problems with the specifications of the CAPM models discussed above. First, we have severely restricted the form of the ARCH equations for expected variances and covariances. Only eight lags of squares and cross-products of residuals are used in updating the variance covariance matrix of returns. This almost inevitably leads to implausibly high variability in estimated market β's. Second, we have not imposed a common price of risk across the models estimated for the four banks. The price of risk is a market variable and should be common across all assets.

The first problem, restricting the memory of agents in forecasting asset volatility, can be handled by making today's expected variances and covariances depend on yesterday's expectations. This gives a generalised ARCH (or GARCH) process[4] of the form

$$E(v_t^2) = \alpha_1 E(v_{t-1}^2) + \alpha_2 v_{t-1}^2 \tag{22}$$

Unravelling (22) shows that the expected variance at time t depends on the complete history of past residuals with declining weights placed on past squares and cross products of errors. The equation has a natural interpretation in terms of Bayesian updating. At the end of each period the variance forecast is updated in the light of the ex-post variability of the asset return within that period. The weight attached to the news is α_2. If no weight is attached to news α_2 is zero and the expected conditional variance is a constant. Clearly for (22) to make sense we require α_1, $\alpha_2 \geq 0$

We re-estimated the CAPM models of share prices using this GARCH specification. We restricted the coefficients α_1 and α_2 (from equation (22)) to be common for the variances and covariances of different assets. That is, we assume that the market attaches the same relative weights to past random forecast errors (v_{t-j} and e_{t-j} $j = 1, 2, ...$) in forming expectations of current conditional variances and covariances across all assets. This restrictive assumption makes the highly non-linear estimation far simpler. There is also some plausibility in the idea that individuals adopt similar forecasting rules for what are similar forecasting problems.

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4 The GARCH specification was first analysed in Bollerslev (1986).

We also imposed the equality of the price of risk across all assets. This gives a very tightly parametised version of the CAPM model with only three unknown parameters: $\hat{\lambda}$ - the price of risk, $\hat{\alpha}_1$, $\hat{\alpha}_2$ - the parameters from the updating equations for conditional variances and covariances (see (22)). The results from estimating the five equation model (4 banks and the market rate) are shown in Table 3.

Table 3:

$\hat{\lambda}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	Log likelihood
2.287	.89	.20	-86.503
(3.74)	(134.1)	(11.9)	

	L1	L2	L3	L4
Market Returns	1.48	3.47	4.95	17.45
Midland Bank	2.01	5.79	6.03	7.44
Barclays	2.75	4.22	5.48	8.78
NatWest	22.21	6.78	8.23	11.88
Lloyds Bank	10.46	0.35	2.89	8.15

L1 = Bera and Jarque test statistic for normality of errors; $\sim\chi^2_2$ under null

L2 = Box Pierce test for up to 1st order serial correlation of errors; $\sim\chi^2_1$, under null

L3 = Box Pierce test for up to 4th order serial correlation of errors; $\sim\chi^2_4$, under null

L4 = Box Pierce test for up to 8th order serial correlation of errors; $\sim\chi^2_8$, under null

The three parameters are highly significant each with the expected sign. The coefficient on news ($\hat{\alpha}_2$) is far enough from zero to strongly reject the hypothesis of constant variances and covariances. The price of risk is very close to the average of the prices shown in Table 1. The estimate of just over two is also close to the average of several US studies (see, in particular, Merton (1980) and references therein). The relatively high value of $\hat{\alpha}_1$ implies that a significant weight is given to past residuals even beyond 12 periods. The mean lag in the conditional variance/covariance equation is 10 months. This gives a correspondingly smoother path both for bank β's and for risk measures. [Charts 5-12.]

With the GARCH specification there now appear some signs of serial correlation. For each of the four banks there is some evidence of negative first order serial correlation in residuals. There are still no signs of higher order correlation for the banks and no indication of error correlation at any order for the market returns. There now also appears some evidence of non-normality in the distribution of residuals. This is not surprising given the finding of Engle and Bollerslev that estimation of the GARCH specification often produces residuals apparently following distributions with fat tails. [See Bollerslev (1985) and Engle and Bollerslev (1986) where results are reported from the estimation of single equation GARCH models assuming residuals follow Student's t distribution. In a multi-equation system maximum likelihood estimation of a GARCH in mean process with errors following Student's t distribution is more difficult.]

Charts 5-8 show the Betas implied by the GARCH specification. They show less variability than with the ARCH specifications, though they follow the broad pattern closely.

Charts 9-12 show time series for each bank of $1/\surd E(\sigma^2_\varepsilon t) = 1/\surd E(e^2{}_t)$. These measures are generated using the parameters of the expectation formation mechanisms for variances and covariances reported in Table 3. As we noted on page 8 these measures show the number of standard deviations the value of capital represents for each bank at each period. Alternatively, the measures show the number of standard deviations away from its expected value that the value of the bank's portfolio would need to be to result in insolvency. This is a natural measure of bankruptcy risk.

The charts reveal that our measure of the one period ahead chances of insolvency are minute. Even allowing for the possibility that residuals come from a distribution with fatter tails than a normal, the probability of being seven standard deviations below the expected return is infinitesimal. Probabilities of insolvency over time periods further than one month into the future will, however, increase significantly and could be calculated using the GARCH equations to dynamically forecast $E(e^2_{t+j})$ for j>1. Given the size of $\hat{\alpha}_1$ amd $\hat{\alpha}_2$ and the fact that the probability of insolvency over, say, a five year horizon depends on the chances of perhaps only one of 60 monthly residuals being very strongly negative, the odds of bankruptcy can quickly become significant.

Charts 9-12 are best seen as indicative of trends in risk; they suggest that there has been significant variation in risk over the past decade. Around 1978 and mid 1986 would appear to have been relatively risky periods for banks.

Chart 13 shows a measure of risk for the whole market. This is $E(v^2_t)$ derived, once again, by using the parameter estimates reported in Table 3. There are large fluctuations in this measure of whole economy risk; 1977 and 1981/82 are revealed as periods of particularly high risk.

Relaxing the Restrictions of Market Efficiency

A crucial assumption underlying the derivation of our measures of the volatility of the value of financial institutions' portfolios was that at each point in time the stock market value of the institution equals the market value of its portfolio of assets and liabilities.

Can this restriction be eased? Suppose that the market only gets the valuation right on average; it makes random errors. We can then replace (1) with the less restrictive.

$$NQ_t = \Sigma P_{jt} X_{jt} + w_t \qquad (23)$$

where $E(w_t) = 0$; w_t we will call 'stock market noise'.

It follows that

$$E(NQ_t) = E(\Sigma P_{jt} X_{jt})$$

and

$$NQ_t - E(NQ_t) = \Sigma P_{jt} X_{jt} - E(\Sigma P_{jt} X_{jt}) + w_t$$

So

$$E(NQ_t - E(NQ_t))^2 = E(\sigma^2_{\Sigma pjt\ xjt}) + E(w_t^2) + 2E(cov(\Sigma P_{jt} X_{jt}, w_t)) \qquad (24)$$

where $\sigma^2_{\Sigma pjtXjt} = (\Sigma P_{jt} X_{jt} - E(\Sigma P_{jt} X_{jt}))^2$

$cov(\Sigma P_{jt}X_{jt}, w_t)$ is the covariance between the market's one step ahead forecast error in predicting the value of the portfolio, and the stock market noise in period t. It is hard to predict what values this covariance might typically have. We assume it is ≥ 0. Using the CAPM with the ARCH processes we derived an estimate of $E(\sigma^2_{et})$ where:

$$E(\sigma^2_{et}) = \frac{E(NQ_t - E(NQ_t))^2}{(Q_{t-1}\ N)^2}$$

By (24), and assuming a non-negative value for the covariance term, this estimate represents an upper bound on the market's perception of the risk of the financial institution's portfolio of assets and liabilities. Thus, the value of our proxy for the market's estimate of the risk of an institution's portfolio may still be informative even if the stock market does not exactly price the value of that portfolio. However, the greater is what we call the stock market noise the less informative is our measure.

A second aspect of our strong definition of market efficiency is that market expectations are rational expectations. A less strict condition is:

$$\underset{mkt}{E}(Q_tN) = E(Q_tN) + z \tag{25}$$

where z is a random variable which is on average zero. In this case market expectations of market values, $(\underset{mkt}{E}(Q_tN))$, are equal to true mathematical expectations plus noise. Plugging (25) into earlier formulae, and assuming once again that $Q_tN = \Sigma P_{jt}\ X_{jt}$, gives a condition analogous to (23):

$$E(NQ_t - \underset{mkt}{E}(NQ_t))^2 = E(\sigma^2 \Sigma P_{jt}\ X_{jt}) + E(z^2) - 2E(cov(z, \Sigma P_{jt}\ X_{jt})) \tag{26}$$

Once again if the expectations noise, z, is not too great, nor too variable, the derived measure is informative. On the assumption that the covariance term in (26) is ≥ 0, the measure represents an upper bound on the true volatility of the financial institution's underlying portfolio.

Summary and Conclusions

What we have tried to do is to use information on stock market valuations of financial institutions to derive measures of the riskiness, or volatility, of the value of the firms' portfolios of assets and liabilities. If one is very sceptical about the market's ability to evaluate institutions' portfolios then the derived measures might be viewed as revealing something about market perceptions, but as having little to do with fundamental risk. At the other extreme is the (Hayekian) view that markets are supremely efficient at aggregating and conveying information and that market perceptions of risk are the best measures available. Either way the derived measures are, we think, of interest. What the measures we obtained suggest is:

(i) perceptions of the risk of the four UK banks have changed significantly over time.

(ii) the relation between the risks of a particular bank and the risks of investing more generally in the stock market (summarised by a bank's 'Beta') is, on the whole, close - Beta's have a mean of around unity. Given the size and diversified nature of the big banks' balance sheets this is not surprising; unexpected events that cause the market to change its view on the economic prospects of the UK are likely to have a significant impact upon bank valuations. At some periods the correlations between general UK risk (as measured by the risk on the market index) and the risk of banks, is, however, low. There could, for example, be shocks to sectors of the economy which are not significantly endebted to banks but which have a high weight in the FT 500 index. The persistence of these types of shock would reduce a bank's Beta (see Charts 1-8).

(iii) although there is significant variation in the perceived chances of insolvency for banks over time those chances, proxied by $1/E(\sigma)$ in Charts 9-12, have always been seen as extremely small, at least over a one month horizon.

What it would be interesting to consider is the determinants of bank (or other financial intermediary) risk. We aim to address this issue in future work by trying to explain the evolution of our measure of insolvency risk in terms of, for example, balance sheet structure, the level of capital or of regulatory innovations. We also aim to analyse the riskiness of other institutions such as securities dealers and to assess how robust results are to different assumptions about how variances and covariances might evolve (eg ARCH vs GARCH). Finally, we are aware that the assumption of market efficiency, in the sense in which it is used here, is so strong that any results we derive from applying our technique have to be treated with care. Our initial feeling is that the assumption that the stock market can efficiently evaluate the risk of various assets and liabilities is such that one should use derived measures as no more than a check on the estimates which are independently made of the variability of the underlying elements of financial institutions' portfolios.

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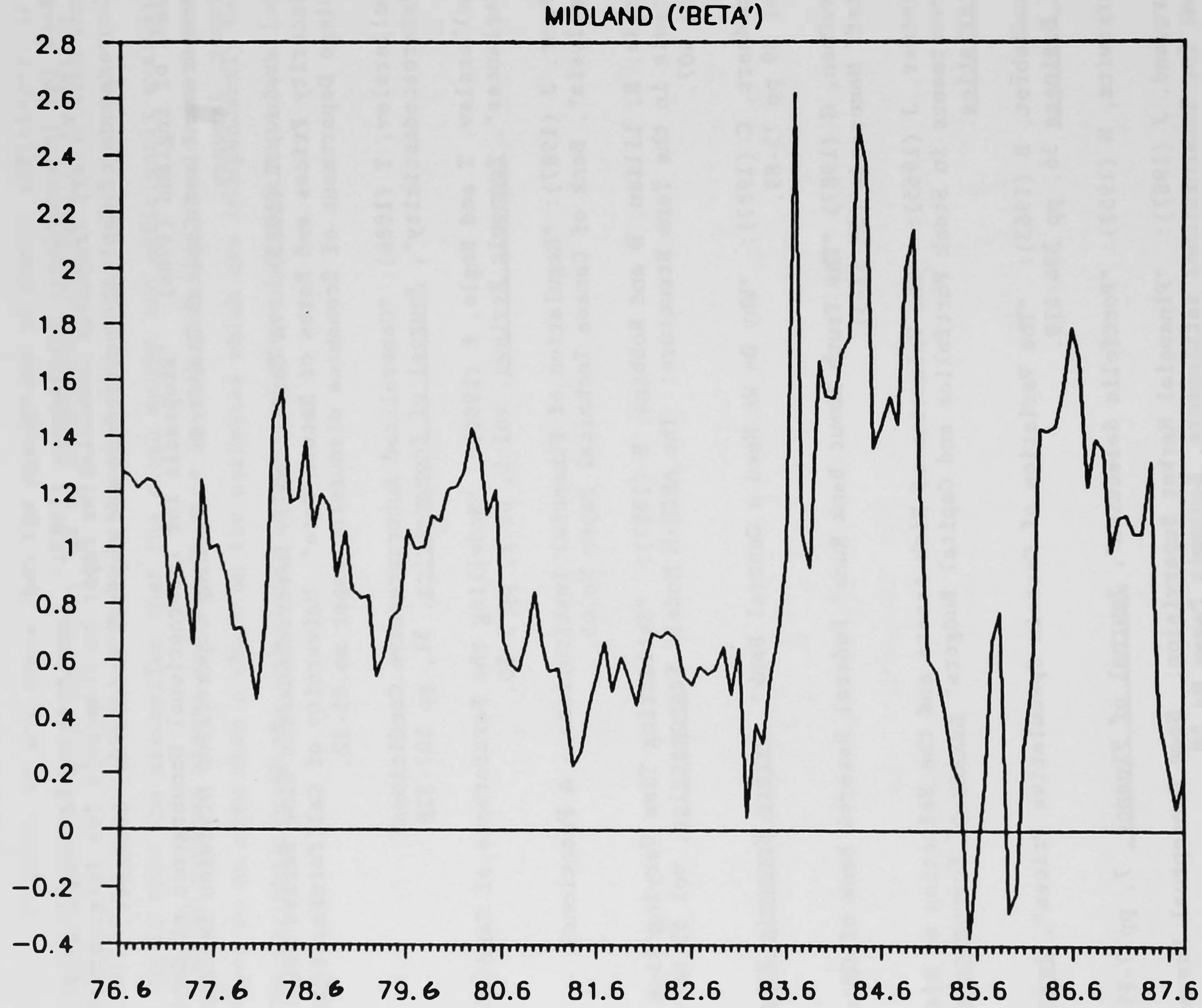
TWO EQUATION MODEL



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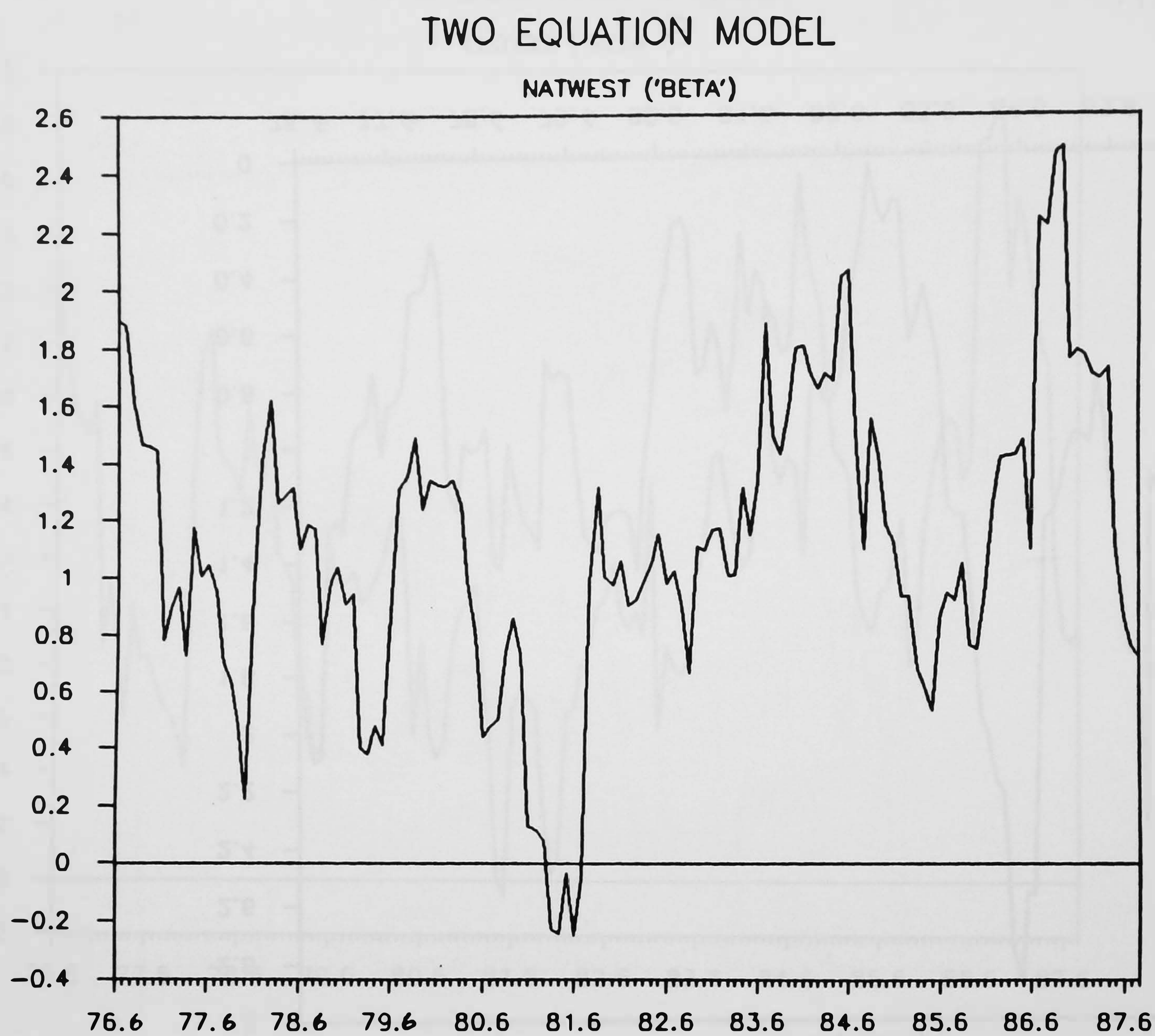
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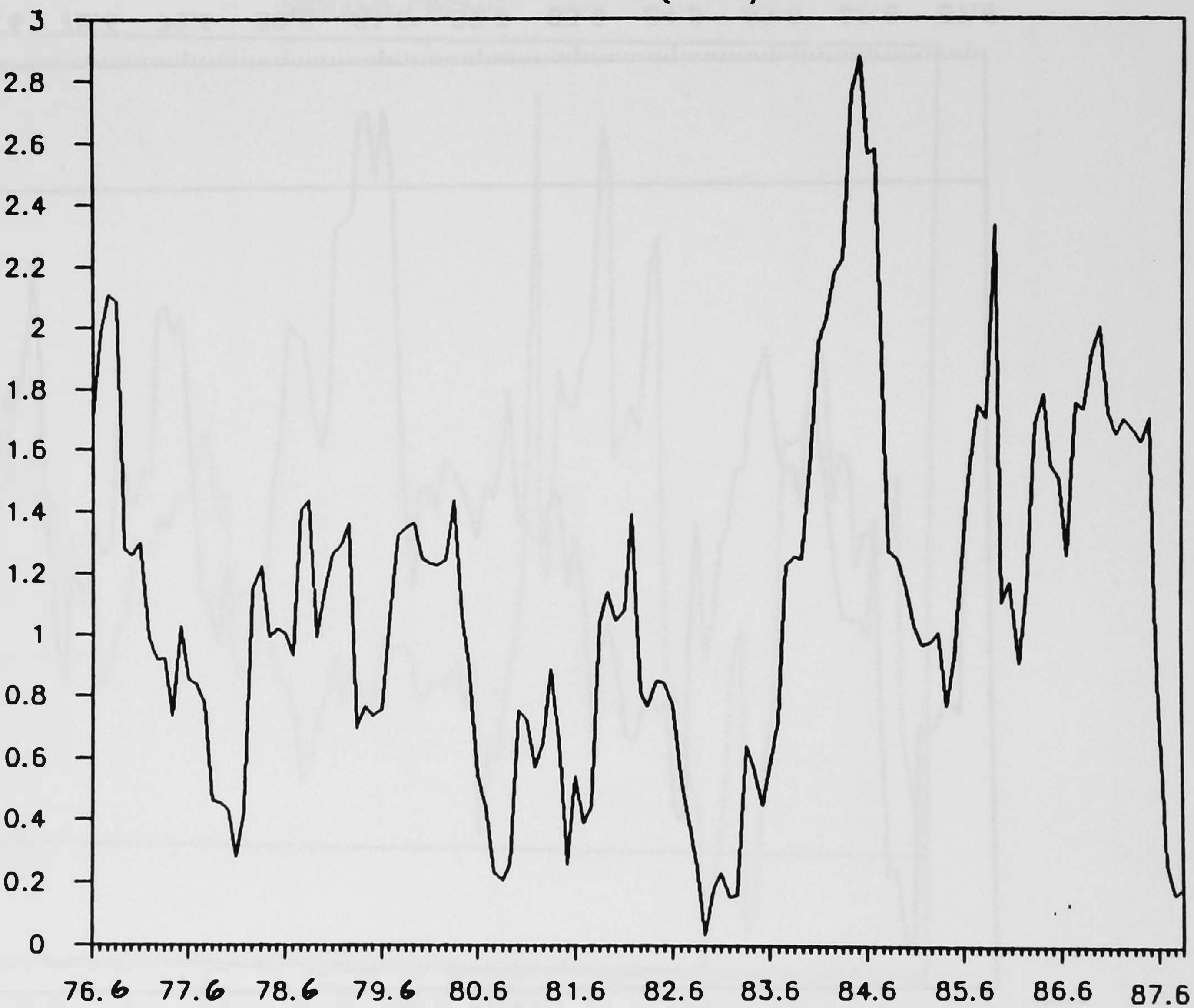

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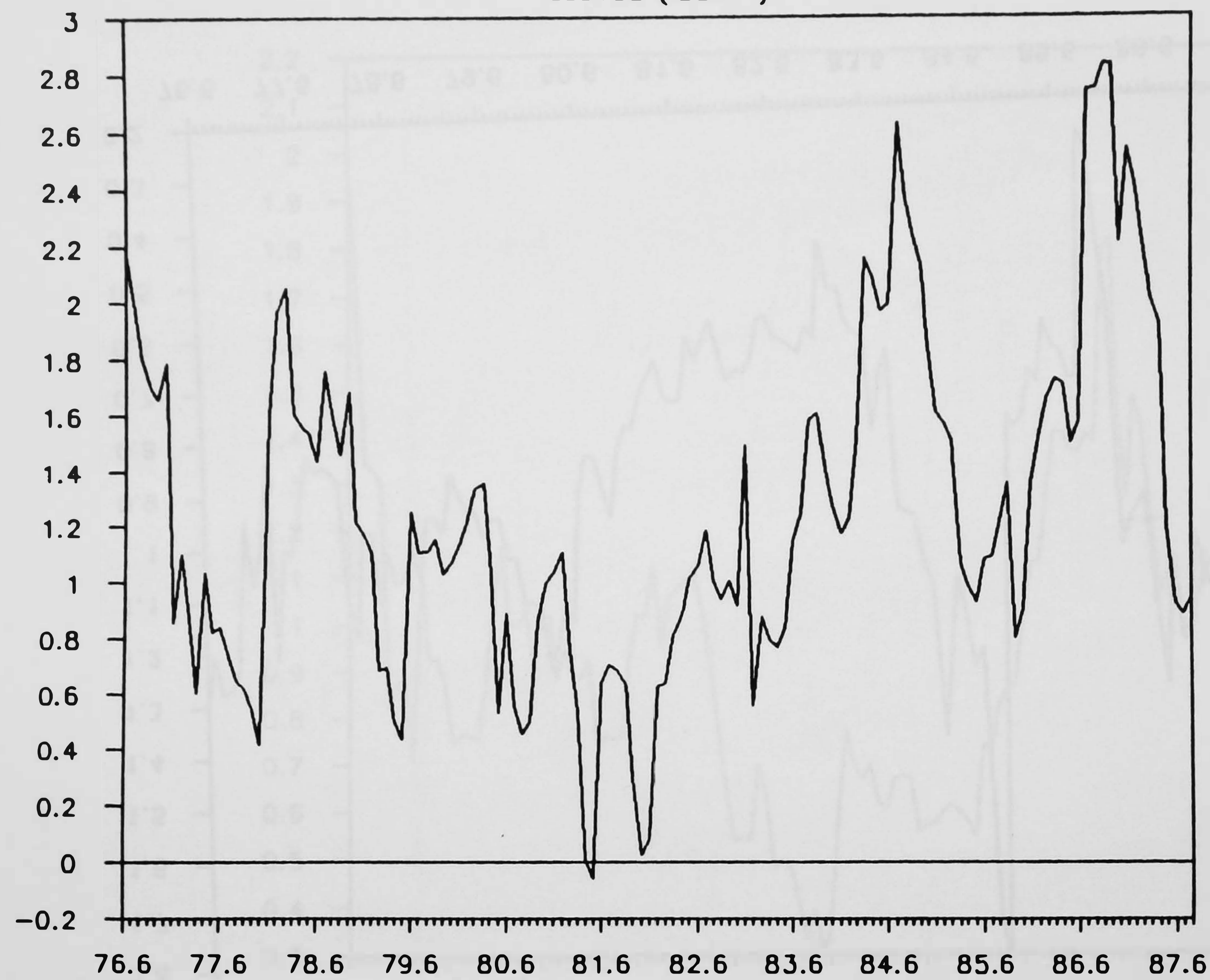
LLOYDS ('BETA ')



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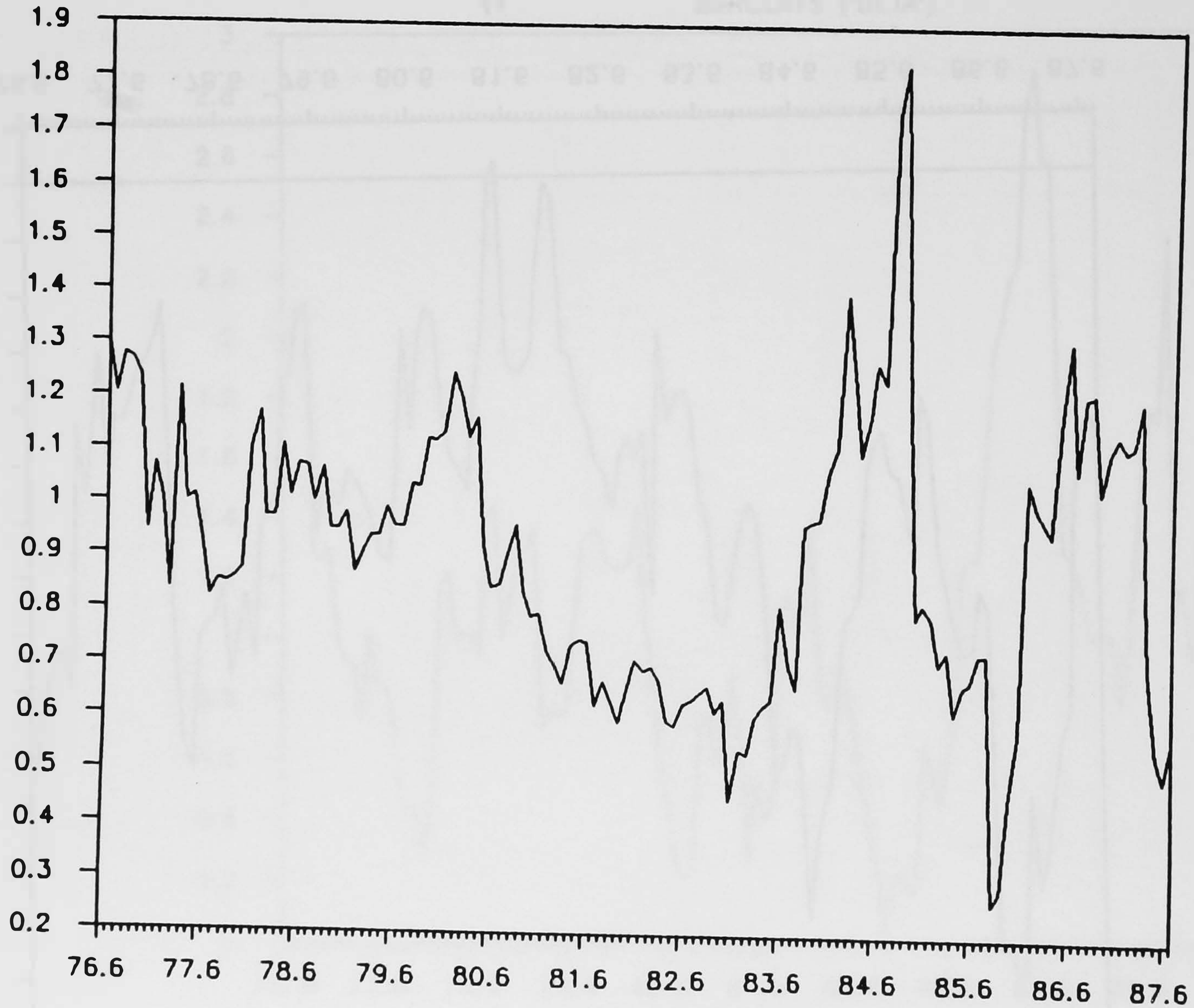
MIDLAND 'BETA'



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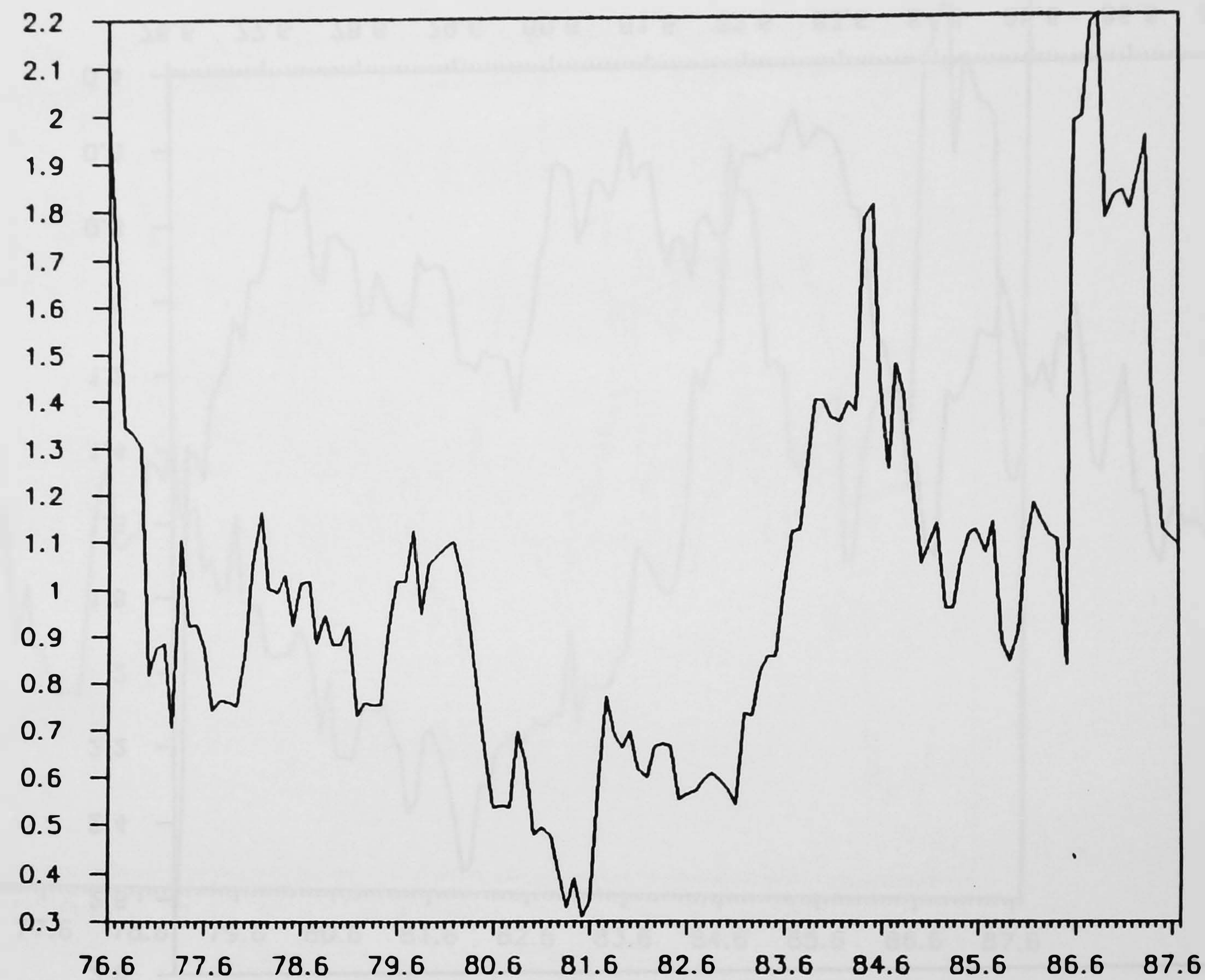

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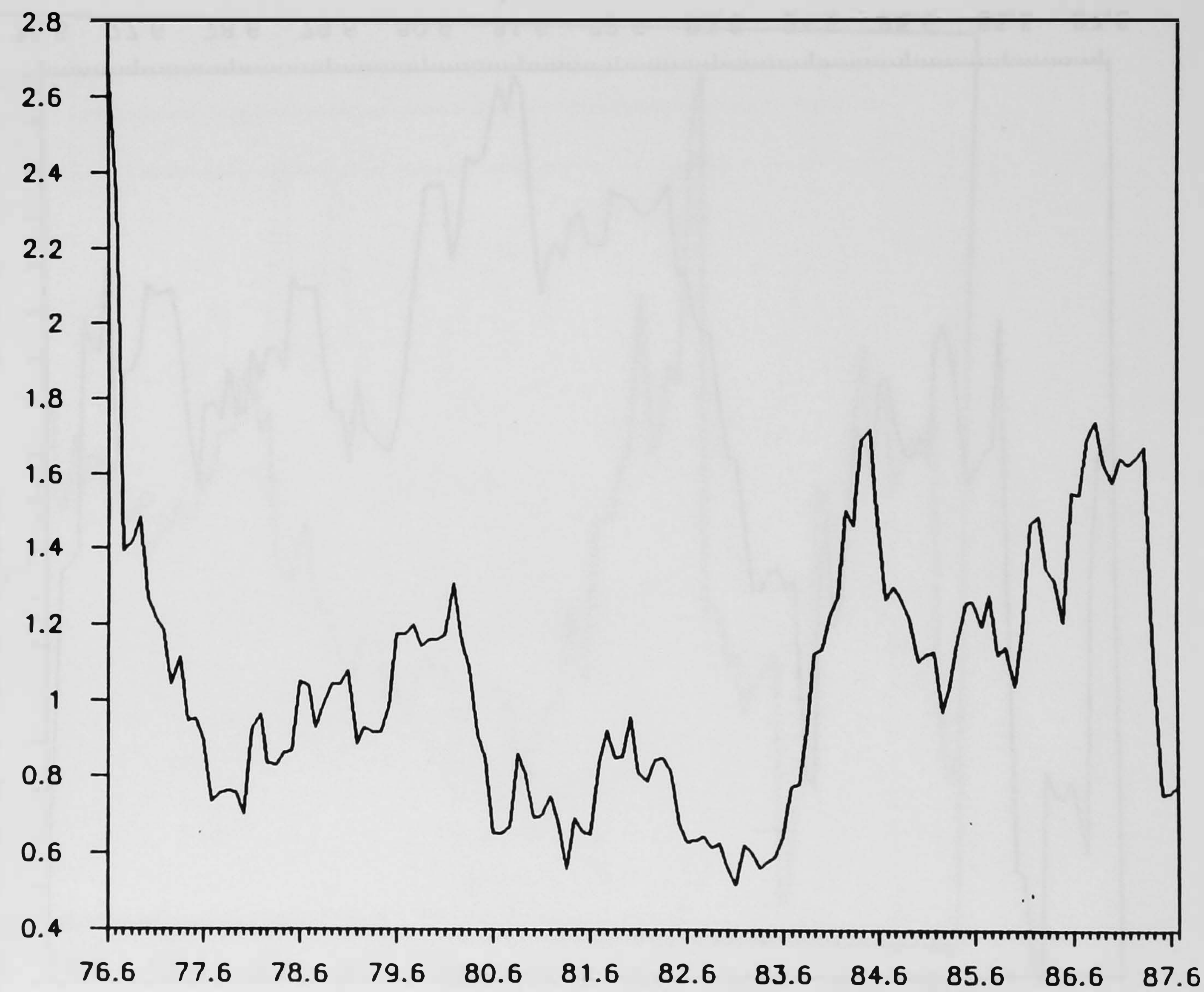

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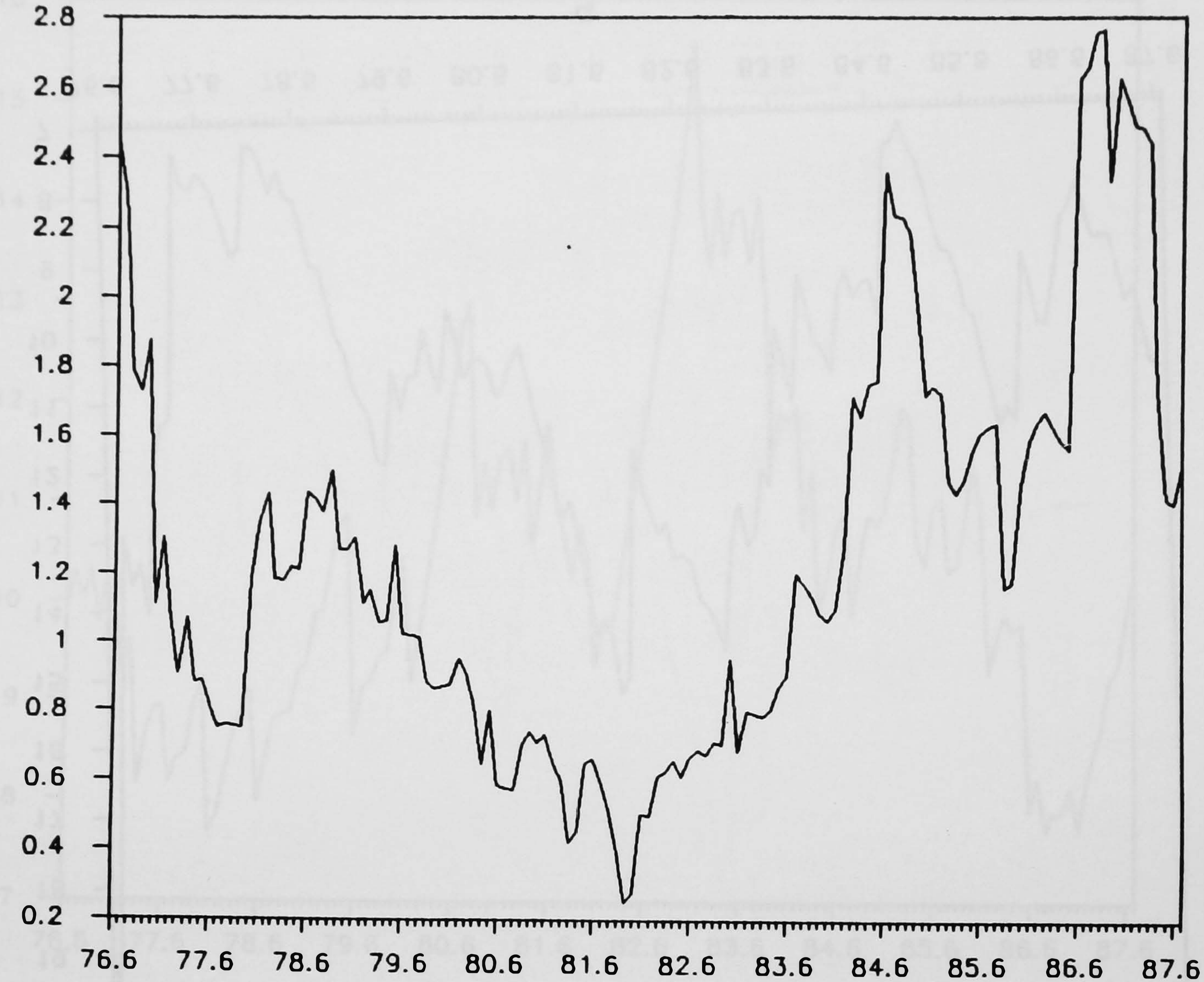
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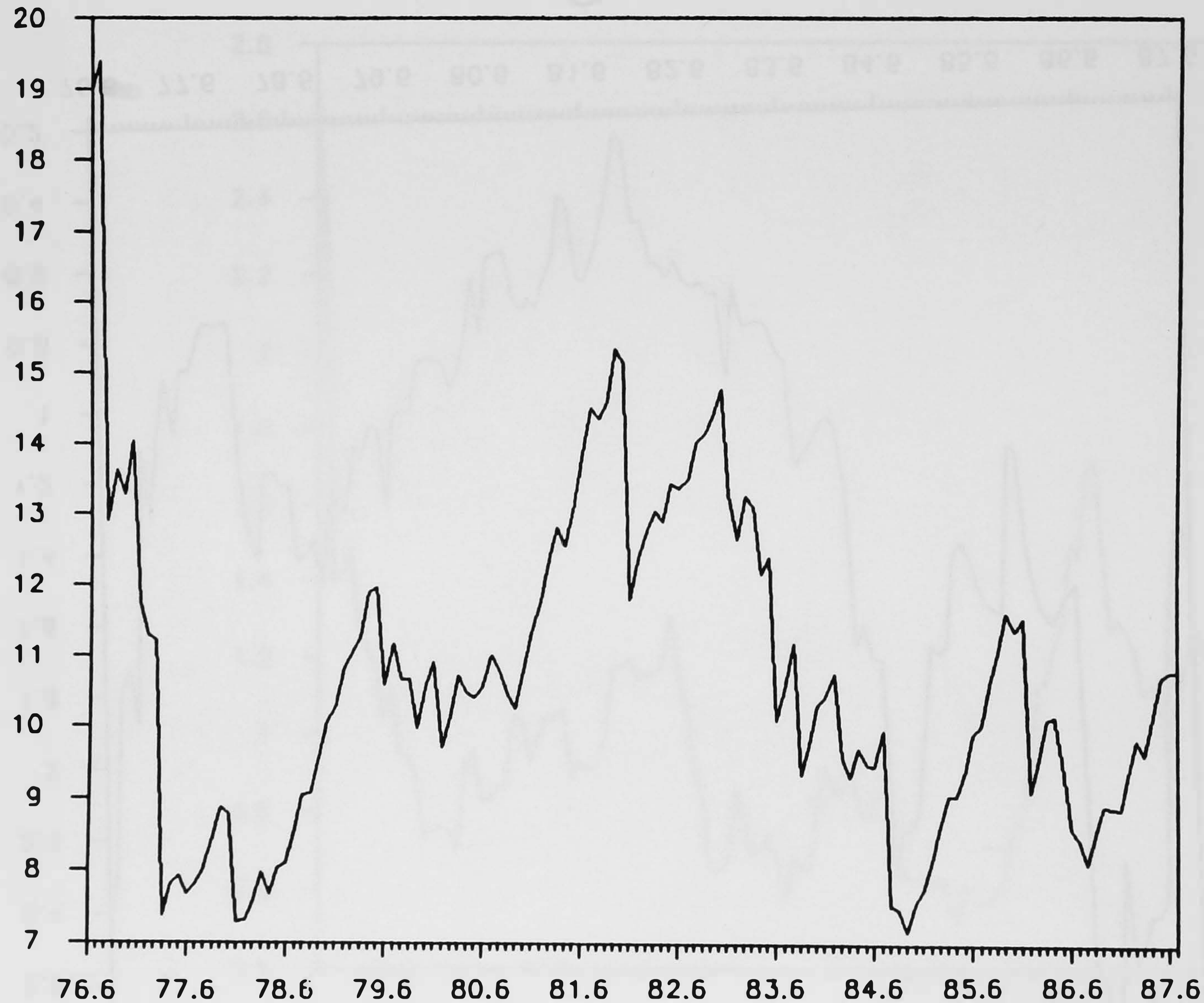
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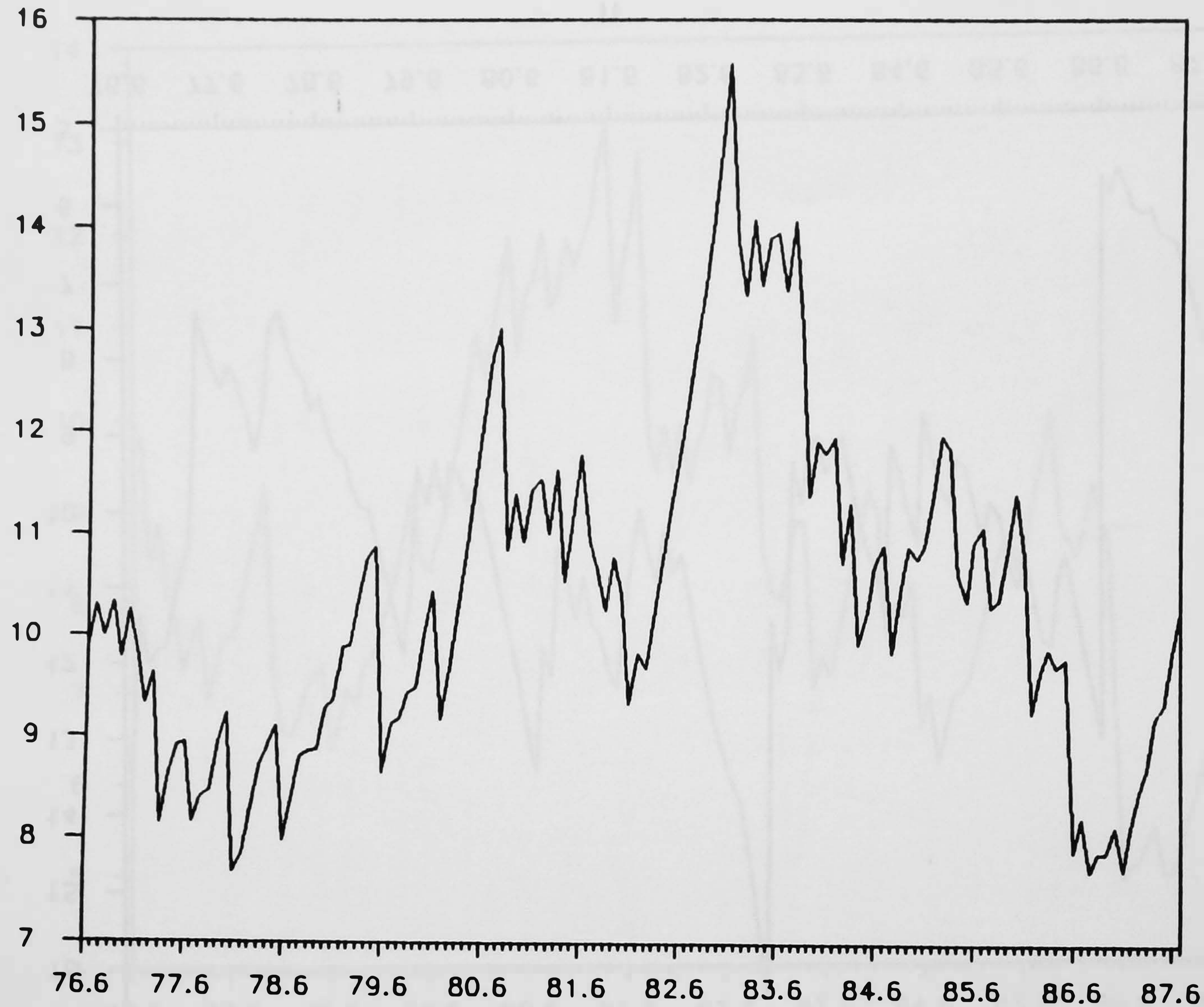

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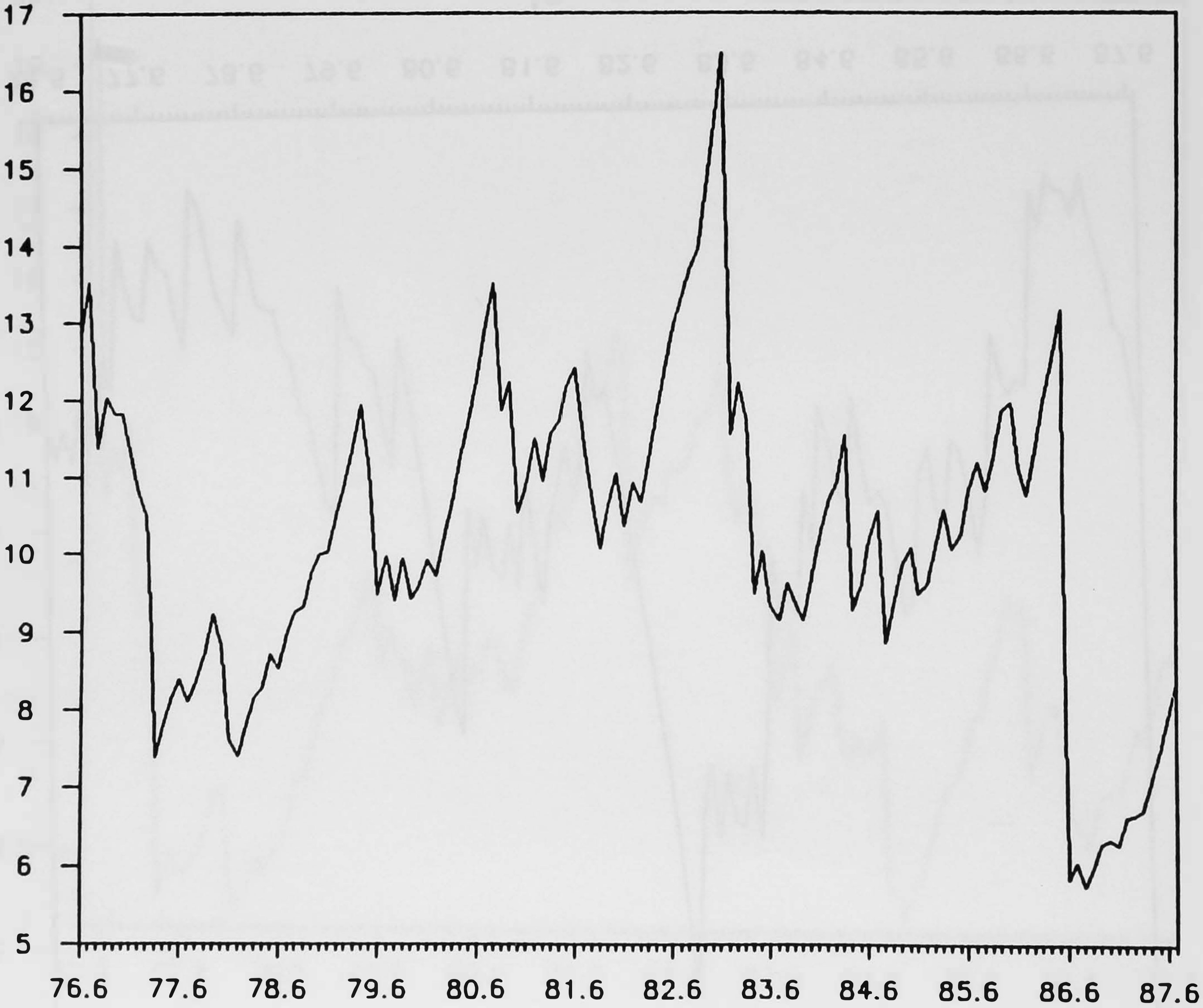

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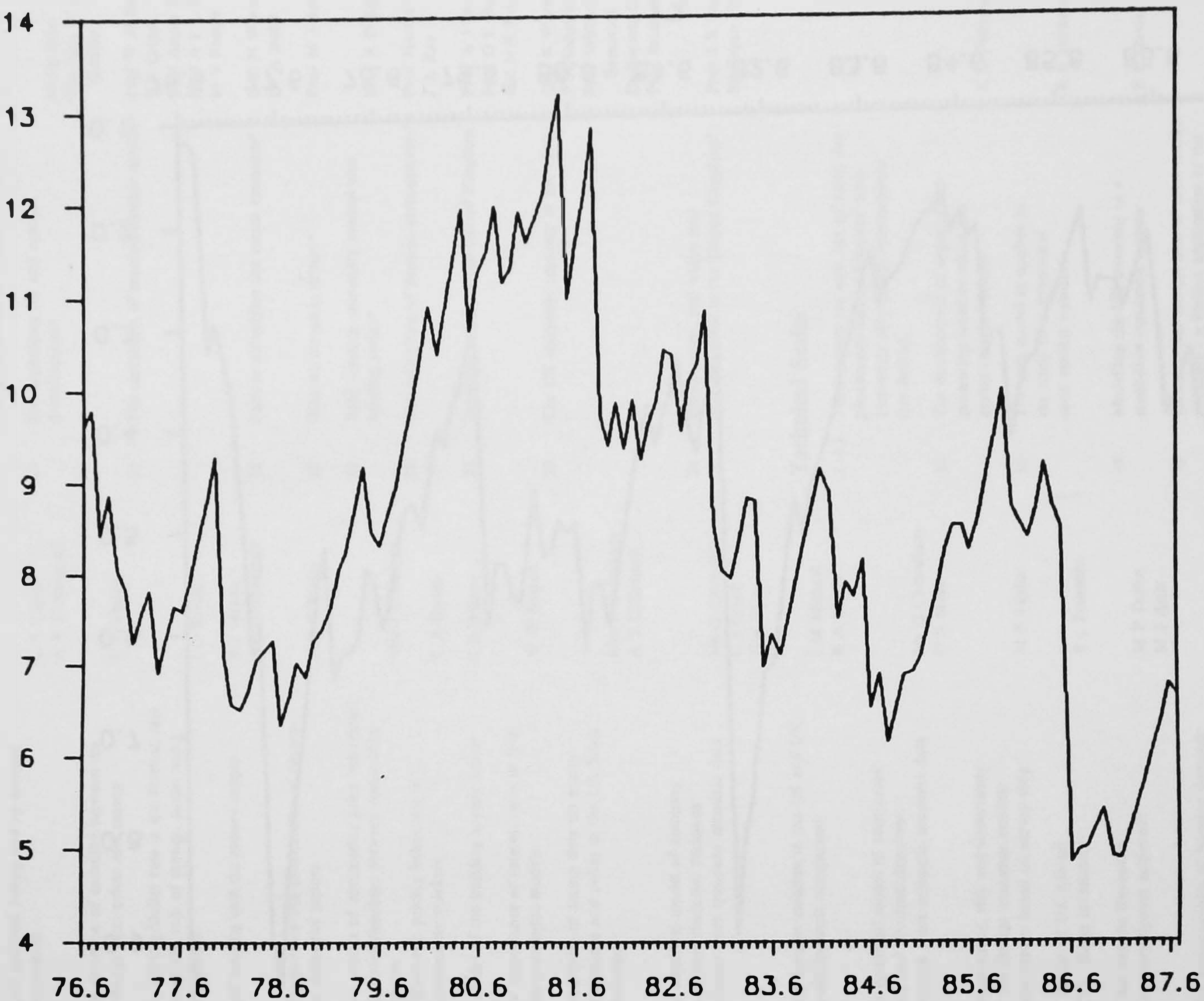
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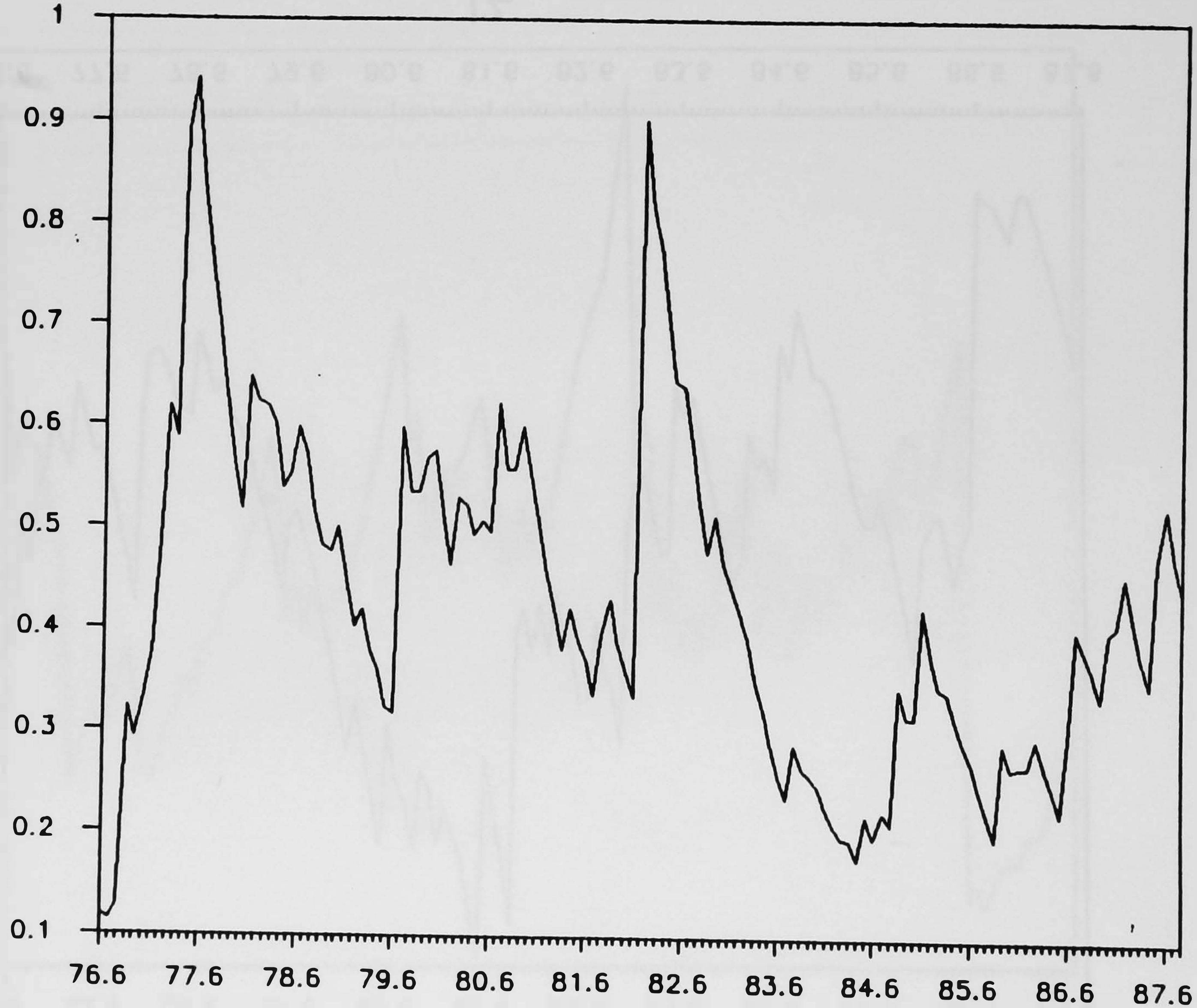
FT500 INDEX (VARIANCE)



13

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