

# **Bank of England**

## **Discussion Papers**

### *Technical Series*

No 21

**Modelling the flow of funds**

by

**D G Barr**

**Keith Cuthbertson**

*February 1989*

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The object of this Technical Series of Discussion Papers is to give wider circulation to econometric research work predominantly in connection with revising and updating the various Bank models, and to invite comment upon it; any comments should be sent to the authors at the address given below. The views expressed are those of the authors and not necessarily those of the Bank of England.

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## (I) INTRODUCTION

The impact of changes in financial variables, on the real sector and inflation are major issues in monetary economics. Broadly speaking changes in financial variables may arise from government policy, from external events and from changes in behaviour by the private sector. Fiscal policy, and open market operations by the authorities cause changes in domestic interest rates which are also strongly influenced by changes in foreign interest rates. Innovation in the provision of financial services initiated either by government legislation (eg Financial Services Act 1986) or by marketing considerations (eg entry of banks into the home loans market) may also lead to a reallocation of portfolios and a change in relative interest rates.

In the theoretical literature one could reasonably take the view that the determination of asset prices is governed by two seemingly contradictory approaches: that is by 'structural' or 'reduced form' models. Applied work on the term structure takes a 'reduced form' approach. For example in the expectations hypothesis long rates are viewed as depending on expected future spot rates, and the determinants of the 'liquidity premium' are often omitted or simply modelled as an additive ad hoc set of variables (Modigliani and Sutch 1967, Modigliani and Shiller 1973). The market segmentation and preferred habitat theories are rather loosely formulated and therefore difficult to test in any precise manner. In the structural approach asset demand and supply functions are estimated and the model is 'solved' for asset prices. However, the equilibrium returns generated by the structural approach will yield an implicit relationship between long rates and short rates. Furthermore, in the structural approach it is clear that the exogenous determinants of asset demands and supply functions may be viewed as influencing the liquidity premium. Hence the two approaches are really complementary.

It is generally accepted that the structural approach in the form of a portfolio balance model is a useful way to analyse the flow of funds and the determination of asset prices. Flow of funds models are essentially

structural models of the movement of funds between government, commercial companies, persons, the overseas sector and financial institutions. Such models are very diverse. A number of macroeconomic models of the financial sector have been built up in a somewhat piecemeal fashion (eg Spencer and Mowl (1978), Patterson et al 1987, Mayes and Savage 1980, Wallis 1984) or are highly aggregative (Minford et al 1980). In general such models provide a reasonable 'fit' to the data but theoretical restrictions are limited to little more than an examination of 'right signs' on certain coefficients. Adding-up constraints are usually met implicitly by a set of 'missing equations' whose structure often remains unexplored. Almost by construction such a piecemeal approach does not consider cross equation restrictions familiar in micro-theory such as homogeneity and symmetry of the coefficients on the 'price' vector.

The Brainard-Tobin (1968) approach fulfills the minimum requirement of adding-up but the long-run asset demands are somewhat ad hoc and are often not based on a choice theoretic model (Backus et al 1980, Cohen 1987, Green 1984, Owen 1986). The mean-variance model has been widely used to model sub-systems of the financial sector, notably the banking sector (eg Berndt et al 1980, Courakis 1975, 1980; Parkin et al 1970, White 1975). However, the theoretical problems associated with this approach (Keating 1985, Courakis 1988) are formidable and attempts to circumvent these can produce implausible outcomes. At an empirical level the mean-variance model and Brainard-Tobin type models have met with very mixed success.

In comparison with the empirical performance of systems models, single equation studies of the demand for assets, particularly aggregate money variables, could be said to have fared better (Hendry 1979, Hendry 1985, Baba et al 1988, Rose 1985). This is in large part due to the application of error-feedback and 'general to specific' modelling strategies (Hendry 1983, Hendry et al 1984). The use of the single equation cointegration techniques (Hendry 1986, Hall 1986, Granger 1986) has given new insights and impetus to the error feedback approach.

In this paper we wish to examine both the theoretical basis of the portfolio balance approach to modelling the flow of funds and 'general to specific' estimation strategies that may yield reasonable search procedures in a systems

framework. The practical results from applying the procedures in this paper are reported for the personal sector in Barr and Cuthbertson (1988a, 1988b) and are of sufficient quality for us to suggest that their wider application in modelling the flow of funds of other sectors could be beneficial.

The rest of this paper is organised as follows. In Section II we examine the applicability of alternative theories of the demand for assets to modelling the flow of funds and argue in favour of adopting an approach based on neo-classical consumer demand theory. Of the several approaches available we suggest development of the Almost Ideal Demand System (AIDS) (Deaton and Muellbauer 1980) to represent long-run asset demands. We are able successfully to interpret results from existing single equation studies in terms of the consumer theory approach. In Section III we discuss the issues of dynamic adjustment, modelling expectations and econometric problems in estimating a system of equations. A final section provides a summary of our main conclusions.

## II THEORETICAL MODELS

The development of theories of the demand for assets has proceeded in a number of directions simultaneously. Friedman (1956) argued that the demand for assets should be based on axioms of consumer choice. He focussed on the demand for money and presents a fairly long list of possible arguments of this function (ie a vector of expected returns, wealth and income) with 'signs' to be determined primarily by the data. Friedman eschews the idea of approaching the theory of demand for assets by considering the motives for holding assets. Friedman (1956) did not present an explicit model of consumer choice and hence his model contains few a priori and potentially refutable restrictions. This perhaps accounts in part for the development of the 'motives approach' which resulted in numerous models which may be classified under the broad headings of transactions, precautionary and risk aversion (or more usually, mean-variance) models. In contrast to Friedman (1956) such models yield precise functional forms and strong a priori restrictions. However, 'precision' is gained at a cost in term of the degree of applicability of the models. Transactions models apply only to narrow money and there appears to be no independent role for inflation (eg Baumol 1952, Tobin 1956) unless one introduces real assets into the choice set (eg



Feige and Parkin 1971). On theoretical grounds the applicability of such models to firms is open to doubt (Sprenkle 1969), and different models yield widely different a priori restrictions (eg the income elasticity in Baumol 1952 is 0.5, while in Akerlof and Milbourne (1980) it may be negative).

Precautionary demand models appear difficult to test because in principle we require a measure of the variance of transactions (Miller and Orr 1966). The mean-variance model (Tobin 1958) considers the demand for a set of assets and allows tests of negativity, symmetry, and homogeneity. However, narrow (non-interest bearing) money is dominated (Tsiang 1972) by other 'capital certain' assets (eg Building Society Deposits). Broad money which is also predominately 'capital certain' cannot be separately identified from other liquid assets. All capital certain assets (or liabilities) are homogenous in this model and hence it does not explain the choice between these assets which are a major element of the flow of funds (Cuthbertson 1985).

We wish to argue that from a theoretical standpoint it appears that it is best to approach the demand for assets, in the context of neoclassical demand theory, rather than from considering the 'motives' for holding money. We argue that this approach provides flexibility, yet allows one to test the basic axioms of rational choice. Of course, like any theory it may not be applicable in all cases. It is a natural extension of Friedman's (1956) ideas but incorporates technical aspects which post-date his contribution. To illustrate this approach we show how a particular (yet tractable) model of consumer demand, namely the AIDS model, may be adapted to the demand for assets. This yields insights into the interpretation of extant assets demand functions. In particular the role of wealth, income, interest rates and the rate of inflation in the demand for assets is clearly analysed within a coherent systems framework.

### **Separability and Two-Stage Budgeting**

The decision as to which portfolio of assets to hold is nested within the very general decision concerning labour supply, consumption and saving (Tobin 1969). A fully specified model would, for example, identify a theoretical relationship between the demand for any particular asset and the willingness

to supply labour. Such a model would be intractable and as a result modelling has developed by assuming some groups of decisions to be taken 'independently' of others. For example, the Keynesian consumption function takes incomes as given and is not concerned explicitly with either labour supply or portfolio allocation.

This separation is an essential part of the flow of funds model. It is assumed that the portfolio allocation decision is independent of all others. A consequence of this assumption is that the total size of the portfolio at the start of any decision period can be taken to be exogenous ie the labour supply and saving decisions are independent of the choice between assets.

This notion of separability (or two stage budgeting) also provides a method of delineating the 'size' of the portfolio allocation model per se.

Separability implies that we can consider, for example, that the personal sector makes decisions on saving, then borrowing, then splits gross wealth between capital certain (liquid) assets, and risky assets (eg gilts, foreign). Separability implies that the choice within the set of liquid assets depends only on returns on the liquid assets and total wealth held in liquid assets. If separability holds in an appropriate form we can ignore asset returns on risky assets when considering the choice between alternative liquid assets. This may reduce collinearity problems. At the outset we will apply the principle of separability to reduce our model to a more manageable size although it is beyond the scope of this paper to discuss non-parameteric tests of separability (Swofford and Whitney 1986).

Separability is also useful when dealing with rationing. If a set of assets is assumed to be weakly separable from the rationed assets (eg personal sector money holdings from rationed mortgage loans for the 1960s and 1970s) then all one needs to do is to deduct the expenditure on the rationed goods (mortgages) from total expenditure and model the demand for this set as a separate system. Alternatively one can include some explicit measure of rationing other than price (eg collateral). Finally, in applied demand analysis it is usual to assume that preferences are weakly intertemporally separable. Demand in the current period depending only on initial endowments (wealth) and expected



prices over a single period: we invoke this assumption in the present paper - (intertemporal models for consumer demand theory have been investigated by Weissenberger 1986 and Rossi 1987).

In the systems approach the precise nature of the interdependences derives from two conditions. The first of these is simply that a number of accounting identities should be satisfied, the second relates to the theoretical model assumed at the outset. The accounting identities must be satisfied if the equations are to form a system (eg Brainard and Tobin (1968)). The advantages of going beyond the accounting stage and imposing a theoretical model are that if the theory is 'correct' the estimated model will be a more accurate representation of the true system and that the properties of the model will be easier to assess and understand.

The basic model used in this study is the Almost Ideal Demand System due to Deaton and Muellbauer although we modify the model in various ways. In particular, we assume a quasi-separable utility function, from which the following separation of the associated cost function can be derived,

$$C(u, p^a, p^b, p^c) = C(u, C_a(u, p^a), C_b(u, p^b), C_c(u, p^c))$$

where a, b, c denote groups of assets;  $C_a$ , etc are the sub-cost functions and  $p^i$  are the vector of prices of 'goods' in group i ( $i=a, b, c$ ).

The cost function denotes the minimum cost of attaining a specified utility level  $u$ . (This is equivalent to the familiar optimum at which  $u$  is the maximum utility available from specified expenditure equal to  $C$ .) By separating the cost function according to groups a, b, c each sub-cost function can be minimised (and in a very loose sense, a form of sub-utility function maximised). The global cost can then be minimised given that each sub group is allocated optimally. This procedure is analogous to two stage budgeting (although further restrictions are needed to ensure this outcome: see below).

Assuming the first stage allocation between groups to have occurred, the total wealth given to each group can be taken as the budget constraint for an optimal allocation within the group. The advantage of this separation form

an econometric point of view is that it allows the demand functions within each sub group to depend (directly) only on those rates of return in the same group (ie there are fewer explanatory variables to be considered). This does not however imply that the shares are totally independent of the other rates since these will influence the allocation of total wealth between the sub-groups. Consequently unless there are no wealth effects (the notional sub-utility functions are homothetic) the shares will depend indirectly on the excluded rates.

A word of caution is in order here concerning the two-stage budgeting procedure proposed. Weak separability is necessary and sufficient for the second stage (ie 'lower level' decisions) of two-stage budgeting, which is the main area of application discussed below. However weak separability has some drawbacks. First, it places quite severe restrictions on the degree of substitutability between goods in different groups (Pudney 1981). For example whole groups will be substitutes or complements with each other. The second problem is, potentially, more serious however. In modelling the 'upper-level' decisions (ie the choice between the broad groups a, b, c etc) it would be extremely useful to be able to establish the maximisation problem in terms of group price and quantity indices (rather than having to utilise all prices that the consumer faces), since this would considerably reduce the number of parameters to be estimated for the 'upper-level' equations. However, strictly speaking, this is only possible under highly restrictive assumptions. If preferences are homothetic which implies all 'expenditure elasticities' are unity or equivalently that the budget shares within each group are independent of total group expenditure, then 'group price indices' can be legitimately used in determining the upper level allocations. If preferences are not homothetic then group price indices can be used providing the utility function is strongly separable and has the Generalised Gorman polar form. However the former implies 'additivity' between groups:

$$u = u_a(q_a) + u_b(q_b) + \dots$$

Additivity is restrictive in that (a) inferior goods are ruled out; (b) goods can only be substitutes given that inferior goods are not allowed; (c) expenditure elasticities are proportional to price elasticities.

Neither of the above restrictive assumptions seems attractive in modelling asset demands. The problem then is that the group price indices  $P_a$  say are given by  $P_a = P_a(u_a, p^a)$  and are dependent on  $u_a$ , which in turn depends on all other prices outside the group. One possibility is not to invoke homotheticity and assume that  $P_a$  does not vary very much with  $u_a$  and hence most of the 'explanation' of  $P_a$  is the sub-set of prices,  $p^a$ . Given the other approximations involved in empirical studies this may be a reasonable expedient to adopt if homotheticity is not found to hold in the data. Having aired the above problems we now return to discuss the 'second stage' or 'lower-level' demand functions.

To derive the demand functions within each sub group we proceed by analogy with consumer demand theory and then present the model in terms of financial asset demands. Initially we deal only with the determinants of long-run static equilibrium asset demands. Considerations of dynamic adjustment are discussed in Section III.

### III CONSUMER DEMAND THEORY

Neoclassical demand theory is usually based either on maximisation of utility subject to an expenditure constraint or the equivalent 'dual' of minimising cost to achieve a given level of utility. For expositional clarity we assume weak intertemporal separability and consider only a one period decision. The general consumer maximisation problem is therefore:

$$\max u = u(q_1, \dots, q_n) \quad (1)$$

$$\text{subject to: } \sum_{i=1}^n \tilde{p}_i q_i = C \quad (1a)$$

or

$$\min C(\tilde{p}, u) \quad (2)$$

$$\text{subject to: } u = \bar{u} \quad (2a)$$

where  $u$  = utility

$q_i$  = quantity of (real) goods  $i$

$\tilde{p}_i$  = price of good  $i$

$C$  = total nominal expenditure

Approaching the problem via the dual is mathematically more tractable but also has the added advantage that we may state the problem without recourse to the underlying utility function if we so wish. The axioms of rational choice in consumer demand theory (ie the existence of consistent preferences) are met providing we choose a cost function that is concave and homogeneous of degree one in prices.<sup>(1)</sup> Of the several flexible functional forms available we select the PIGLOG (Price Independent Generalised Logarithmic) which, in common with others (eg indirect translog, Christensen et al 1975) is a second order approximation but has the advantage of desirable aggregation properties and yields long-run equilibria in the levels of the variables. (The latter is important when we consider co-integration in Section III.) Within the PIGLOG class we choose the AIDS cost function:

$$\ln C = \alpha_0 + \sum_{i=1}^n \alpha_i \ln \tilde{p}_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln \tilde{p}_i \ln \tilde{p}_j + \beta_0 u \left( \prod_k \tilde{p}_k^{\beta_k} \right) \quad (3)$$

Using Shephard's Lemma:

$$s_i = d(\ln C) / d(\ln \tilde{p}_i) \quad (4)$$

the goods expenditure shares  $s_i$  are given by<sup>(2)</sup>

$$s_i = \alpha_i + \sum_{j=1}^n \gamma_{ij} \ln \tilde{p}_j + \beta_i \ln (C/P^*) \quad (5)$$

where

$$\ln P^* = \sum_{i=1}^n s_i \ln \tilde{p}_i \quad (5a)$$

$$\gamma_{ii} = \gamma_{ii}^* \quad (5b)$$

$$\gamma_{ij} = 0.5 (\gamma_{ij}^* + \gamma_{ji}^*) \quad (5c)$$

$$s_i = \tilde{p}_i q_i / C \quad (5d)$$

The adding up constraint (1a) implies:

$$\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = 0, \quad \sum_i \beta_i = 0 \quad (6)$$

Homogeneity requires:

$$\sum_j \gamma_{ij} = 0 \quad (7)$$

Symmetry and negativity (of the Hicksian demand functions) are direct consequences of the axioms of rational choice. From (5c) we see that symmetry implies: (3)

$$\gamma_{ij} = \gamma_{ji} \quad (8)$$

Negativity arises from the concavity of the cost function and implies that the matrix of coefficients  $k_{ij}$ :

$$k_{ij} = \gamma_{ij} + \beta_i \beta_j \ln(C/P^*) - s_i \delta_{ij} + s_i s_j \quad (8a)$$

(where  $\delta_{ij}$  is the Kronecker delta), is negative semi-definite.

Thus our systems approach implicitly imposes data admissibility in the form of adding up constraints and the additional theoretical constraints of symmetry homogeneity and negativity.

### Asset demands

To motivate the application of consumer demand theory to asset demands suppose we make the reasonable assumption that there exists a utility function defined over the expected (one period ahead) value of real assets,  $a_{it+1}^r$ :<sup>(4)</sup>

$$u = u \left( a_{1t+1}^r, a_{2t+1}^r, \dots, a_{nt+1}^r \right) \quad (9)$$

The relationship between real assets in adjacent periods is:

$$a_{it}^r = p_{it}^r a_{it+1}^r \quad (10)$$

$$p_{it}^r = \left[ (1 + r_{it}) (1 - g_{zt}) \right]^{-1} \quad (11)$$

where

$r_{it}$  = expected (proportionate) nominal return on asset  $i$ , between  $t$  and  $t+1$  (including any capital gains).

$z_t$  = goods price index.

$g_{zt}$  = expected (proportionate) rate of goods price inflation between  $t$  and  $t+1$ .

$p_{it}^r$  = 'real price' (ie approximately equal to the inverse of the expected real interest rate,  $(r_{it} - g_{zt})$ ).

The budget constraint here is that nominal (or real) assets sum to nominal (or real) wealth

$$\sum_i a_{it} = w_t \quad (12)$$

$$\sum_i a_{it}^r = w_t^r = (w/z)_t \quad (13)$$

$Z$  = aggregate price level for goods.



Using (10) and (13) the budget constraint becomes

$$\sum_i p_{it}^T a_{it+1}^T = W_t^T \quad (14)$$

Comparing (9) and (14) with the equivalent equations (1) and (1a) from consumer demand theory we can re-formulate the above model in terms of a cost function which by Sheppard's Lemma yields asset share equations

$$s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_{jt}^T + \beta_i \ln (W_t^T / P^{*T})_t \quad (15)$$

where

$$s_i = a_{it} / W_t \quad (15a)$$

$$\begin{aligned} \ln p_{jt}^T &= \ln [(1 + r_{jt}) (1 - g_{zt})]^{-1} = \ln (1 + r_{jt})^{-1} + g_z \\ &= \ln p_{jt} + g_z \end{aligned} \quad (15b)$$

$$\ln P_t^{*T} = \sum_i s_i \ln p_{it}^T = \ln P_t^* + g_{zt} \quad (15c)$$

where

$$\ln P_t^* = \sum_i s_i \ln (1 + r_{it})^{-1} = \sum_i s_i \ln p_{it} \quad (15d)$$

We have separated out the nominal "prices" ( $\ln p_{it}$ ,  $\ln P_t^*$ ) from the rate of goods price inflation  $g_{zt}$ . Note that  $\ln P^{*T}$  may be interpreted as a composite real interest rate.

It is interesting to compare the above approach with the mean-variance model. The latter is consistent with the basic choice axioms of demand analysis but ignores any transactions or hedging demand. It is therefore not appropriate to narrow money or perhaps to the asset structure of life assurance and

pension funds where a 'long-dated' asset may provide a perfect hedge against long-term nominal liabilities. Our consumer demand approach is flexible in that it allows transactions, hedging or perceptions of risk (ie variances and covariances of returns) to be represented in the parameters of the cost function. Should any of the latter change then this will be picked up empirically either by a failure of homogeneity or symmetry or by parameter instability. In principle the cost function could be altered in the light of such empirical results to produce a more general model within this broad framework (eg see the incorporation of a transactions variable, below).

In applied work, we usually assume some form of weak separability, which may be viewed as giving rise to multi-stage budgeting. Hence equation (15) may be applied to a subset of the total choice set.

### Transactions Variable

The perceived cost of achieving a given level of utility may depend in part on which assets are most useful in undertaking transactions. For example this may be important for (gross) liquid assets, and capital certain liabilities. In terms of the AIDS cost function (equation 5) we require an additional term

$$\phi \ln (e_{t+1}/z_{t+1}) \quad (16)$$

where  $e_{t+1}$  = expected level of nominal transactions in period  $t+1$

$z_{t+1}$  = expected price level at  $t+1$

Thus the cost of achieving a given level of utility depends on the level of future transactions.

Using Sheppard's lemma our asset demand (share) equations become

$$s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_{jt}^r + \beta_i \left[ \ln (W/Z)_t - \phi \ln (e_{t+1}/z_{t+1}) - \ln P^* - g_z \right] \quad (17)$$

The terms in real wealth  $\ln(W/Z)_t$  and real transactions  $\ln(e_{t+1}/z_{t+1})$  can be reparameterised into a wealth-transactions (or wealth-income) ratio and a separate real transactions term. It seems reasonable when using discrete time periods to use future expected transactions when deciding on the allocation of assets at the end of the period rather than the more conventional current and lagged values. A further testable restriction concerns the wealth and expenditure terms: the ratio of these coefficients equals  $\phi$  for each equation. A further nested test is  $\phi=1$ : the cost function is homogenous with respect to the level of transactions; equation (17) reduces in this case to:

$$s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i \ln (W_t / e_{t+1} P_t^*) \quad (18)$$

$e_{t+1} P_t^*$  is the discounted present value of future (nominal) transactions and hence  $\ln (W_t / e_{t+1} P_t^*)$  is the current period wealth expected income ratio.

#### Zero Nominal Returns

In cases where one or more of the assets has a zero nominal return (eg notes and coin, non-interest bearing sight deposits) its  $\gamma_{ij}$  coefficient picks up an inflation effect which is discussed further below. All assets yield an unobservable return, (ie utility from non-pecuniary factors) including those with an explicit yield. These implicit returns are incorporated in the form of the utility function. In such cases, the adding up constraints on the  $\gamma_{ij}$  must still hold but symmetry can be imposed only for any sub-block of the  $[\gamma_{ij}]$  matrix for assets with non-zero nominal returns. Homogeneity can also still be tested. To illustrate these points consider a model with notes and coin (ie asset '1') and two interest bearing assets. The  $\gamma_{ij}$  matrix is:

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \begin{bmatrix} g_z \\ \ln p_2^r \\ \ln p_3^r \end{bmatrix}$$

The symmetry restriction then becomes  $\gamma_{23} = \gamma_{32}$  and homogeneity restrictions can be applied to each row, that is,  $\sum_j \gamma_{ij} = 0$ . Note that homogeneity

implies that inflation does not have an effect via the  $[\gamma_{ij}]$  matrix.

### Revaluations

An issue that arises in the case of capital uncertain assets: is the choice between market value wealth and nominal wealth (ie cumulative flows).

Consider first the use of market value wealth,  $w_t^m$ . Increases in  $w_t^m$  consists of revaluations on all assets  $REV_t$  and new net flows into this set of assets  $F_t$ .

$$w_t^m = w_{t-1}^m + REV_t + F_t \quad (19a)$$

A logarithmic approximation to the above is:

$$\ln w_t^m = \ln w_{t-1}^m + (REV_t / w_{t-1}^m) + (F_t / w_{t-1}^m) \quad (19b)$$

or

$$\ln w_t^m = \sum_0^t (REV_{t-j} + F_{t-j}) / w_{t-j-1}^m \quad (20)$$

Total revaluations in period  $t$ , consists of the sum of revaluations for each asset,  $REV_{it}$ , and the latter depends on the return on asset  $a_i$  between  $t-1$  and  $t$ ; hence

$$REV_t = \sum_{i=1}^k REV_{it} \quad (20a)$$

$$REV_{it} = a_{it-1}^m r_{it} \quad (20b)$$

It is clear, therefore, that equation (15) assumes that cumulative revaluations  $REV_t$  and the cumulative flow of funds  $F_t$  into this set of assets have an equal ( $=\beta_i$ ) effect on the asset share  $s_i$ . This may be a reasonable

working assumption for the long-run but in the short-run one may wish to impose the restriction that all (or most) of the revaluations of asset 'i' accrue to asset 'i'. The short-run relationship would therefore be

$$\frac{\Delta a_{it}^n}{W_t^m} = \delta (F_t^T / W_t^m) \quad (21)$$

where  $\Delta a_{it}^n$  = nominal flow into asset i

$F_t^T$  = total nominal flow into all assets in the choice set.

#### IV INTERPRETATION OF EMPIRICAL ASSET DEMAND FUNCTIONS

##### The demand for (nib) M1

What does our model tell us about the appropriate form for the demand for non-interest bearing, (nib), M1, or 'narrow money', which is usually estimated as a single equation rather than as part of a system?<sup>(5)</sup> For expositional purposes assume the demand for liquid assets is weakly separable from other asset and liability choices (and from consumption and leisure).

Equation (15) implies that the demand for M1 may depend on a set of real prices (yields), real wealth and a composite real interest rate ( $\ln P^*$ ). This appears at variance with single equation studies where M1 often depends on only a single nominal interest rate, (Hendry 1985; for exceptions see some of the US studies cited in Judd and Scadding, 1982) a transactions variable and the rate of inflation. Moreover, the impact of the rate of inflation on money demand in steady state growth equilibrium (Currie 1981, Patterson and Ryding 1984) is found to be substantial and one can only interpret this term within the 'motives framework' in a rather ad hoc manner. For example, the transactions model would suggest a move from money into goods. In the precautionary model the level of inflation may be interpreted as a proxy for the variance of inflation (and nominal transactions). However, we argue below that our consumer theory model provides a very clear theoretical rationale for the inclusion of inflation and hence provides a more satisfactory way of explaining the presence of this variable in aggregate demand for money equations.



The (expected) inflation rate appears in (15) in two ways: as part of the rate of return (price) and in the wealth term. We assume asset '1' is nib-M1 with a zero nominal return (ie  $\ln p_1 = 0$ )<sup>(6)</sup>, hence:

$$s_1 = \alpha_1 + \sum_{j \neq 1} \gamma_{1j} \ln p_{jt} + \left[ \sum_{j=1}^n \gamma_{ij} \right] g_z + \beta_1 \ln (W/Z)_t - \beta_1 [(\ln P_t^* + g_z)] \quad (22)$$

where  $Z_t$  is the aggregate goods price level.

The sum of the coefficients on the nominal returns on assets other than narrow money (ie  $\sum_{j \neq 1} \gamma_{ij} \ln p_{jt}$ ) do not equal those on the rate of inflation. Hence a 1% rise in all nominal yields ( $\ln p_{jt}; j \neq 1$ ) and the rate of inflation ( $g_z$ ) will have a direct impact on the demand for narrow money. This would appear to justify the inclusion of nominal rates ( $r_{jt}$ ) and the rate of inflation in aggregate single equation studies of the demand for nib-M1. However, if homogeneity holds  $\sum \gamma_{ij} = 0$ , the separate inflation term disappears and only nominal rates appear to be required (we ignore the wealth term for the moment).

Clearly it is incorrect to test for homogeneity by imposing relative nominal interest rates, that is,  $\sum_{j \neq 1} \gamma_{ij} = 0$  or by running an equation of the form:

$$s_1 = \alpha_1 + \delta_{11} g_z + \sum_{j \neq 1} \delta_{1j} \ln p_{jt} + \text{other terms} \quad (23)$$

and testing,  $\sum_{j \neq 1} \delta_{1j} = \delta_{11}$ , that is, imposing real prices (interest rates). Hence by considering nib-M1 as part of a system of demand equations that obey the axioms of consumer choice, possible errors of interpretation in the single equation approach are clearly highlighted.<sup>(7)</sup>

A further reparameterisation of equation (22) gives

$$s_1 = \alpha_1 + \sum_{j \neq 1} \delta_{1j} \ln p_{jt} + \theta_1 g_z + \theta_2 \ln (W/Z)_t \quad (24)$$



where

$$\delta_{1j} = (\gamma_{1j} - \beta_1 s_j) \quad j \neq 1 \quad (24a)$$

$$\theta_1 = \left( \sum_{j=1}^n \gamma_{1j} \right) - \beta_1 \quad (24b)$$

$$\theta_2 = \beta_1 \quad (24c)$$

If in addition we assume that real wealth (held in liquid assets) and real income are highly correlated then (24) looks like a conventional long-run demand function for (nib) M1, obtained from single equation estimation (eg Hendry 1985). The dependent variable could then constitute the money-income ratio.

If homogeneity holds then any 'inflation effect'  $\theta_1$  comes solely from the wealth coefficient  $\beta_1$ . Thus an 'inflation effect' in our consumer demand approach arises quite naturally from the Hicksian (Hicks 1936) measure of wealth implicit in the  $\ln(W^T/P^{*T})$  term of equation (15). Hicksian measures of income and wealth have proved to be useful theoretical (eg Patterson and Stephenson 1988) and practical (eg Taylor and Threadgold 1979, Hendry and von-Ungern Sternberg 1983) constructs in other areas and they reappear here in an internally consistent model of asset demands.

It is easily demonstrated that the wealth elasticity of demand is:

$$E_w = \left( \frac{\beta_1}{s_1} + 1 \right) \quad (25)$$

Hence given homogeneity:

$$\begin{aligned} E_w < 1 & \quad \theta_1 > 0 \\ E_w = 1 & \quad \text{implies} \quad \theta_1 = 0 \\ E_w > 1 & \quad \theta_1 < 0 \end{aligned} \quad (26)$$

Thus if homogeneity holds, the 'inflation effect', is negative or zero, or positive depending whether the wealth elasticity is greater than or equal to or less than unity. A negative long-run inflation effect found in empirical studies for M1 may therefore be attributed either to a failure of homogeneity, or acceptance of homogeneity but a wealth elasticity greater than unity. If real income is a reasonable proxy for real wealth, then given the empirical result that the income elasticity of the demand for M1 is unity (Hendry 1985) any long-run impact of inflation on money demand may be attributed to a breakdown of homogeneity, if we accept the consumer demand model proposed in this paper.

In empirical aggregate (nib)-M1 equations usually a single nominal interest rate is included on the grounds that this provides a reasonable proxy variable for a set of opportunity cost variables which are likely to be highly collinear. Inflation and a scale variable (usually a transactions variable) are included as separate regressors. Hence estimated M1 equations, broadly speaking have a long-run relationship of the form (24) and therefore the estimated nominal interest (price) response coefficient is

$$(\gamma_{12} - \beta_1 s_2) \ln p_{2t} \quad (27)$$

where asset '1' = nib-M1 and ' $\ln p_{2t}$ ' constitutes the chosen opportunity cost variable. It is immediately apparent that the estimated interest rate response is a combination of the (Hicksian) substitution parameter  $\gamma_{12}$  and the wealth coefficient  $\beta_1$ . Unless  $\beta_1 = 0$ , in the true model then any estimated interest rate response is an incorrect measure of the Hicksian substitution effect. If we accept the consumer demand approach and that  $E_w < 1$  (ie  $\beta_1 < 0$ ) for a 'transactions asset', then we expect conventional demand functions for M1 to understate the Hicksian substitution effect.

It is worth noting that even with homogeneity and  $\beta_1 = 0$ , (ie  $E_w = 1$ ) a one per cent increase in all nominal interest rates and the rate of inflation would lead to a change in the demand for (nib)-M1. This represents a substitution away from an asset (nib)-M1 with a greater negative real rate of return ( $= g_2$ ) to those with an unchanged real return.

The interdependence between the size of the wealth and transactions elasticity in the consumer demand approach has interesting implications for conventional demand for M1 equations. The transactions and wealth elasticities are

$$E_e = -\beta_i \phi / s_i \quad (28a)$$

$$E_w = (\beta_i / s_i) + 1 \quad (28b)$$

If  $\phi = 1$  then  $E_e + E_w = 1$ . If the consumer demand approach provides the correct model and real transactions and real wealth are highly correlated then we would not be surprised to observe a unit transactions (income) elasticity in conventional demand for M1 equations.

We can define 'transactions assets' as those for which  $E_e > 0$  and as this requires  $\beta_i < 0$ , transactions assets also have  $E_w < 1$ . If  $\beta_i > -s_i$  then 'transactions assets' also have a positive wealth elasticity. We might expect transactions assets to have  $E_e > 0$ ,  $0 < E_w < 1$  and if we define luxuries as those assets for which  $E_w > 1$  then 'transactions assets' are "necessities". Hence the relationship between wealth and income elasticities for transactions assets accords with one's intuitive notions. Further, the above results are broadly consistent with those found in Cumming (1981) for aggregate M1, where he finds that  $E_w = 0.42$  and  $E_e = 0.36$ . Thus our theoretical model suggests the inclusion of wealth as well as income in the demand for M1, and indicates that a unit income elasticity may be the result of omitted variables in conventional demand functions (see also empirical results in Hall et al 1988).

#### Other asset demands

Above we have concentrated on the relationship between the consumer demand approach and the aggregate demand for (nib) M1. However the consumer demand approach is very general and can also be used to model capital uncertain assets and liabilities.

For a set of  $k$  weakly separable assets which all have non-zero nominal rates of return equation (15) may be written

$$s_i = \alpha_i + \sum_{j=1}^k \gamma_{ij} \ln p_{jt} + \left( \sum_{j=1}^k \gamma_{ij} \right) g_z + \beta_i [\ln (W/Z)_t - \ln P_t^* - g_{zt}] \quad (29)$$

Homogeneity implies that the separate term in  $g_z$  is zero and that the nominal rates (prices),  $\ln p_{jt}$ , can be reparameterised as relative rates. Thus for a set of assets that are weakly separable from zero yield transactions assets inflation only appears in the (Hicksian) wealth term. If, as is often the case in applied studies, inflation and  $\ln (W/Z)_t$  are entered separately (eg Friedman and Roley 1979) we would not expect a zero 'inflation effect' for all assets in the choice set, unless homotheticity is assumed.

If the Hicksian wealth term is entered correctly (as in equations 15 and 29) then tests of homogeneity and symmetry are straightforward.

The AIDS share equations (29) are derived from a cost function that is a second order approximation, and hence the variance-covariance structure of asset prices does not appear explicitly in (29). If there are major shifts over time in the variance-covariance matrix of asset returns then this would be reflected in temporal parameter instability in the  $\gamma_{ij}$ 's.

The wealth term emphasises the role of expected end of period wealth as a determinant of asset demands:

$$\ln (W^T/P^{*T})_t = \ln (W/Z)_t + \left( \sum_i s_i r_{it} - g_{zt} \right) \quad (30)$$

The term  $\sum s_i r_i$  is a composite expected nominal return between  $t$  and  $t+1$ , and the final term in parentheses is a composite expected real interest rate. Thus  $\ln (W^T/P^{*T})_t$  is expected real wealth at the end of the period. Thus when modelling asset demands the independent variable is expected wealth rather than current wealth.

## Summary

The consumer demand approach and in particular use of the AIDS model applied to asset demands allows one to test the basic axioms of rational choice by directly testing restrictions in the parameters of the AIDS share equations. The approach appears to be less restrictive than the inventory, precautionary and mean-variance models yet utilises a theoretically coherent structure. In principle, the consumer demand approach can be applied to capital certain as well as capital uncertain assets and liabilities and provides insights into extant single equation asset demand functions.

## V DYNAMIC ADJUSTMENT

Our long-run AIDS share equations (15) may be represented in vector notation:

$$s_t^* = \Pi X_t \quad (31)$$

where  $s_t$  =  $k \times 1$  vector of asset shares

$X_t$  =  $m \times 1$  vector of independent variables

$\Pi$  =  $k \times m$  matrix of long-run parameters

In a system of asset demand equations if we include only own-lags, then we must implicitly accept that all assets adjust at the same rate (Smith 1975). To avoid this problem cross-lagged terms must also be included (Brainard and Tobin 1968). The latter can be rationalised by generalising the quadratic cost of adjustment function of Christofides (1976):

$$L^* = (s - s^*)_t' C_1 (s - s^*)_t + \Delta s_t' (C_2) \Delta s_t - (\Delta s_t) C_3 (\Delta s_t^*) \quad (32)$$

where  $C_i$  ( $i = 1, 2, 3$ ) are conformable adjustment matrices. Minimising  $L^*$  with respect to  $s_t$  we obtain generalised error feedback equations, EFE:

$$\Delta s_t = \Pi^* \Delta X_t + L (s - s^*)_{t-1} \quad (33)$$

where the disequilibria in  $(k-1)$  asset shares at time  $t-1$  influence the current period adjustment of any particular asset share. Since



$\sum_{i=1}^k (s_i - s_i^*)_{t-1} = 0$ , only the  $(k-1)$  independent disequilibrium shares are required in (23) (ie  $L$  is  $(k \times (k-1))$ ) (Anderson and Blundell 1983). The adding up restrictions imply that the columns of  $\Pi^*$  and  $L$  sum to zero. In addition on intuitive grounds we might expect the diagonal elements of  $L$  to be negative. However the latter is not required for dynamic stability. As long as the eigen-values of the appropriate adjustment matrix have modulus less than unity, then the system is dynamically stable.<sup>(6)</sup>

The above adjustment equation could be further generalised by including additional lagged own and cross-shares that is, terms of in  $\Delta s_{t-j}$  and  $\Delta x_{t-j}$ ; although the dynamics are then not consistent with the quadratic cost of adjustment function (32). Indeed one can also interpret (33) merely as a reasonably parsimonious method of incorporating dynamics while maintaining the adding up restrictions but without alluding to the cost function (32). We feel that quadratic costs are very unlikely to apply to adjustments in financial assets (eg costs are unlikely to rise with the size of the transaction undertaken) and we therefore interpret equation (33) merely as a convenient method of characterising sluggish adjustment.

Several methods are available to obtain a parsimonious set of dynamic share equations. First, one can substitute for  $s^*$  from (15) in (33) producing a general ADL model and then test down. Second we can reparameterise the ADL model into an error feedback formulation as in (33) and test down using a non-linear estimator (also see Bewley 1979). Third, we can estimate the long-run shares  $s^* = \Pi X$  using co-integration tests to search for a suitable co-integrating set of  $X$  variables. The lagged asset disequilibria  $(s-s^*)_{t-1}$  are substituted in (33) and a search over the short-run parameters  $\Pi^*$  and  $L$  is then undertaken.

The first two methods require a search procedure over a very large number of parameters and the long-run solution alters as one moves to the preferred specification. Also we cannot be sure that the resulting long-run parameters form a co-integrating vector. Although the co-integration technique suffers from small sample bias, it has the major advantage that any sensible long-run



co-integrating vector is held fixed while searching over the short-run parameters. For this reason we suggest the co-integration technique may prove to be most useful. After a preliminary search using co-integration techniques the estimated model could be refined by using non-linear techniques on the equation reparameterised in error feedback form.

## VI ECONOMETRIC ISSUES

The long-run co-integrating regressions (31) for the  $k$  asset shares are run on OLS which yields superconsistent parameter estimates. The lagged residuals from  $(k-1)$  of these regressions, that is,  $(s_j - \hat{s}_j)_{t-1}$  are then included in the EFE (see equation (33)).

Any cross equation restrictions in the system of equations for the co-integrating regressions may be imposed using Zellners SUR model but with a fixed diagonal variance-covariance matrix obtained from running OLS on each equation separately. This method of implementing SUR is equivalent to OLS on each equation separately except for the imposition of any cross equation constraints on the long-run parameters.

An IV estimator may be required when estimating the EFEs because of possible simultaneity between asset prices (yields) and shares. Thus when estimating the EFE, 3SLS may be applied with a fixed variance-covariance matrix obtained from 2SLS on each equation. This produces consistent estimators if there is no serial correlation in the system of equations (Zellner and Theil 1962).

Any hypothesis testing on the EFE using quasi-likelihood ratio tests (eg symmetry of the 'price' coefficient matrix) requires the variance-covariance matrix to be held fixed over the restricted and unrestricted system of equation and the same set of instruments to be used in each equation (Gallant and Jorgensen 1979). Usually, small sample adjustments to the asymptotic test statistics are advisable (Pudney 1981, Bewley 1983).

Expected capital gains (eg for capital uncertain assets) may be treated in a number of ways. Auxiliary equations could be run for each of the expected returns using OLS and the predictions used as instruments for the unobservable expectations (Feige and Pearce 1976). However in view of the possibly large number of expectations terms it may be easier to use the errors in variables method. Like the previous method this may introduce an MA(1) error into the share equations. Cochrane-Orcutt type estimation using IV distorts the RE orthogonality conditions. Systems estimators which are generalisations of Hansen (1982) and Cumby et al (1983) have not yet been developed and therefore serial correlation corrections in the system remain problematic (Berndt and Savin 1975). 3SLS therefore might prove to be the most practical solution available with current software.

## VII CONCLUSIONS

### Conclusion

We have argued that the application of consumer demand theory to the demand for financial assets provides a flexible, tractable approach and allows testing of the key axioms of choice theory. It provides a consistent framework for analysing the interdependent decisions involved in portfolio choice and would seem to offer greater insights into results from aggregate studies of the demand for money and other assets than the 'motives' approach. The AIDS model provides a flexible approach consistent with consumer demand theory and unlike other approaches via the cost function (eg translog etc) yields equations in levels of the variables. The latter makes the approach amenable to the use of cointegration techniques applied to the system. The Engle-Granger (1987) two step procedure can be applied which allows one to model the short-run dynamics while keeping the long-run parameters fixed.

# FOOTNOTES

1 The cost function is also usually assumed to be continuous in prices and that the first and second derivatives with respect to prices exist.

2 From (3) and (4):

$$s_i = \alpha_i + \gamma_{ii} \ln \tilde{p}_i + \sum_{i \neq j} \gamma_{ij} \ln \tilde{p}_j + \beta_i (\beta_0 + \prod_k \tilde{p}_k^{\beta_k}) \quad (3.1)$$

From (3)

$$\beta_0 + \prod_k \tilde{p}_k^{\beta_k} = \ln C - \ln P^{**} \quad (3.2)$$

where

$$\ln P^{**} = \ln C - (\alpha_0 + \sum \alpha_i \ln \tilde{p}_i + \sum_i \sum_j \gamma_{ij}^* \ln \tilde{p}_i \ln \tilde{p}_j) \quad (3.3)$$

If the  $\ln \tilde{p}_i$  variables are reasonably collinear, Deaton and Muellbauer (1980) show that

$$\ln P^{**} \approx \ln P^* = \sum s_i \ln \tilde{p}_i \quad (3.4)$$

Hence using (3.2) (3.4) in (3.1) we obtain our share equation (5) in the text.

3 Note that 'adding up' and symmetry imply homogeneity. (Although homogeneity and 'adding up' do not imply symmetry.)

4 One can argue that, ultimately, utility depends only on current and future consumption. However, in the absence of fully contingent binding contracts, when saving takes place, agents must hold some asset stocks, and it is reasonable to assume that agents are not indifferent to the composition of their assets. Hence asset holdings represent purchasing power over future consumption goods. Any factors not in the

wealth constraint that influence the composition of such assets, will implicitly be captured in the parameters of the utility function (9).

5 In the main we discuss our model and its relationship to nib-M1. In the UK it is only post-1983 that interest bearing M1 accounts began to grow rapidly. Hence prior to about 1983, M1 in the UK is predominantly non-interest bearing. Post-1983 in the UK and perhaps post-1981 in the US (Baba et al 1988) we need to consider the 'own rate' on aggregate M1 or consider the interest-bearing element of M1 as a separate asset. These issues pose no additional theoretical problems for our consumer theory approach to the demand for assets.

6 In the consumer demand approach the implicit return on nib-M1 (eg servicing of current account transactions by low or zero charges), like the perceived non-pecuniary returns on interest-bearing assets is reflected in the parameters of the utility function. Only explicit pecuniary returns are measured by the  $\ln p_j$  variables.

7 Note that homotheticity would require  $\beta_1=0$  for all assets. Clearly for  $n>3$  assets it is possible for some subset (at least two) of the  $\beta_i$  to be non-zero while other  $\beta_i$  are zero and for 'adding up' (ie  $\sum \beta_i = 0$ ) to hold. A conventional demand for (nib) M1 equation could be interpreted as one equation in a (weakly separable) two asset system (ie nib-M1 and 'other' capital certain assets). In this case  $\beta_1 = 0$  and 'adding up' imply homotheticity.

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