

# Bank of England

## Discussion Papers

### *Technical Series*

No 33

**Modelling the sterling effective exchange rate using  
expectations and learning**

by

S G Hall

*July 1990*

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## ABSTRACT

This paper builds on earlier work by Hall (1987) and Currie and Hall (1989) which model the Sterling Effective exchange rate as a structural equation making explicit allowance for the forward looking nature of the Foreign Exchange Markets. This earlier work was based on the assumption of rationality on the part of the agents and the estimation was carried out on the basis of the REH assumption. This paper relaxes the assumption of full information and proposes a learning model of expectation formation. It then develops a stochastic parameter model of expectations formation and discusses how such a model may be estimated by using a Kalman Filter.

These proposals are illustrated by estimating an explicit time varying model of expectations formation and using the resulting estimates in a structural model of exchange rate determination. The resulting model is found to encompass a model which relies solely on the REH assumption and to be structurally stable.

**MODELLING THE STERLING EFFECTIVE EXCHANGE RATE USING EXPECTATIONS AND LEARNING**

Following the seminal paper of MUTH(1961) economists have shown an increasing interest in applications involving the explicit treatment of expectations. While much of the early work both before and after MUTH used simple extrapolative rules for expectation formation (eg adaptive expectations) more recent work has followed MUTH in assuming rational expectation. This assumption can be taken at two possible levels, we can follow MUTH in assuming that agents know the underlying model and use this model to form their expectations, this assumption is generally termed strong rational expectations, alternatively we may assume that agents' expectations are on average correct but not make any specific assumption about how agents arrive at these expectations, this will be called weak rationality, this represents a small generalization of the usual notion of weak rationality eg Feige and Pearce (1976) in that agents are not constrained to use only a univariate model.

While it is perhaps reasonable to assumed that agents do not make consistent errors, the strong form of the rational expectations hypothesis (REH) leaves a number of important questions unanswered. In particular the question of how agents come to know the true model is simply not addressed. So the question of learning has become an important criticism of the strong form assumption.

"Clearly expectations must be based on the agent's observations, which of course is meant to include the history of such observations. But ... the transformation of observation into expectations requires the agent to hold a theory, or, if you like, requires him to have a model. This model itself will not be independent of the history of observations. Indeed, learning largely consists of updating of models of this kind. Although we have Bayes theorem, very little is know about such learning in an economic context. There is thus a great temptation to short-circuit the problem, at least in a first approach, and to consider only economic states in which learning has ceased. There will be states in which the realization of an expected variable provides no disconfirmation of the theory and the beliefs held in the light of that theory and the past realization of the variables. Thus, in such states, the probability distribution over economic variables that agents hold cause them to take actions which in turn generate just this probability distribution. This is the idea of a rational expectations equilibrium."

Hahn (1982)

While we might assume that the weak form REH is consistent with learning the strong form clearly represents a steady state where learning has ceased and can thus be viewed as a highly unrealistic assumption.

In applied estimation work the distinction is often not important as most estimation relies on the errors in variable approach of McCallum (1976) and Wickens (1982) which essentially require only the weak form assumption. When such models are incorporated into a large macro model, however, the situation is rather different as the, now standard, assumption is the strong form REH. That is to say expectations are then formed in a manner consistent with the whole economic model and so the assumption is that agents use that particular model to form their expectations. This assumption is extremely implausible and can often produce highly unrealistic properties in the model (eg Hall (1987a) shows the effect of introducing a consistent expectation on the tracking performance of one large model).

Learning is a natural assumption which both overcomes the objections to many of the early models of expectation such as adaptive expectations where agents make systematic errors and yet also avoids the need for the unrealistic informational assumption of the strong REH. While the theoretical literature on learning has grown recently from the early work of Friedman (1975) and Decanjo (1979) to Bray (1983) and Bray and Savin (1986) the empirical literature has remained very sparse. Struth (1984) provides an example of such an application. This paper will develop an empirical model of the sterling effective exchange rate which will give an important role to exchange rate expectations. These expectations will be assumed to be formed through a learning process. The model of the exchange rate will be closely related to that of Hall (1987b) and Currie and Hall (1989) except that in those papers the expected exchange rate was formed under the REH. Here a separate model of the learning process will be introduced as part of the model. In a recent study of the exchange rate sectors of the main UK macro models undertaken by the Warwick bureau a model virtually identical to that of Hall (1987b) was found to encompass all other models.

### **Learning and Macromodels**

To make these ideas more specific suppose we have a linear model;

$$A Y_t = B X_t + C Y_t^e + U_t \quad (1)$$

where  $Y_t$  is a vector of endogenous variables,  $X$  is a vector of exogenous variables known at time  $t$ , and  $Y_t^e$  is a vector of expectations variables which affect  $Y$ . At any point in time  $Y_t^e$  is formed using the relationship;

$$Y_t^e = D_t Z_t \quad (2)$$

where  $D_t$  is a, possibly time varying, matrix of parameters and  $Z_t$  is the information set used at time  $t$ . Now under model consistent expectations or Muth rationality we make the assumption that  $Y_t^e = Y_t$ , this implies that  $D_t = (A - C)^{-1} B$  and  $Z_t = X_t$ . If  $Y^e$  includes the expectation of a future value of  $Y$  then this is generalized in such a way that  $A$ ,  $C$  and  $B$  are stacked matrices over all time periods and  $X_t$  becomes  $X$  over all time periods. The complexity of  $D_t$  under MCE can therefore be seen to be extreme and quite implausible as a measure of agents expectation formation procedure.

Learning can be modelled on the basis of a number of assumptions about the underlying knowledge which agents possess. The most extreme assumption, underlying much of the theoretical literature, gives rise to the rational learning models, Townsend (1978, 1983) Bray and Kreps (1984) or Frydman (1982). The assumption made here is that agents know the true structure of the model (1) but that some of the parameters of the system are unknown. As the true structural equations are known the agents learning problem is essentially simply one of estimating the parameters of the system and as long as a consistent estimation procedure is used we would expect the system to converge on a full rational expectation equilibria (REE). Thus  $D_t = (A - C)^{-1} B$  as  $t \rightarrow \infty$ , and indeed most of the theoretical investigation of small analytical models have shown this to be the case. The rational learning model then still make very stringent assumptions about the degree of knowledge which agents have of the structure of the system.

A slightly weaker assumption gives rise to the Boundedly rational learning models, here the general assumption is that agents use some 'reasonable' rule of learning to form expectations and that the form of the rule remains constant over time. In fact choosing a rule which all agents regard as reasonable is rather difficult and invariably the choice has fallen to the reduced form of the whole system eg DeCaro (1979), Radner (1982), Bray and Savin (1986). Thus it is assumed that agents know the reduced form of the whole system as it would exist under REE but again do not know some or all of the parameters. The move to bounded rationality may seem to be a small one and yet it has important consequences for the behaviour of the system. The reason for this is that the reduced form of (1) and (2) is

$$Y_t = A^{-1} B X_t + A^{-1} C D_t Z_t + U_t$$

and under bounded rationality  $Z_t = X_t$  so that

$$Y_t = A^{-1} (B + C D_t) X_t + U_t$$

and as  $D_t$  is time varying all the parameters of the true reduced form system will be time varying. The boundedly rational agent is usually assumed to be attempting to parameterise a stable reduced form system and so is actually trying to estimate a misspecified model. Under this assumption Bray and Savin (1986) are able to show that, for a simple cobweb model, the model sometimes converges to the REE and sometimes cycles or diverges from the REE. So even with such a limited form of learning the possibility of never achieving convergence to the REE is created.

When we consider more realistic models a further important complication is obviously that all the structural equations (1) of the model may not be time invariant. Suppose some of the equations in (1) actually represent government decision rules which determine policy instruments such as interest rates. These equations then clearly alter overtime in important ways and if a bounded rationality model is being used this constitutes another source of time variation in the reduced form parameters. So even if the learning process is able to converge on the true model it may be, in effect, chasing a moving target and so not converge to a stable set of parameters.

The normal implementation of the bounded rationality assumption, that agents use the REE reduced form as their 'plausible' expectations formation mechanism also runs up against an important practical problem when it is applied to a large model such as one of the macro economic models used by the forecasting groups. The full reduced form of a dynamic model may entail a very large number of parameters and simply because of the required degrees of freedom estimation of the unknown parameters may be impossible. A more tractable assumption under bounded rationality is therefore that agents use a simple but 'plausible' rule to form expectations which is less complex than the full reduced form of the model. This provides a further possible reason for the non convergence of the parameters of the expectations rule, that the rule is in fact different to the model.

This paper will make the bounded rationality assumption with the expectations rule being chosen as the main determinants of the variable about which

expectations are to be formed. The rule may then be seen as a subset of the full reduced form equation which will contain the main determinants of the variable but which will not contain all possible determinants. This assumption is not only tractable but also intuitively plausible when considering the formation of exchange rate expectations.

The learning process will therefore be modelled as a time varying parameter estimation problem where expectations are formed using (2) subject to a set of equations which govern the evolution of the parameters which takes the following form;

$$\text{VECH } (D_t)' = \text{VECH } (D_{t-1})' + \epsilon_t \quad (3)$$

where  $\epsilon_t$  is a suitably dimensioned vector of white noise error processes with mean zero and covariance matrix  $\Gamma_t$ .

If we then make the weak REH assumption that

$$Y_t = Y_t^e + V_t \quad (4)$$

where  $V_t$  is another suitably dimensioned white noise error process with mean zero and covariance matrix  $\Omega_t$ . Then we may rewrite (2) as;

$$Y_t = D_t Z_t + V_t \quad (4)$$

(4) may then be thought of as a set of measurement equations in a state space representation of the model where (3) constitutes the state equations. The Kalman filter may then be used to produce optimal estimates of the time varying parameters  $D_t$  conditioned on  $\Omega_t$  and  $\Gamma_t$ . Optimal is generally given a minimum mean-squared error interpretation when dealing with the Kalman filter but if the error processes  $V_t$  and  $\epsilon_t$  follow a normal distribution then the Kalman filter may also be thought as a maximum likelihood estimate of  $D_t$ . In order to carry out the Kalman filter we must know the covariance matrix of the measurement and state equation error terms, in fact these may also be estimated by evaluating the likelihood function using the Kalman filter and maximizing this function with respect to the covariance matrices.

Once this is done we may take the forecast of the model (equation 4) as a direct measure of agents expectations. A further complication however is that as we are inevitably still measuring expectation with some degree of error and

so using the constructed expectations series in an OLS estimation process would yield biased parameter estimates. It is necessary therefore to still estimate the final structural equations of the model using instrumental variables.

### A Model of the Exchange Rate

Hall (1987b) derives an equation for the log of the real exchange rate which has the following general form;

$$E_t = A_1 E_{t+1}^e + A_2 E_{t-1} + A_3 r_t + A_4 r_{t-1} + A_5 T_t + A_6 T_{t-1} \quad (5)$$

where  $E_t$  is the log of the real effective exchange rate,  $r_t$  is the real interest rate differential between UK short-term rates and world rates (proxied by the real three-month Eurodollar rate and the real three-month treasury bill rate) and  $T$  is the log of the ratio of exports to imports which is a measure of the real trade balance. The theoretical derivation of this equation will not be repeated here, it may be derived in a number of ways Hall (1987b) uses a capital stock model with government intervention. Currie and Hall (1989) use a model which characterizes capital markets as exhibiting both stock and flow elements in equilibrium. At a pragmatic level it may even be thought of as a general encompassing model of a wide range of models, for example if  $A_0=A_1=A_4=A_5=A_6=0$ ,  $A_2=1$  then the model reduces to the open arbitrage model.

The earlier papers have estimated (5) under the REH using system estimation following Wickens (1982) to correct for the expectation effect of  $E_{t+1}^e$  and also for the endogeneity of  $r_t$  and  $T_t$ . This paper will first construct a learning model for the expected exchange rate and will then enter this into the exchange rate equation.

We may rearrange 5 to give

$$E_{t+1}^e = B_1 E_t + B_2 E_{t-1} + B_3 r_t + B_4 r_{t-1} + B_5 T_t + B_6 T_{t-1}$$

and then we may lag this equation by one period and use it to substitute out the term in  $E_t$ , collecting terms then gives

$$E_{t+1}^e = E_{t-1} \left( B_1^2 + B_2 \right) + B_1 B_2 E_{t-2} + (B_1 B_3 + B_4) r_{t-1} \\ + B_1 B_4 r_{t-2} + B_3 r_t + (B_1 B_5 + B_6) T_{t-1} + B_1 B_6 T_{t-2} + B_5 T_t$$

now if we assume that simple partial reduced form equation for  $r_t$  and  $T_t$  might be

$$r_t = C_1 (L) r_{t-1} + C_2 (L) GDP_{t-1} + C_3 (L) P_{t-1}$$

$$T_t = D_1 (L) T_{t-1} + D_2 (L) GDP_{t-1} + D_3 (L) OP_{t-1} + D_4 (L) E_{t-1}$$

where  $OP$  is the log of real oil prices,  $P$  is the change in the log of the RPI and  $GDP$  is the log of the real output measurement of GDP and  $C_i, D_i$  are polynomial lag operators.

We may then eliminate the terms in  $r_t$  and  $T_t$  and again collect up terms to give

$$\begin{aligned} E_{t+1}^e &= E_{t-1} \left[ B_1^2 + B_2 + B_5 D_4 (L) \right] + B_1 B_2 E_{t-2} + r_{t-1} \left[ B_1 B_2 + B_4 + B_3 C_1 (L) \right] \\ &+ B_1 B_4 r_{t-2} + T_{t-1} \left[ B_1 B_5 + B_6 + B_5 D_1 (L) \right] + B_1 B_6 T_{t-2} + B_3 \left[ C_2 (L) GDP_{t-1} \right. \\ &\left. + C_3 (L) P_{t-1} \right] + B_5 \left[ D_2 (L) GDP_{t-1} + D_3 (L) OP_{t-1} \right] \end{aligned}$$

and this then represents the basic partial reduced form rule which agents use to form their expectation, we further simplify this by dropping any lagged terms which are greater than  $t-3$ . We then further simplify the model by introducing a stochastic constant which allows us to drop all the second lags. This may be seen by considering the following general model

$$Y_t = A_t + B_t X_t' + U_t$$

where  $A_t$  is a stochastic constant generated by the state equation

$$A_t = A_{t-1} + V_t$$

This model may then be rewritten as

$$Y_t = Y_{t-1} + B_t X_t' - B_{t-1} X_{t-1}' + U_t + V_t - U_{t-1}$$

so the presence of a stochastic constant is equivalent to differencing the whole model and estimating it with an MA(1) error process.

This basic structure was then used in fairly limited specification search to produce the following equation for expectation formation.

$$\begin{aligned}
 (E_t - E_{t-2}) = & B_{0t} + B_{1t}(OP_{t-2} - OP_{t-3}) + B_{2t}(r_{t-2}) \\
 & + B_{3t}(\dot{P}_{t-2} - \dot{P}_{t-3}) + B_{4t}(GDP_{t-2} - GDP_{t-3}) \\
 & + B_{5t}(T_{t-2} - T_{t-3}) + B_7 + E_{t-2} \quad (6)
 \end{aligned}$$

Note that all lagged information is dated  $t-2$  or greater so that when this equation is used the forecast  $E_{t+1}$  the information set will still be dated at  $t-1$ . The time varying parameters are then assumed to be generated by the following process;

$$B_{it} = B_{it-1} + \epsilon_t \quad (7)$$

We may then apply the Kalman filter to 6 and 7 conditional on the variance of the error term on (6) and the covariance matrix on (7) which is assumed to be diagonal. In fact the likelihood function may be concentrated so that only the ratio of the variance of each of the state equation to the measurement equation is estimated. This process is carried out using a numerical Hill climbing algorithm and the resulting time varying parameters are given in figures 1-7. The residuals produced by the measurement equation are reasonably well behaved, the Lejung-Box test for serial correlation were  $LB(1)=0.00$ ,  $LB(2)=2.4$ ,  $LB(4)=2.5$ ,  $LB(8)=5.6$ ,  $LB(16)=17.3$  which indicates a lack of serial correlation in the error process.

Perhaps the first notable feature about the parameters is that they all exhibit marked variation over time with no strong tendency to converge on a stable parameter value, they also all generally show a tendency to jump strongly in 1978. Interpreting the movement in the parameter values is not straight forward as we must remember that they reflect market expectations not underlying structural parameters. So for example while in the early part of the period a positive interest rate differential seems to be associated with an expected rise in the exchange rate this effect seems to disappear during the 1980s. Part of the explanation for this may be seen in Figure 7 where we see a corresponding movement in the coefficient on the lagged exchange rate

from zero to nearly minus one. When this coefficient is zero the exchange rate is a first difference formulation so that it is essentially a random walk. When it is minus one the equation determines the level of the exchange rate rather than its change. An interpretation of this might be that as the commitment of the government towards controlling inflation strengthened the market interpreted this as a change in the exchange rate regime such that a particular level of the exchange rate was seen as a target rather than changes in the exchange rate.

The fact that there is no serial correlation in the errors is clearly one requirement for the forecast from the learning model to be weakly rational, but we clearly need to check that the expectations series generated by the model is not consistently biased. We may do this by first generating the one step ahead forecast of the model and then testing this for biasedness relative to the outturn. The one step ahead forecast of the model is generated as;

$$E_{t+1}^e = E_{t-1} + \sum_{i=1}^7 B_{it} X_{it-i} + B_{ot} \quad (8)$$

Where the  $X_i$  are all the variables given in (6), as the  $X_i$  were dated  $t-2$  in 6 they are dated  $t-1$  when forecasting one step ahead. This series of  $E_{t+1}^e$  was then subject to the following tests;

$$E_{t+1} = 1.000898 E_{t+1}^e \quad (9)$$

(0.0017)

$$E_{t+1} - E_{t+1}^e = 0.00451 \quad (10)$$

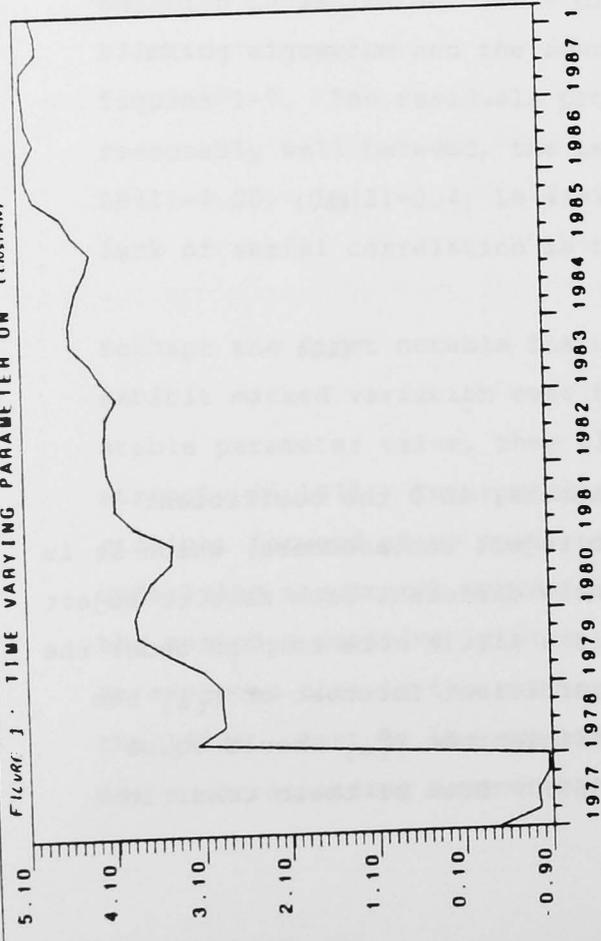
(0.0080)

$$E_{t+1} = 1.49 + 0.678 E_{t+1}^e \quad (11)$$

(0.45) (0.098)

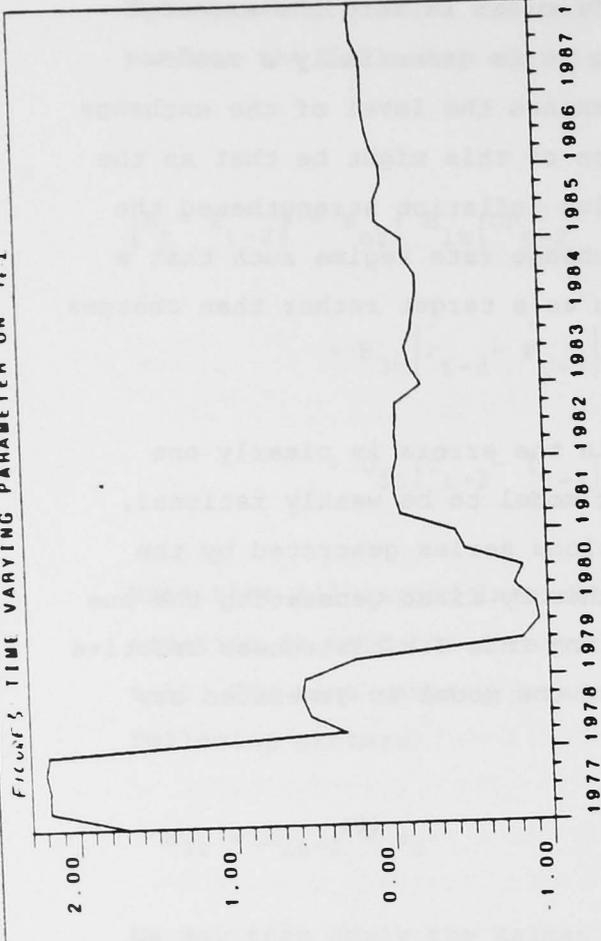
Equation 9 and 10 are simple tests of unbiasedness, in 9 the coefficient should be significantly different from one to reject unbiasedness, which it is not. In 10 the constant should be significantly different from zero to reject unbiasedness, which it is not. Equation 11 is a little more complex under the null hypothesis that  $E_{t+1}^e$  is an unbiased and efficient forecast of  $E_{t+1}$  the constant in 11 should equal zero and the coefficient on  $E_{t+1}^e$  should equal unity (Wallis 1989), Mincer and Zarnowitz (1969). Both of these conditions

FIGURE 1. TIME VARYING PARAMETER ON CONSTANT



C. CJC

FIGURE 5. TIME VARYING PARAMETER ON  $r_{t-1}$



C. CJC

FIGURE 2. TIME VARYING PARAMETER ON  $OP_{t-1} - OP_{t-2}$

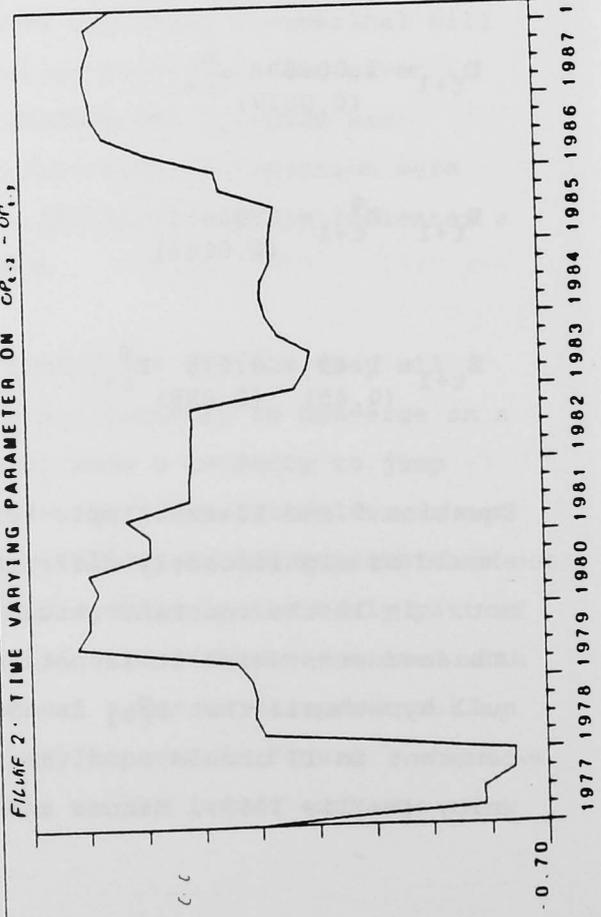


FIGURE 4. TIME VARYING PARAMETER ON  $P_{t-1} - P_{t-2}$

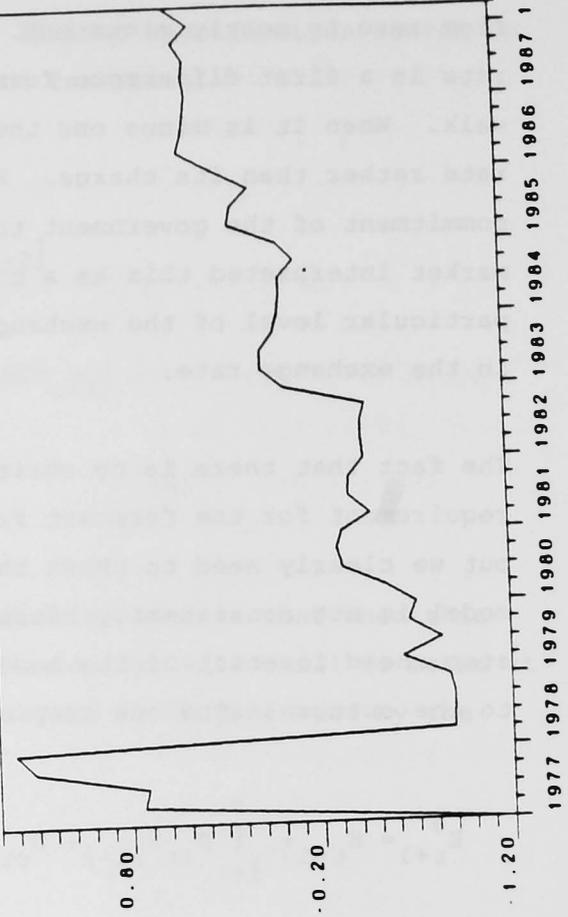
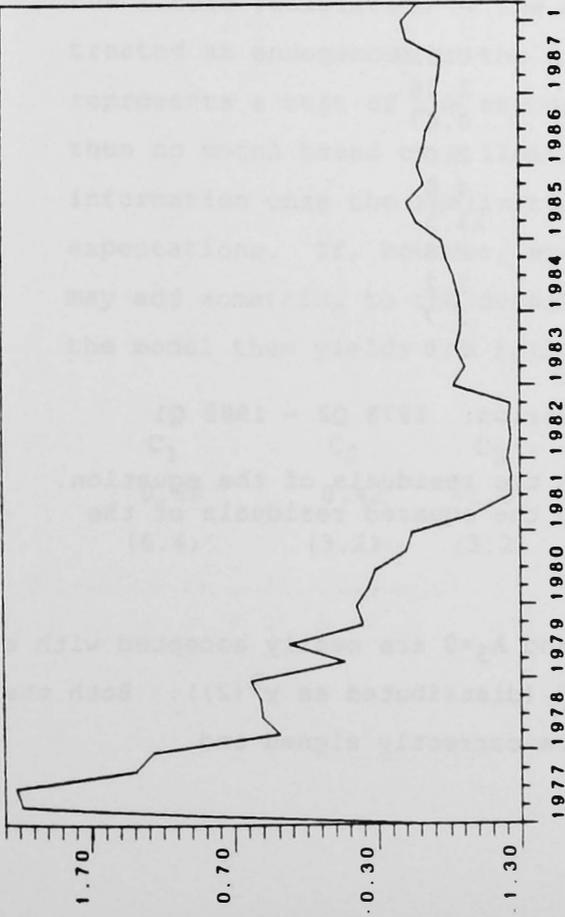


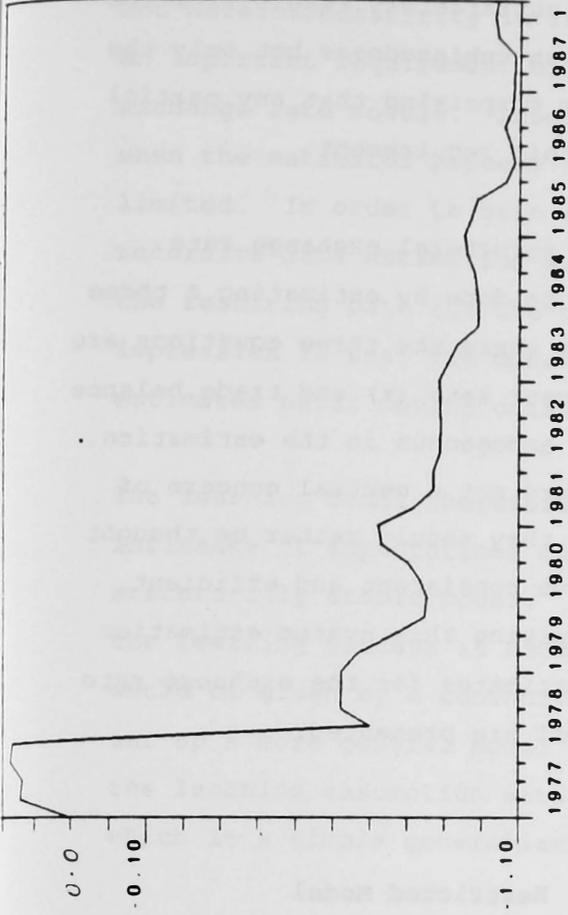
FIGURE 7. TIME VARYING PARAMETER ON  $E_{t-1}$

FIGURE 5 TIME VARYING PARAMETER ON  $CO_2$ ,  $-CO_2$



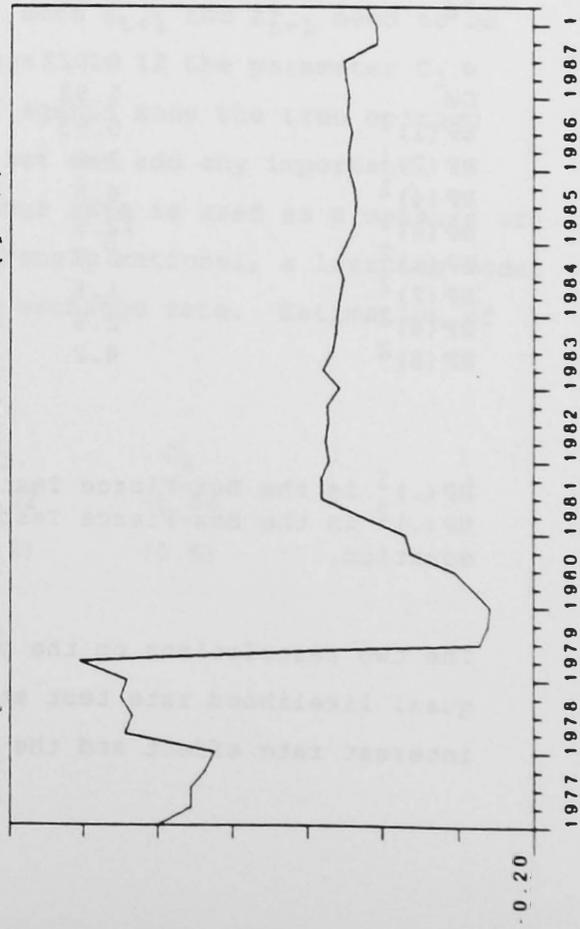
CJC

FIGURE 7 TIME VARYING PARAMETER ON  $E_{p-1}$



C. CJC

FIGURE 6 TIME VARYING PARAMETER ON  $T_{e-3} - T_{e-1}$



are statistically rejected so we may conclude that while the learning model is unbiased it is not fully efficient. This is a satisfactory result from the point of view of this paper the weak REH requires unbiasedness but only the strong REH implies efficiency so it is far from surprising that any partial information learning model would fail to meet this requirement.

Having now derived the expectations series the structural exchange rate equation (5) may then be estimated. This will be done by estimating a three equation system using three stage least squares where the three equations are the exchange rate equation itself, and an interest rate ( $r$ ) and trade balance equation ( $T$ ), in addition  $E_{t+1}^e$  is specified as endogenous in the estimation. The trade balance and interest rate equations are not a central concern of this paper so they will not be discussed here, they should rather be thought of as instrumenting equations which help to give consistent and efficient estimates of the exchange rate parameters. Applying this system estimation technique then gives the following parameter estimates for the exchange rate equation, a restricted and an unrestricted model are presented.

#### Dependent Variable $E_t$

	Unrestricted Model		Restricted Model	
$A_1$	0.55	(4.8)	0.53	(4.8)
$A_2$	0.45	(3.9)	( $1-A_1$ )	
$A_3$	-0.14	(0.4)	-	-
$A_4$	0.73	(2.8)	0.66	(3.8)
$A_5$	0.35	(3.3)	( $T_{-2}-T_{-3}$ )	0.35 (3.6)
$A_6$	-0.20	(1.9)	0.16	(2.9)
$\sigma$	0.022		0.017	
DW	1.92		2.06	
BP (1) <sup>1</sup>	0.03		0.07	
BP (2) <sup>1</sup>	2.8		2.0	
BP (4) <sup>1</sup>	4.5		4.0	
BP (8) <sup>1</sup>	12.9		11.5	
BP (1) <sup>2</sup>	0.8		0.8	
BP (2) <sup>2</sup>	1.6		1.2	
BP (4) <sup>2</sup>	2.9		2.7	
BP (8) <sup>2</sup>	4.2		5.0	

Data Period: 1978 Q2 - 1988 Q1

BP(.)<sup>1</sup> is the Box-Pierce Test carried out on the residuals of the equation.  
BP(.)<sup>2</sup> is the Box-Pierce Test carried out on the squared residuals of the equation.

The two restrictions on the model,  $A_2=1-A_1$  and  $A_3=0$  are easily accepted with a quasi likelihood rate test statistic of 1.32 (distributed as  $\chi^2(2)$ ). Both the interest rate effect and the trade effect are correctly signed and

significant. The summary statistics indicate an absence of serial correlation and Heteroscedasticity in the error process. Structural stability is clearly an important requirement of any equator although it is not often found in exchange rate models. Assessing structural stability is not straight forward when the estimator process is 3SLS and the number of observations is fairly limited. In order to gain some insight into the stability of the model recursive 3SLS estimation was performed over the period 1985 Q1 - 1988 Q1 and the resulting path for the parameters is given in Figures 8-11. The overall impression is that the model is reasonably stable, with the parameter estimates never moving outside their standard error bounds.

The learning model therefore may be seen as one way in which to isolate the influence of expectations on the exchange rate and thereby derive a structurally stable model. However the question still remains as to whether the learning process is adding anything to our understanding beyond that which would be given by a conventional REH model. One way of assessing this is to set up a more general model which contains both a standard REH assumption and the learning assumption within it. This is done in the following equation which is a simple generalization of (5).

$$E_t = C_1 (C_2, E_{t+1}^e + (1-C_2)E_{t+1}) + (1-C_1) E_{t-1} + C_3 R_t + C_4 R_{t-1} + C_5 T_t + C_6 T_t \quad (12)$$

Where  $E_{t+1}^e$  is the expectations series based on the learning model and  $E_{t+1}$  is the future realization of the exchange rate. Both  $E_{t+1}$  and  $E_{t+1}^e$  need to be treated as endogenous in the estimation process. In 12 the parameter  $C_2$  represents a test of the strong form REH. If agents know the true economy then no model based on a limited information set can add any important information once the realization of the exchange rate is used as a measure of expectations. If, however, agents are not strongly rational, a learning model may add something to the determination of the exchange rate. Estimation of the model then yields the following results:

$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
0.68	0.42	-1.0	0.8	0.02	-0.04
(6.4)	(3.2)	(3.2)	(4.5)	(0.2)	(0.5)

FIGURE 8 :RECURSIVE 3SLS

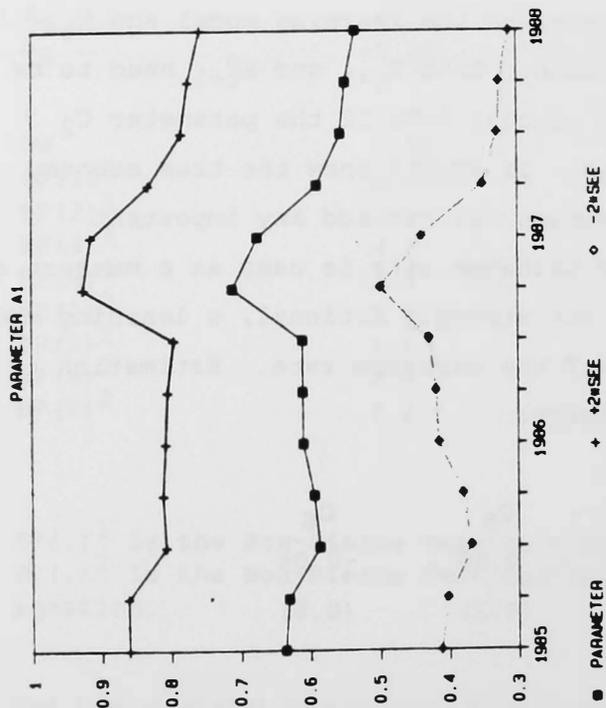


FIGURE 10 :RECURSIVE 3SLS

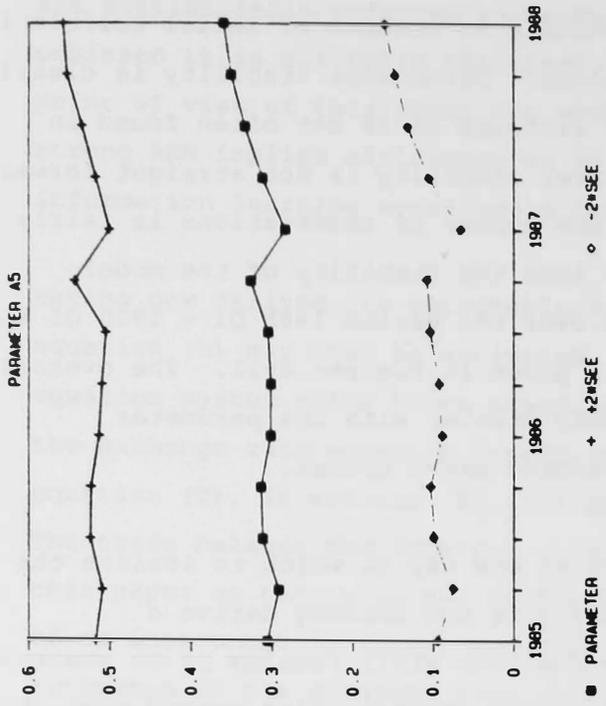


FIGURE 9 :RECURSIVE 3SLS

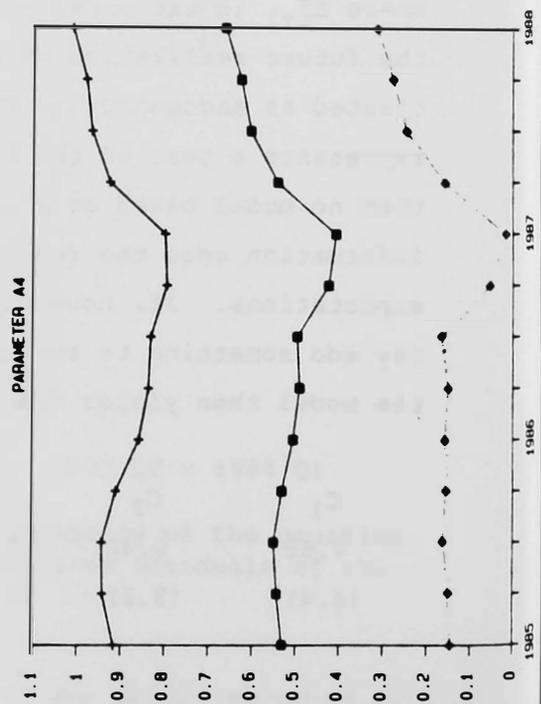
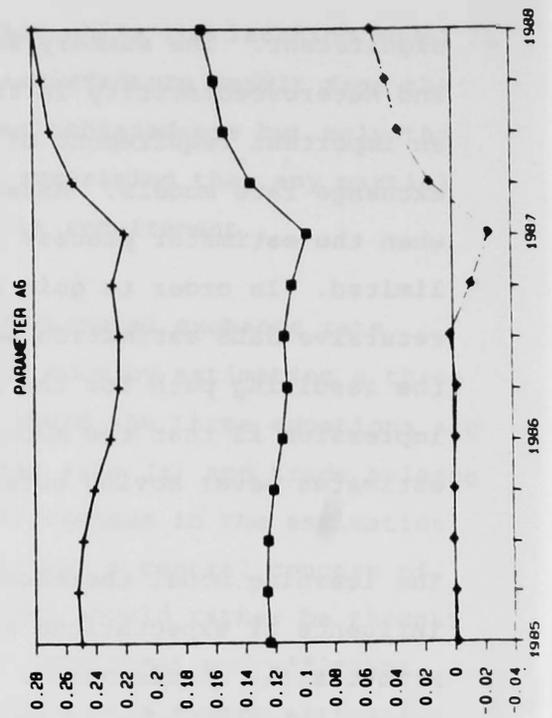


FIGURE 11:RECURSIVE 3SLS



The parameter on  $C_2$  is positive and significantly different from zero which indicates that the market is not fully efficient in its expectation and that there is a role for a learning model. The fact that  $C_2$  is also significantly different from unity also suggests that the particular learning model used here does not capture all relevant information and that the future realization of the exchange rate still contains some important information. This is not really surprising when it is recalled that  $E_{t+1}^e$  is constructed only from data dated  $t-1$  ... so that no current information is used while in the continuous time real world, information lags are much shorter and in terms of quarterly data current period information may often impinge on current expectations of future events.

### Conclusion

This paper has developed a time varying parameter model to emulate the expectations formation procedure of agents in the foreign exchange markets. When this measure of expectations is included in a structural exchange rate model a stable well defined equation emerges. Moreover, when we construct a test of strong rationality we are able to reject this hypothesis by still finding an important role for the forecast of the learning model in the determination of the exchange rate.

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