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No 1

The consumption function in macroeconomic models: a comparative study

by

E P Davis

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Introduction

This paper seeks to assess the importance of the consumer's expenditure equation in macroeconomic models, to consider the theoretical properties of the specifications featured in some well known models, and to test the specifications exhaustively using the latest information available. No novel theoretical developments are carried out in the paper, though some are suggested.

[I would particularly like to thank C J Allsopp, K Cuthbertson, G I Evans, J S Flemming, D F Hendry, N H Jenkinson, J Ryding, I D Saville, A R Threadgold and J C Townend for encouragement, help and suggestions. The errors remain my responsibility.]

2

The consumption function in macroeconomic models

1 The equation relating consumer's expenditure to personal income and other variables is at the centre of the Keynesian income-expenditure analysis of the economy, and remains the centre-piece of most of the, still basically Keynesian, macro models of the UK economy. In the context of a complete model, it determines the effect of changes in government policy and of external economic events on a crucial part of aggregate demand, personal consumption, which is equivalent to 50% of total final expenditure in the economy, and 70% of gross domestic product.

2 The properties of consumption functions are thus a crucial element in the advice and predictions given by macro forecasters, though the judgment of the forecasters is also important. Inaccuracy of the equations can cause huge errors in the predictive power of models, as was found in the early 1970s when equations relating consumption solely to income,[1] failed to predict the increase in the saving ratio that occurred with the rise in inflation, as noted in Townend (1976). In a contemporary context, the success of a policy of stimulating the economy by reducing inflation depends crucially on the likely response of consumers' expenditure. Some equations predict a much smaller or slower response than others, as will be seen below [see also Cuthbertson (1982), Table 2]. Also, the choice between alternative demand injections will be influenced by the properties of consumption functions. The consumption function will show the degree to which an increase in demand which feeds into personal income will be passed on in increased consumption. These responses may vary between alternatives such as income tax cuts and government expenditure increases. The assumed consumption reaction of the economy is also important in the routine assessment of fiscal policy. Knowing the changes that will be made via tax changes to real personal disposable income, the amount of demand that will be removed from, or injected into the economy can be calculated, assuming that changes in taxes do not alter other aspects of behaviour.

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[1] The definition of income used was a national accounts definition of income, including interest receipts but excluding inflationary losses on liquid assets and gains and losses on illiquid assets.

3 One other feature of common practice in macroeconomic models should be noted. They all use seasonally adjusted data, since non-specialist consumers of forecasts find it easier to interpret. Much of the theory underlying existing consumption functions relates to unadjusted data. It will be seen that this causes potentially serious problems [see also Wallis (1974)].

3

Developments in the theory of consumption

4 Keynes (1936) has been interpreted by many economists as suggesting that consumption depended on actual or 'absolute' levels of personal income,[1] whereas later authors [eg Friedman (1957) and Ando and Modigliani (1963)] emphasised longer-run determinants. Friedman suggested that consumption depends on lifetime expected income or permanent income; hence fluctuation in actual or transitory income are not accompanied by changes in consumption, but by savings being increased or drawn down. Ando and Modigliani formulated the life cycle hypothesis, that consumption depends on the stage of the individual in the life cycle, with dissaving in young adulthood, and saving later in working life to cater for retirement when savings are drawn down. These long-run theories are distinct, but hard to disentangle empirically. They both suggest that consumption should empirically depend on current and lagged income and wealth, and were followed in the use of income as a regressor in most of the consumption functions estimated over the 1960s. At the time, these equations were also deemed stable and predicted well. However, in the early 1970s, inflation began to rise and consumption failed to rise with real income (at least as defined by the national accounts); instead the saving ratio rose to record levels and the equations broke down. (This difficulty is discussed in Townend (1976).) An early solution to this problem was proposed by Townend in the same paper, in the form of an equation determining consumption by real liquid assets as well as by income. One rationale is that an increase in inflation reduces the real value of liquid assets and depresses consumption, as individuals save more in an attempt to rebuild their asset stocks in real terms.

5 A further development in explaining consumption is due to Davidson, Hendry et al (1978). Their equation basically relates the fourth difference of the log of consumption to the fourth difference of the log of income and the log of the consumption/income ratio, lagged four quarters.

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[1] This is not strictly correct. Keynes' macroeconomic theory was expressed in terms of wage units, hence his consumption function was $(C/W) = a + (Y/W)$, where C = consumption, Y = income, W = nominal wages. Expressed in real terms, one obtains $(C/P) = a\,(W/P) + (Y/P)$ where the intercept varies with the real wage, P being a price deflator.

6 The lagged levels term corresponds to 'proportional control' [from the optimal control literature, see Phillips (1954, 1957)], while the difference term carries out 'derivative control'. The derivative term gives the effect of changes in income on changes in consumption. The proportional term implies that if consumption a year ago were high relative to income, there would be less consumption now, and vice versa. The proportional term also ensures that the equation has a long-run solution of proportionality between consumption and income in a steady state constant growth equilibrium, with the level of the consumption/income ratio a function of the growth rate of income. This proportionality is in line with the theories outlined earlier, as is the dependence of the ratio on the growth rate of income.

7 Both the Davidson-Hendry and the Townend equations determined the consumption of non-durable, rather than all goods. This is consistent with tests which revealed that durables had very different determinants from non-durables [see Mizon and Hendry (1980), who found an error feedback consumption function inappropriate for explaining purchases of durables in Canada, and Davis (1981 a and b) who found a weaker fit for specifications explaining aggregate consumption than non-durable consumption]. These differences in determinants are related to the different nature of durable and non durable goods. Durables produce a flow of services over time, like a capital good, whereas non durables are assumed to be consumed at the same time as they are purchased.

8 Having given a rough outline of the development of the consumption function from Keynes to the recentt 'state of the art', one may turn to the individual specifications that have been proposed.

4
Specifications of the consumption functions tested

(A) The Townend equation

9 Townend's hypothesis was that the high saving ratios of the mid 1970s were due to the effects of inflation on the real value of net liquid assets, an effect that can be rationalised either by consumers' attempts to rebuild these stocks relative to their incomes or by liquid assets being an indication of the true resources available for an individual's consumption in the long run.

10 He commenced with the equation for non durables that was previously used in the Bank of England short term model, an equation typical of those which forecast consumption well till the early 1970s. This was:

$$C_t = K + a(Y-CG)_t + 0.6CG_t + 0.3CG_{t-1} + 0.1CG_{t-2} \quad (1)$$
$$+ b\,(C_{t-1} - 0.6CG_{t-1} - 0.3CG_{t-2} - 0.1CG_{t-3}) + D_i + U_t$$

where $U_t = U_{t-1} + \varepsilon_t$

where:

C = consumption of non durable goods

Y = income

CG = current grants

D = dummies for the 1968, 1973 budgets

U = autoregressive error term

ε = white noise error term

The equation related real consumption of non durables to lagged real consumption and current real income, subdivided into income from grants and other income. The assumption was that current grants were consumed more rapidly than other income; thus the coefficients on grants were imposed before estimation to achieve a long run propensity was unity. The inclusion of the lagged dependent variable can be rationalised in terms of habit or persistence in spending decisions, as a partial response to current income; or in terms of the adjustment of current to permanent income.

11 After extensive testing, Townend preferred a specification where consumption depends on permanent income, (YP) with the inclusion of a term in the real[1] stock of net liquid assets, (L). This was:

$$Ct = a + bYP_t + cL_{t-1/2} + U_t \qquad (2)$$

where permanent income was defined using the distributed lag:

$$YP_t = (1-d) \sum_{i=0}^{\infty} d^i Y_{t-1} \qquad (3)$$

and hence, combining (2) with (1) and making the 'grant income' distinction noted above:

$$C^*_t = a(1-d) + b(1-d)Y^*_t + cL_{t-1/2} - cdL_{t-1\ 1/2} + dC^*_{t-1} + U_t - dU_{t-1} \qquad (4)$$

where $C^*_t = C_t - 0.6\ CG - 0.3\ CG_{t-1} - 0.1\ CG_{t-2}$

and $Y^*_t = Y_t - CG_t$

The equation (4) required estimation by non-linear means to satisfy the restriction that the coefficient on $L_{t-1\ 1/2}$ is equal to the product of those on $L_{t-1/2}$ and C^*_{t-1}. The moving average error process and associated restrictions were, in practice, proxied by a first order autoregressive process. It was both estimated and used on seasonally adjusted data.

12 This specification was used in the Bank of England quarterly model until mid 1981, when it was felt that the indicated short run marginal propensity to consume had become unrealistically small with the latest data revisions, and a Hendry-von Ungern Sternberg type equation was substituted.

13 Various criticisms of partial adjustment models of this type have been made. Firstly, invalid exclusion of lagged income from (4) may result in a skewed distributed lag relationship between income and consumption with a large mean lag when the coefficient on the lagged dependent variable (ldv) is large (the lag derived from an unrestricted equation need not

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[1] Person's gross holdings of liquid assets minus person's bank borrowing, deflated by the price deflator for consumption of non-durable goods.

have these properties despite having the same coeffcient on the ldv). This may give an unrealistically slow speed of adjustment; in practice, this problem has been experienced with this equation, to a steadily worsening degree. Over the years, the coefficient on Y_t tended to fall and that on C_{t-1} to rise [see Midgley (1981)]. Secondly, as noted, the Townend derivation entails a moving average error term. This means that estimates are inconsistent for the coefficients on the independent variables, and will have inconsistently estimated standard errors, if the common factor restriction implicit in equation (4) is invalid. Deaton (1980) has made a particularly trenchant criticism of this type. In fact, the Townend equation can be shown to be a valid restriction, using a common factor restriction, of a more general model [Davis (1981b)], so this second criticism is not totally valid.

14 All of the rest of the equations to be considered contain the basic features of the Davidson-Hendry (DHSY) equation noted above; viz an error correction formulation with long-run proportionality of income to consumption, specification in log form, choice of non-durable consumption as the variable to be explained and (usually) four period differencing.

(B) The Davidson-Hendry equation (DHSY)

15 As described above, this equation was basically a difference equation with a levels term, relating consumption (C) to income (Y), ie

$$\Delta_4 \ln C_t = a\Delta_4 \ln Y_t + b\ln (C/Y)_{t-4} + V_t \tag{5}$$

where Δ_i is the difference operator, ie $\Delta_i x_t = (x_t - x_{t-i})$

V_t is a white noise error term

This specification can be derived from a more general equation:

$$\ln C_t = \beta_1 \ln Y_t + \beta_2 \ln Y_{t-4} + \alpha_1 \ln C_{t-4} + V_t \tag{6}$$

by applying the restriction

$$\beta_1 + \beta_2 + \alpha_1 = 1 \tag{7}$$

16 Davidson, Hendry et al (1978) added terms in the log of consumer prices ($\ln P_t$) to this specification to account for the correlation of the saving ratio with inflation (though not necessarily explaining it). The inclusion of these terms alters the long-run constant growth equilibrium to

$$C/Y = B(g,\dot{P}) \qquad (8)$$

where the consumption/income ratio is related to steady state inflation ($\dot{P}$) as well as growth (g), having price level homogeneity as well as unit income elasticity. Neither of these restrictions were rejected by the data.

17 The inclusion of 'acceleration terms' in income and prices, defined as $\Delta_1\Delta_4$) could also not be rejected when included in a general model. These gave the following equation:

$$\Delta_4 C_t = a\Delta_4 \ln Y_t + b\Delta_1\Delta_4 \ln Y_t + c\Delta_4 \ln P_t + d\Delta_1\Delta_4 \ln P_t + e\ln\left(\frac{C}{Y}\right)_{t-4} + D_t \quad (9)$$

18 This equation can be rationalised by a 'feedback' theory according to which consumers plan to spend in each quarter of the year the same as they spent in the same quarter of the previous year ($\ln C_t = \ln C_{t-4}$) modified by a proportion of the annual change in income ($\Delta_4 \ln Y_t$), and whether income growth was increasing or decreasing ($\Delta_1\Delta_4 \ln Y_t$); these form the 'short-run' consumption decision, which is altered by $\ln\left(\frac{C}{Y}\right)_{t-4}$, the feedback from the previous C/Y ratio, ensuring coherence with the long run 'target' outcome $C_t = KY_t$. The addition of inflation and acceleration of inflation terms improved the 'fit' of the equation to the data and its forecasting performance. It was admitted that the price terms were consistent with several theories of inflation effects on consumption, for example Deaton's (1977) suggestion that consumers mistake increases in absolute prices for changes in relative prices when sequentially purchasing goods, and the erosion of the real value of liquid assets as suggested by Townend (1976). Further light was cast on price effects in consumption functions by later work; in particular, Bean (1978) and Hendry and von Ungern Sternberg (1979).

19 Other features of the equation are of note. The equation is in logs; this choice was partly determined by goodness of fit rather than theoretical

considerations[1], though also a log formulation has more tractable mathematical properties in a steady growth equilibrium.

20 The equation is specified in fourth differences, and seasonal dummies are excluded, implying the assumption that seasonality is not evolving. Such a formulation facilitates use of non-seasonally adjusted (sua) data, which are likely to convey more economic information than seasonally adjusted (sa) data, and are more econometrically acceptable because the different filters that are used for seasonally adjusting different variables can be shown to bias the resulting estimates [see Wallis (1974)]. This problem is discussed further below (Section 6 and Appendix 1).

(c) The Treasury consumption function (HMT)

21 Bean (1978) modified the initial Davidson-Hendry (1978) equation. He found that $\Delta_4 \ln(L_t/Y_t)$ (where L is the real liquid asset stock) had no statistically significant effect when added to the basic DHSY equation (9) but, like DHSY, found a strong effect of $\Delta_4 \ln P_t$. He explained his specification in terms of an orthodox portfolio adjustment model with real and money-fixed assets.

22 Equation (9) was thus interpreted as:

$$\Delta_4 \ln C_t = a\Delta_4 \ln Y_t + b\Delta_1\Delta_4 \ln Y_t + \eta\Delta_4 \ln W_t \qquad (10)$$

with $\Delta_4 \ln W$ replacing $\ln (C/Y)_{t-4}$ and $\Delta_4 \ln P_t$. For Bean's proof of this explanation and definition of η, see footnote [2] below.

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[1] The choice was based on Sargan's (1964) approximation to the likelihood ratio test [see Aneurin Evans and Deaton (1980)].

$$r = \frac{s.\bar{Y}g}{S} \qquad (11)$$

where s is the standard error (se) of the log equation; S is the se of the natural equation; and $\bar{Y}g$ is the geometric mean of the dependent variable. If r>1 the linear 'fit' is better, if r<1 the log 'fit is better though no significance tests can be applied.

[2] Bearing in mind the long-run dynamic equilibrium of DHSY (11),

$$C = KY \qquad (12)$$

when $K = \exp \quad [((1-a)\Delta_4 \ln Y - c\Delta_4 \ln P)/^{e}]$ (cont) (13)

23 Bean assessed the effect of the fourth difference of real net liquid assets but again found it insignificant, as was 'rational money illusion' ie the Deaton (1977) effect of unanticipated saving occurring for as long as actual inflation exceeds the expected rate. Interest rate and credit effects were also found insignificant or wrong-signed.

24 The acceleration of the unemployment rate $\Delta_1\Delta_4 \ln U$ was also included. The significance of this variable can be explained in several ways; for example greater uncertainty when unemployment accelerates might lead to increased precautionary saving. Alternatively the involuntarily unemployed might be viewed as 'income constrained' in a Clower (1965) sense and hence might behave differently from agents who are on their labour supply function and are thus unconstrained. [This hypothesis was first explored in Flemming (1973)]. Bean found that one could not reject the hypothesis that those constrained (as proxied by the percentage rate of unemployment) obeyed a consumption function C = KY where K = 1, while the unconstrained followed the portfolio adjustment model noted above.

(cont)
a similar relation can be derived for the wealth/income ratio in dynamic equilibrium. If W = real wealth at the start of the period, W* = nominal wealth, θ = the fraction of the portfolio in money fixed assets and k = the wealth income ratio, then one can hypothesise that:

$$\Delta W \approx S_{-1} - \theta W_{-1} \Delta \ln P \tag{14}$$

where S is the level of saving (income minus consumption, where income includes capital gains), and thus

$$k(1 + \Delta \ln Y) = (1 - \theta \ln P) k_{-1} + (S/Y)_{-1} \tag{15}$$

If $|(1 - \theta\Delta \ln P)/(1 + \Delta \ln Y)| < 1$, then using equation (13) and the approximation $\ln(C/Y) \approx -S/Y$, in equilibrium

$$k = [(a-1)\Delta_4 \ln Y + c\Delta_4 \ln P]/e(\Delta \ln Y + \theta \Delta \ln P) \tag{16}$$

so near equilibrium, equation 9 can be expressed

$$\Delta_4 \ln W \approx 4(S/kY)_{-4} - \theta\Delta_4 \ln P \tag{17}$$

$$\approx -4\ln(C/Y)_{-4}/k - \theta\Delta_4 \ln P \tag{18}$$

and equation 9 can be interpreted as an equation of the following form

$$\Delta_4 \ln C = a\Delta_4 \ln Y + b\Delta_1\Delta_4 \ln Y + \eta\Delta_4 \ln W \tag{19}$$

(ie with $\Delta_4 \ln W$ replacing $\ln(C/Y)_{t-4}$ and $\Delta_4 \ln P$)

where $\eta = -c/\theta = -ek/4$ (20)

Solving equations 9 and 14 for k, assuming $\Delta_4 \ln Y \approx 4\Delta \ln Y$, gives

$$k = 4(a-1)/e \tag{21}$$

which yields a value for the equilibrium wealth/quarterly income ratio of around 20 and a value of θ of 33% for typical values of the parameters. These were felt to be reasonable approximations to the actual values (ie the average over historic data).

25 The above considerations led Bean to conclude that the equation:

$$\Delta_4 \ln C_t = \sum_{i=0}^{3} a_i \Delta_4 \ln Y_{t-i} + b \ln\left(\frac{Y_t}{C_t}\right)_{t-4} + c\Delta_4 \ln P_t + d\Delta_1\Delta_4 \ln U_t + e_i \Delta_4 D_i \qquad (22)$$

adequately described the data, where D_t are dummy variables and U_t is the percentage rate of unemployment. The Almon polynominal on the fourth difference of the log of income was used to smooth the first year response.

26 It should be noted that Bean used unadjusted data to test and to find a preferred specification then re-estimated it using seasonally adjusted data for use in the Treasury model. This may not be the best technique to adopt, see Appendix 1.

(D) The London Business School consumption function (LBS)

27 Like the DHSY and HMT equations, this specification is based on the error-correction formulation of aggregate consumer behaviour, with the additional dependence of consumption on prices, though with the omission of the fourth lag of consumption. The specification, as given in the latest London Business School Manual (1981) is:

$$\Delta_1 \ln C_t = a\Delta_1 \ln C_{t-1} + b\Delta_1 \ln C_{t-2} + c\Delta_1 \ln Y + d\Delta_2 \ln Y_{t-2} + e \ln\left(\frac{C_{t-3}}{Y_{t-4}}\right) + f\Delta_1 \ln P_t + g\Delta_1\Delta_1 \ln P_{t-1} + h\Delta_1 \ln P_{t-3} + j_i D_t \qquad (23)$$

28 No details are available of the derivation of this equation. The dependent variable is specified in first difference form, in contrast to the other equations. This might be a more appropriate formulation than fourth differences on the seasonally adjusted data which is used (see below, p 26), as the danger of higher order autocorrelation may be reduced.

29 The functional form used, unlike the simpler rational lag of HUS, does not guarantee a smooth response to income with a superimposed seasonal pattern (see Figure 1 p68). The proportional adjustment term does ensure a long-run solution to the equation. The LBS equation was both estimated and used on seasonally adjusted data.

(E) The Hendry and von Ungern Sternberg consumption function (HUS)

30 The results discussed in (B-D) above found no role for liquid assets in consumption analysis, ("...the positive results found by Townend, may be a consequence of specification errors" (Bean (1978 p12)). However, Hendry and von Ungern Sternberg (1979) suggested that inflation had been dealt with wrongly in most earlier work, leading to the insignificance of liquid assets in equations. Since inflation erodes real wealth, then if agents have a target ratio of wealth to income, a part of nominal incomes needs to be devoted to restoring the real value of wealth. A new measure of income, denoted Y*, was defined, as the accrual which would leave wealth intact. This is of course lower than cash flow when there is inflation, as inflation causes the real value of assets to fall, offsetting the increases in interest receipts which occur in inflationary periods and cause apparent boosts to incomes and savings ratios. This should imply that the true saving ratio (with Y* as the denominator) would be lower than the measured ratio. The problem is that one cannot observe either the 'true' Y* or wealth. The simplest solution is to substitute real net liquid assets for wealth. If no nominal capital gains are made on liquid assets (L_t) then erosion by inflation ($\dot{p}$) is measured by $\dot{p}Lt$. If all private wealth were liquid assets then $Y^* = Y - \dot{p}L_t$. The problem is of course, that there are other assets such as housing, gilts and equities whose nominal and real values may rise when inflation occurs. Also, to the extent that the corporate and banking sectors achieve a gearing gain in inflation, (as the value of their liquid debt falls) the value of their shares will be higher, and households who hold them will be better off (the same argument might apply to government nominal debt and the discounted present value of future taxes). These considerations suggest that the correction $\dot{p}L$ exaggerates the erosion of personal sector net wealth, and hence

$$Y^* = Y - \lambda \dot{p}L \qquad (24)$$

was used, with λ found by grid search to minimise the residual sum of squares. Using this method HUS found λ to be 0.5. Using Y* instead of Y meant that expressions in P_t were no longer required in the consumption function. Hence a criticism of the earlier work was removed, namely that inflation had an unbounded effect on the saving ratio. Instead, all one could lose from inflation was liquid assets (= 0.75 of annual Y); so if $\dot{p} \to \infty$ there would still be a quarter of income left to consume per annum, even if λ were 1.

31 The second major development over the earlier equations was the addition of a third control mechanism to the proportional and derivative mechanisms present in DHSY (see paragraph 6). It was felt that equation 9 had an incomplete dynamic specification because it omitted the effect of changes in the latent asset stock, A_t (including consumer durables) when expenditure is unequal to income. Proof is given below[1] that allowing for such considerations implies addition of an 'integral control mechanism', the assets/income ratio to the other control mechanisms.

32 If one allows liquid assets (as defined in footnote [1], p11) to be a proxy for all assets, these considerations suggest that one should add

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[1] Let A_t be the latent asset stock. Define A_t by:

$$A_t = A_{t-1} + (Y_t - C_t) \qquad (25)$$

A_t is the sum (or 'integral') of all past saving Assume that agents wish to maintain constant ratios of $A^e = B^*Y$ and $C^e = K^*Y$ (as before) in dynamic equilibrium (denoted e) where for consistency with (25) $K^* = 1 - [g/(1 + g)]B^*$. Of course, the former may be particularly unreasonable, especially if assets are proxied by liquidity, as it is likely that desired liquidity/ income ratios are variable, and depend on such factors as real personal disposable income (RPDI), wealth, interest rates, inflation and uncertainty, and similar determinants will affect the relationship between other assets and income. Also, the possible conflict of two targets, both based on transformations of the same absolute distance between two variables, Y and C, may cause conflict in the model [see Salmon (1982)]. In logs, (25) can be written as a steady state approximation:

$$\Delta_1 \ln A^e_t = H^* (\ln Y_t - \ln C^e_t) \quad \text{(where } H^* = (1 + g)/B^*\text{)} \qquad (26)$$

$$\text{where } \ln C^e_t = K^* + \ln Y_t \qquad (27)$$

giving desired levels,

$$\text{and } \ln A^e_t = B^* + \ln Y_t \qquad (28)$$

These targets would not be exactly achieved as the outcomes are stochastic, and equation 26 is only a steady state approximation to equation 25. To model agents assigning priorities to removing disequilibria, assume they have a quadratic loss function where the first two terms are the relative costs attached to discrepancies between planned values (superscript p) and their steady state outcomes (superscript e). Also, to penalise violent responses to random fluctuations assume agents attach costs to changing C^p_t from C_{t-1}, though with an offset term to allow more adjustment at a

(cont)

$\ln(L_{t-4}/Y_{t-4})$ to the equation. It would be expected to have a positive sign, so that, if agents have excessive asset holdings, they will consume more, and vice-versa.

(cont)
given cost when C_t^e has changed compared with when it is constant, so as not to penalise quadratically changes in C_t^p when it is known that C_t^p has changed. This is in contrast to partial adjustment models, which enforce quadratic adjustment costs irrespective of how much the target is known to have changed. Thus one can choose the one period loss function

$$\ln qt = \lambda_1 (\ln A_t^p - \ln Y_t - B^*)^2 + \lambda_2 (\ln C_t^p - \ln Y_t - K^*)^2$$
$$+ \lambda_3 (\ln C_t^p - \ln C_{t-1}) + 2\lambda_4 (\ln C_t^p - \ln C_{t-1})(\ln Y_t - \ln Y_{t-1}) \qquad (29)$$

where $\lambda_i \geqslant 0$. Since one might expect Y_t to be uncertain, the objective should be to minimise $E(q_t)$ (the expected value of the loss) with respect to C_t^p, taking account of equation 26 holding for planned quantities. Then, setting $\partial E(q_t)/\partial C_t^p$ to zero gives a 'servomechanism' solution:

$$\Delta_1 \ln Ct = \theta_0 + \theta_1 \Delta_1 \ln \tilde{Y}t + \theta_2(\ln Y_{t-1} - \ln C_{t-1}) + \theta_3(\ln A_{t-1} - \ln Y_{t-1}) + Ut \qquad (30)$$

where $\Delta_1 \ln \tilde{Y}_t = E(\ln Y_t) - \ln Y_{t-1}$,

$$\ln C_t - \ln C_t^p = U_t \sim NI(0, \sigma^2 u)$$

independently of C_t^p and $\theta_i \varepsilon (0,1)$; also

$$\theta_0 = (\lambda_2 K^* - \lambda_1 H^* b^*)/\Psi,$$
$$\theta_1 = [H^*\lambda_1 (H^*-1) + \lambda_2 + \lambda_4)/\Psi, \qquad (31)$$
$$\theta_2 = (H^{*2}\lambda_1 + \lambda_2)/\Psi,$$
$$\theta_3 = H^*\lambda_1/\Psi \text{ and}$$
$$\Psi = (H^{*2}\lambda_1 + \lambda_2 + \lambda_3)$$

The variables in equation 30 correspond to derivative, proportional and integral control mechanisms (though the derivation here has only rested on one period optimisation).

33 The first difference of lagged real liquid assets is also included (as in Townend) to "strengthen derivative control and dampen any potential oscillatory behaviour generated by the integral correction mechanism" [Hendry and von Ungern Sternberg (1979) p18].

34 Seasonality is treated differently in HUS, with seasonal dummies included to give a seasonally varying MPC. The full equation is:

$$\Delta_4 \ln Ct = \sum_{i=0}^{2} (3-i) a_i \Delta_4 \ln Y_t^* + b\Delta_1 \ln L_{t-1} + c\ln \left(\frac{L}{y}\right)_{t-4} \tag{32}$$

$$+ d\ln \left(\frac{C}{y^*}\right)_{t-4} + cnst + D_t + Q_{it}$$

where Q_i are quarterly dummies, and $\sum_{i=0}^{2} (3-i)$ is an imposed linear "Almon" with declining weights.

35 The long-run equilibrium solution of this specification is

$$(C/Y^*) = f(g, L/Y^*) \tag{33}$$

$$\text{or } (C/Y) = f(g, p, L/Y) \tag{34}$$

It hence differs from the specification described in (B) above in that the liquid assets/income ratio affects the consumption/income ratio. The long-run solutions are only equivalent when (L/Y) is constant.

36 It should be borne in mind that HUS was estimated using unadjusted data throughout.

37 The following two equations are more recent developments of the HUS specification.

(F) The NIESR consumption function

38 The HUS function may be criticised in several ways. As noted, it merely proxies possible capital gains made elsewhere; second, it allows only crudely (via λ) for the possibility that the desired stock of real liquid assets falls during inflationary periods; third, the imposition of a constant equilibrium liquid assets to adjusted income ratio is very restrictive, as is the assumption of an identical lag response of C to Y and $\dot{p}$.L. In Cuthbertson (1981) inflation losses were split from income, to

allow the lag response of consumption to 'normal' income to differ from the response to 'inflation loss' income.

39 In this 'unrestricted' HUS equation, the chosen specification for inflation losses was

$$IL = (0.25(\Delta_4 \ln P_{t-3})(L_{t-4}/Y_{t-3}) + (0.25(\Delta_4 \ln P_{t-4})(L_{t-5}/Y_{t-4}) \qquad (35)$$

40 If one makes this adjustment to the HUS specification and estimates using seasonally unadjusted data, the liquid asset to unadjusted income ratio $(L/Y)_{t-4}$ becomes insignificant, thus removing the imposed equilibrium liquid asset ratio from the specification, while 'inflation losses' (IL) and the lagged difference of real gross liquid assets remain significant, giving the equation [see NIESR (1981)]:

$$\Delta_4 \ln Ct = a \ln \left(\frac{C}{Y}\right)_{t-4} + b\Delta_4 \ln Y_t + c\Delta_1 \ln L_{t-1} + dIL + D_t + Q_{it} \qquad (36)$$

(G) Hendry's more recent HUS formulation

41 Some development has been carried out by Hendry on this equation since his original article. First, it was felt that the "Almon" (3-i) applied to $\Delta_4 \ln Y^*_t$ in HUS was less data coherent than the DHSY parameterisation $\Delta_4 \ln Y^*_t$ and $\Delta_1 \Delta_4 \ln Y^*_t$ (though the latter omits Y^*_{t-2}, Y^*_{t-3} and Y^*_{t-6}). Second, the dating of the (L/Y) integral correction term was changed to a one period lag, rather than a four period lag. Third, the 'real' interest rate was allowed to affect C/Y, in the form $(\ln r_t - \Delta_4 \ln p_{t-1})$ where $r_t = (1 + roi/100)$ and the chosen interest rate (roi) was the rate on 2 1/2% consols, a measure influencing liquid assets and proxying general interest rates.[1] Such a variable might also capture the 'wealth effect' of rising (falling) real interest rates, ie increases (reductions) in the value of illiquid assets and liabilities of the personal sector such as equities.[2] 'Permanent income' theories of

[1] This choice of interest rate and deflator can be criticised, as they relate to wholly different time periods.

[2] The nominal interest rate is also relevant to the value of gilts, but not the real interest rate. It is not known whether the constraint that the coefficients on $\ln r_t$ and $\Delta_4 \ln p_{t-1}$ are equal and opposite was tested.

consumption would agree that such changes, <u>if sustained</u>, should influence consumption. Other researchers (Bean, Townend) have also tried to introduce interest rates to their specifications, without success. It is possible that the real liquid asset/inflation variables in such equations have captured this wealth effect in such a way that real interest rates have been unable to exert a discernable distinct influence during estimation.

42 The specification is thus

$$\Delta_4 \ln C_t = a\Delta_4 \ln Y^*_t + b\Delta_1\Delta_4 \ln Y^*_t + c\ln\left(\frac{L}{Y}\right)_{t-1} + d\ln\Delta_1 L_{t-1}$$

$$+ e\ln\left(\frac{C}{Y}\right)_{t-4} + f(\ln r_t - \Delta_4 \ln p_{t-1}) + D_t + Q_{it} + cnst \qquad (37)$$

and the long run solution is of the form

$$(C/Y) = f(g,\ p,\ r^*,\ L/Y) \qquad (38)$$

(H) <u>The Pesaran-Evans specification</u>

43 The most recent contribution in the field of the consumption function is a paper by Pesaran and Evans (1982). Since their specification is annual, determines aggregate consumption, and is not for use in a forecasting model it is not analysed in detail in the following section. However, the approach of the paper should be noted.

44 Their fundamental point is a criticism of HUS for omitting capital gains; it is argued that gains and losses on gilts and equities as well as the 'inflation tax' on liquid assets, should be added to raw income to produce 'adjusted income'.

45 Their preferred equation is linear, in levels:

$$S_t - \Theta_t S_{t-1} = a(1-\Theta_t) - b\left(\frac{\dot{p}_t}{1-\dot{p}_t}\right) Y_t - c\,\Theta_t(S_{t-1} + (h/y)_{t-1})$$

$$- d\,[(h/y)_t - \Theta_t(h/y)_{t-1} - (q/y)_t + \Theta_t(q/y)_{t-1}] \qquad (39)$$

where S_t is the saving ratio

Θ_t is the inverse of the rate of growth of nominal income

$\dot{p}_t$ is the inflation rate

y_t is real personal disposable income (RPDI)

$(h/y)_t$ are gains on illiquid assets as a proportion of RPDI

$(q/y)_t$ are losses on liquid assets as a proportion of RPDI

This equation is obtained from the life cycle consumption function

$$C_t = a + b\,(Y^*_t) + c(W/P) + V_t \qquad (40)$$

[where $Y^*_t = Y_t + (h/p)_t - (q/p)_t$ and W_t is wealth] by use of the stock-flow relation,

$$W_t - W_{t-1} = P_t Y^*_t - P_t C_t \qquad (41)$$

and various restrictions on the reduced form coefficients.

46 One of their restrictions, which is accepted by the data, is that the signs on inflation losses and capital gains should be equal and opposite. They argue that this should in any case apply on a priori grounds, an argument which seems to ignore the holding of personal sector gilts and equities by proxy (via life and pension funds), a revaluation of whose portfolios might be expected to have a small impact on current consumption decisions compared with the losses of purchasing power due to inflation on current directly held liquid asset holdings. This asymmetry is perhaps a result of imperfect capital markets, which make it difficult to borrow against one's future income.

47 Pesaran and Evans report testing this specification against HUS, DHSY and an earlier Deaton specification using the criteria of maximised log likelihood and an 'index of parsimony' ie the number of estimated parameters. Their equation was found to be superior to the alternatives on both counts. However, it might be felt that such a conclusion should not be regarded as robust till it has faced the test of data revision. Experience suggests that it is easy to find a superior specification of the consumption function when using newer data than the alternative, and subjecting them to tests on the newer data. (The corollary of this, of course, is that there may be evolving behaviour which is not captured in <u>any</u> of the equations.)

48 On points of detail too, their criticisms of HUS seem ill-founded. As noted, the variable λ in the HUS specification is designed to allow for an offset from capital gains. Secondly, in carrying out comparative tests, they annualise the quarterly specifications proposed by HUS and DYSY, thus omitting the richness of their dynamic effects. Thirdly, terms proposed by Hendry in the change of real liquid assets and the level of real interest rates are left out of the HUS specification.

49 The next section tests specifications (A) to (G) on a common dataset.

5

Testing the specifications

50 The approach employed in this section was to re-estimate the equations using the latest data (as first published in Economic Trends, September 1981.[1] This approach might be felt to give 'equal treatment' to the specifications, none of which were originally estimated on this data. If the specifications adequately describe the data generation process, they should be little altered by this re-estimation. However, in order also to test the equations as they stand, some tests were carried out using the most recent published coefficients from the bodies concerned.

51 Testing is organised in five sections:

(a) Estimation and test statistics
(b) Stability
(c) Non nested testing
(d) Forecasting
(e) Simulations

Sections (a) to (c) consider estimates on both the seasonally adjusted (sa) and unadjusted (sua) data. Since macro models use only seasonally adjusted data (see paragraph 3), the main concentration is on sa data, however. The specifications have been left as published except where this is impractical

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[1] The magnitude of the changes in consumption and RPDI over the data revisions that have occured in successive 'Blue Books' can be gauged from the following table:

	October 1978		October 1979		October 1980		October 1981		
	Saving ratio	RPDI index	Saving ratio	RPDI index	Saving ratio	RPDI index	Saving ratio	RPDI index	Inflation
1970	8.9	100	9.0	100	9.3	100	9.3	100	6.3
1971	8.7	103	7.8	102	7.6	101	7.7	101	9.1
1972	10.4	111	9.5	110	9.7	110	10.1	110	7.2
1973	11.9	118	11.0	117	11.7	118	11.9	118	9.1
1974	14.4	118	14.2	118	13.5	118	12.3	116	16.0
1975	15.0	118	14.7	118	12.7	116	12.4	116	24.2
1976	14.6	117	14.6	118	11.8	115	11.6	115	16.5
1977	14.2	116	13.9	116	10.5	113	10.8	114	16.7
1978			15.2	124	12.4	122	13.0	123	8.3
1979					13.8	129	14.7	131	13.4
1980							15.8	133	18.0

Source: "Blue Books" 1978-1981

(for example, the Townend and LBS equations break down when estimated on sua data, except when seasonal dummies are added).

52 Since the Townend equation is in levels rather than in logs and features a non-linear restriction, it cannot be subjected to some of the tests described below given the programmes available (Lagrange multiplier test for autocorrelation, post sample parameter stability test, 'F' test for linear restrictions, recursive stability testing, non nested testing). In some analysis of this equation in the past [see Davis (1981b)] a log linear analogue to the equation has been created for such tests. However, because in this project the aim has been to re-estimate and test the equations as proposed, the equation has been omitted from these tests.

(a) <u>Estimation and test statistics</u>

53 The results of estimation are shown on Tables 1-7 in Appendix 2. All the equations were estimated using OLS. Instrumental variables (IV) estimation was not used; evidence from Davidson-Hendry et al (1978) suggests that estimation of non durables equations using IV makes little difference to the resulting parameter estimates. In addition to the coefficients, the $\bar{R}^2$, standard error, Durbin-Watson, residual sum of squares, the Ljung Box χ^2 (8) and Lagrange multiplier χ^2(8) tests for autocorrelation, the chi squared test for post sample parameter stability, and an 'F' test of the equations' linear restrictions are shown. All of the equations were estimated over 66Q3-75Q4 and 66Q3-80Q4 (the late starting date being necessitated by limitations on some of the data series, and the long lags implicit in the income adjustment equations).

54 On seasonally adjusted data, the Townend equation, Table 1, exhibits a short run response of consumption to non-grant income of 0.15 over the latest data period. It can be seen how this has fallen since the 1966-75 data period and the original estimate. The lagged dependent variable has correspondingly risen while the coefficient on real liquid assets, 0.04 has again fallen. The autoregressive term is not strongly determined over either data period. The available autocorrelation indicators (which are not strictly valid with a lagged dependent variable) indicate that the specification is not autocorrelated after fitting the autoregressive term.

On unadjusted data, the impact response to income is higher, and the lagged dependent variable lower, giving a possibly more plausible response pattern. The 1966-80 estimate is shown by the Ljung Box statistic to be autocorrelated, however.

55 All the 'error correction' equations show reasonable parameter consistency over the different data sets and periods. The derivative adjustment terms ($\Delta_4 \ln Y_t$ and variants) are always the best determined, and imply a short-run elasticity of consumption with respect to income of about 0.4. The proportional adjustment terms $(\frac{C}{Y})_{t-4}$ are also generally well determined (except for the LBS), and stable (except the LBS and HUS). They imply an elasticity of consumption with respect to disequillibria in the C/Y ratio of about -0.1, and a higher absolute value when adjusted income is used. The integral adjustment terms in HUS and Hendry's equations give an elasticity of consumption with respect to disequilibria in the L/Y* ratio of about 0.08, though this effect is always less well determined than the proportional adjustment effect when both are entered. The elasticity with respect to a change in real liquid assets in HUS, NIESR and Hendry is around 0.1 for adjusted data, 0.25 for unadjusted. The 'price rise' terms which substitute for these effects in HMT, DHSY and LBS generally have coefficients of 0.13 and are well determined. The equations tend to have $\bar{R}^2$ of around 0.9, standard errors of 0.008.

56 This overall assessment suggests that the theory behind the various error correction equations reasonably describes the data generation process; at least until one comes to consider autocorrelation. The following table of Lagrange multiplier (LM) statistics tells its own story:

Lagrange multiplier test for residual autocorrelation

	Sua 66Q3-75Q4 LM(8)	Sua 66Q3-80Q4 LM(8)	Sa 66Q3-75Q4 LM(8)	Sa 66Q3-80Q4 LM(8)
DHSY	12.9	10.4	19.1	22.4
HMT	5.9	10.4	10.7	20.6
LBS	23.0	25.4	9.2	8.5
HUS	10.4	9.7	10.8	21.3
NIESR	16.3	20.1	13.8	15.2
Hendry	15.3	17.9	15.5	17.2

where $\chi^2(8)$ = 15.5 at the 95% level

and 13.4 at the 90% level

57 Attention is directed especially to the last column; the LM test suggests that all of the fourth difference specifications feature autocorrelation at the 90% level when estimated using seasonally adjusted data up to the end of 1980, though the NIESR equation is not autocorrelated at 95%. Only the first difference LBS equation is indicated to be free of it. Most of the fourth difference equations are not autocorrelated over the same data period using unadjusted data, perhaps implying a filtering problem, as discussed in Appendix 1. This implies a considerable inefficiency of estimates, and probable weakness in forecasting, with the equations estimated as they stand on sa data. Solutions to this problem are discussed below.

58 What of the explanatory power of the individual equations? The lowest standard errors are for the Hendry equation; the averages over the estimation periods are:

	Average standard error		
	sa data	sua data	Original standard error (type of data used)
DHSY	0.007113	0.008665	0.0062 (sua)
HMT	0.008126	0.009368	0.0078 (sua)
LBS	0.007888	0.009693	0.0062 (sa)
HUS	0.006852	0.007059	0.0055 (sua)
NIESR	0.009482	0.009812	0.0075 (sua)
Hendry	0.006381	0.006765	0.0074 (sua)
Townend[1]	114.7	83	28.8 (sa)

59 It can be seen that the sizes of the standard errors on both types of data support the income adjustments and 'integral control' concepts as discussed above, and present in the HUS and Hendry equations. Some drift seems to have occurred since the original estimates except in the Hendry case (though this may partly be because he used more recent data).

60 χ^2 tests for post sample parameter stability were run for the eight quarters after 1975 Q4, using equations estimated up to that date. The results are shown below:

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[1] These numbers are not, of course, comparable because this equation is in levels while the others are in logs.

	Seasonally adjusted	Seasonally unadjusted
DHSY	3.0	8.1
HMT	4.8	4.6
LBS	2.8	4.4
HUS	25.5	7.1
NIESR	6.8	5.9
Hendry	15.8	18.3

The critical level for the χ^2 is 15.5, so it can be seen that the Hendry equation fails the test on both types of data, HUS on seasonally adjusted. The failure of Hendry may be a result of its recent estimation: the specification might not have been chosen had it been estimated up to 1975 only. In fairness to HUS, the equation was designed for use on sua data, and there the test was passed. It will be seen that the ranking of the results for seasonally adjusted data is similar to the accuracy of the dynamic simulation tracks discussed below (p31).

61 F tests were also conducted, comparing the residual sum of squares (RSS) of each equation without linear restrictions against the RSS of the equations as estimated. The results suggested that the LBS equations' restrictions were rejected in each case, while the NI equations' restrictions are not accepted in the seasonally adjusted estimation up to 1980, the Hendry equation fails the test on unadjusted data up to 1975, and the Treasury equation fails in each case when estimated up to 1975. These results suggest that the specifications whose restrictions are most commonly accepted are DHSY and HUS: a tribute to the seminal specifications in the field of empirical consumption analysis of the UK economy.

62 One may now consider the 'distinctive' features of the individual equations. In the DHSY equation the acceleration of income term is generally well determined (though less well than other effects) with a coefficient of -0.1. The acceleration of prices is more weakly determined, and changes sign in one estimate. These disequilibrium effects are obviously weaker and less stable than the basic error correction mechanisms. Their weakness would seem to justify the exclusion of the acceleration of prices effect as in the HMT equation, while inclusion of unemployment seems justified by its consistency (at around -0.025 over the four data samples). The third term in the HMT's income Almon seems very weak, with a maximum 't' value of 0.4 and a sign change.

63 The LBS equation is perhaps over-parameterised, as several of its coefficients are poorly determined and change sign or have large variations in their size over the different data sets. This comment applies even if one compares only the two adjusted or unadjusted estimates. The specification also produces a proportional adjustment term which is only weakly determined.

64 In the HUS equation, all of the coefficients are somewhat unstable over the estimation periods except for the income term. The NIESR's equation is a great deal more stable over the four data sets, and Cuthbertson's 'liquid asset loss' term is always significant, with an effect centred on -0.1. Hendry's equation is an extremely consistent specification. The real interest rate term always has a 't' value of 1.5, and an effect centred on -0.1, though declining since 1975. However, this equation might be expected to be one of the most stable and best determined as it was estimated most recently. The discussion of stability is continued below.

(b) Stability

65 A more searching test of the stability of the equations was carried out using 'recursive OLS', which adds an observation at a time to the seaonally adjusted regression period, and plots the graph of the resulting coefficients [see Saville (1977)]. Some comments on the graphs reproduced in figures 2-7 are worth noting.

66 The derivative adjustment (income) terms on the top left of the charts generally show a monotonic increase and a marked degree of stability since 1975. The proportional adjustment terms on the bottom left are less stable, and tend to increase absolutely where price is used as an inflation effect, and fall where liquid assets are used, though all seem to converge on a value (allowing for income adjustment) of -0.1. The 'price' effects in DHSY and HMT show a decline over time, and stability since 1976. The 'change in real liquid assets' effect in HUS, NIESR and Hendry is much less stable, but has again been roughly constant since 1978. The integral adjustment terms in HUS and Hendry have tended to fall monotonically over time. The income acceleration terms in DHSY and Hendry show similar patterns, of a rising then falling effect, whether it is income or adjusted income that is being employed, as does HMT's disequilibrium 'acceleration of unemployment'. Liquid asset losses in the NIESR equation show some instability until 1976, and stability thereafter. Hendry's real interest

rate term shows a monotonic decline. There are two sign changes, in the DHSY price term and the LBS proportional adjustment term, though in fairness to the former, the earlier estimation period is very short, and it shows great stability later.

67 The summary statistics for the stability analysis are as folows:

(1) the predictive χ^2 test for static forecast residuals being larger than over the estimation period.

(2) the Chow test for structural change;

(3) Harvey and Collier's t-test for functional mis-specification;

(4) the Von Neumann ratio, testing for serial correlation.

The results may be expressed in terms of the number of significant values observed over the period over which the recursive test were made, ie 71Q3-80Q4;

	Forecast χ^2	Sequential chow: number of observations in each block				Von Neumann ratio	't' test
		1	2	3	4		
Hendry	1	1	1	0	0	0	1
LBS	0	2	2	2	1	0	0
NI	3	4	2	1	1	3	2
DHSY	0	1	1	0	0	0	0
HUS	0	1	1	0	0	8	30
HMT	2	4	3	2	0	18	0

These results suggest that on the basis of these tests, the most stable and best-specified equations for use on seasonally adjusted data are the Hendry and DHSY.

(c) Non-nested testing

68 The equations were tested against each other using the Davidson-McKinnon (1981) method, in which the estimated values of the dependent variable in each specification is entered as an independent variable in the other specifications. A significant value may show that one equation explains the

data better than another. There is a certain amount of controversy about the interpretation of such tests; a great deal is likely to depend on whether the alternative hypotheses are wholly distinct (eg Monetarist and Keynesian aggregate price equations) or in principle derivable from a super-general 'encompassing' equation (eg types of consumption functions). For expositions of each side of the debate, see Pesaran (1982) and Mizon and Richard (1982). The approach suggested by Hendry is to supplement the tests by measures of the equations' goodness of fit; if an equation is dominated (ie fits much worse) and fails the test, then it should be rejected. If an undominated equation is rejected, it may imply that more information should be incorporated in the equation, which in turn should improve the 'fit'.

69 The results are shown in Tables 8 and 9, giving estimates for seasonally adjusted and unadjusted data. The results for both are basically similar; the estimated values of the dependent variable from the new Hendry specification is always significant in the other equations, except in the LBS equation. The estimated values of the dependent variable from HUS is significant for the LBS equation in the unadjusted estimate, though it is not significant in the new Hendry equation. The same applies to the DHSY estimated values in the seasonally adjusted case. These results suggest that the ordering from the test should be:

1 Hendry

2 HUS, DHSY

3 LBS, HMT, NIESR

It is notable that this ordering is similar to that in the 'standard errors' table (see above p27) so Hendry's approach would give the same ordering as the Davidson-McKinnon approach on its own.

70 Some support is thus lent to income adjustment and integral control (especially with the inclusion of real interest rates) as a means of best explaining the data.

(d) Forecasting

71 The estimates on sa data over the period 66Q3-75Q4 were used to forecast dynamically over 76Q1-80Q4 as they would be in macro-economic models. This is a severe test, beginning at a depressed period with high inflation, and encompassing a fall and rise in inflation and considerable variations in the growth of consumption. Dynamic simulation tracks such as these should not,

however, be seen as tests of the 'truth' or otherwise of the models, ie as giving a selection criterion. This is because they only reveal how much of the explanation is attributed to outside factors. For a test of forecasting ability that can be used to select, attention is directed to the post sample parameter stability χ^2 discussed above (p28).

72 The exercise is fairly straightforward in the LBS, HMT and DHSY equations, although it ignores the endogeneity of income and prices to consumption in the whole economy. Actual values of these variables are used, ie assuming exogeneity, and, since the forecast is dynamic, one feeds in computed lagged values of consumption when they appear on the right hand side. However, for the other equations, liquid assets present a problem, as

$$\Delta L_t = f(Y_t - C_t) \qquad (42)$$

in any quarter, so that, if one omits the determination of liquid asset stocks from the system, a very direct feedback is being omitted, and liquid assets are falsely assumed to be strongly exogenous. Some attempt was made to rectify this omission by running the NIESR, HUS and Hendry equations in harness with a liquid asset equation. The form of this equation was as follows:

$$\Delta_4 \ln L_t = \Delta_4 \ln C_t + \ln \left(\frac{L_t}{C_{t-4}}\right) + \Delta_4 \ln P_t + \Delta_4 \ln R_t + cnst + Q_i \qquad (43)$$

73 If one assumes long-run proportionality of consumption to income in a stationary steady state, C = aY and (more tenuously) assume similar proportionality of real net liquid asset stocks to income flow L = bY (this is implicit to some extent in the HUS formulation), then long-run proportionality of consumption to liquid asset stocks is also implied: $L = {}^{b}/_{a}\, C$.

74 This property has been imposed on a simple Davidson-Hendry type equation, where the presence of C ensures a feedback from the consumption function to the liquid assets equation, and the presence of the level of a nominal interest rate on all government securities, not merely consols, ensures that the system is identified when the equation is run together with the consumption functions. The estimate was:

$$\Delta_4 L_t = \underset{(4.4)}{0.461}\ \Delta_4 \ln C_t - \underset{(6.2)}{0.17239} \ln \left(\frac{L_t}{C_{t-4}}\right) - \underset{(13.1)}{0.596}\Delta_4 \ln P_t$$

$$+ \underset{(2.2)}{0.0332}\Delta_4 \ln R_t + \underset{(7.2)}{0.255} - \underset{(0.4)}{0.00162}Q1 - \underset{(0.2)}{0.0007}Q2 - \underset{(0.2)}{0.001}Q3 \qquad (44)$$

$\bar{R}^2 = 0.857$, DW = 0.6, SE = 0.01397, RSS = 0.001171, LB(8) = 77

The severe autocorrelation should be noted; it may mean the 't' statistics are biased away from zero, but bear in mind that this equation is designed purely to keep liquid assets 'on track' through the dynamic simulation periods of the consumption function, rather than being used for prediction itself.

75 A second problem in choosing equations for forecasting is the value of λ, the amount by which losses on liquid assets due to inflation reduce real income in the HUS and Hendry equations. In more recent estimates, a value of '1' had been imposed by Hendry, but in their earlier paper Hendry and von Ungern Sternberg (1979) found 0.5 to be an appropriate value. It was decided to test this, using their criterion of minimising the residual sum of squares. The results were as follows:

Residual sums of squares and values of

	Hendry	HUS	Hendry	HUS
Estimation over:	66-80	66-80	66-75	66-75
λ = 0.0	0.002646	0.003197	0.001298	0.001018
0.1	0.002523	0.003005	0.001262	0.001014
0.2	0.002405	0.002832	0.001228	0.001018
0.3	0.002294	0.002679	0.001197	0.001032
0.4	0.002191	0.002547	0.001168	0.001054
0.5	0.002098	0.002437	0.001142	0.001083
0.6	0.002017	0.00235	0.001119	0.00112
0.7	0.001947	0.002287	0.0011	0.001163
0.8	0.001891	0.002247	0.001084	0.001211
0.9	0.00185	0.00223	0.001072	0.001263
1.0	0.001824	0.002236	0.001063	0.001317

76 On this basis the choice of $\lambda=1$ in the estimates to 1980 is justified. For the Hendry equation $\lambda=1$ also seems to be borne out by the data for the 1966 -75 estimate, but for HUS a much lower number seems appropriate. Initially 0.1 was chosen, then 0.5, but in each case the forecast proved worse than when 1 was imposed. The results reported are thus for $\lambda=1$.

77 The dynamic simulation tracks are shown in Table 10. The DHSY, HMT and Townend equations track this period particularly well, and the NI equation also 'returns to track' in 1980 after some discrepancy in the intervening period. The Hendry equation follows a similar track, though with a greater terminal error. The LBS and HUS equations go completely 'off track' over this period.

78 The summary statistics (Theil's U[1], UM[1] mean absolute error and RMS error) in Table 10 tell much the same story over this period.

(e) Simulations

79 The 'simulation' (or 'shocking') routine involves changing the magnitude of a variable or coefficient in a model, making a dynamic run, then subtracting this from a 'base' dynamic run, featuring no shock. The models consist of the equations described above, viz the seasonally adjusted estimates of each equation up to 1980, supplemented in 'alternative' tests by the liquid assets equation (44). The simulations were also performed using the latest published coefficients of each equation. The shocks imposed were as follows:

(i) A shock to inflation, increasing it by one percentage point per annum throughout the test period, 1970-80.

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(1) Theils (1966) 'U' can be defined as:

$$U = \frac{\sum_{t=1}^{T-1} (P_t - A_t)^2}{\sum_{t=1}^{T-1} A_t^2} \qquad (45)$$

where A_t is the actual percentage change in the dependent variable and P_t is the corresponding predicted change. The measure is zero in the case of perfect forecasts. UM measures the errors in central tendency or bias proportion.

$$UM = \frac{(\bar{P} - \bar{A})^2}{\frac{1}{T-1} \sum_{t=1}^{T-1} (P_t - A_t)^2} \qquad (46)$$

If it is large, it means the average predicted change deviates substantially from the average actual change. For further details see Theil (1966) and Maddala (1977), page 38.

(ii) A corresponding one percentage point increase in the rate of real income growth.

(iii) A one-period 5% increase in the level of prices.

(iv) A one-period £200 million increase in real income.

(v) A continuing increase in the level of money income by
   (a) a constant amount (5% of the 1970 level) and
   (b) a constant percentage.

(vi) A constant percentage increase in the level of prices.

80 The effects of these shocks could have been calculated analytically for some of the equations by finding the rational lag profile, and operating on the cumulative responses or analysing the equations in terms of growth rates.[1] However, this would be intractable in the HUS and Hendry equations because prices, incomes and liquid assets enter adjusted income linearly in a log equation, and hence the separate effects require building into a small model to disentangle. Also the 'small model' technique allows the liquid assets feedback to be built in, though of course the feedback effects on income are still ignored.

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[1] This procedure has been carried out by Cuthbertson (1982). However, the results from this method appear to differ from the dynamic simulations over actual data shown here; for example, Cuthbertson found that the response of the saving ratio to a one percentage point rise in inflation in the Treasury model was 1 1/4%, in contrast to 0.6% in this paper. The reason for the discrepancy seems to be that the analytical method compares two equilibrium steady states, while in fact adjustment of the equations is slow and the simulation method may be base dependent. To illustrate the first point, I constructed a steady state, with 1% inflation and 2 1/2% real income growth. When inflation was increased by one percentage point, the resulting increase in the saving ratio after 56 years (1.1%) was closer to the analytical result, but was still in the process of converging. To illustrate base dependence, I commenced one such steady state in disequilibrium (a saving ratio of 20% when the 'warranted' rate was around 13%), and another in rough equilibrium (13%). When the inflation rate was increased, the speed of response of the saving ratio was slower in the disequilibrium case (by 0.2 percentage points after 10 years). Such discrepancies would plausibly be greater when estimated over a 'real world' data period with many disequilibria. However, the method chosen here remain defensible. It shows the responses of consumption in a real world situation, and has been adopted as such by many economists testing forecasting models.

81 The dependent variable assessed in the simulations is the saving ratio, including expenditure on durables assumed to be given exogenously. As the denominator, income, is the same for each equation in each case, the relative effects of the shocks to consumption in each equation can also be assessed.

82 All of the shocks are fairly symmetric and the response is linear to a reasonable approximation.

(i) Effects of one percentage point higher inflation

83 The effect of faster inflation on the saving ratio should be to increase it, as the value of accumulated saving is reduced and more accumulation must occur in order to maintain a desired level of assets, with incomes held constant.

84 The results are shown in Table 12. With the published coefficients, most of the equations increase the saving ratio by 0.5-0.6 percentage points after five years and hold it at this level. The exceptions are the HUS, Townend and Hendry formulations which increase the ratio by over 1 percentage point. There is more variety among the 'new' estimates, with the NI equation showing a small rise of 0.3 percentage points, and the LBS a large 1.1 percentage point increase. When the liquid asset feedback is introduced, the Hendry and HUS equations revert to the norm for the rest of the equations, around 0.6 percentage points, while the Townend equation only increases the saving ratio by 0.2 percentage points. These results imply, assuming that one can reverse the effects, that, if the new estimates were used, the NI and Townend equations would suggest the least beneficial effects to consumption of a fall in inflation, and the LBS the most. If the original specifications were used in a complete model, the policy advice from the different equations would be remarkably similar.

(ii) Effects of one percentage point higher real income growth

85 Higher economic growth might be expected to increase consumption, but by less than the increase in income, hence increasing the saving ratio. This result is borne out by the simulations shown in Table 13; the results for the new data show an increase in the saving ratio of 2 1/2-4 1/2 percentage points over ten years. There is some asymmetry here, as the NI, DHSY and HMT equations all give a rise of around 2.6 percentage points, with the increase tapering off towards the end as the economy tends to a new equilibrium.

However, the other equations give rises of 3 1/2-4 1/2 percentage points, and, in the case of the LBS, the increases show little sign of tapering off towards the end.

86 With the liquid assets equation switched in, the Hendry equation returns to a similar path to the NI, DHSY and HMT specifications, viz a tapering increase to 2.8 percentage points. The HUS and Townend equations continue to show a greater rise, though it is lower in each case than when the feedback to liquidity is ignored.

87 Testing the original specifications revealed roughly similar results to the new estimates, except in the case of the LBS equation which raised the saving ratio by a more plausible 2.45 percentage points over the period. This may indicate some breakdown of the specification over the new data period.

(iii) Increasing money incomes for one period by £430 million at given prices

88 This shock shows the effects of a government 'give away' Christmas bonus to the poor (assuming they have similar savings propensities to the rest of the population) or any other windfall that does not continue. The effect is to increase consumption, but by less than income, so the savings ratio increases in the period concerned. In later periods, there are small reactions to the initial shock as the effects feed through the long lag structures of the equations.

89 Using the new estimates (Table 14) the equations all show an increase of around 0.6 percentage points in the saving ratio in the first quarter, which is offset by small reductions in saving in the years that follow. This effect is in line with the long run proportionality of income to consumption imposed on the equations (except Townend) the saving ratio being altered only by changes in the equilibrium growth or inflation rates. The equation that least follows this pattern is the LBS, which gives a larger increase of 0.825 percentage points in the first year, followed by further increases totalling 0.255 percentage points over the next three years, which are not offset. This equation thus suggests that an increase in income is less expansionary than in the other cases. The offset of higher saving is also less marked for the Townend specification.

90 The results of the simulation on the different specifications are more similar when the original coefficients are used. In each case, the saving

ratio rises by 0.55-0.77 percentage points in the first quarter, an increase which is later offset by falls in saving in later years. The Hendry and Townend equations follow this pattern least, as there is a great deal less 'offset' in these cases.

91 The results with the liquid assets equation incorporated are similar to those without it, except that the increased consumption in later years, when income is unchanged, is greater. This is a result of increased accumulation of liquid assets in the 'shock' period, which provides an additional expansionary force later.

(iv) A 5% increase in the level of prices

92 One is modelling here an unexpected price increase which is subsequently reversed (with an assumption of no reaction of money incomes), for example an increase in specific indirect taxes over and above the inflation rate, or an oil price increase. The increase in prices is assumed to reduce real incomes, and consumption thus also falls, but by a smaller amount, as savings are drawn down to cushion the fall. This means that the saving ratio falls in the first period, but rises later as Savings are somewhat rebuilt in reaction, see Table 15. Equations estimated on the new data show a fall of 0.4%-0.625% in the first quarter, with the decline reversed in later years, remembering that the level of real savings equivalent to a 1% change in the saving ratio increased over the years. The exception is again the LBS equation, which gives the largest initial fall in the saving ratio, and no later offset. The results are similar with the original coefficients, except now there is some reaction in the LBS case.

(v) Increasing money incomes (a) by £430 million per quarter, and
(b) by a constant proportion (5% per quarter)

93 These two types of shocks might be compared with increases in personal income tax allowances and reductions in income tax rates respectively. In each case, real incomes increase, but, in the former case, the value of the concession is eventually eroded by inflation, while, in the latter, the real annual increase continues at roughly the same amount, <u>ceteris paribus</u>. In fact, in 1975 prices, the former shock starts at £730 million per quarter in 1970, and ends in 1980 with £203 million per quarter, while the latter starts as £730 million per quarter and ends as £1,070 million per quarter (given real income growth).

94 The constant money shock of £430 million per quarter, shown in Table 16, gives an initial increase in the saving ratio of around 2.5 percentage points, as income has increased more than consumption. As the increase in real income falls off, the increase in the saving ratio compared with base declines, until after around 7 years (when the value of the concession has fallen by around two thirds), the saving ratio becomes lower than base. This occurs in each case, both with the new and original estimates, except for the Townend, Hendry and HUS. The crucial effect of feeding back liquid assets in these equations, and the false inferences possible if these effects are ignored, are shown when the accumulation effects are allowed to feed back. In all cases, except Townend, the results then converge with the other cases. The magnitude of the effects are similar for each equation when these 'extended' Hendry and HUS estimates are included in the comparison; the LBS case is again the outlier, with the greatest initial increase in the saving ratio (and hence the lowest short run marginal propensity to consume).

95 When real incomes are increased by roughly 5% throughout as shown in Table 17, the saving ratio initially increases sharply by about 2.5%, but then tails off to an increase of around 0.5% after ten years. These results are broadly similar for each specification, when the liquid assets extension to the 'adjusted income' equations is carried out. Again the LBS gives the highest and most durable increase in the saving ratio.

(vi) An increase in prices by 5% in each period

96 This simulation could be interpreted as successive increases in the rates of value of added tax. It is not plausible to assume that there will be no reaction of money incomes in the long run, though a decline in real incomes for 2-3 years might be plausible result of this policy. The saving ratio falls by around 2% at once, but after two years this decline is reduced to just over 1%. Consumption falls in this case, but by less than the fall in real incomes, as shown in Table 18.

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97 The results of the section above, where the various equations have been shocked in various ways, are remarkably similar across equations, so long as liquidity is fed back in the Hendry and HUS cases, though the LBS and Townend equations have always tended to differ slightly in their effects from the rest of the equations considered. These results imply that the long run behaviour implicit in the specifications are very similar and are easy to model. The short run dynamics are where the equations differ. However, it should be borne in mind that the above simulations do not describe what would

actually happen to consumption if these shocks were applied. The feedbacks that would be present in a full model, especially the feedbacks from consumer demand to income (via the multiplier-accelerator process) and prices (via demand inflation) are lacking. Also, of course, there are the myriad of other effects that policy measures have on other sectors of the economy and external effects, that might indirectly affect personal consumption.

6

Conclusions, the seasonal adjustment problem and further work

(1) Conclusions

98 This study has not attempted to select a 'best' consumption function, nor could one be developed without weighting each individual test and statistic, including some tests, for example the simulations, where the 'right answer' is unknown. Instead, it has examined the approach embodied in the various specifications, and perhaps has shown something about the behaviour of the personal sector. The latter may be particularly true of the simulations which demonstrate a remarkable convergence of results from diverse specifications, whether coefficients were those derived from the different estimation periods of the original estimates or those from the new estimates that have been made for this study. The differences between the equations have been brought out more in the other sections of results; in particular, the results of dynamic simulations have been found to be extremely diverse, though, as pointed out, these tracks only show how much of the explanation is attributable to 'outside' factors and cannot test the truth of the model. The estimation and non-nested testing revealed some superiority of the equations featuring income adjusted for losses on liquid assets. All of the fourth difference equations were found to have autocorrelated residuals when estimated on seasonally adjusted data at the 90% level, however. This indicates some fundamental problems, discussed further below.

(2) Seasonally adjusted and unadjusted data: the problem

99 Wallis (1974) showed how the use of seasonally adjusted data can induce autocorrelation in equations. Proof of this proposition is shown in Appendix 1.

100 The fact that the sa results are almost all autocorrelated and differ in coefficient values from the sua results suggests that this critique may be well founded. Possible solutions seem to be:

(a) Change the entire macro model to unadjusted data, perhaps seasonally adjusting the results, using a common filter.

(b) Try to model the error polynomial with a type of ARIMA process, or to at least smooth out the biases caused by seasonal adjustment, using sa data to build a specification. Some attempts at the latter are discussed below.

(c) Accept inconsistent and biased estimates.

(3) A suggested specification for use on seasonally adjusted data

101 A possible solution to the seasonal adjustment problem may be to impose restrictions on the coefficients of a general equation such that the lagged variables are averaged over the previous year, undoing the seasonal adjustment, while retaining a basic HUS specification. The formulation would imply that consumers look back over all of the previous year when considering their future consumption plans; they are presumed to compare adjusted income over the last year with current adjusted income, and to consider the average levels of the liquid assets/adjusted income and consumption/adjusted income ratios over the same period. This seems a great deal more plausible than asserting that consumers compare their current income etc with the levels in specific earlier quarters, eg the first or the fourth, given the distortions caused by seasonal adjustment. This is not a new technique. In the original HUS article, the integral control mechanism was originally specified as

$$\ln(\overline{L/Y^*})_{t-1} = \ln\left(\sum_{i=1}^{4} L_{t-i} \Big/ \sum_{i=1}^{4} Y^*_{t-i}\right) \qquad (57)$$

This was said to give a variable that is 'self-seasonally adjusted' with unadjusted data.

102 The specification suggested is:

$$\begin{aligned} \ln C_t - \ln C_t^+ &= (\ln Y^*_t - \ln Y^{*+}_t) \qquad (58) \\ &+ \Delta_1(\ln Y^*_t - \ln Y^{*+}_t) \\ &+ (\ln C_t^+ - \ln Y^{*+}_t) \\ &+ (\ln L_t^+ - \ln Y^{*+}_t) \\ &+ \Delta_1 \ln L_{t-1} \\ &+ \text{CNST} \\ &+ (D_t - D_t^+) \end{aligned}$$

where X_t^+ implies $\dfrac{(\ln X_{t-1} + \ln X_{t-2} + \ln X_{t-3} + \ln X_{t-4})}{4}$

for variable X (ie the geometric mean) and the other variables are as defined in the description of the HUS equation above.

103 This process is tantamount to admitting that the process of seasonal adjustment makes the imposition of more precision on the lag profile implausible, and accepting that there is not enough information in the data to give a more accurate pattern of response. It should also minimise the biases caused by any inaccuracies in data measurement; instead of a one-period error being able to knock the equation off track (a frequent problem in forecasting), an equal error in four quarters would be required to give the same bias. This is particularly desirable because of the equations importance; harmonics in it can cause irregular patterns in the whole model's forecast.

104 Estimation started with a freely estimated general equation estimated on seasonally adjusted data. This equation was successfully restricted to specification (58) (see table below).

Nested testing of the general equation

	RSS	t-k	R	F	F critical
(1) Most general (4 lags of C, 6 lags of L and Y)	.001318	36			
(2) Restricted to lags included in this equation freely estimated	.00146	40	4	1.07	2.61
(3) ΔC and C/Y restrictions imposed	.001515	43	3	0.54	2.8
(4) Δ_1L restriction imposed	.001567	44	1	1.51	4.04
(5) All restrictions imposed	.001612	49	5	0.62	2.4
(6) Compared with (2)	.001612	49	9	0.56	2.1

105 The results [using Δ^+ to imply $X_t - X_t^+$] and estimating over 1966-1980 were:

$$\Delta^+ \ln C_t = \underset{(10.3)}{0.34893}\,\Delta^+ \ln Y_t^* - \underset{(3.0)}{0.10882}\,\Delta_1 \Delta^+ \ln Y_t^* \tag{59}$$

$$- \underset{(3.2)}{0.10046}\ln(C/Y^*)^+ + \underset{(3.5)}{0.03259}\ln(L/Y^*)^+ + \underset{(1.3)}{0.10686}\,\Delta_1 \ln L_{t-1} - \underset{(3.2)}{0.04021}$$

$$+ \underset{(2.2)}{0.01156}\Delta^+ D68 + \underset{(3.7)}{0.01969}\Delta^+ D73 + \underset{(3.3)}{0.01714}\Delta^+ D79$$

$\bar{R}^2 = 0.845$ SE = 0.005736 RSS = 0.001612 LM(8)=15.3

1966 Q3 - 1980 Q4

The standard error is lower than for <u>any</u> of the other specifications estimated over this period. The LM test for this specification shows a lack of residual autocorrelation at the 95% level.

106 A real interest rate similar to that used by Hendry (ie $(\ln r - \Delta_4 \ln P_{-1})^+$) was tested in this specification but it failed the significance tests, and induced autocorrelation, so it was omitted. The specification of the difference of liquid assets as $(\ln L_{t-1} - (\ln L_{t-2})^+)$ was also unsuccessful, though inclusion as a first difference (a common feature of other equations) was felt to be acceptable.

107 The equation has a much smoother lag response than those featuring specific lags of the dependent variable (see figure 1). Since the equation is accepted compared with a general equation, this feature is empirically warranted as well as being desirable in an equation.

108 A complete analysis of the performance of this equation (on seasonally adjusted data, given that it was designed for use on such data) is shown in Table 19. Of particular note is that the equation is shown to be superior by the non nested test; since the equation also has a lower standard error than the others, this result would also be accepted on Hendry's criterion (paragraph 68). Both of the estimations pass all of the diagnostic tests, and parameter stability is marked. The latter point is also brought out by the results of the stability analysis (see table 19 and figure 8).

109 Of course, the fact that this equation outperforms others on a new data set and has not as yet shown systematic bias need not imply that it will always do so; the ultimate test of a specification is robustness over changes in the data as noted in paragraph 49. For this reason, it is necessary to be cautious in assessing the specifications' apparent superiority.

110 A variant of this specification has currently been adopted for use in the Bank of England quarterly model. Although it is still early days, it has shown very small and unsystematic errors over the five data points (1981 Q1 - 1982 Q1) that it has passed since estimation, and, as such, its forecast path required no residual adjustment in the summer 1982 short term forecast.

(4) Further work

(a) Some long run properties and growth elasticities of the equations have not yet been estimated (though indications are that the specifications are rather similar).

(b) Further research into a more appropriate definition of income for consumption decisions might be appropriate, for example deduction of owner occupied rent and net contributions to life and pension funds, 'labour' income as opposed to income from other sources, or the reintroduction of Townend's grant/nongrant concept.

(c) Even the inclusion of a liquid asset identity does not fully approximate the behaviour of the equations in a complete macroeconomic model. As noted, no response of income to consumption has been modelled above, which would increase, for example, the response of consumption to an initial increase in income. The responses of the equations when fitted into a complete model might be very different, and should be tested.

(d) Debate is likely to continue on the best definition of wealth to use in consumption functions.

APPENDIX 1

Wallis Critique

Suppose that one has estimated the specification

$$C_t = \alpha + \beta Y_t + U_t \qquad (47)$$

on seasonally unadjusted data, as one might if, with Hendry, one has argued that there is more economic information in unadjusted than sa data. Then, suppose one wishes to estimate it using sa data, given this is the type of data required to give acceptable answers to consumers of economic forecasts. Assume that the data has been filtered using a linear lagged and led function, so

$$C^a_t = \eta(L)\ C_t \quad \text{where (L) is a rational lag polynomial } \frac{\alpha(L)}{\beta(L)} \qquad (48)$$

and

$$Y^a_t = \xi(L)\ Y_t \qquad (49)$$

Now, one is trying to estimate

$$C^a_t = \alpha + \beta Y^a_t + V_t \qquad (50)$$

but

$$V_t = I_t + (C_t - C^a_t) - \beta(Y_t - Y^a_t) \qquad (51)$$

$$= U_t + (I - \eta(L)\ C_t - \ (1 - \xi(L)\ Y_t \qquad (52)$$

Suppose the <u>same</u> filters are used for each variable, that is

$$\eta(L) = \xi(L)$$

then one can extract the common factor, so:

$$V_t = [1 + (1 - \eta(L)]\ U_t \qquad (53)$$

a moving average error process that can in principle be taken account of in estimation, and where if it were perfectly modelled, the estimates of and would not be altered. However suppose different filters are used, as is normal, that is:

$$\eta(L) \neq \xi(L)$$

and assuming that one can express:

$$\eta(L) = \xi(L) * \theta(L) \qquad (54)$$

then one can still eliminate (L) and extract the common factor

$$\frac{V_t}{(1- (L))} = \frac{U_t}{(1- (L))} + C_t - \beta_t \frac{(1-\eta(L))(1-\theta(L))Y_t}{(1-\eta(L))} \qquad (55)$$

but one has an extra term in Y_t included in the error process:

$$V_t = (1 - ((1 - \eta(L)) U_t + \beta\theta(L) Y_t \qquad (56)$$

This biases the coefficient of β_t and implies that one is no longer estimating the same model.

APPENDIX 2

STATISTICAL RESULTS

TABLE 1
TOWNEND EQUATION

	C^*_{t-1}	L_{t-1}	Y^*_t	68L	68	73	79	CNST	U-1
66Q3-80Q4 sa	.641 (7.0)	.0394 (2.6)	.149 (3.9)	-308 (3.1)	131 (1.3)	355 (3.5)	270 (3.4)	1444 (2.8)	-.0033 (0.0)
66Q3-75Q4 sa	.542 (5.4)	.0598 (3.7)	.1959 (4.0)	-249 (2.9)	89 (1.0)	296 (3.2)	-	1345 (2.8)	-0.27 (1.5)
66Q3-80Q4 sua	0.419 (3.3)	0.0399 (2.5)	0.248 (4.6)	-159 (1.0)	142 (0.8)	179 (1.1)	- 50 (0.4)	2901 (3.8)	-0.005 (0.3)
66Q3-75Q4 sua	0.445 (3.1)	0.057 (2.9)	0.243 (3.5)	-171 (1.2)	99 (0.7)	131 (0.9)	-	2203 (3.1)	-0.359 (1.9)
Original est 65-81 sua	0.596 (9.9)	0.054 (5.4)	0.18 (6.0)	-308	131	355	269	996	-0.68 (3.4)

	Q1	Q2	Q3	$\bar{R}^2$	SE	RSS	DW	LB(8)
66Q3-80Q4 sa	-	-	-	.999	96.8	459017	1.9	3.6
66Q3-75Q4 sa	-	-	-	.999	83	206450	2.0	6.2
66Q3-80Q4 sua	-181.5 (19.1)	-58.4 (7.7)	-48.1 (7.7)	.999	153	1080351	2.0	8.3
66Q3-75Q4 sua	-171.9 (11.5)	-45.7 (3.1)	-40.9 (4.9)	.999	126	435425	2.1	8.3
Original est 65-81 sua	-	-	-	.993	28.8	-	-	-

TABLE 2

DHSY EQUATION

	$\Delta_4 Y$	$\Delta_1 \Delta_4 Y$	$\Delta_4 P$	$\Delta_1 \Delta_4 P$	CY4	68	73	79	$\bar{R}^2$	SE	DW	RSS	LB(8)	LM(8)	PPS(8)	F(t-k,R)
66Q3-80Q4	0.392	-0.051	-0.16	0.16	-0.128	0.012	0.019	0.021	0.937	.007219	1.6	.002606	11.9	19.1	-	0.71
sa	(12.5)	(1.1)	(5.8)	(2.2)	(7.2)	(2.4)	(3.5)	(4.0)								(43,7)
66Q3-75Q4	0.411	-0.104	-0.154	0.038	-0.126	0.012	0.018	-	0.936	.007007	1.9	.001522	9.5	22.4	3.0	0.67
sa	(10.6)	(2.0)	(5.3)	(0.3)	(7.0)	(2.4)	(3.4)	-								(25,6)
66Q3-80Q4	0.422	-0.197	-0.12	0.06	-0.1	0.011	0.017	0.018	0.906	.00878	1.6	.003854	7.3	10.4	-	1.58
sa	(11.6)	(1.9)	(4.0)	(0.7)	(5.3)	(1.7)	(2.7)	(2.9)								(43.7)
66Q3-75Q4	0.431	-0.153	-0.119	-0.058	-0.104	0.011	0.016	-	0.906	.00855	1.9	.002266	5.5	12.9	8.1	0.83
sua	(9.5)	(2.4)	(3.7)	(0.4)	(5.3)	(1.7)	(2.6)	-								(25.6)
Original																
est 58-75	0.48	-0.23	-0.12	-0.31	-0.09	0.06	0.06	-	0.85	.0062	2.0			$23.0^{(12)}$		-
sua	(16.0)	(5.8)	(3.0)	(3.1)	(9.0)	(3.0)	(3.0)									

TABLE 3

HMT EQUATION

	Y/C_4	$\Delta_4 P$	$\Delta_1\Delta_4 U$	68	73	79	$\Delta_4 Y$	$\Delta_4 Y_1$	$\Delta_4 Y_2$	$\Delta_4 Y_3$	$\Sigma\Delta_4 Y_i$	$\bar{R}^2$	SE	DW	RSS	LB(8)	LM(8)
66Q3-80Q4	0.113	-0.142	-0.03	0.011	-	0.004	0.32	0.073	-0.012	0.064	0.445	0.909	.008652	1.4	.003743	27.9	20.6
sa	(5.5)	(4.5)	(2.2)	(2.4)	-	(0.8)	(7.3)	(2.3)	(0.4)	(1.4)	(10.7)						
66Q3-75Q4	0.113	-0.137	-0.024	0.011	-	-	0.308	0.075	-0.006	0.065	0.442	0.925	.007599	1.6	.00179	13.7	10.7
sa	(5.4)	(4.6)	(1.4)	(2.8)	-	-	(6.2)	(2.2)	(0.2)	(1.4)	(7.5)						
66Q3-80Q4	0.094	-0.112	-0.022	0.009	-	0.003	0.339	0.087	-0.014	0.037	0.449	0.884	.009757	1.6	.00476	10.8	10.4
sua	(4.5)	(3.4)	(1.4)	(1.8)	-	(0.6)	(6.7)	(2.3)	(0.4)	(0.7)	(10.2)						
66Q3-75Q4	0.095	-0.111	-0.023	0.009	-	-	0.309	0.083	0.001	0.064	0.457	0.896	.008979	1.8	.0025	6.0	5.9
sua	(4.3)	(3.4)	(1.1)	(2.1)	-	-	(5.2)	(1.9)	(0.0)	(1.1)	(7.0)						
Treasury estimate[1]	0.1223	-0.1451	-0.0246	-	-	-	0.2605	0.1697	0.0769	0.0741	0.4274	-	-	-	-	-	-

	PPS(8)	F(t-k,R)
66Q3-80Q4	-	1.44
sa		(35,15)
66Q3-75Q4	4.8	2.7
sa		(19,12)
66Q3-80Q4	-	1.76
sua		(35,15)
66Q3-75Q4	4.6	2.7
sua		(19,12)
Treasury estimate[1]	-	-

[1] This is the last estimate published in the 1981 Treasury manual.

TABLE 4

LBS EQUATION

	$\Delta_1 C_1$	$\Delta_1 C_2$	$\Delta_1 Y$	$\Delta_2 Y_2$	C_3/Y_4	$\Delta_1 P$	$\Delta_1 \Delta_1 P_1$	$\Delta_1 P_3$	68	73	79
66Q3-80Q4	-0.079	0.004	0.179	0.019	-0.019	-0.014	-0.3569	-0.219	0.02	0.014	0.02
sa	(0.07)	(0.9)	(3.1)	(0.4)	(0.4)	(0.1)	(2.5)	(1.9)	(3.5)	(2.4)	(3.4)
66Q3-75Q4	-0.096	0.027	0.182	0.001	0.028	-0.044	-0.248	-0.098	0.02	0.013	-
sa	(0.6)	(0.2)	(2.0)	(0.1)	(0.3)	(0.2)	(1.2)	(0.5)	(2.9)	(2.0)	-
66Q3-80Q4	-0.084	-0.354	0.216	0.082	-0.008	-0.059	-0.434	-0.161	0.017	0.009	0.009
sua	(0.7)	(2.6)	(3.2)	(1.1)	(0.1)	(0.4)	(3.7)	(1.1)	(2.0)	(1.2)	(1.1)
66Q3-75Q4	-0.101	-0.326	0.089	0.133	-0.013	-0.371	-0.237	0.182	0.022	0.014	-
sua	(0.8)	(2.6)	(1.2)	(1.9)	(0.1)	(2.1)	(1.3)	(1.0)	(2.9)	(2.0)	-
Original											
est 62-80	-0.47	-0.344	0.208	0.151	-0.096	-0.201	-0.213	-0.166	0.017	0.01	0.017
sa	(4.6)	(3.0)	(4.5)	(3.0)	(1.7)	(1.8)	(2.0)	(1.7)	(3.4)	(2.9)	(3.4)

	Q1	Q2	Q3	CNST	$\bar{R}^2$	SE	RSS	DW	LB(8)	LM(8)	PPS(8)	F(t-k,R)
66Q3-80Q4	-	-	-	0.006	0.519	.007502	.002589	2.0	4.1	8.5	-	4.34
sa	-	-	-	(0.8)								(39,7)
66Q3-75Q4	-	-	-	0.011	0.422	.008273	.001826	2.0	2.3	9.2	2.8	3.42
sa	-	-	-	(0.8)								(21.6)
66Q3-80Q4	-0.149	-0.014	-0.067	0.065	0.974	.010272	.004537	2.2		23.0	-	2.36
sua	(20.6)	(0.7)	(3.2)	(4.0)								(36,7)
66Q3-75Q4	-0.153	0	-0.061	0.061	0.98	.009113	.003155	2.2		25.4	4.4	4.71
sua	(14.2)	(0.0)	(3.0)	(3.3)								(18,7)
Original												
est 62-80	-	-	-	0.0045	0.671	.0062						
sa	-	-	-	(0.6)								

TABLE 5

HUS EQUATION

	C/Y^*_4	L/Y^*_4	ΔL_{11}	$DL\Delta Y^*_4$	CNST	68	73	79	Q1	Q2	Q3
66Q3-80Q4 sa	-0.126 (4.0)	0.052 (4.6)	0.21 (2.3)	0.063 (13.8)	-0.06 (4.2)	0.0127 (2.6)	0.021 (4.1)	0.019 (3.8)	0.00061 (0.2)	-0.00081 (0.2)	-0.00063 (0.2)
66Q3-75Q4 sa	-0.243 (2.9)	0.14 (2.8)	0.101 (0.7)	0.8607 (9.1)	-0.171 (2.7)	0.0143 (2.9)	0.09787 (3.3)	- -	-0.0008 (0.0)	-0.0033 (0.4)	-0.9864 (0.2)
66Q3-80Q4 sua	-0.0117 (3.7)	0.0495 (4.3)	0.262 (2.9)	0.060 (13.4)	-0.05 (3.8)	0.0111 (2.2)	0.02 (4.0)	0.0176 (3.4)	-0.0097 (2.6)	-0.0083 (2.5)	-0.004 (1.4)
66Q3-75Q4 sua	-0.254 (3.3)	0.151 (2.9)	0.151 (1.1)	0.057 (8.9)	-0.172 (2.9)	0.01356 (2.6)	0.01857 (3.4)	- -	-0.0247 (3.0)	-0.0917 (3.0)	-0.01 (3.2)
Original estimate 62Q4-73Q1 sua	-0.2 (4.0)	0.074 (4.1)	0.24 (3.4)	0.082 (16.4)	-0.098 (3.9)	0.009 (4.5)	0.009 (4.5)	0.099 (4.5)	-0.017 (4.2)	-0.007 (2.3)	-0.003 (1.0)

	$\bar{R}^2$	SE	DW	RSS	LB(8)	LM(8)	PPS(8)	F(t-k,R)
66Q3-80Q4 sa	0.899	.00687	1.7	.002218	15.4	21.7	-	1.3 (38,9)
66Q3-75Q4 sa	0.833	.006833	1.7	.001307	7.0	11.2	-	0.8 (20,8)
66Q3-80Q4 sua	0.892	.00709	1.6	.002363	12.6	21.1	-	1.1 (38,9)
66Q3-75Q4	0.881	.007027	1.5	.001383	8.3	11.3	7.1	0.9 (20,8)
Original estimate 62Q4-73Q1 sua	0.928	.0052				11.2(6)		

TABLE 6

NI EQUATION

	ΔY_4	C/Y_4	ΔGL_{1} $_{1}$	IL	68	73	79	Q1	Q2	Q3	$\bar{R}^2$	DW	SE	RSS	LB(8)	LM(8)
66Q3-80Q4	0.409	-0.098	0.188	-0.081	0.007	0.004	0.013	0.001	-0.001	-0.000	0.879	1.7	.00999	.00479	6.9	15.2
sa	(8.6)	(4.6)	(1.4)	(3.6)	(0.8)	(0.5)	(1.5)	(0.4)	(0.3)	(0.1)						
66Q3-75Q4	0.448	-0.131	0.056	-0.133	0.008	0.005	-	-0.001	-0.001	-0.000	0.895	2.2	.008973	.002335	5.8	13.8
sa	(5.9)	(4.8)	(3.5)	(3.5)	(1.1)	(0.7)	-	(0.3)	(0.2)	(0.1)						
66Q3-80Q4	0.377	-0.104	0.333	-0.061	0.005	0.002	0.01	-0.006	-0.005	-0.003	0.873	1.6	.010227	.00502	10.9	20.1
sua	(8.1)	(3.9)	(2.8)	(2.7)	(0.6)	(0.3)	(1.2)	(1.2)	(1.1)	(0.8)						
66Q3-75Q4	0.378	-0.154	0.298	-0.107	0.007	0.005	-	-0.013	-0.009	-0.005	0.887	2.1	.009396	.002561	9.7	16.3
sua	(5.2)	(4.3)	(1.6)	(2.8)	(0.6)	(0.8)	-	(2.0)	(1.7)	(1.1)						
Original estimate 65Q1-75Q4	0.31	-0.15	0.45	-0.12									.0075			10.6
sa	(7.0)	(5.6)	(4.2)	(4.9)												

	PPS(8)	F(t-k, R)
66Q3-80Q4 sa	-	5.25 (40, 8)
66Q3-75Q4 sa	6.8	1.12 (23, 6)
66Q3-80Q4 sua	-	1.52 (40, 8)
66Q3-75Q4 sua	5.9	0.85 (23, 6)
Original estimate 65Q1-75Q4 sa		

TABLE 7
HENDRY EQUATION

	$\Delta_4 Y^*$	$\Delta_1\Delta_4 Y^*$	L/Y^*_1	C/Y^*_4	$\Delta_1 L_1$	$R-\Delta_4 P_1$	CNST	Q1	Q2	Q3
66Q3-80Q4 sa	0.454 (11.7)	-0.154 (4.0)	0.066 (4.4)	-0.177 (5.5)	0.092 (1.0)	-0.074 (1.5)	-0.082 (4.6)	0.001 (0.4)	-0.000 (0.1)	-0.001 (0.3)
66Q3-75Q4 sa	0.5 (9.5)	-0.157 (3.4)	0.11 (2.6)	-0.28 (3.9)	0.02 (0.1)	-0.139 (2.0)	-0.142 (2.8)	-0.000 (0.0)	-0.002 (0.7)	-0.001 (0.2)
66Q3-80Q4 sua	0.441 (11.4)	-0.168 (4.2)	0.066 (4.4)	-0.156 (4.8)	0.176 (1.9)	-0.09 (1.8)	-0.071 (4.2)	-0.011 (3.2)	-0.012 (3.6)	-0.006 (2.2)
66Q3-75Q4 sua	0.457 (9.3)	-0.166 (3.3)	0.096 (2.3)	-0.258 (3.7)	0.23 (1.5)	-0.155 (2.0)	-0.115 (2.4)	-0.017 (2.8)	-0.016 (3.2)	-0.006 (1.6)
Original est 65-81 sua	0.52 (13.0)	-0.26 (6.5)	0.072 (5.5)	-0.13 (3.3)	0.18 (3.3)	-0.21 (4.2)	-0.078 (5.2)	-0.01 (2.5)	-0.09 (2.3)	-0.003 (1.0)

	68	73	79	$\bar{R}^2$	SE	RSS	DW	LB(8)	LM(8)	PPS(8)	F (t-k, R)
66Q3-80Q4 sa	0.012 (2.7)	0.017 (3.4)	0.018 (3.8)	0.913	.006366	.001824	2.0	11.4	17.2	-	1.13 (40, 5)
66Q3-75Q4 sa	0.013 (2.7)	0.015 (2.8)	- -	0.898	.006395	.001063	2.5	11.4	15.5	15.8	2.62 (22, 4)
66Q3-80Q4 sua	0.011 (2.3)	0.017 (3.4)	0.016 (3.3)	0.906	.006628	.001977	2.0	9.8	17.9	-	1.63 (40, 5)
66Q3-75Q4 sua	0.012 (2.4)	0.018 (3.4)	- -	0.885	.006902	.001239	2.4	10.2	15.3	18.3	3.3 (22, 4)
Original est 65-81 sua	0.01 (2.5)	0.01 (2.5)	0.01 (2.5)	0.900	.0074				(6) 1.4		

TABLE 8

DAVIDSON McKINNON TEST - SEASONALLY ADJUSTED DATA

Tested Equation	Value of edv from: Hendry	LBS	NI	DHSY	HUS	HMT
Hendry	-	-0.126 (0.6)	0.044 (1.2)	-0.295 (0.5)	0.149 (0.2)	0.098 (0.3)
LBS	0.02 (0.0)	-	0.028 (0.2)	0.58 (1.8)	0.195 (0.4)	-0.254 (1.1)
NI	0.763 (1.7)	0.157 (0.7)	-	-0.03 (0.1)	0.351 (0.9)	-0.127 (0.5)
DHSY	1.1 (2.5)	-0.109 (0.7)	-0.053 (0.2)	-	0.056 (0.1)	0.317 (1.1)
HUS	0.968 (2.4)	-0.098 (0.7)	0.056 (0.2)	-0.208 (0.6)	-	0.097 (0.4)
HMT	0.832 (2.0)	-0.202 (1.2)	0.071 (0.3)	-0.092 (0.3)	0.34 (0.8)	-

TABLE 9

DAVIDSON McKINNON TEST - SEASONALLY UNADJUSTED DATA

Tested Equation	Value of edv from: Hendry	LBS	NI	DHSY	HUS	HMT
Hendry	-	-0.19 (1.3)	0.039 (0.1)	-0.585 (1.0)	-0.173 (0.3)	0.36 (0.9)
LBS	-0.864 (1.2)	-	0.243 (0.8)	0.397 (0.8)	1.04 (1.9)	-0.179 (0.4)
NI	1.03 (2.8)	-0.037 (0.5)	-	-0.026 (0.1)	0.082 (0.3)	-0.064 (0.2)
DHSY	1.1 (3.5)	-0.004 (0.3)	-0.085 (0.4)	-	0.02 (0.07)	0.344 (0.9)
HUS	1.05 (3.3)	0.001 (0.1)	0.001 (0.0)	-0.395 (0.9)	-	0.229 (0.7)
HMT	1.05 (3.5)	-0.004 (0.3)	-0.052 (0.2)	-0.097 (0.3)	0.086 (0.3)	-

TABLE 10

FORECASTS, errors in £mn, 1975 prices

		1976	1977	1978	1979	1980
Hendry,	1a off	226	502	934	1,328	711
	1a on	226	381	993	1,169	595
LBS		397	747	2,164	4,101	4,737
NI,	1a off	677	1,047	1,360	1,052	46
	1a on	673	1,034	1,379	1,081	73
HUS,	1a off	612	1,158	2,445	3,130	3,070
	1a on	611	1,128	2,295	3,016	3,113
DHSY		- 191	8	105	155	- 532
HMT		- 353	- 98	165	242	- 429
Townend	1a off	46	876	741	307	- 570
	1a on	159	653	625	302	- 647

TABLE 11

SUMMARY STATISTICS

		Theils 'U'	UM	Mean absolute error	% RMS error
Hendry,	1a off	1.201	0.002	215	1.7
	1a on	1.154	0.001	168	1.5
LBS		0.79	0.172	608	4.7
NI,	1a off	0.967	0.003	241	1.8
	1a on	0.989	0.003	242	1.8
HUS,	1a off	0.919	0.045	521	3.7
	1a on	0.979	0.037	508	3.6
DHSY		0.727	0.001	100.7	0.8
HMT		0.685	0.000	98	0.8
Townend	1a off	0.757	0.0024	157	1.2
	1a on	0.938	0.0032	146	1.4

TABLE 12

EFFECTS ON THE SAVING RATIO OF A 1% INCREASE IN THE RATE OF INFLATION (PERCENTAGE POINTS)

(a) 6680 estimate

	Townend	HUS	NI	Hendry	DHSY	HMT	LBS
70	0.1	0.125	0.05	0.125	0.075	0.1	0.125
71	0.2	0.275	0.175	0.25	0.175	0.2	0.275
72	0.3	0.4	0.275	0.35	0.275	0.3	0.425
73	0.4	0.525	0.325	0.425	0.375	0.4	0.55
74	0.45	0.6	0.4	0.55	0.425	0.4	0.65
75	0.5	0.675	0.4	0.55	0.475	0.5	0.75
76	0.525	0.8	0.375	0.725	0.55	0.5	0.85
77	0.6	1.025	0.375	0.95	0.575	0.6	0.975
78	0.65	1.2	0.325	1.125	0.6	0.6	1.025
79	0.7	1.4	0.325	1.325	0.6	0.6	1.025
80	0.7	1.525	0.325	1.45	0.675	0.6	1.1

(b) Original estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	0.1	0.175	0.1	0.125	0.05	0.1	0.175
71	0.3	0.25	0.275	0.3	0.175	0.2	0.275
72	0.4	0.35	0.4	0.45	0.175	0.3	0.35
73	0.5	0.35	0.525	0.575	0.225	0.4	0.425
74	0.6	0.425	0.65	0.675	0.25	0.4	0.475
75	0.625	0.45	0.775	0.65	0.325	0.5	0.5
76	0.775	0.525	0.85	0.575	0.475	0.5	0.525
77	0.8	0.55	1.175	0.575	0.725	0.575	0.575
78	0.9	0.625	1.375	0.525	0.975	0.6	0.6
79	0.95	0.625	1.525	0.55	1.2	0.6	0.6
80	1.0	0.625	1.7	0.5	1.375	0.6	0.6

(c) 6680 estimate with liquidity feedback

	Townend	HUS	NI	Hendry
70	0.1	0.125	0.025	0.1
71	0.125	0.275	0.15	0.225
72	0.2	0.325	0.25	0.25
73	0.2	0.375	0.3	0.3
74	0.225	0.45	0.375	0.375
75	0.225	0.45	0.375	0.375
76	0.225	0.5	0.375	0.375
77	0.225	0.625	0.425	0.45
78	0.225	0.65	0.35	0.5
79	0.225	0.725	0.35	0.55
80	0.225	0.75	0.375	0.55

TABLE 13

EFFECTS ON THE SAVING RATIO OF A 1% INCREASE IN REAL INCOME GROWTH (IN PERCENTAGE POINTS)

(a) 6680 estimate

	Townend	Hendry	LBS	NI	HUS	DHSY	HMT
70	0.55	0.575	0.675	0.5	0.55	0.5	0.475
71	1.0	0.975	1.325	0.9	0.95	0.975	0.875
72	1.325	1.25	1.775	1.2	1.35	1.225	1.125
73	1.625	1.575	2.175	1.5	1.725	1.525	1.4
74	2.05	1.9	2.675	1.8	2.125	1.8	1.7
75	2.45	2.15	3.175	2.0	2.4	2.075	1.9
76	2.825	2.45	3.55	2.125	2.775	2.3	2.125
77	3.25	2.875	4.025	2.375	3.225	2.5	2.375
78	3.4	3.075	4.125	2.4	3.45	2.525	2.4
79	3.525	3.275	4.25	2.475	3.675	2.575	2.475
80	3.875	3.45	4.475	2.575	3.875	2.7	2.6

(b) Original estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	0.475	0.475	0.45	0.55	0.45	0.5	0.625
71	0.775	0.875	0.775	1.025	0.925	0.875	1.125
72	1.125	1.125	1.025	1.325	1.225	1.175	1.375
73	1.425	1.375	1.325	1.6	1.6	1.425	1.625
74	1.7	1.675	1.6	1.875	1.975	1.725	1.9
75	2.05	1.9	1.75	2.025	2.3	1.975	2.125
76	2.4	2.175	2.0	2.1	2.7	2.175	2.3
77	2.775	2.4	2.375	2.225	2.225	2.375	2.45
78	2.925	2.475	2.55	2.2	3.575	2.4	2.425
79	3.05	2.55	2.8	2.25	3.875	2.45	2.4
80	3.35	2.7	2.975	2.3	4.225	2.6	2.45

(c) Liquidity Feedback (6680 estimate)

	Townend	HUS	Hendry	NI
70	0.575	0.55	0.5	0.475
71	0.95	0.975	0.925	0.875
72	1.225	1.35	1.175	1.175
73	1.55	1.65	1.475	1.45
74	1.925	2.025	1.75	1.75
75	2.25	2.275	1.95	1.975
76	2.65	2.6	2.275	2.15
77	3.05	2.95	2.5	2.4
78	3.15	3.075	2.6	2.4
79	3.25	3.25	2.7	2.475
80	3.575	3.45	2.8	2.55

TABLE 14

EFFECTS ON THE SAVING RATIO OF A ONE-PERIOD SHOCK OF £430 MILLION TO MONEY INCOME [EQUAL TO £430 MILLION INCREASE IN REAL INCOME] (PERCENTAGE POINTS)

(a) 6680 estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	0.6	0.6	0.625	0.6	0.6	0.55	0.825
71	-0.05	-0.1	-0.025	-0.066	-0.05	-0.075	-0.1
72	0	-0.075	-0.025	-0.05	-0.025	-0.075	-0.1
73	0	-0.075	-0.05	-0.05	-0.025	-0.05	-0.025
74	0	-0.05	-0.025	-0.05	-0.025	-0.05	0
75	0	-0.05	0	-0.05	0	-0.05	0
76	0	-0.05	0	-0.025	-0.025	-0.05	0
77	0	-0.025	0	-0.025	-0.025	-0.025	0
78	0	-0.025	0	-0.025	0	-0.025	0
79	0	-0.025	0	-0.025	0	-0.025	0
80	0	-0.025	0	-0.025	0	-0.025	0

(b) Original estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	0.425	0.55	0.5	0.7	0.55	0.575	0.675
71	-0.05	-0.05	-0.025	-0.125	0 5	-0.075	-0.125
72	0	-0.025	0	-0.1	-0.025	-0.05	-0.1
73	0	-0.025	-0.025	-0.075	0	-0.05	-0.1
74	0	-0.025	-0.025	-0.075	0	-0.025	-0.1
75	0	-0.025	0	-0.05	0	-0.05	-0.1
76	0	-0.025	-0.025	-0.05	0	-0.05	0
77	0	-0.05	-0.025	-0.05	0	-0.05	0
78	0	-0.025	0	-0.025	-0.025	-0.025	0
79	0	-0.025	0	-0.025	0	-0.025	0
80	0	-0.025	0	-0.025	0	-0.025	0

(c) Liquid Assets Fed Back (6680 estimate)

	Townend	HUS	Hendry	NI
70	0.6	0.625	0.575	0.65
71	-0.075	-0.05	-0.005	-0.1
72	0	-0.025	-0.05	-0.05
73	0	-0.025	-0.05	-0.05
74	0	-0.025	-0.05	-0.05
75	0	-0.025	-0.05	-0.05
76	0	-0.025	-0.05	-0.05
77	0	-0.025	-0.025	-0.05
78	0	-0.05	0	-0.025
79	0	-0.025	0	0
80	0	-0.025	0	-0.025

TABLE 15

EFFECTS ON THE SAVING RATIO OF INCREASING PRICES BY 5% IN ONE PERIOD
(IN PERCENTAGE POINTS)

(a) 6680 estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	-0.4	-0.525	-0.5	-0.55	-0.475	-0.575	-0.625
71	0.05	0.75	0.1	0.125	0.15	0.75	0
72	0	0.05	-0.05	0.05	-0.25	0.05	0
73	0	0.05	0.05	0.05	0.075	0.05	0
74	0	0.025	0.075	0.05	0.05	0.05	0
75	0	0.025	0.05	0.025	0.05	0.025	0
76	0	0.025	0.025	0.025	0.05	0.025	0
77	0	0	0.05	0.025	0.025	0.025	0
78	0	0	0.05	0.025	0.025	0.025	0
79	0	0	0.025	0.025	0.025	0.025	0
80	0	0	0.025	0	0.025	0.025	0

(b) Original estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	-0.275	-0.4	-0.325	-0.65	-0.55	-0.45	-0.45
71	0.05	0.025	0.15	0.175	-0.125	0.05	0.025
72	0	0.025	-0.1	0.075	-0.075	0.05	0.1
73	0	0.025	0.05	0.075	0.075	0.025	0.1
74	0	0.025	0.05	0.05	0.05	0.025	0.05
75	0	0.025	0.025	0.05	0.05	0.05	0
76	0	0.025	0.025	0.05	0.05	0.025	0
77	0	0.025	0.025	0.025	0.05	0.025	0
78	0	0.025	0	0.025	0.025	0.025	0
79	0	0.0	0.025	0.025	0.025	0.025	0
80	0	0.0	0.025	0.025	0.025	0.025	0

(c) With Liquid Assets Fed Back (6680 estimate)

	Townend	HUS	Hendry	NI
70	-0.45	-0.5	-0.475	-0.575
71	0.025	0.1	0.15	0.1
72	0	-0.075	-0.025	0.025
73	0	0.05	0.025	0.025
74	0	0.05	0.025	0.025
75	0	0	0.05	0.025
76	0	0.05	0.05	0.025
77	0	0.05	0.025	0.025
78	0	0.05	0.05	0.025
79	0	0.05	0.05	0.025
80	0	0.025	0	0.025

TABLE 16

EFFECTS ON THE SAVING RATIO OF INCREASING MONEY INCOMES BY £430 MN PER QUARTER (IN PERCENTAGE POINTS)

(a) 6680 estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	2.675	2.6	2.6	2.275	2.375	2.375	3.175
71	1.85	1.85	1.975	1.825	1.825	1.675	2.7
72	1.425	1.275	1.55	1.3	1.375	1.175	2.025
73	1.15	8.75	1.225	0.9	1.075	0.8	1.5
74	1.0	0.55	0.975	0.575	0.875	0.55	1.125
75	0.8	0.275	0.7	0.025	0.55	0.25	0.725
76	0.725	0.075	0.55	-0.025	0.4	0.1	0.45
77	0.65	-0.075	0.475	-0.1	0.35	0	0.325
78	0.575	-0.2	0.35	-0.2	0.25	-0.1	0.1
79	0.375	-0.3	0.275	-0.3	0.15	-0.2	-0.5
80	0.325	-0.3	0.225	-0.4	0.1	-0.3	-1.75

(b) Original estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	2.35	2.225	2.175	2.675	2.25	2.475	2.95
71	1.61	1.625	1.5	2.025	1.75	1.725	2.075
72	1.225	1.15	1.15	1.3	1.475	1.175	1.2
73	1.0	0.85	0.975	0.775	1.275	0.8	0.625
74	0.775	0.625	0.725	0.375	1.125	0.525	0.25
75	0.7	0.325	0.475	-0.025	0.875	0.225	-0.1
76	0.625	0.2	0.375	-0.25	0.775	0.05	-0.25
77	0.55	0.075	0.35	-0.4	0.725	-0.05	-0.35
78	0.45	0	0.3	-0.5	0.625	-0.175	-0.4
79	0.35	-0.1	0.2	-0.525	0.525	-0.275	-0.475
80	0.3	-0.2	0.175	-0.575	0.45	-0.3	-0.5

(c) Liquidity fed back (6680 estimate)

	Townend	HUS	Hendry	NI
70	2.625	2.575	2.35	2.275
71	1.775	1.925	1.75	1.825
72	1.3	1.45	1.25	1.275
73	1.05	1.1	0.9	0.875
74	0.875	0.825	0.6	0.575
75	0.7	0.525	0.35	0.25
76	0.625	0.375	0.175	0.075
77	0.55	0.25	0.075	-0.05
78	0.425	0.125	-0.025	-0.075
79	0.325	-0.025	-0.125	-0.25
80	0.3	-0.1	-0.2	-0.3

TABLE 17

EFFECTS ON THE SAVING RATIO OF INCREASING MONEY INCOMES BY 5% EACH QUARTER (IN PERCENTAGE POINTS)

(a) 6680 estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	2.8	2.5	2.725	2.4	2.5	2.575	3.35
71	2.175	2.125	2.3	2.175	2.125	2.0	3.125
72	2.025	1.8	2.15	1.875	1.95	1.7	2.75
73	1.925	1.55	2	1.6	1.775	1.5	2.475
74	1.9	1.375	1.925	1.4	1.7	1.3	2.35
75	1.875	1.2	1.8	1.2	1.55	1.175	2.175
76	1.925	1.05	1.75	1.0	1.475	1.05	2.05
77	1.925	0.9	1.775	0.8	1.475	0.975	1.925
78	1.85	0.8	1.7	0.7	1.4	0.8	1.675
79	1.775	0.675	1.65	0.6	1.375	0.7	1.575
80	1.725	0.6	1.6	0.5	1.3	0.6	1.4

(b) Original estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	2.5	2.35	2.325	2.8	2.375	2.575	3.15
71	1.925	1.9	1.725	2.375	2.05	2.0	2.425
72	1.725	1.65	1.625	1.9	2.025	1.7	1.875
73	1.625	1.5	1.525	1.5	2.0	1.475	1.425
74	1.65	1.35	1.4	1.25	1.975	1.525	1.125
75	1.6	1.275	1.275	0.95	1.9	1.15	0.9
76	1.625	1.175	1.225	0.65	1.95	1.0	0.75
77	1.675	1.025	1.275	0.5	2.05	0.9	0.625
78	1.575	0.9	1.25	0.3	2.0	0.8	0.45
79	1.5	0.8	1.275	0.2	2.0	0.675	0.35
80	1.5	0.7	1.175	0.1	1.975	0.6	0.275

(c) Liquidity feedback (6680 estimate)

	Townend	HUS	Hendry	NI
70	2.75	2.7	2.45	2.375
71	2.075	2.25	1.975	2.1
72	1.875	2.025	1.75	1.8
73	1.775	1.85	1.55	1.525
74	1.7	1.75	1.475	1.4
75	1.7	1.575	1.25	1.2
76	1.725	1.5	1.175	1.0
77	1.775	1.4	1.1	0.875
78	1.65	1.325	0.975	0.725
79	1.575	1.2	0.875	0.625
80	1.55	1 1	0.75	0.55

TABLE 18

EFFECTS ON THE SAVING RATIO OF INCREASING PRICES BY 5% EACH QUARTER (PERCENTAGE POINTS)

(a) 6680 estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	-2.0	-2.1	-2.125	-2.225	-1.95	-1.975	-2.825
71	-1.375	-1.575	-1.375	-1.525	-1.275	-1.5	-2.375
72	-1.175	-1.35	-1.5	-1.325	-1.45	-1.3	-2.05
73	-1.125	-1.15	-1.425	-1.15	-1.225	-1.1	-1.875
74	-1.1	-1.0	-1.25	-1.075	-0.975	-1.0	-1.75
75	-1.1	-0.875	-1.075	-0.95	-0.825	-0.875	-1.6
76	-1.1	-0.775	-0.95	-0.875	-0.675	-0.8	-1.5
77	-1.125	-0.675	-0.825	-0.8	-0.575	-0.7	-1.475
78	-1.05	-0.6	-0.725	-0.675	-0.475	-0.6	-1.25
79	-0.975	-0.5	-0.6	-0.625	-0.375	-0.5	-1.15
80	-1.0	-0.4	-0.55	-0.55	-0.3	-0.5	-1.0

(b) Original estimate

	Townend	DHSY	HUS	NI	Hendry	HMT	LBS
70	-1.5	-1.525	-1.525	-2.325	-2.1	-2.05	-2.475
71	-0.85	-1.475	-0.55	-1.275	-1.575	-1.55	-1.8
72	-0.7	-1.3	-0.85	-1.025	-1.85	-1.3	-1.4
73	-0.725	-1.125	-0.8	-0.8	-1.675	-1.1	-1.075
74	-0.7	-1.105	-0.65	-0.75	-1.5	-1.0	-1.85
75	-0.7	-0.975	-0.5	-0.75	-1.3	-0.875	-0.675
76	-0.7	-0.9	-0.4	-0.525	-1.125	-0.775	-0.45
77	-0.7	-0.8	-0.325	-0.45	-1.025	-0.7	-0.45
78	-0.625	-0.7	-0.25	-0.375	-0.85	-0.6	-0.375
79	-0.575	-0.6	-0.175	-0.3	-0.775	-0.5	-0.25
80	-0.6	-0.6	-0.175	-0.275	-0.65	-0.4	-0.2

(c) Liquidity feedback (6680 estimate)

	Townend	HUS	Hendry	NI
70	-2.075	-2.15	-2.0	-2.275
71	-1.45	-1.4	-1.325	-1.55
72	-1.25	-1.625	-1.5	-1.3
73	-1.225	-1.525	-1.325	-1.15
74	-1.225	-1.4	-1.15	-1.05
75	-1.2	-1.2	-0.95	-0.95
76	-1.225	-1.1	-0.85	-0.85
77	-1.275	-1.025	-0.75	-0.725
78	-1.15	-0.9	-0.65	-0.625
79	-1.075	-0.85	-0.55	-0.525
80	-1.1	-0.75	-0.475	-0.5

TABLE 19

SUGGESTED SPECIFICATION

Estimation	$\Delta^{+}Y^{*}$	$\Delta\Delta_{1}{}^{+}Y^{*}$	$(C/Y^{*})^{+}$	$(L/Y^{*})^{+}$	$\Delta_{1}L_{t-1}$	CNST	68^{+}	73^{+}	79^{+}	$\bar{R}^{2}$	SE	RSS	DW	LB(8)	LM(8)
66Q3-80Q4 sa	0.349 (10.3)	-0.109 (3.0)	-0.1 (3.2)	0.033 (3.5)	0.107 (1.3)	-0.04 (3.2)	0.012 (2.2)	0.02 (3.7)	0.017 (3.3)	0.845	.005736	.001612	2.0	6.8	15.3
66Q3-75Q4 sa	0.352 (7.9)	-0.098 (2.1)	-0.17 (2.2)	0.064 (1.8)	0.004 (0.1)	-0.083 (1.8)	0.012 (2.3)	0.018 (3.3)	-	0.834	.005742	.000989	2.2	3.1	11.2

	PPS(8)	F(t-k,r)
66Q3-80Q4 sa	-	1.83 (28,21)
66Q3-75Q4 sa	11.4	1.46 (13,17)

Stability analysis

	Forecast χ^{2}	Sequential chow 1	2	3	4	Von Neumann test	't' test
No of significant values	0	2	0	0	0	0	8

Forecast

	la off	la on
76	+ 298	+ 311
77	+ 653	+ 622
78	+1,459	+1,330
79	+1,742	+1,598
80	+1,380	+1,268
U	0.766	0.758
UM	0.0169	0.015
MAE	277	258
% RMS	2.2	1.9

Simulations

	Increased inflation		Increased growth		One-period price rise		One-period income rise	
	la off	la on	la off	la on	la off	la on	la off	la on
70	0.125	0.125	0.525	0.525	-0.4	-0.425	0.6	0.625
71	0.3	0.275	0.975	0.975	0.15	0.125	-0.025	-0.075
72	0.425	0.35	1.325	1.275	-0.025	-0.025	0	-0.05
73	0.525	0.4	1.65	1.575	0.1	0.05	0	-0.025
74	0.625	0.45	1.975	1.9	0.05	0.05	0	0
75	0.625	0.5	2.2	2.175	0	0.025	0	0
76	0.75	0.5	2.575	2.375	0	0	0	0
77	0.95	0.6	2.95	2.725	0	0	0	0
78	1.1	0.65	3.125	2.8	0	0	0	0
79	1.225	0.7	3.325	2.925	0	0	0	0
80	1.35	0.7	3.5	3.075	0	0	0	0

Sustained increases in income

	by constant amount		by constant %		constant % rise in prices	
	la off	la on	la off	la on	la off	la on
70	2.475	2.475	2.6	2.6	-1.975	-2.0
71	1.975	1.9	2.3	2.25	-1.35	-1.425
72	1.45	1.4	2.025	1.975	-1.475	-1.55
73	1.125	1.0	1.9	1.775	-1.275	-1.375
74	0.825	0.7	1.75	1.55	-1.05	-1.175
75	0.525	0.35	1.55	1.35	-0.9	-1.05
76	0.35	0.225	1.5	1.25	-0.75	-0.9
77	0.275	0.075	1.475	1.15	-0.65	-0.8
78	0.175	-0.05	1.35	1.075	-0.525	-0.7
79	0.075	-0.15	1.275	0.975	-0.45	-0.6
80	0	-0.2	1.25	0.875	-0.4	-0.525

TABLE 19 (cont)

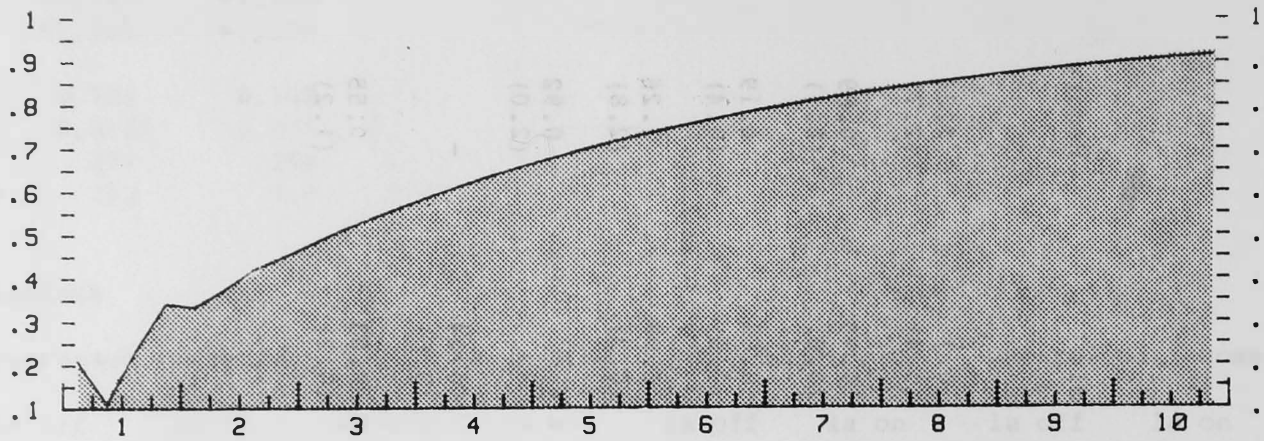
DAVIDSON McKINNON TEST - SEASONALLY ADJUSTED DATA

Tested Equation	Value of edv from: DHSY	HMT	LBS	HUS	NI	Hendry	NEW
DHSY	-	0.22 (0.8)	0.06 (0.4)	0.16 (0.4))	0.1 (0.4)	1.24 (2.7)	-0.42 (1.8)
HMT	-0.08 (0.3)	-	0.05 (0.3)	0.63 (1.6)	0.15 (0.6)	0.89 (2.1)	-0.78 (3.0)
LBS	0.4 (1.3)	-0.39 (1.6)	-	-0.13 (0.3)	-0.05 (0.3)	0.19 (0.4)	0.79 (2.3)
HUS	-0.06 (0.1)	0.05 (0.2)	0.05 (0.3)	-	0.08 (0.3)	1.26 (2.8)	-0.53 (1.8)
NI	0.03 (0.1)	-0.13 (0.6)	0.28 (1.4)	0.31 (0.8)	-	0.92 (2.0)	-0.36 (1.4)
Hendry	-0.44 (0.7)	0.09 (0.4)	-0.02 (0.1)	-0.37 (0.6)	0.24 (0.6)	-	-0.66 (2.0)
NEW	0.17 (0.6)	-0.28 (1.4)	0.26 (1.4)	-0.2 (0.5)	-0.03 (0.2)	0.55 (1.2)	-

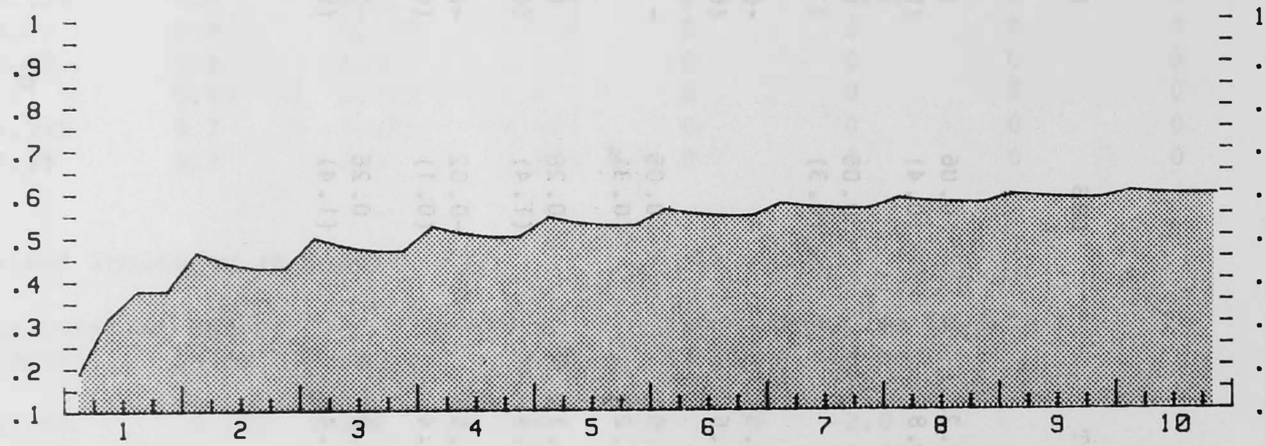
APPENDIX 3

FIGURE 1: Cumulative rational lags on income

(a) LBS



(b) HUS



(c) New equation

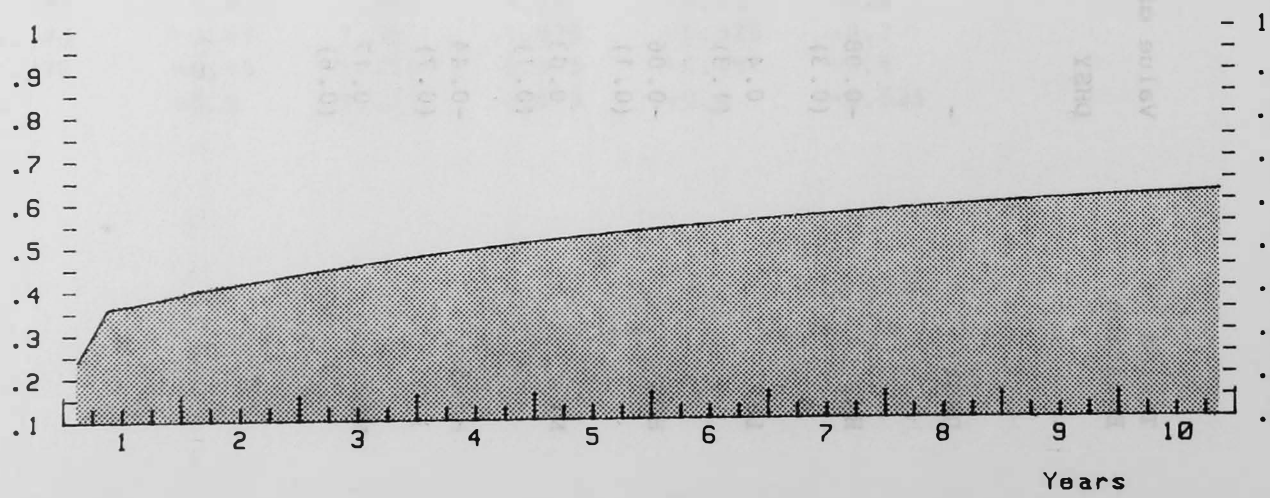
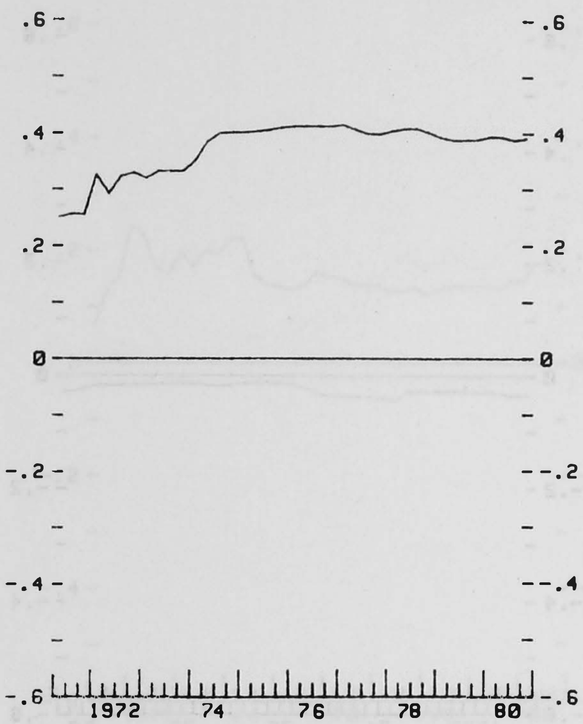
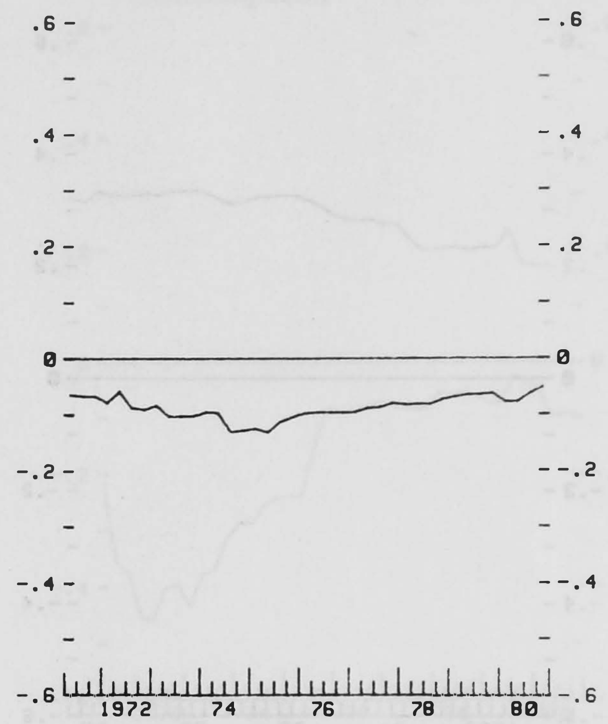


FIGURE 2: Coefficient values from recursive OLS, DHSY equation

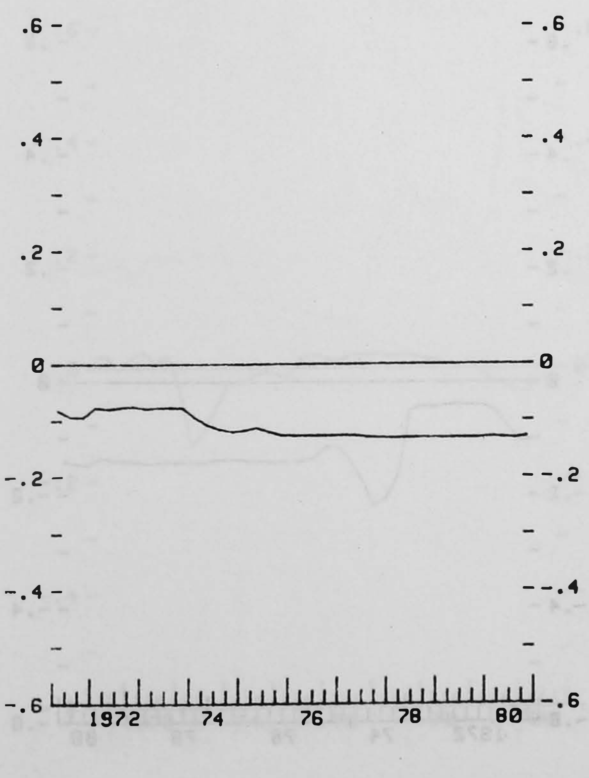
(a) Fourth difference of income



(b) Acceleration of income



(c) Lagged consumption/income ratio



(d) Fourth difference of prices

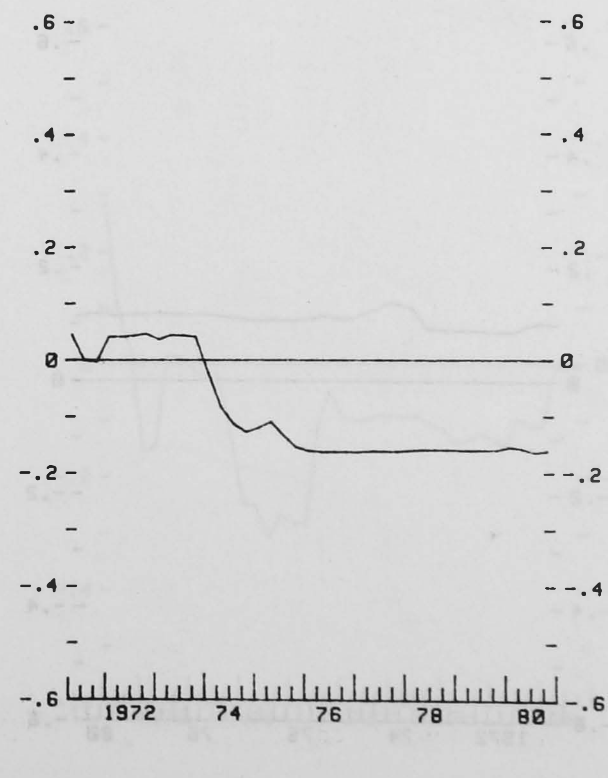


FIGURE 3: Coefficient values from recursive OLS, HMT equation

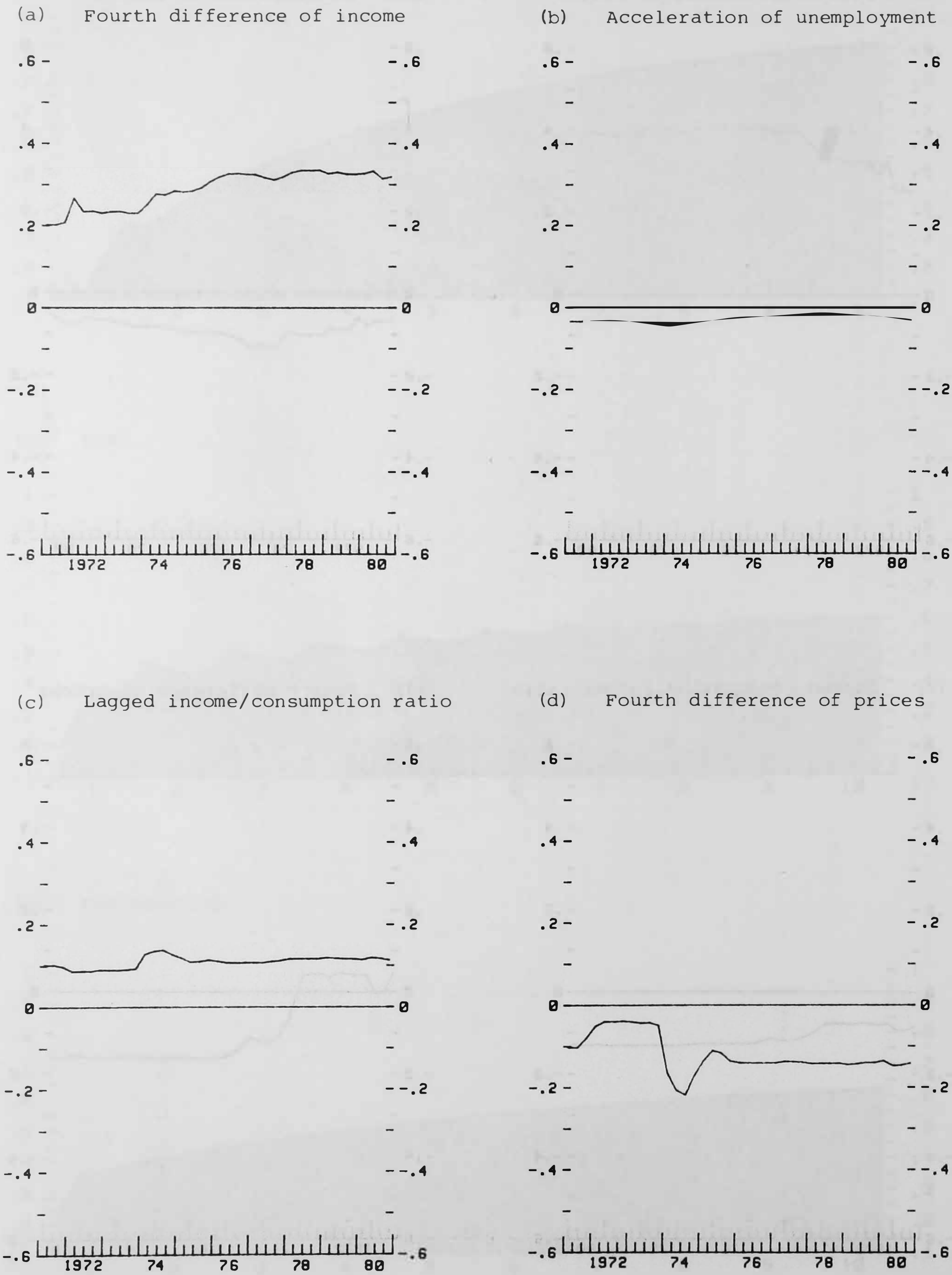
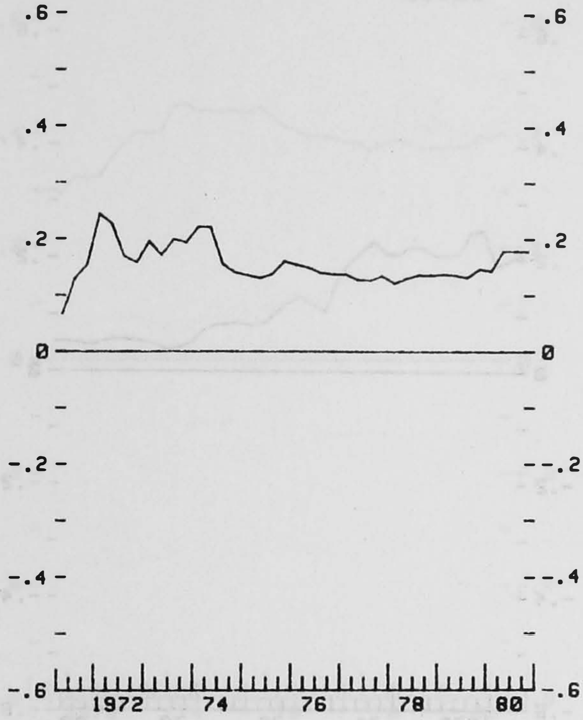


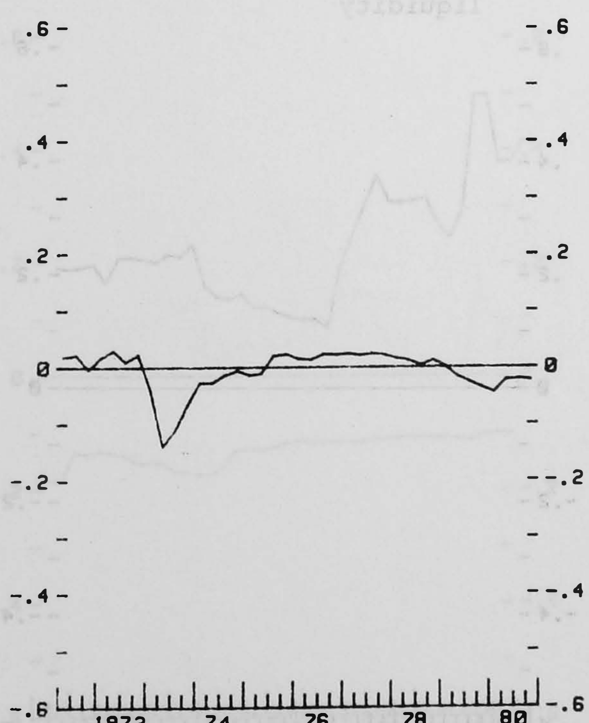
FIGURE 4: Coefficient values from recursive OLS, LBS equation

(a) First difference of income



(b) Lagged first difference of consumption

(c) Lagged consumption/income ratio



(d) First difference of prices

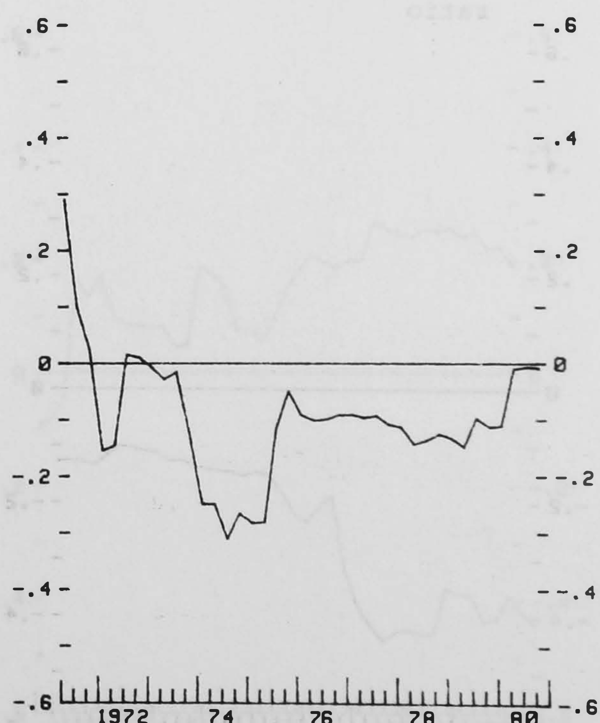
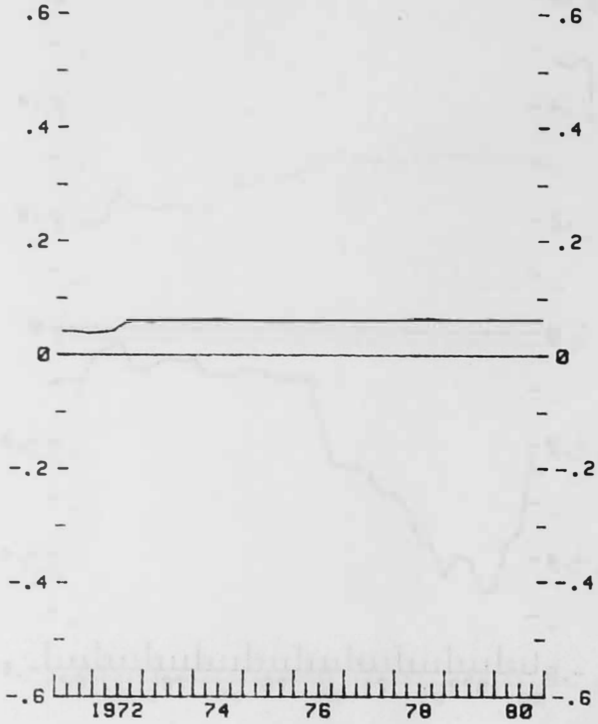
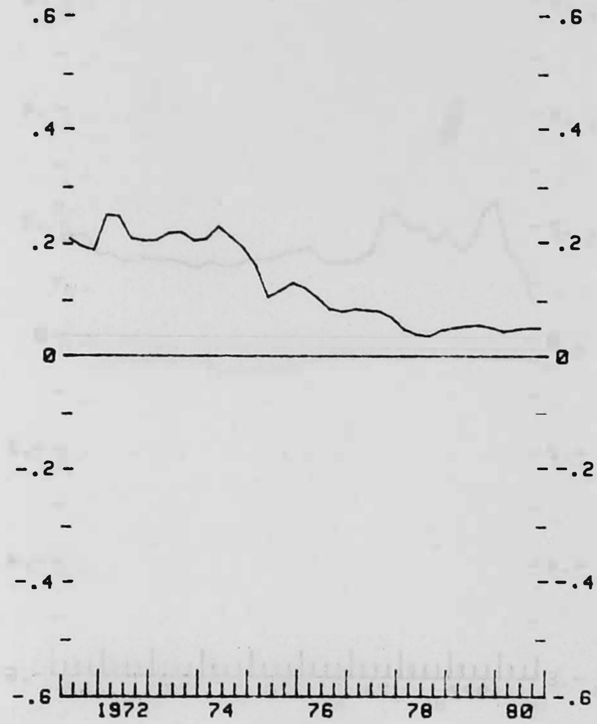


FIGURE 5: Coefficient values from recursive OLS, HUS equation

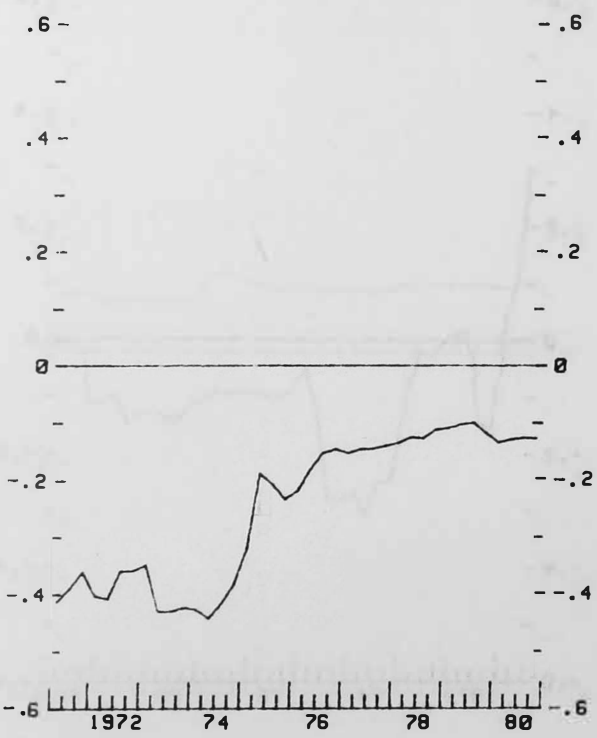
(a) Distributed lag of adjusted income



(b) Lagged liquidity/adjusted income ratio



(c) Lagged consumption/adjusted income ratio



(d) Lagged first difference of liquidity

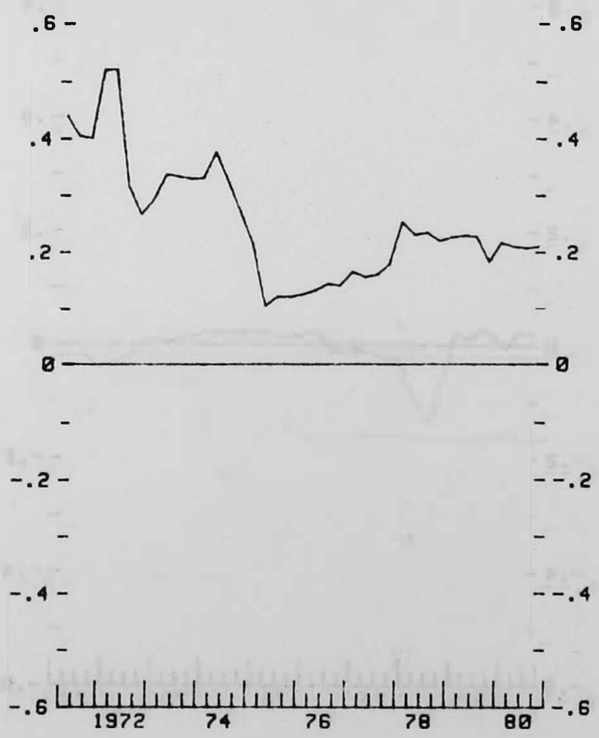
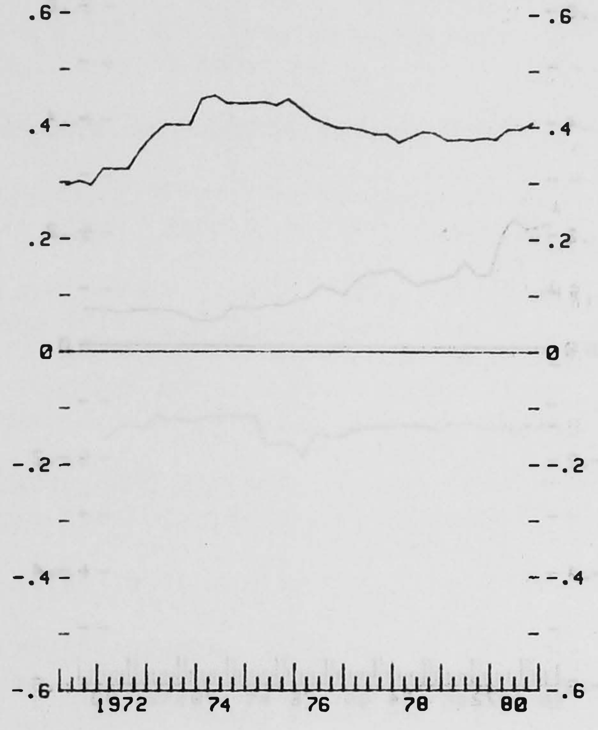
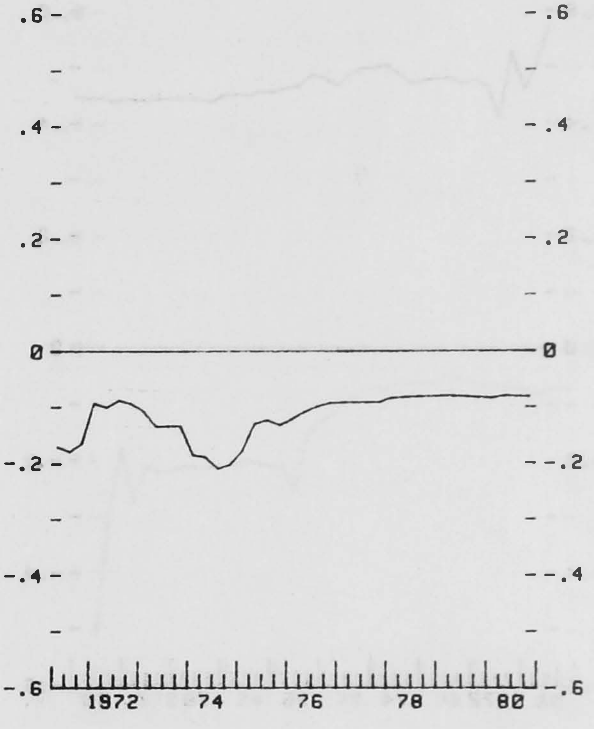


FIGURE 6: Coefficient values from recursive OLS, NI equation

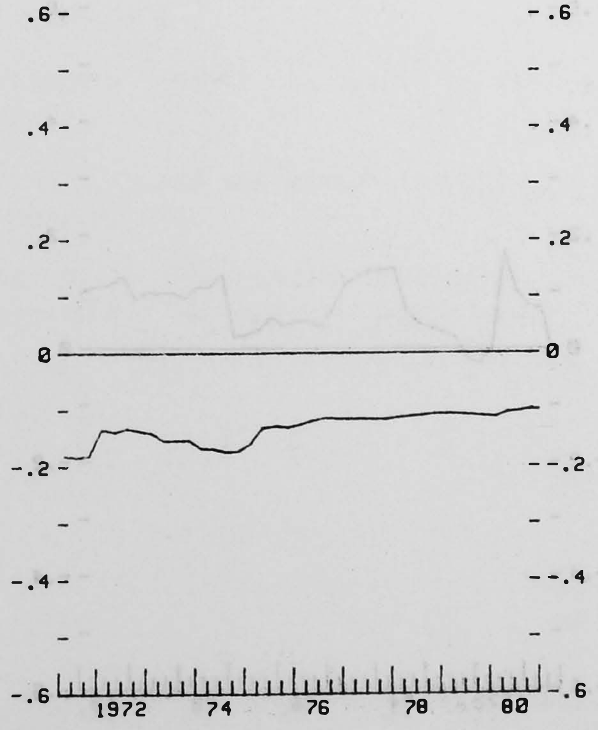
(a) Fourth difference of income



(b) Losses on gross liquid asset stocks



(c) Lagged consumption/income ratio



(d) Lagged first difference of gross liquidity

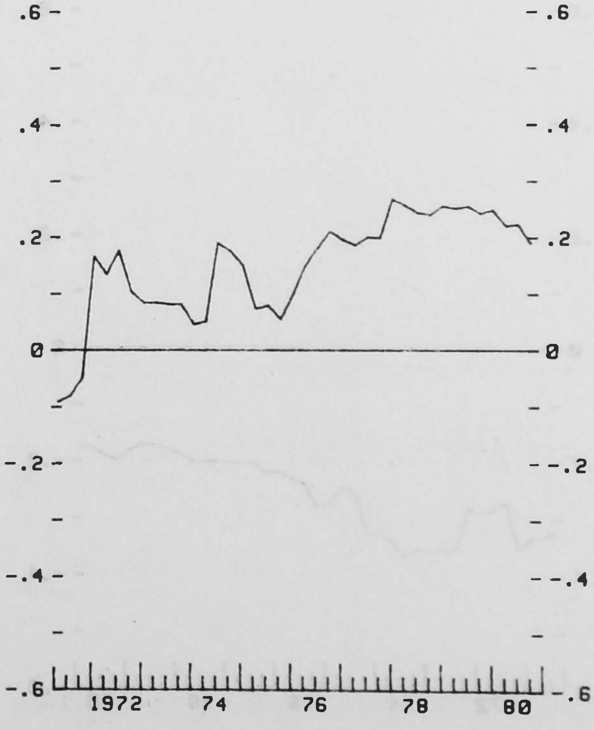
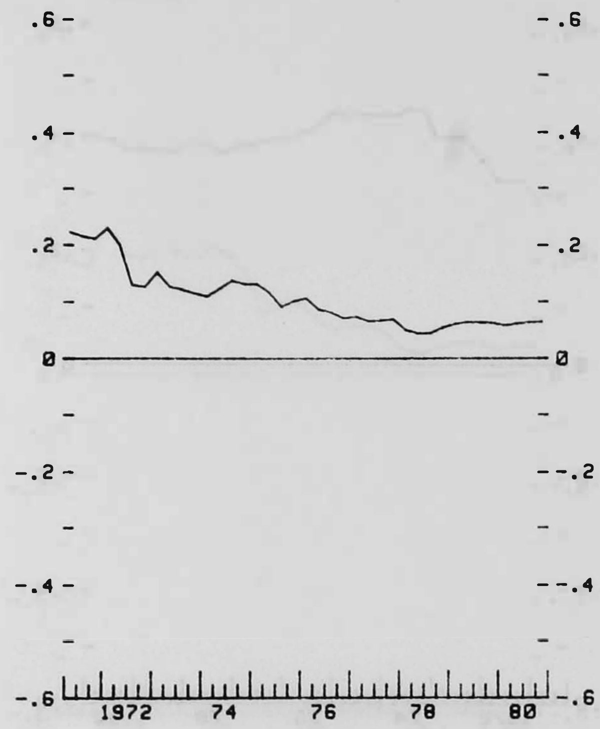


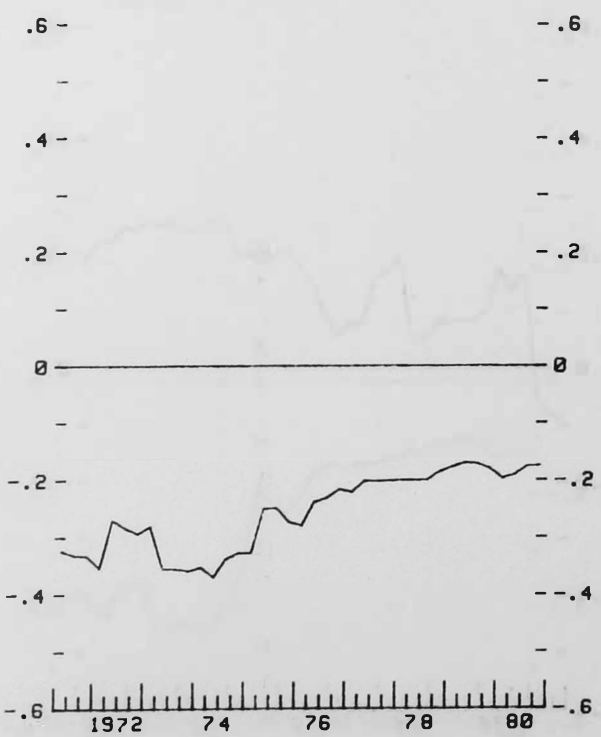
FIGURE 7: Coefficient values from recursive OLS, Hendry equation

(a) Fourth difference of adjusted income

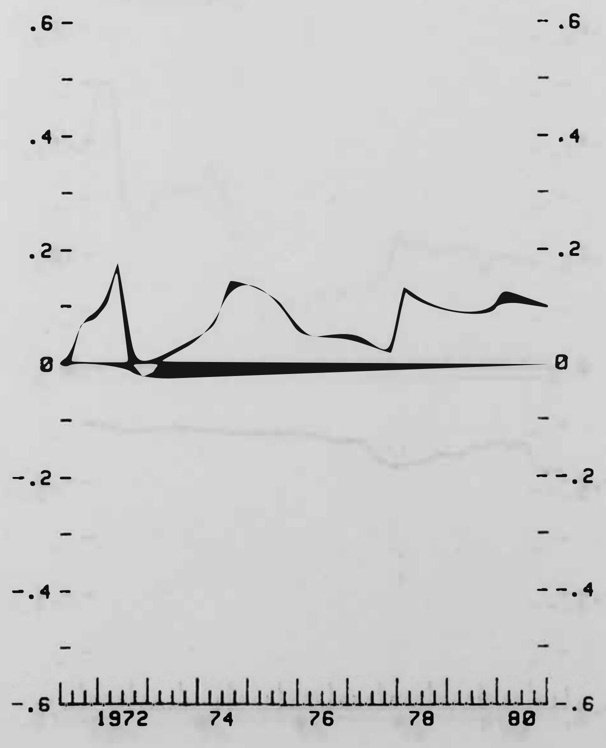
(b) Lagged liquidity/adjusted income ratio



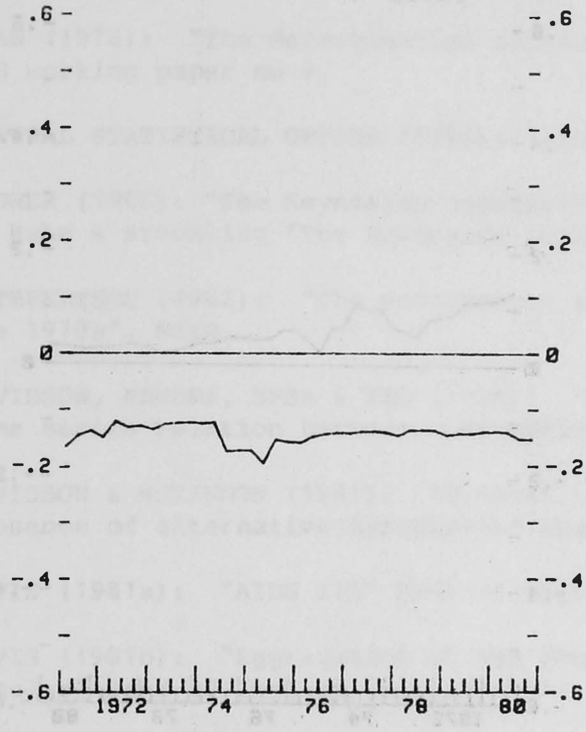
(c) Lagged consumption/adjusted income ratio



(d) Lagged first difference of liquidity



(e) Acceleration of adjusted income



(f) Real interest rates

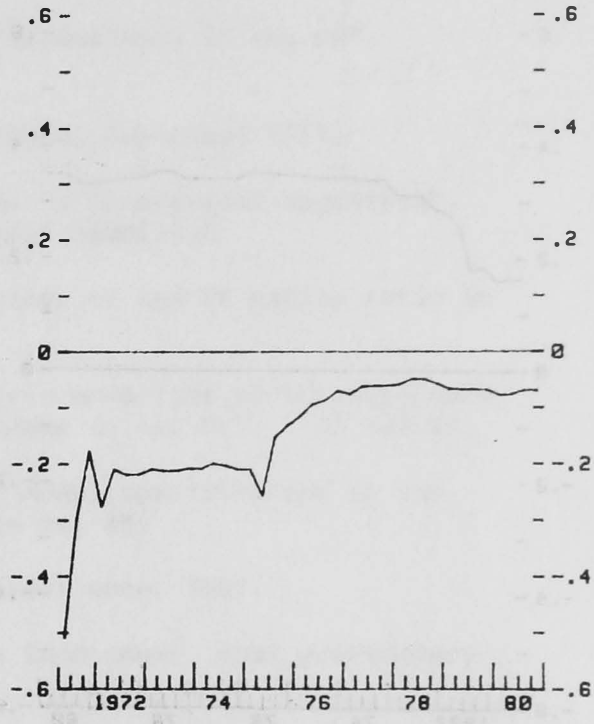
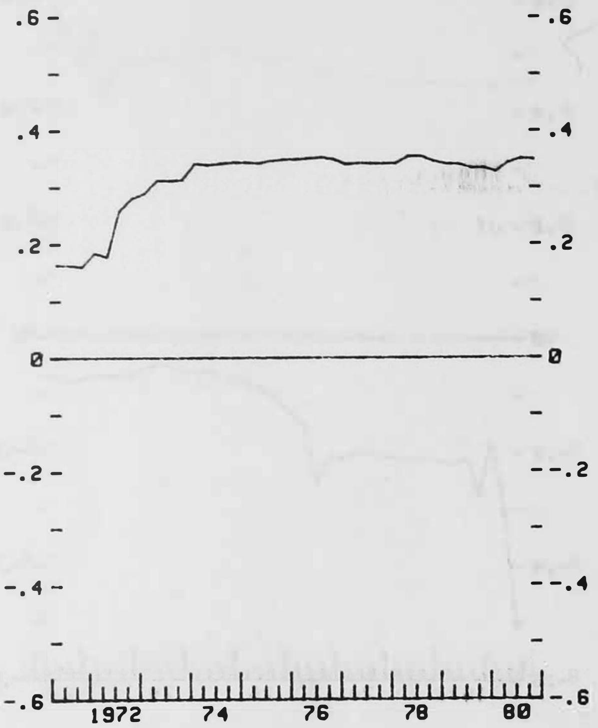
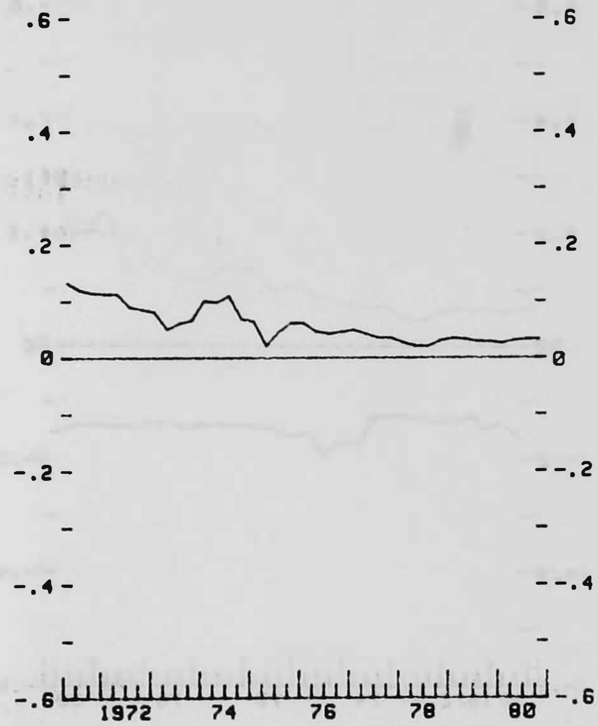


FIGURE 8: Coefficient values from recursive OLS; new equation

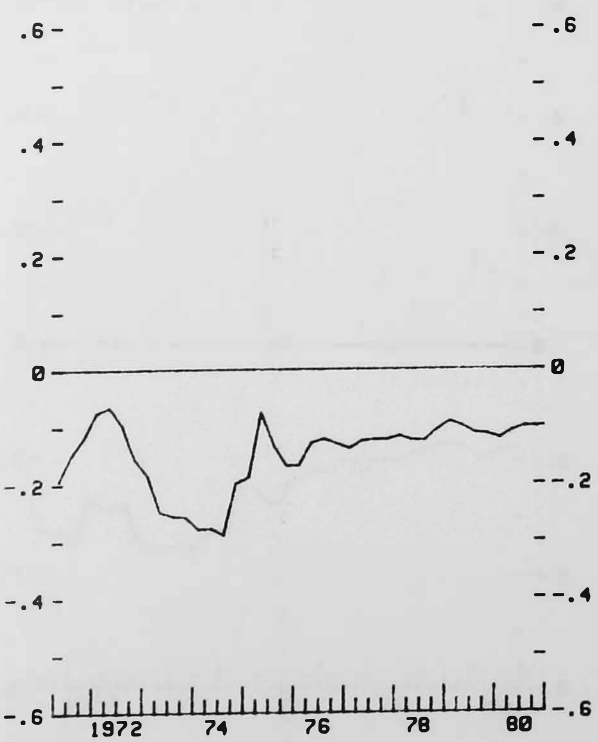
(a) Difference of adjusted income



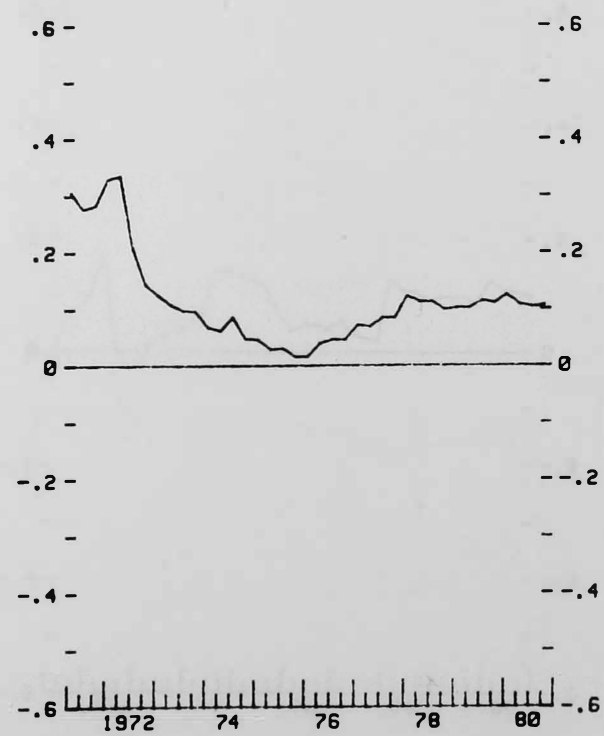
(b) Lagged liquidity/adjusted income ratio



(c) Lagged consumption/adjusted income ratio



(d) Lagged first difference of liquidity



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