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The development of expectations generating schemes which are asymptotically rational

by

K D Patterson

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The object of this Technical Series of Discussion Papers is to give wider circulation to econometric research work predominantly directed towards revising and updating the various Bank models. Any comments should be sent to the author at the address given below.

This paper develops the concept of asymptotically rational expectations generating schemes and applies it to the problem of model design. Easily applicable conditions are derived to ensure asymptotic rationality and several models are developed which satisfy these conditions. The connections are shown between developments of the adaptive expectations hypothesis and, *inter alia*, error correction mechanisms. The ideas are illustrated with the Livingston price expectations data.

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THE DEVELOPMENT OF EXPECTATIONS GENERATING SCHEMES WHICH ARE ASYMPTOTICALLY RATIONAL

1 INTRODUCTION

The hypothesis that expectations are formed rationally has largely supplanted the hypothesis that expectations are formed adaptively (though in some very simple cases the two hypotheses coincide) as the expectations generating scheme which forms part of the standard apparatus of macroeconomics and econometrics. However, although there has been a great deal of theoretical work developing the implications of incorporating the rational expectations hypothesis, the REH, into macroeconomic theory and a lesser, but still substantial, amount of work attempting to test the empirical implications of the REH there is a notable line of critical comment on the assumptions and implications of the REH. For example, tests of the REH on directly reported price expectations have led to widely differing conclusions - see, inter alia, Pesando (1975), Mullineaux (1978), and on related matters see also the debate generated by Figleswki and Wachtel (1981, 1983); for a critical analysis of the information assumptions underlying the REH see Friedman (1979), and for a rejection of the empirical implications of the REH on UK data see Pesaran (1982).

The development of the REH does however suggest some dissatisfaction with the adaptive expectations hypothesis, the AEH. This may have arisen because of the inappropriate use of the AEH in situations which could be characterised by data which was not stationary. In such cases the simple form of the AEH which operated on the level of the variable would lead to a continual underprediction of that level. It was this kind of consideration which lead Flemming (1976) to suggest that if the level of the variable, on which an expectation was being formed, displayed a trend, then to avoid continual underprediction agents would use the AEH on the rate of change of the variable. In contrast to some forms of the REH this suggestion can be interpreted as implying some learning behaviour on the part of agents. In a related way the examination by Davidson and Hendry (1981) of the relative merits of Hall's (1978) REH consumption function and the Hendry and von Ungern-Sternberg (1981), hereafter HUS, consumption function and their finding in favour of the latter can be interpreted as evidence of learning behaviour on the part of agents, since the foundation of the HUS consumption function is in the error correction philosophy associated with Davidson et al (1978) and Phillips (1957). Indeed the latter article and the developments associated with it by Salmon (1982) provide a strong justification for the use of control rules with error feedback properties; and it is along these lines that this paper develops the concept of asymptotic rationality, denoted ASR.

An expectations generation scheme, denoted EGS, which is ASR avoids the continual under (or over) prediction associated with, for example, the simple AEH on a trended variable. The concept of ASR is however a conditional one in the sense that an EGS which is ASR for constant growth in a variable will not necessarily be ASR if that variable undergoes a sustained period of accelerating growth. Thus, the design of an EGS will not, if it is to be ASR, be invariant to the data characteristics under which it will operate.

This paper is organised as follows. In the next section the concepts necessary to understand the definition of asymptotic rationality are explained, and the constraints which are required for an EGS to be ASR of a given order are derived. It is shown that these constraints are linear, and hence should not provide undue difficulties for imposition and testing. In the third section we consider three important parameterisations (or models) which are developed as potential expectations generating schemes. The relationships between these models is also shown, which should serve to clarify the taxonomy of model specification in this area. In the fourth section we consider and apply the concepts and developments of the earlier sections on the Livingston price expectations data - see, for example, Carlson (1977); the results given here are likely to be of interest in their own right, but they also show the application of the ASR concept in a situation of some practical interest. The final section contains some concluding remarks. It should be noted that whilst the development which follows considers the construction of expectations generating schemes for prices, it will also be of relevance to other variables on which expectations are formed - for example, income, output, exchange rates and so on.

A central concept in understanding the definition and implications of asymptotic rationality is that of growth coefficients. Further detail on the derivation and proofs of results given here is to be found in Patterson and Ryding (1982, 1984a) and Patterson (1984b), and illustrations of relevant applied analyses can be seen in Patterson and Ryding (1984b), Patterson (1984a) and Patterson (1985).

Consider the general autoregressive distributed lag model, ADM, relating expectations of the price level to actual prices

$$(1-\gamma(L))p_t^e = \beta(L)p_t \tag{1}$$

where p_t^e is the expectation of p_t formed at t-1, $\gamma(L)=\sum_{\ell=1}^{K}\gamma_\ell L^\ell$ and $\beta(L)=\sum_{j=1}^{J}\beta_j L^j$. (A stochastic disturbance term would be added to (1) if the circumstances warranted it.) The model of (1) is denoted ADM(K,J) to indicate the order of the lag polynomials; and it implies a distributed lag response of p_t^e to p_t with the lag generating function given by $W(L)=\beta(L)/(1-\gamma(L))$. An alternative way of writing (1) which emphasises the dependence of p_t^e on the level and rates of change of p_t is,

$$p_t^e = \lambda(L)p_t \text{ where } \lambda(L)=\sum_{i=o}^{\infty}\lambda_i(1-L)^i \tag{2}$$

The λ_i have been termed growth coefficients because they summarise the impact of growth (or the level for λ_o) in the explanatory variable on the dependent variable. For the discrete case the growth coefficients are obtained from

$$\lambda_i = (-1)^i(i!)^{-1}\left[\frac{\partial^i w(Z)}{\partial Z^i}\right]_{Z=1} \tag{3}$$

and a table of the λ_i in terms of the coefficients of $\gamma(L)$ and $\beta(L)$ is given in Patterson and Ryding (1982, page 10).

The steady state for (1) and (2), is defined in the following way. Suppose p_t follows a path described by a polynomial of degree n in time, t; that is,

$$p_t = \sum_{g=o}^{n} \alpha_g t^g \qquad (4)$$

for example if n=1, and we assume that p_t is the log of the price level, then α_1 is the constant rate of growth. If n is finite the number of terms in the summation of (2) is reduced since $\Delta^{n+1}p_t=o$, and higher order differences are, by hypothesis, zero; (2) then gives the value of p_t^e in a steady state of order n, ie

$$p_t^{e,s} = \sum_{i=o}^{n} \lambda_i \Delta^i p_t \qquad (5)$$

The difference between p_t and p_t^e in steady state defines the <u>steady state error</u>, ie $e_t^s \equiv p_t - p_t^{e,s}$. Two other concepts will also be of interest; they are the <u>integral error</u> defined by

$$e_{IT} \equiv \sum_{t=o}^{T} (p_t - p_t^e), \text{ regarding } t=o \text{ as the starting date of the process}$$

and the <u>steady state integral error</u> defined by $e_I^s \equiv \text{limit } e_{IT}$ as $T \to \infty$.

We now define asymptotic rationality of order n, n>o, denoted ASR-n, in the context of an expectations generating scheme:

the EGS defined by $p_t^e = \beta(L)p_t + \Upsilon(L)p_t^e$ is ASR-n if (and only if) the steady state error is zero for p_t on a growth path of order n.

Thus, if a particular EGS is able to track prices when the latter display a constant growth rate it would be denoted as ASR-1. Notice that if an EGS is ASR-n it is also ASR-(n-1); so if an EGS was designed to track hyperinflation, ie n=2, it would also track constant inflation. Asymptotic rationality of order 1 would seem to be an essential requirement of an EGS which is intended either as part of a macroeconomic model or as an explanation of directly observed price expectations data - eg the Livingston series, see Carlson (1977), Figleswki and Wachtel (1981). The use of an EGS which was ASR-2 might also be justified for some part of the post-war period, perhaps with a switch point from an ASR-1 process - see Flemming (1976) for an exposition of related ideas.

A necessary and sufficient condition for an EGS to be ASR-n is that $\lambda_o=1$, and $\lambda_i=o$ for $i=1...n$. If the steady state error is non-zero but finite it is given by $-\alpha_n n!\lambda_n$, which will be the case for a process which is ASR-(n-1). For the steady state integral error to be zero, for growth of order n, the EGS must be ASR-(n+1); and if the EGS is ASR-n the steady state integral error is finite and given by $-\alpha_n n!\lambda_{n+1}$. Thus, the growth coefficients allow a numerical evaluation of both the steady state error and the steady state integral error. The magnitude of the steady state integral error could be a useful measure to distinguish between, or among, model specifications which are alike in their steady state error characteristics.

To illustrate some of these points consider Figure 1 which graphs the (log of the) following time path for prices: a step change from one static level to another followed, in due course, by a change to a growth rate of 5% which is then increased to 15%. The tracking of this time path is shown for a model, denoted model 1, which is ASR-O. Model 1 tracks the step change but <u>not</u> the changes to positive growth rates when it starts to incur a constant steady state error, which can be numerically evaluated, in this case, from $-\alpha_n\lambda_n$; and since the models are in the logs of the variables the absolute divergences in the unlogged levels is increasing. Model 2, however, has been designed to be ASR-1, and hence will track the change from zero to non-zero growth rates without incurring a steady state error. (To anticipate section 3, Model 1 is derived from the adaptive expectations hypothesis and Model 2 is a 'differences plus disequilibrium' model).

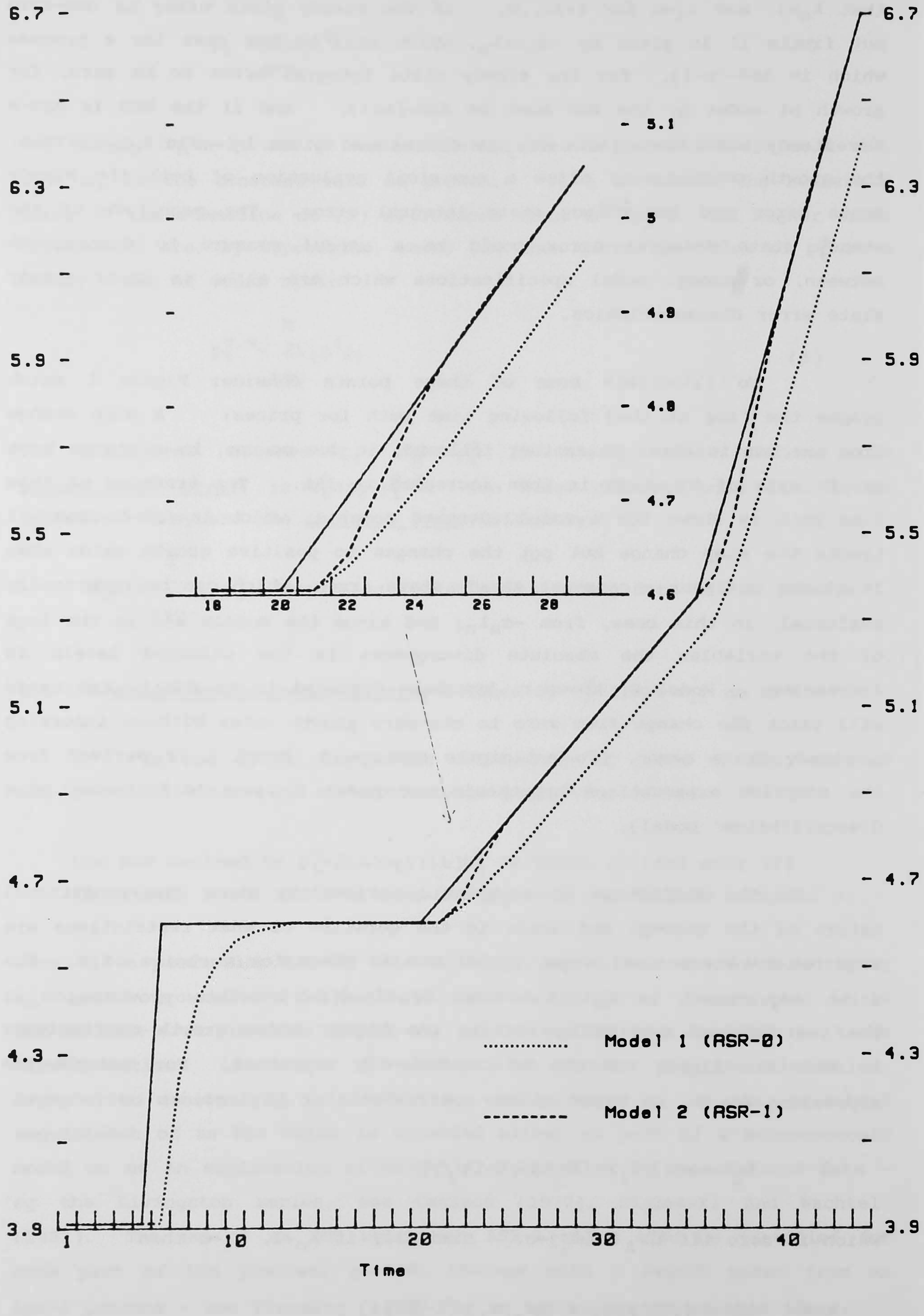
The definition of asymptotic rationality shows the conditional nature of the concept and leads to the question of what restrictions are required to ensure that a particular EGS is ASR-n for a choice of n. The first requirement is $\lambda_o=1$, so that $Y(L=1)=\beta(L=1)$; then providing $\lambda_o=1$ the restrictions implied by setting the higher order growth coefficients to zero are <u>linear</u> and can be conveniently expressed. For example the expression for λ_1 in terms of the coefficients of (1) reduces to

$$\lambda_1 = -[(\Sigma Y_\ell \ell+\Sigma\beta_j j)/(1-\Sigma Y_\ell)]$$

which is zero iff $\Sigma Y_\ell \ell+\Sigma\beta_j j=o$. Similarly if $\lambda_o=1$, $\lambda_1=o$ then

$$\lambda_2 = [(\Sigma Y_\ell {}_\ell C_2 + \Sigma\beta_j {}_j C_2)/(1-\Sigma Y_\ell)]$$

Figure 1.The tracking ability of some expectations generating schemes



which is zero iff $\Sigma Y_{\ell\ell}C_2 + \Sigma\beta_{jj}C_2 = 0$. In general, if $\lambda_0=1$, and $\lambda_i=0$ for $i=1...h$ then $\lambda_{h+1}=0$ if, and only if,

$$\Sigma Y_{\ell\ell}C_h+\Sigma\beta_{jj}C_h=0$$

where ${}_jC_h$ is the binomial coefficient for choosing h from j. These restrictions are easily expressed in the standard $R\beta=r$ form; for example for an EGS to be ASR-2 the restrictions are,

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 3 & \dots & J & 1 & 2 & 3 & \dots & L \\ 0 & 1 & 3 & \dots & {}_JC_2 & 0 & 1 & 3 & \dots & {}_KC_2 \end{bmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_J \\ Y_1 \\ \vdots \\ Y_K \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

In the next section we consider some models which have been proposed, or are likely to be of interest, as expectations generating schemes. It will be shown that some of these models are convenient reparameterisations of an autoregressive distributed lag model which satisfies asymptotic rationality of a certain order.

3 MODEL SPECIFICATION AND ASYMPTOTIC RATIONALITY

3(i) Differences plus disequilibrium model - DDE

Consider the specification given by

$$\Delta p_t^e = \sum_{j=1}^{r}\theta_j\Delta p_{t-j} + \sum_{i=1}^{r}\mu_i(p_{t-i}-p_{t-i}^e) \qquad (6)$$

where P_t is the price level, and P_t^e is the point forecast of P_t at t-1 and lower case letters indicate logarithms. This consists of two sets of terms: the lagged differences in the target variable, and the lagged disequilibrium terms, $p_{t-i}-p_{t-i}^e$. This model will be referred to by the mnemonic DDE(r)-1. A simple version of this model with r=1 has become very popular in empirical work, particularly in the estimation of consumption functions, see Davidson et al (1978) and Hendry (1983). Part of the usefulness of the arrangement of terms in (6) is that contrasted to the ADM, of which it is a re-arrangement, the explanatory variables are likely to be relatively less collinear; it is also, by design, characterised by long run homogeneity, that is λ_o is identically equal to unity in (6), so that p_t^e will be able to track step changes in p_t. It is the case, however, that without further restrictions $\lambda_1 \neq 0$, and hence the DDE(r)-1 is ASR-o but not ASR-1.

The further restriction which is necessary (and sufficient) to ensure that $\lambda_1=o$ is easy to derive given the developments in section 2: if $\lambda_o=1$ then $\lambda_1=o$ requires $\Sigma Y_\ell \ell = -\Sigma \beta_j j$. Collecting like terms in (6) this condition reduces to the simple linear constraint, $\Sigma \theta_j=1$. One way of imposing this constraint is to substitute out for θ_1 to give

$$\Delta p_t^e - \Delta p_{t-1} = \sum_{j=2}^{r} \theta_j(\Delta p_{t-j} - \Delta p_{t-1}) + \sum_{i=1}^{r} \mu_i(p_{t-i} - p_{t-i}^e) \qquad (7)$$

This version of the DDE model will be referred to as DDE(r)-2, which will be ASR-1 and hence be able to track constant growth in the price level.

Using the results of section 2 the DDE model can quite easily be constrained to achieve higher orders of asymptotic rationality. One case likely to be of interest relates to hyperinflations - or even fairly sustained spurts in the inflation rate. To ensure that the DDE model is ASR-2, and hence able to track the price level in cases of hyperinflation, then given $\lambda_o=1$ and $\lambda_1=o$ it is necessary and sufficient to have $\lambda_2=o$; in turn this requires from section 2, $\Sigma Y_{\ell} {}_{\ell}C_2 = -\Sigma \beta_j {}_jC_2$, and it is not difficult to show that this is translated into $\Sigma \theta_j j = o$ for the DDE model. Once again this is a linear constraint, which can either be imposed explicitly in estimation or by expressing θ_2, for example, in terms of other other coefficients. This result can be generalised in the following way: for the DDE model to be ASR-n it is necessary and sufficient to have $\Sigma \theta_j=1$ and $\Sigma \theta_j {}_jC_h=0$, for h=1...n-1; these constraints are linear and ensure that $\lambda_o=1$ and $\lambda_i=o$ for i=1...n. The simplicity of these constraints and the interpretation of the coefficients of the DDE model suggest that it is an attractive candidate for empirical work.

3(ii) The adaptive expectations hypothesis - AEH

(a) AEH on the price level

The adaptive expectations hypothesis was, prior to the rational expectations hypothesis, probably the most widely used expectations generation scheme. The AEH on the price level states that expectations are revised in proportion to the most recently available forecast error, ie

$$p_t^e - p_{t-1}^e = \gamma(p_{t-1}-p_{t-1}^e) \quad (8)$$

The ADM form of (8) is $p_t^e=\gamma p_{t-1}+(1-\gamma)p_{t-1}^e$, which shows that the expected price level is a weighted average of the previous expectation and the actual price level. It is easy to verify in (8) that $\lambda_o=1$ but $\lambda_1\neq o$, hence such a model is ASR-o but not ASR-1. A numerical assessment of the steady state error for constant growth in the price level is obtained by evaluating $e^s=-\alpha_n n!\lambda_n$, see section 2, for n=1. This gives a steady state error of $-\alpha_1/\gamma$ where α_1 is the growth rate; for example, if $\gamma=.5$ and $\alpha_1=.1$ then $e^{ss}=-.2$, so that in steady state the ratio of expected prices to actual prices is .818. Clearly this kind of result is unsatisfactory (even as $\gamma\to 1$), and prompted Flemming (1976) to suggest that if the price level has a trend agents would change to adaptive expectations on the inflation rate.

(b) AEH on the rate of change of the price level

The simplest version of this hypothesis is the extension of (8) to operate on the rates of change of the variables, ie

$$\Delta^+p_t^e-\Delta^+p_{t-1}^e = \rho_1(\Delta p_{t-1}-\Delta^+p_{t-1}^e) \quad (9)$$

where $\Delta^+p_t^e=p_t^e-p_{t-1}$.

This extension of the basic AEH has been suggested by Flemming (1976) and estimated on the Livingston price expectations data by Figlewski and Wachtel (1981). It can easily be verified from the analysis of section 2 that (9) is an AD(1,2) model with $\lambda_o=1$ <u>and</u> $\lambda_1=o$, and it is therefore AR-1. Thus, if the price level displays constant growth, (9) ensures that agents 'learn' on the appropriate difference of the price level. The degree of learning could be characterised by the integral error which summarises the error agents incur in 'catching' the trend in the price level. The steady state integral error for (9) is $e_I^{ss}=-\alpha_1\lambda_2$, where $\lambda_2=-1/\rho_1$ with limits $-\infty<\lambda_2\leq-1$ for $o<\rho_1\leq 1$; thus, the closer ρ_1 is to unity the smaller the total error in picking up the trended price level.

Using the definition of $\Delta^+p_t^e$ we can rewrite (9) in a revealing way as

$$\Delta p_t^e=\Delta p_{t-1}+\rho_1(p_{t-1}-p_{t-1}^e) \quad (10)$$

which can now be seen to be a particular example of the DDE(1)-2 model. Such a connection can be developed by considering the generalisation of (9) to include further lags of the explanatory variable, ie

$$\Delta^{+}p_t^e-\Delta^{+}p_{t-1}^e = \sum_{i=1}^{s} \rho_i(\Delta p_{t-i}-\Delta^{+}p_{t-i}^e) \tag{11}$$

This model will be referred to by the mnemonic AEH(s)-2 to indicate that it is a second order adaptive expectations hypothesis with s lags - for example (9) is AEH(1)-2. On similar lines the general AEH on the level of the variable will be referred to as the first order AEH and denoted AEH(s)-1, in general. Now (11) can be re-arranged to become,

$$\Delta p_t^e = \Delta p_{t-1} + \sum_{i=1}^{s} \rho_i(p_{t-i}-p_{t-i}^e) \tag{12}$$

and it is easily verified that, for this model, $\lambda_o=1$ and $\lambda_1=o$. Notice that (12) is a restricted form of the DDE(r)-2 model with $\theta_1=1$; and a progression from (6), for s=r, to (12) forms a nested sequence.

3(iii) Error correction mechanisms - ECMs

The error correction mechanism has become familiar through the work of Phillips (1954, 1957) and is developed by Salmon (1982) and Patterson (1984 b). The general form of an ECM of order r, denoted ECM-r, as an expectations generating scheme is,

$$\Delta^{r}p_t^e= A(L)(p_{t-1}-p_{t-1}^e) \tag{13}$$

where $A(L)=a(L)/(1+b(L))$ is a rational polynomial with $a(L)$ and $b(L)$ polynomials in L of orders m and f-r, respectively. A fundamental characteristic of this model specification is that the growth coefficients, λ_i, for i=1...r-1 are identically zero; hence, such models are automatically ASR-(r-1). For more details on the derivation of these results see Salmon (1982) and Patterson (1984 b).

To see how ECMs are related to some of the models outlined earlier consider the ECM with r=2 and m=f=2, ie

$$\Delta^{2}p_t^e=(a_1+a_2L)(p_{t-1}-p_{t-1}^e) \tag{14}$$

Now by design in (14) $\lambda_0=1$ and $\lambda_1=0$, hence (14) is ASR-1 but not ASR-2. Following Patterson (1984 b) this can be re-arranged into the (restricted) DDE form as

$$\Delta p_t^e = \Delta p_{t-1}+(a_1-1)(p_{t-1}-p_{t-1}^e)+(1+a_2)(p_{t-2}-p_{t-2}^e) \qquad (15)$$

Notice that (15) is identical to the model given by AEH(2)-2, see (12) with s=2. Hence, in this case, the ECM and AEH formulations lead to the same model. The steady state integral of this model for constant growth in the price level is obtained on noting that $\lambda_2=-1/(a_1+a_2)$, or in terms of the AEH coefficients $\lambda_2=-1/(\rho_1+\rho_2)$. The necessary condition for stability of the difference equation implicit in (14) implies $0<a_1+a_2<2$, and therefore $-\infty<\lambda_2<-.5$; thus, this model can achieve a smaller steady state integral error compared with the AEH on the rate of change of the price level given by (9).

We can generalise the ECM of (14) to allow for further lags on the target error, $p_t-p_t^e$, to give

$$\Delta^2 p_t^e = \sum_{i=1}^{k} a_i L_i (p_t-p_t^e) \qquad (16)$$

This will be denoted as ECM(k)-2 to indicate that it is a second order ECM with k lags on the target error; (16) can also be written in (restricted) DDE form as

$$\Delta p_t^e = \Delta p_{t-1}+(a_1-1)(p_{t-1}-p_{t-1}^e) + (a_2+1)(p_{t-2}-p_{t-2}^e) + \sum_{i=3}^{k} a_i(p_{t-i}-p_{t-i}^e) \qquad (17)$$

It is also a simple step to show that if k=s then (17), and hence (16), is equivalent to the AEH(s)-2 of (12) with $\rho_1=a_1-1$, $\rho_2=a_2+1$ and $\rho_i=a_i$ for $i>2$. The equivalence holds for $k>2$. For k=1(=s), however, the models of (16) and (12) are different even though both have $\lambda_1=0$ and hence are ASR-1; in particular they imply very different dynamic responses as (16) will, in this case, be a 2nd order difference equation with complex roots whereas (12) will be a first order difference equation with a delayed geometric lag. This serves to emphasis that whilst an ECM-2 is sufficient to ensure the ASR-1 property, it is <u>not</u> necessary.

This section has introduced a number of models which are convenient parameterisations of auto-regressive distributed lag models with constraints on the growth coefficients to ensure asymptotic rationality of the required order, and some of these models were shown to be related or equivalent. It may therefore be helpful if we give a tabular representation of the nested structure of models which are ASR-1.

TABLE 1: NESTED STRUCTURE OF EXPECTATIONS GENERATING SCHEMES WHICH ARE ASYMPTOTICALLY RATIONAL FOR CONSTANT GROWTH IN THE PRICE LEVEL (ASR-1)

ADM(K,J) with $K \geq r$, $J \geq r+1$, $\beta_0 = 0$, $\lambda_0 = 1$, $\lambda_1 = 0$

↓ (K+J-2r-1)

DDE(r)-2 $\equiv$ AD(r,r+1) with $\beta_0 = 0$, $\lambda_0 = 1$, $\lambda_1 = 0$

↓ (r-1)

r>1 ECM(r)-2 $\equiv$ AEH(r)-2

↓ (r-1) ↓ (r-1)

r=1 ECM(1)-2 $\neq$ AEH(1)-2 $\equiv$ DDE(1)-2

Nb. Number by arrow indicates restrictions imposed in going to next level

ADM(K,J) is $p_t^e = \sum_{j=0}^{J} \beta_j p_{t-j} + \sum_{\ell=1}^{K} \gamma_\ell p_{t-\ell}^e$

DDE(r)-2 is $\Delta p_t^e = \sum_{j=1}^{r} \theta_j \Delta p_{t-j} + \sum_{i=1}^{r} \mu_i (p_{t-i} - p_{t-i}^e)$ subject to $\Sigma\theta_j = 1$

AEH(r)-2 is $\Delta^+ p_t^e - \Delta^+ p_{t-1}^e = \sum_{i=1}^{r} \rho_i (\Delta p_{t-i} - \Delta^+ p_{t-i}^e)$ equivalent to

$$\Delta p_t^e = \Delta p_{t-1} + \sum_{i=1}^{r} \rho_i (p_{t-i} - p_{t-i}^e)$$

ECM(r)-2 is $\Delta^2 p_t^e = \sum_{i=1}^{r} a_i L^i (p_t - p_t^e)$

The use of expectations generating schemes which are asymptotically rational has implications for macroeconomic and econometric modelling, since they will, in general, imply non-linear restrictions in the equations including variables on which expectations are formed. If there is more than one equation which includes the same expectational variable there will also be cross equation constraints. Consider a simple example: suppose that a behavioural equation specifies

$$y_t=\alpha p_t^e+u_t$$

and the EGS is specified to be the AEH(1)-2 of (9) then, on substitution for p_t^e, we obtain

$$y_t=\alpha(1+\rho_1)p_{t-1}-\alpha p_{t-2}+(1-\rho_1)y_{t-1}+(1-(1-\rho_1)L)u_t$$

The final equation implies a non-linear constraint on the coefficients of p_{t-1}, p_{t-2}, and y_{t-1} as well as a constraint on the coefficients of u_{t-1} and y_{t-1}. If another equation also depended on p_t^e then consistency would require that ρ_1 is the same in both equations.

4 APPLICATION

4(i) The Livingston data

This section illustrates the application of the concepts and models of sections 2 and 3 to the Livingston expectations data on the consumer price index, the CPI. These data are by now widely known and a number of studies have used it to test hypotheses relating to the formation of expectations. Descriptions of the data and its collection are to be found, inter alia, in Pesando (1975), Mullineaux (1978), Brown and Maital (1981), with the best single reference being Carlson (1977).

Briefly, the data is obtained as follows. Livingston publishes the responses to a semi-annual survey he conducts of a panel of economists. Amongst the questions asked the survey elicits forecasts of the CPI over different forecast horizons. For example in the first survey of a year the respondents are asked to forecast the level of the CPI for December; and as Carlson (1977) shows this is effectively an 8 month forecast as the latest available information at the time of making the forecast is the April index. It is this data which is used in the estimation results reported below, with the forecast horizon converted to six months on the fairly mild assumption, initially used by Mullineaux (1978), that respondents are forecasting a path corresponding to a constant rate of change of the price level.

4(ii) Empirical Results

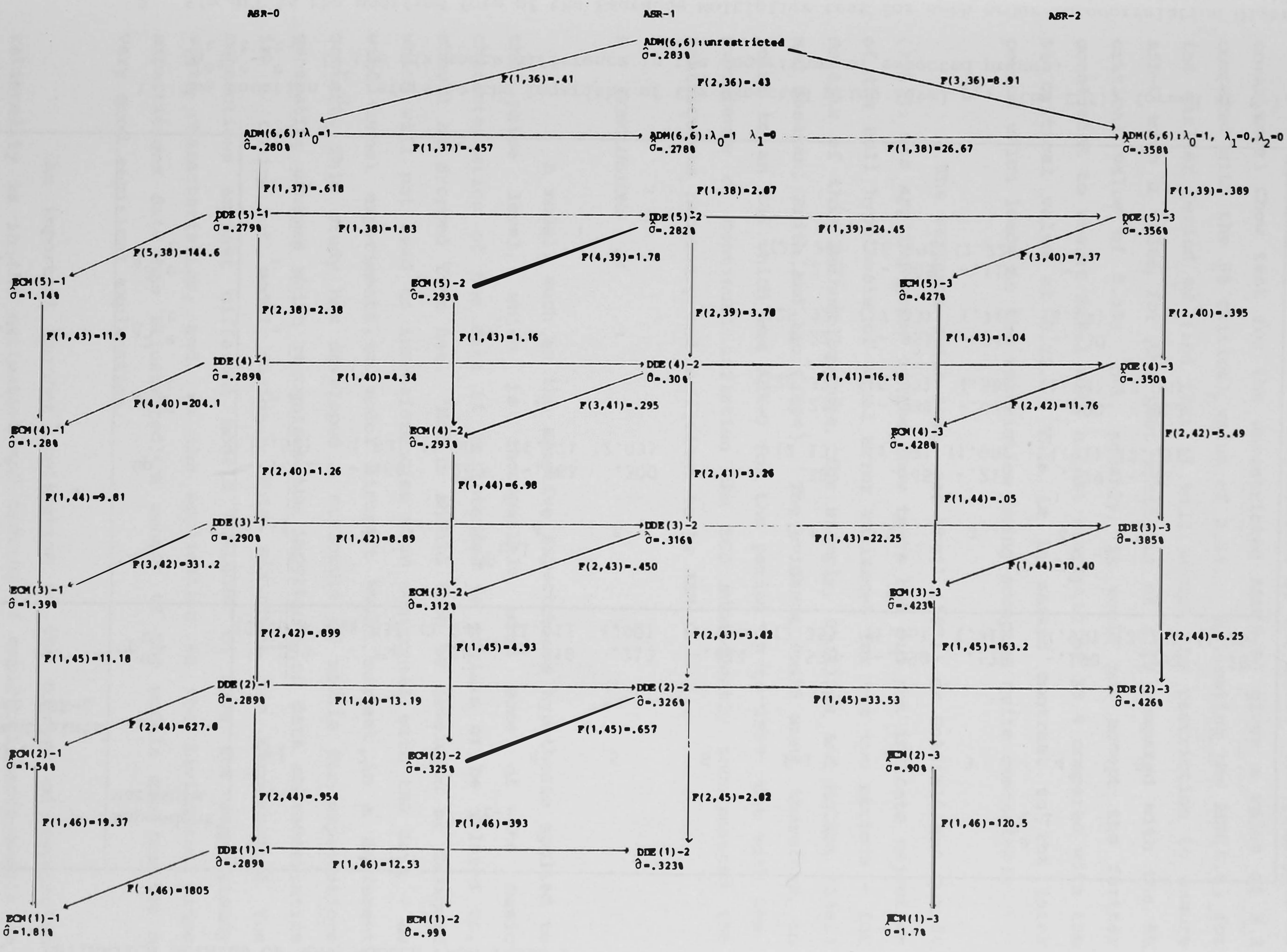
Some previous research on the Livingston price expectations series has suggested that a structural break in their accuracy occurred around 1959 - see Gibson (1972) and Turnovsky (1970), and note that the sample period used by Pesando (1975) and Mullineaux (1978) did not include observations up to 1959, Initially, therefore in reporting the empirical results the sample period 1959-1 to 1982-2 is considered. Table 2 sets out the structure of the set of estimated results. The most general model estimated is the ADM(6,6) without any restrictions, and all of the other models shown lead from (are nested in) this model. For each model the estimated (percentage) standard error is given; and the value of the F statistic to go from one model to the next in the direction indicated is given as F(a,b), where a is the number of restrictions and b is the degrees of freedom in the less restricted model. In table 3 we give more detailed results of the estimation for those models suggested by adopting a sequential approach to the choice of models from those given in table 2.

It is clear from table 2 that whichever route is taken the hypothesis that the EGS is ASR-2 is rejected at reasonable significance levels. The hypothesis that the EGS is ASR-1 is, however, certainly not rejected; for example we may take the route from the unrestricted ADM(6,6) to the imposition of $\lambda_o=1$ and $\lambda_1=0$, or to the imposition of $\lambda_o=1$ and then to $\lambda_o=1$ and $\lambda_1=0$. From this model we could then go along the nested sequence to DDE(5)-2, but not further to DDE(4)-2 unless a marginal significance level of less than 2% is used. ECM simplifications could be entertained as far as ECM(4)-2; a marginal significance level of less than 1.1% would have to be used to go from the ECM(4)-2 to the ECM(3)-2. (For more on the principles of ordering hypotheses into a nested sequence and subsequent testing see Anderson (1971) and Mizon and Hendry (1980).) The estimated standard errors of, for example, the ADM(6,6) with $\lambda_o=1$ and $\lambda_o=1$, or DDE(5)-2, are only of the order of .28% (ie close to one quarter of one per cent); the detailed estimates of 4 of the models are given in table 3.

4(iii) A Structural Break?

Given the suggestions in previous research that there was a structural break around 1959, a separate analysis was also conducted for

TABLE 2: STRUCTURE OF EMPIRICAL MODELS



NB: 5% critical values - $F(1,40)=4.08$, $F(2,40)=3.23$, $F(3,40)=2.84$, $F(4,40)=2.61$, $F(5,40)=2.45$

TABLE 3. ESTIMATION DETAILS OF SOME MODELS FROM TABLE 2

	Dependent variable[1]	p_{t-i}						p^e_{t-i}						$\%\hat{\sigma}$	F(n,df)
		i=1	2	3	4	5	6	i=1	2	3	4	5	6		
ADM(6,6) with constraints to ensure ASR-1	p^e_t	1.239 (24.67)	-.756 (3.93)	.281 (1.24)	-.318 (1.41)	.213 (.90)	-.444 (1.53)	.578 (3.35)	-.258 (1.36)	.178 (.91)	-.190 (.99)	.392 (2.77)	.284 (1.49)	.278	F(8,30)=1.76
		Δp_{t-i}						$(p_t-p^e_t)_{-i}$							
		i=1	2	3	4	5		i=1	2	3	4	5			
DDE(5)-2	Δp^e_t	.863 (5.52)	-.168 (.83)	.267 (1.30)	-.263 (1.57)	.300 (2.03)		.362 (2.13)	.245 (1.27)	-.213 (1.08)	.179 (.91)	-.37 (2.60)		.282	F(8,31)=1.52
ECM(5)-2	$\Delta^2 p^e_t$	-	-	-	-	-		1.216 (24.37)	-.939 (16.97)	-.033 (.58)	-.098 (1.73)	-.053 (1.08)		.292	F(8,36)=1.30
ECM(4)-2	$\Delta^2 p^e_t$	-	-	-	-	-		1.232 (25.92)	-.951 (17.33)	-.041 (.74)	-.130 (2.64)	-		.293	F(8,37)=1.21
ECM(3)-2	$\Delta^2 p^e_t$	-	-	-	-	-		1.230 (24.29)	-.965 (16.59)	-.116 (2.22)	-	-		.312	F(8,38)=1.08

1 The notation p^e_t refers to the logarithm of the expected price level p_t, (the CPI), formed at t-1, and hence $\Delta p^e_t = p^e_t - p^e_{t-1}$ is the six month difference in the logarithms of expected prices.

2 F(n,df) is the modified form of the Lagrange Multiplier test for n-th order autocorrelation distributed, approximately, as F(n,df) under the null of no autocorrelation - see Kiviet (1981) and Harvey (1981);

the period 1949-2 to 1958-2 and compared with the later period. These results corroborate the previous evidence of a structural break though we are able to offer here a new interpretation of the data. The (analysis of covariance) Chow test for the unrestricted ADM(6,6) gives a value of 5.1 compared with the 5% critical value of 2.64. Estimating the ADM(6,6) for the earlier period we find that it will accept the restriction to ensure ASR-O with a value for the test statistic of 1.19 compared with the 5% critical value of 5.59; but, notably, it would not accept the further constraint to ensure ASR-1 with a test statistic of 39.4 compared with the 5% critical value of 5.32. This is in marked contrast to the later period which leads to the restriction being accepted quite comfortably.

The variance ratio test, see Theil (1971) or Goldfeld and Quandt (1965), was applied prior to the Chow tests but did not indicate rejection of the null hypothesis of equal error variances for the two periods - for details of this methodology see, for example, Phillips and McCabe (1983) and Pesaran, Smith and Yeo (1984). The evidence would seem, therefore, to point to an EGS which was ASR-O for the period up to 1959, but with the experience of continual inflation the EGS subsequently incorporated the restrictions necessary and sufficient to be ASR-1.

5 CONCLUSIONS

A model, such as the adaptive expectations hypothesis applied to the price level, which is incompatible with some of the basic characteristics of the data it is intended to explain or be related to, should be dropped from use. Models should then be developed or designed which will not lead to inconsistencies when confronted with the data - as wind tunnel experiments on model aircraft would suggest in a different context. This study has developed a structure of models for expectations generating schemes which recognises the importance of data characteristics in a choice of model design. This structure has demonstrated the connections amongst different models designed to have the same steady state characteristics; and, as the application to the Livingston price expectations data has illustrated, a subset of the models can provide a very good empirical explanation.

An important further application of the concept of asymptotic rationality is in the estimation and testing of empirical macro-models to see if the constraints, outlined in sections 2 and 3 above, which ensure asymptotic rationality of an appropriate order, are satisfied. This line of research is analogous to developments of the rational expectations hypothesis suggested by, for example, Wallis (1980) to which it could usefully be compared in an empirical context.

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