

Bank of England

Discussion Papers

No 41

**The long-run determination of the UK
monetary aggregates**

by

S G Hall

S G B Henry

J B Wilcox

August 1989

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The object of this series is to give a wider circulation to research being undertaken in the Bank and to invite comment upon it; and any comments should be sent to the authors at the address below.

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Contents page

1	Introduction	1
2	Cointegration and long-run properties	3
3	An exercise for M0	7
3.i	Financial innovation and the demand for cash	9
3.ii	An alternative approach to innovation	12
4	A Cointegration exercise for M1	16
5	A model of M4	24
6	A Model of M3	29
7	Conclusion	33
	Appendix: The Johansen procedure	34
	References	38

1 Introduction

This paper presents the results of an empirical study of the demand for the monetary aggregates M0, M1, M3 and M4. The approach adopted in the study is a two stage one: at the first stage the main determinants of the long-run demand for money are examined; and in the second stage we concentrate on specifying the dynamic structure.

Modelling the demand for money has received considerable attention over many years. Recently a range of new approaches have been tried. The buffer stock model has received considerable attention -see, for example Carr and Darby (1981), Cuthbertson (1986), Cuthbertson and Taylor (1987); the estimation of complete systems has been considered by Davidson (1987) and Davidson and Ireland (1987); a Bayesian approach to modelling the monetary aggregates has been used by Lubrano, Pierce and Richard (1985); and more 'conventional' approaches have been adopted by Hendry and Mizon (1978), Hendry (1979), Trundle (1982), Hendry and Ericson (1983), Johnston (1984), Artis and Lewis (1984) and Patterson (1987). Despite the fact that many of these studies claim to offer structurally stable demand functions, subsequent studies often proceed by demonstrating the inadequacies of their predecessor. Indeed the whole area of monetary modelling may be characterised as one of structural breakdown. This comes as no surprise to economists actually working in the monetary sector where the anecdotal orthodoxy is that general innovations in cash management and the financial sector generally preclude the possibility of a stable money demand function of the simple text book form.

We make use of the cointegration tools provided by Engle and Granger (1987) and Johansen (1988), and illustrated by Hall (1986) and Hall (1988), to approach the question of modelling the monetary aggregates from a new perspective. This

procedure enables us to concentrate at the first stage on testing that the set of explanatory variables used are sufficient to adequately model the series. When this procedure is used on a simple demand for money equation for each of the aggregates, which has only real income, prices and an interest rate as explanatory variables, the inability of this limited set of variables to capture the major movements in the series becomes transparent. Attempting to model the dynamics at this stage may merely serve to obscure this basic problem. If a valid dynamic model of money demand, of any form including an error correction model or a forward looking buffer stock model, is to exist it must contain a set of variables which satisfy the tests of cointegration which are applied at the first stage. If this is not the case then the model will be subject to the Granger and Newbold (1974) spurious regression problem and we would not expect it to be structurally stable. So preliminary test for cointegration may rule out many models as inadmissible.

There are a number of practical difficulties with the approach we will use, which is based on the Engle and Granger two step estimation procedure. The well known problem of small sample bias highlighted by Banerjee et al (1986) is one such problem. Another and perhaps a more serious problem in our view is the problem of multiple cointegrating vectors. In general if we are considering N non-stationary variables there may exist anything up to $N-1$ distinct cointegrating vectors. So, in anything more complex than a two variable case we cannot know that we are dealing with a unique cointegrating vector and using OLS to estimate a cointegrating vector may simply produce a complex linear combination of all the distinct cointegrating vectors which exist in the system. If this happens we cannot interpret the resulting equation in any meaningful economic way. Johansen (1988) has offered a solution to both these problems by providing a maximum likelihood estimation technique for all the distinct cointegrating vectors which exist amongst a set of variables. This technique, therefore, provides numerical confirmation of the OLS estimates as well as checking on the number of other cointegrating vectors which may exist. The maximum likelihood procedure is discussed in an appendix to this paper. As the technique is still relatively new we will rely mainly on the conventional OLS procedures and use the new one as a test of the properties of the old one.

Below we consider the monetary aggregates M0, M1, M3 and M4. We model each of the aggregates in turn following a similar procedure for each. First we examine the possibility of forming a cointegrating regression for the aggregate using only the price level, real income and an interest rate as explanatory variables. For all the aggregates this fails to provide a cointegrating vector. This original set of variables is then augmented by a wealth term and by variables capturing financial innovation. It is shown that this larger set of variables is capable of providing a cointegrating set for each of the aggregates. Dynamic equations are also provided for each aggregate.

This paper has the following plan. Section 2 discusses the approach of cointegration and defines the time series properties of the data to be used in subsequent sectors. Then each of the monetary aggregates, M0, M1, M3, and M4, are tested and examined in individual sections (sections 3-6, respectively). The final section makes inter-aggregate comparisons and draws some general conclusions.

2 Cointegration and Long-Run Properties

The concept of cointegration, first proposed in Granger and Weiss (1983) and extended in Engle and Granger (1987), is a fundamental one to the use of the ECM formulation. In particular, the Granger Representation theorem establishes that for a valid ECM to exist the set of variables must cointegrate and if the variables do cointegrate then a valid ECM form of the data must exist. The importance of this result to general estimation procedures is that if an ECM model is estimated for a set of variables which do not cointegrate then this regression will be subject to all the well known problems of 'spurious' regression outlined in Granger and Newbold (1974). This suggests that tests for cointegration be a necessary component of estimation exercises conducted with ECM models. Further, the 'super convergence' proof due to Stock (1985) and generalised by Phillips and Durlauf (1986) and Park and Phillips (1986) suggests that very precise estimates of the levels terms can be obtained in the cointegrating

regression (although some doubts about this are raised by Banarjee, et al, (1986), which may in fact be relevant here).

We will not attempt to summarise the background theory of cointegration here, as recent surveys are provided by Hendry (1986) and Granger (1986) and an application by Hall (1986). Some of the analysis will rely on the maximum likelihood approach of Johansen (1988) which is not yet widely known and so we include an appendix which summarises the technique. Before any estimation work can properly begin within this framework we first need to establish the properties of the series we are dealing with. This is because, in principle, it is only possible for certain combinations of series to cointegrate, so if the set of series under consideration does not fall within this set there is simply no point in proceeding with estimation. Table 2.1 presents the Dickey-Fuller (DF) and the Augmented Dickey-Fuller (ADF) tests for the series we will be considering throughout this paper. The tests for integration of order zero $I(0)$ are tests carried out on the level of the variables; the tests for integration of order one $I(1)$ are carried out on the first difference of the variables; and the tests for integration of order two $I(2)$ are carried out on the second differences of the variables.

TABLE 2.1: THE TIME SERIES PROPERTIES OF THE VARIABLES

	Test for I(0)		Test for I(1)		Tests for I(2)	
	DF	ADF ⁽⁶⁾	DF	ADF ⁽⁶⁾	DF	ADF ⁽⁶⁾
LMO ⁽¹⁾	-2.3	-2.4	-6.6	-1.8	-16.4	-5.7
LNMI ⁽³⁾	-0.5	-0.6	-8.6	-3.8	-16.8	-7.4
LMi ⁽¹⁾	5.2	2.6	-7.1	-1.7	-19.0	-6.5
LM3 ⁽¹⁾	3.1	0.9	-6.1	-2.5	-16.0	-5.9
LM4 ⁽¹⁾	3.1	0.9	-3.6	-2.6	-14.0	-6.5
LGDP ⁽¹⁾	-1.2	-1.1	-11.9	-5.4		
LPGDP ⁽¹⁾	0.9	-0.4	-4.1	-2.6	-14.6	-4.8
LQCE ⁽¹⁾	0.3	0.4	-11.7	-3.7		
LCPI ⁽¹⁾	0.9	-0.9	-2.9	-1.7	-11.5	-5.18
LTFE ⁽¹⁾	-0.9	-0.6	-10.8	-4.9		
LPTFE ⁽¹⁾	0.7	-0.8	-2.9	-1.7	-12.6	-4.3
RTB ⁽¹⁾	-2.4	-2.4	-8.9	-5.3		
Σ BSSR ⁽¹⁾	-1.6	-1.6	-8.5	-4.6		
BDR ⁽¹⁾	-2.4	-2.6	-7.9	-4.5		
CONS ⁽¹⁾	-1.8	-1.8	-8.2	-4.6		
CC ⁽²⁾	1.3	1.9	-4.9	-2.4		
CDA ⁽²⁾	11.5	2.5	-1.7	0.5		
CAP ⁽²⁾	2.7	0.9	-2.1	-1.9		
BSSR ⁽¹⁾	13.9	1.8	-1.6	-1.6		
LFW ⁽¹⁾	0.8	-0.7	-278	-2.6		
LTW ⁽⁴⁾	0.7	1.4	-6.1	-3.1		
SND ⁽⁴⁾	-1.2	-2.7	-4.6	-3.7		
LSM ⁽⁵⁾	-1.58	0.65	-11.57	-3.84		

(1) 1963 Q2 - 1987 Q2 (2) 1966 Q1 - 1986 Q4

(3) 1975 Q2 - 1987 Q2 (4) 1968 Q1 - 1986 Q4

(5) 1963 Q1-1987 Q2

(6) ADF uses fourth-order correction

Definition of variables

LMO - log of M0

LNMI - log of non-interest bearing M1

LMI - log of M1

LM3 - log of M3

LM4 - log of M4

LGDP - log of real gross domestic products

LPGDP - log of the GDP deflator

LQCE - log of real total consumption

LCPI - log of consumer price index

LTFE - log of total final expenditure

LPTFE - log of TFE deflator

RTB - three-month treasury bill rate

BSSR - building society average share rate

Σ BSSR - cumulated interest rate term defined in section 3

BDR - clearing banks' 7 day deposit rate

CONS - 20-year consul yield

CC - number of credit cards issued

DA - number of cash dispensers in use

LCAP - log of the number of current accounts per head of the population

LFW - log of financial wealth of the personal sector

LTW - log of non-financial wealth of the personal sector

LPW - log of total financial and non-financial wealth of the personal sector

LFTI - log of the Financial Times ordinary share index

SND - defined in section 5.

LSM - log of real stock market turnover

The DF and ADF tests are constructed as t-tests with a non-standard distribution which is tabulated in Dickey and Fuller (1979). In broad terms the conclusions of Table 1 are that the measures of real output (LGDP, LQCE and LTFE) and the interest rate variables (RTB, BSSR, BDR, CONS) are clearly I(1) variables. The various measures of money (M0, M1, M3, M4) and prices (LPGDP, LCPI and LPTFE) are probably I(2)

variables, although they are often close to the critical value of the $I(1)$ test and so might be $I(1)$. This conforms well with our theoretical priors as it suggests that money and prices must cointegrate $I(2,1)$, that is money and prices are $I(2)$ and combine to be $I(1)$, and then this series can cointegrate with the remaining variables (income interest rates etc) to produce a stationary residual process. The implication of this is that we might well be able to work in terms of real money which is $I(1)$ rather than nominal money and prices.

3 An Exercise For M0

Our starting point is to illustrate the major features of real M0 over the period end-1969 to end-1986 (see Figure 3.1). Real M0 has fallen by about 30% over this period but the fall has been far from uniform. There is a correspondence between periods for which real M0 falls more rapidly and periods of sluggish growth in real consumers expenditure (see Figure 3.2). Thus the ratio of M0 to consumption shows a smoother downward trend than real M0 (Figure 3.3). The theory of the transactions demand for money suggests that the price level times the square root of real expenditure may be the appropriate deflator; M0 deflated in this way is shown in Figure 3.4.

This casual empiricism leads us to the widely held belief that real expenditure and price movements cannot by themselves explain the demand for cash over the past 20 years. This assertion can be examined by running tests of cointegrability on the variables concerned (see Table 3.1)

FIGURE 3.1: REAL M0 (log scale)

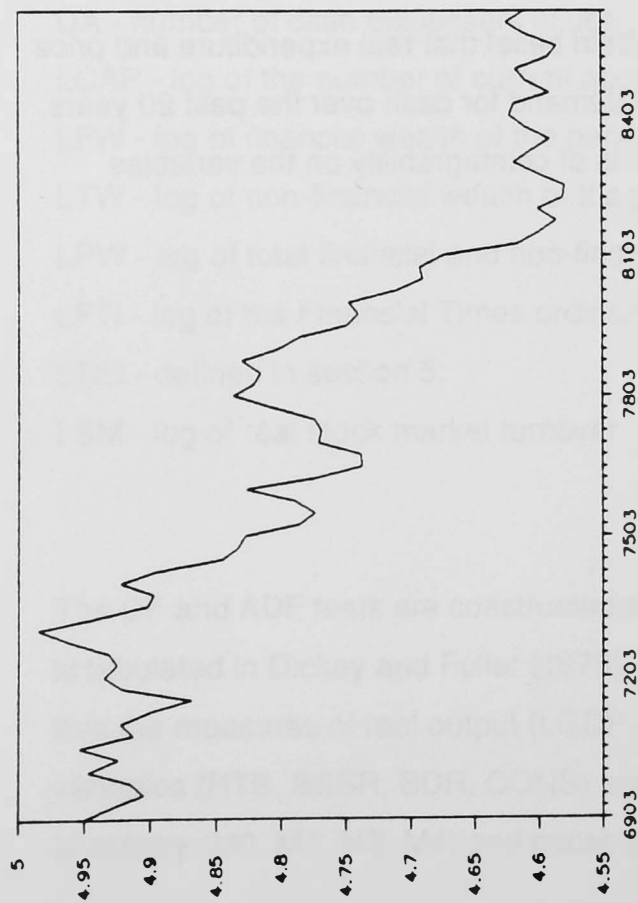


FIGURE 3.2: REAL CONSUMPTION (log scale)

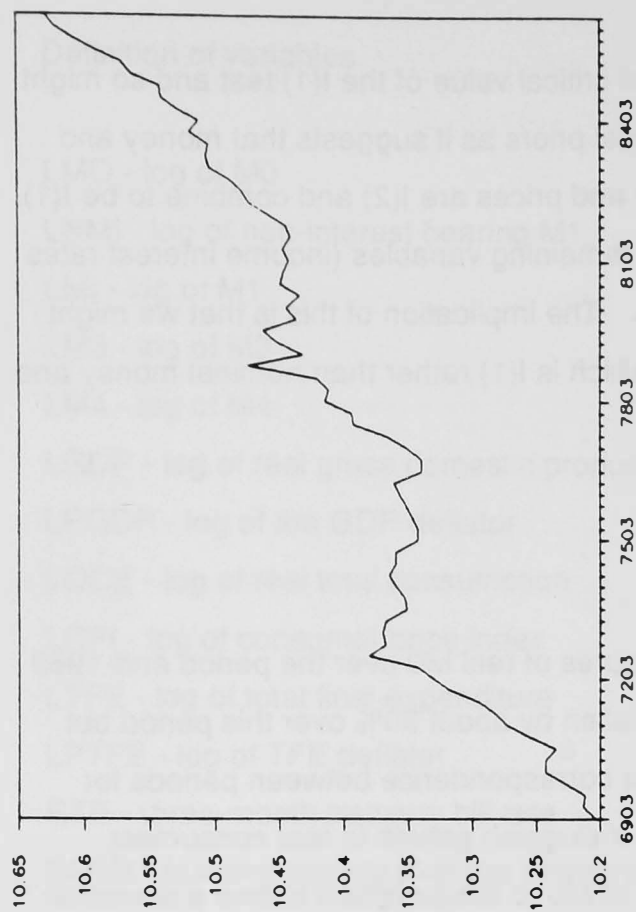


FIGURE 3.3: M0 RELATIVE TO CONSUMPTION (log scale)

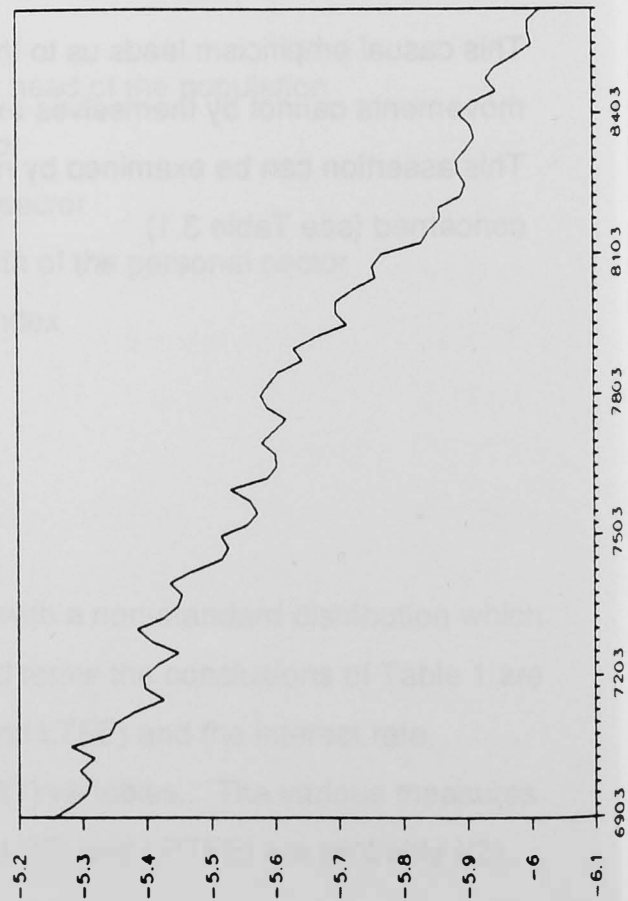


FIGURE 3.4: M0 RELATIVE TO THE SQUARE ROOT OF CONSUMPTION (log scale)

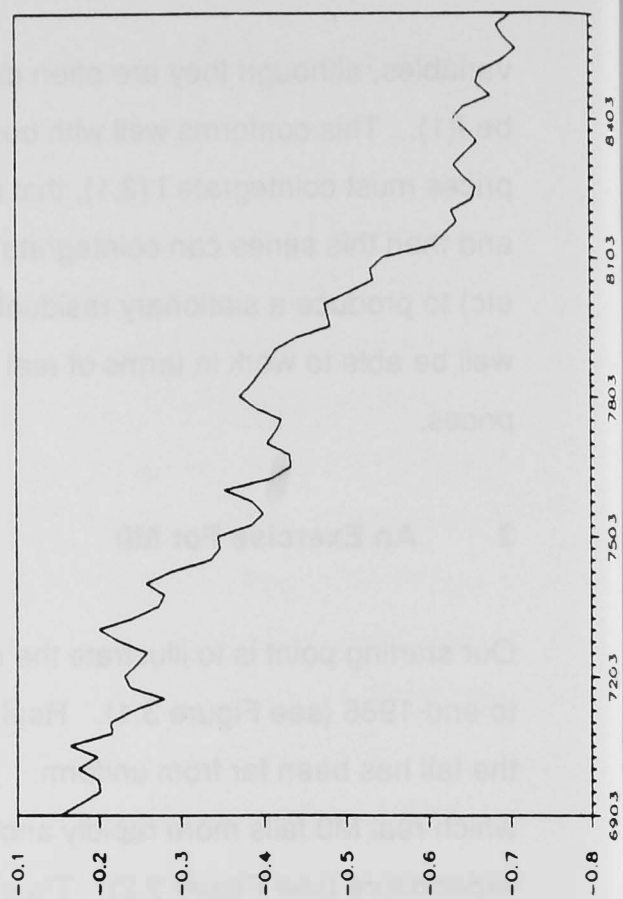


Table 3.1: Tests of Cointegrating Vectors Involving LMO

	(1)	(2)	(3)	(4)
LCPI	0.7	1.00	1.00	1.00
LQCE	0.47	0.88	1.00	0.50
BSSR	0.007	-0.009	-0.039	-0.031
CRDW	0.32	0.14	0.06	0.06
DF	-2.3	-1.7	1.1	0.79
ADF	-2.1	-2.6	1.3	0.87
R ²	0.99	0.71	0.12	0.13

Sample 1969: Q3 to 1986 Q4

CRDW is the cointegrating regression Durbin Watson statistic.

When the simple unrestricted transactions demand for money equation is run (equation 1), the upward trend in velocity is partly captured by an unacceptably low price elasticity. When the price elasticity is restricted to one, the income elasticity becomes unacceptably low (equation 2). When both the price and income elasticities are restricted (equations 3 and 4) the equations fail the cointegrability tests even more noticeably than the first two equations.

The OLS results strongly suggest that no cointegrating vector exists amongst this set of variables. The Johansen procedure produces the following results for this set of variables.

<u>Eigenvalue</u>	<u>Eigenvector</u>			
	<u>LMO</u>	<u>LCPI</u>	<u>LQCE</u>	<u>BSSR</u>
0.18	-30.27	26.67	-6.14	-0.59
0.16	-44.35	31.91	27.83	0.28
0.05	18.89	-10.14	-22.74	0.04
0.0007	-39.22	27.91	-0.53	1.57

The likelihood ratio test that there are at most r cointegrating vectors gives the following results:

r	LR	5% critical value
0	30.2	38.6
1	16.0	23.8
2	3.8	12.0
3	0.05	4.2

The likelihood ratio test suggests that we are unable to reject the hypothesis that there are 0 cointegrating vectors. So the Johansen procedure confirms the OLS suggestion that cointegration does not exist between this set of variables. It is interesting to note that the eigenvectors corresponding to the two largest eigenvalues produce parameter estimates which are similar to the OLS results in columns (1) and (2) of Table 3.1.

3.i Financial Innovation and the Demand for Cash

The most plausible explanation for the decline in the ratio of $M0$ to nominal consumers' expenditure is financial innovation. Over the sample period transactions technology has changed considerably with the widespread introduction of cash dispensers, and credit cards, and a noticeable increase in the number of bank current accounts. Johnston (1984) provides a further description of these changes and attempts to assess their impact on narrow money holdings. Our next attempt to find a cointegrating solution involved the three innovation variables indicated above. The results are reported in Table 3.2.

Table 3.2: Tests of Cointegrating Vectors Involving LMO - LCPI - LQCE

	(5)	(6)	(7)	(8)	(9)	(10)	(11)
BSSR	-0.002	0.004	0.01	-0.015	0.012	-0.015	0.011
LCC	0.08		0.02	0.001	-0.28		
LCDA	-0.11	-0.07		-0.25		-0.25	
LCAPOP	-0.78	-0.68	-1.02				-0.97
CRDW	0.64	0.48	0.45	0.27	0.15	0.27	0.44
DF	-2.9	-2.6	-2.4	-2.4	-1.5	-2.3	-2.45
ADF	-1.96	-2.1	-2.1	-2.2	-2.2	-2.2	-2.23
R ²	0.99	0.99	0.98	0.96	0.92	0.96	0.98

Sample: 1969 Q3 1986 Q4.

Even with these trended innovation variables the equations fail the tests of cointegration. A further problem arises because of the log form of the innovation variables. As these variables have low values at the beginning of the sample the effect of an extra unit implied by the log form is large. An alternative possibility is to consider entering the innovation variables linearly.

Table 3.3: Tests of Cointegrating Vectors Involving LMO - LCPI - LQCE

	(12)	(13)	(14)	(15)	(16)	(17)	(18)
BSSR	-0.0002	-0.015	0.002	0.001	-0.02	-0.05	0.0008
CC(10^{-5})	0.69	-3.9	0.19	-	-2.4	-	-
CDA($\times 10^{-5}$)	-0.96	0.26	-	0.05	-	-6.2	-
CAPOP	-0.02	-	-0.02	-0.02	-		-0.02
CRDW	0.45	0.38	0.40	0.38	0.26	0.25	0.38
DF	-0.94	-2.8	-1.06	-1.1	-2.3	-2.1	-1.1
ADF	-1.13	-2.5	-1.64	-1.6	-2.5	-2.1	-1.6
R ²	0.98	0.93	0.98	0.98	0.93	0.89	0.98

None of the equations passes the cointegrability tests. The equations that come closest to passing are (13), which has a perversely signed coefficient on the cash dispenser variable, and (16). In this case the Johansen procedure produces results which are at slight variance with Tables 3.2 and 3.3. If we include all three innovation variables (as in equation 12) then the likelihood ratio tests suggests that there may be up to four cointegrating vectors in the data. The explanation for this discrepancy is that the Johansen procedure treats all the variables in a similar fashion whereas the OLS procedure normalises on one variable (money in this case) and so treats this variable in a special way. The Johansen procedure will therefore reveal cointegrating vectors which do not involve money while the OLS procedure is barred from this. It would seem therefore that a number of cointegrating vectors exist between the innovation variables and interest rates but none of these may be the relevant one for determining money.

If we apply the Johansen procedure to a case where there is only one innovation variable (we chose CC) then we obtain the following results.

<u>Eigenvalue</u>	<u>Eigenvector</u>		
	<u>LMO-LCPI-LQCE</u>	<u>CC</u>	<u>BSSR</u>
0.302	-7.13	-0.0004	-0.52
0.15	18.17	0.0004	0.72
0.023	12.29	0.0007	-0.57

Likelihood ratio test that there are at most r cointegrating vectors

<u>r</u>	<u>LR</u>	<u>5% critical value</u>
0	36.5	23.8
1	12.3	12.0
2	1.6	4.2

We can reject the hypothesis that there are no cointegrating vectors and there may be two cointegrating vectors in this set (12.3 is on the borderline). The coefficients implied by the first cointegrating vector are plausible and conform with the parameters given in Table 3.3 reasonably well.

Our interpretation of these conflicting results is that financial innovation is indeed a promising explanation of the movement in $M0$ but that our measures of innovation are not satisfactory. So in the next section we turn to a more flexible alternative approach.

3.ii An Alternative Approach to Innovation

An alternative approach to direct measures of innovation is to attempt to model the innovation process. This approach separates two distinct interest rate effects.

- (i) For a given transactions technology a rise (fall) in the rate will lead to an increase (decrease) in velocity.

- (ii) An increase in the rate provides an incentive for cash saving technology and payments methods to be introduced. Such changes are likely to be asymmetric and once implemented unlikely to be repeated exactly or reversed, eg the introduction of cash dispensers; furthermore, such changes alter the transactions technology and hence the likely magnitude of responses under (i).

We now consider a model of the effect described in (ii). The money demand function considered is of the form

$$LMO - LCPI - LQCE = a + br_t + I_t + e_t$$

where I_t is a variable capturing the innovation process.

The benefits to innovation considered are those which arise from a reduced need to hold assets in the form of cash. It is assumed that the opportunity cost of holding cash is the nominal rate of interest. A general model of the process might assume that the benefits from innovating in any period can be proxied by an expected future rate of interest r^e . The costs of introducing cash saving payment methods are not explicitly modelled but are represented by a time varying constant $A_2(t)$. Note that the gains to innovation will be positive if $[r_t^e - A_2(t)]$ is positive; if this term is less than or equal to zero we assume that no innovation takes place. A possible functional form relating the rate of introduction of cash savings payments methods to the expected rate of interest might be:

$$\Delta I_t = \max \left[\left(r_t^e - A_2(t) \right) : 0 \right]$$

This implies that unless the expected interest rate is above a certain minimum level no innovation takes place. This would suggest using a term of the form:

$$I_t = A_1 \sum_{s=0}^t \max \left[\left(r_s^e - A_2(s) \right) : 0 \right]$$

Or allowing for the possibility of other non-linearities in the response.

$$I_t = A_1 \left(\sum_{s=0}^t \max \left[\left(r_s^e - A_2(s) \right)^{A_4} : 0 \right] \right)^{A_3}$$

where A_3 and A_4 allow for possible non linearities.

A particularly simple form which assumes the function is linear ($A_3 = A_4 = 0$), that the cost of innovation is constant ($A_2(t) = A_2$), that the expected rate may be proxied by the actual rate, and that over the sample period the rate has always exceeded A_2 is

$$I_t = A_1 \sum_{s=0}^t (r_s - A_2) = A_1 \sum_{s=0}^t r_s - A_1 A_2 t$$

Table 3.4 uses this restricted equation as the innovation variable.

Table 3.4: Tests of Cointegrating Vectors Involving LMO - LCPI - LQCE

	(19)	(20)	(21)
BSSR	-0.0012	0.0012	-
Σ BSSR	-0.0013	-0.0008	-0.0013
Time	-	-0.004	-
CRDW	0.67	0.73	0.65
DF	-3.8	-3.8	-3.7
ADF	-2.9	-2.9	-2.8
R^2	0.99	0.99	0.99

Noting that in this case, because the additional lags in the ADF test statistic were not significant, the sample DF statistic is relevant hence these results indicate that the simple form of innovation variable provides a cointegrating vector with the log of the ratio of M0 to nominal consumers' expenditure. We have estimated equations using

forms of the innovation variable ($A_4, A_3 \neq 0$) but so far this shows little improvement on equation (21).

If we apply the Johansen procedure to the two variables LMO-LCPI-LQCE and BSSR we get the following result.

<u>Eigenvalue</u>	<u>Eigenvector</u>	
	<u>LM0-LCPI-LQCE</u>	<u>ΣBSSR</u>
0.16	58.26	0.0769
0.017	-20.28	-0.037

Test that there are at most r cointegration vectors

<u>r</u>	<u>LR test</u>	<u>5% critical value</u>
0	13.3	12.0
1	1.14	4.2

This, then, also suggests that a cointegrating vector exists and further if we normalise the first eigenvector on money we get a coefficient of -0.0013 on BSSR which is identical to the OLS estimate in (21).

Given the agreement between the OLS results and the Johansen results we will proceed to estimate a dynamic model based on equation (21). At this second stage the dependent variable used is the difference of the log of $M0$, the explanatory variables are past changes in the log of consumers' expenditure and the consumer price deflator together with the level of the interest rate. The lagged residuals from the cointegrating equation (RES_{t-1}) are entered into the dynamic equation and should appear with a negative sign. Initial parameter estimates suggested a restriction between the coefficients on the lagged residuals and changes in the exogenous variables; hence, the equation below contains a term of the form $(M0_t^\wedge - M0_{t-1})$ where $M0_t^\wedge$ is the long-run equilibrium level of $M0_t$ suggested by the cointegrating equation. Following a conventional general to specific search methodology we arrive at the following dynamic equation for $M0$.

FIGURE 3.5: RECURSIVE ESTIMATION TIME SERIES OF $\Delta LMO(t-1)$

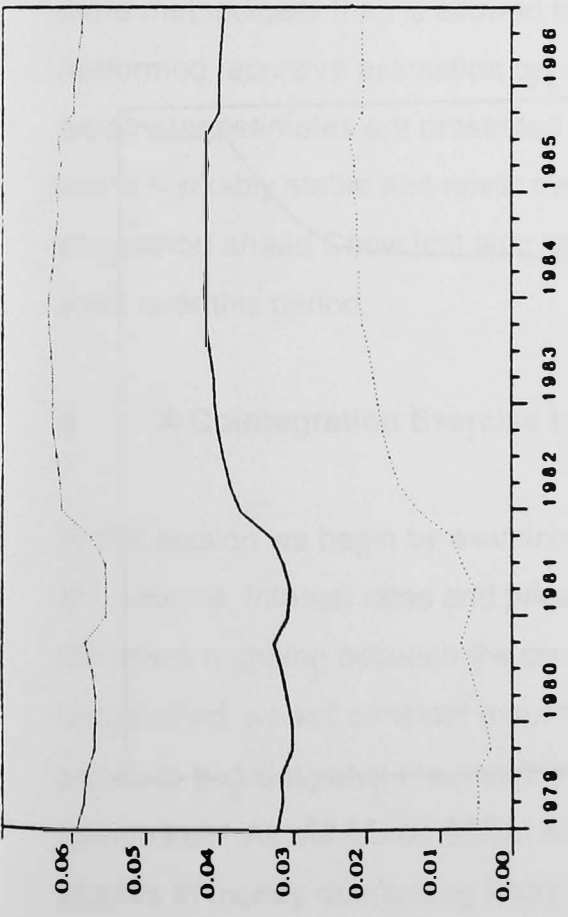


FIGURE 3.6: RECURSIVE ESTIMATION TIME SERIES OF $\Delta \Delta LCP(t-1)$

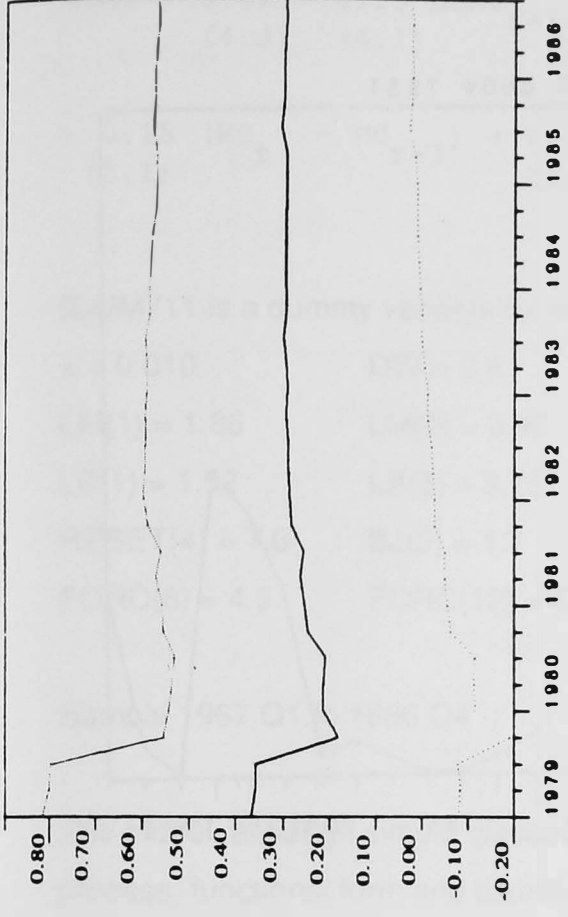


FIGURE 3.7: RECURSIVE ESTIMATION TIME SERIES OF $\hat{MO}(t-1) - \hat{MO}(t)$

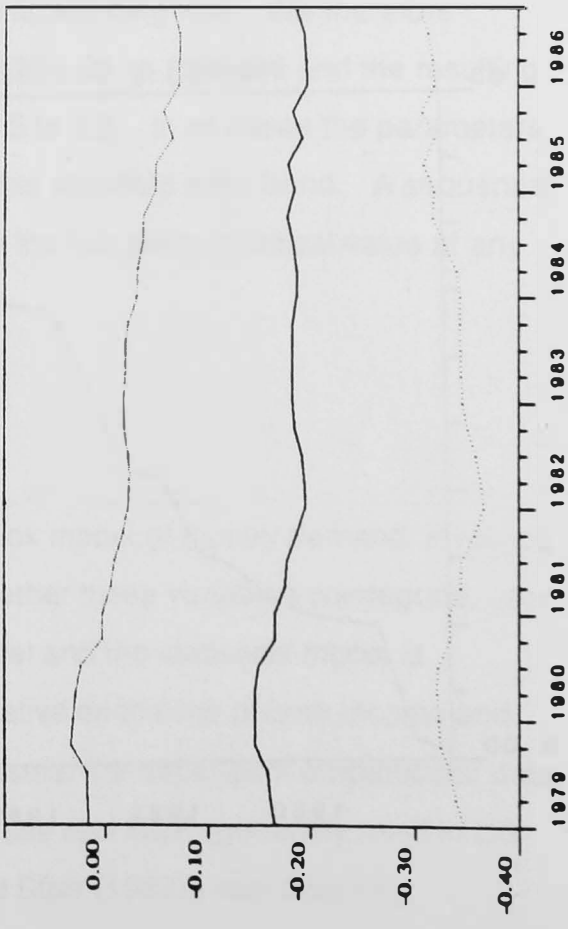
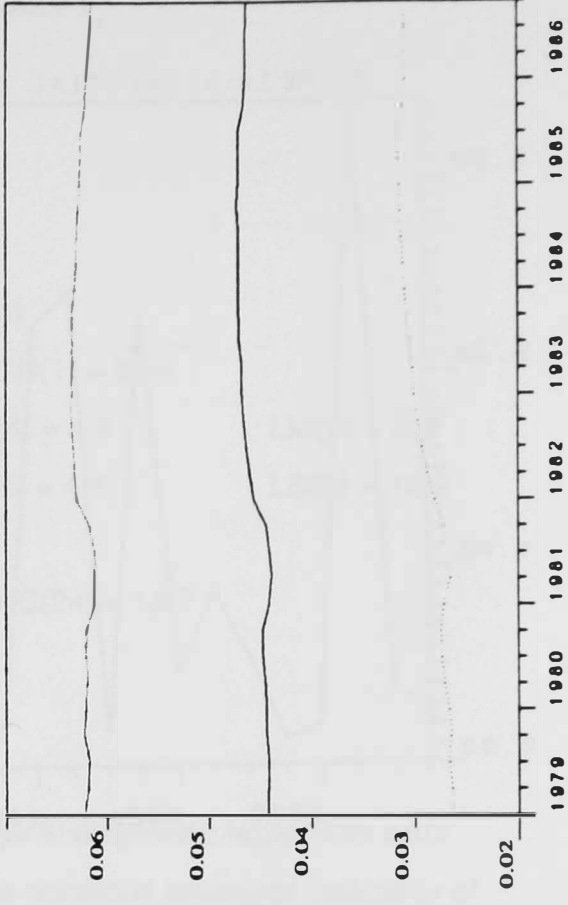


FIGURE 3.8: RECURSIVE ESTIMATION TIME SERIES OF DUM711



DOTTED LINES SHOW TWO STANDARD ERRORS AROUND THE COEFFICIENT VALUES

TEST STATISTICS FOR RECURSIVE ESTIMATION OF THE DYNAMIC M0 EQUATION

FIGURE 3.9: SEQUENTIAL ONE PERIOD CHOW TEST

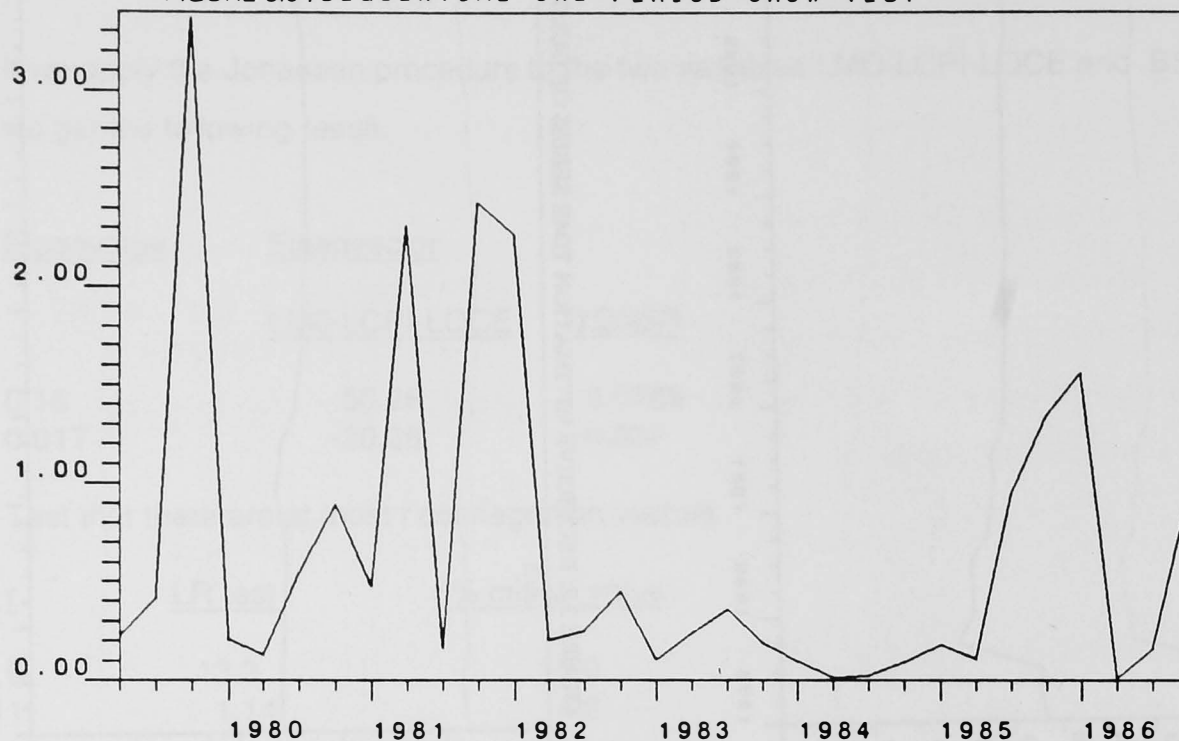
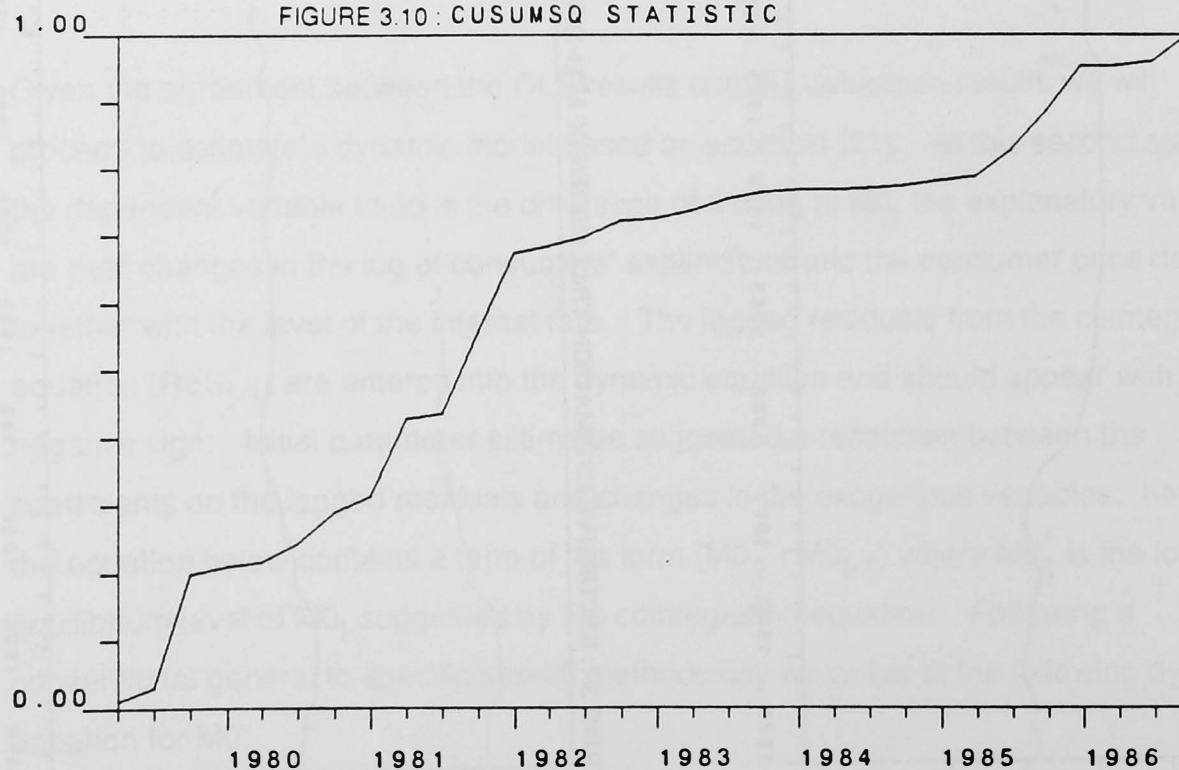


FIGURE 3.10: CUSUMSQ STATISTIC



$$\begin{aligned} \Delta LM0 = & 0.01 + 0.39 \Delta LM0_{t-1} + 0.30 \Delta \Delta LCPI_{t-1} \\ & (4.4) \quad (4.1) \quad (2.2) \\ & + 0.18 (M0_t^\wedge - M0_{t-1}) + 0.05 DUM711 \\ & (3.1) \quad (6.1) \end{aligned}$$

(DUM711 is a dummy variable for decimalisation)

$\sigma = 0.010$	DW = 2.3	ARCH(1) = 0.09	
LM(1) = 1.86	LM(2) = 3.90	LM(4) = 4.6	LM(8) = 9.3
LB(1) = 1.52	LB(2) = 3.75	LB(4) = 4.4	LB(8) = 12.9
RESET(4) = 7.0	BJ(2) = 1.7		
FORC(8) = 4.9	FORC(12) = 4.8	FORC(24) = 16.7	

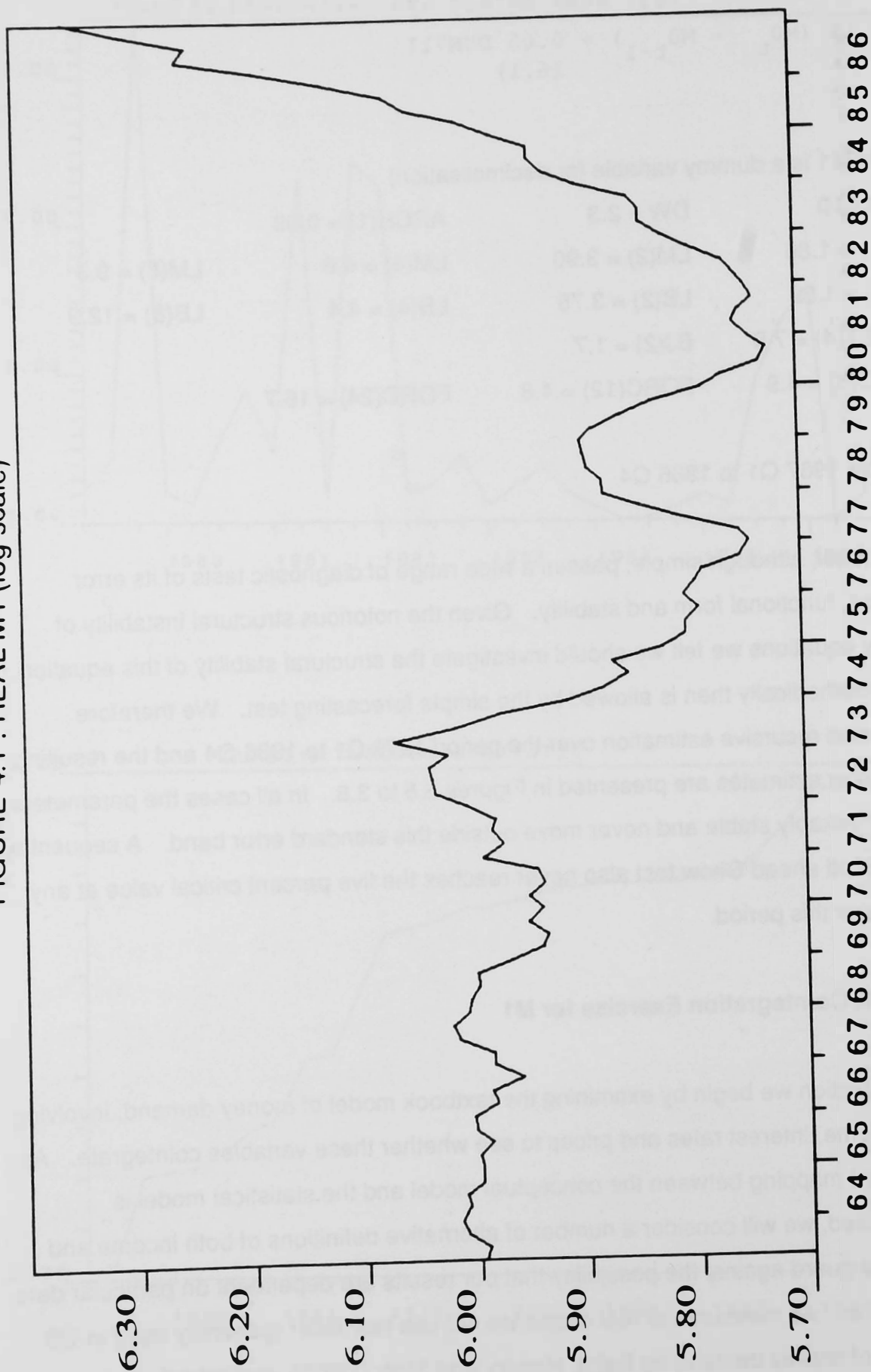
Sample 1967 Q1 to 1986 Q4

The model, although simple, passes a wide range of diagnostic tests of its error process, functional form and stability. Given the notorious structural instability of money equations we felt we should investigate the structural stability of this equation more methodically than is allowed by the simple forecasting test. We therefore performed recursive estimation over the period 1979 Q1 to 1986 Q4 and the resulting parameter estimates are presented in Figures 3.5 to 3.8. In all cases the parameters are remarkably stable and never move outside this standard error band. A sequential one period ahead Chow test also never reaches the five percent critical value at any point over this period.

4 A Cointegration Exercise for M1

In this section we begin by examining the textbook model of money demand, involving M1, income, interest rates and prices to see whether these variables cointegrate. As the latent mapping between the conceptual model and the statistical model is unspecified, we will consider a number of alternative definitions of both income and prices to guard against the possibility that our results are dependent on particular data definitions. As measures of real output we will use real GDP (generally used in US studies of money demand eg Baba, Hendry and Starr (1982)), real total final

FIGURE 4.1 : REAL M1 (log scale)



expenditure TFE (generally used in UK studies eg Hendry (1979)) and real consumption. As measures of prices we will use the three corresponding price deflators.

Table 4.1 presents estimates of the parameters and test statistics for a set of possible cointegrating equations. The parameter estimates are quite sensible; in the final three equations involving prices income and interest rates, the price elasticity varies between 0.91 and 1.04, the income elasticity varies between 0.81 and 1.58 and the interest rate effect is -0.03. However, it is clear from the ADF statistics that none of these equations represents a cointegrating set of variables. So we can immediately rule out any simple model which implies a long-run relationship between M1, income, prices and interest rates only. This set does not cointegrate and so we would not expect any dynamic equation based solely on this set to be stable. This then confirms the anecdotal view that innovation over the past twenty years has been crucial in the determination of money demand.

TABLE 4.1: TESTS OF COINTEGRATING VECTORS INVOLVING M1

	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
LPGDP	0.96			0.84			0.88		
LPTFE		0.96			0.81			0.84	
LCPI			0.99			0.69			0.74
LGDP				0.62			0.85		
LTFE					0.70			0.91	
LQCE						1.5			1.6
RTB							-0.02	-0.03	-0.02
CRDW	0.05	0.05	0.05	0.05	0.05	0.08	0.15	0.14	0.22
DF	2.5	2.7	2.8	1.2	1.8	0.15	-0.5	-0.5	-1.6
ADF	-0.5	-0.4	-0.2	-0.05	0.08	-0.03	-0.7	-0.7	-0.9
R ²	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.99

Sample 1966 Q1 to 1986 Q4

A number of studies have attempted to incorporate measures of financial innovation. Johnston (1984) used data on the number of cash dispensing machines, the number of credit cards and the number of current accounts per head of the population in his study for the UK. Baba, Hendry and Starr (1982), when studying US money demand functions, use direct data on the interest rates on new types of interest bearing accounts in M2. We will begin investigating the financial innovation story by including a range of variables used by Johnston and the cumulated interest rate effect used in the last section.

This is done in the first 4 columns of Table 4.2, where CC, LCDA, LCAP and BSSR are added to the basic money demand function which was considered in Table 4.1. We now only consider the use of LTFE and LPTFE; this choice is rather arbitrary as the data is not really able to choose between the various measures of output and prices. However, none of the subsequent results are sensitive to this choice. Three of the four regressions reported then pass the cointegration tests once the innovation variables are included. However, we would still reject these as suitable cointegrating vectors as the price elasticity has an implausibly low coefficient. If we impose a unit coefficient on the price effect (columns 5-8) then cointegration may be rejected in two of the three cases. In the remaining case, when CC is used, the income elasticity is only 0.01 and again we regard this result as implausible.

If we consider the time series of real M1, shown in figure 4.1, we can quickly gain on insight into the nature of the problem. Real M1 is virtually constant from 1963-1982, with no discernible trend. This pattern changes sharply in 1982 and a rapid growth in real M1 begins. None of the explanatory variables in Table 4.2 exhibit this type of behaviour and so it seems likely that some other important effect is missing. The early 1980s saw two important developments with regard to the behaviour of M1. First, the removal of the 'corset' in 1981 had a general effect on the monetary sector and, second, the early 1980s saw the wide scale introduction of interest-bearing cheque accounts. During the 1970s less than 10% of M1 was interest bearing, with the rapid

growth of the financial sector and with the introduction of the new accounts in the early 1980s this proportion rose rapidly so that by the end of 1982 almost 30% of M1 was interest bearing and by the middle of 1987 65% was interest bearing. A reasonable interpretation therefore is that large amounts of this new M1 would actually represent a growth in the real M1 stock which was due, almost solely to the growth in the new interest bearing element.

TABLE 4.2: TESTING THE EFFECT OF INNOVATION

Dependent Variables	LMI			LM1-LPTFE		
	(31)	(32)	(33)	(34)	(35)	(36)
LTFE	0.70	1.7	0.2	-0.5	1.4	1.2
LPTFE	0.55	0.76	0.1	-	-	-
RTB	-0.008	-0.02	-0.02	-0.02	-0.03	-0.02
CC(10^{-5})	3.2	-	-	1.5	-	-
LCDA	-	-0.05	-	-	-0.13	-
LCAP	-	-	2.1	-	-	-0.5
CRDW	0.39	0.20	0.15	0.17	0.21	0.15
DF	-2.4	-1.2	-2.4	-0.7	-1.3	-0.4
ADF	-2.1	-0.5	-2.7	-1.9	-0.9	-0.9
R ²	0.99	0.98	0.99	0.36	0.32	0.32

TABLE 4.3 A COINTEGRATING VECTOR FOR M1

DEPENDENT VARIABLE	LM1 (37)	LM1- LPTFE (38)	LM1-LPTFE -LTFE (39)	LM1-LPTFE -LM/N-LTFE (40)
CONSTANT	-2.9	-4.9	-4.8	-4.8
LTFE	0.88	1.005		-
LPTFE	0.78	-	-	-
CONS	-0.02	-0.02	-0.02	-0.03
Σ BSSR	-0.003	-0.001	-0.001	-0.001
LM/N	0.98	1.2	1.2	-
CRDW	0.76	0.71	0.76	0.74
DF	-4.4	-4.4	-4.4	-4.2
ADF	-3.8	3.5	-3.5	-3.2
R ²	0.99	0.92	0.98	0.98

In order to remove these effects we include a variable $LM/N = \text{LOG}(M1/NIBM1)$ which shows the rising proportion of interest-bearing M1. At this stage we make no attempt to model the introduction of interest-bearing M1 accounts by economic variables. This is partly because the data period of the growth of these deposits is still quite short, about 5 years, and partly because data such as the number of high interest accounts is not available.

Table 4.3 then considers the possibility of a cointegrating vector existing between this full set of variables. The first column in 4.3 performs a completely unrestricted estimate of the cointegrating vector. The tests for cointegration are all passed and the parameter values are reasonable; however, the price elasticity is a little low at 0.9. The next three columns restrict one coefficient at a time and test for cointegration. With this full set of variables imposing price homogeneity does not cause a breakdown of cointegration nor does the income elasticity become implausible, as was the case in

Table 4.2. The remaining columns of 4.3 restrict the cointegrating vector to have a unit effect from LM/N and LTFF; neither restriction prevents cointegration nor does it substantially alter the remaining parameters.

We now proceed to verify these results using the Johansen procedure. If we apply the technique to the full set of unrestricted variables the conclusion is that there are four cointegrating vectors within the set of variables. We will not present the full matrix here, but one of the four is very similar to our final restricted equation having a near unit coefficient on LPTFE, LTFE and LM/N. We therefore repeated the Johansen procedure using the restricted variable in equation 40. This produced the following results.

<u>Eigenvalue</u>	<u>Eigenvector</u>		
	LMI-LPTFE -LM/N-LTFE	CONS	Σ BSSR
0.202	21.7	0.013	0.034
0.17	35.9	0.046	1.38
0.03	16.3	0.023	-0.01

Likelihood ratio test of maximum of r cointegrating vectors

r	<u>LR</u>	<u>5% critical value</u>
0	37.5	23.8
1	18.6	12.0
2	2.9	4.2

In this case we can reject the hypothesis that there is only one cointegrating vector ($r=1$) in favour of two cointegrating vectors. In fact the second vector (associated with the eigenvalue 0.17) is virtually identical to that estimated in equation 40 table 4.3, and so we will use this equation as our cointegrating vector for M1.

Having now achieved a suitable cointegrating model of the long-run determination of M1 we may then use the residuals from this equation to build a dynamic model. We proceed along a conventional path of specifying a high order dynamic model and nesting down from this general model to a parsimonious representation of the data.

This should then produce a structurally stable, satisfactory model for M1.

Following this procedure produces the model.

$$\Delta LMI = 0.01 + 0.14 \Delta LMI_{t-1} + 0.39 \Delta LMI_{t-2}$$

(3.7) (1.6) (4.3)

$$-0.3 \Delta LTFE + 0.36 \Delta LM/N$$

(2.7) (2.4)

$$+ 0.36 (\Delta LPTFE_{t-2} - \Delta LPTFE_{t-4})$$

(2.0)

$$- 0.004 (\Delta CONS + \Delta CONS_{t-1})$$

(2.4)

$$- 0.20 RES_{t-1}$$

(3.5)

(RES - Residuals from cointegrating regression, Table 4.3 column 4)

$\sigma = 0.0157$	$R^2 = 0.50$	DW = 1.98
ARCH = 0.005	RESET(4) = 1.2	BJ(2) = 7.6
LM(1) = 0.04	LM(4) = 0.7	LM(8) = 8.1
LB(1) = 0.0	LB(4) = 3.0	LB(8) = 7.8
CHISQ(8) = 10.8	CHISQ(12) = 10.6	CHISQ(24) = 22.8

The Model passes a wide range of diagnostics, the ARCH statistic is close to the 5% cut-off region, and easily passes the Hendry forecast test. The recursive residuals are reasonably random and the model easily passes the CUSUMSQ test over the period 1979 Q1 - 1986 Q4. Over this period the cumulated sum of residuals (CUSUM) is only -4.7 after 24 quarters, and a recursive one period ahead Chow test reaches its 5% critical value only once. The recursive parameter estimates are all highly stable. The one failure in the sequential Chow test occurs in the third quarter of 1986, this seems to be due to a marked distortion of the money figures possibly caused by the flotation of the trustee savings bank.

TEST STATISTICS FOR RECURSIVE ESTIMATION OF THE DYNAMIC M1 EQUATION

FIGURE 4.2: SEQUENTIAL ONE PERIOD CHOW TEST

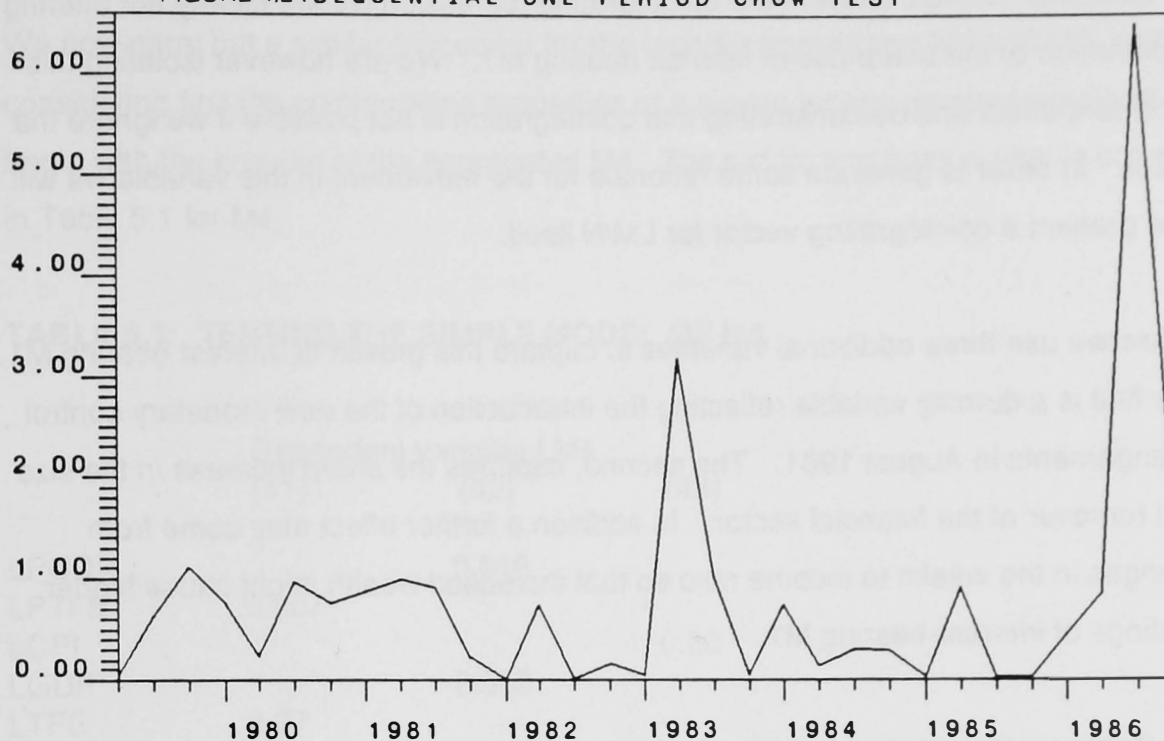
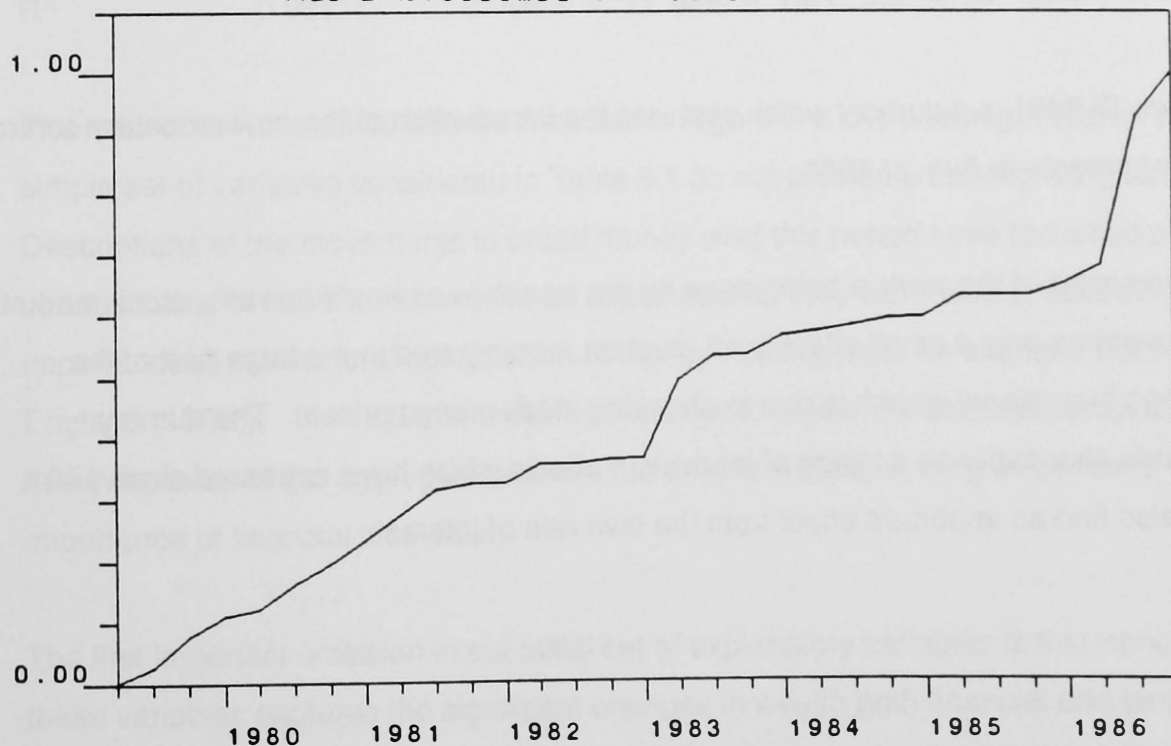


FIGURE 4.3: CUSUMSQ STATISTIC



By using the variable LM/N in the cointegrating regression we are clearly not offering an explanation of the sharp rise of interest bearing M1. We are however isolating this important effect and demonstrating that cointegration is not possible if we ignore the effect. In order to generate some rationale for the movement in this variable we will now present a cointegrating vector for LM/N itself.

Below we use three additional variables to capture this growth of interest bearing M1. The first is a dummy variable reflecting the introduction of the new monetary control arrangements in August 1981. The second, captures the sharp increase in the size and turnover of the financial sector. In addition a further effect may come from changes in the wealth to income ratio so that increased wealth might cause higher holdings of interest-bearing M1.

A suitable cointegrating regression for LM/N is

$$LM/N = 2.7 + 0.76 (LFW - LTFE) + 0.04 LSM - 0.04 DUM81 + 0.003 BDR$$

$$CRDW = 0.87 \quad DF = -3.5 \quad ADF = -3.5 \quad R^2 = 0.90$$

Where DUM81 is a dummy which captures the introduction of the new monetary control arrangements in August 1981.

Clearly most of the work is being done by the wealth to expenditure ratio, stock market turnover has only a small effect but it must be remembered that a large part of the change in personal wealth is due to changing stock market prices. The dummy variable also captures a range of innovation effects which have appeared since 1981. We also find an important effect from the own rate of interest.

5 A Model of M4

We now carry out a similar procedure for the broader aggregates M4 and M3, again considering first the cointegrating properties of a simple money demand equation. We begin with the broader of the aggregates M4. The simple text book model is considered in Table 5.1 for M4.

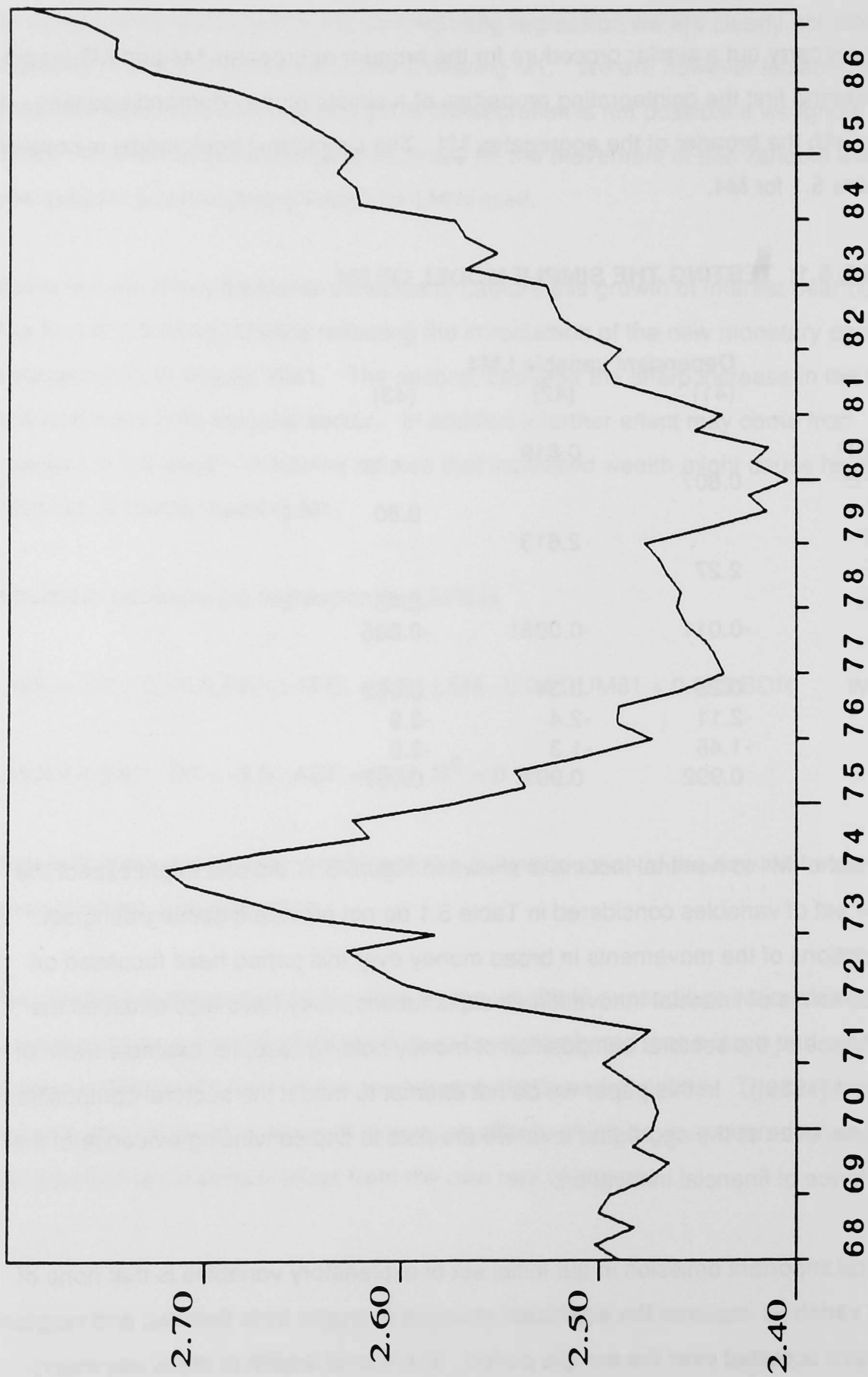
TABLE 5.1: TESTING THE SIMPLE MODEL OF M4

	Dependent variable LM4		
	(41)	(42)	(43)
LPGDP		0.819	
LPTFE	0.807		
LCPI			0.80
LGDP		2.613	
LTFE	2.27		
LQCE			2.43
RTB	-0.011	-0.0081	-0.005
CRDW	0.25	0.34	0.665
DF	-2.11	-2.4	-3.9
ADF	-1.46	-1.3	-2.6
R ²	0.992	0.991	0.997

The ratio of M4 to nominal income is shown in Figure 5.1. As one might expect the simple set of variables considered in Table 5.1 do not provide a cointegrating set. Descriptions of the movements in broad money over this period have focussed on various forms of financial innovation as explanations; they have also stressed the importance of the sectoral composition of money holding (see, for example Bank of England (1986)). In this paper we do not attempt to model the sectoral composition; however, even at the aggregate level we are able to find convincing evidence of the importance of financial innovation.

The first important omission in our initial set of explanatory variables is that none of these variables captures the significant changes in wealth both financial and tangible that have occurred over the sample period. The role of wealth in many key macro economic relationships is at present receiving renewed attention. Although wealth

FIGURE 5.1 : M4 RELATIVE TO GDP (log scale)



might be thought the crucial scaling variable if the demand for money is seen as part of a wider portfolio decision, many demand for money equations have omitted such a variable (for an exception see Grice and Bennett (1984)). Wealth plays a crucial role in determining the demand for money in the equations below, this is particularly important in explaining developments since 1980 when both wealth and broad money have grown rapidly. The wealth variable we will use is personal sector gross wealth both financial and tangible, the latter mainly reflects the value of the owner-occupied housing stock.

A second variable added to our initial list is inflation. A number of studies have found a significant impact on money holdings of inflation, for an example and further references on this see Taylor (1987). This effect could be due to a number of influences including the tendency for nominal rates not to move in line with expected inflation. In part, it could reflect front-end loading which occurred in debt markets during periods of high inflation and high nominal interest rates. During such periods the real value of debt fell noticeably and the constraints associated with front-end loading could have resulted in consumers running down liquid assets to maintain consumption levels.

Variables reflecting rates of return on assets were found to be unimportant with one exception. A two year moving average of the quarterly change in the log of stock market prices was used as a proxy for expected capital gains. Although this term proved to be unimportant, a moving average of the falls in the index proved to be useful in capturing the movements in money holdings, helping, in particular, to capture movements in liquid asset holdings following the large fall in the stock market index in the early 1970s.

It is perhaps worth commenting further on the failure to identify significant interest rate effects. To the extent that the interest rate differentials that enter are themselves stationary we should not expect them to enter the cointegrating regression. It is, however, a little more surprising that no such effects were found in the dynamic equations at the second stage. This may be a reflection of problems of measuring

appropriate own and competing rates for a period of financial innovation. It may also reflect the fact that different rates are appropriate for different sectors of the economy, and a disaggregated approach may be able to identify interest rate effects.

The final variable added to the set was a dummy variable to capture the introduction of competition and credit control. With these additions it proved straightforward to form a cointegrating vector.

Table 5.2: A Cointegrating Vector for M4

<u>Dependent variables</u>	<u>LM4</u>	<u>LM4-LPGDP</u>	<u>LM4-LPGDP-LGDP</u>
	(44)	(45)	(46)
LPGDP	0.99		
LGDP	1.07	1.05	
Δ_4 LPGDP	-0.71	-0.71	-0.71
LPW-LGDP-LPGDP	0.71	0.71	0.72
SND	-0.20	-0.20	-0.20
DCCC	0.09	0.09	0.10
CRDW	1.30	1.30	1.30
DF	-6.00	-6.00	-6.00
ADF	-5.1	-5.1	-5.0
R ²	0.99	0.99	0.96

Sample period 1968 Q1 to 1987 Q2 where

PW is personal sector real and financial wealth

LFTI is the log Financial Times ordinary share index and

ND = min (Δ LFTI:0)

$$SND = \sum_{i=0}^7 ND(t-i)$$

DCCC has the value 0 prior to 1971 Q4 and 1 thereafter.

These results provide a plausible cointegrating vector for M4. The parameter values show only minor variation as the restriction of unit elasticity are applied. The demand for M4 has a long-run income elasticity of 0.28 and a wealth elasticity of 0.72. The inflation effect plays a powerful role with an increase of 1 percentage point in the inflation rate leading to a reduction in desired money holdings of 0.7%. The stock market term implies that if falls in the index over the previous two years amounted to 1% then money holdings would be 0.2% larger than if no falls in the index had occurred.

We now verify that this is indeed a cointegrating vector, as well as considering its uniqueness by applying the Johansen procedure to this set of variables. The final equation in Table 5.2 contains five variables, the Johansen procedure suggested that there may exist two cointegrating vectors but that there is unlikely to be more than two. The test statistics are:

<u>r</u>	<u>LR</u>	<u>5% critical value</u>
0	78.9	57.2
1	42.1	38.6
2	17.7	23.8
3	7.3	12.0
4	1.14	4.2

The Eigenvalues and Eigenvectors corresponding with the two significant test statistics are:

TEST STATISTICS FOR RECURSIVE ESTIMATION OF THE DYNAMIC M4 EQUATION

FIGURE 5.2: SEQUENTIAL ONE PERIOD CHOW TEST

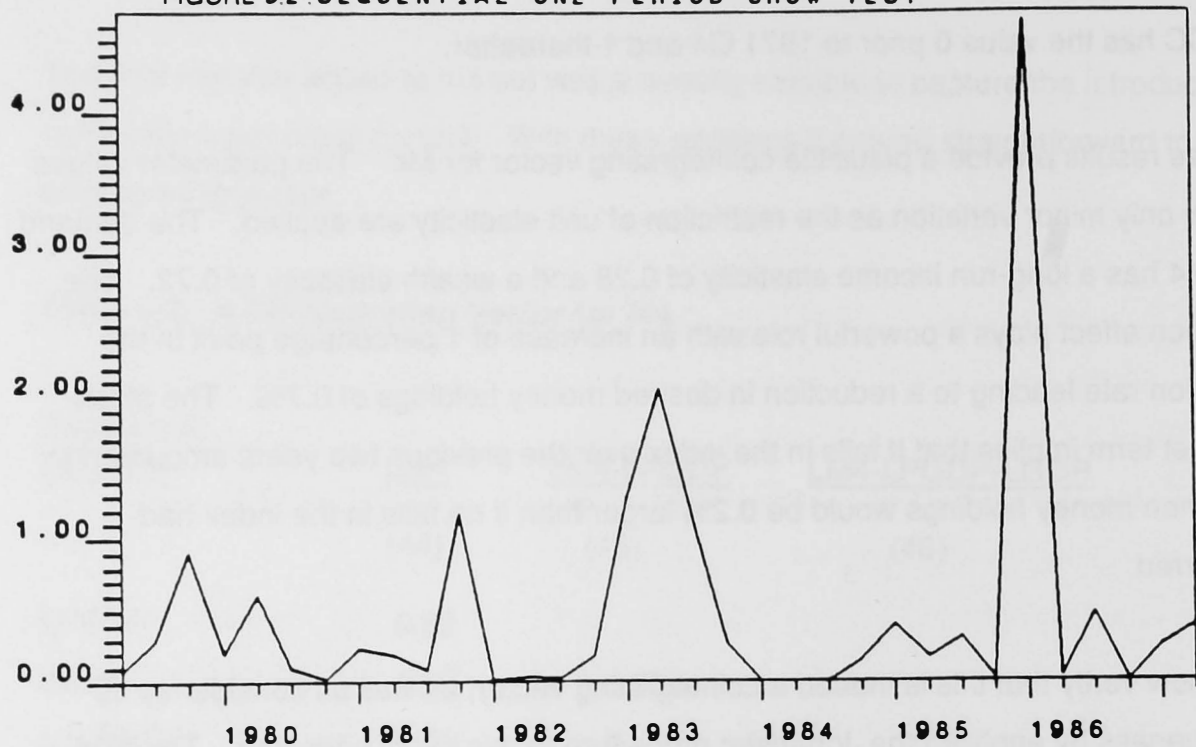
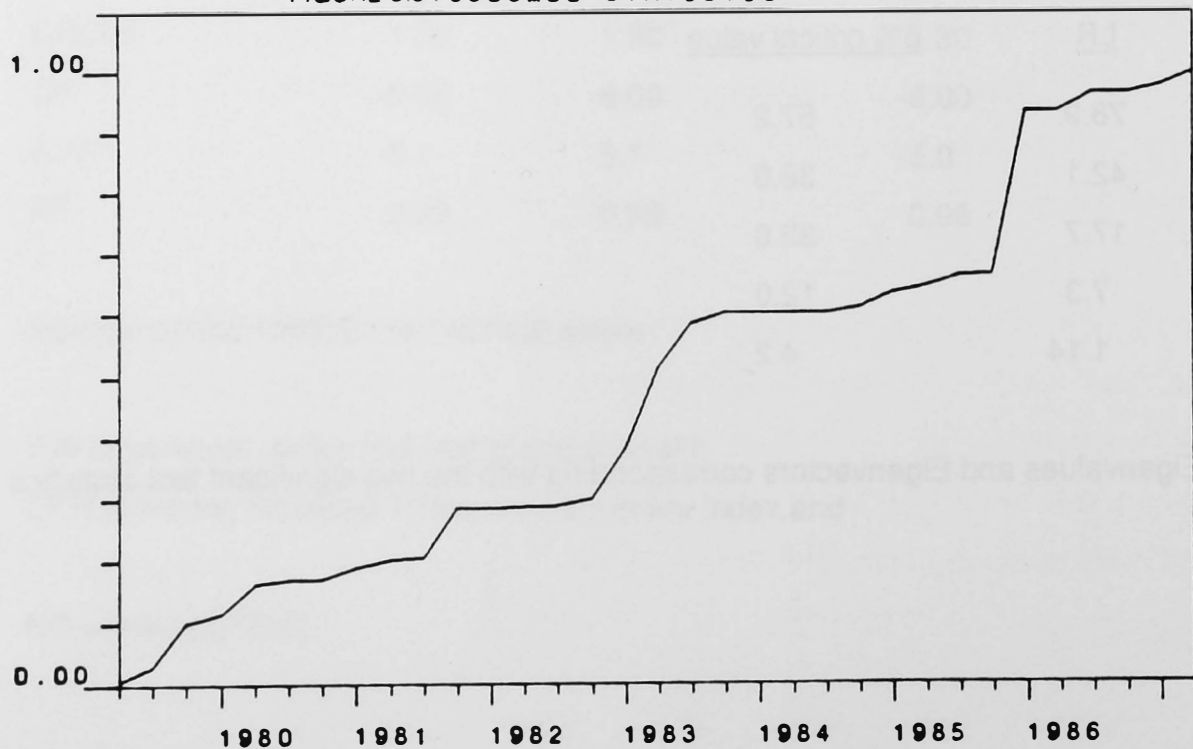


FIGURE 5.3: CUSUMSQ STATISTIC



<u>Eigenvalue</u>	<u>Eigenvector</u>				
	LM4V	Δ_4 LPCDP	LPW-LGDP-LPGDP	SND	DCCC
0.391	4.82	25.6	-4.05	-1.53	-2.23
0.281	-92.5	-80.33	66.68	11.03	10.13

In this case the eigenvector corresponding to the second eigenvalue is almost identical to the OLS results given in Table 5.2. The eigenvector corresponding to the largest eigenvalue is quite implausible as a causal relationship for M4 and given the large coefficient on inflation it seems likely that this cointegrating vector is actually determining inflation. So this procedure again finds a set of results which broadly support our OLS findings.

We now proceed to a dynamic model for M4 which is based on the final cointegrating regression given in Table 5.2. Once again we proceed along a conventional path of general to specific modelling, beginning with an error correction model containing four lags in the differences of all the variables. The final parsimonious model is given below.

$$\Delta LM4 = 0.016 + 0.512 \Delta LM4_{t-1} - 0.15 RES_{t-1}$$

(4.8) (5.2) (2.4)

(RES - residuals from the cointegrating regression in Table 5.2 column 3)

(Data period 1969,1-1987,2)

$\sigma = 0.0096$	$R^2 = 0.30$	DW = 1.98
ARCH(1) = 0.27	RESET(4) = 6.5	BJ(2) = 0.32
LM(1) = 0.57	LM(4) = 10.8	LM(8) = 10.7
LB(1) = 0.12	LB(4) = 2.45	LB(8) = 3.02
CHISQ(8) = 6.1	CHISQ(12) = 6.4	CHISQ(24) = 11.4

Recursive estimation over the period 1979 Q1 to 1987 Q2 again shows these parameters to be very stable. CUSUMSQ statistic and a sequential one period ahead Chow test clearly suggest that the equation is stable (see Figures 5.2 and 5.3).

6 A Model of M3

We now repeat the exercise for M3 following much the same procedure as used for the other aggregates. We begin by examining the cointegrating properties of the simple text book model for M3, involving interest rates output and prices. This is done in Table 6.1.

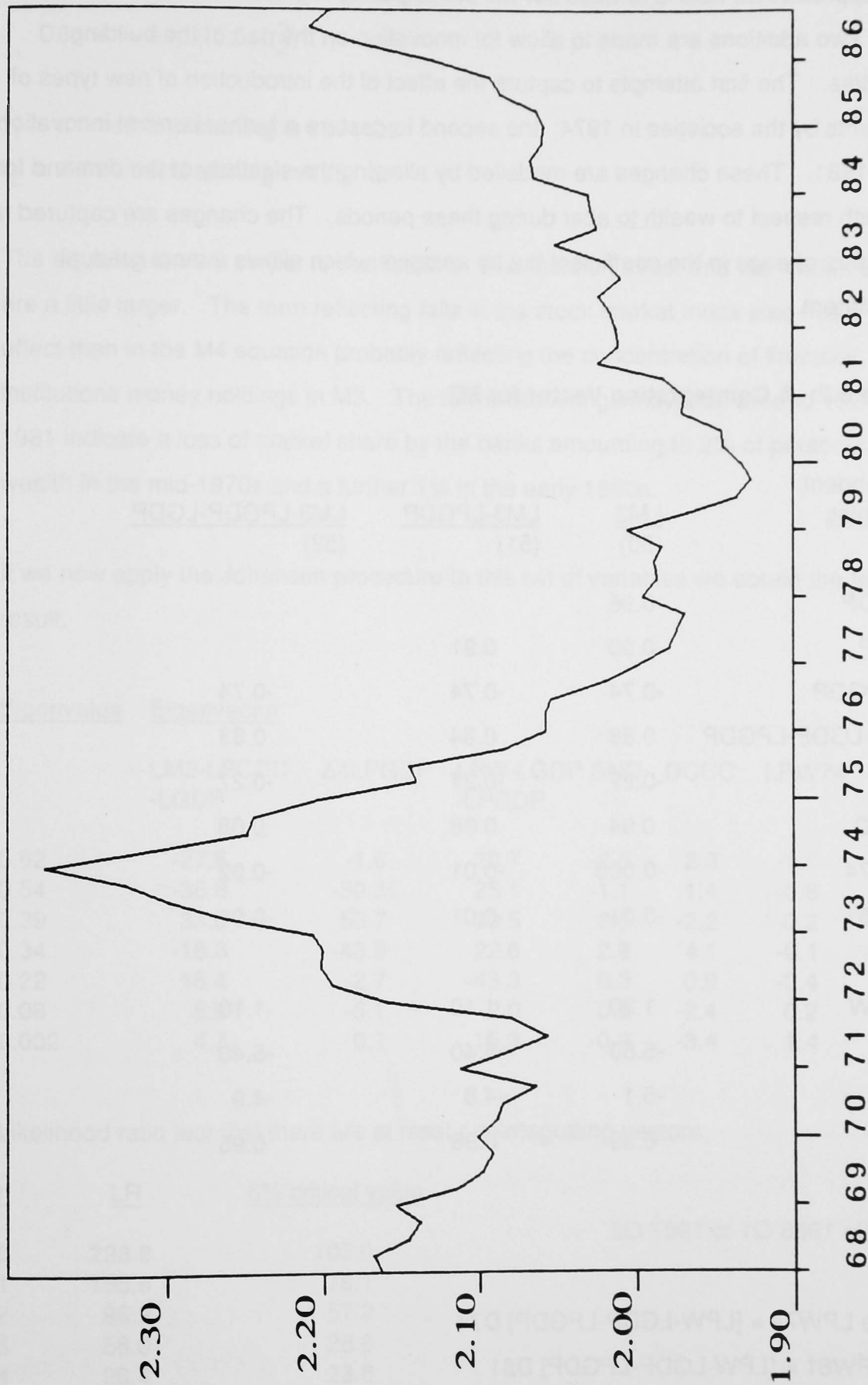
Table 6.1 Testing the Simple Model of M3

	(47)	(48)	(49)
LPGDP	-	0.84	-
LPTFE	0.82	-	-
LCPI	-	-	0.74
LGDP	-	1.5	-
LTFE	1.4	-	-
LQCE	-	-	2.0
RTB	-0.005	-0.002	0.0
CRDW	0.13	0.16	0.33
DF	-1.3	-1.5	-2.9
ADF	-1.9	-1.8	-2.5
R ²	0.99	0.99	0.99

Sample period 1968 Q1 to 1987 Q2

This table shows, quite decisively, that again this simple set of variables do not constitute a cointegrating vector. Figure 6.1 shows the path of M3 relative to total final expenditure and it is clear from this that there have been large changes in M3 holdings which must be explained by something other than nominal income.

FIGURE 6.1 : M3 RELATIVE TO GDP (log scale)



The approach we take is to base our M3 cointegrating regression around that used for M4. Two additions are made to allow for innovation on the part of the building societies. The first attempts to capture the effect of the introduction of new types of accounts by the societies in 1974; the second to capture a further burst of innovation after 1981. These changes are modelled by allowing the elasticity of the demand for M3 with respect to wealth to alter during these periods. The changes are captured not by a step change in the coefficient but by an ogive which allows a more gradual adjustment.

Table 6.2: A Cointegrating Vector for M3

Dependent variables	<u>LM3</u> (50)	<u>LM3-LPGDP</u> (51)	<u>LM3-LPGDP-LGDP</u> (52)
LPGDP	0.96		
LGDP	0.90	0.91	
Δ_4 LPGDP	-0.74	-0.74	-0.74
LPW-LGDP-LPGDP	0.88	0.84	0.83
SND	-0.27	-0.27	-0.27
DCCC	0.94	0.08	0.08
LPW74	-0.003	-0.01	-0.02
LPW81	-0.01	-0.01	-0.01
CRDW	1.20	1.10	1.10
DF	-5.80	-5.40	-5.40
ADF	-5.1	-4.8	-4.9
R ²	0.99	0.96	0.95

Sample 1968 Q1 to 1987 Q2

where $LPW74 = [LPW-LGDP-LPGDP] D74$

$LPW81 = [LPW-LGDP-LPGDP] D81$

$$\text{and } D74 = 1 - \exp(-0.01 \cdot t_1^2)$$

$$D81 = 1 - \exp(-0.01 \cdot t_2^2)$$

t_1 is a time trend starting in 1974 Q2

t_2 is a time trend starting in 1981 Q4

The equation is very similar to that for M4. The inflation effect and the wealth effect are a little larger. The term reflecting falls in the stock market index also has a larger effect than in the M4 equation probably reflecting the concentration of financial institutions money holdings in M3. The terms capturing innovation around 1974 and 1981 indicate a loss of market share by the banks amounting to 2% of personal sector wealth in the mid-1970s and a further 1% in the early 1980s.

If we now apply the Johansen procedure to this set of variables we obtain the following result.

<u>Eigenvalue</u>	<u>Eigenvector</u>						
	LM3-LPCDD -LGDP	Δ 4LPGDP	LPW-LGDP -LPGDP	SND	DCCC	LPW74	LPW81
0.62	-27.6	-1.6	20.7	-2.5	2.3	-1.1	0.6
0.54	-35.8	-30.36	25.1	-1.1	1.4	-0.6	-0.5
0.39	33.0	53.7	30.5	8.5	-2.2	-0.2	1.2
0.34	-18.8	-43.9	22.6	2.9	4.1	-0.1	-0.8
0.22	18.4	-2.7	-43.3	0.3	0.9	-0.4	1.94
0.08	5.9	-6.1	2.0	0.6	-2.4	0.2	-0.2
0.002	4.7	0.1	15.3	-0.9	-3.4	1.4	-0.7

Likelihood ratio test that there are at most r cointegrating vectors

<u>r</u>	<u>LR</u>	<u>5% critical value</u>
0	228.8	103.0
1	155.6	78.1
2	96.2	57.2
3	58.0	28.6
4	26.4	23.8
5	7.2	12.0
6	0.2	4.2

On the basis of this test procedure there may be five cointegrating vectors amongst these seven variables. In this case the eigenvector corresponding to the second largest eigenvalue is very similar to the OLS result. We will therefore proceed to the dynamic modelling stage on the basis of the OLS results.

Once again we use a conventional general to specific methodology to derive a dynamic model. As in the case of M4 this produced a simple dynamic model which is given below.

$$\Delta LM3 = 0.016 + 0.48 \Delta LM3_{t-1} - 0.28 RES_{t-1}$$

(4.6) (4.8) (3.7)

(RES is the residuals from the cointegrating regression in Table 6.2 column 3)

(Sample period 1969:1-1987:2)

$\sigma = 0.015$	$R^2 = 0.32$	DW = 2.04
ARCH(1) = 0.18	RESET(4) = 4.5	BJ(2) = 0.84
LM(1) = 0.25	LM(4) = 9.5	LM(8) = 10.5
LB(1) = 0.05	LB(4) = 7.6	LB(8) = 8.0
CHISQ(8) = 13.4	CHISQ(12) = 13.6	CHISQ(24) = 20.3

This equation is again well behaved in terms of a wide range of diagnostic tests which include the standard structural stability tests. Recursive estimation over the period 1979 Q1 to 1986 Q4 show that the parameters are highly stable. The CUSUM and CUSUMSQ statistics both show no sign of underlying mis-specification. Finally the sequential Chow test detects instability in 1987 Q1 and 1986 Q1 only (with a critical value of 4 neither failure is a large one).

TEST STATISTICS FOR RECURSIVE ESTIMATION OF THE DYNAMIC M3 EQUATION

FIGURE 6.2: SEQUENTIAL ONE PERIOD CHOW TEST

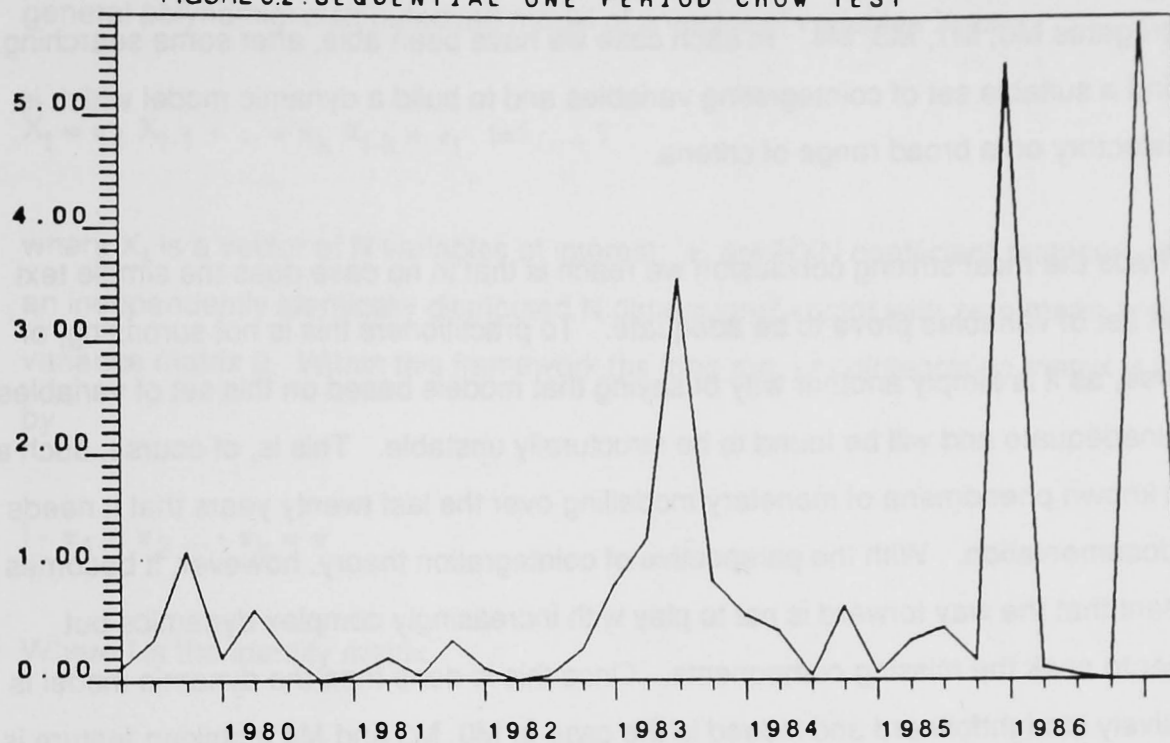
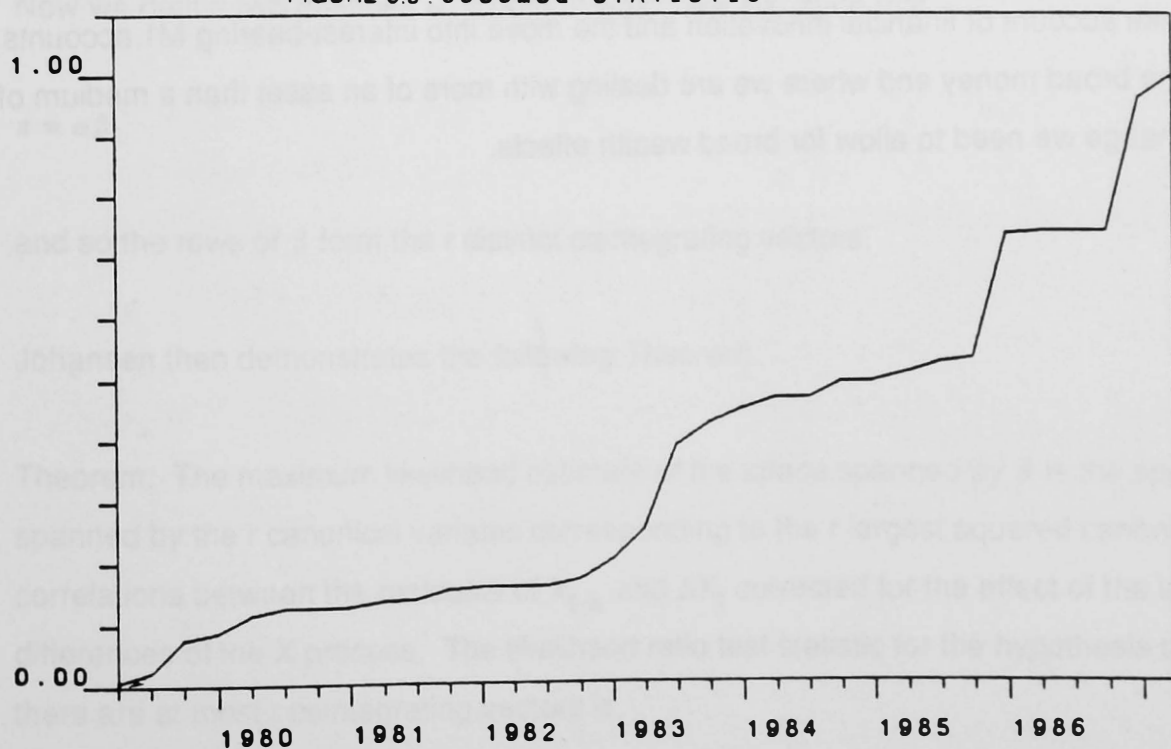


FIGURE 6.3: CUSUMSQ STATISTIC



7 Conclusion

In this paper we have applied cointegration techniques to model a range of monetary aggregates M0, M1, M3, M4. In each case we have been able, after some searching, to find a suitable set of cointegrating variables and to build a dynamic model which is satisfactory on a broad range of criteria.

Perhaps the most striking conclusion we reach is that in no case does the simple text book set of variables prove to be adequate. To practitioners this is not surprising, of course, as it is simply another way of saying that models based on this set of variables are inadequate and will be found to be structurally unstable. This is, of course, such a well known phenomena of monetary modelling over the last twenty years that it needs no documentation. With the perspective of cointegration theory, however, it becomes evident that the way forward is not to play with increasingly complex dynamics but rather to seek the missing components. Once this is done then the dynamic model is relatively straightforward and indeed in the case of M0, M3 and M4 a striking feature is just how simple the resulting dynamics are.

As we move from narrow money M0 to broad money M4 it is not surprising that the types of extra effects we need to include change. At the narrow money end we need to take account of financial innovation and the move into interest-bearing M1 accounts. At the broad money end where we are dealing with more of an asset than a medium of exchange we need to allow for broad wealth effects.

APPENDIX: THE JOHANSEN PROCEDURE

Johansen (1988) sets his analysis within the following framework. Begin by defining a general polynomial distributed lag model of a vector of variables X as

$$X_t = \pi_1 X_{t-1} + \dots + \pi_k X_{t-k} + \epsilon_t \quad t=1, \dots, T \quad (A1)$$

where X_t is a vector of N variables of interest; π_i are $N \times N$ coefficient matrices, and ϵ_t is an independently identically distributed N dimensional vector with zero mean and variance matrix Ω . Within this framework the long run, or cointegrating matrix is given by

$$I - \pi_1 - \pi_2 \dots - \pi_k = \pi \quad (A2)$$

Where I is the identity matrix.

π will therefore be an $N \times N$ matrix. The number, r , of distinct cointegrating vectors which exists between the variables of X , will be given by the rank of π . In general, if X consists of variables which must be differenced once in order to be stationary (integrated of order one or $I(1)$) then, at most, r must be equal to $N-1$, so that $r \leq N-1$. Now we define two matrices α , β both of which are $N \times r$ such that

$$\pi = \alpha \beta'$$

and so the rows of β form the r distinct cointegrating vectors.

Johansen then demonstrates the following Theorem.

Theorem: The maximum likelihood estimate of the space spanned by β is the space spanned by the r canonical variates corresponding to the r largest squared canonical correlations between the residuals of X_{t-k} and ΔX_t corrected for the effect of the lagged differences of the X process. The likelihood ratio test statistic for the hypothesis that there are at most r cointegrating vectors is

$$-2 \ln Q = -T \sum_{i=r+1}^N \ln(1 - \hat{\lambda}_i) \quad (A3)$$

where $\hat{\lambda}_{r+1} \dots \hat{\lambda}_N$ are the $N - r$ smallest squared canonical correlations. Johansen then goes on to demonstrate the properties of the maximum likelihood estimates and, more importantly, he shows that the likelihood ratio test has an asymptotic distribution which is a function of an $N - r$ dimensional Brownian motion which is independent of any nuisance parameters. This means that a set of critical values can be tabulated which will be correct for all models. He demonstrates that the space spanned by β is consistently estimated by the space spanned by $\hat{\beta}$.

In order to implement this Theorem we begin by reparameterising (A1) into the following error correction model.

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Gamma_k X_{t-k} + \epsilon_t \quad (A4)$$

where

$$\Gamma_i = -I + \pi_1 + \dots + \pi_i; \quad i=1 \dots k$$

The equilibrium matrix π is now clearly identified as $-\Gamma_k$.

Johansen's suggested procedure begins by regressing ΔX_t on the lagged differences of ΔX_t and defining a set of residuals R_{ot} , then regressing X_{t-k} on the lagged differences and defining R_{kt} . The likelihood function, in terms of α , β and Ω is then proportional to

$$L(\alpha, \beta, \Omega) = |\Omega|^{-T/2} \text{EXP} \left[-\frac{1}{2} \sum_{t=1}^T (R_{ot} + \alpha \beta' R_{kt})' \right] \quad (A5)$$

$$\Omega^{-1} (R_{ot} + \alpha \beta' R_{kt})]$$

If β were fixed we could maximise over α and Ω by a regression of R_{0t} on $-\beta'R_{kt}$ which gives

$$\hat{\alpha}(\beta) = -S_{0k} \beta (\beta'S_{kk}\beta)^{-1} \quad (A6)$$

and

$$\hat{\Omega}(\beta) = S_{00} - S_{0k} \beta (\beta'S_{kk}\beta)^{-1} \beta'S_{k0} \quad (A7)$$

where

$$S_{ij} = T^{-1} \sum_{t=1}^T R_{it} R'_{jt} \quad i, j = 0, k$$

and so maximising the likelihood function may be reduced to minimising

$$|S_{00} - S_{0k} \beta (\beta'S_{kk}\beta)^{-1} \beta'S_{k0}| \quad (A8)$$

It may be shown that (A8) will be minimised when

$$|\beta'S_{kk}\beta - \beta'S_{k0} S_{00}^{-1} S_{0k}\beta| / |\beta'S_{kk}\beta| \quad (A9)$$

attains a minimum with respect to β .

We now define a diagonal matrix D which consists of the ordered eigenvalues

$\lambda_1 > \dots > \lambda_N$ of $S_{k0} S_{00}^{-1} S_{0k}$ with respect to S_{kk} . That is λ_i satisfies

$$|\lambda S_{kk} - S_{k0} S_{00}^{-1} S_{0k}| = 0 \quad (A10)$$

Define E to be the corresponding matrix of eigenvectors so that

$$S_{kk} E D = S_{k0} S_{00}^{-1} S_{0k} E \quad (A11)$$

where we normalize E such that $E'S_{kk}E = I$

The maximum likelihood estimator of β is now given by the first r rows of E , that is, the first r eigenvectors of $S_{ko}S_{oo}^{-1}S_{ok}$ with respect to S_{kk} . These are the canonical variates and the corresponding eigenvalues are the squared canonical correlations of R_k with respect to R_o . These eigenvalues may then be used in the test proposed in (3) to test either for the existence of a cointegrating vector $r = 1$ or the number of cointegrating vectors $N > r > 1$.

Johansen (1988) calculates the critical values for the likelihood ratio test for the cases where $m \leq 5$, where $m = P - r$, P is the number of variables in the set under consideration and r is the maximum number of cointegrating vectors being tested for. This limits us to testing only within sets of five or less variables. Johansen provides an approximation formulae for the critical values and we have used this formulae to compute the critical values for all cases up to $m=10$. For $m \leq 5$ we may compare the approximation with the numerically derived numbers to gauge the accuracy of the approximation.

Critical Values for the Likelihood Ratio Test

(95% critical value)

m	Johanson Critical Values	Approximation Values
1	4.2	3.25
2	12.0	12.1
3	23.8	23.6
4	38.6	38.4
5	57.2	56.6
6		78.1
7		103.0
8		131.1
9		162.8
10		197.7

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