Bank of England

Discussion Paper No 19

Unobserved components, signal extraction and relationships between macroeconomic time series

by

T C Mills

December 1981

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The object of this series is to give a wider circulation to research work being undertaken in the Bank and to invite comment upon it; and any comments should be sent to the authors at the address given below. The views expressed are theirs, and not necessarily those of the Bank of England.

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Introduction[1]

1 The idea that an observed time series can be decomposed into two or more unobserved components is one that has a long history in the analysis of economic time series; two early contributions to the literature being Babbage (1856) and Jevons (1866). The traditional decomposition has usually been in terms of trend, cycle, seasonal and irregular components and this classification of economic fluctuations formed the basis for the early analysis of the business cycle undertaken by Persons (1919), which may be considered the forerunner of the subsequent work on business cycles undertaken by the National Bureau of Economic Research - see, for example, Burns and Mitchell (1947). More recently, the division of a time series into unobserved components has played an important role in the analysis of seasonal adjustment [see Nerlove, et al. (1979, Chapter 8) for a convenient survey] and, more generally, Sims (1974) has shown that if different components of two time series are linked by different distributed lag relationships, then the estimation of a distributed lag model between the two observed series will lead to biased coefficients and misleading inferences.

2 Economic theorists have also become increasingly interested in unobserved component models, particularly in the context of modelling long-run or equilibrium behaviour. Friedman (1957), in his study of the consumption function, introduced the concepts of permanent and transitory income and these may be regarded as two unobserved components of actual income. In his analysis of the natural rate of unemployment hypothesis, Friedman (1968) also considers the effect of anticipated and unanticipated inflation on labour market behaviour and such a decomposition of an observed series has subsequently played an important role in the rational expectations approach to the theory of economic policy - see, for example, the models developed by Barro (1976) and Sargent and Wallace (1976). The interpretation of an unobserved component of an economic time series as an expectational variable has also been considered by Nerlove et al. (1979, Chapter 13),

[1] This paper was prepared for and presented at the Money Study Group Conference, Brasenose College, Oxford, in September 1981.

who assume that agents react not to observed variables but, rather, to estimates of the current values of unobserved components, terming such estimates 'quasi-rational' expectations.

3 As just stated, being unobserved by economic agents, these individual components must be estimated before the decompositions discusssed above can become fully operational and, in an attempt to give empirical content to his decomposition of observed income, Friedman (1957) assumed that permanent income was generated by an exponentially weighted distributed lag of income with the 'smoothing parameter' arbitrarily set at 0.33. In a series of papers, Barro has constructed an empirical proxy for anticipated money growth as the predicted value obtained by regressing actual money growth on lagged money growth and various other regressors, determining unanticipated money growth by residual. [See, for example, Barro (1977) and for an application to the United Kingdom, Attfield et al. (1981).] A further approach to modelling the permanent or anticipated components of economic time series has been followed by Lucas (1980), who uses two-sided symmetrical moving averages of observed money growth, inflation and interest rate-series to investigate two important propositions of the quantity theory. The use of two-sided filters has important implications , for it implies that agents not only use past data in forming their expectations, but, as they care primarily about the future, incorporate forward looking behaviour in their optimal estimates of the current values of the components.

4 This latter approach is explicitly derived using a well-developed statistical theory for the estimation of unobserved components, classic references being Wiener (1949) and Whittle (1963) with Nerlove et al.(1979) providing a comprehensive exposition, and is similar in spirit to the methodology to be presented in this paper. However, as is discussed in detail later, the approach of Lucas (1980) may be criticised for being conditional upon the adequacy of certain simple stochastic processes that are assumed to generate the components. This 'adhocery' may be avoided by utilising the methodology and algorithm of Burman (1980), which enables estimates of trend, seasonal and irregular components to be obtained for observed time series that permit an autoregressive - integrated - moving average (ARIMA) representation.

5 Section 2 of the paper therefore presents an exposition of unobserved component models, discusses the problems associated with dynamically modelling observed time series, and summarises Burman's technique and algorithm for estimating the individual components. Section 3 utilises this technique to estimate the trend (or permanent) components of three familiar macroeconomic time series and attempts to bring out the salient features of the methodology. Sections 4 and 5 present two empirical applications of the unobserved component model approach; the relationship between money and prices and the construction of permanent income and expected inflation variables for subsequent insertion in an equation modelling bank lending. Finally, Section 6 presents some tentative conclusions and suggestions for further research.

Unobserved component models and signal extraction

6 Suppose that y_t and x_t are two observed time series, each of which can be decomposed into two components, viz:

$$y_t = T_{yt} + S_{yt} \tag{1}$$

$$x_t = T_{xt} + S_{xt} \tag{2}$$

all sequences being assumed to be jointly covariance stationary. It is further assumed that the T and S components, which may be thought of as, for example, trend-seasonal or permanent-transitory decompositions, are related by the distributed lag models:

$$T_{yt} = C(B)T_{xt} + u_t \tag{3}$$

$$S_{yt} = D(B)S_{xt} + v_t \tag{4}$$

where u_t and v_t are white noise sequences uncorrelated with each other or with T_x and S_x, and C(B) and D(B) are one-sided, possibly infinite, polynomials in the lag operator B. The empirical lag distribution between the observed y and x is defined as γ, where:

$$y_t = \gamma(B)x_t + w_t \qquad E(w_t x_s) = 0 \text{ for all } t, s \tag{5}$$

7 If $g_{ab}(z)$ is the cross covariance generating function between the jointly covariance stationary series a_t and b_t then from equation 2:

$$g_{xx}(z) = g_T(z) + g_S(z) \tag{6}$$

and from equations 1, 3 and 4:

$$g_{yx}(z) = C(z)\, g_T(z) + D(z)g_S(z). \tag{7}$$

Using a result due to Whittle (1963) we obtain:

$$\gamma(z) = \frac{g_{yx}(z)}{g_{xx}(z)} = \frac{C(z)g_T(z)}{g_{xx}(z)} + \frac{D(z)g_S(z)}{g_{xx}(z)} \quad (8)$$

$$= C(z)[1-h(z)] + D(z)\ h(z)$$

where $h(z) = g_S(z)/g_{xx}(z)$. So in the frequency domain it is seen that the Fourier transform of γ is simply a weighted average of the transforms of C and D with weights varying with frequency. Thus, if S_y and S_x are indeed seasonal components, then it would seem reasonable to assume that h(z) is close to unity near the seasonal frequencies and close to zero (though positive) away from them. Hence $\gamma(z)$ will tend to be close to D(z) near seasonal frequencies and closer to C(z) elsewhere. However, the effects in the time domain will be quite different, for Sims (1974) shows that lag distributions fitted between unadjusted seasonal series will often show spurious seasonal fluctuations in the coefficients as well as being two-sided. This latter phenomenon results whenever different lag distributions link different unobserved components, irrespective of whether a seasonal/non-seasonal classification has been made. This general result is of obvious importance, for it immediately flags that something is wrong, but unfortunately two-sided lag distributions can result from a variety of causes, for example, simultaneity or time aggregation [Sims (1972)], so that the precise nature of the misspecification may be difficult to ascertain.

8 Of course, unconstrained two-sided lag distributions are not usually estimated and typically γ will be constrained to be one-sided with parametric forms being adopted to either truncate or smooth the pattern of the lag coefficients. Sims (1974) also points out that such restrictions on γ are likely to both conceal and aggravate the bias caused by differences in the C and D polynomials and therefore such biases could be potentially important in applied macroeconomic research.

9 Two special cases are of interest. If $C \equiv D$, then $\gamma \equiv C$, and any decomposition of the observed series is irrelevant, whereas if $D \equiv 0$, so that S_y and S_x are independent:

$$\gamma(z) = C(z)[1-h(z)] = \frac{C(z)g_T(z)}{g_{xx}(z)}$$

and Sims (1974) shows that the correct lag distribution between T_y and T_x, C(B), will be obtained by investigating the relationship between y and T_x.

10 This discussion therefore emphasises that, if there are different distributed lag relationships linking pairs of components of two time series, and if we are primarily interested in the relationship between one particular pair, for example, the trend or permanent components, then estimating an unrestricted two-sided distributed lag model between the observed series will produce biased lag coefficients and hence misleading inferences. Moreover, the possible presence of non-zero future coefficients may result in the incorrect conclusion that simultaneity or time aggregation problems are present. If, as is more likely in practice, smoothness constraints are placed on the distributed lag coefficients, the problem of bias will still be present but could be obscured. Thus it may be very important to investigate relationships not between observed economic time series but between certain pairs of components, bearing in mind that the behaviour of such components vis-à-vis the observed series will also often be of considerable interest, particularly as many theories suggest that variables should appear in permanent or expected form.

11 Of course, such components are, by their very nature, unobserved, and hence must be estimated if analysis using them is to be attempted. The problem of estimation may best be illustrated by considering the classical approach to predicting (or extracting) T_x, say. This assumes that T_x and S_x are generated by the following processes.

$$T_{xt} = \alpha T_{xt-1} + \varepsilon_t, |\alpha| < 1 \tag{9}$$

$$S_{xt} = \eta_t \tag{10}$$

where ε_t and η_t are mutually uncorrelated white noise sequences with variances σ_ε^2 and σ_η^2 respectively. Thus the 'permanent' component is assumed to follow a first order autoregressive process whereas the 'transitory' component is white noise. The problem is then to estimate T_{xt} given the record of observations on $x_t = T_{xt} + S_{xt}$, and a standard criterion is the minimisation of $E[T_{xt} - \hat{T}_{xt}]^2$ where:

$$\hat{T}_{xt} = \psi(B)x_t = \sum_{j=-\infty}^{\infty} \psi_j x_{t-j}.$$

Following Whittle (1963, Chapter 5), the least squares estimator $\hat{T}_{xt}$ based on the entire history of the observed x_t series is given by:

$$\hat{T}_{xt} = \left(\frac{\alpha - \beta}{\alpha}\right) \sum_{j=-\infty}^{\infty} \beta^{|j|} x_{t-j} \tag{11}$$

where: $\beta = \dfrac{(1 + \lambda + \alpha^2) - \sqrt{(1 + \lambda + \alpha^2)^2 - 4\alpha^2}}{2}$; and

$\lambda = \sigma^2_\varepsilon/\sigma^2_\eta$.

12 For values of the 'signal to noise' variance ratio λ near zero, β is approximately equal to α and $\hat{T}_{xt}$ will then be given by a very long moving average of future and past values of x. On the other hand, if λ is large, β will be close to zero, in which case $\hat{T}_{xt}$ will be almost equal to the most recently observed value x_t. This behaviour seems quite reasonable since, when λ is small, most of the period-to-period variation in the observed series is, on average, due to variation in the transitory component, and vice versa for large values of λ.

13 If α equals unity (that is, T_x follows a random walk), then the optimal estimator is given by:

$$\hat{T}_{xt} = (1 - \beta^*) \sum_{j=-\infty}^{\infty} \beta^{*|j|} x_{t-j} \tag{12}$$

where: $\beta^* = \dfrac{2 + \lambda - \sqrt{\lambda^2 + 4\lambda}}{2}$

and this is equivalent to the models considered by Lucas (1980).

14 The above estimators will only be optimal, in the minimum mean square error sense, if the unobserved components T_x and S_x are actually generated by the processes assumed in equations 9 and 10. To check the adequacy of these assumptions, the implied model for x_t may be tested against the data. Combining equations 2, 9 and 10 yields:

$$(1 - \alpha B)x_t = \varepsilon_t + \eta_t - \alpha\eta_{t-1}$$
$$= (1 - \beta B)a_t \text{ say.} \qquad (13)$$

15 Hence, if the classical signal extraction model characterised by equations 9 and 10 is appropriate, then the observed series should be adequately modelled by an ARIMA (1,0,1) process. As all series are assumed to be covariance stationary, various transformations and differences will typically have been performed on the actually observed series X_t. If, say, x_t is obtained after differencing $f(X_t)$ d times [that is, $x_t = (1-B)^d f(X_t)$], then $f(X_t)$ must be adequately modelled by an ARIMA (1, d, 1) process for the classical signal extraction model to be appropriate. Restricting α to equal unity therefore requires $f(X_t)$ to be generated by an ARIMA (0, d + 1, 1) process and Granger and Newbold (1977, page 203) suggest that this model may be a useful representation of many non-seasonal economic series. In fact, if $\psi(B)$ is restricted to be one-sided, the optimal extraction equation 12 corresponds to the use of simple exponential smoothing [cf Friedman (1957)]. However, if S_x corresponds to a seasonal, rather than a transitory, component, then the assumpton that it is white noise is clearly unrealistic, and hence the derived ARIMA (1, d, 1) model for the observed series will be inadequate. Nerlove _et al_. (1979) consider unobserved component models having components generated by particular ARIMA forms, implying that the observed series follows a high order but highly restricted ARIMA model. However, a modelling approach that removes this arbitrariness of _a priori_ specification of the ARIMA forms generating the components is to identify and estimate the appropriate model for the observed series and, conditional upon this model, extract optimal estimates of the underlying components. Such an approach has been proposed by Burman (1980), whose algorithm allows trend, seasonal and irregular components to be estimated for any series that can be

adequately characterised by an ARIMA seasonal model. Only a brief exposition of the technique, termed Minimal Signal Extraction (MSX), is given here - full details may be found in Burman (1980).

16 Thus, consider the general ARIMA (p, d, q)(P, D, Q)$_s$ seasonal model for the observed series x_t:

$$x_t = f(B)a_t = \frac{\theta(B)}{(1-B)^d(1-B^s)^D \phi(B)\Phi(B^s)} a_t \tag{14}$$

where $\theta(B)$ is of degree $q^* = q + Qs$ and $\phi(B)$ and $\Phi(B^s)$ are of degrees p and P respectively. By noting that $(1 - B^s)^D$ and $\Phi(B^s)$ can, in principle, be factored into seasonal and non-seasonal parts, for example $(1 - B^s)^D = (1 - B)^D (1 + B + .. + B^{s-1})^D$, equation 14 can be rewritten with its denominator rearranged into components having no common factor:

$$x_t = \frac{\theta(B)}{\psi_m(B)\ \psi_s(B)} a_t \tag{15}$$

where $\psi_m(B)$ and $\psi_s(B)$ may be regarded as trend and seasonal components respectively.

17 If it is also assumed that x_t can be decomposed into trend (T), seasonal (S) and irregular (R) components such that:

$$x_t = T_t + S_t + R_t \tag{16}$$

with $T_t = f_T(B)b_t$; $S_t = f_S(B)c_t$; $R_t = f_R(B)d_t$, b_t, c_t and d_t being independent white noises with variances σ_b^2, σ_c^2 and σ_d^2, then Whittle (1963) showed that the optimal linear estimator of T_t is given by:

$$\hat{T}_t = \frac{\sigma_b^2 f_T(B) f_T(F)}{\sigma_a^2\ f(B)\ f(F)} x_t \tag{17}$$

where $F = B^{-1}$, with similar expressions being obtained for $\hat{S}_t$ and $\hat{R}_t$. Burman (1980) shows that equation 17 can be rewritten as:

$$\hat{T}_t = \frac{C_T(B,F)}{\theta(B)\,\theta(F)} x_t \qquad (18)$$

where $C_T(B,F)$ is a symmetrical polynomial in B and F of degree $p^* = p + d + sD + sP$.

18 The MSX algorithm estimates $\hat{T}_t$, $\hat{S}_t$ and $\hat{R}_t$ by partitioning the spectrum of x_t into trend, seasonal and irregular components and then generating the filters required. Although the original formulation of this 'signal extraction' technique was developed for a doubly infinite x series and filter the expected values of the signal series can be obtained by extending the observed series with backcasts and forecasts. Only a limited number of these predictions are, in fact, required in practice because of the particular algorithm employed.

19 This algorithm, primarily designed for seasonal adjustment purposes, also has a procedure for modifying extreme residuals after preliminary estimates of the components have been made and has a further refinement to deal with bias in multiplicative models, ie models in which $x_t = \log(X_t)$.

20 Having thus obtained estimates of the unobserved trend, seasonal and irregular components, these can then be used for a variety of purposes. The earlier analysis has emphasised distributed lag relationships between components and therefore techniques such as transfer function methodology can be employed to model such interactions. Alternatively, the estimated trend, if it is classified as the 'permanent' component, may be immediately compared with the observed series, the difference between the two being the transitory component. The trend component could also be regarded as an estimate of the 'expected' value of the variable, on the Nerlove et al. (1979) view that economic agents react not to observed values but rather to estimates of the current values of the unobserved components. In this case the estimated component, being constructed using future as well as past values of the observed series, will incorporate forward looking behaviour, although both types of information enter symmetrically. On either interpretation, the estimated trend component can be used to empirically model such variables.

Some examples of the estimation of unobserved components

21 This section discusses the estimation of certain unobserved components of three quarterly economic time series using the MSX methodology and algorithm developed by Burman (1980). The series investigated are seasonally unadjusted £M3, the retail price index and the local authority three-month rate.

22 Conventional identification and estimation techniques led to the following ARIMA(1,1,0)$(0,1,1)_4$ model for $x \equiv \ln(£M3)$, the sample period being 1963 Q1 to 1980 Q4.

$$(1 - B)\ (1 - B^4)x_t = \frac{\underset{(.07)}{[1 - .92\ B^4]}}{\underset{(.10)}{[1 - .54\ B]}} a_t \qquad (19)$$

$\hat{\sigma} = .0159 \qquad Q_1(10) = 10.1 \qquad T = 67.$

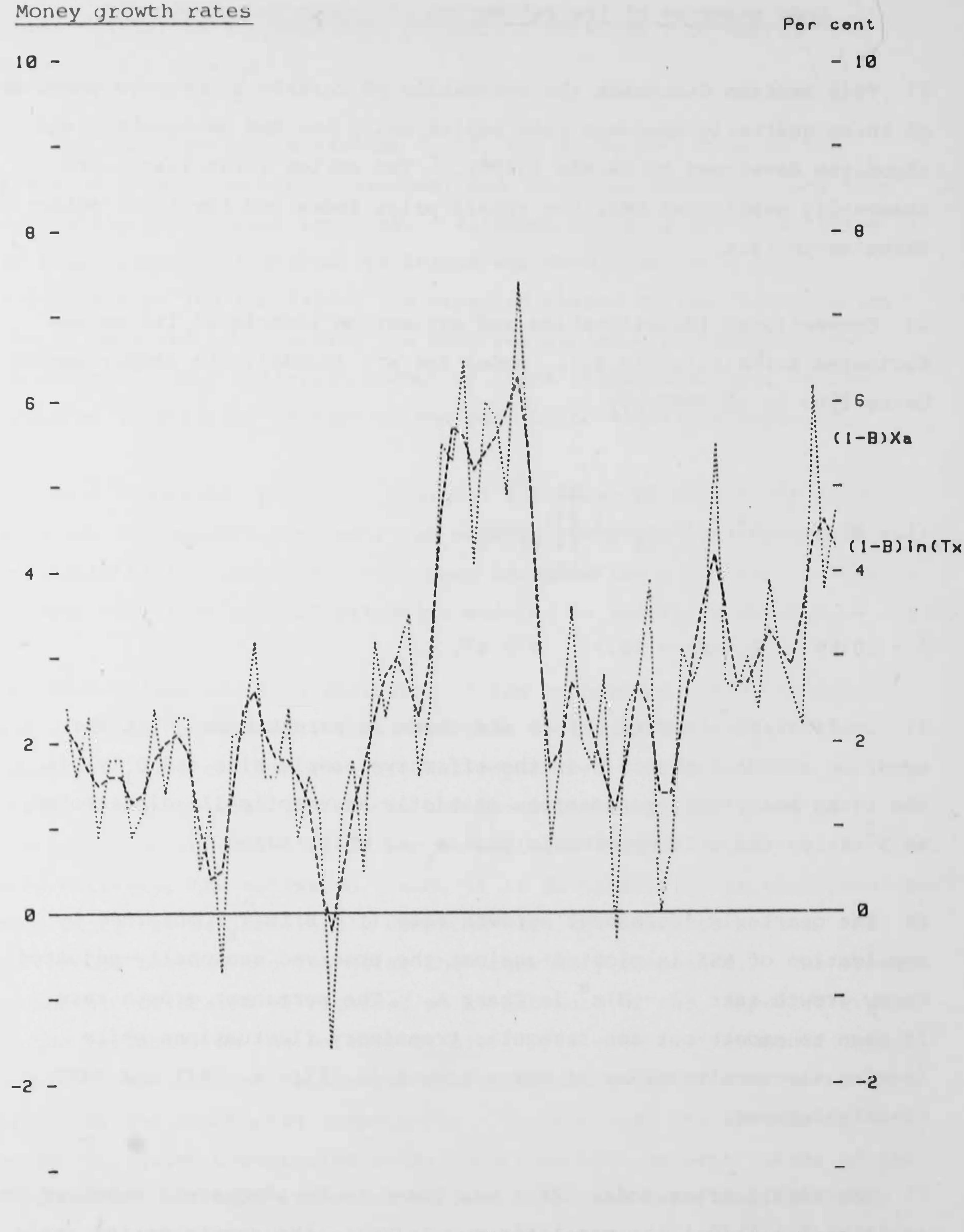
23 Coefficient standard errors are shown in parentheses, $\hat{\sigma}$ is the equation standard error, T is the effective sample size and $Q_1(v)$ is the Ljung-Box (1978) portmanteau statistic asymptotically distributed as $\chi^2(v)$ on the null hypothesis that a_t is white noise.

24 The quarterly 'permanent' growth rate $(1 - B)\ln(\hat{T}_x)$ obtained by application of MSX is plotted against the observed seasonally-adjusted money growth rate $(1 - B)x^a$ in Chart A. The permanent growth rate is seen to smooth out the irregular transitory fluctuations while leaving the accelerations of money growth in 1972-73, 1977 and 1980 clearly defined.

25 The retail price index (RPI) was found to be adequately modelled by an ARIMA(1,1,1)$(0,1,1)_4$ model for $y \equiv \ln(RPI)$, the sample period again being 1963 Q1 to 1980 Q4.

Chart A

Money growth rates



$$(1 - B)(1 - B^4)y_t = \frac{\underset{(.15)}{[1 - .30\ B]}\ \underset{(.08)}{[1 - .96\ B^4]}}{\underset{(.08)}{[1 - .85\ B]}} a_t \tag{20}$$

$\hat{\sigma} = .0116 \quad Q_1(10) = 4.6 \quad T = 67$

26 Permanent or 'anticipated' inflation, $(1 - B)\ln(\hat{T}_y)$ is plotted against observed inflation $(1 - B)y$ in Chart B. Again the irregular transitory fluctuations are smoothed out and the inflation peaks in 1975 and 1980 are clearly emphasised. Furthermore, the (annualised) permanent inflation series was found to be almost identical to the corresponding permanent inflation series obtained using monthly data, thus providing evidence of the robustness of the MSX algorithm.

27 Both series are seasonal and the models generating them are therefore more complicated than the classical signal extraction models discussed in the previous section. However, the local authority rate (RLA) is non-seasonal, and was found to be adequately modelled by an ARIMA(0,1,1) process for the longer sample period 1957 Q1 to 1980 Q2:

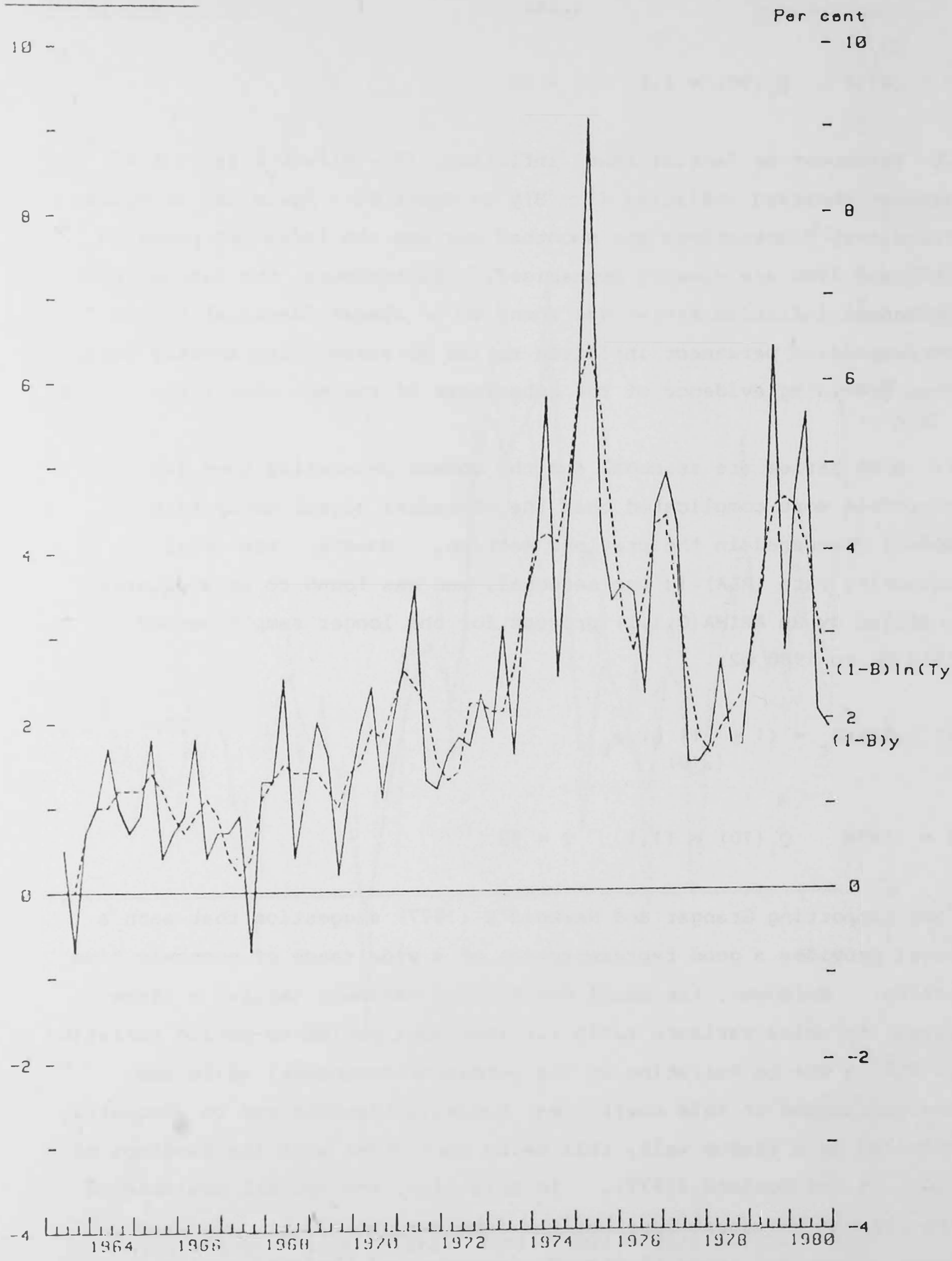
$$(1 - B)RLA_t = (1 + \underset{(.10)}{.11}\ B)\ a_t \tag{21}$$

$\hat{\sigma} = .1409 \quad Q_1(10) = 11.1 \quad T = 93$

thus supporting Granger and Newbold's (1977) suggestion that such a model provides a good representation of a wide range of economic time series. Moreover, the small coefficient estimate implies a large signal to noise variance ratio (ie that most period-to-period variation in RLA is due to variation in its permanent component) while the insignificance of this coefficient suggests that RLA can be adequately modelled by a random walk, this being consistent with the findings of Goodhart and Gowland (1977). In this case, the optimal estimate of the current permanent level of the interest rate is the observed level itself and hence the trend component and observed series should be almost identical on applying MSX to equation 21 and indeed this was found to be the case.

Chart B

Inflation rates



28 Thus, in all three examples, the extracted trend components are found to be sensible and to bear ready economic interpretations.

The relationship between money and prices in the United Kingdom

29 This section considers in some detail the bivariate relationship between two of the time series investigated in the previous section, £M3 and the RPI, for the period 1963 Q1 to 1980 Q4.

30 Assuming that the relationship between the two series is unidirectional, running from £M3 to RPI, the following transfer function was developed:

$$\underset{(.08)}{(1 - .57\,B)}(1 - B)y_t = \underset{(.003)}{.004} + (\underset{(.05)}{-.10} + \underset{(.05)}{.37}\,B^6)(1 - B)x_t + a_t \qquad (22)$$

$\hat{\sigma} = .0106 \qquad Q_1(12) = 6.4 \qquad T = 63$

31 From this model it can be seen that there is a negative contemporaneous relationship and a six-quarter delay before a positive response from RPI to a change in £M3 occurs. The long-run response is 0.62, significantly smaller than the quantity theory hypothesis of unity. However, this contemporaneous correlation plus the presence of significant correlations at three and five-quarter leads provides evidence of feedback from RPI to £M3. While such feedback is certainly plausible, the analysis of Section 2 suggests that these correlations could be flagging the possibility of bias due to different distributed lags linking the underlying components of the two series, in which case the long-run response may be substantially biased. This misspecification appears equally plausible for it is unlikely that the permanent and transitory components of money and prices would be related by identical lag distributions. Moreover, many economists would feel that it is the relationship between the permanent components of the two series that is of fundamental importance, thus arguing that the investigation of the relationship between £M3 and RPI should be carried out in terms of the trend components T_x and T_y.

32 Modelling the extracted trend components $\hat{T}_x$ and $\hat{T}_y$ yields the following transfer function:

$$(1 - \underset{(.10)}{2.03}\ B + \underset{(.17)}{1.78}\ B^2 - \underset{(.09)}{.67}\ B^3)(1 - B)\ \ln(\hat{T}_y)_t = \underset{(.02)}{.08}\ B^5(1 - B)\ \ln(\hat{T}_x)_t$$

$$+ \frac{a_t}{(1 + \underset{(.14)}{.38}\ B)} \qquad (23)$$

$\hat{\sigma} = .0030 \qquad Q_1(11) = 13.8 \qquad T = 62$

33 No evidence of model inadequacy was found for this model, thus suggesting that the deficiencies of equation 22 were due to 'component' bias rather than feedback effects. Following Jenkins (1979), equation 23 can be rewritten as:

$$(1 - 2.03\ B + 1.78\ B^2 - .67\ B^3)\ \ln(\hat{T}_y)_t = .08\ B^5\ \ln(\hat{T}_x)_t + N_t \qquad (24)$$

$$(1 + .38B)(1 - B)N_t = a_t \qquad (25)$$

from which it can now be seen that there is a five-quarter delay in the reaction of permanent prices to a change in permanent money, the response peaking at seven quarters after which the response path is cyclically damped with a long-run response of 1.09 and a mean lag of eleven quarters. This dynamic response is therefore consistent with quantity theory implications, unlike that of the relationship between the observed series, which has the additional disadvantage that it might be erroneously respecified as a feedback model rather than a model having different distributed lags linking different components. It may also be noted that the relationship between the permanent components is somewhat stronger than the relationship between the observed series, the use of $\hat{T}_x$ to explain $\hat{T}_y$ reduces the unexplained variation in $\hat{T}_y$ by ten per cent, whereas the use of £M3 to explain RPI reduces the unexplained variation in RPI by only five per cent.

34 It could also be hypothesised that the transitory components of £M3 and RPI are independent, in which case the appropriate relationship to be modelled would be that between RPI and T_x. Cross correlating the approximately white noise series $(1 - B^4)\ \hat{S}_{xt}$ and $(1 - B^4)\ \hat{S}_{yt}$,

where $\hat{S}_{xt} = £M3_t - \hat{T}_{xt}$ and $\hat{S}_{yt} = RPI_t - \hat{T}_{yt}$, produced the significant Haugh (1976) test statistic of $Q_2(24) = 96.6$, this being asymptotically distributed as χ^2 on the null hypothesis of independence. Hence this confirms that the appropriate analysis of the £M3 and RPI relationship is between the 'trend' or 'permanent' components of the two series.

35 This application of unobserved component models therefore lends support to the view of Lucas (1980) that the quantity theory is essentially a characteristic of 'long-run average' behaviour and that such behaviour can be empirically captured by analysing the relationship between the permanent components of money and prices.

Modelling bank lending

36 In the Bank of England (1979) model of the UK economy, bank lending to persons is assumed to be a function of nominal transitory income and the real cost of such lending. Within the model, nominal transitory income is defined as the difference between real personal disposable income (net of current grants to persons from the public sector), RY, and real permanent non-grant personal income, YDPM, inflated by the price deflator for total consumption, PC. The permanent income series YDPM is generated by a twelve-quarter approximation to a simple exponential smoothing algorithm with 'smoothing constant' set at 0.3, that is:

$$YDPM_t \simeq 0.3\ RY_t + 0.7\ YDPM_{t-1} \qquad (26)$$

As Granger and Newbold (1977) show, this is equivalent to assuming that RY is generated by the ARIMA(0,1,1) model:

$$(1 - B)RY_t = (1 - .70\ B)a_t \qquad (27)$$

Estimation of such a model yields:

$$(1 - B)RY_t = (1 - \underset{(.10)}{.02}\ B)a_t \qquad (28)$$

$\hat{\sigma} = 279 \qquad Q_1(11) = 17.6$

The parameter estimate is clearly insignificantly different from zero and indeed RY is adequately characterised as a random walk:

$$(1 - B)RY_t = a_t \qquad (29)$$

$\hat{\sigma} = 278 \qquad Q_1(12) = 17.8$

Since this model means that the smoothing constant in equation 26 is zero, this implies that the optimal estimate of real permanent income $YDPM_t$ is current observed real income RY_t, hence transitory real income, and therefore in this context transitory nominal income, is always zero.

37 A more satisfactory method of constructing transitory nominal income would be to extract the permanent component of nominal personal disposable income, NY, and hence obtain the transitory component by subtraction. Univariate analysis of this income series yielded the model:

$$(1 - B)(1 - B^4) \ln (NY_t) = (1 - \underset{(.09)}{.56} B^4) a_t \qquad (30)$$

$\hat{\sigma} = .0184 \qquad Q_1(10) = 10.1$

and the transitory income series TYB thus derived is plotted in Chart C along with the model based alternative TYA = PC(RY - YDPM). It is seen that for the early part of the data period, 1965-69, both series follow similar paths but from 1970 onwards the fluctuations in TYA become much greater than those in TYB, this being confirmed by the standard deviations of the two series. It is interesting to note that both series can be adequately modelled by MA(1) processes as befits variables having only transient characteristics.

38 The real interest rate variable, measuring the cost of bank lending, is constructed in the Bank model as the difference between the local authority three-month rate, RLA, and an inflation expectations variable PEXP, based mainly on past movements in prices but also including expectations of any change in the exchange rate. Given the theme of this paper, an alternative expectations series could be obtained by extracting the permanent component of the price series used in the construction of PEXP. The following univariate model for this series, the imputed wholesale price index of manufacturing output (net of tax), PIMN, was obtained:

$$(1 - B)(1 - B^4) \ln (PIMN)_t = \frac{[1 - \underset{(.08)}{.95} B^4]}{(1 - \underset{(.11)}{.50} B - \underset{(.12)}{.39} B^2)} a_t \qquad (31)$$

$\hat{\sigma} = .0096 \qquad Q_1(9) = 11.0$

Chart C

Transitory income

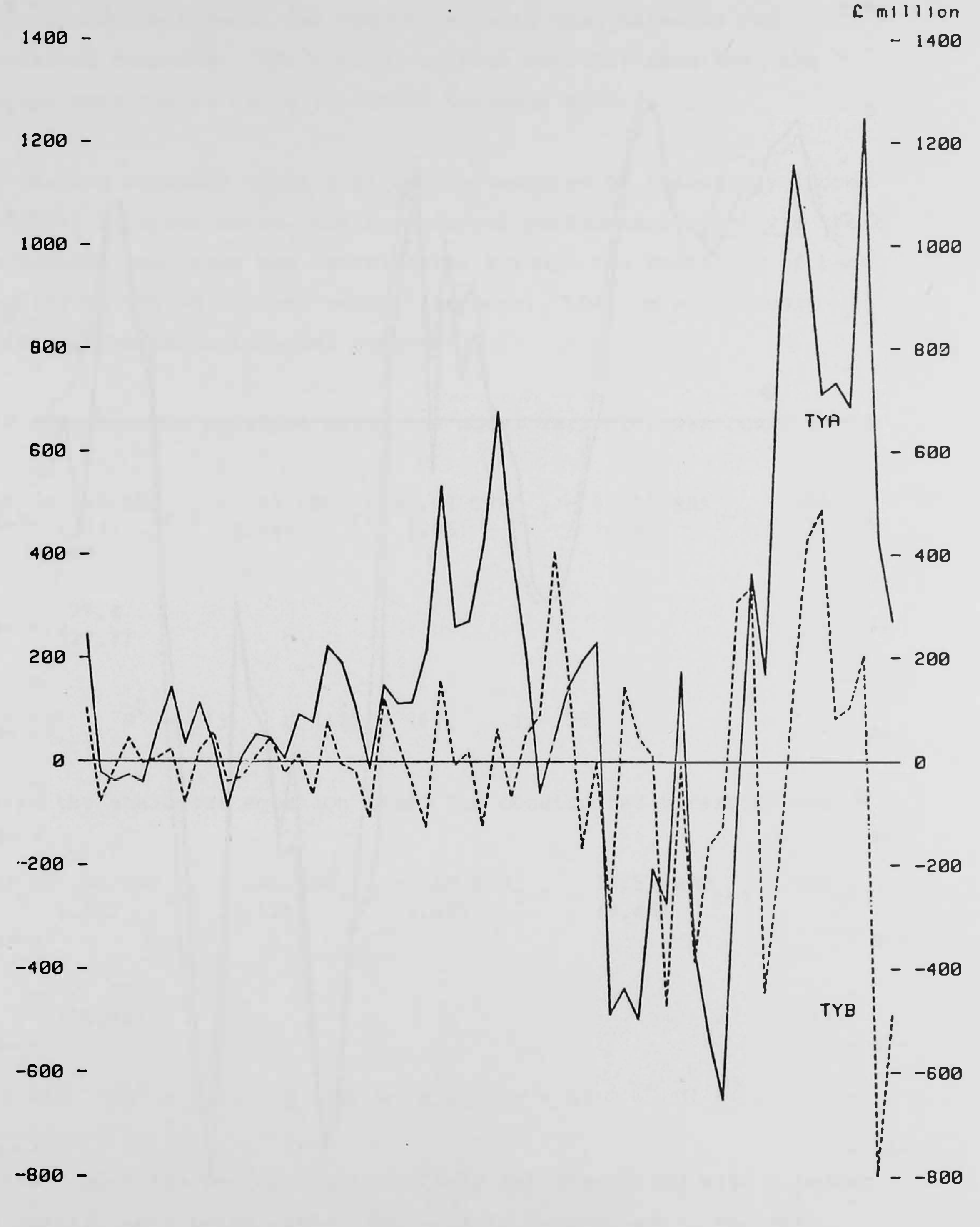
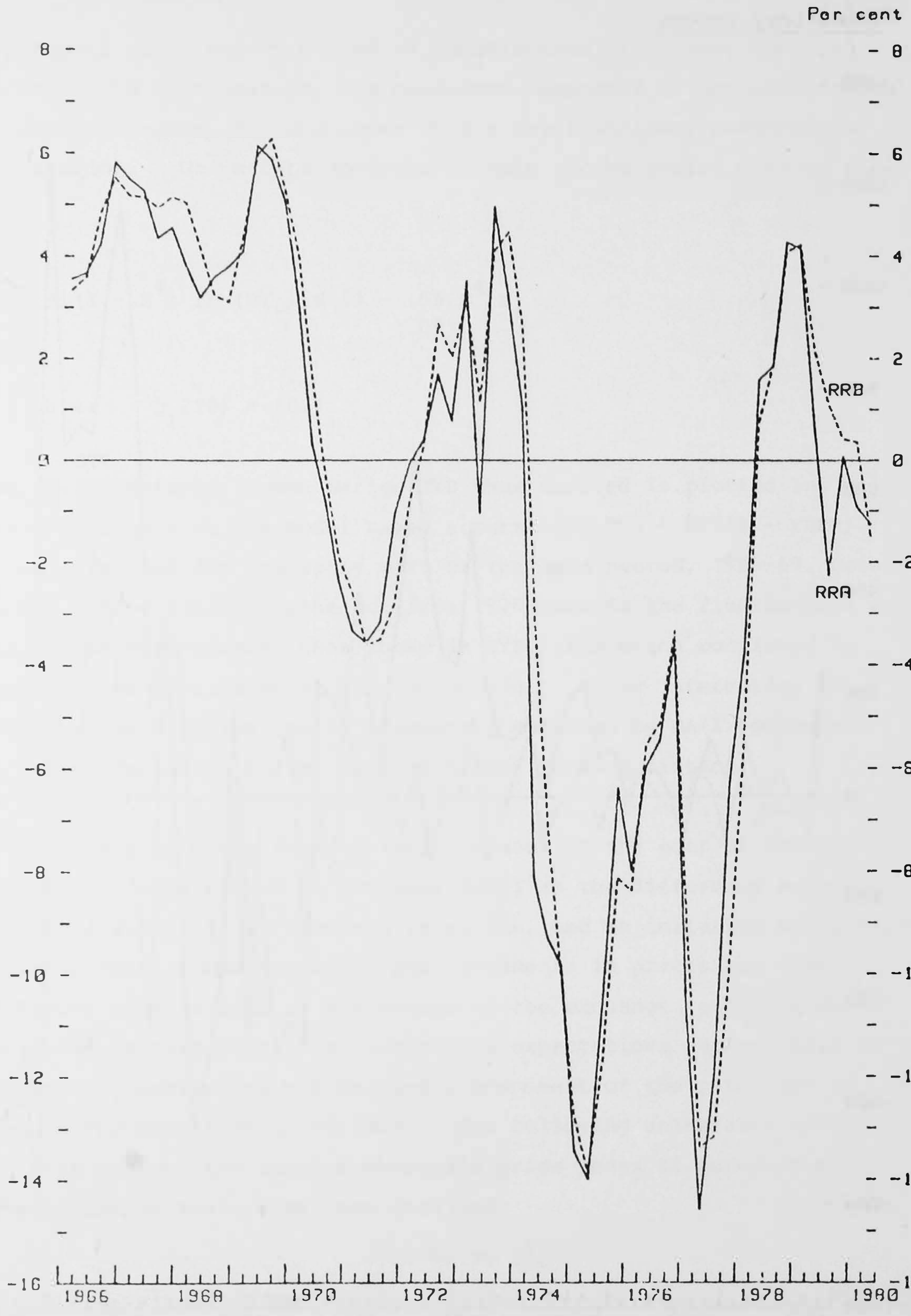


Chart D

Real interest rates



39 The extracted permanent component was then converted into an annual expected inflation series, ISIG, analogous to PEXP. The two real interest rate series constructed using these alternatives are shown in Chart D. There is seen to be a reasonably close correspondence between the two series with that based on the extracted component, RRB, being somewhat smoother than RRA, the series constructed using the model variable PEXP.

40 Having obtained these alternative measures of transitory income and real interest rates, their empirical performance vis-à-vis their Bank model analogues was investigated through the modelling of bank lending to the UK private sector (persons), LDJ; a notoriously difficult variable to model adequately.

41 A reasonable equation using the model variables was found to be:

$$LDJ_t = \underset{(.13)}{.53}\, LDJ_{t-1} + \underset{(.14)}{.43}\, LDJ_{t-2} + \underset{(.06)}{.01}\, TYA_{t-1} - \underset{(6.96)}{10.16}(RRA_{t-1} - RRA_{t-2}) - \underset{(21.7)}{29.4} \qquad (32)$$

$\hat{\sigma} = 137 \quad \bar{R}^2 = .73 \quad Q_1(12) = 16.2 \quad T = 59$

while the analogous equation using the constructed variables was:

$$LDJ_t = \underset{(.12)}{.48}\, LDJ_{t-1} + \underset{(.12)}{.46}\, LDJ_{t-2} - \underset{(.09)}{.09}\, TYB_{t-1} - \underset{(8.46)}{15.87}(RRB_{t-1} - RRB_{t-2}) - \underset{(20.96)}{27.01} \qquad (33)$$

$\hat{\sigma} = 138 \quad \bar{R}^2 = .72 \quad Q_1(12) = 14.9 \quad T = 63$

Neither equation performs particularly satisfactorily with a number of coefficients being rather imprecisely determined. For both equations changes in real interest rates have a negative impact effect on bank lending but a zero overall effect, with the impact effect being stronger for RRB, based on the extracted inflation expectations series. The coefficient of this latter variable is also significantly different from zero at a marginal level of less than .10.

42 Although both transitory income variables are insignificant, the 'optimally extracted' alternative is at least correctly signed, with the long-run response implying that a £100 million increase in transitory income will lead to a £157 million decrease in bank lending, although this response is rather slow, the mean lag being almost six years.

43 These results, although not entirely satisfactory, do suggest that the use of such optimally extracted components to proxy permanent or expected variables may be of potential importance in the modelling of various macroeconomic relationships. It may be argued, however, that the use of two-sided filters to construct trend components having permanent or expectational connotations requires economic agents to process information that they cannot possess. An alternative procedure would be to extract the current value of the trend component by successively truncating the observed series before applying the MSX algorithm recursively, thus utilising no future information other than optimal forecasts. Initial experimentation suggests that such a method leads to only minor changes in the values of the extracted component, but further research into this important area is continuing.

6

Conclusions

44 This paper has considered how the idea of unobserved components of economic time series may be of use in certain areas of macroeconomic modelling. The estimation of such components in a general, model-based framework has now become feasible using the methodology and algorithm of Burman (1980), although it should be emphasised that the results presented here are still tentative and exploratory.

45 It has been argued that the decompositions thus considered may be interpreted in a variety of ways that might be of interest to economists, particularly given the increasing role of expectations and permanent or anticipated variables in macroeconomic models. The technique advocated here enables ad hoc approaches to the construction of such variables to be avoided and also allows forward looking behaviour to be incorporated by the use of future as well as past values of the observed series in the calculation of these components. Moreover, long-run relationships between economic variables may be investigated along the lines suggested by Lucas (1980), but avoiding the restricted framework employed by that author.

46 The presence of unobserved components has also been shown to have important potential consequences for empirical macroeconomic practice, for the presence of different distributed lags linking different components of time series has been shown to produce biased coefficients and possibly misleading inferences when distributed lag models are estimated between observed time series. This has been amply demonstrated by the investigation of the relationship between £M3 and the RPI.

47 Finally, it should be pointed out that the decomposition of an observed time series used in this paper is not unique and, for example, Beveridge and Nelson (1981) have developed an alternative decomposition designed specifically for identifying turning points in business cycles. Investigation into the comparison of alternative methods for decomposing time series is therefore an important area of further research.

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