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No 49

Yield curves for gilt-edged stocks:

a new model

by

Katerina Mastronikola

December 1991

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The object of this Technical Series of Discussion Papers is to give wider circulation to research work in the Bank, and to invite comment upon it; any comments should be sent to the author at the address given below.

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Abstract

The paper describes the construction of the Bank of England par yield curve model which is currently in use. In the current model a curve fitting approach has been adopted. No assumptions are made about investor expectations on future levels of interest rates. Investors are assumed to be rational, ie seeking to maximise their post-tax return (net yield) by trading lost capital gain for additional coupon income and vice versa. It is assumed that their investment decisions are determined by the maturity, coupon, FOTRA status of gilts (stocks that are Free Of Tax to Residents Abroad) and whether gilts are in their ex-dividend (XD) period. The par yield curve is modelled using a smoothly spliced structure of cubic polynomials. The yield-coupon relation is studied using capital-income diagrams. The model's fit is superior to that of its predecessor.

1. Introduction

The Bank of England advises the Treasury on the interest rates that local authorities and some nationalised industries are charged when they borrow funds from the Public Works Loan Board (PWLB) and the National Loans Fund (NLF). The PWLB and NLF rates are reset regularly in order to be as close to market rates as possible. So the Bank relates those rates, quoted by PWLB and NLF, to interest rates derived from the daily quoted prices of gilts. These interest rates are the **par yields** of gilt-edged stocks¹ at different maturities. Par yields are calculated from the daily prices on gilts, and hence can be viewed as being implied collectively by investors in the gilts market. They are the yields on hypothetical stocks whose price stands at par (ie stocks priced at £1 per £1 nominal). An important consequence of the price of a stock standing at par is that the yield of such a stock is equal to its coupon. In other words, the return (par yield) from such a stock is equal to the interest (coupon) that it pays to its holder. This critical property has a very significant consequence, namely, that the par yield bypasses the effect of the variation of coupon with yield (this is the coupon effect, which is discussed, in detail, in section 3). This feature of the par yield together with the absence of default risk in the gilts market make the par yield curve (plot of par yields against time to maturity) a reasonable choice to represent the term structure of interest rates.

As par yields can, hence, be treated as representations of the term structure of interest rates, they are used for research purposes. Par yield curves are also published in the Bank of England *Quarterly Bulletin* (BEQB).

This paper explains the way in which the Bank calculates par yields. It contains an extended presentation of the par yield curve model discussed in the BEQB article of February 1990 (BEQB 1990). It is organised in the following way. Section 2 is an outline of the history of past yield curve models. Sections 3 and 4 follow, explaining the theory behind the current model and its mathematical formulation. In section 5 the fit and convergence properties of the model are discussed and compared with those of its predecessor. The conclusion appears in section 6.

¹ The reason these interest rates are calculated from gilts-edged stock prices is because of the absence of default-risk in the gilts market.

2. History

For many years the Bank's par yields were based on the model described in the December 1972 BEQB (J P Burman and W R White, 1972) and its successors. In that model (and its successors) it was explicitly assumed that different investors operate within different (preferred) maturity habitats and that their investment decisions are based on their expectations of the future level of interest rates for different time horizons. A result of this assumption about term segmentation in the gilts market was, that the maturity structure of the old model consisted of two components; one covering short maturities the other covering the long maturities. In other words, the short and long maturities were each treated separately to allow for the different investors alleged to be operating in each of them. In 1973 (J P Burman, 1973) the model was modified to take account of the work of R S Clarkson (R S Clarkson, 1972). Clarkson's paper described the effect on the relationship between coupons and yields of the relative attractiveness of capital gains compared with coupon income. There was a further modification in 1976 to improve the splicing of the two components of the par yield curve (J P Burman and O Page, 1976). Observed yields fitted this model until the autumn of 1981, when the calculated yields reached levels well above the highest coupons in the market. Par yields were therefore fairly distant from of yields on actual stocks and some users found them difficult to accept. Moreover, especially at maturities around five years, they were in fact not well-determined: variations in the model's parameters could lead to sharp changes in the par yields but no real change in how well the model, overall, fitted yields on actual stocks. Modifications were made which overcame this problem (Bank of England *Quarterly Bulletin*, June 1982) but the model was still not fully satisfactory. The restricted range of shapes for the longer maturities could give a poor fit to stocks at the long end, particularly when the yield curve was downward sloping.

It was, therefore, decided that a new method of estimating par yields should be adopted. Various other models were reviewed and experimented upon (M Arnold and G Pepper, 1979, R S Clarkson, 1978, S M Schaefer, 1981, S M Schaefer and E S Schwartz, 1983). It was decided, however, that a model, that would suit the Bank's needs best, would be one where, by adopting a curve fitting approach, less reliance was placed on hypotheses about investors' expectations on future levels of interest rates. The new model was described in a consultation paper (Bank of England 1986) and comments were invited from interested people outside the Bank. These led to some further modifications. The resulting model has been in use since March 1990. This model is explained here more fully.

3. The new model

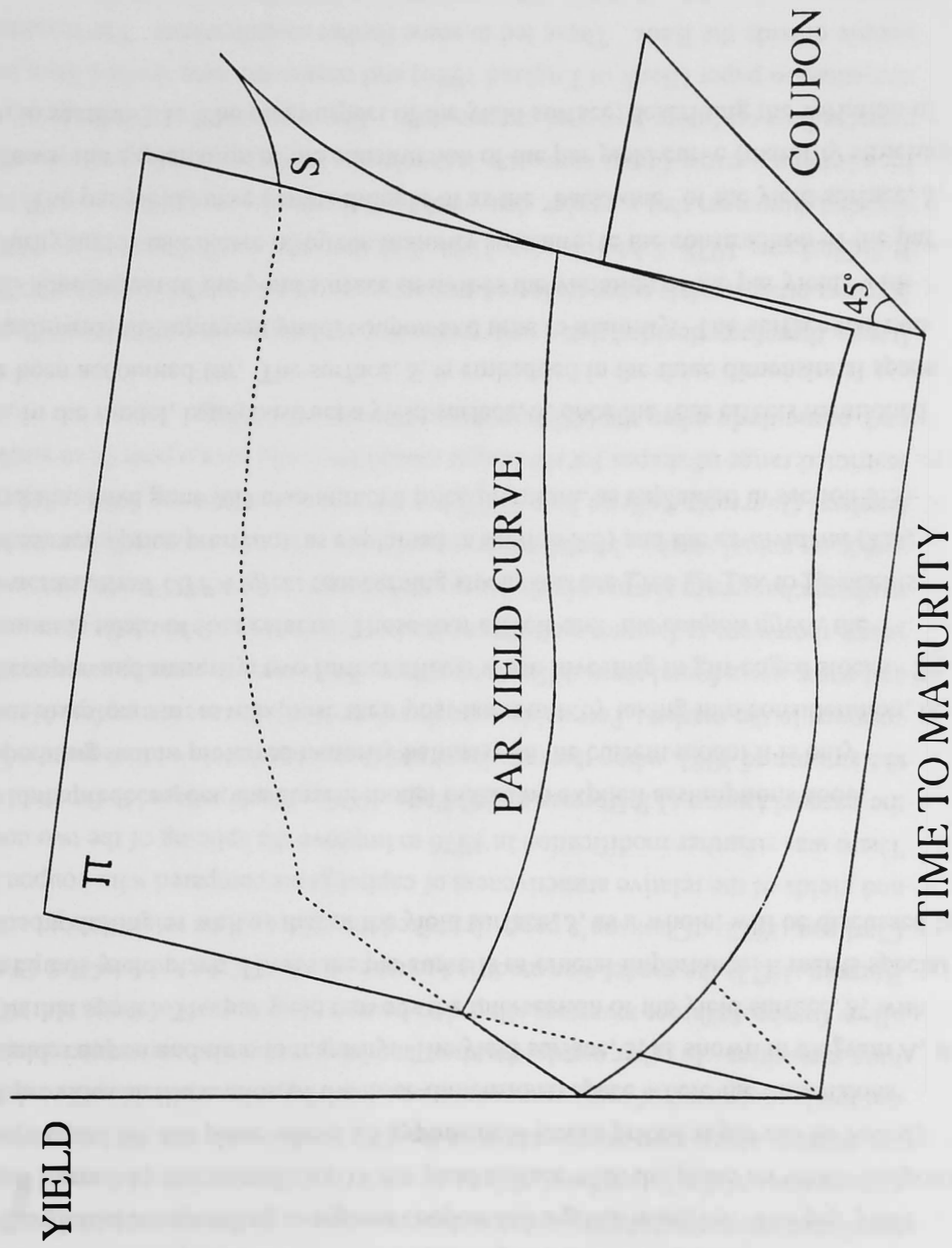
The yield on fixed interest rate securities such as gilts depends on both coupon and time to maturity. Because yield is a function of two components, coupon and time to maturity, it is a yield *surface* that is under consideration rather than a yield *curve*. A point on this surface represents the yield corresponding to a given coupon and a given maturity. The **par yield curve** is then formed by the intersection of the yield surface with the plane on which coupons and yields are equal (ie, the plane where all hypothetical stocks priced at par can be found). Diagram A provides an illustration of the three dimensional space where the dimensions represent yield, coupon and time to maturity. The yield surface, S , as shown in diagram A, is embedded in that space. The par yield curve is the intersection of the yield surface, S , with the coupon-equals-yield plane, Π . As the par curve is of crucial importance, it merits special attention. Its modelling as well as that of the yield surface, S , as a whole, will be discussed in detail here.

In contrast to its predecessors, the current model makes no explicit assumptions about investors operating within preferred maturity habitats. In the current model it is only assumed that investors aim to maximise their post-tax return by taking into consideration, in addition to coupon and maturity, two further effects when investing in gilt-edged stocks. So, explicit account is taken of four effects. These four effects are: the *coupon effect*, the *maturity structure*, the *FOTRA effect* (concerning stocks that are Free Of Tax to Residents Abroad and attract a price premium, as explained in section 3.3) and the *ex-dividend (XD) effect* (stocks that have gone XD also attract a price premium, as explained in section 3.3).

So, the aim, in the model, is to construct a yield surface, S , once the four effects mentioned above have been accounted for. The surface, S , is embedded in the three dimensional space, whose three dimensions represent yield, coupon and time to maturity. The surface has two aspects to it. One aspect of the yield surface describes the variation of the par yields with time to maturity and is taken care of by the maturity structure, ie the construction of the par yield curve. The par yield curve can be thought of as the "backbone" of the yield surface, S . In what follows, the explanation of the construction of the par yield curve (maturity structure) comes first, in section 3.1. The other aspect of the yield surface, describing the variation of

DIAGRAM A

THE YIELD SURFACE AND PAR YIELD CURVE



yields with coupon, is portrayed by curves representing the price-coupon² relationship. These curves are called capital-income curves and they are discussed below, where the coupon effect is explored. These curves can be thought of as the "ribs" of the yield surface, S . The coupon effect is discussed, at length, in section 3.2 as it is the most complex. The FOTRA and XD effects are discussed last, in section 3.3; they can be treated in a relatively straightforward fashion once the maturity and coupon effects have been dealt with.

The four effects, discussed earlier, are represented directly by four separate functions. Modelling is done using 12 parameters, so each of the four functions depends on a distinct subset of the 12 model parameters. The nature of these 12 model parameters and their role in the model will be discussed, in detail, in what follows. The approach here is to model the price of each stock and from it calculate the corresponding fitted yield (using an initial set of estimates - to be discussed in section 6.2 - of the 12 parameters). A search routine is then used to identify an optimal set of parameter values, ie the set for which the sum of the squared weighted yield residuals (ie differences between observed and fitted yields) is a minimum. In the description of the model it will be assumed that the optimal parameter values are given. The way in which these optimal parameter values are estimated will be discussed more fully in section 4. Each of the 12 parameters used in our model will from now be referred to as $x_1, x_2, x_3, \dots, x_{12}$.

3.1. Maturity structure

As a result of the alleged term segmentation in the gilts market, the maturity structure of the old par yield curve model consisted of two components at the short and long ends of the par yield curve, as already discussed in section 2. The range of shapes of each of the two components was limited. This meant that the range of shapes that the par yield curve could assume was limited too. Various attempts to improve this structure did not eliminate the problem (see section 2). It was recognised, therefore, that in order to remove this handicap a new finer structure was needed. It was decided to adopt a curve fitting approach, and after a lot of experimentation the maturity structure was given the form of a smoothly spliced structure consisting of cubic polynomials (Lancaster and Salkauskas, 1986). This is achieved in the following way.

² The yield-coupon relation can be investigated through curves representing the price-coupon dependency because, there is a one-to-one correspondence between the price of a stock and its yield.

Let t_1, \dots, t_6 be six maturities (time, t , is measured in years) with $t_1=0$, and $t_6=\infty$ corresponding to the irredeemables. These six maturities are fixed by the model (their choice is discussed, later, in section 4). Six of the 12 model parameters, ie x_1, \dots, x_6 are par yields corresponding to these maturities. The optimal values of these six par yields will be achieved once the sum of the squared weighted yield residuals has been minimised. However, for the sake of this presentation, it will be assumed that these optimal values of these six parameters are given. The par yields at maturities other than the six maturities above can be found through a method of interpolation along the cubic spline structure to be described here.

First, time t is transformed using a discounted present value function, ie

$$u = 1 - \exp(-\sigma t) \quad (1)$$

The discount rate σ is one of the model's fixed parameters and its choice will be discussed in section 4. The rationale behind this transformation is that each future year is given a weight proportional to the present value of £1 to be received in that year.

The values of u, u_1, \dots, u_6 , correspond to the six maturities t_1, \dots, t_6 . Note that when $t=0$ then $u=0$ and when $t=\infty$ then $u=1$ and that $0 \leq u \leq 1$.

The **par yield curve** is set up in the following way. A curve is constructed, which passes through the points with coordinates $(u_1, x_1), \dots, (u_6, x_6)$. This curve is made up of cubic polynomials in u , one polynomial to each interval $u_i \leq u \leq u_{i+1}$, and it is designed to be as smooth as possible where the separate polynomials meet. These cubic polynomials are the cubic splines or spline functions. The par yield curve has zero second derivative when $u=0$ ($t=0$) and is flat when $u=1$ ($t=\infty$, corresponding to the irredeemables). This constraint at the long end allows the model to give a good approximation to the unfitted irredeemable stocks. Irredeemables are not fitted because their recorded prices can change from day to day, and can lead to spurious jumps in the long end of the curve. Points on the par yield curve corresponding to maturities t are denoted $p(t)$ and are the **par yields** at these maturities.

The par yield curve built using this cubic spline structure is the "backbone" of the yield surface, discussed earlier (see diagram A). Next the curves describing the yield-coupon

relation that form the "ribs" of the yield surface, are investigated. As the yield-coupon relation is equivalent to the price-coupon relation, it is the price-upon-coupon dependency that is studied in the following section using capital-income diagrams.

3.2. Coupon effect

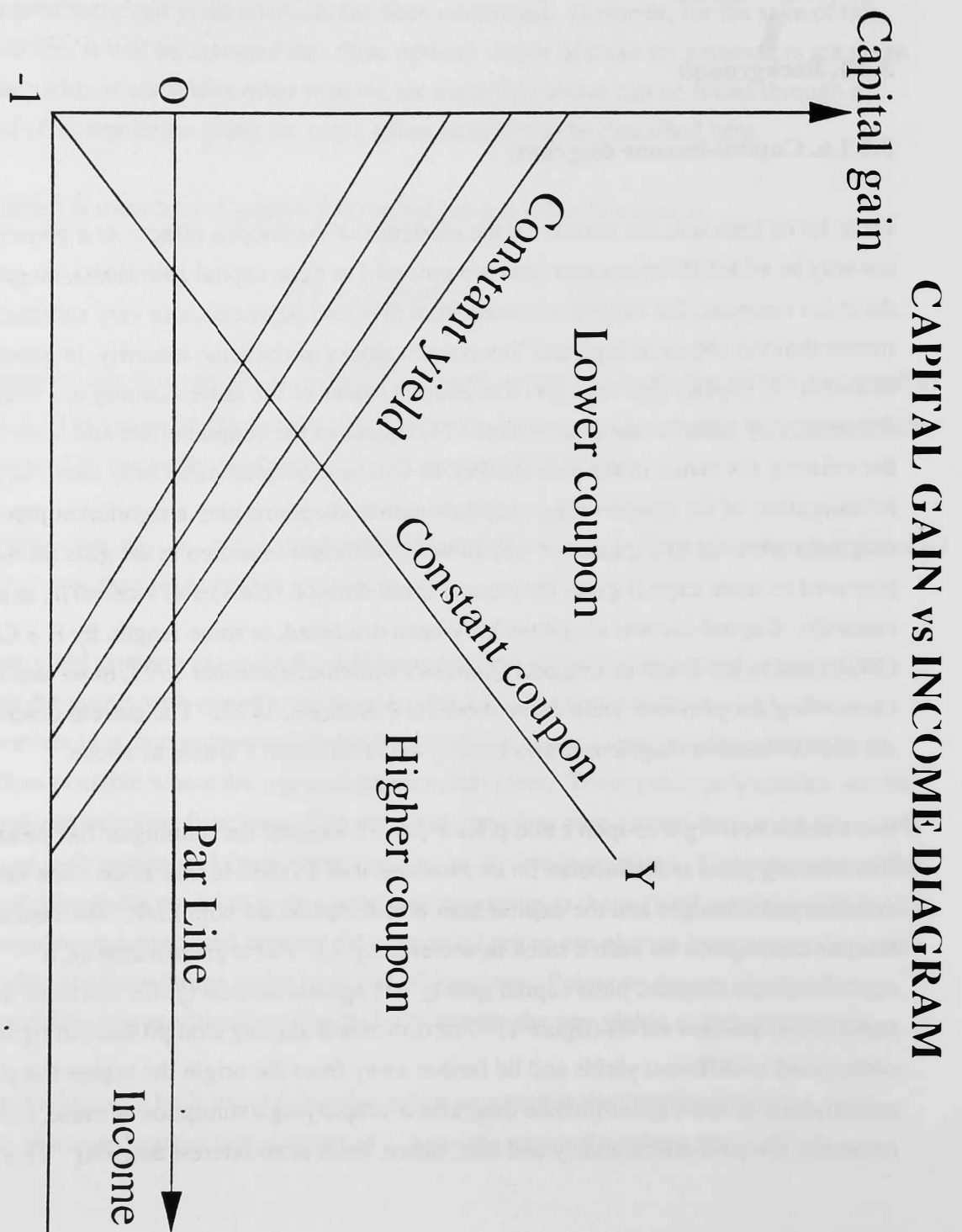
3.2.1. Background

3.2.1.a. Capital-income diagrams

First, let us look into the reason for the existence of the coupon effect. *At a given maturity*, the way in which different investors are prepared to trade capital gain (loss), on gilts bearing different coupons, for income obtained from dividend payments, can vary substantially. This means that the return of high and low coupon stocks of the same maturity, in general, is different. It implies that high and low coupon stocks of the same maturity are valued differently by those in the gilts market. This is called the coupon effect and is the result of the existing tax status in the gilts market, as will be explained more fully later. In the investigation of the coupon effect **capital-income** diagrams play a significant part. These diagrams are used to explore the way in which different investors in the gilts market are prepared to trade capital gains for income (both defined below) and vice versa, *at a given maturity*. Capital-income diagrams have been discussed, at some length, by R S Clarkson (1978) and in the Bank of England *Quarterly Bulletin*, September 1973, in an article concerning the previous yield curve model (J P Burman, 1973). The current description of the capital-income diagrams draws heavily on J P Burman's article as above.

For a stock bearing a coupon c and price P per £1 nominal the running or flat yield is $\frac{c}{P}$. The running yield is the **income** on an investment of £1 cash in that stock. The value of the nominal stock bought and the **capital** sum at redemption are both $\frac{1}{P}$. The capital gain (or loss) to redemption on such a stock is, therefore, $\frac{1}{P} - 1$. *For a given maturity*, a capital-income diagram plots capital gain ($\frac{1}{P} - 1$) against income ($\frac{c}{P}$) for stocks of that maturity, as coupon varies (figure 1). The downward sloping straight lines in figure 1 correspond to different yields and lie further away from the origin the higher the yield. In the construction of the capital-income diagrams a simplifying assumption is made, that dividends on stocks are paid continuously and that, hence, there is no interest accruing. This means that

FIGURE 1



capital-income curves vary smoothly with time to maturity, an essential property for the yield surface to be smooth³. The horizontal line where capital gain is zero, which coincides with the horizontal axis, is the **par line** (figure 1). A given coupon in a capital-income diagram of a certain maturity is represented by a straight line (line Y in figure 1). The slope of this line is the inverse of the coupon. So, in a capital-income diagram, coupon lines corresponding to low coupon stocks can be found close to the vertical axis, and those corresponding to high coupon stocks can be found close to the horizontal axis.

R S Clarkson (1978) investigated the cheapness and dearness of gilts with respect to a fitted price surface upon which the gilts market was assumed to be in "equilibrium under switching". The market is in "equilibrium under switching" if and only if an investor cannot switch from one stock to any combination of other stocks if such a switch results in:

a higher capital sum at maturity and maintained income, or
higher income and maintained capital sum at maturity, or
higher income and a higher capital sum at maturity.

For the market to be in such an equilibrium, the curve representing the capital-income relation for the whole market should be downward sloping and concave towards the origin (for a justification of this see R S Clarkson, 1978, and J P Burman, 1973). These are the Clarkson conditions. In our yield curve model, it is postulated that investors are rational⁴ and hence that the market reaches an "equilibrium under switching" and therefore that the Clarkson conditions are satisfied. This gives rise to capital-income curves that are downward sloping and concave to the origin.

The redemption yield, y , on a stock having a given maturity is defined as the rate at which all future dividend payments and redemption proceeds must be discounted, in order for the sum of their present values to equal the price of the stock. For a stock paying dividends semi-annually, this can be expressed mathematically, as follows:

³ A significant implication of this is that a capital-income curve can be specified, corresponding to each and every point along the par yield curve. This is important because, as explained above, capital-income curves describe the price-coupon (and hence yield-coupon) relation and thus take care of the yield surface, in the yield-coupon direction.

⁴ The postulate that investors are rational implies that those in the gilts market seek to maximise the post-tax return (ie net yield) of their investment and that lost capital gain can be compensated with additional income and vice versa.

$$P = \sum_{i=1}^{2t} \frac{\frac{c}{2}(1-\tau)}{\left(1 + \frac{y}{2}(1-\tau)\right)^i} + \frac{1}{\left(1 + \frac{y}{2}(1-\tau)\right)^{2t}} \quad (2)$$

where t is the time to maturity in half-years (this explains why the sum in equation (2) is taken up to $2t$) and τ is the tax rate. The coupon c and yield y (both per £1) are annual rates. For an investor facing a rate of income tax τ , at a given time to maturity, the yield y is *constant* for all coupons. So, for a constant redemption yield, y , and a given tax rate, τ , the relationship between the price P and coupon c is linear. From this it can be deduced that the capital gain $\left(\frac{1}{P} - 1\right)$ satisfies the following equation, ie

$$\frac{1}{P} - 1 = A(y - r) \quad (3)$$

with r the running yield, y the redemption yield and

$$A = \frac{\left(1 + \frac{1}{2}y(1-\tau)\right)^{2t} - 1}{y} \quad (4)$$

The coefficient A , which determines the slope of the straight line in equation (3), is a function of the tax rate τ (since, at a given maturity, t , the yield, y , is constant). Equation (3) shows that the relation between capital gain $\frac{1}{P} - 1$ and income r is linear. A line defined by equation (3) corresponds to a constant yield y . The higher that yield, the further away the line lies from the origin. The slope of each such straight line represents the degree of substitution of capital gain for income and vice versa, which is acceptable to an investor paying income tax at a rate τ . Straight lines defined by equation (3) are, therefore, indifference lines.

3.2.1.b. The role of taxation

A fundamental feature of the British government stock market is the distinction which investors draw between the investment return obtained as income, which is liable to tax, and capital gains, which are tax free. The amount of additional income required to compensate investors paying income tax for lost capital gain, therefore, increases with their tax rate. This makes the indifference lines, discussed above, steeper the lower the tax rate an investor is

facing. Figure 2 illustrates the indifference curves corresponding to investors paying no income tax (line GG') and higher rate tax payers (line HH')⁵. Depending on their tax status, therefore, different investors' indifference lines have different slopes and are hence expected to cross (point D in figure 2). Higher rate tax payers' preference for capital gains makes them offer more for low coupon stocks, thus rendering those stocks less attractive to investors paying no income tax (gross investors). It is assumed that the latter (ie the gross investors) are consequently not driven out of the gilts market, but are instead directed towards the higher coupon stocks. In general, the cross over point of the indifference lines HH' and GG' can be expected to be above the par line (figure 2). This feature is due to the higher rate tax payers' strong preference for capital gains in the region of high coupons⁶.

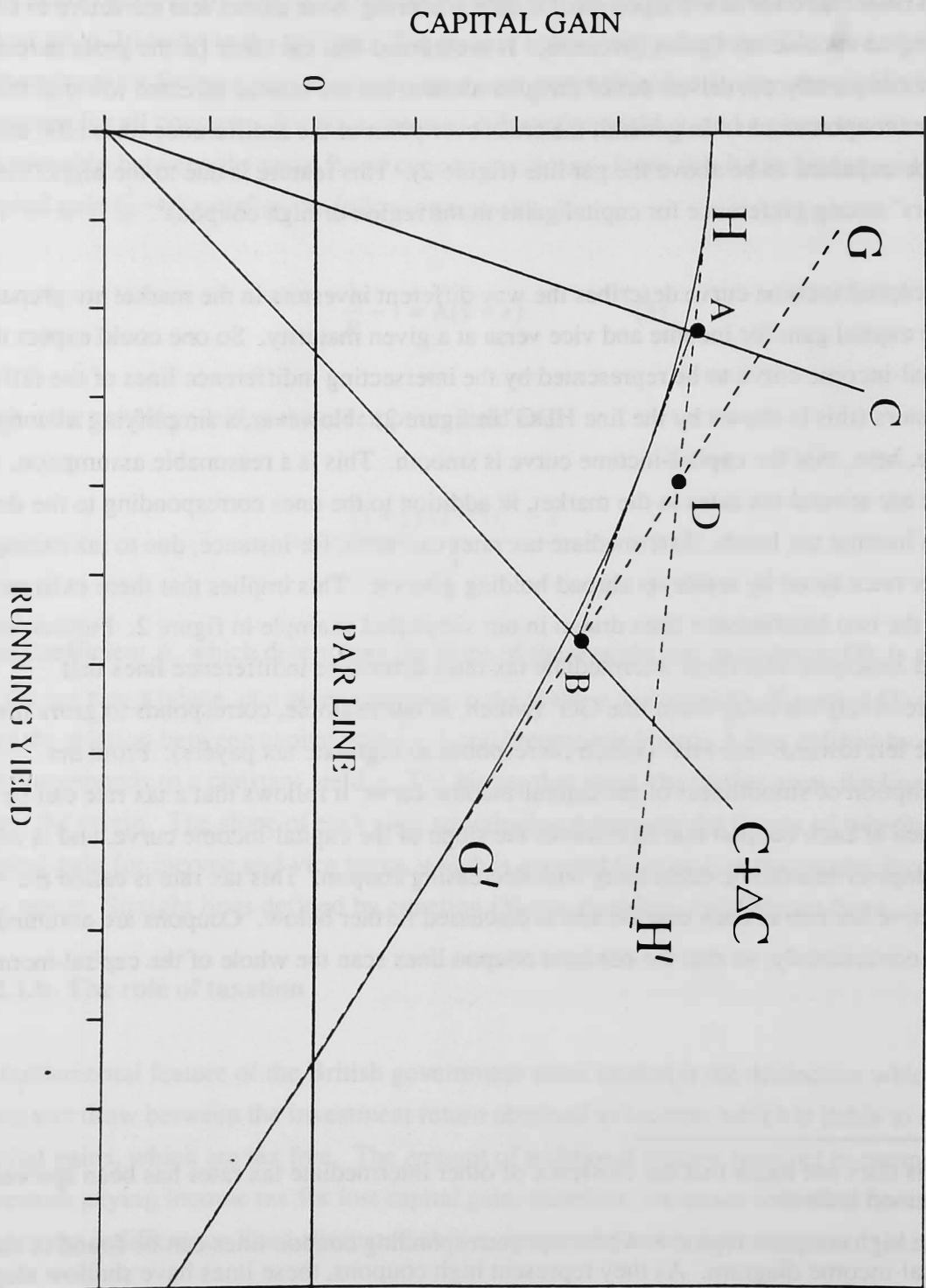
The capital-income curve describes the way different investors in the market are prepared to trade capital gain for income and vice versa at a given maturity. So one could expect the capital-income curve to be represented by the intersecting indifference lines of the different investors (this is shown by the line HDG' in figure 2). However, a simplifying assumption is made, here, that the capital-income curve is smooth. This is a reasonable assumption, as there are several tax rates in the market, in addition to the ones corresponding to the three main income tax bands. Intermediate tax rates can arise, for instance, due to tax exemptions or tax rates faced by residents abroad holding gilts etc. This implies that there exist more than the two indifference lines drawn in our simplified example in figure 2. Further, one could anticipate that these intermediate tax rates determine indifference lines that progressively tilt away from line GG' (which, in our example, corresponds to gross investors) to the left towards line HH' (which corresponds to high rate tax payers). From the assumption of smoothness of the capital-income curve, it follows that a tax rate can be defined at each coupon that determines the slope of the capital-income curve, and is such that the slope is monotonic decreasing with decreasing coupon. This tax rate is called the effective tax rate at each coupon and is discussed further below. Coupons are assumed to vary continuously, so that the constant coupon lines scan the whole of the capital-income

⁵ This does not mean that the existence of other intermediate tax rates has been ignored, as explained below.

⁶ The high coupons region is where the corresponding coupon lines can be found in the capital-income diagram. As they represent high coupons, these lines have shallow slopes and can, hence, be found close to the horizontal axis.

FIGURE 2

ILLUSTRATION OF THE EFFECTIVE TAX RATE



plane. An effective tax rate is defined continuously at each of all the coupons, ensuring that the slope of the capital-income curve also varies continuously. The resulting capital-income curve is smooth and it satisfies the Clarkson conditions.

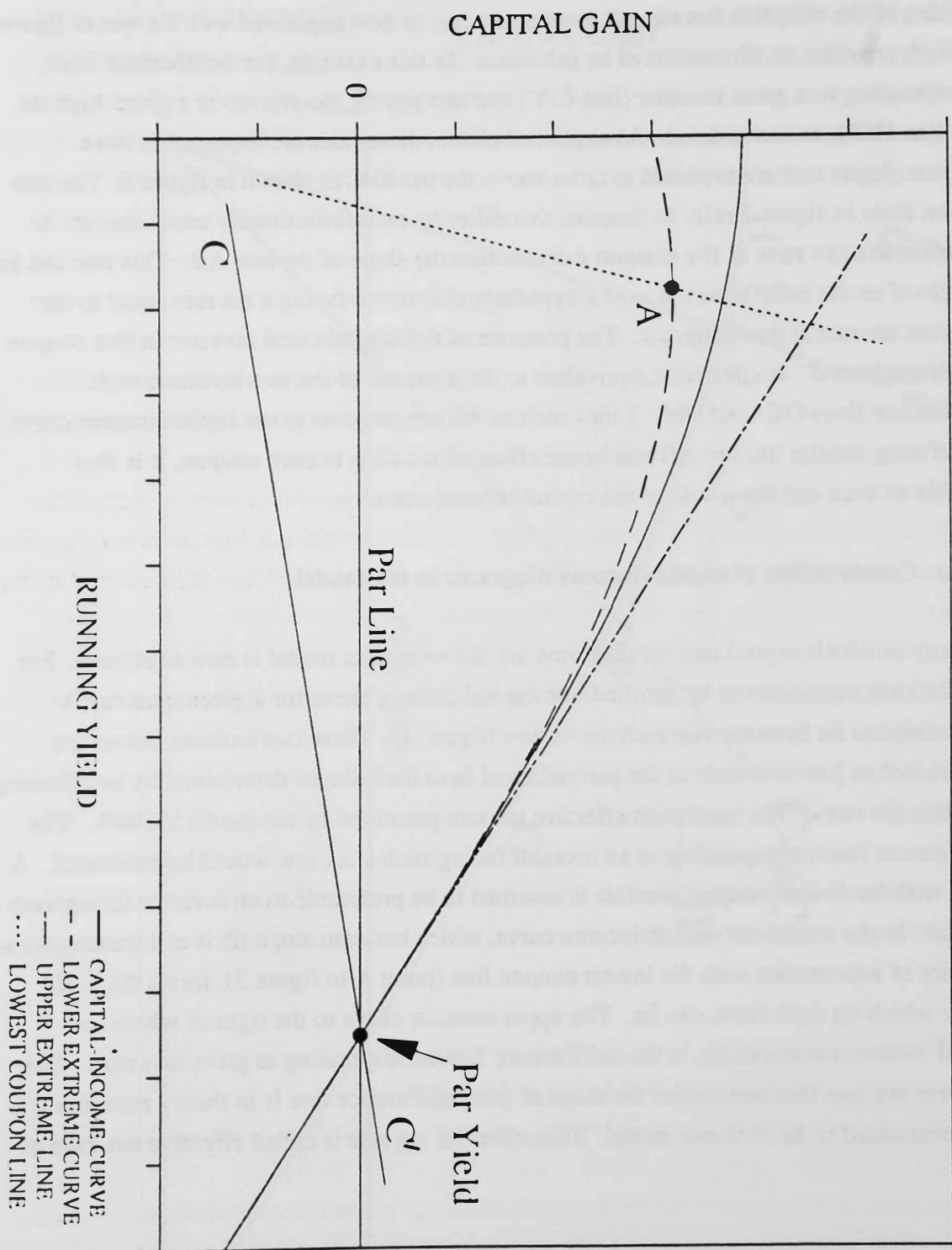
The idea of the **effective tax rate at a coupon c** can be best explained with the use of figure 2, which provides an illustration of its definition. In this example, the indifference lines corresponding to a gross investor (line GG') and one paying income tax at a given high tax rate (line HH'), are considered. As explained above, these lines are expected to have different slopes and are expected to cross above the par line, as shown in figure 2. The two coupon lines in figure 2 refer to coupons that differ by an infinitesimally small amount Δc . The **effective tax rate at the coupon c** determines the slope of the line AB . This line can be thought of as the indifference line of a hypothetical investor facing a tax rate equal to the effective tax rate at that coupon c . The presence of this hypothetical investor in that coupon "neighbourhood" is *effectively* equivalent to the presence of the two investors with indifference lines GG' and HH' . Lines such as AB are tangents to the capital-income curve. By defining similar lines to AB and hence effective tax rates at each coupon, it is thus possible to trace out the whole of the capital-income curve.

3.2.1.c. Construction of capital-income diagrams in the model

The way in which capital-income diagrams are drawn for our model is now explained. For the Clarkson conditions to be satisfied, the capital-income curve for a given maturity is constrained to lie between two extreme curves (figure 3). These two extreme curves are constrained to join smoothly at the par yield and have their slopes determined by two limiting effective tax rates. The maximum effective tax rate permitted by the model is 100%. The indifference line corresponding to an investor facing such a tax rate would be horizontal. A stock with the lowest coupon possible is assumed to be preferable to an investor facing such a tax rate. In the model the capital-income curve, which has zero slope (ie is at a maximum) at its point of intersection with the lowest coupon line (point A in figure 3), forms the limit below which no such curve can lie. The upper extreme curve to the right of which no capital-income curve can lie, is the indifference line corresponding to gross investors. The effective tax rate that determines the slope of that indifference line is in theory zero, but it is not constrained to be so in our model. That effective tax rate is called **effective tax rate at**

CAPITAL-INCOME DIAGRAM

FIGURE 3



par⁷. The effective tax rate at par depends on time to maturity and the two model parameters x_7 and x_8 which represent tax rates. In our model, the effective tax rate at par defines the slope of the segment of the capital-income curves, where running yield or income is greater than the par yield at that maturity. It is the segment of our capital-income curve in the region of high coupon stocks and lies below the par line. For running yields less than the par yield, in the region of lower coupons, the capital-income diagram is curved (concave to the origin) and lies between the two extremes discussed earlier (figure 3). The curved segment has its shape determined by time to maturity and the model parameters x_9 and x_{10} . The two segments are constrained to join smoothly at the par yield at that maturity (figure 3).

It is assumed that all stocks of equal maturity are represented by points in the capital-income curve of that maturity. By finding the coordinates of the point where each constant coupon line intercepts the capital-income curve of a given maturity, one can work out the fitted price P_i (ie the one predicted by the model) of any stock i bearing that coupon and having that maturity. A consequence of the simplifying assumption made earlier (ie that dividends are paid continuously, implying that no interest is accruing) in the formulation of the capital-income diagrams, is that the fitted price is best interpreted as a "clean" price (ie one that excludes accrued interest) of a stock whose dividends are paid at discrete intervals (this still involves an approximation error). From that price the fitted yield, y_i^{fitted} , predicted by the model can then be calculated.

The following section contains the technical description of the coupon effect and may be omitted by the non-technical reader.

3.2.2. Technical description of the coupon effect

In this technical description of the coupon effect the optimal and final values of the model parameters x_7 , x_8 , x_9 , and x_{10} are assumed to be all given. The nature and role of these parameters will be discussed here.

⁷ The reason for that name is the following: In our model, the point of intersection of that upper extreme straight line with the constant coupon line corresponding to a coupon equal to the par yield at that maturity (line CC' in figure 3), is fixed to lie on the par line (the line where capital gain is zero). That point is the par yield at that maturity. Hence the effective tax rate that determines the slope of that upper extreme straight line (upon which gross investors find themselves) has been given the name **effective tax rate at par**.

A stock of maturity t , bearing a coupon c and having price P per £1 nominal, may be represented by the point in the capital-income plane whose coordinates are $((1/P)-1, c/P)$ and vice versa. We assume that all stocks of residual maturity t lie on a curve $(f(r), r)$ in the capital-income plane, where the function $f(r)$ determines capital gain associated with running yield or income r .

In our model, as already discussed, the capital-income curve consists of a straight line (whose slope is determined by the **effective tax rate at par**) for running yields greater than the par yield at that maturity ($r \geq p(t)$) joining smoothly to a curved segment for running yields less than the par yield ($r < p(t)$). The function $f(r)$ determining the relation between capital gain and running yield, can be assumed to take the form:

$$f(r) = \begin{cases} \alpha(p(t) - r), & \text{for } r \geq p(t) \\ \alpha(p(t) - r) + \lambda(p(t) - r)^\delta, & \text{for } r < p(t) \end{cases} \quad (5)$$

Note that here α is given by:

$$\alpha = \frac{\left(1 + \frac{1}{2}p(t)(1 - \tau_{par})\right)^{2t} - 1}{p(t)} \quad (6)$$

where τ_{par} is the **effective tax rate at par** and $p(t)$ is the par yield at that maturity. This tax rate, in our model, depends on the time to maturity t and the model parameters x_7 and x_8 (both these parameters are tax rates) and is defined as follows:

$$\tau_{par} = \begin{cases} x_7, & \text{for } t \leq 1 \\ x_7 \frac{1-u}{1-\bar{u}} + x_8 \frac{u-\bar{u}}{1-\bar{u}}, & \text{otherwise} \end{cases} \quad (7)$$

where $\bar{u} = 1 - e^{-\sigma}$.

The values of α and λ are allowed to vary with different settings of the parameters, but δ is a fixed parameter (its value will be discussed in section 4).

For running yields below $p(t)$ ($r < p(t)$) the capital-income curve is not a straight line and it has its curvature determined by λ . The curve is differentiable if $\delta > 1$. Differentiability of the curve is important as it implies smoothness: an essential property of our capital-income curves.

For the capital-income curve to satisfy the Clarkson conditions λ is defined in the following way. There is a value $\tilde{\lambda}$ corresponding to the capital-income curve (lower extreme curve in figure 3), which has zero slope at its point of intersection with the line of constant coupon for $c=c_{\text{low}}$ (point A in figure 3), where c_{low} is the lowest coupon in the market (typically 2.5% or 3%). As explained earlier this zero slope corresponds to an effective tax rate of 100%. This value $\tilde{\lambda}$ can thus be determined by setting the first derivative of the equation for the curved segment ($f(r)$ for $r < p(t)$) equal to zero, ie

$$\frac{df(r)}{dr} = 0 = -\alpha - \lambda \delta (p(t) - r)^{\delta-1} \quad (8)$$

For the lower extreme curve in figure 3 this condition holds at point A. So $\tilde{\lambda}$ is the value of λ that solves equation (8) above, when $r=r_A$, ie

$$\tilde{\lambda} = -\frac{\alpha}{\delta(p(t) - r_A)^{\delta-1}} \quad (9)$$

By substituting this expression for $\tilde{\lambda}$ back into equation (5) above, one can obtain the equation for the lower extreme curve in figure 3, ie

$$f(r) = \begin{cases} \alpha(p(t) - r), & \text{for } r \geq p(t) \\ \alpha(p(t) - r) \left(1 - \frac{1}{\delta(p(t) - r_A)^{\delta-1}} (p(t) - r)^{\delta-1} \right), & \text{for } r < p(t) \end{cases} \quad (5a)$$

Point A (which exists on $f(r)$ where $r < p(t)$) is the point where this curve, given by equation (5a), is at a maximum and is intercepted by the constant coupon line corresponding to the lowest coupon in the market. The constant coupon lines in a capital-income diagram obey the equation:

$$f(r) = \frac{r}{c} - 1 \quad (10)$$

So, next make use of the constant coupon line equation, when $c = c_{low}$, ie

$$f(r) = \frac{r}{c_{low}} - 1 \quad (11)$$

in order to obtain the running yield corresponding to point A, ie

$$r_A = \frac{\delta c_{low} + c_{low} \alpha (\delta - 1) p(t)}{\delta + \alpha (\delta - 1) c_{low}} \quad (12)$$

This is the point where the lower extreme curve in figure 3 has its slope determined by an effective tax rate of 100% and is intercepted by the constant coupon line when $c = c_{low}$.

The upper extreme line in our capital-income diagrams is the straight line discussed earlier, where gross investors find themselves. This corresponds to $\lambda = 0$, where the slope α is determined by the effective tax rate at par, τ_{par} , at the given maturity, ie

$$f(r) = \alpha(p(t) - r) \quad (13)$$

From this it can be deduced that $\tilde{\lambda} \leq \lambda \leq 0$ and is given by

$$\lambda = \xi \tilde{\lambda} \quad (14)$$

where $0 \leq \xi \leq 1$. ξ depends on time to maturity t and the curvature parameters x_9 and x_{10} and is explicitly defined as follows:

$$\xi = \begin{cases} x_9, & \text{for } t \leq 1 \\ x_9 \frac{1-u}{1-\bar{u}} + x_{10} \frac{u-\bar{u}}{1-\bar{u}}, & \text{otherwise} \end{cases} \quad (15)$$

with $\bar{u} = 1 - e^{-\sigma}$.

Once the capital-income diagram has been drawn, a fitted price P_i can be determined for each stock i as explained in section 3.2.1. From that price the corresponding fitted yield y_i^{fitted} for each stock i can then be calculated.

This completes the technical description of the coupon effect.

3.3. XD and FOTRA effects

In the description of these two effects it is assumed that the final and optimal values of the parameters x_{11} and x_{12} are given. As noted above, the fitted price deduced from the capital-income curve is a "clean" price. The corresponding "dirty" price (one that includes the accrued interest) is obtained by adding the accrued interest to the "clean" price. Accrued interest is negative when a stock is in its ex-dividend (XD) period and positive when the stock is not XD. Ex-dividend (XD) is the term given to stocks that are sold without the rights of the next dividend passing to the buyer. Accrued interest on a stock i bearing a coupon c per £1, is calculated as follows:

$$AI_i = \frac{c}{2} d \quad (16)$$

where c is paid semi-annually and d is the time in half-years since the last dividend. Once a stock has gone XD equation (16) still holds, but in this case the time since the dividend previous to the first payable is negative and hence so is the accrued interest AI_i . The buyer of an XD stock is not entitled to receive its forthcoming dividend, but must wait until the dividend date following. Hence, since dividends are taxed as income, investors paying income tax prefer to avoid dividends and favour buying stocks in their XD periods. This implies that the "clean" price of XD stocks is bid up by investors paying income tax. This is the XD effect. In the model, this is allowed for by adding a proportion of the accrued interest to the fitted price of all stocks; this proportion is a model parameter (x_{11}).

Now suppose P_i (i refers to each stock used in the model) is the fitted price corresponding to each stock derived from the capital-income diagram, which is interpreted as a "clean" price. A proportion of the accrued interest AI_i must, therefore, be added to P_i to give the corresponding fitted "dirty" price, ie

$$P_i + x_{11}AI_i \quad (17)$$

From this price a fitted gross redemption yield y_i^{fitted} may be found. For a non-FOTRA stock, this is taken to be the predicted yield. FOTRA stocks, however, attract a price premium as their dividends are tax free to residents abroad. Consequently, the fitted yield on such a stock is reduced by an amount x_{12} to allow for the anticipated premium in the price.

This completes the description of the construction of the curve.

4. Estimation of parameter values - Parameter bounds and settings

So far, it has been assumed that the parameters used by the model were all, somehow, given. The model, however, is equipped with an estimator which, when supplied with an initial set of parameter estimates, it is designed to search for the parameters' final values. The 12 parameters will assume their final and optimal values once the sum of the squared weighted yield residuals (differences between observed and fitted yields) is minimised, ie

$$\min \sum_i \{w_i(y_i^{observed} - y_i^{fitted})\}^2 \quad (18)$$

Here i refers to each stock used in the model. For each stock $y_i^{observed}$ is the yield calculated using its observed market price and y_i^{fitted} is the fitted yield derived from the model (section 4.3). The factor w_i is the weight attributed to each such stock. The weight of each stock is typically unity but, there are exceptions. Any stock with maturity of less than a year will have its weight reduced to ensure smooth fading out as the model is fitted from day to day. Outliers have their weight reduced in the course of the estimation. The following types of stocks are given zero weight: *partly-paid stocks, stocks with small amounts in issue, stocks*

with conversion options (convertibles) and the irredeemables. Index-linked stocks are excluded from the estimation altogether. The procedure used to perform this minimisation is a non-linear least squares optimisation routine (section 5.2).

Some parameters can be allowed to take unlimited values, while others may be bounded to avoid such things as degenerate solutions and ill-defined models, or to prohibit tax rates which are considered impossible.

The parameters x_1, \dots, x_6 , which are par yields (section 3.1), are unbounded. The other parameters are bounded. Natural boundaries have been identified and used whenever appropriate. x_7 and x_8 represent tax rates and so are deemed to have natural boundaries at 0% and 100%. x_9 and x_{10} , which determine the shape of the capital-income curve, have natural boundaries at 0 and 1, for outside these boundaries the Clarkson conditions will be transgressed. x_{11} and x_{12} (the XD and FOTRA parameters) have no natural boundaries, but are expected to be close to unity and fairly close to zero respectively. Furthermore, bounding the possible values of x_{11} and x_{12} can help the model's estimator find the final optimal parameter values rapidly. x_{11} is bounded by 0 and 2, and x_{12} is bounded by -10% and +10%.

For the model to be precisely specified, certain constants used in the construction need to be given fixed settings. These settings have been determined by trial and error (ie they were the ones that provided the best fit) and they take the following values: $\delta = 1.75$ (see equation (5)), $\sigma = 0.1$ (see equation (1)).

The maturities of the par yields that serve as joints in the cubic spline structure are: $t_1 = 0$, $t_2 = 2$, $t_3 = 5$, $t_4 = 10$, $t_5 = 15$, $t_6 = \infty$, where time, here, is measured in years. They were chosen so that they are nearly evenly spaced apart in transformed time (see equation (1) in section 3.1).

5. Results

5.1. Diagnostic statistics

In order to examine how well the model fits the data, various test statistics were calculated and compared with those of the previous yield curve model. Three sets of statistics are used to measure the adequacy of the model: the \bar{R}^2 , the RMS (Root Mean Square) and the Durbin-Watson statistic. The \bar{R}^2 and RMS statistics have been modified to take account of

the weighting of the residuals. The \overline{R}^2 measures the percentage of the variation in yield explained by the model. The RMS is the root mean square of the weighted residuals multiplied by 100 to put them in percentage terms. The RMS measures the average model error. The Durbin-Watson statistic gives a measure of the correlation between residuals of adjacent maturities.

For testing purposes, the model was fitted to gilts data for each of 32 dates ranging from 1974 to 1989. For dates before and during 1977 the FOTRA parameter x_{12} in the model has been fixed at zero because there were few non-FOTRA stocks at the time. Tables 1 to 3 show some statistics for the fit of the yield curve.

Note that the \overline{R}^2 and RMS are nearer to 1 and 0 respectively for the new model than they are for the old one. There appears to be an improvement in fit, which suggests that the new model gives a closer estimate of the par yields than the previous one.

The par yield curve derived from the new model was plotted against the one derived using the old model. The plots refer to the same set of 32 dates as the statistics discussed above. On the same charts the yields of actual stocks have also been plotted, differentiated between those standing above and those standing below par. The yield on a stock standing above or below par should lie above or below the par curve, so that the curve separates the stocks above and below par. The new par yield curve, on the whole, appears to be separating stocks above and below par more distinctly, which suggests that the par yield curve is more accurately determined by the current model.

5.2. Convergence

The convergence properties of a model, are as important as the fitting but, they are more difficult to quantify and describe. Poor convergence may indicate instability (a tendency for large changes in the parameters from day to day) and ultimately the possibility of the model failing to converge at all, leading to meaningless fitted yields. The model described here has good convergence properties, as did the old one. As stated above, the function to be minimised by the model is the sum of the squares of the weighted yield residuals, given the values of the initial parameter estimates. These initial parameter estimates supplied to the

model have been chosen in order to accelerate convergence (the model's estimator, a search routine, is designed to search and reach a point of convergence regardless of what are the values of the initial parameter estimates)⁸.

There are various methods of minimising a non-linear function. The method used in the model now is based on work by A Jones (A Jones, 1970) but differs from it in some respects: upper and lower limits are set for each variable, which act as reflecting barriers. This means that, if a variable persistently moves towards a limit, it is fixed and the search continues in a smaller number of dimensions. This ensures rapid convergence.

6. Conclusion

Recapitulating, the main difference between the theoretical foundation of the current model and the old one is that now no assumptions are made about investors making investment decisions based on return expectations up to certain time horizons. Investors are, therefore, not assumed to operate within any preferred maturity habitats. The model simply postulates that investors in the market will invest rationally, seeking to maximise their post-tax return, on the basis of the four effects (maturity, coupon, FOTRA and ex-dividend) discussed earlier.

The assumption about the term segmentation in the gilts market, in the old model, lead to a maturity structure consisting of two components. This old form of the maturity structure proved a handicap as the range of shapes the par yield curve could, thus, assume was limited (already explained in sections 2 and 3.1). Lifting that assumption about investor expectations meant that the maturity structure could be given a more continuous form. This was done by the introduction of the cubic splines, that replaced the two components at the short and long end of the par yield curve in the old model. The current maturity structure is less rigid and endows the par yield curve with a more extensive range of shapes.

⁸ In order to facilitate rapid convergence these initial parameter estimates have been chosen to be not far from the expected final optimal values of the parameters or well within the bounds of the bounded parameters (section 4). So initial parameter estimates for x_1, \dots, x_6 (as explained in section 3.1, these are par yields) have been set equal to 14%. Parameters x_7 and x_8 represent tax rates and their initial estimates have been set equal to 25%. The curvature parameters x_9 and x_{10} initial estimates have been set equal to 0.25. Finally the ex-dividend parameter (x_{11}) initial estimate has been set equal to 1 and the FOTRA parameter (x_{12}) initial estimate has been set equal to 0.

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Table 1

R-BAR SQUARED

Date	old model	new model
04.01.74	.8804	.9344
30.12.74	.9659	.9780
20.03.75	.9405	.9695
09.01.76	.9384	.9455
03.10.77	.9684	.9835
04.01.80	.9194	.9708
09.01.81	.9484	.9786
02.06.81	.9518	.9799
26.10.81	.9152	.9434
10.03.82	.9138	.9655
20.12.82	.9487	.9614
18.04.83	.9583	.9795
18.07.83	.9552	.9647
17.10.83	.9399	.9827
30.01.84	.9458	.9730
21.05.84	.9274	.9818
20.08.84	.9017	.9669
29.10.84	.8794	.9512
29.07.85	.9521	.9800
11.11.85	.9622	.9883
24.02.86	.9616	.9860
21.04.86	.9449	.9747
23.05.86	.9472	.9809
29.09.86	.9426	.9643
30.03.87	.9629	.9906
14.10.87	.9280	.9889
21.10.87	.9272	.9826
28.10.87	.9257	.9858
30.03.88	.8945	.9900
30.09.88	.9387	.9678
31.03.89	.9541	.9830
04.10.89	.9645	.9887

Table 2

RMS

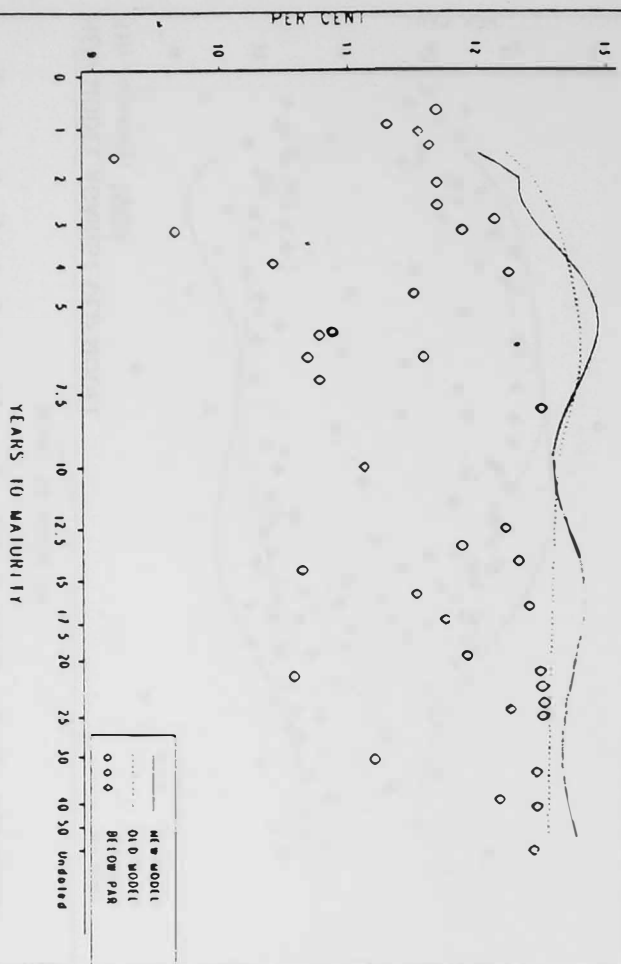
Date	old model	new model
04.01.74	.2705	.2153
30.12.74	.4602	.3741
20.03.75	.3680	.2637
09.01.76	.4849	.4770
03.10.77	.3338	.2369
04.01.80	.3123	.1907
09.01.81	.2410	.1535
02.06.81	.2343	.1532
26.10.81	.3414	.2877
10.03.82	.2380	.1525
20.12.82	.2493	.2162
18.04.83	.1708	.1187
18.07.83	.1620	.1452
17.10.83	.1742	.0941
30.01.84	.1954	.1429
21.05.84	.1839	.0981
20.08.84	.2238	.1277
29.10.84	.1977	.1277
29.07.85	.1337	.0936
11.11.85	.1202	.0745
24.02.86	.1189	.0799
21.04.86	.1208	.0852
23.05.86	.1529	.0995
29.09.86	.1574	.1276
30.03.87	.0925	.0520
14.10.87	.1239	.0525
21.10.87	.1207	.0630
28.10.87	.1257	.0582
30.03.88	.1520	.0540
30.09.88	.1042	.0758
31.03.89	.1438	.0870
04.10.89	.1579	.0885

Table 3

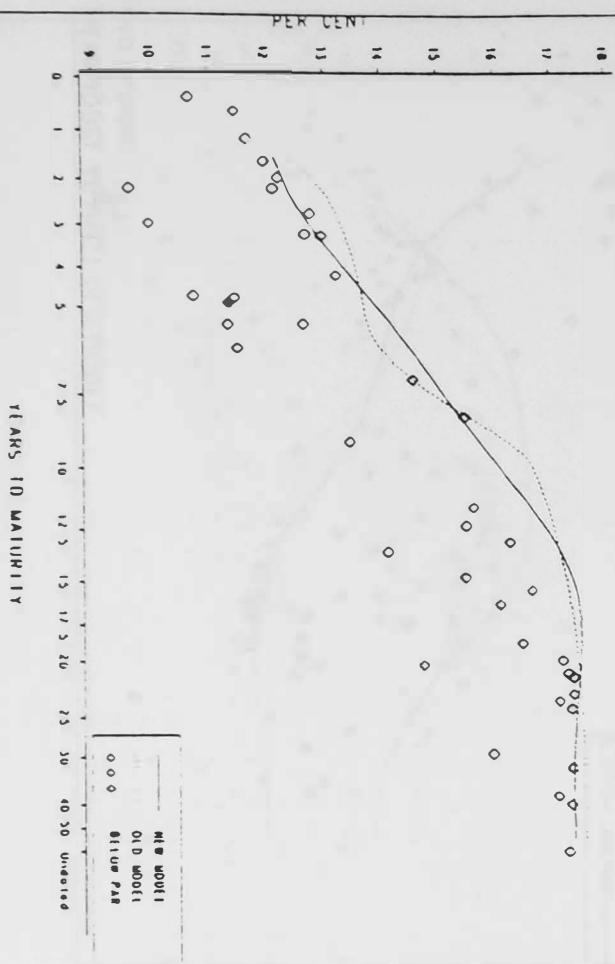
DURBIN-WATSON

Date	old model	new model
04.01.74	2.066	2.487
30.12.74	1.615	2.330
20.03.75	1.351	2.050
09.01.76	1.347	2.162
03.10.77	2.008	2.513
04.01.80	1.324	2.146
09.01.81	1.214	2.161
02.06.81	1.300	2.036
26.10.81	1.199	2.041
10.03.82	1.635	2.140
20.12.82	2.086	2.144
18.04.83	1.615	1.854
18.07.83	1.270	2.381
17.10.83	1.381	2.250
30.01.84	1.701	2.240
21.05.84	1.007	2.211
20.08.84	1.171	2.083
29.10.84	0.861	1.807
29.07.85	1.507	2.132
11.11.85	1.181	2.074
24.02.86	0.925	1.161
21.04.86	1.759	2.345
23.05.86	1.772	1.623
29.09.86	1.590	2.202
30.03.87	1.982	2.202
14.10.87	1.941	2.195
21.14.87	1.471	1.783
28.10.87	1.812	2.156
30.03.88	1.353	1.555
30.09.88	1.502	2.336
30.03.89	1.111	2.610
04.10.89	1.185	2.249

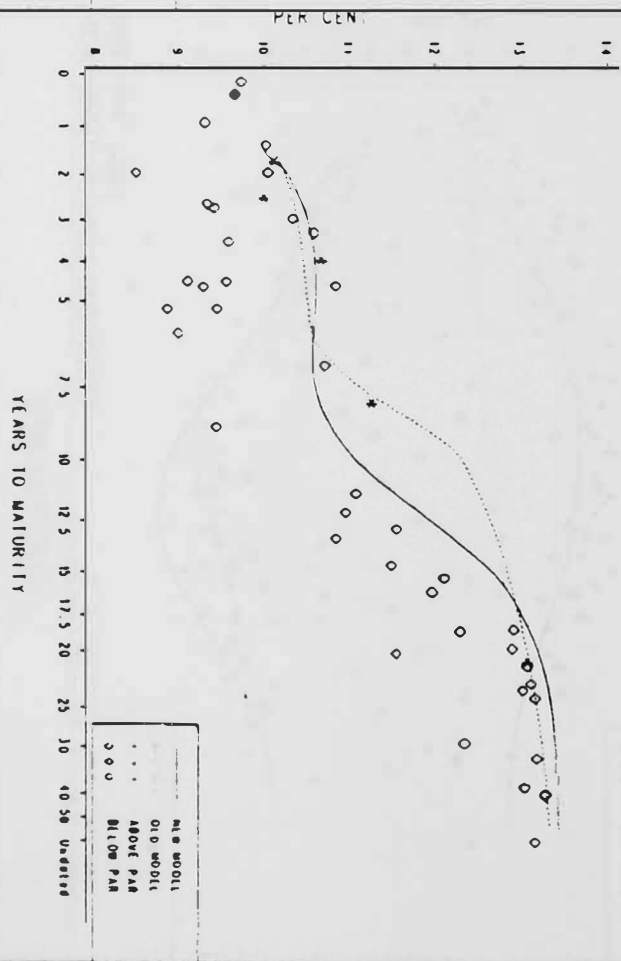
NEW MODEL AGAINST OLD MODEL 4th January, 1974



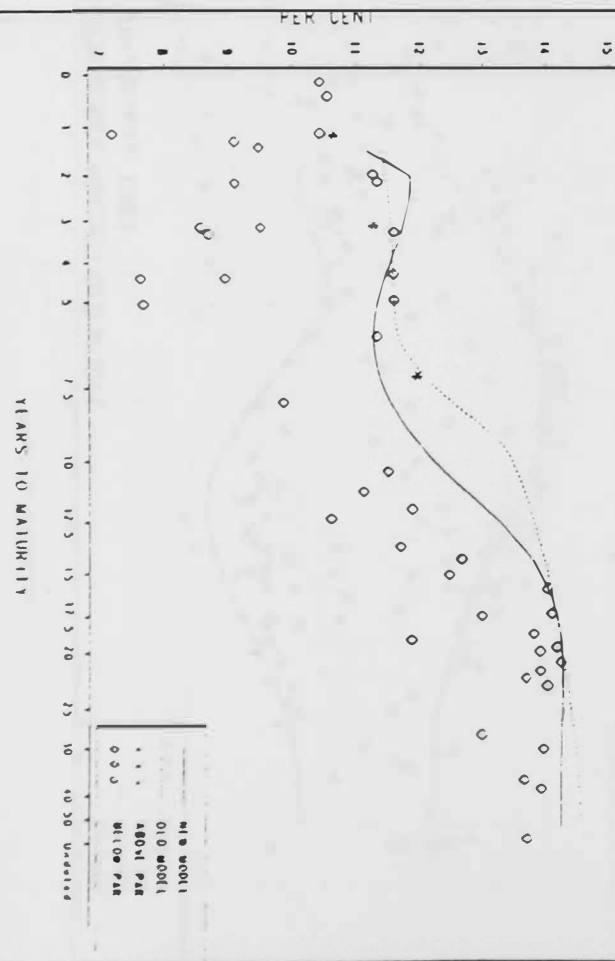
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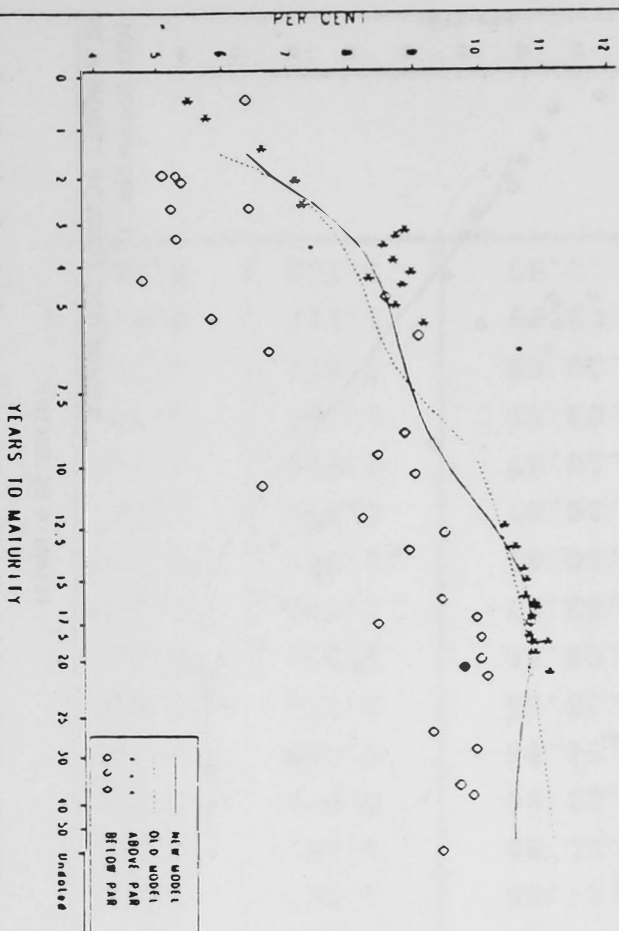
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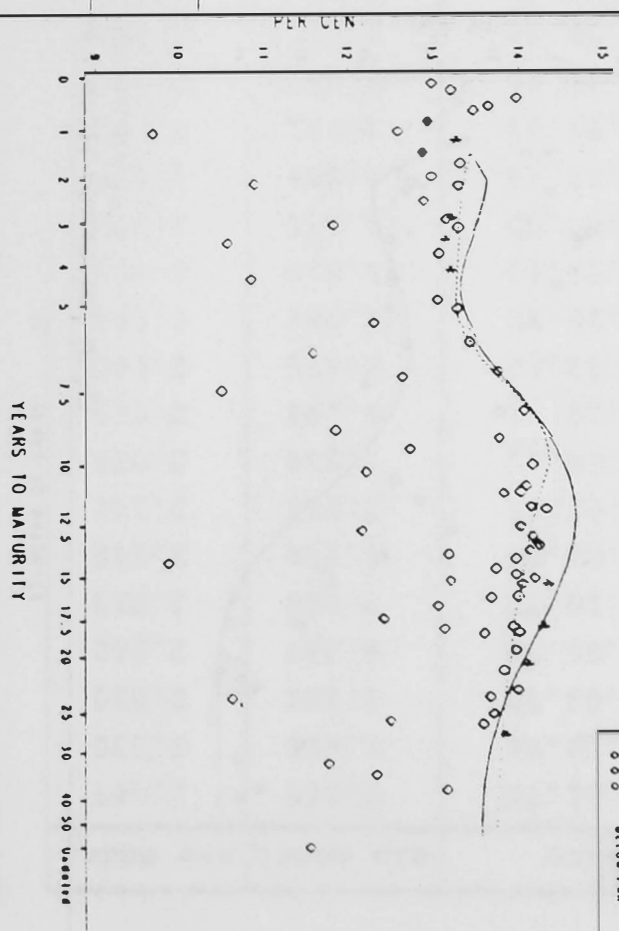
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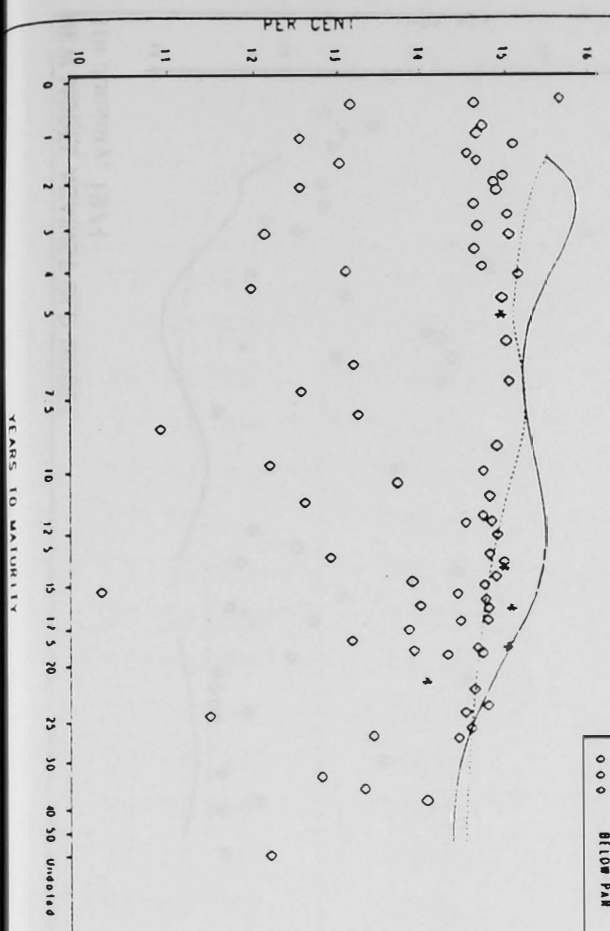
NEW MODEL AGAINST OLD MODEL
3rd October, 1977



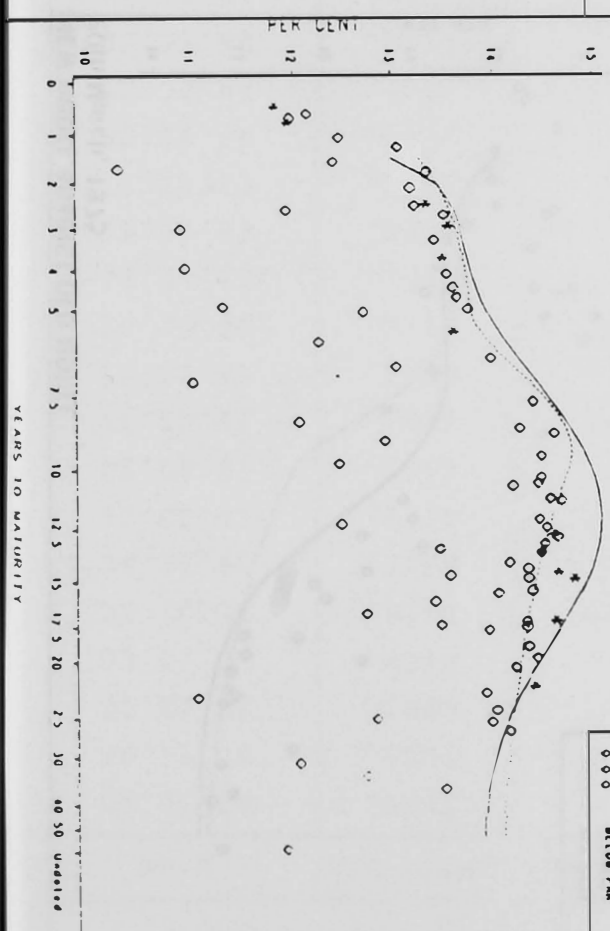
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5th January, 1981



NEW MODEL AGAINST OLD MODEL
4th January, 1980

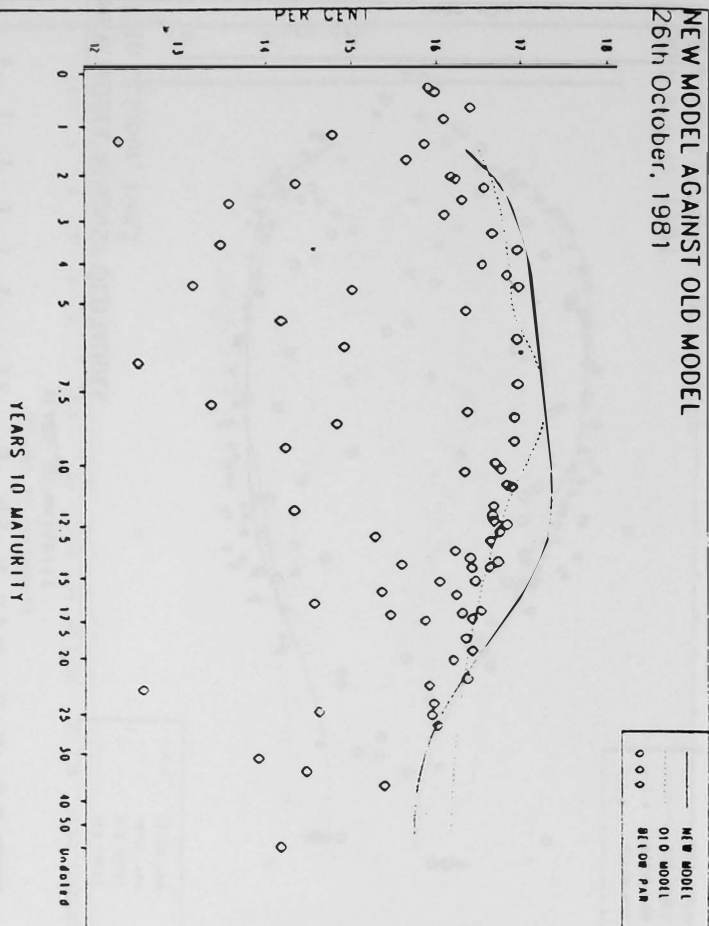


NEW MODEL AGAINST OLD MODEL
2nd June, 1981



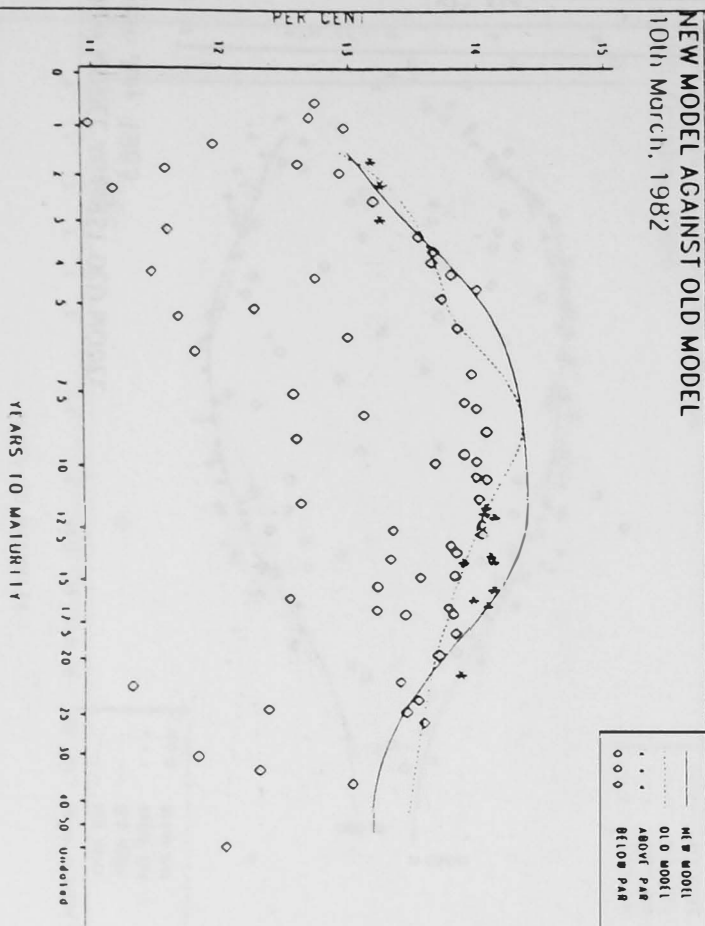
NEW MODEL AGAINST OLD MODEL

26th October, 1981



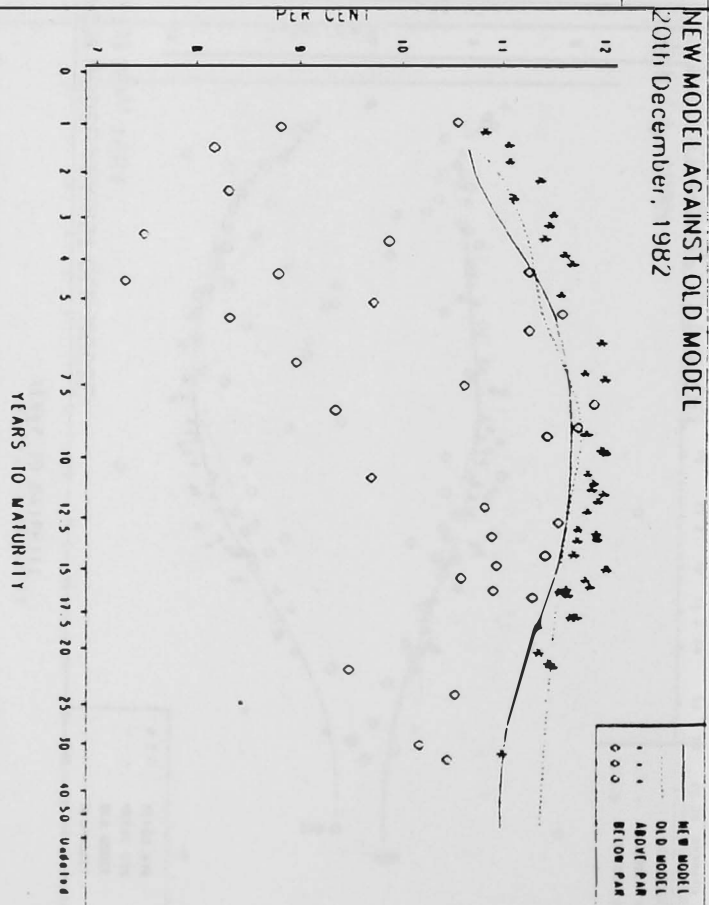
NEW MODEL AGAINST OLD MODEL

10th March, 1982



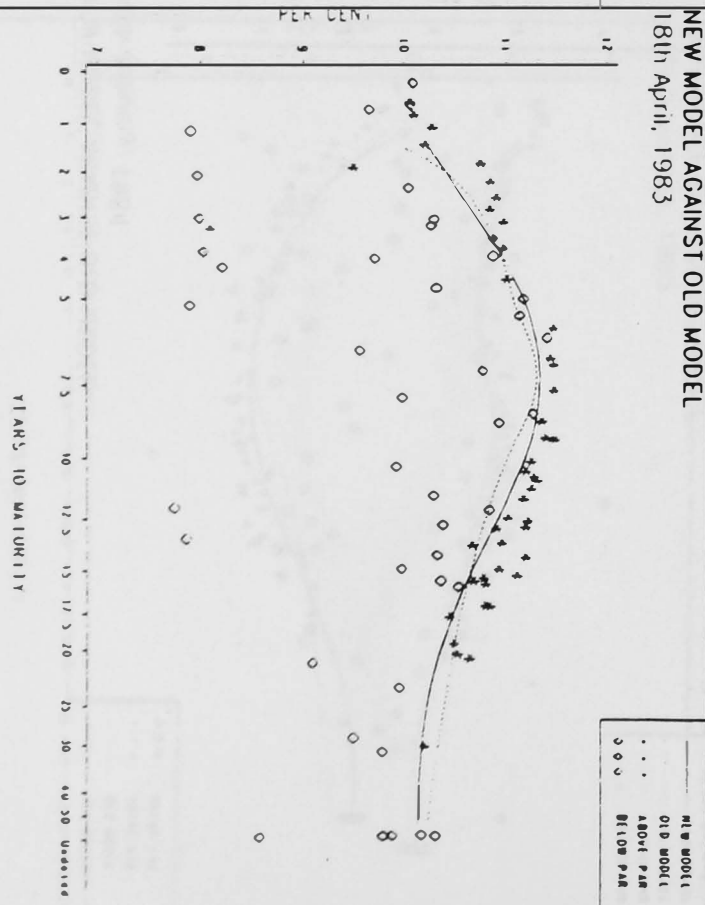
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20th December, 1982

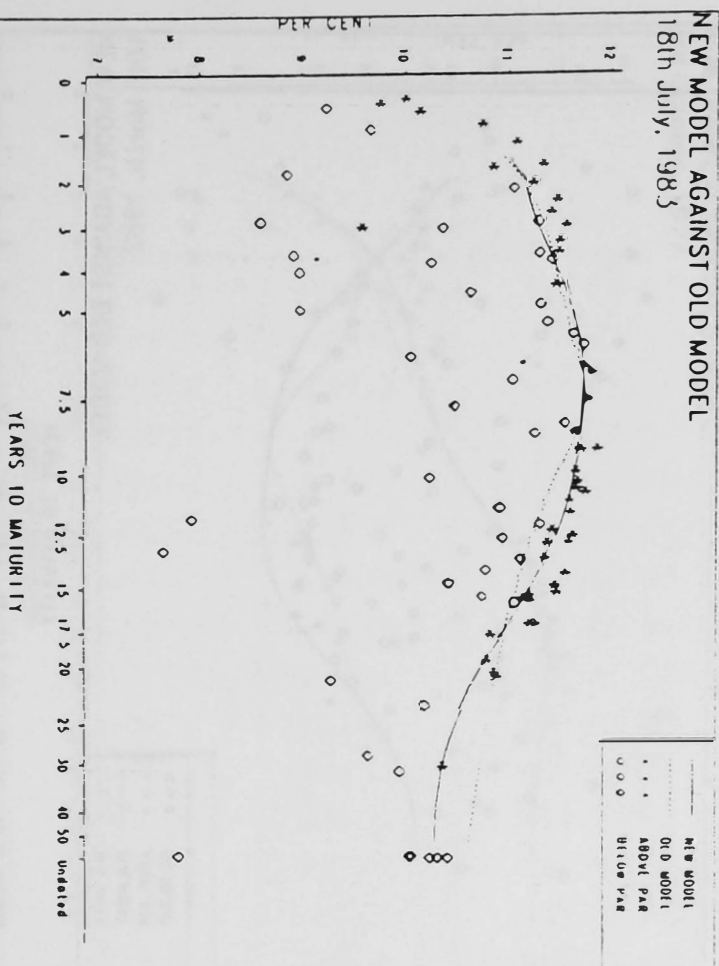


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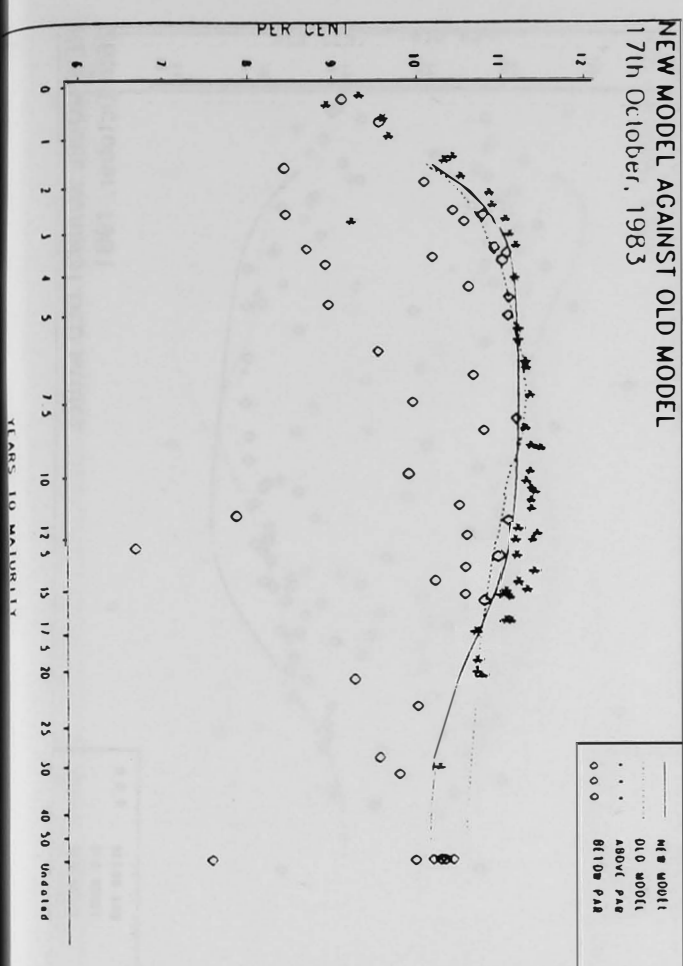
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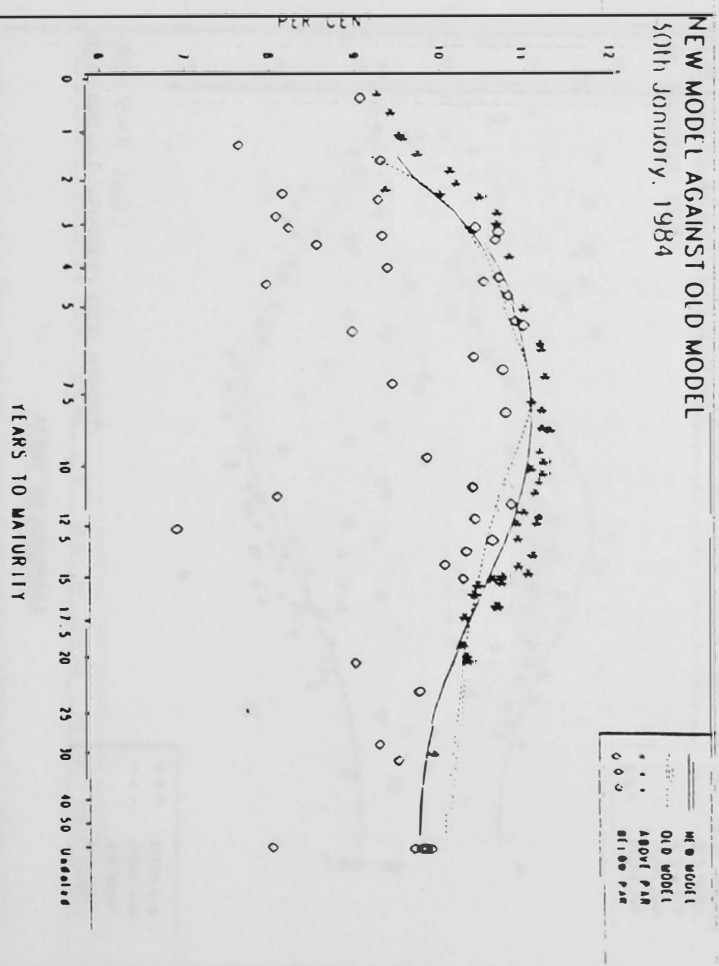
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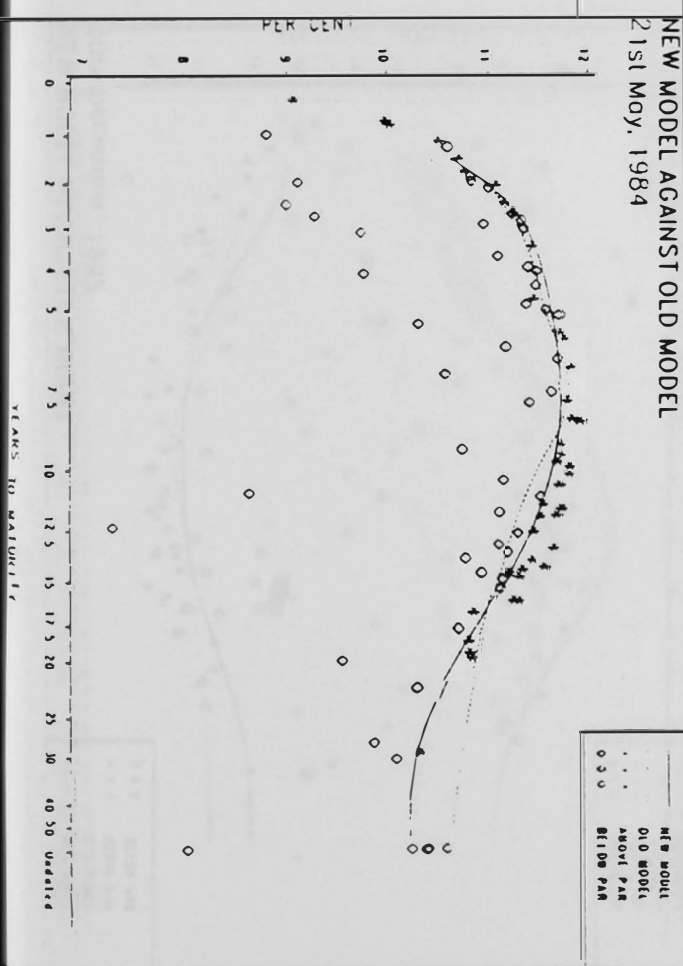
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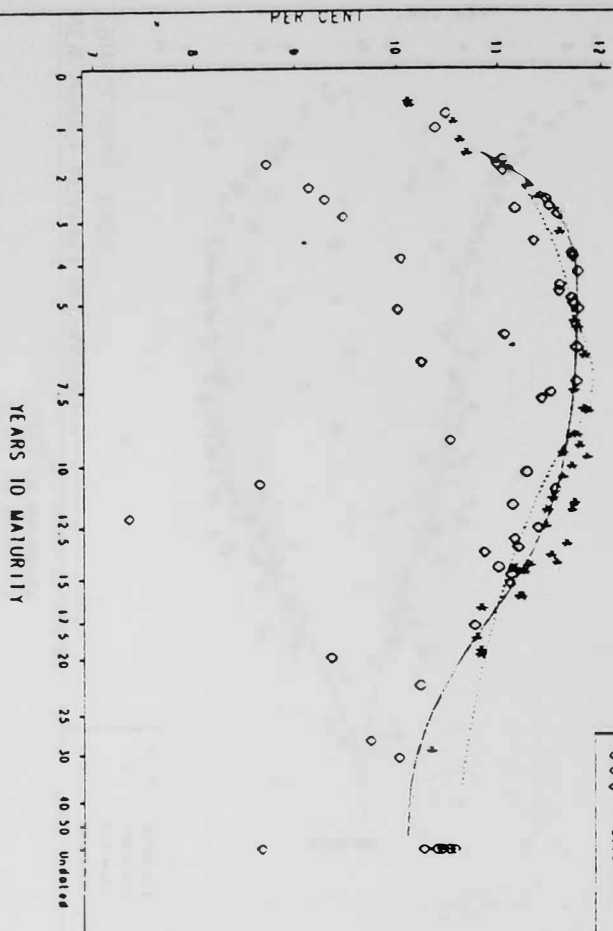
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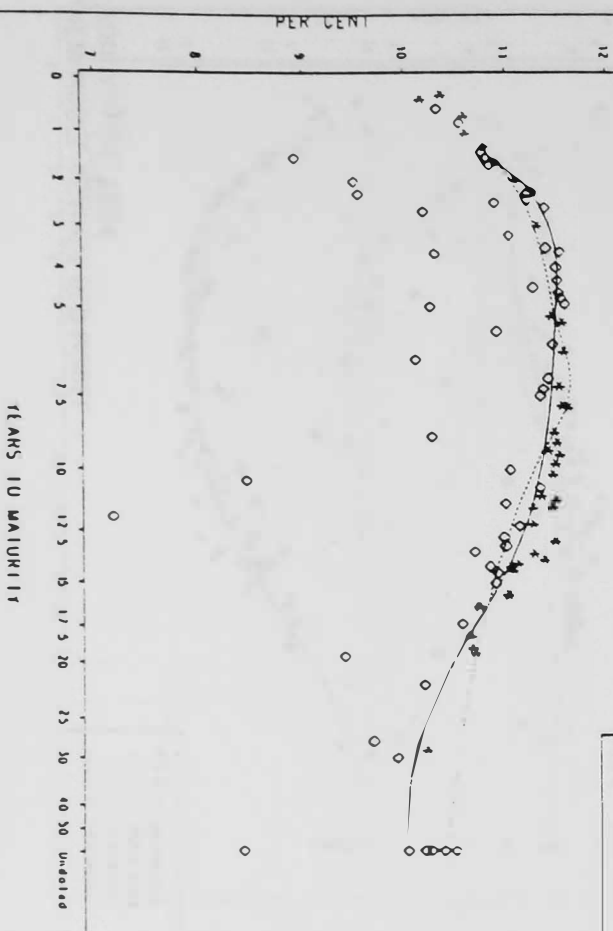
NEW MODEL AGAINST OLD MODEL 21st May, 1984



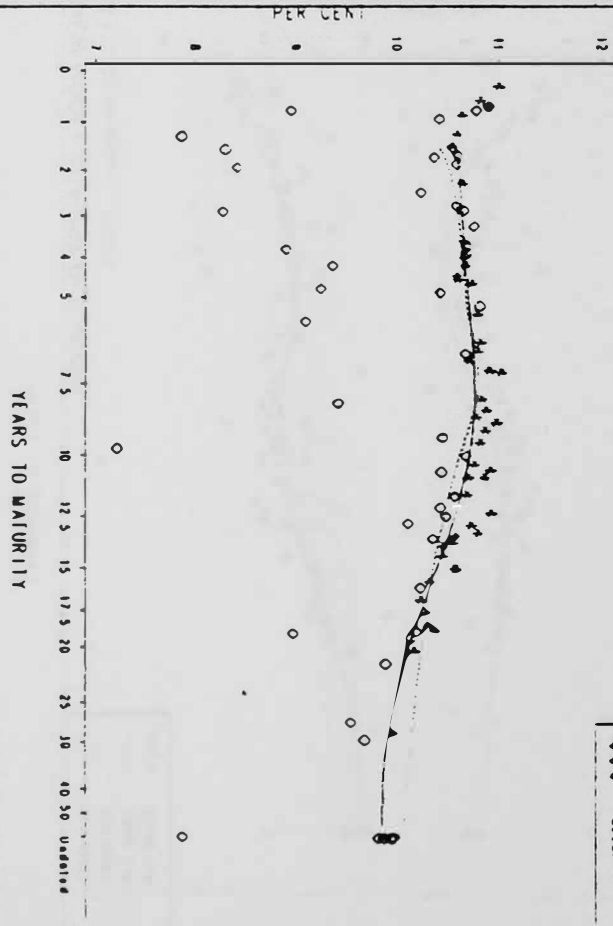
NEW MODEL AGAINST OLD MODEL 20th August, 1984



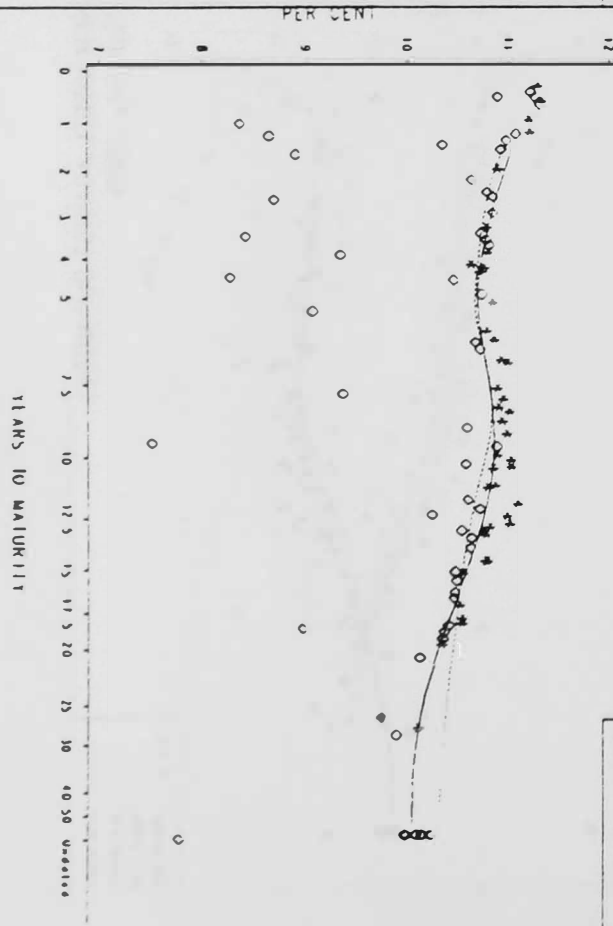
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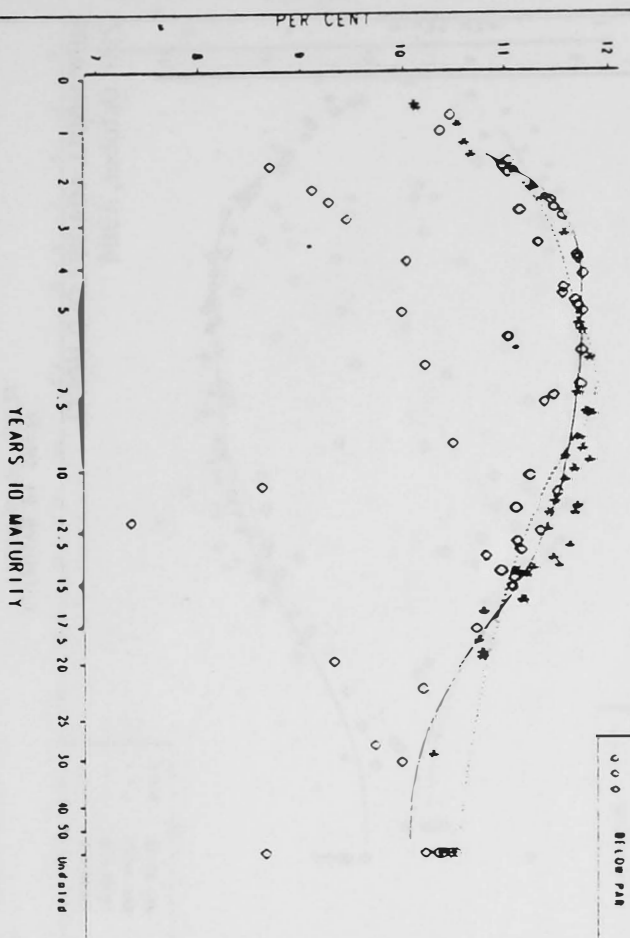
NEW MODEL AGAINST OLD MODEL 29th July, 1985



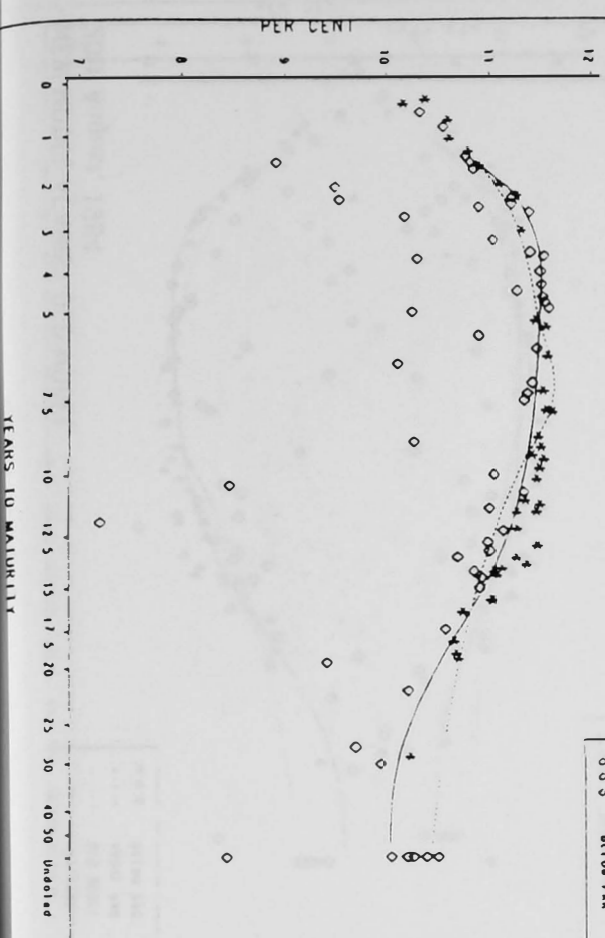
NEW MODEL AGAINST OLD MODEL 11th November, 1985



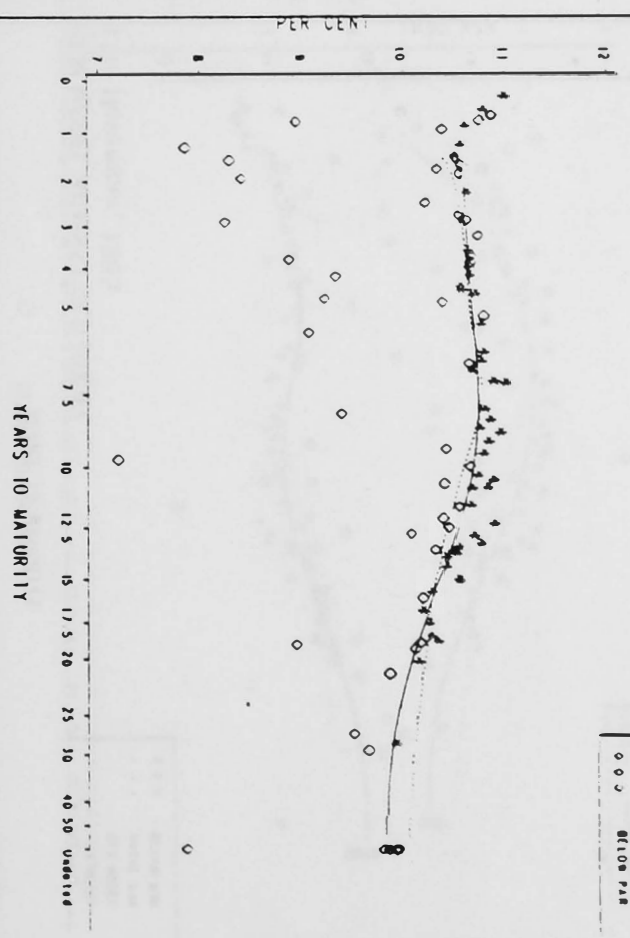
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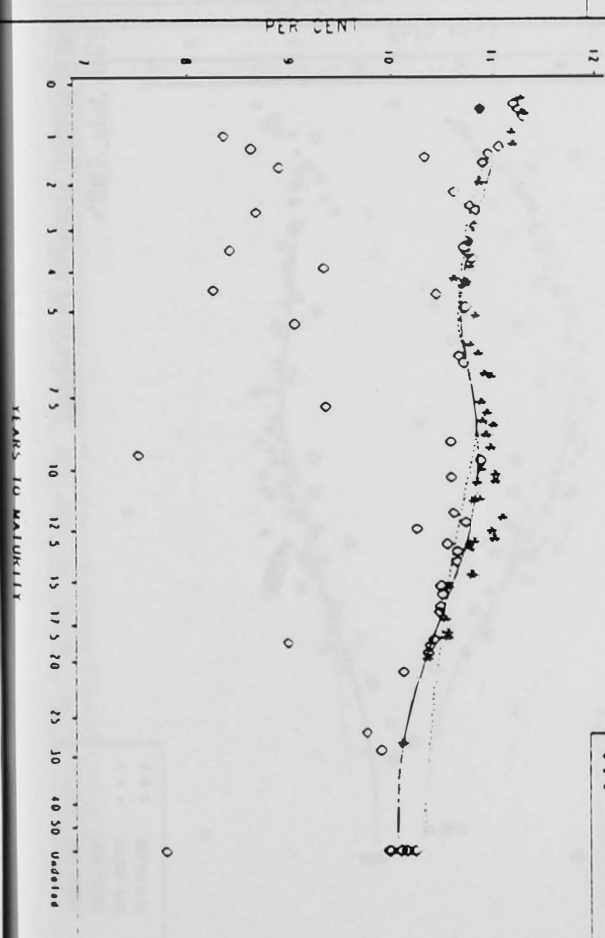
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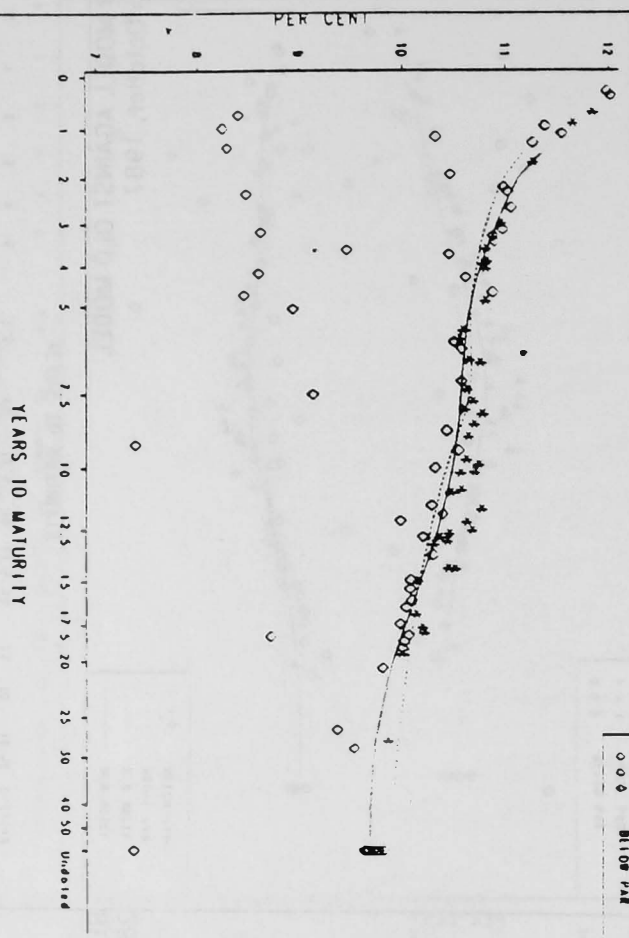
NEW MODEL AGAINST OLD MODEL 29th July, 1985



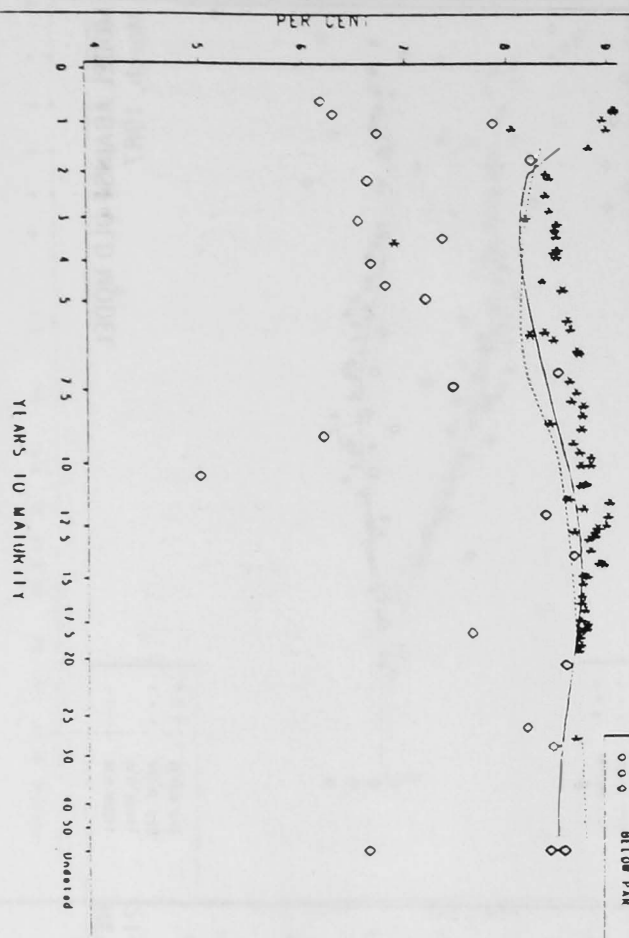
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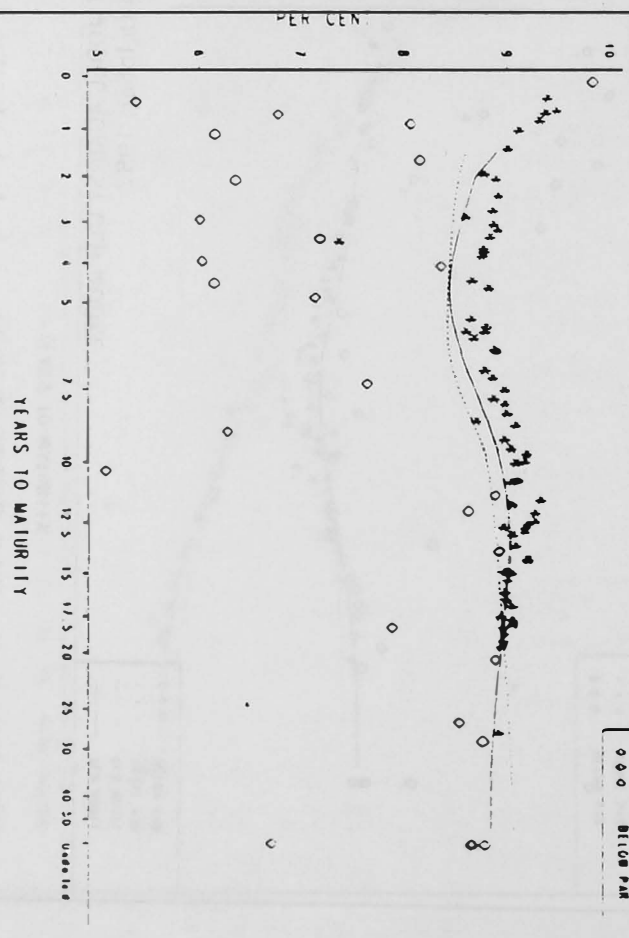
NEW MODEL AGAINST OLD MODEL
24th February, 1986



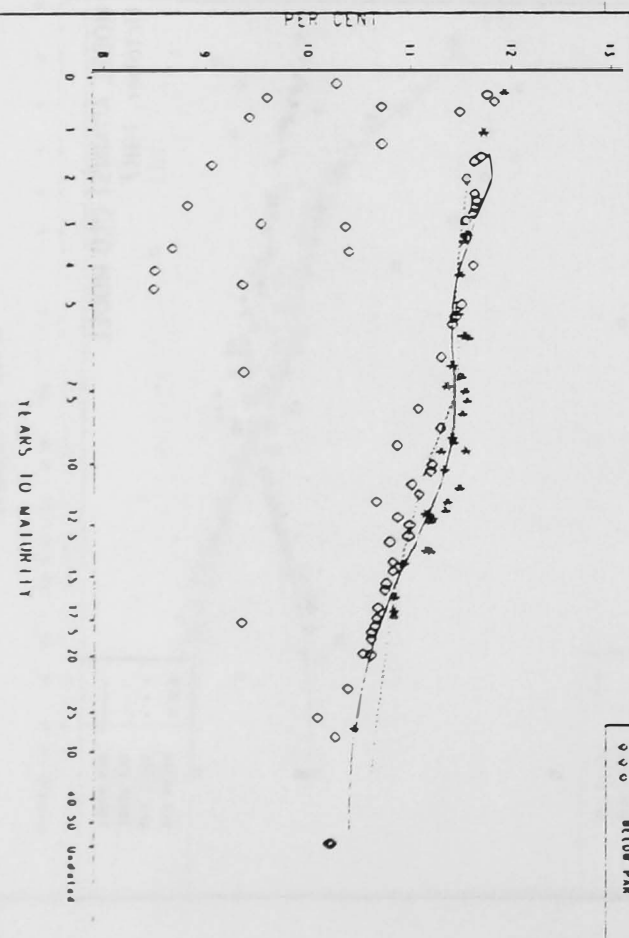
NEW MODEL AGAINST OLD MODEL
21st April, 1986



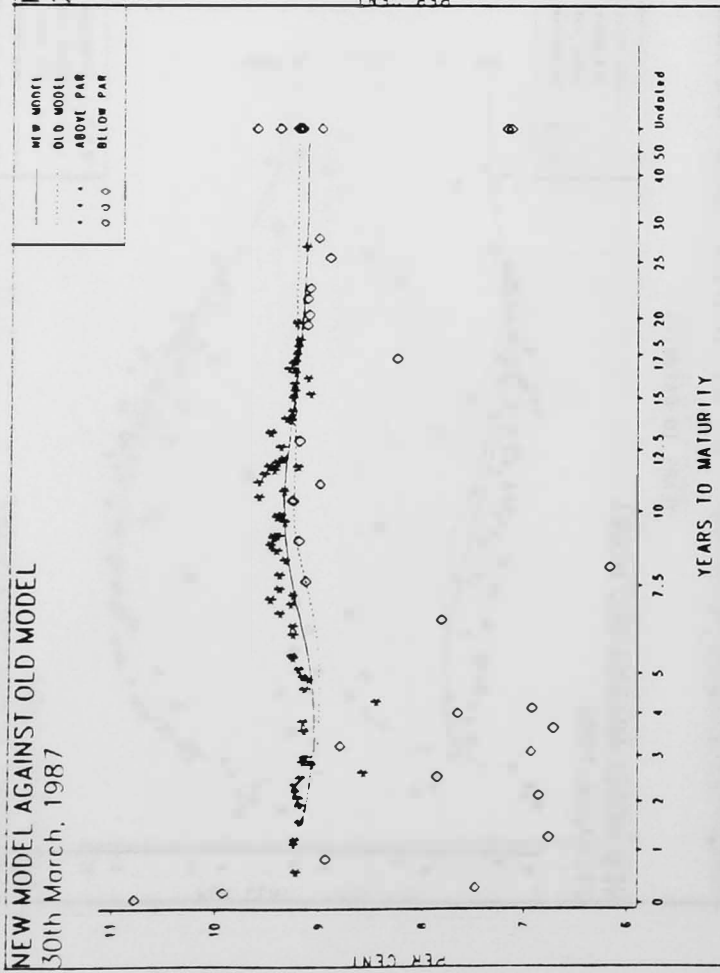
NEW MODEL AGAINST OLD MODEL
23rd May, 1986



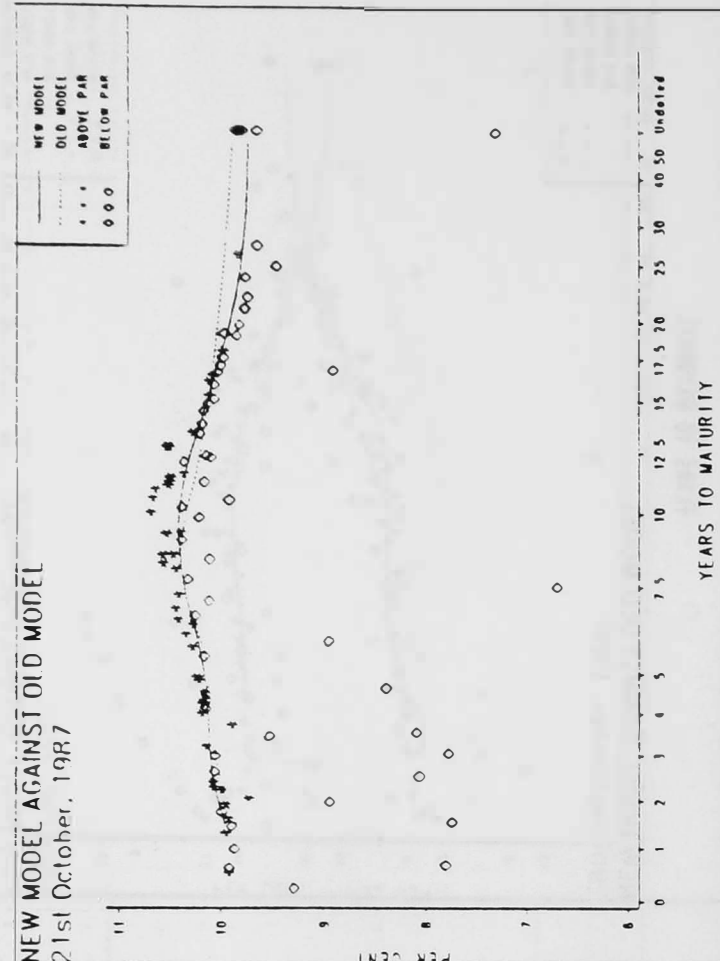
NEW MODEL AGAINST OLD MODEL
29th September, 1986



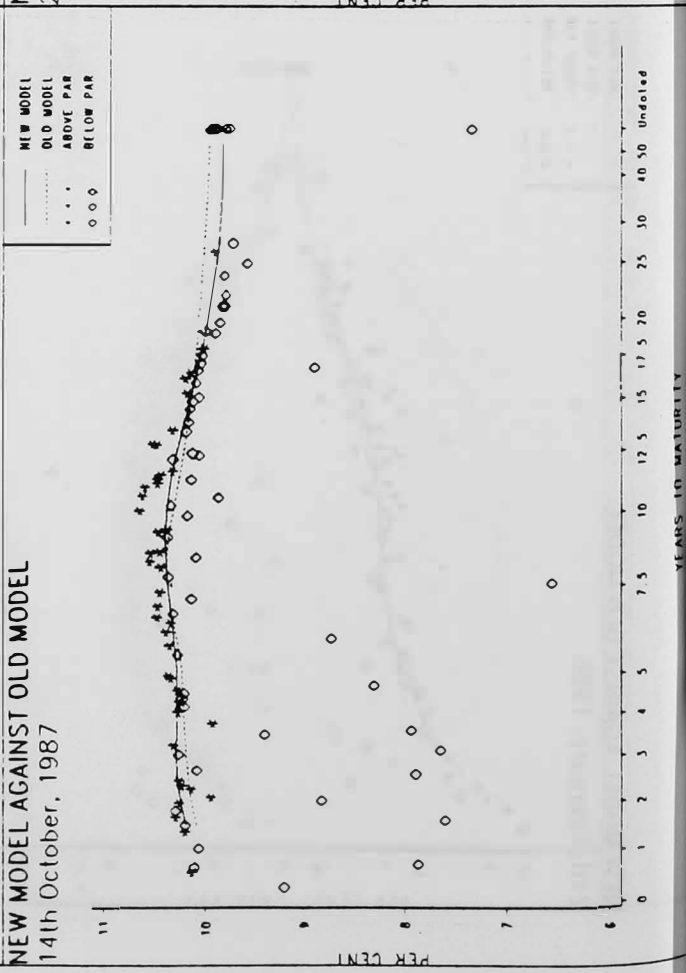
NEW MODEL AGAINST OLD MODEL
30th March, 1987



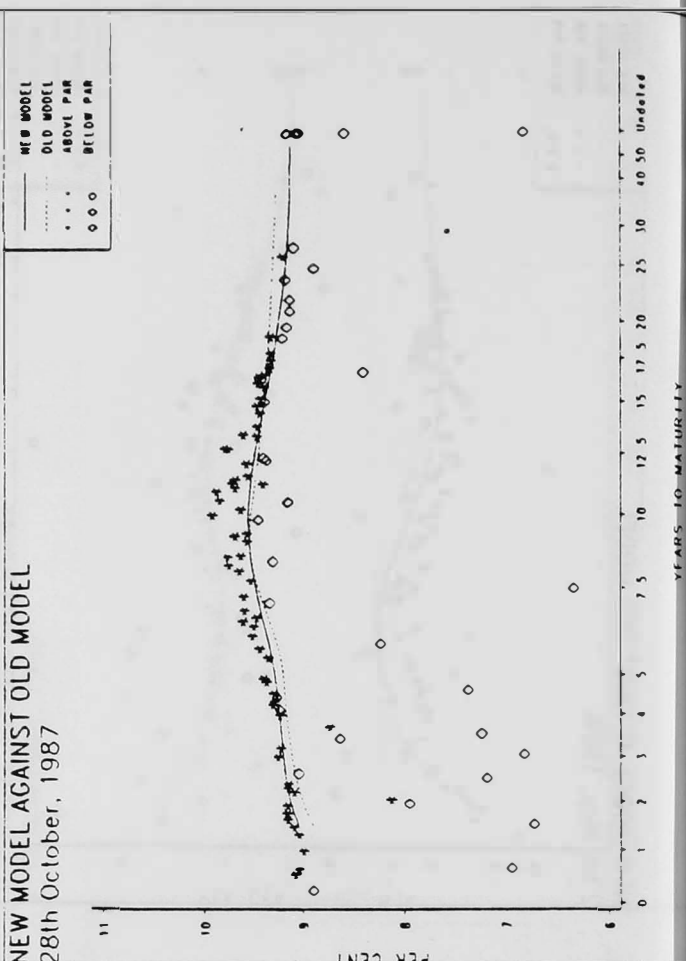
NEW MODEL AGAINST OLD MODEL
21st October, 1987



NEW MODEL AGAINST OLD MODEL
14th October, 1987

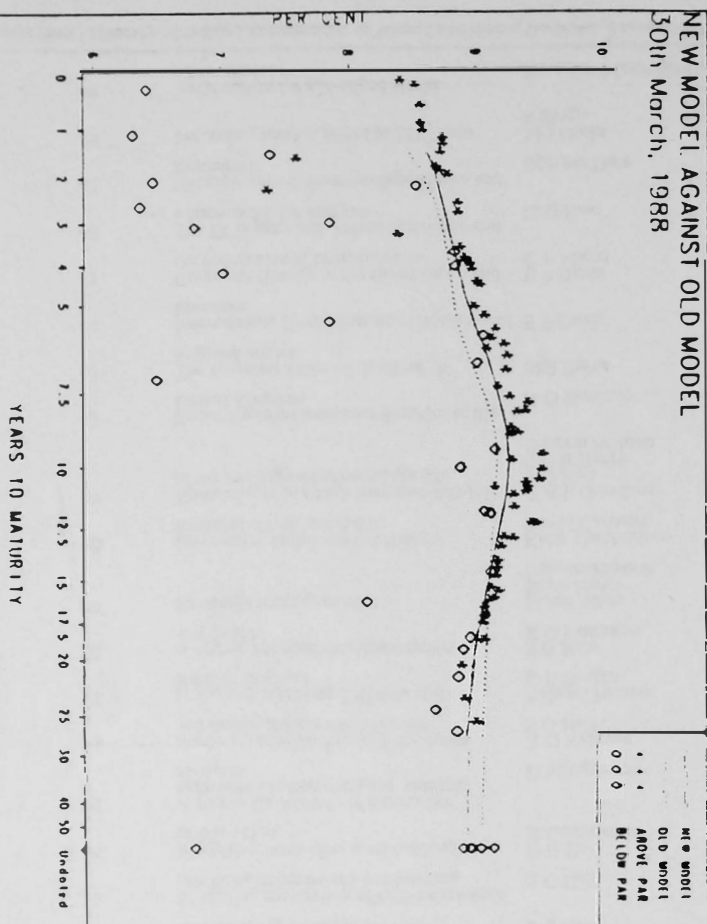


NEW MODEL AGAINST OLD MODEL
28th October, 1987



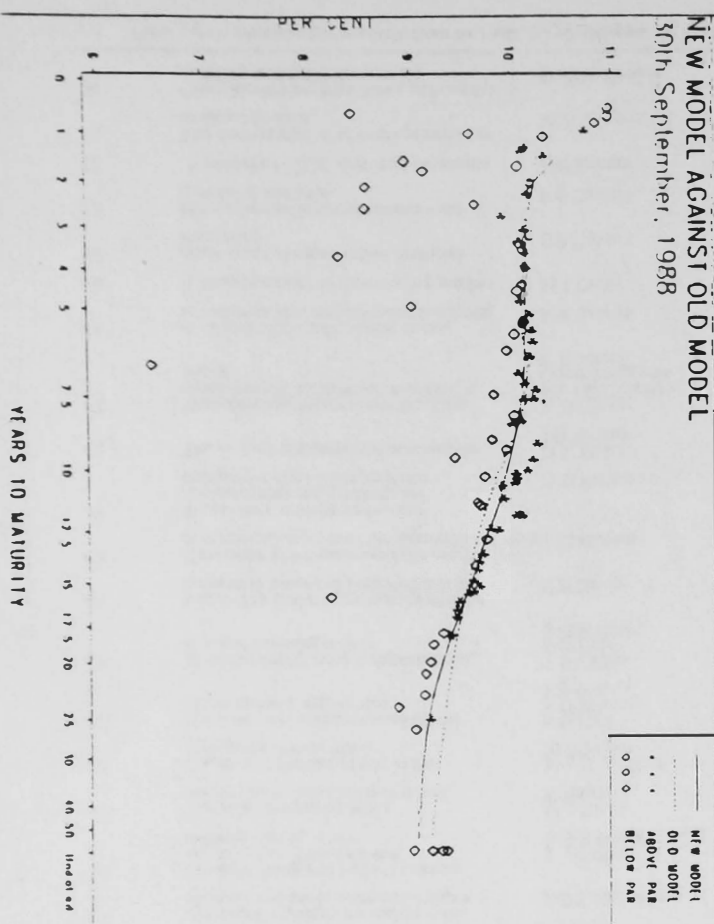
NEW MODEL AGAINST OLD MODEL

30th March, 1988



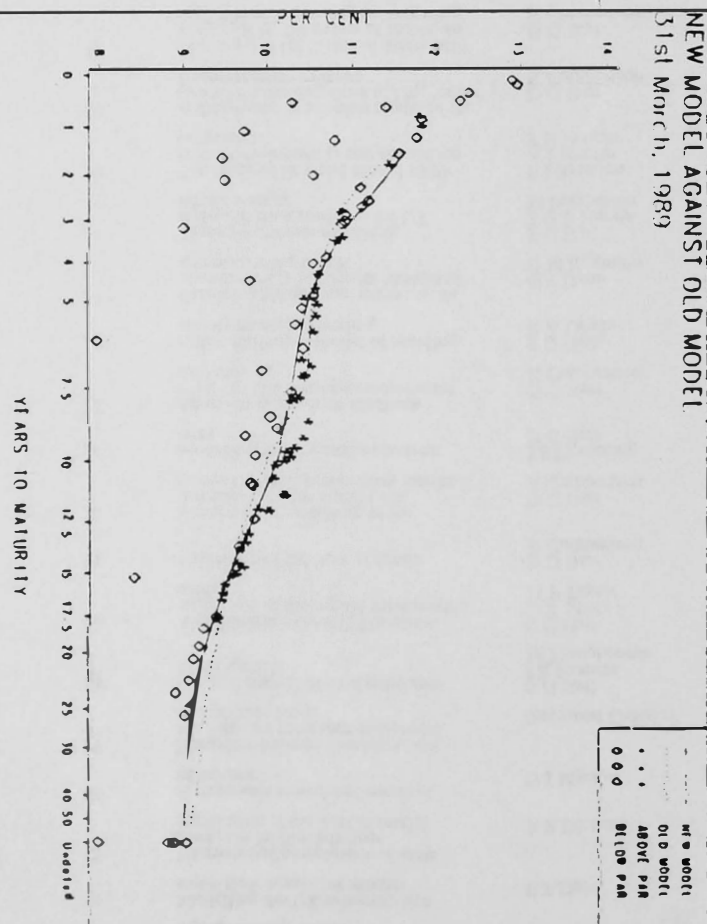
NEW MODEL AGAINST OLD MODEL

30th September, 1988



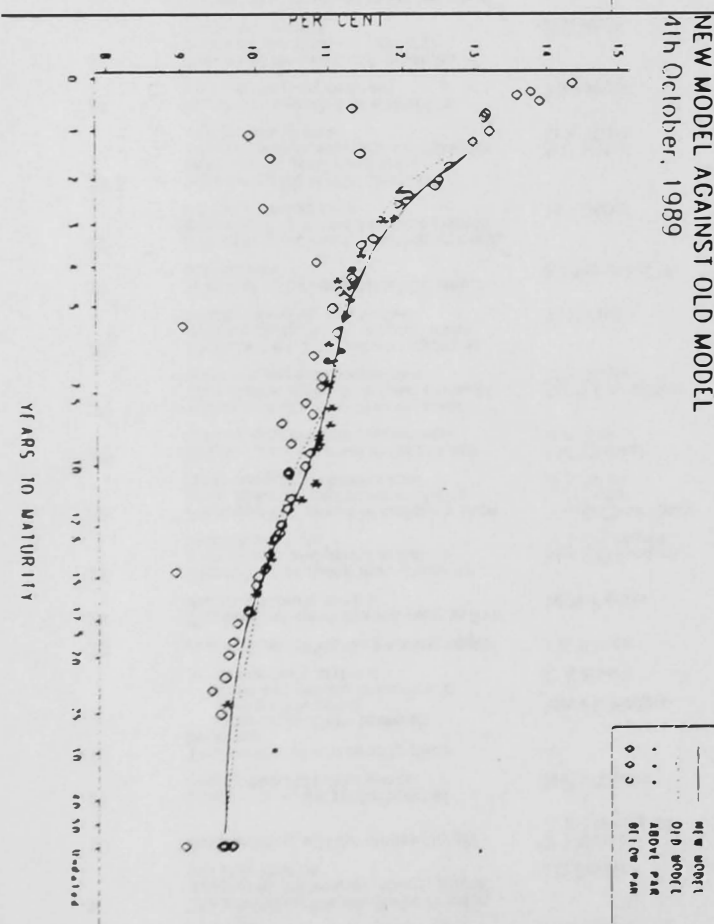
NEW MODEL AGAINST OLD MODEL

31st March, 1989



NEW MODEL AGAINST OLD MODEL

4th October, 1989



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Title		Author
1-5, 8, 11-14, 16-17, 19-22, 31 <i>These papers are now out of print, but photocopies can be obtained from University Microfilms International</i>		
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