A New Approach to Nowcasting with Mixed-Frequency Bayesian VARs

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Nowcasting and Mixed-Frequency Models

- Nowcasting is important for practitioners and policymakers: key statistics only collected on a quarterly basis, and often published with a significant lag.
- Lots of higher-frequency information can be exploited for a more timely assessment of the state of the economy, so great interest in efficient modelling of the information flow
- Successful nowcasting requires exploiting data at different frequencies
- Recent surveys: Bok et al. (2017), Luciani et al. (2017).
  - Initial focus of literature on factor models
  - VAR alternatives, since large BVARs with appropriate shrinkage have been shown to be competitive against factor models
  - (Mostly) state-space models for handling mixed-frequency

State-space models for handling mixed-frequency

Key idea: specify model at highest frequency, treat lower frequency data as missing and handle optimally using the appropriate filter.

- Factor models: Mariano and Murasawa (2003), Giannone et al. (2008), Aruoba et al., (2009), Banbura et al. (2011, 2013); many others
- DSGE models: Giannone, Monti, Reichlin (2010, 2016), Foroni and Marcellino (2014)
- BVARs - Long tradition:
  - Zadrozny (1988), Mittnik & Zadrozny (2005), Eraker et al. (2015), Schorfheide & Song (2015), Brave et al. (2016), Anderson et al. (2016), Cimadomo et al. (2018).

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- Stacking or blocking: treat higher-frequency data as multiple lower-frequency series.
  - Bittanti et al. (1988), Carriero, Clark and Marcellino (2015), Mogliani et al. (2017).
  - McCracken et al. (2015) for VARs.

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  - McCracken et al. (2015) for VARs.
- **This paper**: model specified at the highest frequency, but estimated at lower frequency.
  - We map the quarterly model into a carefully selected monthly counterpart.
  - Can turn many existing quarterly VARs into nowcasting models!

This Paper

New approach to deal with mixed-frequency data in VARs:

- Estimate a VAR(p) at quarterly frequency.
- Map it into a monthly model:

  1. Posit $VAR(3p-2)$ structure for the monthly model
  2. Many models will map into the quarterly VAR: we find the ones with real coefficients.
  3. Among these, choose the model that has the highest likelihood.

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(Pseudo-)Real-time forecast evaluation against Survey of Professional Forecasts and available alternatives.

From a Quarterly to a Monthly Model

Univariate Case

Quarterly $AR(1)$ model:

$$y_{t_q} = \phi y_{t_q - 1} + \varepsilon_{t_q} \qquad \varepsilon_{t_q} \sim \mathcal{N}\left(0, \sigma_\varepsilon\right)$$

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- Latent monthly variable x_{t_m} (and ε_{t_m}), corresponds to quarterly variable y_{t_q} when observed at the end of each quarter.
- In March, June, September and December:

$$x_{t_m} = \phi x_{t_m - 3} + \varepsilon_{t_m}$$

$$y_{t_q} = x_{t_m}, \quad \varepsilon_{t_q} = \varepsilon_{t_m}$$

Univariate Case

Posit a *monthly* model:

$$x_{t_m} = \phi_m x_{t_m - 1} + \varepsilon_{m,t_m} \quad \varepsilon_{m,t_m} \sim \mathcal{N}\left(0, \sigma_{\varepsilon_m}\right)$$

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- Iteration implies:

$$x_{t_m} = \phi_m^3 x_{t_m-3} + \underbrace{\varepsilon_{m,t_m} + \phi_m \varepsilon_{m,t_m-1} + \phi_m^2 \varepsilon_{m,t_m-2}}_{\varepsilon_{t_q}}$$

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- $\phi_m = \sqrt[3]{\phi}$: 3 roots: identification problem.
- Choose **real** cube root and recover σ_{ε_m} from

$$var\left(\varepsilon_{m,t_m}\right) = \left(1 + \phi_m + \phi_m^2\right)^{-1} var\left(\varepsilon_{t_q}\right)$$

Multivariate Case

Quarterly $VAR(p)$ (in companion form):

$$Y_{t_q} = \Phi Y_{t_q - 1} + \nu_{t_q}$$

with $Y_{t_q} = \left(y'_{t_q,\ldots,}y'_{t_q-p+1}\right)'$, $\nu_{t_q} = \left(\varepsilon'_{t_q}, 0'\right)'$, $\nu \sim \mathcal{N}(0, \Omega)$.

- Monthly (possibly latent) counterpart of Y_{t_q}: $X_{t_m} = \left(x'_{t_m}, \ldots, x'_{t_m-3p+3}\right)'$. In the last month of each quarter

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Posit a *monthly* model:

$$X_{t_m} = \Phi_m X_{t_m - 1} + \nu_{m, t_m}$$

where $\nu_m \sim \mathcal{N}(0, \Omega_m)$ and

$$\Phi_m = \begin{bmatrix} \Phi_{m11} & \Phi_{m12} & \ldots & \Phi_{m1p} \\ \Phi_{m21} & \ddots & & \\ \vdots & & \ddots & \\ \Phi_{mp1} & & & \Phi_{mpp} \end{bmatrix} \qquad \Omega_m = \begin{bmatrix} \Sigma_{\varepsilon_m} & 0_n & \ldots & 0_n \\ 0_n & \ddots & & \\ \vdots & & \ddots & \\ 0_n & & & 0_n \end{bmatrix}$$

Multivariate Case

- The first n rows correspond to a restricted monthly $VAR(3p-2)$:

$$x_{t_m} = \Phi_{m11} x_{t_m-1} + \Phi_{m12} x_{t_m-4} + \cdots + \Phi_{m1p} x_{t_m-3p+2} + \varepsilon_{m,t_m}$$

- The remaining $n(p-1)$ rows impose restrictions on the monthly disturbances.
  - Intuition: monthly states need to match quarterly observables at the end of each quarter.

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- Caveat: if data is averaged over the 3 months of the quarter, a VAR might not be a good approximation (empirical matter).

Multivariate Case

$\Phi = VDV^{-1}$. Then $\Phi^{\frac{1}{3}} = VD^{\frac{1}{3}}V^{-1}$.

- **Multiple real solutions** are possible: 3^k if k pairs of complex conjugate eigenvalues of Φ.
- Anderson, Deistler, et al. (2016) show g-identifiability when (enough) high frequency data is available.
- We evaluate likelihood of solutions using the Kalman filter and pick the one with the highest likelihood.

$\rightarrow$ corresponds to the one with the roots with the smallest argument.

Likelihood and smoothness

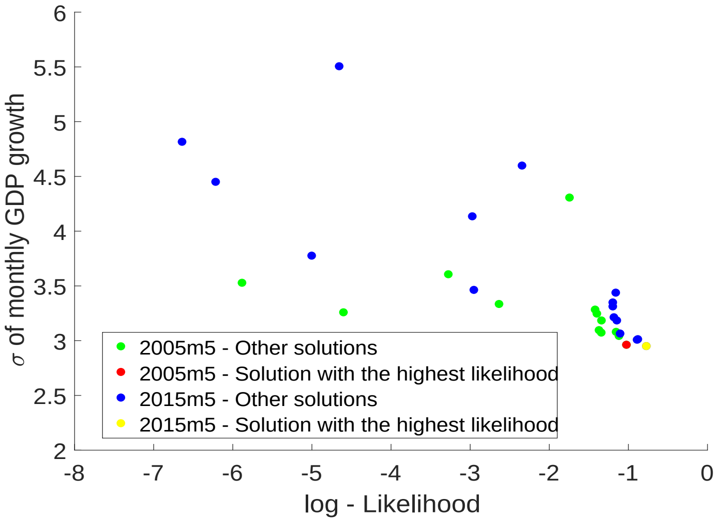


Figure : Likelihood and "smoothness" of the cube root solutions for two sample dates

Forecast Evaluation

Nowcasting US GDP

Pseudo-real-time dataset 1965M1-2016M2, forecast evaluation window: 1990M1:2015M2. Point forecast evaluation.

- Nowcasts perform only slightly worse than SPF.
- Both stacking and full state-space approach shown to perform similarly to our method.

Table : Data and Release Calendar.

Series	Frequency	Release Date
ISM Manufacturing Index	Monthly	End of current month
Total Nonfarm Payrolls	Monthly	Beginning of subsequent month
Industrial Production	Monthly	Middle of subsequent month
Retail Sales (ex Food Services)	Monthly	Middle of subsequent month
Disposable Income	Monthly	End of subsequent month
GDP	Quarterly	End of first subsequent month

Note: The ISM Manufacturing index was since removed from the FRED and ALFRED databases, but was still available in the 2016M2 data vintage at the time of the data download.

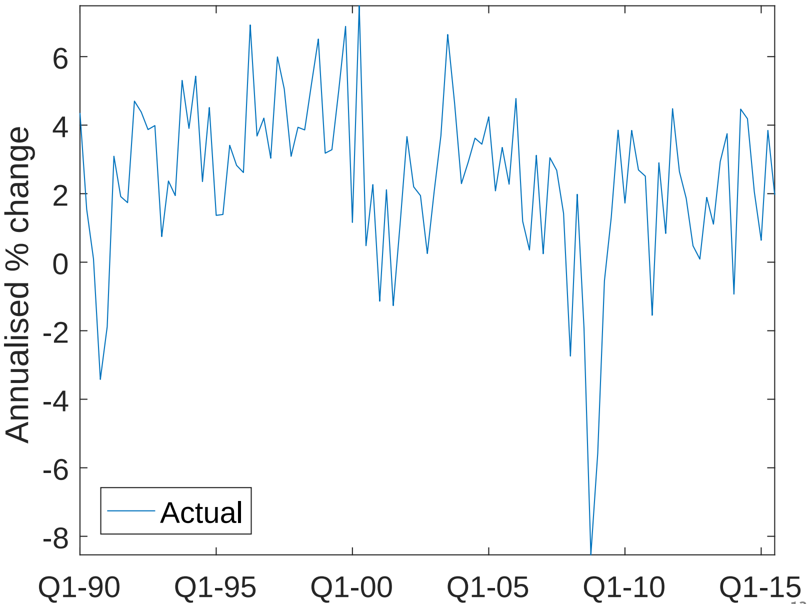
Point Forecast Evaluation: Nowcast

Table : MSFE of Nowcasts vs Naive Model

	m1	**m2**	**m3**	
Q balanced VAR	naïve	7.263	-1.929	-1.929
			(1.301)	(1.301)
SPF	SPF		-3.666	
			(1.336)	
This paper	Q2M-BVAR	-0.950	-2.781	-3.793
		(0.4207)	(1.150)	(1.145)
Stacking	Stack-BVAR	-1.698	-2.504	-3.746
		(0.932)	(1.557)	(1.548)
Full estimation	MF-BVAR	0.281	-1.200	-2.600
		(0.855)	(1.251)	(1.394)

Note: Mean squared forecast errors (MSFE), differences from naive benchmark, with HAC-adjusted standard deviations reported in brackets.

Nowcasts vs Outturns: GDP growth



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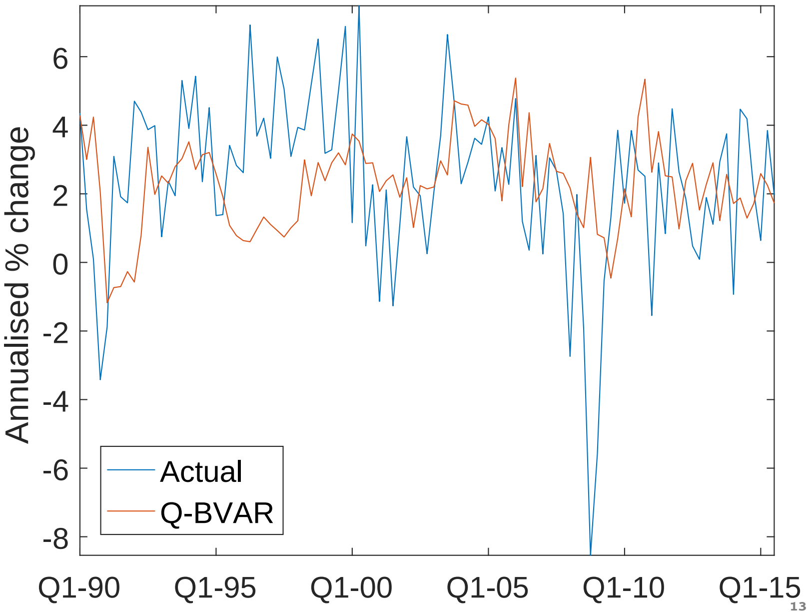

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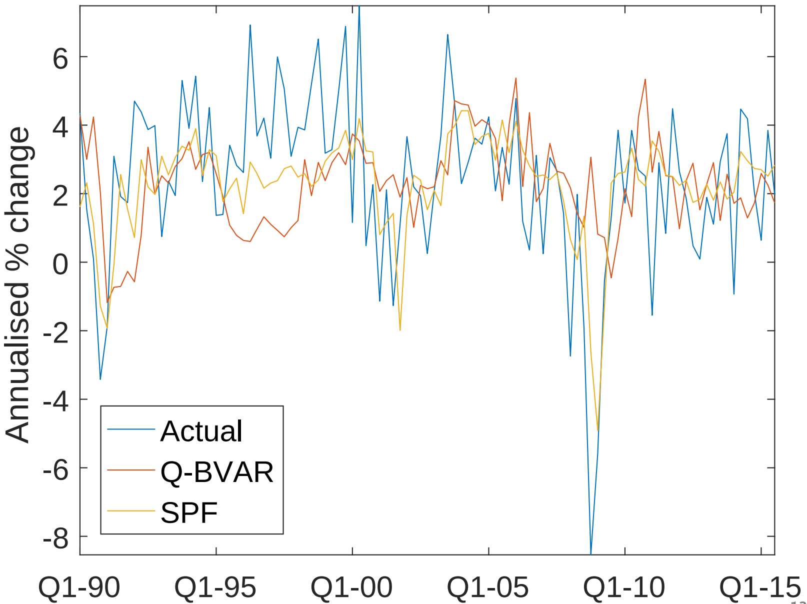

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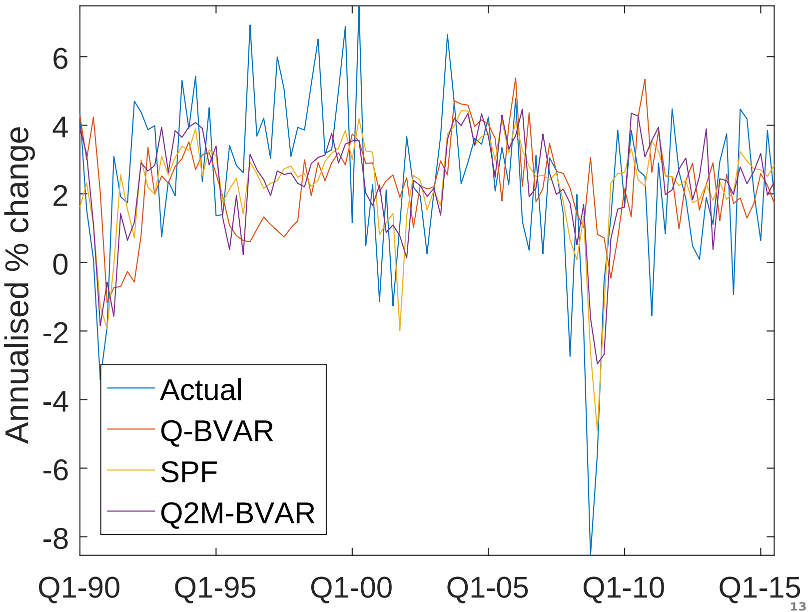

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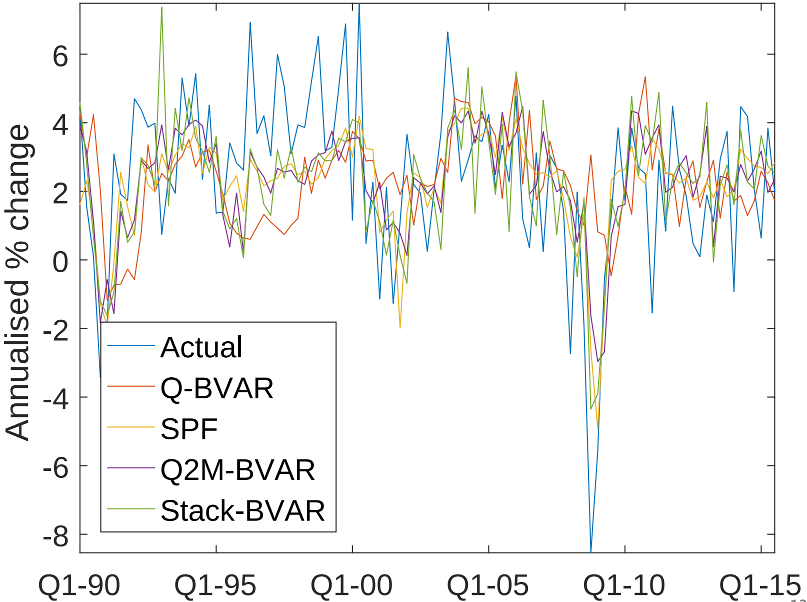

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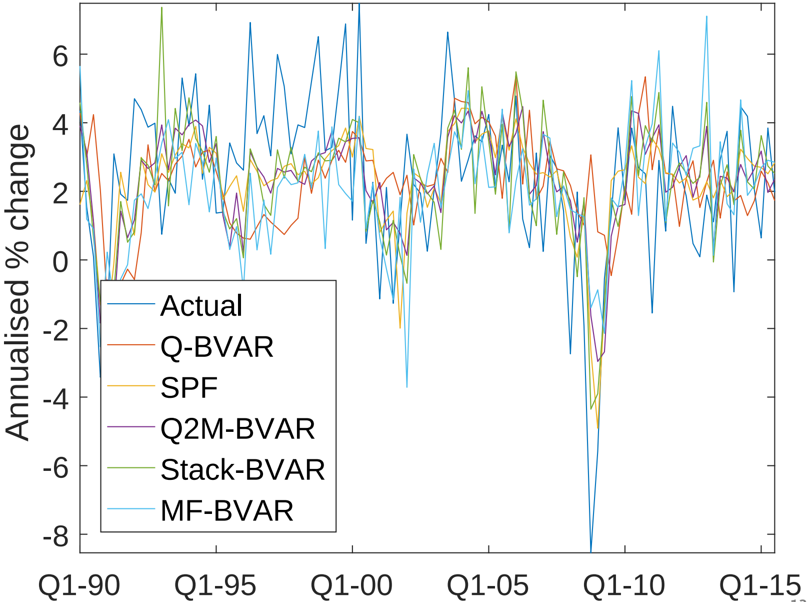

Conclusion and next steps

- Mixed-frequency VAR using a model estimated at low frequency.
- Accurate and scalable approximation.
- Can be used to easily produce nowcasts with existing models estimated at lower frequency.
- Works as well as alternative approaches to deal with mixed frequency data for nowcasts.

Next steps

- Extend to a large dataset
- Real-time analysis

Appendix

MF-BVAR vs Q-BVAR

Table : Relative RMSFE of Nowcasts, Mixed-Frequency BVAR versus Quarterly BVAR.

	MF-BVAR m1	**MF-BVAR m2**	**MF-BVAR m3**
Full sample	0.931∗∗∗	0.915 ∗∗	0.807∗
Pre-crisis	0.935∗∗∗	0.902 ∗∗	0.778∗
Post-crisis	0.927∗∗∗	0.933	0.848 ∗∗

Note: Relative RMSFE of forecasts for the current quarter made on the first Friday of each month of the current quarter against forecasts made using the quarterly model. Note that MF-BVAR m1 is compared against Q-BVAR m1, while MF-BVAR m2 and m3 are compared against Q-BVAR m2, which has one more quarter of data than Q-BVAR m1 due to the intervening GDP release. (*), (**) and (***) denote statistical significance at the 10, 5 and 1% level, respectively.

Q-BVAR and MF-BVAR vs SPF

Table : Relative RMSFE of Nowcasts, Quarterly and Mixed-Frequency BVAR versus Survey of Professional Forecasters.

	Q-BVAR m1	**MF-BVAR m1**	**Q-BVAR m2**	**MF-BVAR m2**	**MF-BVAR m3**
Full sample	1.421∗∗∗	1.325∗∗∗	1.218∗∗∗	1.116∗∗∗	0.982
Pre-crisis	1.347∗∗∗	1.261 ∗ ∗	1.181 ∗ ∗	1.069∗	0.917∗
Post-crisis	1.544	1.431	1.281∗∗∗	1.196∗∗∗	1.089

Note: The columns report relative RMSFE between forecasts for the current quarter from our models and the SPF. The informaton set available at the time of the SPF forecast is comparable to that of the Q-BVAR and MF-BVAR estimated at the beginning of the second month. (*), (**) and (***) denote statistical significance at the 10, 5 and 1% level, respectively.

Nowcast: Different Approaches vs SPF

Table : Relative RMSFE of Nowcasts, Alternative approaches to Mixed-Frequency BVAR versus Survey of Professional Forecasters.

	MF-BVAR m2	**Block-BVAR m2**	**CGL-BVAR m2**
Full sample	1.116***	1.150 **	1.298***
Pre-crisis	1.069*	1.164 **	1.273***
Post-crisis	1.196***	1.126	1.341 **

Note: The columns report relative RMSFE between forecasts for the current quarter from the models and the SPF. The informaton set available at the time of the SPF forecast is comparable to that of the models. (*), (**) and (***) denote statistical significance at the 10, 5 and 1% level, respectively.