Combining Survey Long-Run Forecasts and Nowcasts with BVAR Forecasts using Relative Entropy[1]

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Introduction

- VARs are popular tools for forecasting; produce accurate forecasts
- Banbura, Giannone, and Reichlin (2010) showed large VARs work ok
  - Resurgence in use of VARs for forecasting and policy analysis
- Fancier VARs: time-varying parameters, regime switching
  - Good forecasting properties but not necessarily better
    - Competitive to fixed-parameter VARs est.1985+ sample
      e.g. Aastveit, Carriero, Clark, and Marcellino, 2017
    - Outperforms simple VARs est. 1960+ for inflation and interest rates; mixed-evidence for real variables
      e.g. D'Agostino, Gambetti, and Giannone, 2013; Barnett, Mumtaz, and Theodoridis, 2014; Aastveit et al, 2014
  - Additional computational demands and complexity
- Constant parameter VARs remain popular for forecasting

Introduction

- Unrestricted long-run forecasts converge to ergodic mean of sample
  - Problematic as at times ergodic mean overlooks external forces (e.g. inflation target; demographic factors) that informs economists' view
  - Poses communication challenge for Monetary Policy, e.g. inflation 3-year out 3.5% from a model estimated with 1960+ data

- Beyond 4 quarters, forecasts increasingly influenced by model's implied steady-state (Clements and Hendry, 1999; Clark and McCracken, 2008)
  - Inflation forecasts 1 to 3 years out likely biased upwards
  - Why not then estimate using a shorter sample that provides more reasonable trend forecast? One possible route

- Some may prefer longer-sample when interest in **forecasts of multiple variables** using a **single multivariate model**
  - Recent popular papers on VAR (e.g. Banbura et al, 2010; Koop, 2013; Carriero et al, 2015) all focused on longer sample

Introduction

- Survey Long-horizon projections reasonable proxy for underlying trends, such as potential growth, natural rate of unemployment, r-star (e.g. Faust and Wright, 2013)
  - adjust more rapidly in response to changes in underlying fundamentals such as demographic factors not featured in VARs
  - knowledge of inflation target, central bank communications

In this paper

- Propose a systematic approach to influence forecasts of implied trends from VAR models to values informed from external surveys
- Utilize the technique of relative entropy
  - To tilt the long-horizon VAR forecast of **select** variables towards the long-horizon survey expectations
  - fixed-parameter VARs (short and long sample) and time-Varying VAR
  - Survey of Professional Forecasters (SPF) as it is publicly available
- Implications on forecast accuracy of **all** VAR variables over forecast horizon of interest to monetary policy makers (i.e. 1 to 12 quarters)
- Previous research highlights role of nowcasts to improve multi-horizon forecast accuracy (Kruger et al. 2017; Knotek and Zaman, 2017)
  - also tilt VAR one-quarter ahead forecasts to survey nowcasts

Preview of results

- Improvements in forecast accuracy of VAR forecasts tilted to survey long-run forecasts **and** nowcasts (**hybrid forecast**)
  - All models benefit; gains largest for fixed-parameter VAR est. with longer sample and smallest for time-varying VAR

- Time-Varying VAR: significant gains for inflation but small for others

- Constant parameter VAR with longer sample
  - Notable improvements for many variables with biggest gains for price inflation, wage inflation, and interest rates
  - Forecast accuracy for inflation competitive to univariate benchmarks
  - And rivals forecast accuracy from time-varying VAR

- These gains are made possible because our proposal mitigate misspecification issues arising from structural breaks

Related Research

- Incorporating Survey Long-Run Projections into VAR models
  - Wright (2013) uses steady-state VAR of Villani (2009) and sets prior values for steady states informed from Blue Chip survey; stationary VAR and MCMC
  - Modeling in Gaps, i.e. deviation from time-varying trends informed from survey (e.g. Clark and McCracken, 2010; Clark, 2011, Zaman, 2013)
    - Requires the history of survey as long as the estimation sample
- Relative Entropy (RE) to Combine Survey information
  - Applied to forecasting by Robertson, Tallman and Whiteman (2005)
  - Altavilla, Giacomini and Ragusa (2017) tilt segments of term-structure forecasts to survey expectations
  - Kruger, Clark, and Ravazzolo (2017) tilt one-step ahead forecasts from TVP-VAR toward survey nowcasts
- This paper: uses RE to tilt VAR forecasts toward survey Long-Run projections in addition to survey nowcasts

Empirical Model and Data

In our examination, we consider following quarterly VAR models:

- Small VAR consisting of five variables (i.e. n=5)
    - Core variables of interest to monetary policy makers: Real GDP growth, CPI Inflation, unemployment rate, federal funds rate
    - Add a financial variable: credit spread (BAA rate - 10yr Treasury rate)
    - Several papers on VAR forecasting employ it as a benchmark VAR

- Medium VAR consisting of ten variables (builds on Small VAR by five additional variables; n=10)
    - Productivity growth, wage inflation, nonfarm payroll employment growth, real consumption growth, core CPI inflation
    - Shown to be useful in improving forecasts of core variables
    - Forecasts of these additional variables maybe of their own interest

- Time-Varying VAR (real GDP growth, CPI inflation, unemployment rate); along the lines of Primiceri (2005)

Empirical Model and Data

The usefulness of Stochastic Volatility

- We also evaluate results of allowing for stochastic volatility (SV) in our Small and Medium VARs
    - past research provides strong evidence of the importance of SV (e.g. Clark, 2011; D'Agostino, Giannone, and Gambetti, 2013)
    - implements the computationally convenient approach of Carriero, Clark, and Marcellino (2016); a phenomenal contribution
    - SV helps significantly improve the calibration of the density forecasts
    - But gains in relative accuracy are marginal because density forecasts from hybrid approach are centered around a more accurate mean
- Presentation focus on results from Small VAR without stochastic volatility

Empirical Model and Data

- High-dimensional VARs susceptible to overfitting, estimate using Bayesian methods
  - Employ conjugate Normal-Inverse Wishart prior
  - Prior has computational advantage and competitive forecasting properties (Koop, 2013; Carriero et al, 2015)
  - Allows us to conveniently generate multi-step predictive densities
  - Hyper parameters that govern the tightness of Minnesota and Sum of Coefficients prior are set based on optimizing the marginal likelihood over the pre-forecast evaluation sample

Empirical Model and Data

Forecast details

- Forecasts generated recursively with real-time data and evaluated with real-time data (third release); robust to using revised data
- Estimation start 1959.Q4; and 1985.Q1
- Real-time vintages as of SPF date
- Forecasts 1 to 40 quarters ahead but focus on 1 to 12 quarters ahead
- Forecast evaluation samples: 1994.Q1 to 2016.Q4 (and 1994 - 2006)
- MSE for point forecasts and CRPS metric for density forecasts
- Following Kruger, Clark, and Ravazzolo (2017) statistical significance using Diebold, Mariano and West test using two-sided tests of standard normal
  - HAC variance estimator with lag h-1 truncation parameter; finite sample correction proposed by Harvey et al (1997)

Methodology: Relative Entropy

- Start with a predictive density **p(Y)** corresponding to an n-dimensional random variable Y generated by our VAR model
- Modify it to obtain a new predictive density **g(Y)** such that it satisfies a given set of moment conditions (e.g. survey forecasts)
- But in doing so minimizes the relative entropy (i.e. Kullback-Liebler Information Criterion) between the two predictive densities; that is $g(Y)$ is as close as possible to the original density $p(Y)$ in the information-criterion sense
- Density $g(Y)$ is essentially a re-weighted original density $p(Y)$
  - to work there needs to be support in $p(Y)$ for the moment conditions

Methodology: Relative Entropy

- An effective and flexible conditional forecasting method (KCR, 2017)
  - allows to combine both mean condition and the confidence in it
  - an important advantage if the interest is in density forecasts
- In a VAR, conditioning or tilting on some future horizon will influence the forecast starting from the jumping-off point all the way to the tilted horizon
  - e.g. tilt real GDP growth at h=6 then tilting it will impact the forecast trajectory from h=1 to h=5 for all the variables
  - simultaneously tilting on multiple variables result in forecast trajectories that reflect cumulative effect of those conditions
- Easily adapted to any VAR that is able to generate predictive densities

Determining the forecast horizon for tilting

- Natural inclination to combine at some very distant future horizon
- Some macroeconomic variables more persistent than others
  - Unemployment rate very persistent while GDP growth on other extreme
  - Inflation is in between
- Accounting for this is important when combining the forecasts
- **Proposed approach: Informed from the BVAR model estimates**
  At each forecast origin t, retrieve the persistence estimates (i.e. slope parameters), corresponding to variable i from equation i of the VAR.

$$\rho_{i,t}^{+,BVAR} = \sum_{l=1}^{p} \bar{A}_{i,l}^{(i,i)}$$

where $\bar{A}_{i,l}^{(i,i)}$ is posterior estimate of the slope coefficient of variable i in equation i of the VAR system.

Determining the forecast horizon for tilting

- The corresponding metric that roughly determines the number of quarters it takes to revert back to BVAR's implied steady state

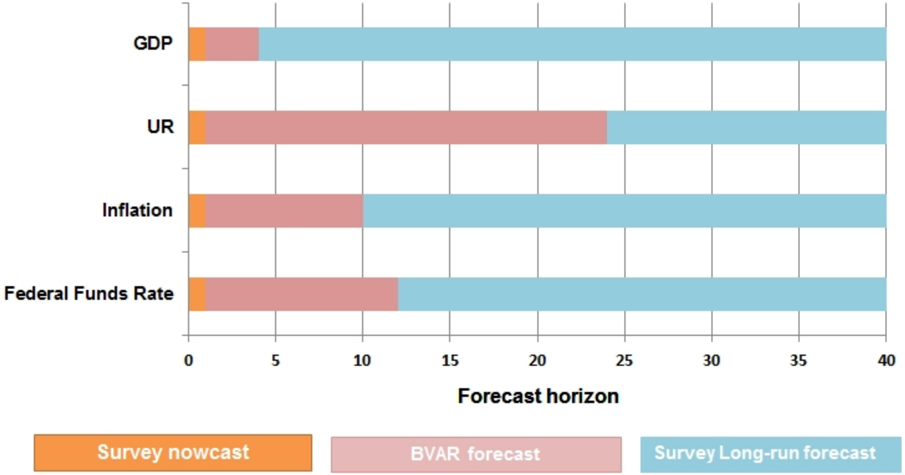
$$h_{i,t}^{+,BVAR} = \frac{1}{1 - \rho_{i,t}^{+,BVAR}}$$

The horizon, $h^*_{i,t}$ at which the survey long-run forecast is combined with the BVAR forecast for variable i is set as

$$h^*_{i,t} = \max \left\{P_t^Q, h_{i,t}^{+,BVAR}\right\}$$

  - P_t^Q provide control to override $h_{i,t}^{+,BVAR}$
  - $P_t^Q = 5$ to reflect our preference to have a VAR forecast from h=2 to h=4
  - $P_t^Q = 0$ similar results because this choice only binds on real GDP growth

Hybrid Forecast: Components



Forecast Accuracy Comparison

Hybrid VAR Forecasts

versus

Baseline VAR Forecasts

Baseline forecast tilts Raw BVAR on survey nowcasts only

Hybrid forecast tilts Raw BVAR on both survey nowcasts and long-run forecasts

Results I: Point Forecast Accuracy

Full Sample (1994.Q1 - 2016.Q4) Small BVAR (est. 1960+)

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative MSE: Hybrid / Baseline						
Real GDP	1.00	1.01	0.77*	0.80*	0.88	0.93
CPI Inflation	1.00	0.83*	0.78***	0.70**	0.60***	0.62***
Unemployment rate	1.00	1.16	1.07	0.98	0.94	0.92
Federal funds rate	1.00	0.92	0.90	0.84*	0.75**	0.69***
Credit Spread	1.00	0.94	0.90***	0.84***	0.81***	0.79***

Baseline forecast tilts Raw BVAR on survey nowcasts only

Hybrid forecast tilts Raw BVAR on both survey nowcasts and long-run forecasts

Results II: Density Forecast Accuracy

Full Sample (1994.Q1 - 2016.Q4) Small BVAR (est. 1960+)

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative CRPS: Hybrid - Baseline						
Real GDP	0.00	0.02	-0.17*	-0.11*	-0.08	-0.05
CPI Inflation	0.00	-0.10*	-0.12***	-0.19**	-0.28***	-0.24***
Unemployment rate	0.00	0.02	0.02	-0.01	-0.03	-0.04
Federal funds rate	0.00	-0.01	-0.03	-0.07	-0.16**	-0.27***
Credit Spread	0.00	-0.01	-0.04***	-0.07***	-0.09***	-0.10***

Baseline forecast tilts Raw BVAR on survey nowcasts only

Hybrid forecast tilts Raw BVAR on both survey nowcasts and long-run forecasts

Results III: Point Forecast Accuracy

Full Sample (1994.Q1 - 2016.Q4) Small BVAR (est. 1985+)

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative MSE: Hybrid / Baseline						
Real GDP	1.00	1.04	0.95	0.86*	0.87	0.90
CPI Inflation	1.00	0.98	0.92*	0.91***	0.87**	0.85**
Unemployment rate	1.00	1.16	1.22	1.21	1.15	1.08
Federal funds rate	1.00	0.88***	0.85	0.83	0.79	0.73
Credit Spread	0.98	1.00	0.93	0.87	0.83*	0.80*

Baseline forecast tilts Raw BVAR on survey nowcasts only

Hybrid forecast tilts Raw BVAR on both survey nowcasts and long-run forecasts

Results IV: Density Forecast Accuracy

Full Sample (1994.Q1 - 2016.Q4) Small BVAR (est. 1985+)

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative CRPS: Hybrid - Baseline						
Real GDP	0.00	0.01	-0.05	-0.12**	-0.11*	-0.09**
CPI Inflation	0.00	-0.02	-0.06*	-0.06***	-0.10**	-0.12**
Unemployment rate	0.00	0.02	0.04	0.05	0.04	0.03
Federal funds rate	0.00	-0.03**	-0.06	-0.11	-0.17	-0.24*
Credit Spread	0.00	0.00	-0.02	-0.04*	-0.06*	-0.08*

Baseline forecast tilts Raw BVAR on survey nowcasts only

Hybrid forecast tilts Raw BVAR on both survey nowcasts and long-run forecasts

Results V: Time-Varying VAR

Full Sample (1994.Q1 - 2016.Q4) TVP-VAR SV

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative MSE: MSE Hybrid TVP-VAR SV / MSE Baseline TVP-VAR SV						
Real GDP	1.00	0.93	0.88	0.86	0.90	1.00
CPI Inflation	1.00	1.00	0.87***	0.81***	0.78***	0.81***
Unemployment rate	1.00	1.02	1.04	1.05	1.06	1.08

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative CRPS: CRPS Hybrid TVP-VAR SV - CRPS Baseline TVP-VAR SV						
Real GDP	0.00	-0.05	-0.06	-0.07	-0.04	0.02
CPI Inflation	0.00	-0.01	-0.03	-0.05	-0.06	-0.04
Unemployment rate	0.00	-0.01	-0.01	-0.01	-0.01	-0.01

Baseline forecast tilts TVP-VAR SV on **survey nowcasts only**

Hybrid forecast tilts TVP-VAR SV on both survey nowcasts and long-run

Forecast Accuracy Comparison

Hybrid VAR Forecasts

versus

Univariate Benchmarks

Results VI: Horse race 1

CPI Inflation Forecast Accuracy: Hybrid 1960+ vs. Univariate Benchmarks

Point Accuracy (1994.Q1 - 2016.Q4)

	h=2Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative MSE: MSE Hybrid from Medium BVAR / MSE Univariate						
RW (Atkeson and Ohanian)	0.82**	0.87**	0.86**	0.77	0.82*	0.90***
UCSV (Stock and Watson)	0.96	0.99	0.97	0.94	0.91	1.00
AR Gap (Faust and Wright)	1.02	0.98	0.98	0.94**	0.94**	0.94**
SPF	1.00	1.03				

Density Accuracy (1994.Q1 - 2016.Q4)

	h=2Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative CRPS: CRPS Medium BVAR - CRPS UCSV						
Baseline - UCSV	0.05	0.15**	0.15***	0.19***	0.24***	0.24**
Hybrid - UCSV	0.00	0.03	0.03	0.01	0.04	0.10**

Results VII: Horse race 2

Forecast Accuracy: Hybrid 1960+ vs. TVP-VAR SV

Point Accuracy (1994.Q1 - 2016.Q4)

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative MSE: MSE Hybrid Small BVAR / MSE Baseline TVP-VAR SV						
Real GDP	1.00	1.03	0.93	0.93*	0.95	1.03
CPI Inflation	1.00	0.99	0.94***	0.87***	0.83**	0.82**
Unemployment rate	1.00	1.06	1.05	1.02	1.02	1.02

Density Accuracy (1994.Q1 - 2016.Q4)

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative CRPS: CRPS Hybrid Small BVAR - CRPS Baseline TVP-VAR SV						
Real GDP	0.11***	0.08*	0.02	0.02	0.04	0.07**
CPI Inflation	-0.05***	0.01	0.03	-0.01	-0.03	-0.03
Unemployment rate	0.01***	-0.01	-0.04	-0.06	-0.08	-0.08

Additional Benefits

- Compare and assess implications on forecast of a range of values
  - Across surveys including with shorter history e.g. Summary of Economic Projections (SEP); median and range as mean and variance restrictions
  - These days policy makers communicate their view of the underlying trend rates
  - Compare how model's forecast of core variables change, Policymaker A vs. B
- Does not require survey history to match estimation sample
  - Could be beneficial for developing and emerging countries
- Interpolate survey forecasts for missing quarters
  - Well-established survey forecasts hard to outperform (e.g. Croushore,2010)
  - But they cover smaller number of variables and forecast horizons; infrequent
  - SPF and Blue Chip report forecast values for five quarters and 10-year out
- Taylor-rule restriction over the forecast horizon (e.g. Robertson et al, 2005)

Conclusion

- Approach to construct Hybrid forecast consisting of survey nowcast, VAR forecast, and long-run survey forecast
  - Use Relative Entropy; easily adapt to existing VARs
- Meaningful gains in forecast accuracy in all VAR models
  - Gains largest for fixed parameter VARs estimated with longer sample
  - An important practical result; lends credibility to the use of simple VARs for production of forecasts under strict time constraints
- Inflation hybrid forecasts rival univariate benchmark models
  - A useful practical contribution for monetary policy makers
- Hybrid forecasts' accuracy from simple VARs rivals TVP-VARs
- Extent of improvements suggest a post-estimation method to accommodating structural change and moving end points

Extra Slides

Figure: Real-Time Long Run Forecasts: GDP and Unemployment Rate

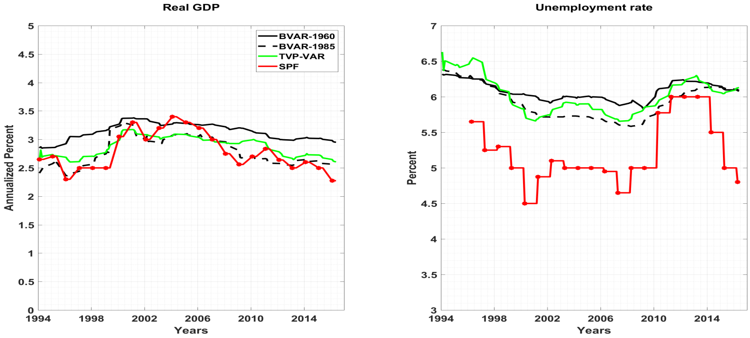


Figure: Real-Time Long Run Forecasts: CPI and Short-Term Interest Rate

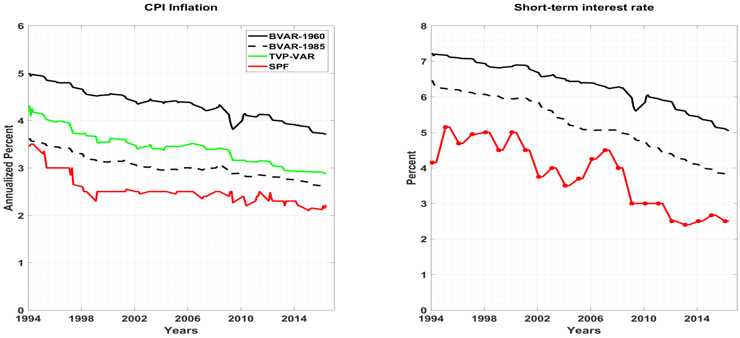
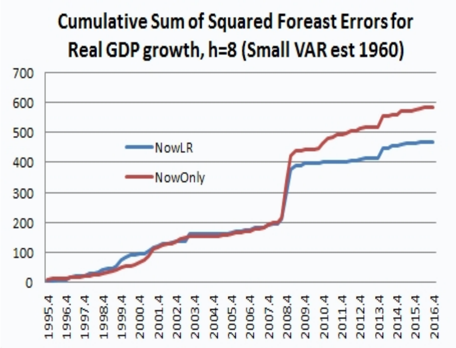


Figure: Cumulative Squared Error



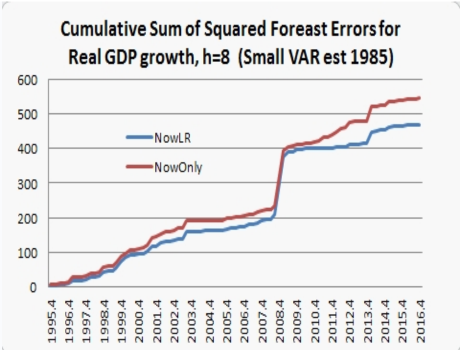
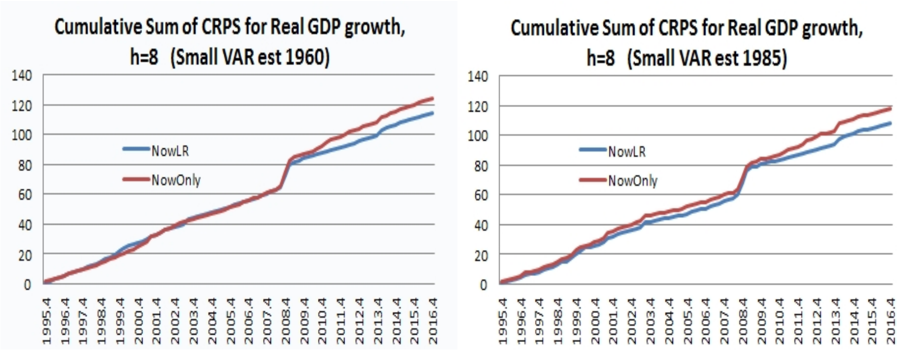


Figure: Cumulative CRPS



Appendix Result: Point Forecast Accuracy

Full Sample (1994.Q1 - 2006.Q4) Small BVAR (est. 1960+)

	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative MSE: Hybrid / Baseline						
Real GDP	1.00	0.96	0.91	1.02	1.05	1.07
CPI Inflation	1.00	0.86**	0.68***	0.63***	0.46***	0.49***
Unemployment rate	1.00	1.01	0.94	0.87	0.90	0.95
Federal funds rate	1.00	1.11*	1.07	0.92	0.79	0.73
Credit Spread	1.02	1.10	0.93*	0.82*	0.78*	0.79*

Baseline forecast tilts Raw BVAR on survey nowcasts only

Hybrid forecast tilts Raw BVAR on both survey nowcasts and long-run forecasts

Appendix Result: Point Forecast Accuracy

Full Sample (1994.Q1 - 2006.Q4) Small BVAR (est. 1985+)

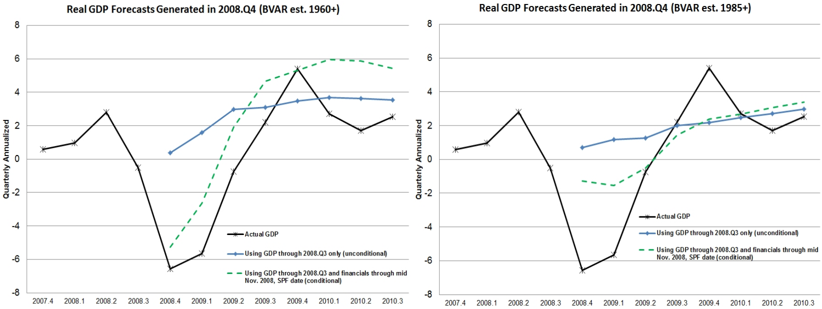
	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q
Relative MSE: Hybrid / Baseline						
Real GDP	1.00	0.98	1.00	0.86**	0.88**	0.92
CPI Inflation	1.00	0.89**	0.79**	0.85***	0.80*	0.92
Unemployment rate	1.00	0.91	1.00	1.03	0.98	0.92
Federal funds rate	1.00	0.88***	0.83	0.81*	0.79*	0.78**
Credit Spread	1.00	1.03	0.94	0.89*	0.86*	0.89*

Baseline forecast tilts Raw BVAR on survey nowcasts only

Hybrid forecast tilts Raw BVAR on both survey nowcasts and long-run forecasts

Figure: More on Shock Uncertainty

Knotek and Zaman (2017, IJF forthcoming)



Methodology: Relative Entropy

- Start with a predictive density **p(Y)**
  - D draws each with a weight $w_i = 1/D$, where $i = 1, ...D$
- Modify it to obtain a new predictive density **g(Y)**
  - such that it satisfies a given set of moment conditions $\bar{g}$ (e.g. survey forecasts)
  - $\mathbb{E}g(Y) = \sum_{i=1}^{D} w_i^* p(Y_i) = \bar{g}$
- Minimizes the relative entropy (i.e. Kullback-Liebler Information Criterion)
  - $g(Y)$ as close as possible to $p(Y)$ in the information-criterion sense
  - equivalent to solving for new weights

$$K(w^* : w) = \sum_{i=1}^{D} w_i^* log(\frac{w_i^*}{w_i})$$

  - satisfies the following constraints

$$w_i^* \geq 0, \ \sum_{i=1}^{D} w_i^* = 1, \ \sum_{i=1}^{D} w_i^* p(Y_i) = \bar{g}$$

Methodology: Relative Entropy

- Density $g(Y)$ is essentially a re-weighted original density $p(Y)$
  - to work there needs to be support in $p(Y)$ for the moment conditions

- The solution to the minimization problem using method of Lagrange

$$w_i^* = \frac{w_i \ \exp(\acute{\gamma}\ p(Y_i))}{\sum_{i=1}^{D} w_i \ \exp(\acute{\gamma}\ p(Y_i))}$$

where γ is the vector of Lagrange multipliers associated with the constraints

- γ can be obtained as a solution to the following minimization problem

$$\gamma = arg \ \min_{\tilde{\gamma}} \sum_{i=1}^{D} w_i \ \ \exp(\acute{\tilde{\gamma}}\ [p(Y_i) - \bar{g}])$$