Measuring GDP Growth Data Uncertainty

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Macroeconomic Uncertainty

- Many measures of uncertainty rise during recessions: stock market volatility, macroeconomic forecasting uncertainty, professional forecasters disagreement and economic policy uncertainty (Bachman et al, 2013; Jurado et al, 2015; Baker et al, 2016; Rossi et al, 2016; surveyed by Bloom (2014)).
Data Uncertainty

- Economic statistics uncertainty has two components (Manski, 2014). *Transitory statistical uncertainty*, early data releases that are revised as new information arrives; and *permanent statistical uncertainty* from data incompleteness or the inadequacy of data collection which does not diminish over time.

For GDP growth, we are mainly concerned about the transitory uncertainty. We expect this *epistemic* uncertainty, that is uncertainty due to lack of knowledge about current and past data, to diminish as additional data are collected and both corrections and improvements to these data are made by the statistics office.

We show that GDP data uncertainty may affect policy decisions by adding a layer of uncertainty on the measurement of the current state of the economy.
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We show that GDP data uncertainty may affect policy decisions by adding a layer of uncertainty on the measurement of the current state of the economy.
While the Office for National Statistics emphasise the uncertainty of early GDP data releases by indicating that their data will be revised, it is the Bank of England that provide quantitative estimates of GDP data uncertainty, as perceived by their Monetary Policy Committee.
Communication of UK GDP Growth Data Revision
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Uncertainty

- The fan charts show "how the MPC’s best collective judgement of the most likely path for the mature estimate of GDP growth, and the uncertainty around it, both over the past and into the future."
- “To the left of the first vertical dashed line, the centre of the darkest band of the fan chart gives the Committee’s best collective judgement of the most likely path for GDP growth once the revisions process is complete. The estimate is based on an analysis of business surveys and the past pattern of official data revisions.”
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- We propose a generic, loss-function based approach to measuring uncertainty; and show how density forecast calibration tests can then be constructed from this.

- We first show that UK GDP revisions matter, emphasising that they are time-varying and contain ‘news’. Then we provide evidence that our measure of GDP data uncertainty increased at the onset of the 2018-2019 recession and is correlated with measures of macroeconomic uncertainty.
• GDP growth values published after 3 years, $y_{t+13}$, are a popular measure of ‘mature’ data in the US data revision literature.
• But UK GDP growth revisions between $y_{t+13}$ and $y_{t+17}$ are mainly news (not correlated with the earlier release) and imply a non-zero mean revision suggest that $y_{t+17}$ might be a better measure of ‘mature’ data for the UK.
• The revisions $y_{t+17} - y_{t+1}$ have a 0.4 sample mean (1993Q2-2013Q1), which is statistically different from zero, and are uncorrelated with $y_{t+1}$. Mean absolute revision is 0.7.
Display of $y_{t+1}$ and $y_{t+17}$.
The model describes “mature” data, $y_{t+l}$ with respect to an earlier estimate $y_{t+b}$ ($l > b, b \geq 1$) as:

$$y_{t+l} = y_{t+b} + \mu_t + u_t,$$

where $\mu_t$ is a time-varying local mean revision. The disturbance, $u_t$, characterises the measurement error assumed mean zero, so that $\text{Var}(u_t)$ measures the degree of measurement error in the initial release.

The time-varying local mean, $\mu_t$, follows a random walk process implying that the average revision moves slowly over time:

$$\mu_t = \mu_{t-1} + e^{0.5(h_0 + \omega h_t)} \zeta_{b,t}$$

$$\tilde{h}_t = \tilde{h}_{t-1} + \zeta_{h,t};$$

$\zeta_{b,t}, \zeta_{h,t}$ are both iid $N(0,1)$. 

Galvao/Mitchell Data Uncertainty
And the measurement error also has a time-varying volatility:

\[
(y_{t+l} - y_{t+b}) = rev_t^{(l-b)} = \mu_t + e^{5(g_0+w_g\tilde{g}_t)} \zeta_{u,t}
\]

We use a Bayes factor approach to check whether we need both stochastic volatility process. The results support the model above for \( l = 17 \) and \( b = 1, 4, 8, 12 \).
Time-Varying Local Mean Revision

Estimates for $y_{t+17} - y_{t+1}$. 
Estimates for $y_{t}^{t+17} - y_{t}^{t+1}$.
Local Mean Volatility

Estimates for $y_{t+17} - y_{t+1}$
Measuring Uncertainty

- We are interested in the unforecastable component of data uncertainty.
- Uncertainty is the difference between the *ex post* (or realised) and the *ex ante* (or expected) values of the chosen loss function (or scoring rule):

\[ Unc_t = L(f_t, y_t) - E_{f_{i-h}}[L(f_t, Y_t)] , \]

- \( E_{f_{i-h}}[L(f_t, Y_t)] \) are in expectations computed by (honest and loss-minimising) agents who assume (*ex ante*) that their forecast is as good as it can be.
- \( Unc_t \) is a (realised) shock to confidence, the definition of ambiguity in Ilut and Schneider (2014), with confidence measured, in our framework, by \( E_{f_i}[L(f_t, Y_t)] \) i.e. the expected “risk” of the forecast.
Given loss function $L(f_t, y_t)$, correct unconditional average calibration of the forecast $f_t$ with respect to the realisation $y_t$, is defined as when

$$H_0^U : E(Unc_t) = 0.$$ 

If we view $Unc_t$ as capturing Knightian uncertainty, under $H_0^U$ while there may be data risk there is no data uncertainty - as users of the forecast, $f_t$, are correctly capturing all aspects of $Y_t$ that are relevant given $L(f_t, Y_t)$. 
Uncertainty Measures for Specific Loss (score) functions: I

1. Mean squared error loss:

\[ Unc_t^{MSE} = (y_t - \hat{y}_t)^2 - \sigma_t^2 \]

2. Interval loss:

\[ Unc_t^{Int} = x_t(\alpha) - \alpha \]

where \( x_t(\alpha) = 1(y_t \in J(\alpha)); J(\alpha) = [lower_t, upper_t] \).

3. Logarithm score (-1) for Gaussian densities:

\[ Unc_t^{\log S} = 0.5(z_t^2 - 1) \]

as \( E_{f_{t-h}} \left[ L^{\log S}(f_t, Y_t) \right] = 0.5(1 + \log 2\pi) + \log (\hat{\sigma}_t) \) and

\[ z_t = \left( \frac{(y_t - \hat{y}_t)}{\hat{\sigma}_t} \right). \]
CRPS for Gaussian densities:

\[ \text{Unc}^\text{CRPS}_t = ((y_t - \hat{y}_t)) (2\Phi(z_t) - 1) + 2\hat{\sigma}_t \phi(z_t) - 2(\hat{\sigma}_t / \sqrt{\pi}) \]

as \( E_{ft}(L^\text{CRPS}(f_t, y_t)) = (\hat{\sigma}_t / \sqrt{\pi}) \).
Let $\hat{y}_t^{t-B}, ..., \hat{y}_t^{t-2}, \hat{y}_t^{t-1}$ denote the MPC’s point estimate of growth ending in quarter $t - 1, ..., t - B$, as announced by the MPC in quarter $t$; and let $\hat{\sigma}_t^{t-B}, ..., \hat{\sigma}_t^{t-2}, \hat{\sigma}_t^{t-1}$ denote the corresponding set of standard deviation estimates.
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Because estimates are computed after ONS first release, the predicted revision is $\hat{y}_{t-1} - y_{t-1}$. 
Bank of England’s Probabilistic Backcasts

- Let $\hat{y}_t^{t-B}, ..., \hat{y}_t^{t-2}, \hat{y}_t^{t-1}$ denote the MPC’s point estimate of growth ending in quarter $t-1, ..., t-B$, as announced by the MPC in quarter $t$; and let $\hat{\sigma}_t^{t-B}, ..., \hat{\sigma}_t^{t-2}, \hat{\sigma}_t^{t-1}$ denote the corresponding set of standard deviation estimates.
- Because estimates are computed after ONS first release, the predicted revision is $\hat{y}_t^{t-1} - y_t^{t-1}$.
- In general the predicted revision is $\hat{y}_t^{t+b} - y_t^{t+b}$, where $y_t^{t+b}$ is ONS earlier $b^{th}$ estimate and $\hat{y}_t^{t+b}$ is a MPC’s prediction for mature GDP values that uses information up to $t + b$. 

"Mature" values are observed (released by the ONS) at $t+l$: the ONS GDP estimation error is $y_t^{t+l} - y_t^t$ and the MPC’s backcast error is $\hat{y}_t^{t+l} - \hat{y}_t^t$. We set $l=17$ so we evaluate backcasts for $t = 2007Q3, 2008Q1, ..., 2013Q4$. 

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Data Uncertainty
Bank of England’s Probabilistic Backcasts

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We set $l=17$ so we evaluate backcasts for $b = 1, ..., 16$ for $t = 2007Q3 - 2013Q1$. 
MPC’s Predicted Revision with $b=1$

Computed as $\hat{y}_{t+1} - y_{t+1}$
MPC’s Predicted Standard Error (Uncertainty) with \( b=1 \)

\[ se(y_{t+1} - \hat{y}_{t+1}) \]
MPC’s Backcasts 90% Predicted Interval with $b=1$
Because time-series models of stationary data converge to the unconditional mean and variance at long horizons, we compute predictions using the recursively updated (in real-time) unconditional mean and standard deviation of the revisions.

The probabilistic backcasts for the observation $t$, using information up to $t + b$, which includes a time series of ONS revisions between the $l$th data release and the $b$th release for observations up to $t - l + 1$ is

$$N(\hat{y}_{t}^{t+b,unc}, \hat{\sigma}_{t}^{2,t+b,unc})$$
where

\[
\hat{y}_{t}^{t+b,unc} = y_{t}^{t+b} + \hat{\mu}_{t}^{t+b,unc} \quad \text{for } t = T - (b - 1) + 1, ..., T + P
\]

\[
\hat{\mu}_{t}^{t+b,unc} = \frac{1}{t - l + 1} \sum_{\tau=1}^{t-l+1} \text{rev}_{\tau}^{(l-b)}
\]

\[
\hat{\sigma}_{t}^{t+b,unc} = \sqrt{\frac{1}{t - l + 1} \sum_{\tau=1}^{t-l+1} \left( \text{rev}_{\tau}^{(l-b)} - \hat{\mu}_{t}^{t+b} \right)^2}
\]

\[
\text{rev}_{\tau}^{(l-b)} = y_{\tau+l} - y_{\tau+b}.
\]
Note that ONS average revision with $b = 1$ is 0.33 over the period.

\begin{table}
\centering
\begin{tabular}{|l|c|c|c|}
\hline
 & Bias & RMSE & RMSE Ratio to ONS est. \\
\hline
MPC, $b=1$ & -0.046 & 1.243 & 0.931 \\
Uncond., $b=1$ & -0.223 & 1.335 & 1.000 \\
MPC, $b=4$ & -0.024 & 1.138 & 0.951 \\
Uncond., $b=4$ & -0.034 & 1.173 & 0.980 \\
MPC, $b=8$ & -0.070 & 0.953 & 0.973 \\
Uncond., $b=8$ & -0.067 & 0.971 & 0.991 \\
MPC, $b=12$ & 0.092 & 0.770 & 0.948 \\
Uncond, $b=12$ & 0.139 & 0.806 & 0.992 \\
MPC, $b=16$ & -0.025 & 0.283 & 0.997 \\
Uncond, $b=16$ & -0.005 & 0.283 & 0.997 \\
\hline
\end{tabular}
\end{table}
## Accuracy of Revisions Predictive (or Backcasts) Densities

<table>
<thead>
<tr>
<th></th>
<th>Logscore</th>
<th>CRPS</th>
<th>90%Cov.</th>
<th>75%Cov.</th>
<th>50%Cov.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MPC, b=1</strong></td>
<td>2.070</td>
<td>0.702</td>
<td>83%</td>
<td>65%</td>
<td>43%</td>
</tr>
<tr>
<td><strong>Uncond., b=1</strong></td>
<td>2.118</td>
<td>0.735</td>
<td>74%</td>
<td>61%</td>
<td>43%</td>
</tr>
<tr>
<td><strong>MPC, b=4</strong></td>
<td>1.898</td>
<td>0.655</td>
<td>87%</td>
<td>61%</td>
<td>39%</td>
</tr>
<tr>
<td><strong>Uncond., b=4</strong></td>
<td>2.631</td>
<td>0.687</td>
<td>62%</td>
<td>38%</td>
<td>23%</td>
</tr>
<tr>
<td><strong>MPC, b=8</strong></td>
<td>1.513</td>
<td>0.559</td>
<td>74%</td>
<td>65%</td>
<td>35%</td>
</tr>
<tr>
<td><strong>Uncond., b=8</strong></td>
<td>2.055</td>
<td>0.595</td>
<td>67%</td>
<td>37%</td>
<td>20%</td>
</tr>
<tr>
<td><strong>MPC, b=12</strong></td>
<td>1.215</td>
<td>0.520</td>
<td>74%</td>
<td>74%</td>
<td>39%</td>
</tr>
<tr>
<td><strong>Uncond, b=12</strong></td>
<td>2.533</td>
<td>0.483</td>
<td>62%</td>
<td>44%</td>
<td>29%</td>
</tr>
<tr>
<td><strong>MPC, b=16</strong></td>
<td>0.571</td>
<td>0.196</td>
<td>96%</td>
<td>91%</td>
<td>91%</td>
</tr>
<tr>
<td><strong>Uncond, b=16</strong></td>
<td>1.166</td>
<td>0.110</td>
<td>89%</td>
<td>89%</td>
<td>84%</td>
</tr>
</tbody>
</table>
Expected (ex ante) and Realised (ex post) Uncertainty: MSE

As in Clements (2014).
Expected (ex ante) and Realised (ex post) Uncertainty: CRPS
Expected (ex ante) and Realised (ex post) Uncertainty: Logscore

![Expected vs Realised Uncertainty Graph](image-url)
Test for calibration of MPC’s backcasts (for a given loss function) for $t = 2007Q3 – 2013Q1$.

<table>
<thead>
<tr>
<th>$b$</th>
<th>MSE</th>
<th>CRPS</th>
<th>Logscore</th>
<th>Cov, 90%, $pv$</th>
<th>Cov, 75%, $pv$</th>
<th>Cov, 50%, $Pv$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.34</td>
<td>1.38</td>
<td>1.61</td>
<td>0.28</td>
<td>0.30</td>
<td>0.53</td>
</tr>
<tr>
<td>2</td>
<td>1.22</td>
<td>1.36</td>
<td>1.47</td>
<td>0.28</td>
<td>0.14</td>
<td>0.53</td>
</tr>
<tr>
<td>3</td>
<td>1.29</td>
<td>1.45</td>
<td>1.53</td>
<td>0.10</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>4</td>
<td>1.19</td>
<td>1.31</td>
<td>1.40</td>
<td>0.64</td>
<td>0.14</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>0.92</td>
<td>0.98</td>
<td>1.20</td>
<td>0.64</td>
<td>0.30</td>
<td>0.83</td>
</tr>
<tr>
<td>6</td>
<td>0.75</td>
<td>0.82</td>
<td>1.11</td>
<td>0.71</td>
<td>0.30</td>
<td>0.83</td>
</tr>
<tr>
<td>7</td>
<td>0.86</td>
<td>1.12</td>
<td>1.36</td>
<td>0.90</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>8</td>
<td>1.19</td>
<td>1.57</td>
<td>1.90</td>
<td>0.03</td>
<td>0.30</td>
<td>0.14</td>
</tr>
<tr>
<td>9</td>
<td>1.20</td>
<td>1.57</td>
<td>1.89</td>
<td>0.03</td>
<td>0.30</td>
<td>0.14</td>
</tr>
<tr>
<td>10</td>
<td>1.23</td>
<td>1.60</td>
<td>2.00</td>
<td>0.01</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>11</td>
<td>1.08</td>
<td>1.38</td>
<td>1.80</td>
<td>0.01</td>
<td>0.56</td>
<td>0.30</td>
</tr>
<tr>
<td>12</td>
<td>0.74</td>
<td>1.04</td>
<td>1.43</td>
<td>0.03</td>
<td>0.90</td>
<td>0.30</td>
</tr>
<tr>
<td>13</td>
<td>0.03</td>
<td>0.41</td>
<td>0.95</td>
<td>0.10</td>
<td>0.71</td>
<td>0.83</td>
</tr>
<tr>
<td>14</td>
<td>-0.19</td>
<td>0.09</td>
<td>0.79</td>
<td>0.10</td>
<td>0.71</td>
<td>0.30</td>
</tr>
<tr>
<td>15</td>
<td>-1.20</td>
<td>-1.02</td>
<td>0.07</td>
<td>0.64</td>
<td>0.38</td>
<td>0.02</td>
</tr>
<tr>
<td>16</td>
<td>-5.49</td>
<td>-4.82</td>
<td>-2.90</td>
<td>0.31</td>
<td>0.04</td>
<td>0.00</td>
</tr>
</tbody>
</table>
CRPS deviations for \( b=1,\ldots,7 \)
CRPS deviations for $b=8,\ldots,16$
UK GDP Growth Data Uncertainty and other Macro Uncertainty Measures

Max correlation is at the time of first release and approx. 0.8.
We propose a measure of data uncertainty. The measure is calculated for UK GDP growth data; and is based on identifying the component of future data revisions that the Bank of England is unable to predict correctly after observing earlier ONS growth estimates.

We find that UK data uncertainty rises at the onset of recessions; and is positively correlated with measures of UK macroeconomic uncertainty, such as the measures computed by Redl (2017) and Bank of England (2016).

Data uncertainty might be interpreted as an additional source and layer of uncertainty relative to the more traditional macroeconomic uncertainty measures discussed in Bloom (2014).
Conclusions II

- We find that the MPC’s point estimates of historical GDP growth are more accurate measures of revised ONS data than the equivalently timed estimates from the ONS themselves.

- We find that the MPC’s probabilistic backcasts for GDP growth are, on average, well-calibrated and perform well relative to a benchmark model; but the MPC do appear to have over-estimated (ex ante) data uncertainty for observations in the 2010-13 period. Data revisions to mature ONS data ($b > 8$) are harder to predict, because of the unknown impact of future benchmark revisions.
We commend the Bank of England for communicating the \textit{(ex ante)} predictable component of data revisions in their published fan charts;

and we recommend the ONS reconsiders if and how they communicate the uncertainty associated with their early GDP estimates.

We do believe, however, that the Bank of England would improve communication further if they stated explicitly what data vintage they seek to forecast.