Selecting Models with Judgment

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European Central Bank

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Question

Start from a judgmental forecast.

How can it be statistically improved?

Outline

The frequentist statistical decision rule

Model selection under misspecification

Empirical illustration

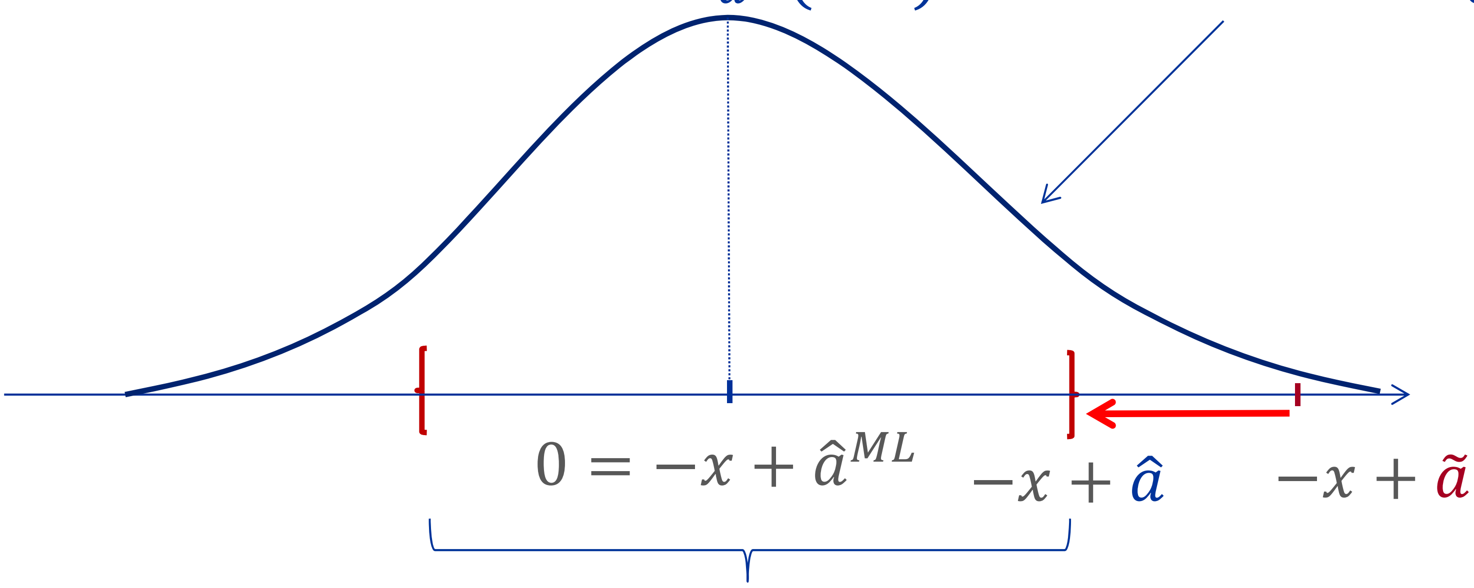
- Preliminaries
- Results

The frequentist decision rule with judgment

1) $X \sim N(\theta, 1)$ 2) $L(\theta, a) = 0.5(\theta - a)^2$
3) Judgment: $\{\tilde{a}, \alpha\}$ 4) $X = x$

FOC: H_0: $\nabla_a L(\theta, \tilde{a}) = -\theta + \tilde{a} = 0$

Distribution of $\nabla_a L(\hat{\theta}, \tilde{a}) = -X + \tilde{a}$ under $H_0: \theta = \tilde{a}$



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Test statistic: $-\mathrm{X} + \tilde{a}$

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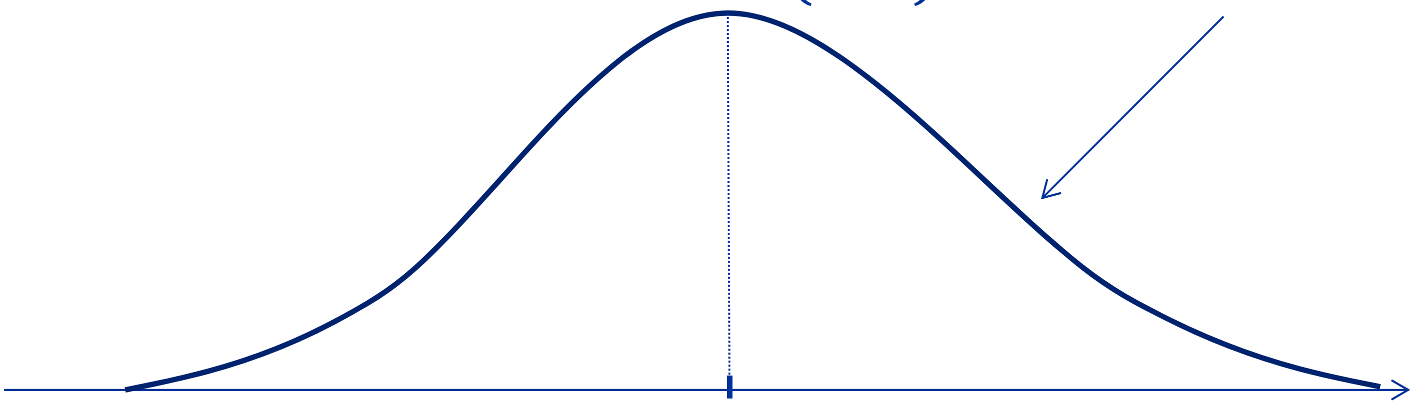

$0 = -x + \hat{a}^{ML}$

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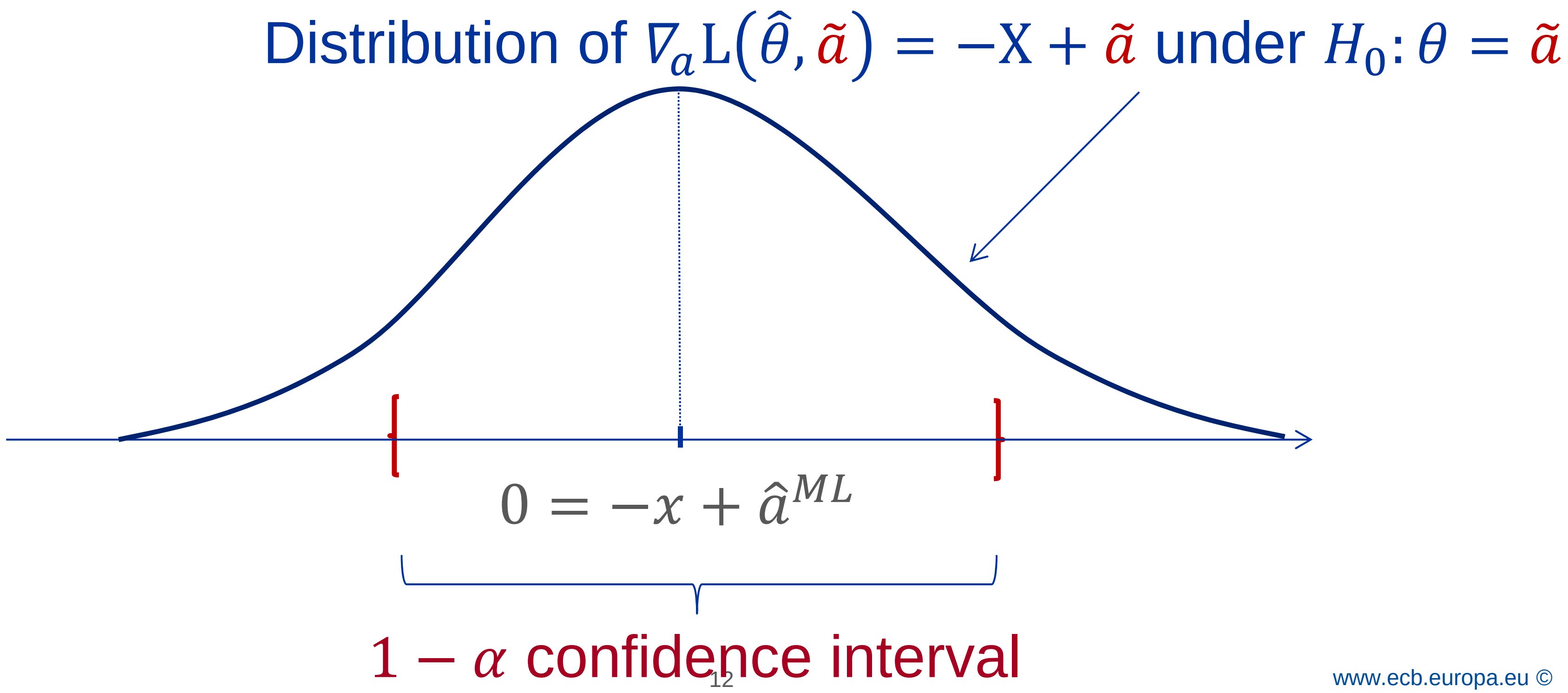

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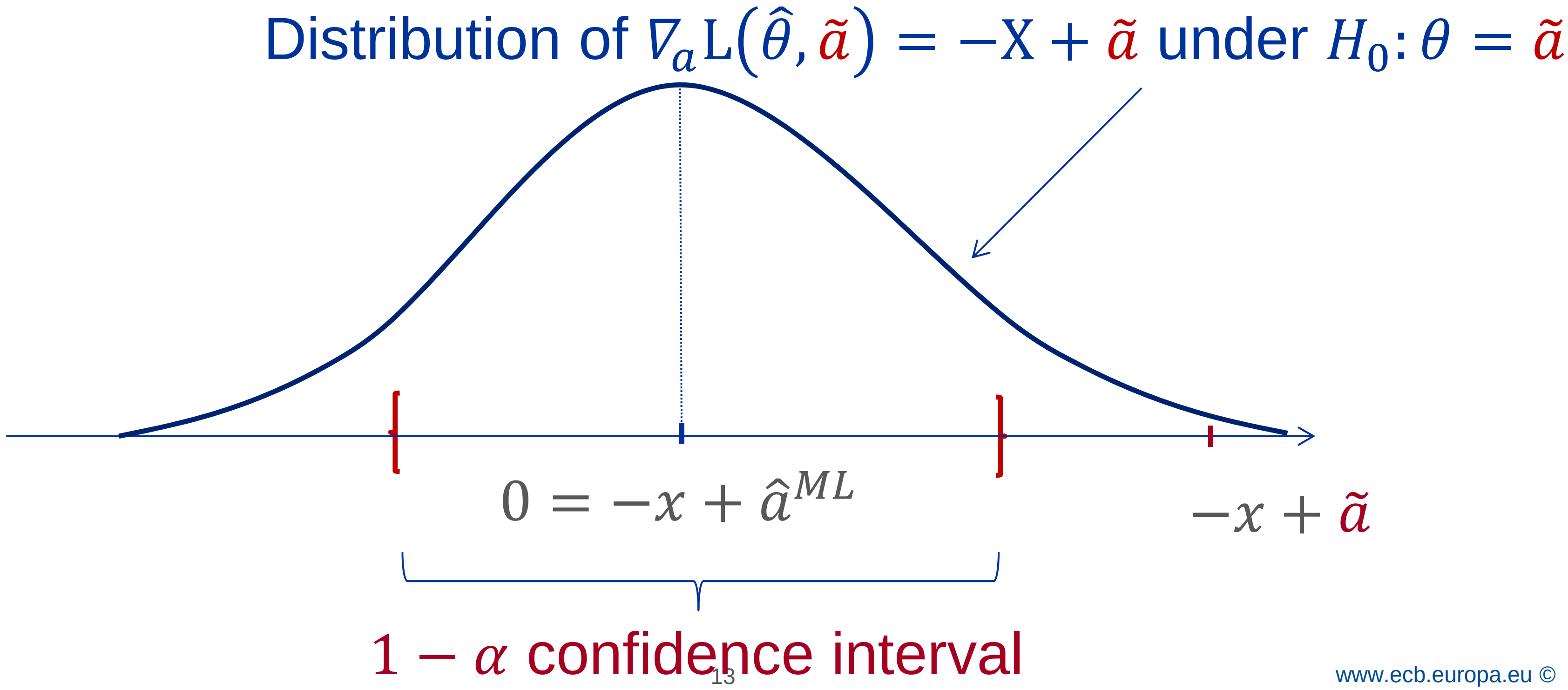

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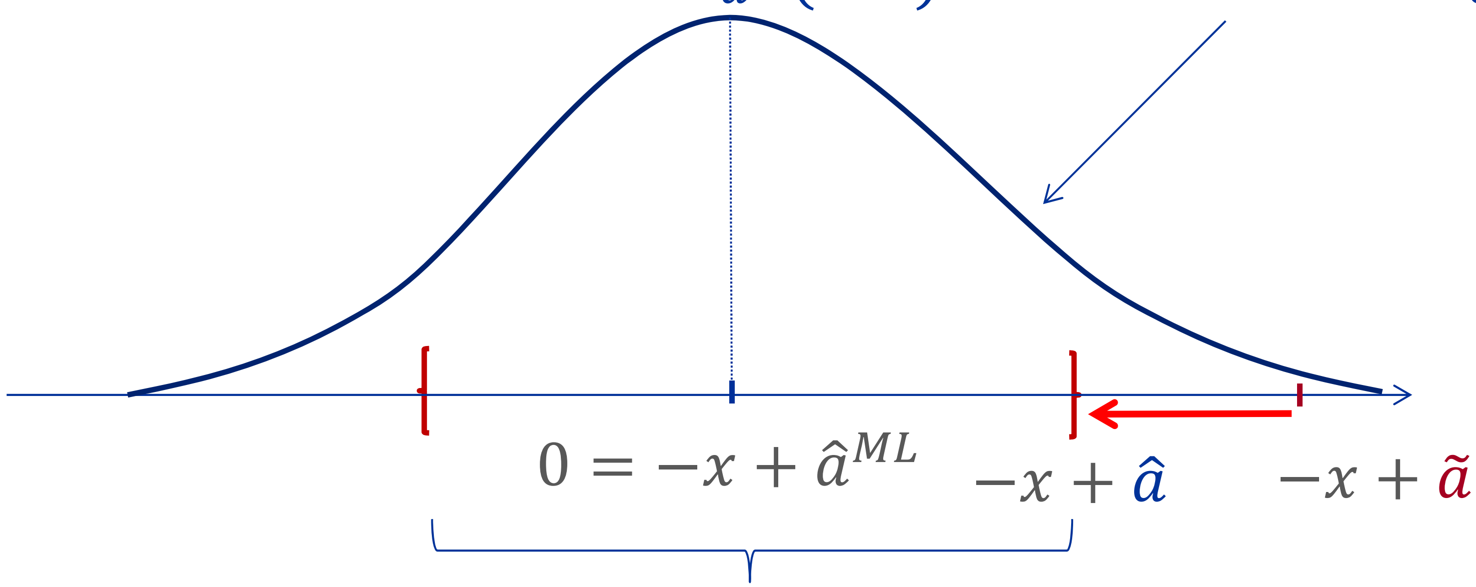
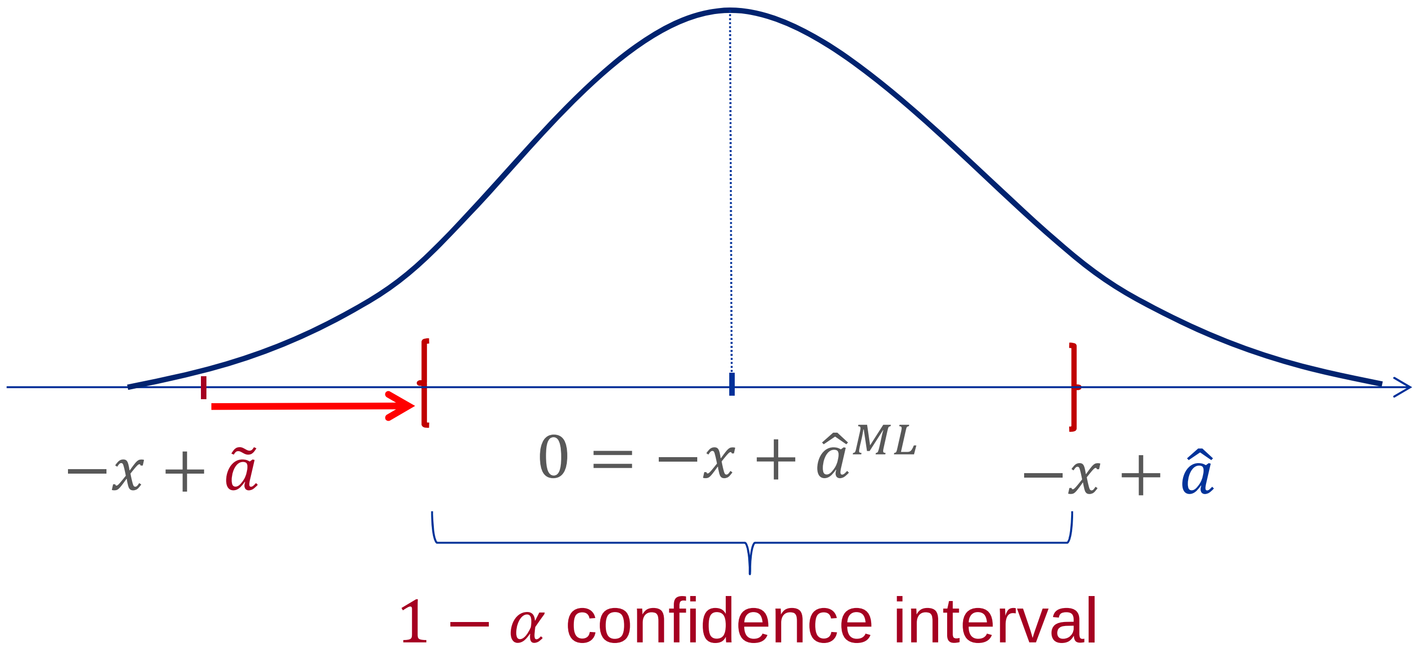


$1 - \alpha$ confidence interval

The frequentist decision rule with judgment



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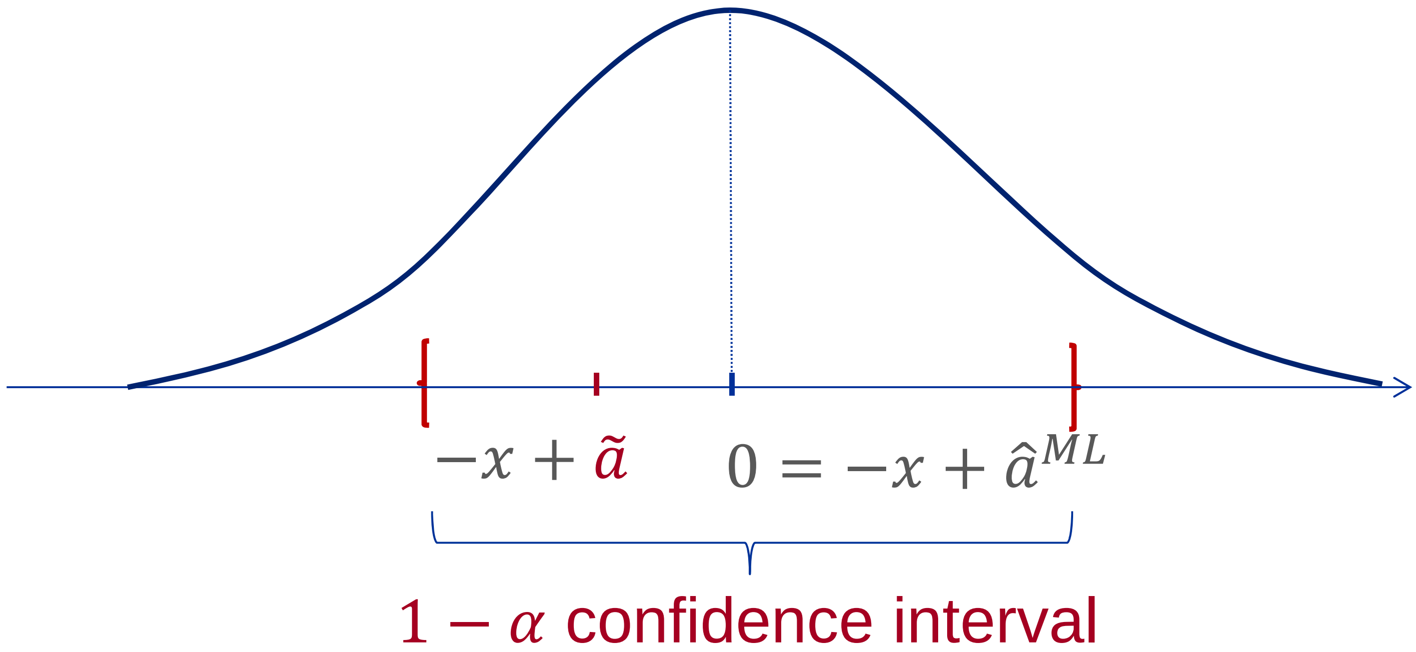

The frequentist decision

Frequentist decision rule:

$$\delta(X) = (x + c_{1-\alpha}) \cdot I(-X + \tilde{a} > c_{1-\alpha}) + \\ + (x + c_{\alpha}) \cdot I(-X + \tilde{a} < c_{\alpha}) + \\ + \tilde{a} \cdot I(c_{\alpha} \leq -X + \tilde{a} \leq c_{1-\alpha})$$

Economic interpretation:

$$P_{\theta}\left[L\left(\theta, \delta(X)\right) > L(\theta, \tilde{a})\right] \leq \alpha$$

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Assumptions so far:

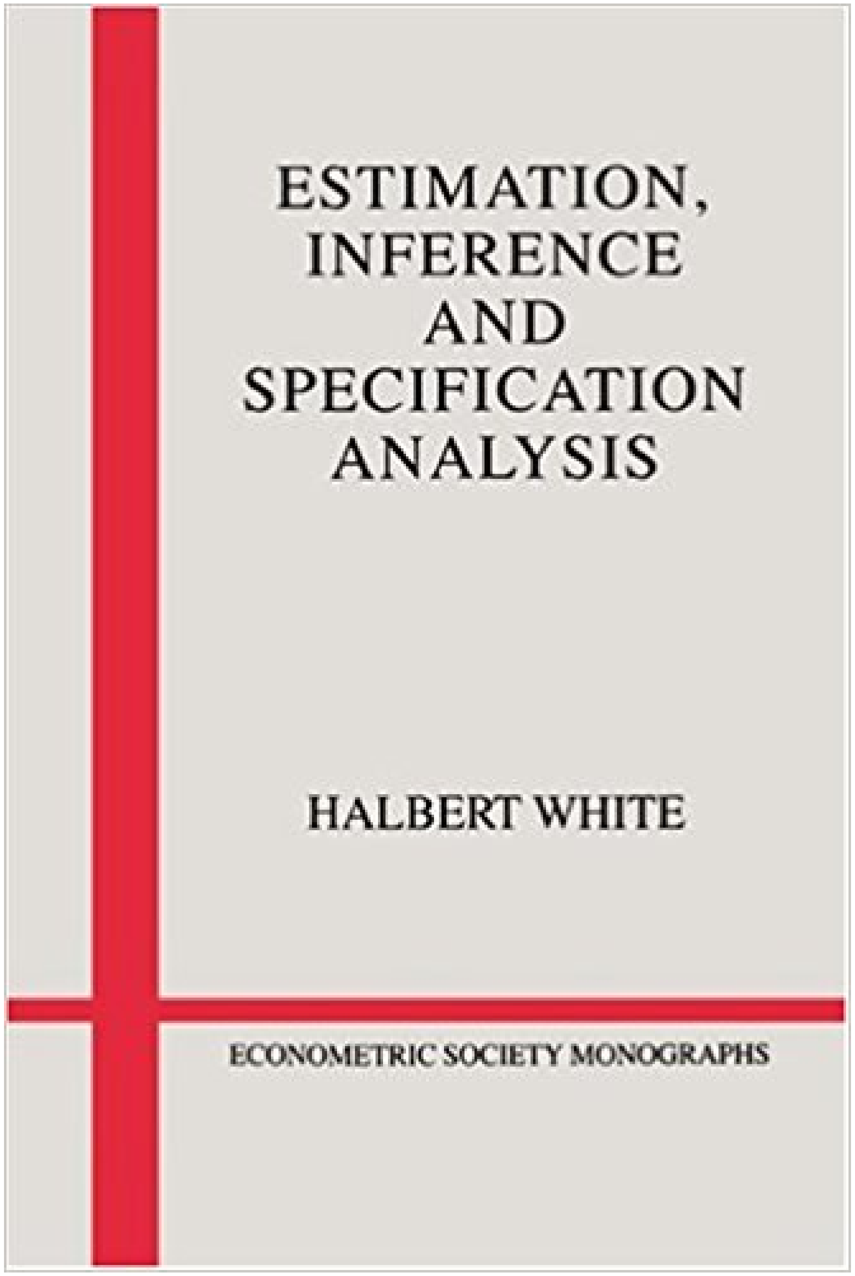
1) Quadratic loss function
   Easy to generalise to convex loss functions

2) A single observation normally distributed
   Easy to generalise, appealing to asymptotic approximations

3) Correctly specified model
   This is hard…

Halbert L. White, Jr., 1950-2012

Theory for misspecified models



Quasi-maximum likelihood estimation

$$\max_{\theta} n^{-1} \sum_{t=1}^{n} \log f_t(X^t, \theta)$$

$$B^{*-\frac{1}{2}} A^* \sqrt{n}(\hat{\theta}(X^n) - \theta^*) \xrightarrow{A} N(0, I_p)$$

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Econometrics boils down to the choice of $f_t(X^t, \theta)$.

Special cases:

- Macroeconometrics (VAR, DSGE, …)
- Financial econometrics (GARCH, CAViaR, …)
- Machine learning (neural nets, random forests, …)

Quasi-maximum likelihood estimation

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$$B^{*-\frac{1}{2}} A^{*} \sqrt{n} (\hat{\theta}(X^n) - \theta^{*}) \xrightarrow{A} N(0, I_p)$$

The asymptotic VC matrix is consistently estimated by:

$$\hat{A}^{-1} \hat{B} \hat{A}^{-1}$$

$$\hat{A} = n^{-1} \sum_{t=1}^{n} \nabla^2 \log f_t\, (X^t, \hat{\theta}(X^n))$$

$$\hat{B} = n^{-1} \sum_{t=1}^{n} \nabla \log f_t\, (X^t, \hat{\theta}(X^n)) \nabla' \log f_t\, (X^t, \hat{\theta}(X^n))$$

Judgment and asymptotics

The decision problem for inflation forecasting:

$$\min_{a} L(\theta, a) = 0.5(x_{n,h}(\theta) - a)^2$$

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$$\min_a L(\theta, a) = 0.5(x_{n,h}(\theta) - a)^2$$

Null hypothesis that the judgment is optimal:

$$H_0: \nabla L(\theta^*, \tilde{a}) = -x_{n,h}(\theta^*) + \tilde{a} = 0$$

Use classical Wald, LR or LM tests.

Back to the simple case

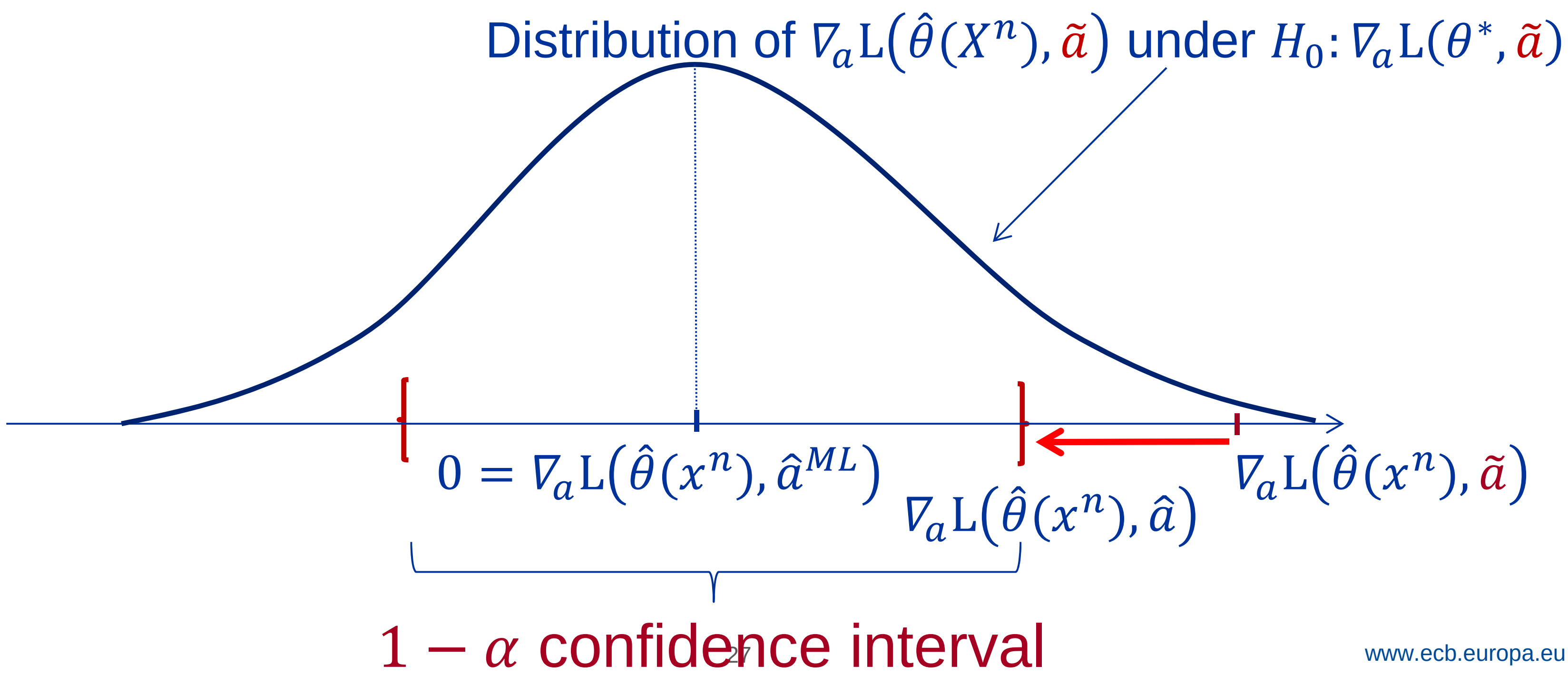
1) $x_{n,h}(\hat{\theta}(X^n)) \sim N\left(x_{n,h}(\theta^*), \sigma^2\right)$

2) $\mathrm{L}(\theta^*, a) = 0.5(x_{n,h}(\theta^*) - a)^2$

3) Judgment: $\{\tilde{a}, \alpha\}$

4) $X^n = x^n$

FOC: H_0: $\nabla_a \mathrm{L}(\theta^*, \tilde{a}) = -x_{n,h}(\theta^*) + \tilde{a} = 0$



Model selection

Select the least misspecified model.

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Minimise the Kullback-Leibler distance:

$$\Pi(a^0 : \delta^m) = n^{-1} \sum_{t=1}^{n} \left(L({\delta_t}^m(x^n)\ ) - L({a_t}^0) \right) \geq 0$$

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Asymptotic approximation:

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Asymptotic approximation:

$$\hat{L}^m = n^{-1} \sum_{t=1}^{n} 0.5({\delta_t}^m(x^n) - x_{t,h})^2$$

Best model:

$$m^+ = \arg\min_m \hat{L}^m$$

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Judgment – Inflation forecast

Trusting that the ECB will deliver on its mandate:

$$\tilde{a} \equiv 24^{-1} E(\sum_{i=1}^{24} \pi_{n+i} \,|X^n) = 1.9\%$$

Judgment – Confidence level

$\alpha = 10\%$

Ingredients for forecasting

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Quadratic loss function

$$L(a) = 0.5E((\pi_{n,24} - a)^2 | X^n)$$

where

$$\pi_{n,24} \equiv 24^{-1} \sum_{i=1}^{24} \pi_{n+i}$$

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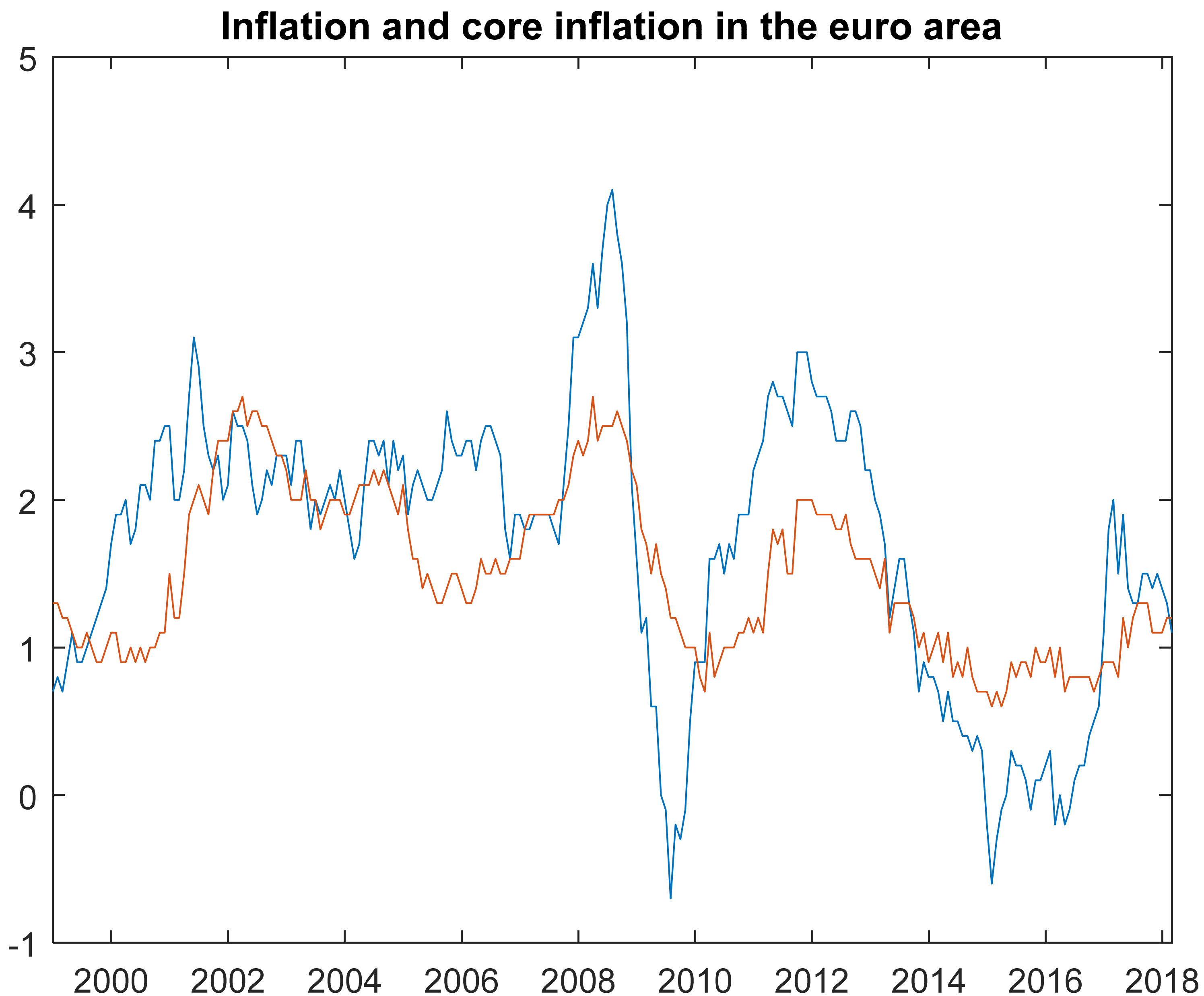
Data

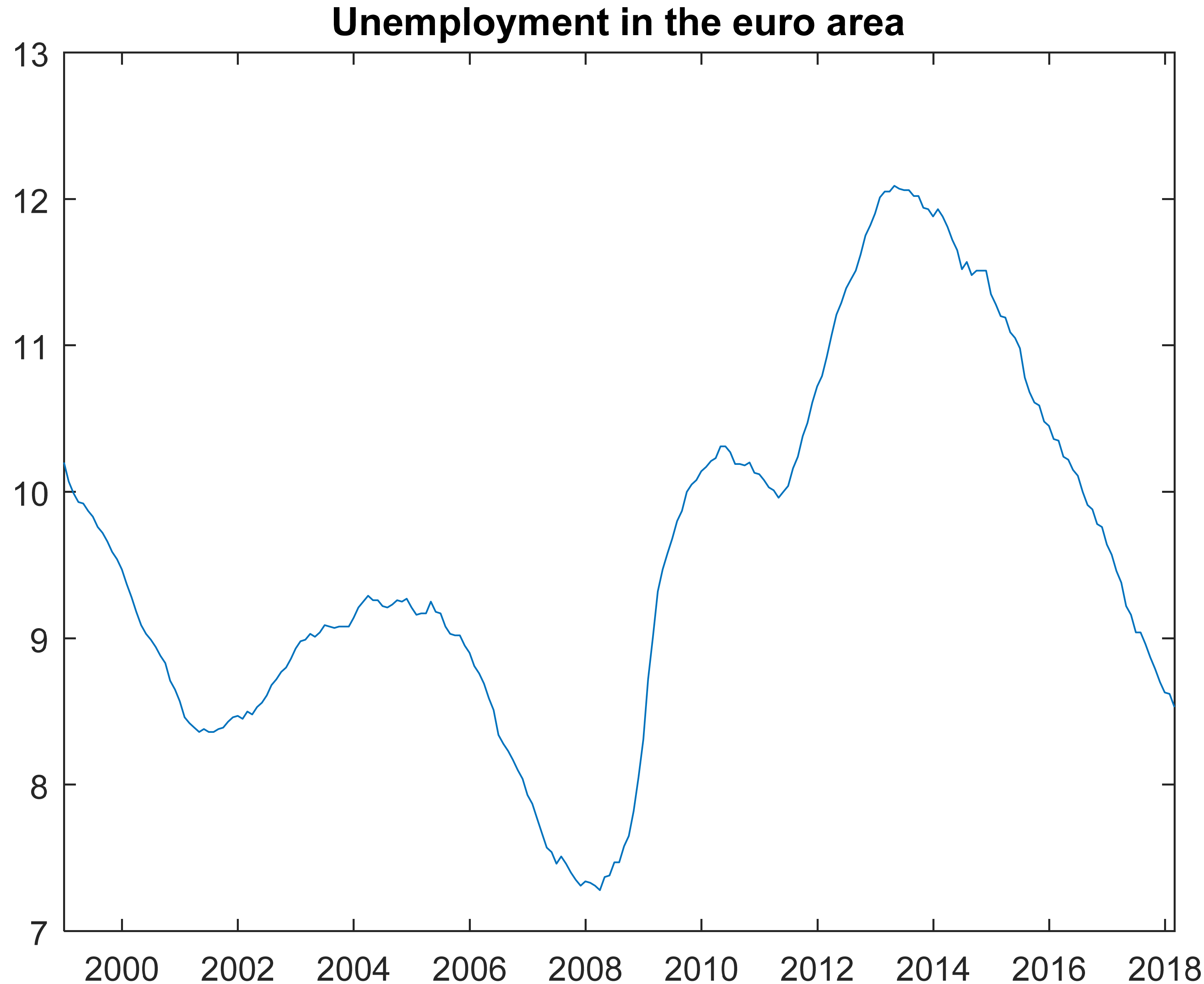
1) HICP - Overall index, Annual rate of change, Eurostat
2) HICP - All-items excluding energy and unprocessed food, Annual rate of change, Eurostat
3) Unemployment, Eurostat
4) Industrial Production Index, Eurostat

Source: https://sdw.ecb.europa.eu

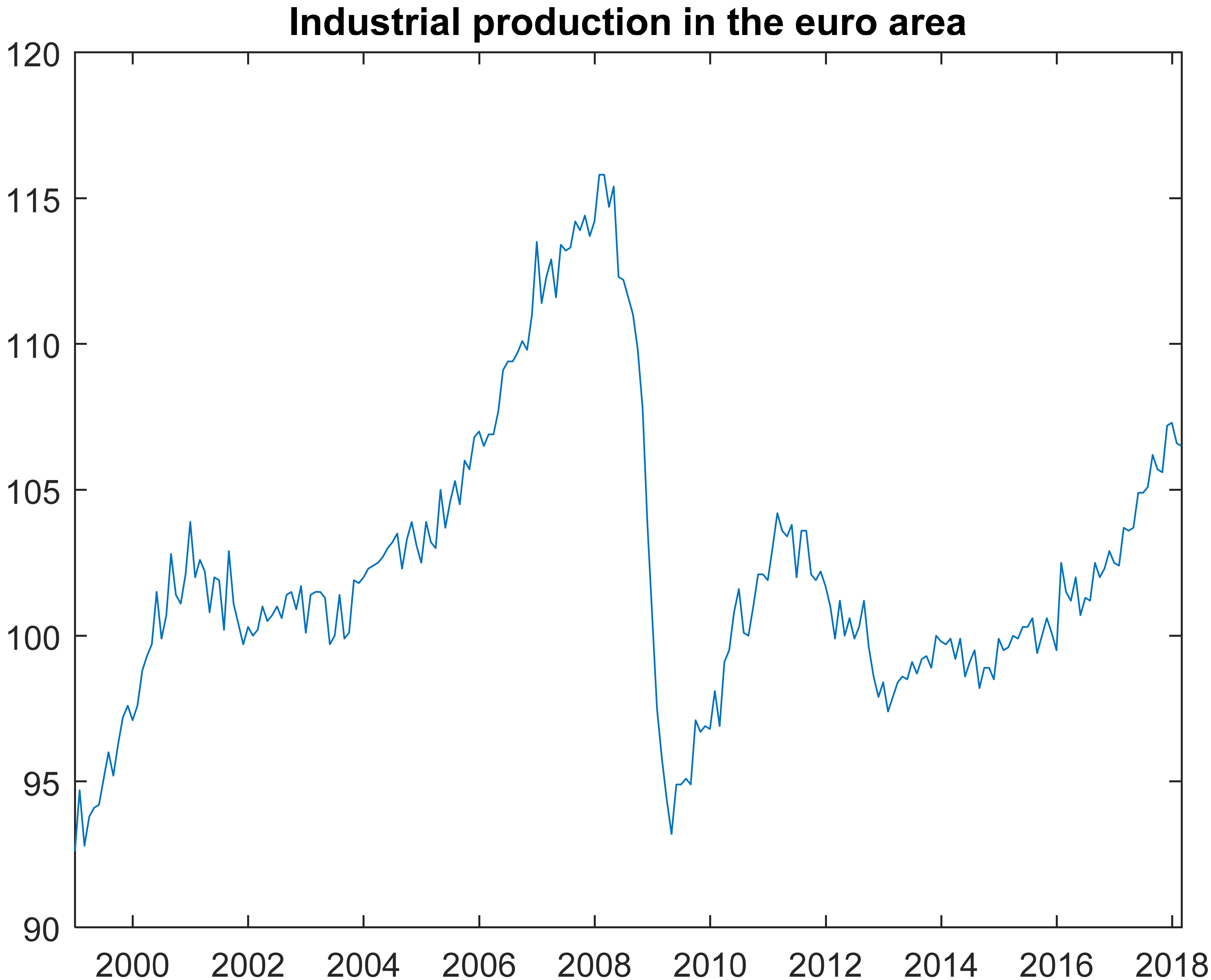
Annualised inflation



Unemployment



Industrial production



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VAR model

$$X_t = \upsilon + A_1 X_{t-1} + \cdots + A_p X_{t-p} + u_t$$
$$u_t \sim iid(0, \Sigma)$$

with zero restrictions:

→ ratio of parameters to data not too large

Choice of $f_t(X^t, \theta)$:

$$f_t(X^t, \theta) = \exp(-0.5 u_t{}' \Sigma^{-1} u_t)$$

Example

	c	1	2	3	4	5	6
HICP	1	1 0 1 0	0 0 0 1	0 0 0 0	0 0 0 0	0 0 0 0	1 0 0 0
HICPX	0	1 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 1 0 0
UN	0	1 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
IP	1	0 0 0 0	0 0 0 1	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
	2	4	2	0	0	0	2

	7	8	9	10	11	12	
	0 0 0 0	0 0 0 0	0 1 0 0	0 0 0 0	0 0 0 0	0 0 1 1	8
	0 0 0 0	0 0 0 0	1 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	3
	0 0 0 0	0 0 0 0	0 0 1 0	0 0 0 0	0 0 0 0	0 0 0 0	2
	0 0 0 0	0 0 0 1	0 0 1 0	0 0 0 1	0 0 0 0	0 0 0 0	5
	0	1	4	1	0	2	18

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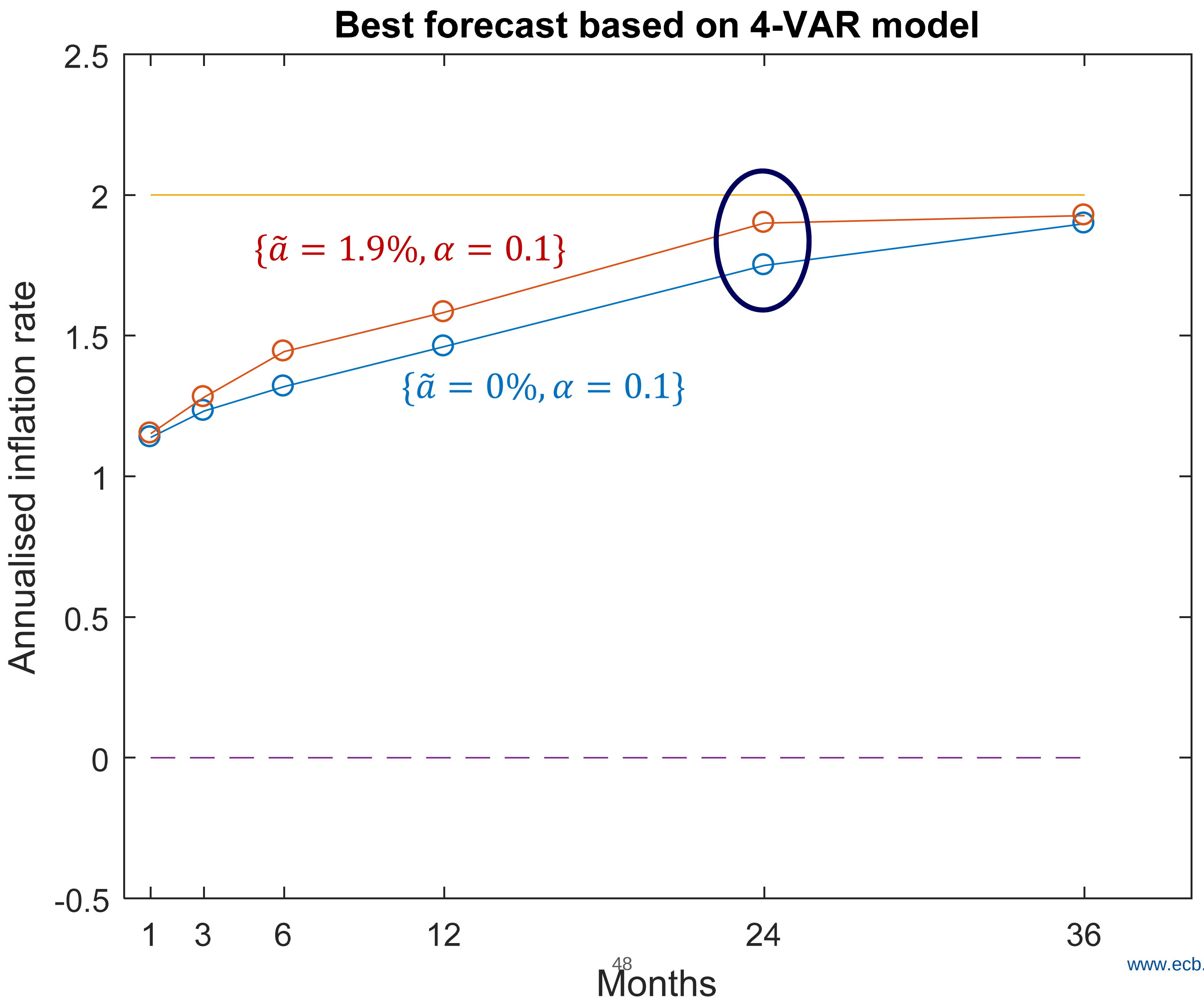
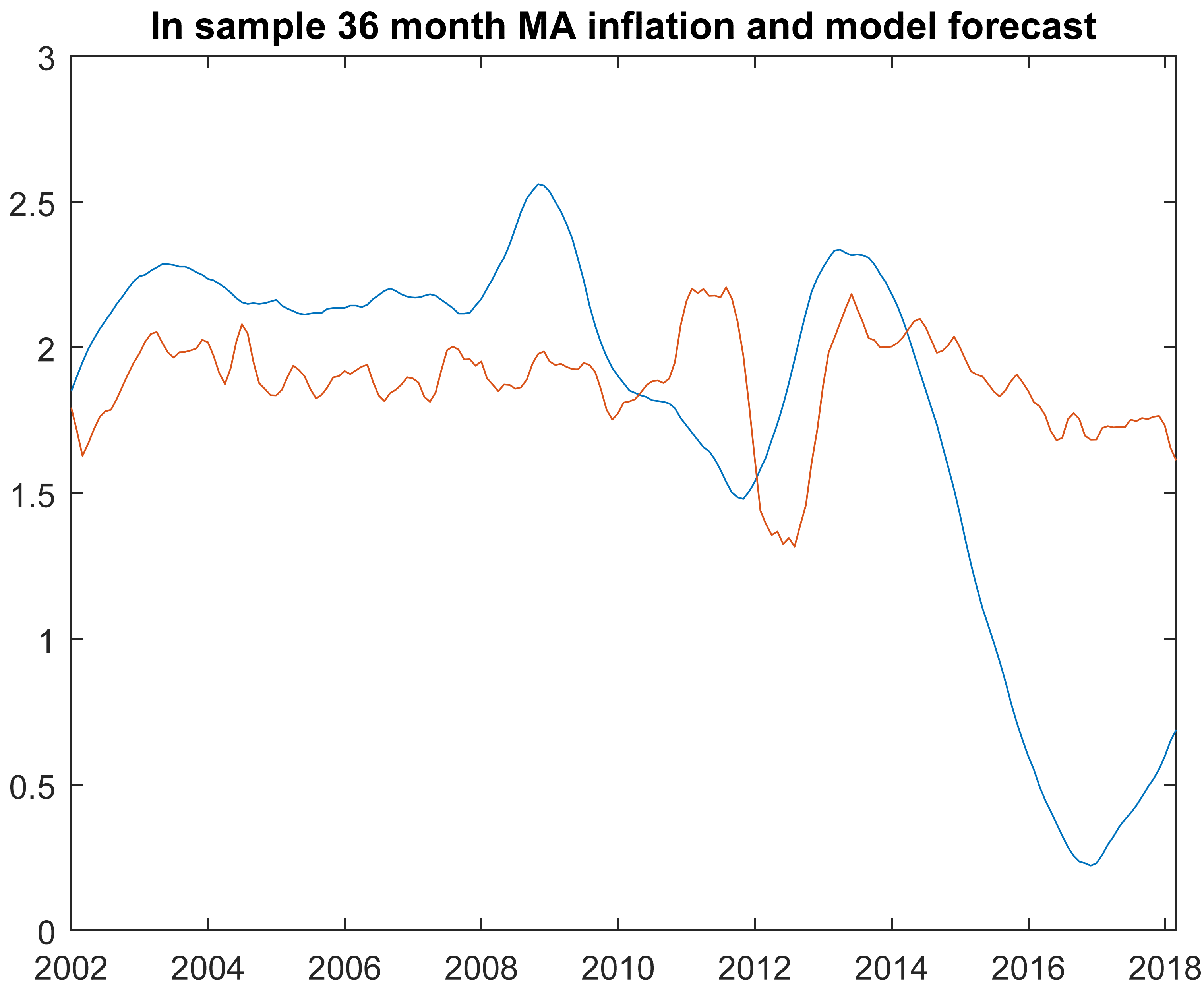
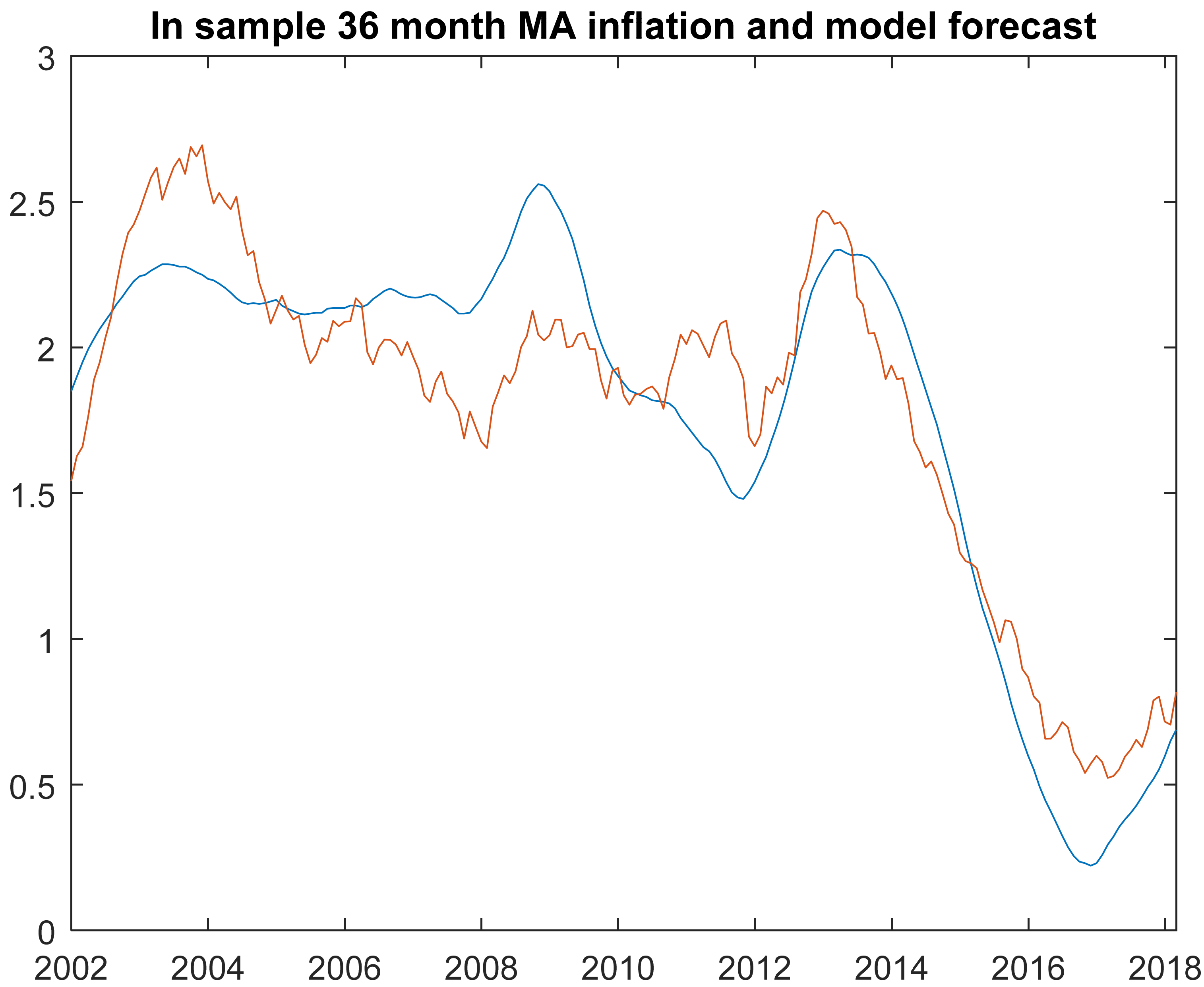
Euro area inflation forecast as of February 2018



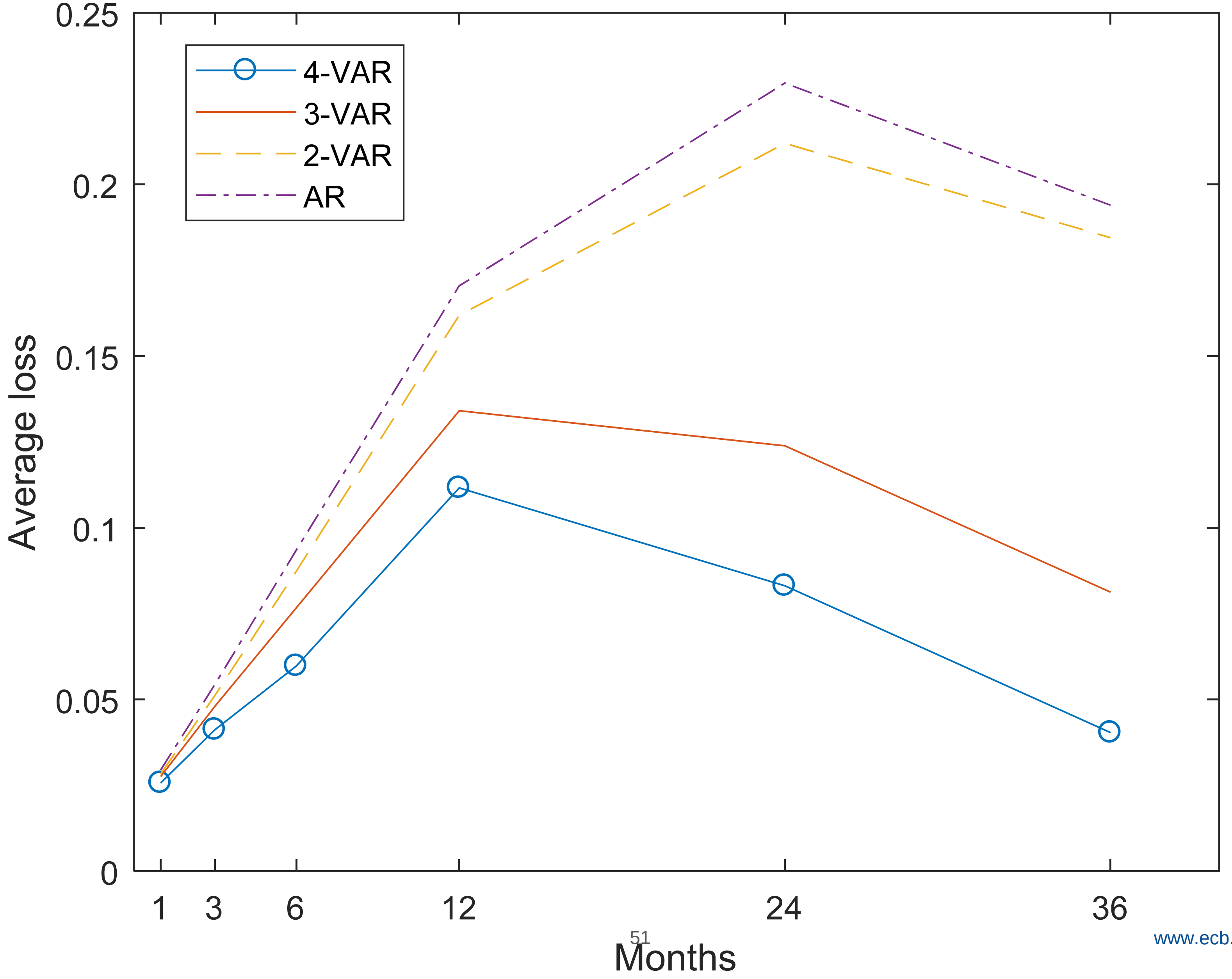
In sample performance of AR model



In sample performance of 4-VAR model



Average in sample loss



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3. Pay attention to unemployment, rather than core inflation