Structural Scenario Analysis with SVARs

Juan Antolín-Díaz[1] Juan Rubio-Ramirez[2] Ivan Petrella[3]

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[1]London Business School
[2]Emory University
[3]Warwick Business School and CEPR

Conditional Forecast and Scenario Analysis

A key question in applied macroeconomics and policy analysis is:

"if, for the next few years, variable y follows alternative paths, how do the forecasts of other macroeconomic variables change?"

Conditional Forecast and Scenario Analysis

Common uses:

- Assessing the path of macroeconomic variables to **alternative policy scenarios** (e.g. alternative monetary or fiscal policy)
- **Incorporating external information** such as data from futures prices (to condition on the path of oil prices or other financial variables)
- **"Stress Testing"**: assessing the reaction of asset prices or bank profits to economic recessions

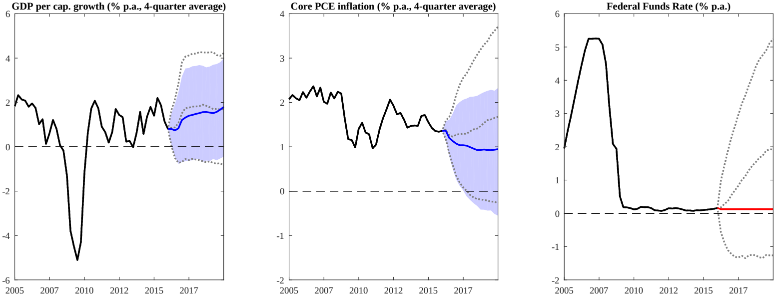
Conditional forecasts with VARs

- Waggoner and Zha (1999) provide methods for computing conditional density forecasts in the context of VAR models
- Andersson et al. (2010) extend their results to the case when there is uncertainty about the paths of the conditioning variables.

- The answer provided by these papers is *statistical* in nature... does **NOT** require to identify the underlying economic shocks in a Structural VAR!

A simple example: Forward Guidance

"what is the likely path of output an inflation, given that the fed funds rate is kept at zero for two years?"



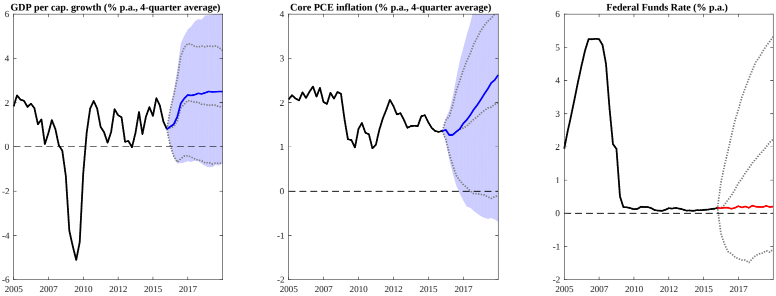
The meaning of conditional forecasts

- What is the most likely set of circumstances that will keep the Federal Funds Rate at zero for two years?
- But recall: monetary policy is endogenous! Most of the movements in the fed funds rate are the systematic reaction of the Fed to economic developments.
- Statistically, if the Fed is keeping the Fed Funds rate low for two years, some negative shock must be lowering output and inflation.

Structural Scenario Analysis

An alternative question

"what is the likely path of output and inflation, **if a sequence of monetary policy shocks keeps the federal funds rate at zero** *for two years?"*



Outline of the Talk

1. Reminder: Structural VARs and the identification problem
2. General Framework for Conditional Forecasting
   2.1 Relation to entropic tilting
3. Three special cases (with MP example)
   3.1 Conditioning on Observables
   3.2 Conditioning on Shocks
   3.3 Structural Scenarios
4. Assessing the plausibility of the scenarios
5. Stress testing example

Reminder: The Structural VAR

Consider the structural vector autoregression (SVAR) with the general form

$$\mathbf{y}_t'\mathbf{A}_0 = \sum_{\ell=1}^{p} \mathbf{y}_{t-\ell}'\mathbf{A}_\ell + \mathbf{d} + \varepsilon_t' \text{ for } 1 \leq t \leq T, \tag{1}$$

where

- $\mathbf{y}_t$ is an $n \times 1$ vector of variables
- $\mathbf{A}_\ell$ is an $n \times n$ matrix of parameters for $0 \leq \ell \leq p$ with $\mathbf{A}_0$ invertible
- $\mathbf{d}$ is a $1 \times n$ vector of parameters, p is the lag length, and T is the sample size.
- The vector of structural shocks ε_t is Gaussian with mean zero and covariance matrix $\mathbf{I}_n$, the $n \times n$ identity matrix.

VAR: REDUCED FORM REPRESENTATION

ORTHOGONAL REDUCED-FORM PARAMETERIZATION

- The reduced-form representation implied by Equation (1) is

$$\mathbf{y}_t' = \sum_{\ell=1}^{p} \mathbf{y}_{t-\ell}' \mathbf{B}_\ell + \underbrace{\boldsymbol{\varepsilon}_t' h(\boldsymbol{\Sigma}) \mathbf{Q}^{-1}}_{=\mathbf{u}_t'} \text{ for } 1 \leq t \leq T, \qquad (2)$$

  where the $n \times n$ matrix $h(\boldsymbol{\Sigma})$ is any decomposition of the covariance matrix $\boldsymbol{\Sigma}$ satisfying $h(\boldsymbol{\Sigma})' h(\boldsymbol{\Sigma}) = \boldsymbol{\Sigma}$.

- Given the reduced-form parameters and a decomposition h, one can consider each value of the orthogonal matrix $\mathbf{Q}$ as a particular choice of structural parameters.

- An identification scheme is a set of restrictions on the admissible $\mathbf{Q}$ matrices.

Identification with Sign Restrictions

- Starting with Faust (1998), Canova and Nicolo (2002), Uhlig (2005), and Rubio-Ramirez et al. (2010) it has become prominent to identify SVARs with sign restrictions
- Sign restrictions as an alternative to other conventional approaches
- Based on a handful of uncontroversial sign restrictions on either the IRFs or the structural parameters
- Attractive
  - Likely to be agreed upon by a majority of researchers
  - Robust across the set of SVARs that satisfy the restrictions
- We label these as Traditional Sign Restrictions

Problems with Traditional Sign Restrictions

- The small number of Traditional Sign Restrictions results in a large set of structural parameters with very different implications
- Best case: difficult to arrive to meaningful conclusions
- Worst case: retain in the admissible set structural parameters with implausible implications
- Challenge: find additional uncontentious sign restrictions that shrink the set of admissible structural parameters
- In a previous paper (Antolin-Diaz and Rubio-Ramirez, AER 2018), we develop the idea of **Narrative Sign Restrictions**.

A Monetary Policy VAR

Specification and Identification

- For the next few slides of the talk we will work with a highly stylized monetary policy VAR with three variables: Real GDP growth, Core PCE inflation, and the Federal funds rate.
- We can think of the correlation between these three series as arising from the materialization of (primitive) demand, supply, and monetary policy shocks.

A monetary policy VAR

Specification and Identification

The structural parameters are identified with a combination of traditional and narrative sign restrictions.

	Impact			Long Run		
Variable / Shock	MP	AD	AS	MP	AD	AS
Real GDP	$-$	$-$	$-$	0	0	
Core PCE inflation	$-$	$-$	$+$			
Federal funds rate	$+$	$-$	$+$			

Narrative Sign Restriction 1. *The monetary policy shock for the observation corresponding to the fourth quarter 1979 must be of positive value.*
Narrative Sign Restriction 2. *For the observation corresponding to the fourth quarter of 1979, a monetary policy shock is the overwhelming driver of the unexpected movement in the federal funds rate.*

Unconditional Forecast

- Assume that we want to forecast (uncoditional) the observables for h periods ahead: $\mathbf{y}'_{T+1,T+h} = (\mathbf{y}'_{T+1} \dots \mathbf{y}'_{T+h})$
- Conditional on past realizations, the unconditional forecast can be rewritten as the sum of a deterministic and a stochastic component

$$\mathbf{y}'_{T+1,T+h} = \mathbf{b}'_{T+1,T+h} + \varepsilon'_{t+1,t+h}\mathbf{M}. \tag{3}$$

where the stochastic component reflects the shocks unfolding over the forecast horizon

Unconditional Forecast

Forecast distribution

- Given equation (3) the unconditional forecast $\mathbf{y}'_{T+1,T+h}$ is distributed as

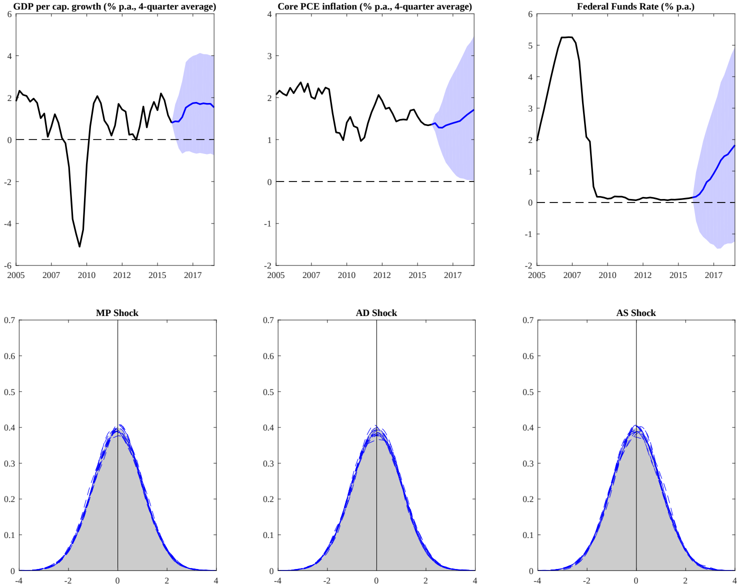
$$\mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{b}_{T+1,T+h}, \mathbf{M}'\mathbf{M}\right). \tag{4}$$

- The distribution of $\varepsilon_{T+1,T+h}$ compatible with the unconditional forecast distribution of $\mathbf{y}_{T+1,T+h}$ is

$$\varepsilon_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{0}_{nh}, \mathbf{I}_{nh\times nh}\right). \tag{5}$$

- Unconditional forecast does **NOT** depend on the structural parameters!

Unconditional Forecast



General Framework for Linear Restrictions

- Assume that we want to forecast the observables, while imposing (linear) restrictions on the path:

$$\mathbf{C}\mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{f}_{T+1,T+h}, \mathbf{\Omega}_f\right), \tag{6}$$

- where $\mathbf{C}$ is any given $k \times nh$ matrix (with k denoting the number of restrictions and h the number of periods ahead).
- The matrix $\mathbf{\Omega}_f$ captures the uncertainty associated with the forecast scenario.

The restriction in Equation (6) implies a restriction on the distribution of the future structural shocks. To see this, combine Equation (6) with Equation (3) to obtain

$$\mathbf{C}\mathbf{y}_{T+1,T+h} = \mathbf{C}\mathbf{b}_{T+1,T+h} + \mathbf{D}\tilde{\boldsymbol{\varepsilon}}_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{f}_{T+1,T+h}, \boldsymbol{\Omega}_f\right), \quad (7)$$

where $\mathbf{D} = \mathbf{C}\mathbf{M}'$ and $\tilde{\boldsymbol{\varepsilon}}_{T+1,T+h}$ are the restricted future shocks.

This implies that $\tilde{\boldsymbol{\varepsilon}}_{T+1,T+h} \sim \mathcal{N}\left(\boldsymbol{\mu}_{\varepsilon}, \boldsymbol{\Sigma}_{\varepsilon}\right)$. Define $\boldsymbol{\Sigma}_{\varepsilon} = \mathbf{I}_{nh} + \boldsymbol{\Psi}_{\varepsilon}$, $\boldsymbol{\mu}_{\varepsilon}$ and $\boldsymbol{\Psi}_{\varepsilon}$ as the deviation of the mean and covariance matrix of $\tilde{\boldsymbol{\varepsilon}}_{T+1,T+h}$ from the mean and covariance matrix of the unconditional shocks, $\boldsymbol{\varepsilon}_{T+1,T+h}$.

Following Penrose (1955, 1956) we will choose the following expression for $\boldsymbol{\mu}_{\varepsilon}$ and $\boldsymbol{\Psi}_{\varepsilon}$,

$$\boldsymbol{\mu}_{\varepsilon} = \mathbf{D}^{\star}\left(\mathbf{f}_{T+1,T+h} - \mathbf{C}\mathbf{b}_{T+1,T+h}\right) \tag{8}$$

$$\boldsymbol{\Psi}_{\varepsilon} = \mathbf{D}^{\star}\boldsymbol{\Omega}_{f}\mathbf{D}^{\star\prime} - \mathbf{D}^{\star}\mathbf{D}\mathbf{D}^{\prime}\mathbf{D}^{\star\prime}, \tag{9}$$

where $\mathbf{D}^{\star}$ is the Moore-Penrose inverse of $\mathbf{D}$.

When $k \leq nh$, the above equations characterize the solution that implies the smallest deviation of the mean and covariance matrix of $\tilde{\boldsymbol{\varepsilon}}_{T+1,T+h}$ from the mean and covariance matrix of $\boldsymbol{\varepsilon}_{T+1,T+h}$. In this case, $\mathbf{D}^{\star} = \mathbf{D}^{\prime}\left(\mathbf{D}\mathbf{D}^{\prime}\right)^{-1}$.

Relation with Entropic Tilting

We show in the paper the equivalence between the framework introduced above and entropic forecast tilting, popularized in the macroeconomic literature by Robertson et al. (2005) (see also Giacomini and Ragusa, 2014)

Proposition 1. Denote with $\mathcal{N}_{UF}$ the distribution of the unconditional forecast represented by Equation (4). Then $\boldsymbol{\mu}_y$ and $\boldsymbol{\Sigma}_y$, given by Equations (8) and (9), are the solution to the following relative entropy problem

$$\min_{\boldsymbol{\mu},\boldsymbol{\Sigma}} D_{KL}\left(\mathcal{N}\left(\boldsymbol{\mu},\boldsymbol{\Sigma}\right)||\mathcal{N}_{UF}\right)$$

subject to $\mathbf{C}\boldsymbol{\mu} = \mathbf{f}_{T+1,T+h}$ and $\mathbf{C}\boldsymbol{\Sigma}\mathbf{C}' = \boldsymbol{\Omega}_f$.

PROOF.
See Appendix B. $\square$

Three special cases

- The preceding equations define a general framework for imposing linear density restrictions on the forecast of the form:

$$\mathbf{C}\mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{f}_{T+1,T+h}, \mathbf{\Omega}_f\right), \tag{10}$$

- We will now consider three useful special cases:
  - Conditioning on Observables
  - Conditioning on Shocks
  - Structural Scenario Analysis
- These cases will be implemented by appropriate choices of $\mathbf{C}$, $\mathbf{f}_{T+1,T+h}$ and $\mathbf{\Omega}_f$.

Conditional-on-Observables Forecasting

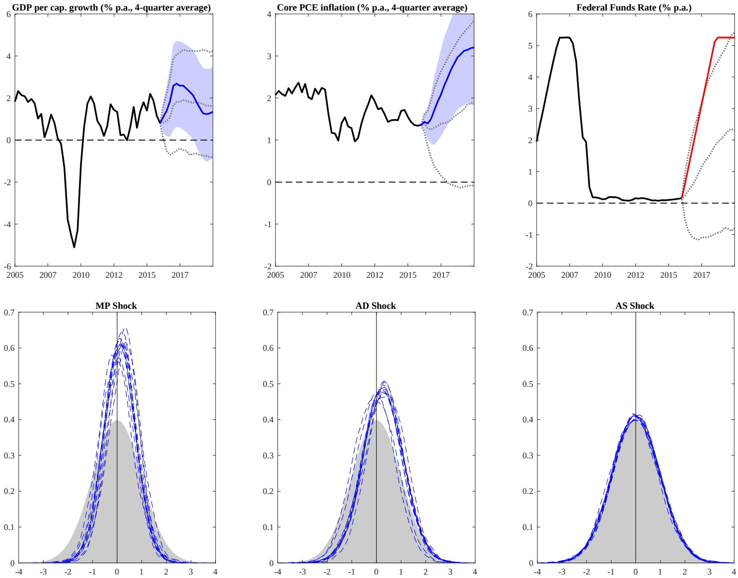
Classic conditional forecasting á la Waggoner and Zha (1999)

- Assume that we want to forecast the observables for some periods ahead, but restricting the forecasts for a subset of the observables for some of the periods ahead

$$\overline{\mathbf{C}}\mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{f}_{T+1,T+h}, \boldsymbol{\Omega}_f\right), \tag{11}$$

- Let $\overline{\mathbf{C}}$ be a $p \times nh$ selection matrix formed by ones and zeros (with p denoting the number of restrictions and h the number of periods ahead, and $p \leq nh$).
- The matrix $\boldsymbol{\Omega}_f$ captures the uncertainty associated with the forecast scenario (e.g. $\boldsymbol{\Omega}_f = 0$ corresponds to hard conditioning).

Conditioning on a monetary policy tightening



Conditional-on-Shocks Forecasting

- The restriction on the structural shocks is implemented by imposing that $\Xi \varepsilon_{T+1,T+h}$ is

$$\Xi \varepsilon_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{g}_{T+1,T+h}, \mathbf{\Omega}_g\right), \tag{12}$$

where Ξ be a $k \times nh$ selection matrix formed by ones and zeros

- Note that Equation (3) means that

$$\varepsilon_{t+1,t+h} = \left(\mathbf{M}'\right)^{-1} \mathbf{y}_{t+1,t+h} - \left(\mathbf{M}'\right)^{-1} \mathbf{b}_{t+1,t+h}.$$

- Therefore eq. (12) implies that

$$\underline{\mathbf{C}} \mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\underline{\mathbf{C}} \mathbf{b}_{T+1,T+h} + \mathbf{g}_{T+1,T+h}, \mathbf{\Omega}_g\right), \tag{13}$$

where $\underline{\mathbf{C}} = \Xi(\mathbf{M}')^{-1}$.

Structural Scenario Analysis

A structural scenario is defined by the combination of:

1. k_o restrictions on the path for one or more of the variables
2. k_s restrictions that only a subset of the shocks can deviate from their unconditional distribution.

The conditional-on-observables method implied restrictions on all structural shocks... here only the shocks that are assumed to be drivers of the scenario are altered, whereas the rest are restricted to retain their unconditional distribution!

Structural Scenario Analysis

Formally

1. the restriction on the observables is implemented by imposing that $\overline{\mathbf{C}}\mathbf{y}_{T+1,T+h}$ is distributed as follows

$$\overline{\mathbf{C}}\mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{f}_{T+1,T+h}, \boldsymbol{\Omega}_f\right),$$

2. while the restriction on the structural shocks is implemented by imposing that

$$\boldsymbol{\Xi}\boldsymbol{\varepsilon}_{T+1,T+h} \sim \mathcal{N}\left(\mathbf{0}_{k_s}, \mathbf{I}_{k_s}\right),$$

which in turn implies

$$\underline{\mathbf{C}}\mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\underline{\mathbf{C}}\mathbf{b}_{T+1,T+h}, \mathbf{I}_{k_s}\right),$$

where $\underline{\mathbf{C}} = \boldsymbol{\Xi}(\mathbf{M}')^{-1}$.

Structural Scenario Analysis

- If we define

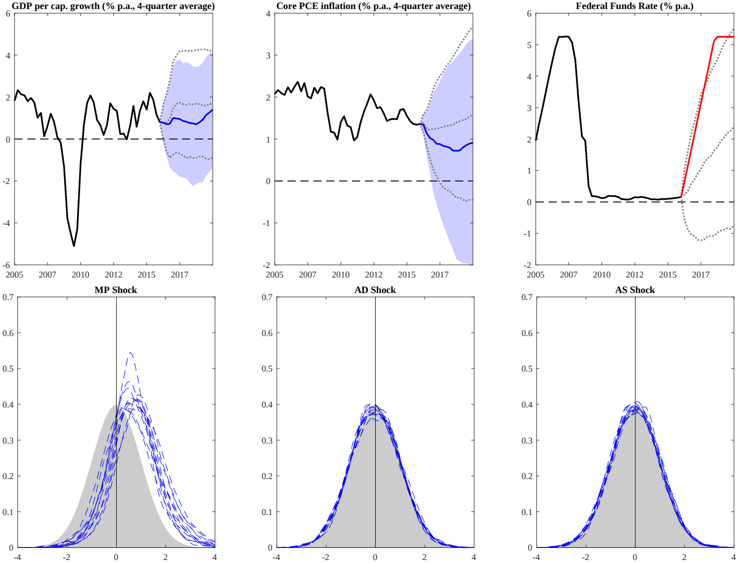
$$\mathbf{C} = \begin{bmatrix} \overline{\mathbf{C}} \\ \boldsymbol{\Xi}(\mathbf{M}')^{-1} \end{bmatrix},$$

- We get that

$$\mathbf{C}\mathbf{y}_{T+1,T+h} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{f}_{T+1,T+h} \\ \underline{\mathbf{C}}\mathbf{b}_{T+1,T+h} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Omega}_f & \mathbf{0}_{k_o \times k_s} \\ \mathbf{0}_{k_s \times k_o} & \mathbf{I}_{k_s} \end{bmatrix} \right).$$

- therefore we can use our results to simulate from the distribution of the conditional forecasts.

A monetary policy tightening (structural) scenario



How plausible is the structural scenario?

- We quantify how implausible a structural scenario is by determining how "far" the distribution of the structural shocks $\varepsilon_{T+1,T+h}$ compatible with the structural scenario from the unconditional distribution of structural shocks $\varepsilon_{T+1,T+h}$.
- We will use the Kullback-Leibler (KL) divergence as a measure of how different the two distributions of structural shocks are.
- Since the unconditional distribution of the shock ($\mathcal{N}_{UF}$) and under the scenario ($\mathcal{N}_{SS}$) are both multivariate Normal, and the unconditional distribution is a standard Normal:

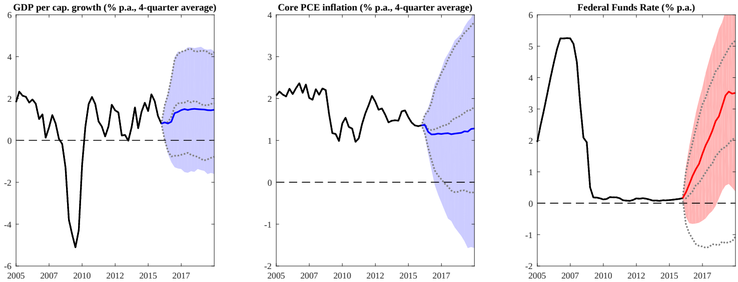
$$D_{\mathrm{KL}}(\mathcal{N}_{SS} \| \mathcal{N}_{UF}) = \frac{1}{2}\left(\operatorname{tr}\left(\boldsymbol{\Sigma}_{\varepsilon}\right) + \boldsymbol{\mu}_{\varepsilon}'\boldsymbol{\mu}_{\varepsilon} - nh - \ln(\det \boldsymbol{\Sigma}_{\varepsilon})\right)$$

How plausible is the structural scenario?

- Fine for relative comparisons, but what about plausibility in absolute sense?
- To ease the interpretation of the the KL divergence, McCulloch (1989) proposes to calibrate the KL divergence from P to Q to the the KL divergence between two Bernoulli distributions: likens the comparison between P and Q to a comparison between the flips of a fair and a biased coin (with probability $q(z)$).
- We obtain a number between 0.5 and 1, where 0.5 means very similar to the unconditional forecast, and 1 very different.

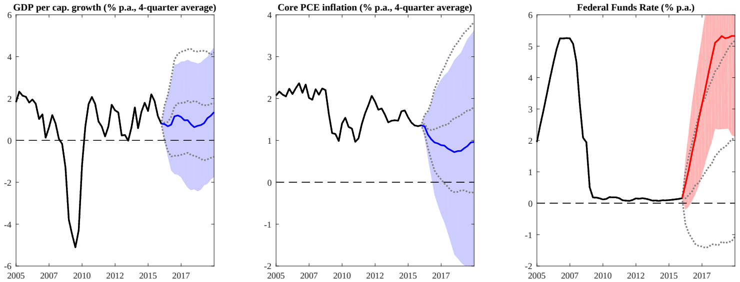
Evaluating Alternative Scenarios

Baseline from Summary of Economic Projections



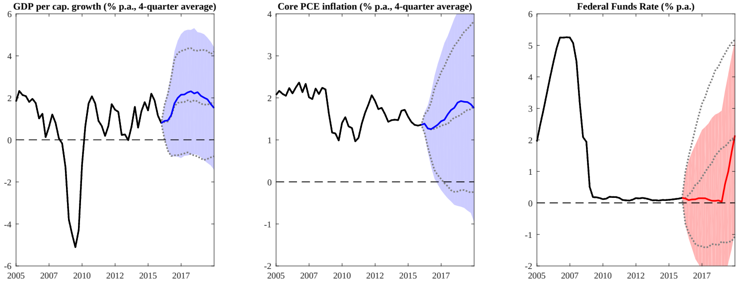
Evaluating Alternative Scenarios

A Tighter Policy Path



Evaluating Alternative Scenarios

A Tighter Policy Path



Evaluating alternative scenarios

	Certain FFR		Uncertain FFR	
	KL div.	Calib. KL	KL div.	Calib. KL
Baseline SEP	5.57×10^4	1	4.45	0.70
Lower for Longer	5.92×10^4	1	4.69	0.71
Tighter	1.01×10^5	1	5.08	0.72

Note: We report the mean of the KL divergence across draws of the posterior.

Application to Stress Testing

- Stress test is an analysis conducted under unfavorable economic scenarios (e.g. a recessionary episode) designed to determine whether a bank has enough capital to withstand the impact of adverse developments.
- Not all recessions are alike! So one should be careful when setting up the scenario in stress test exercises
- Recession of similar size can have very different impact on bank profitability depending on what originates the recession
- Example: **Financial vs non-financial recessions**

Identification of a financial shock

Bernanke (2016) highlights how the collapse of Lehman brothers in September 13, 2008 caused *"short-term lending markets to freeze and increase the panicky hoarding of cash" (p. 268) "... fanned the flames of the financial panic" (p. 269), and "directly touched off a run on money market funds" (p. 405), " ... and triggered a large increase in spreads" (p. 405).*

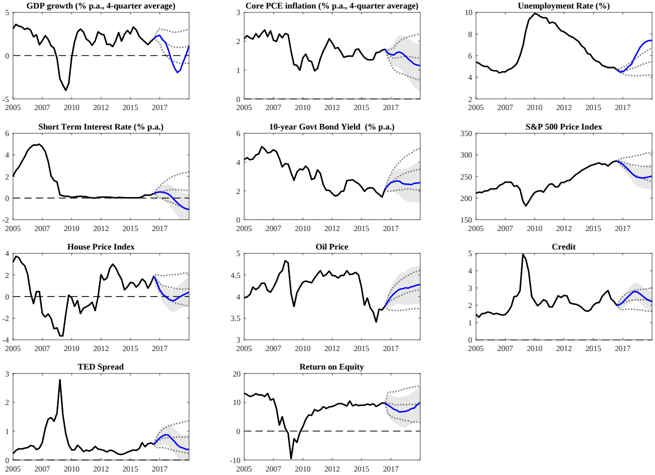
Identification of a financial shock

Sign Restrictions. *The financial shock is restricted to have a negative impact on stock prices and bank profitability, and to increase the BAA and TED spreads.*

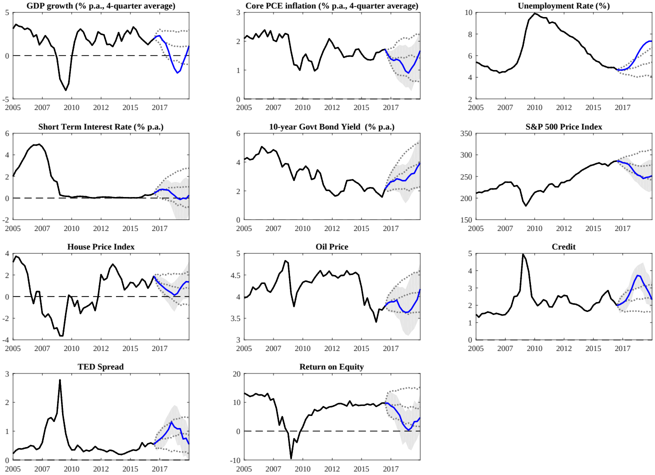
Narrative Sign Restriction 1. *The financial shock for the observation corresponding to the fourth quarter of 2008 must be of positive value.*

Narrative Sign Restriction 2. *In the fourth quarter of 2008, the financial shock is the overwhelming driver of the unexpected movement in the TED spread and credit spread. In other words, the absolute value of the contribution of financial shocks to the unexpected movement in these variables is larger than the sum of the absolute value of the contributions of all other structural shocks.*

(Non Financial) Recession Scenario



Financial Recession Scenario



Conclusions

- Conditional forecasting is usually a statistical exercise, involving the dynamic correlations among endogenous variables, and remaining silent about the underlying economic causes behind the forecast.
- For the same path of the conditioning variables, the result can be very different depending on which shock is assumed to drive the scenario.
- More often than not, the researcher would like to analyze the conditional forecast through the lens of a structural model, and assess the future value of the variables given some path for other variables which is driven by *a specific set of structural shocks:* **Structural scenario analysis**
- Structural scenario analysis can be a useful complement to conditional forecasting when an economic interpretation of the forecasts is sought.

Thanks!

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