The Global Component of Inflation Volatility

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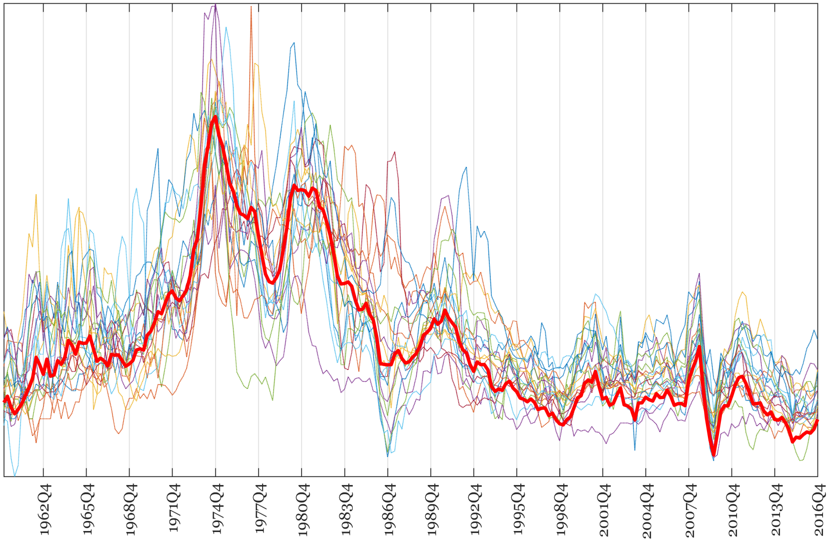
How much global is the Inflation process?

- Rogoff (2003), Rogoff (2006), Carney (2017), Miles, Paniza, Reis and Ubide (2017): **Globalisation, Inflation and Central Banks**.
- Borio and Filardo (2007), Bianchi and Civelli (2015) and Auer, Borio and Filardo (2017): **effects of global economic conditions on inflation.**
- Ciccarelli and Mojon (2010), Mikolajun and Lodge (2016): **a substantial amount of variation** in a large set of national inflation rates **is explained by global factors** that capture the most persistent component (slow moving trends).
- Engle (1982), Stock and Watson (2007), Mumtaz and Surico (2008): **including changing volatility when modeling inflation**.

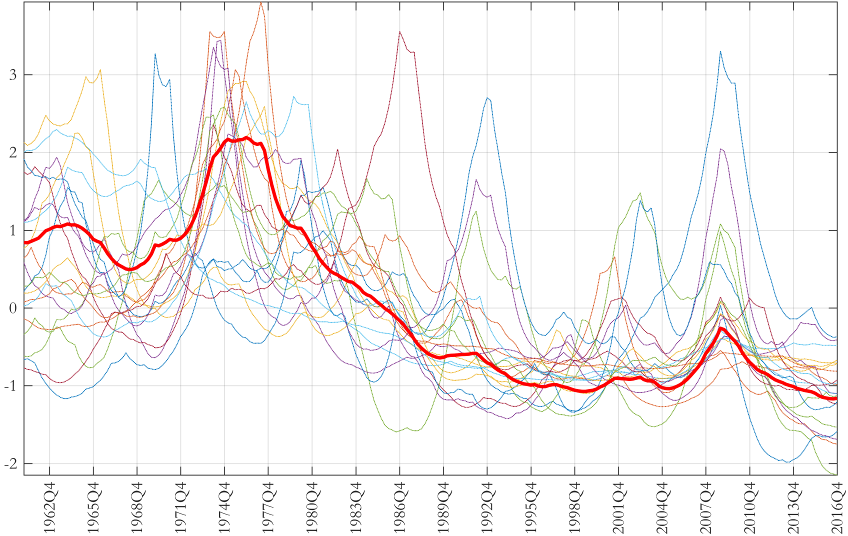
CPI inflation rates and PCA

Data for 20 OECD countries. The first PC explains almost 75%



CPI inflation rates SVs from univariate AR-SV

Data for 20 OECD countries. The first PC explains almost 60%



What this paper does

- We include **Stochastic Volatility in modeling multi-country inflation rates** (20 OECD countries since the 1960s).
- We investigate cross-country commonality **not only in inflation levels, but also in inflation volatilities**.
- We build a Multivariate Autoregressive Index model with Autoregressive components and Stochastic volatility (**MAI-AR-SV**), and derive a fully-fledged **Bayesian MCMC algorithm**.
- We decompose both levels and volatilities so to disentangle **contributions of a single global component** and the idiosyncratic components.
- We run a **point and density forecasting evaluation** to test the out of sample performance of the model.

Main Results

- The estimated **global factor explains roughly 70%** of the variability **of CPI inflation levels**.
- Significantly time-varying **global inflation volatility** since the 1960s.
- **Important evidence of commonality in volatilities, increased in the last two decades.** A large fraction of headline CPI inflation volatility can be attributed to the global factor.
- The same decompositions conducted on **Non-Food&Non-Energy inflation** show a **smaller and more stable** degree of commonality.
- Point and density forecasting evaluation shows that the MAI-AR-SV model has **very good out of sample performance** for inflation rates.

The MAI-AR-SV model

Introducing SV in the Multivariate Autoregressive Index with AR components

- Reinsel (1983), Carriero Kapetanios and Marcellino (JoE, 2016)

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} \underbrace{A_\ell \cdot B_0}_{\displaystyle\downarrow} \cdot \; y_{t-\ell} + \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + u_t$$

$$\underbrace{A_\ell}_{n\times r} \cdot \underbrace{B_0}_{r\times n}$$

- Rank reduction from n to r

$$F_t \equiv \underbrace{B_0}_{r\times n} \cdot y_t$$

- F_t, i.e. the "Index", will be interpreted as Global Inflation ($r = 1$)

The mai-AR-sv model

Introducing SV in the Multivariate Autoregressive Index with AR components

- Cubadda and Guardabascio (2017)

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot \; y_{t-\ell} + \sum_{\ell=1}^{q} \underbrace{\Gamma_\ell}_{\downarrow} \cdot y_{t-\ell} + u_t$$

$$\underbrace{\Gamma_\ell}_{n\times n} = \begin{bmatrix} \gamma_{1,\ell} & 0 & \dots & 0 \\ 0 & \gamma_{2,\ell} & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \gamma_{n,\ell} \end{bmatrix}$$

- q Univariate AutoRegressive Coefficients (q potentially larger than p) in diagonal Γ_ℓ

The MAI-AR-SV model

Introducing SV in the Multivariate Autoregressive Index with AR components

- Cogley and Sargent (2005) and Primiceri (2005)

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot\ y_{t-\ell} + \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + \underbrace{u_t}$$

$$u_t \overset{i}{\sim} \mathcal{MN}\,(\,\mathbf{0},\,\Omega_t\,), \qquad \underbrace{\Omega_t}_{n\times n} = G^{-1}\Sigma_t\Sigma_t\left(G^{-1}\right)'$$

- Log-volatilities law of motion

$$\Sigma_t = Diag(\sigma_t), \qquad \log\sigma_t = \log\sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{Q_\sigma}_{n\times n}\right)$$

Decomposition of SVs and levels

- Decompose innovations in **two orthogonal sets of components**:

$$u_t = \Omega_t B_0' \Xi_t^{-1} \cdot \underbrace{\omega_t}_{\text{Common}} + B_{0\perp}' \Xi_{\perp,t}^{-1} \cdot \underbrace{\psi_t}_{\text{Idiosyncratic}}$$

$$\begin{bmatrix} \omega_t \\ \psi_t \end{bmatrix} = \begin{bmatrix} B_0 u_t \\ B_{0\perp} \Omega_t^{-1} u_t \end{bmatrix} \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix} \Xi_t & 0 \\ 0 & \Xi_{\perp,t} \end{bmatrix} \right)$$

- Exploit the orthogonality of ω_t and ψ_t to decompose the SV...

$$\Omega_t = \Omega_t^{com} + \Omega_t^{idio} \Leftrightarrow \begin{cases} \Omega_t^{com} = \Omega_t B_0' \Xi_t^{-1} B_0 \Omega_t \\ \Omega_t^{idio} = B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \end{cases}$$

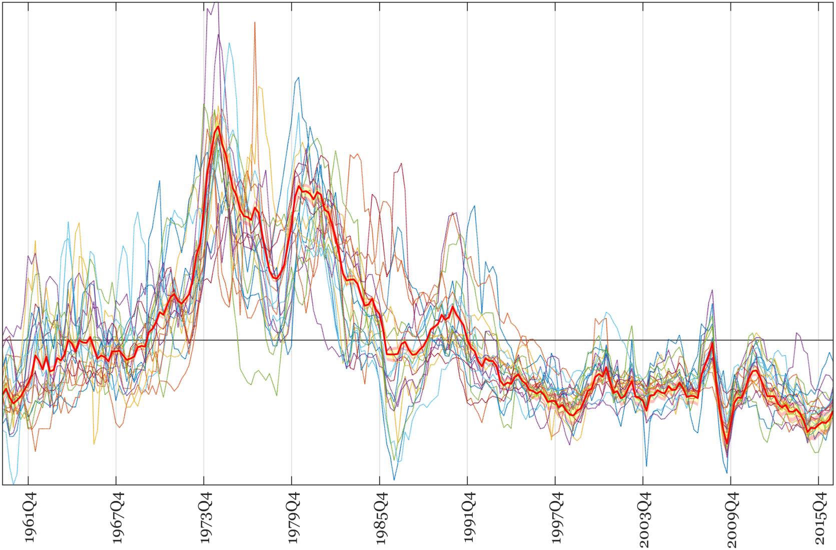
- ...and the observables y_t by regressing on contemporaneous and lagged values of ω_t:

$$y_t = B_1(L)\omega_t + B_2(L)\psi_t.$$

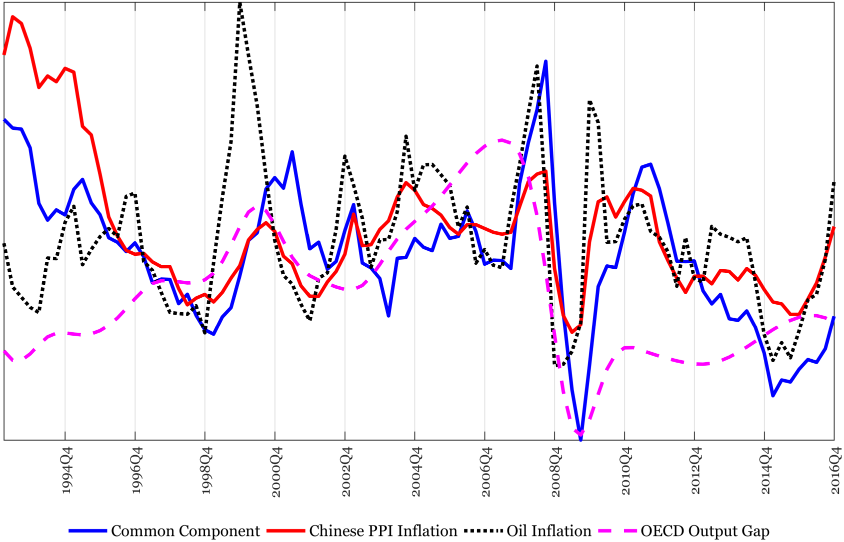
Specification and Dataset

- **CPI inflation: Consumer Price Index, year on year growth**
- The analysis is performed for **both headline and core CPIs changes**
- Source: OECD *Main Economic Indicators*
- Quarterly frequency dataset:
  - **All Items**: 228 observations, 1960-Q1 → 2016-Q4
  - **Non-food & non-energy items**: 152 observations, 1979-Q1 → 2016-Q4
- Data for 20 OECD countries:
  USA, Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK
- Single Index (global common factor), as Ciccarelli and Mojon (2010)
- 4 lags used

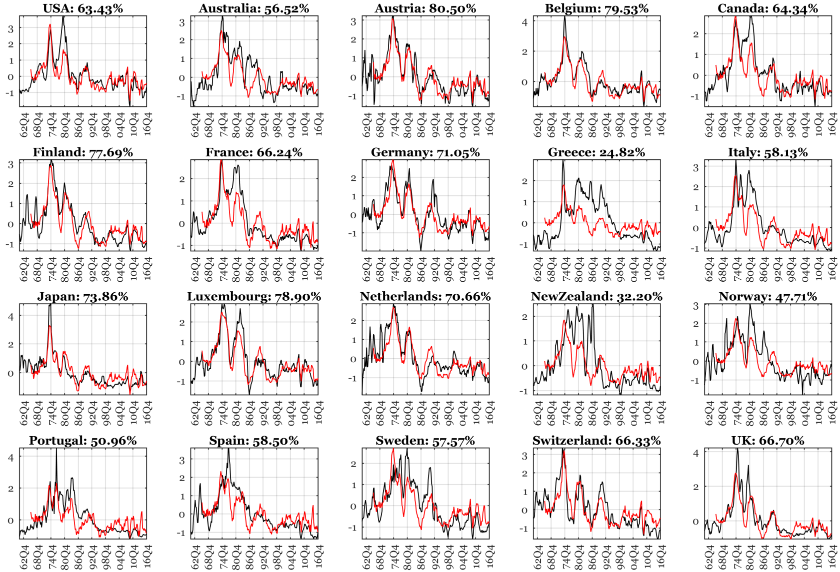
Data Vs Global Factor, Posterior bands



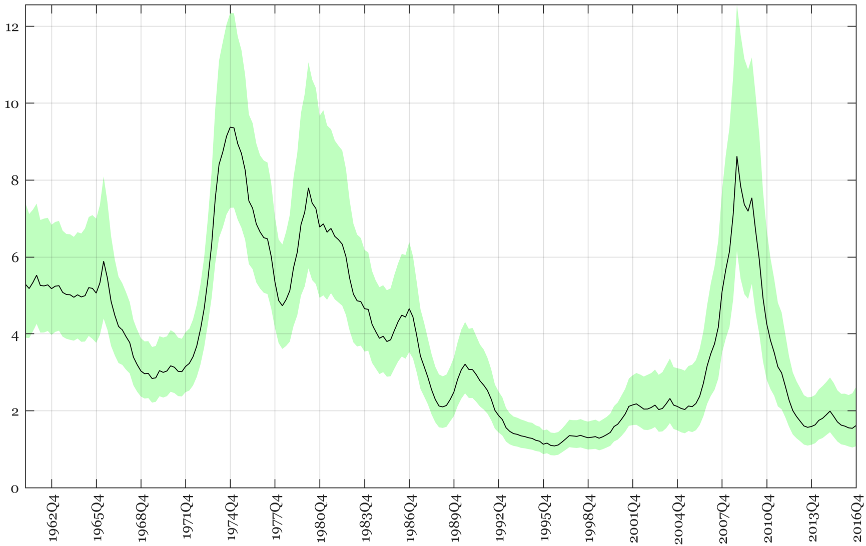
Global Inflation Factor Vs Oil, Chinese PPI, OECD Output Gap



Projections on the Common Component ω_t

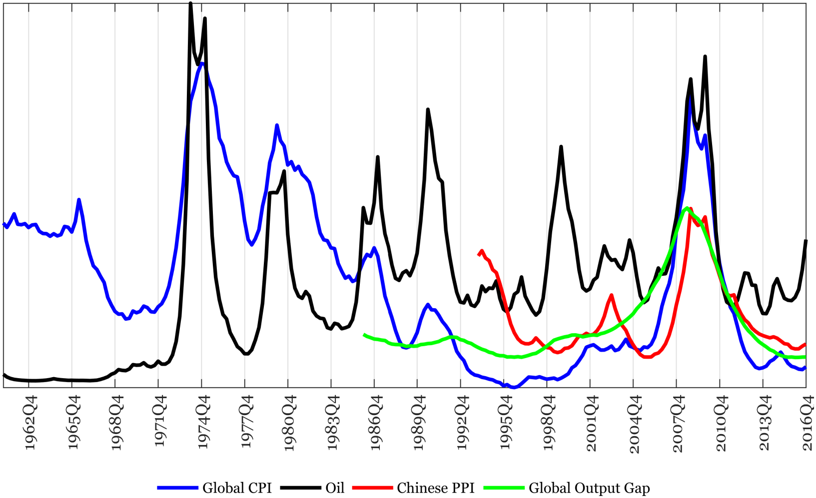


Global Inflation Volatility, $\mathbb{E}\left(\omega_t^2\right)$



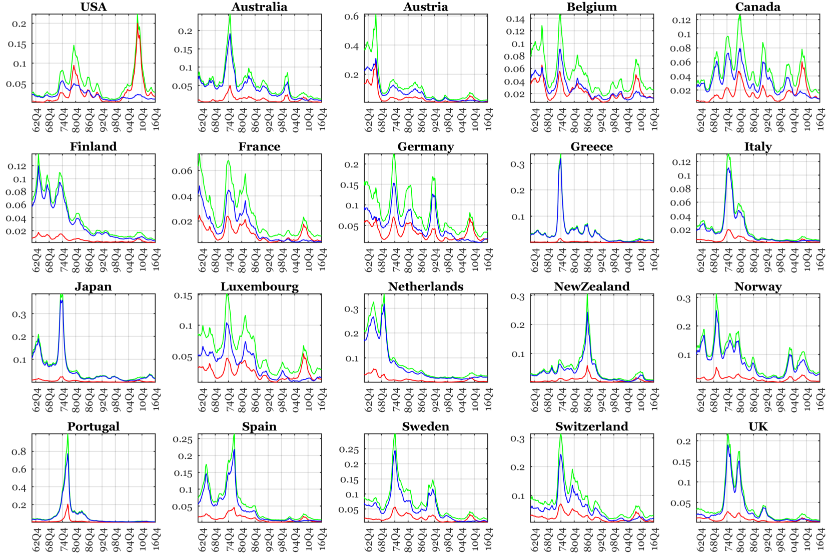
Global Inflation SV & Oil and Chinese PPI SVs

Global Inflation SV is correlated with: Oil SV (0.6), Chinese PPI SV (0.8) and Global Output Gap SV (0.6).



Residuals Volatility Decomposition

total (green), common (red), idio (blue)



Residuals Volatility Decomposition, % shares

common (red), idio (blue)

CPI Inflation Pseudo Out of Sample Forecasting

- Recursive Estimation and Out of Sample Forecasting
- **101 Quarterly Vintages**
  (estimation window endpoint spanning from 1989Q4 to 2014Q4)
- **From 1 quarter to 2 years ahead:** $h \in \{1, \ldots 8\}$
- Specifications with 4 lags
- **Six Models Evaluated:**
  MAI-AR-SV (benchmark), MAI-AR, Univariate AR, Univariate AR-SV, VAR, VAR-SV
- Prior distributions calibrated as Univariate Random Walks across models
- **Extensive usage of parallelization** to perform MCMC estimation of several vintages simultaneously

Forecasting Point and Density Diagnostics

- Forecasting diagnostics framework of Clark and Ravazzolo (2015)
- For each model $m \in \{1, \ldots, M\}$, variable $j \in \{1, \ldots, n\}$ and horizon $h \in \{1, \ldots, H\}$
  - **Root Mean Squared Forecast Error** (RMSFE):

$$RMSE^{m}_{j,h} = \sqrt{\frac{1}{T^*} \sum_{t=T+1}^{T+T^*} \left(y_{j,t+h} - \widehat{y}^{m}_{j,t+h} \right)^2}$$

  - **Log Predictive Scores** obtained via non-parametric kernel smoothing density estimators:

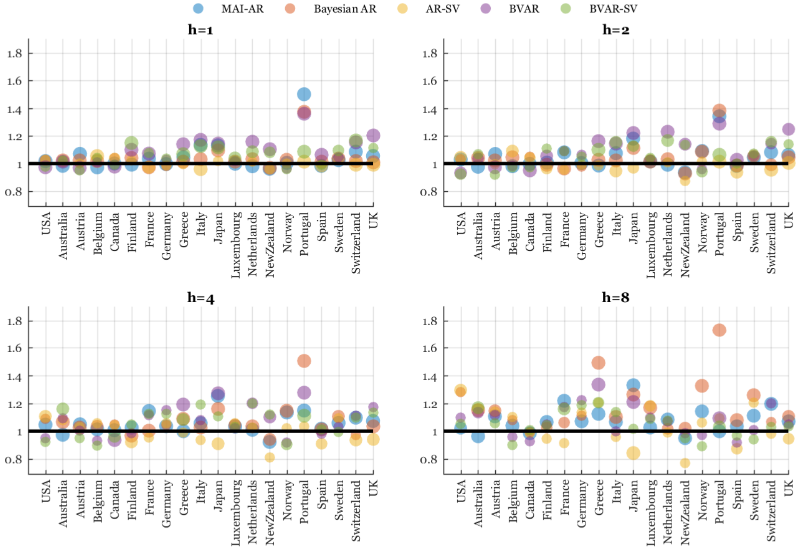
$$\overline{logScore}^{m}_{j,h} = \frac{1}{T^*} \sum_{t=T+1}^{T+T^*} \log \left(\frac{1}{\widehat{\mathcal{H}} \cdot L_c} \sum_{i=1}^{L_c} \mathcal{K}_{\mathcal{N}} \left(\frac{y_{j,t+h} - \widehat{y}^{m,i}_{j,t+h}}{\widehat{\mathcal{H}}} \right) \right)$$

  - **Continuous Ranked Probability Score** (CRPS):

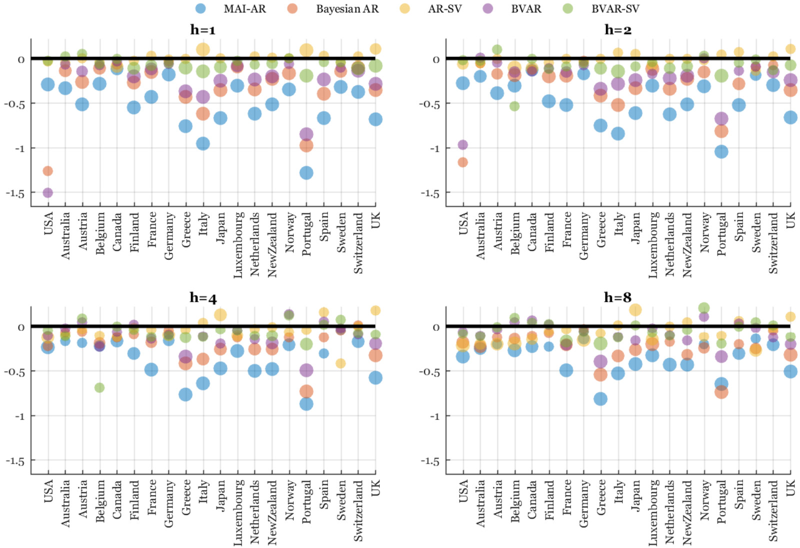
$$\overline{CRPS}^{m}_{j,h} = \frac{1}{T^*} \sum_{t=T+1}^{T+T^*} \left(\frac{1}{L_c} \sum_{i=1}^{L_c} \left| \widehat{y}^{m,i}_{j,t+h} - y_{j,t+h} \right| - \frac{1}{2 \cdot L_c} \sum_{i=1}^{L_c} \left| \widehat{y}^{m,i}_{j,t+h} - \widehat{y}^{m,i'(i)}_{j,t+h} \right| \right)$$

- To test for significantly different performances: Diebold and Mariano (1995) t-tests for equality are computed for all diagnostics.

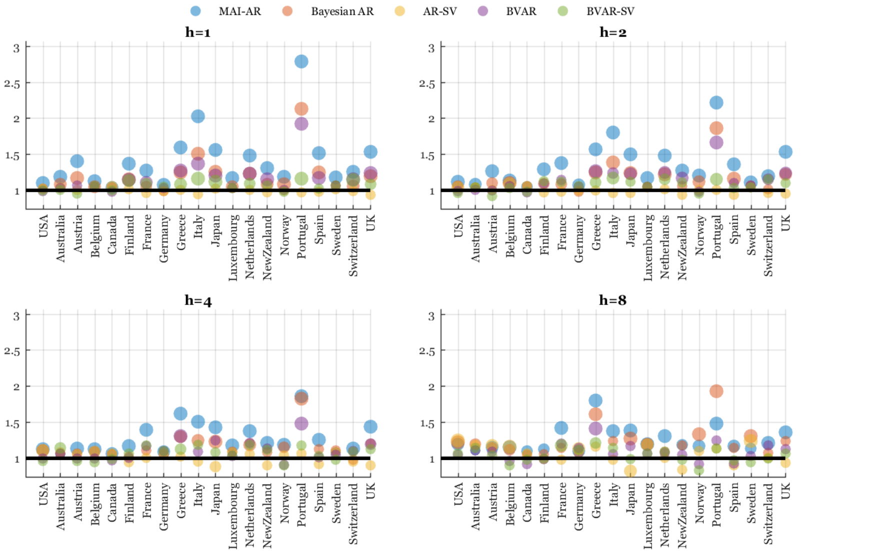
Relative RMSFE (ratios with MAI-AR-SV)



Relative Log-Scores (differences with MAI-AR-SV)



Relative CRPS (ratios with MAI-AR-SV)



THANK YOU

1 Extras and Appendix

Multivariate Autoregressive Index + AR components + SV

- Reinsel (1983), Carriero Kapetanios and Marcellino (2016), Cubadda and Guardabascio (2017)

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} \underbrace{A_\ell}_{n\times r} \cdot \underbrace{B_0}_{r\times n} \cdot y_{t-\ell} + \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + u_t$$

- A **Global Inflation "Index"** $(r = 1) \rightarrow \qquad F_t = B_0 \cdot y_t$
- Cogley and Sargent (2005) and Primiceri (2005)

$$u_t \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0},\, \Omega_t \right), \qquad \Omega_t = G^{-1} \Sigma_t \Sigma_t \left(G^{-1} \right)'$$

$$\Sigma_t = Diag(\sigma_t), \qquad \log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\, Q_\sigma \right)$$

MAI-AR-SV, Gibbs Sampler

1. **Draw the AR coefficients** $\gamma \Big| A, B_0, G, (\sigma_t)_{t=1}^T$

   Transform the model, standardize, and perform a Bayesian Regression.

2. **Draw the loadings** $A \Big| B_0, G, \gamma, (\sigma_t)_{t=1}^T$

   Bayesian Multivariate Regression with heteroskedastic innovations. Use the orthogonalization approach of CCM (2016) to handle large n.

3. **Draw the factor weights elements** in $B_0 \Big| \gamma, A, G, (\sigma_t)_{t=1}^T$

   Metropolis step similar to CKM2016 but adapted to take into account SV.

4. **Draw the off-diagonal elements** in $G \Big| \gamma, A, B_0, (\sigma_t)_{t=1}^T$

   Transform the model as in Primiceri (2005) and perform a Bayesian Regression with heteroskedastic innovations.

5. **Draw a history of volatilities** $(\sigma_t)_{t=1}^T \Big| \gamma, A, B_0, G$

   As amended by Del Negro and Primiceri (2013), and using the Omori, Chib, Shephard and Nakajima (2007) approximation for the log χ_1^2.

Prior on B_0

- Block structure, r blocks of variables

$$\underbrace{y_t}_{n\times 1} = \left[y_t^{1\,\prime} \quad y_t^{2\,\prime} \quad \dots \quad y_t^{r\,\prime} \right]', \qquad \forall j \in \{1,\dots,r\} \; \underbrace{y_t^j}_{n_j\times 1}, \qquad n = \sum_{j=1}^{r} n_j$$

- Normalization of the first variable of each block (identifying restriction)

$$\underbrace{B_0}_{r\times n} = \begin{bmatrix} 1 & \widetilde{B}_{0,1} & 0 & \mathbf{0}_{1\times(n_2-1)} & \dots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 1 & \widetilde{B}_{0,2} & \dots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 0 & \mathbf{0}_{1\times(n_2-1)} & \dots & 1 & \widetilde{B}_{0,r} \end{bmatrix}, \quad \forall j \quad \underbrace{\widetilde{B}_{0,j}}_{1\times(n_j-1)}$$

- $n - r$ separate univariate regressions to calibrate independent priors of $B_{0,j,k}$ using the first principal components of each j-th block $\left(S_t^j\right)_{j=1}^{r}$

$$\forall j \in \{1,\dots,r\}, \quad \forall k \in \left\{2,\dots,n_j\right\}, \quad S_t^j = B_{0,j,k} \cdot y_{t,k}^j + u_{j,k,t}, \quad u_{j,k,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{j,k}^2\right)$$

Prior on other elements

- The prior on $a = vec\,(A')$ is $a \sim \mathcal{MN}\,(\,\mathbf{0},\ V_a\,)$:

$$V_a = \begin{bmatrix} \widehat{\sigma}^2_{y,1} & 0 & \dots & 0 \\ 0 & \widehat{\sigma}^2_{y,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \widehat{\sigma}^2_{y,n} \end{bmatrix} \otimes \begin{bmatrix} \Upsilon_1 & 0 & \dots & 0 \\ 0 & \Upsilon_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \Upsilon_p \end{bmatrix}, \quad \Upsilon_\ell = \frac{\lambda_a^2}{\ell^d} \cdot I_r$$

- Prior on SV are calibrated as in Primiceri(2005).
- Prior on the AR coefficients

$$\bar{\gamma} = \begin{bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \\ \vdots \\ \bar{\gamma}_q \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{n\times 1} \\ \mathbf{0}_{n\times 1} \\ \vdots \\ \mathbf{0}_{n\times 1} \end{bmatrix}, \qquad V_\gamma = \lambda_\gamma \cdot \begin{bmatrix} 1^{-d} & 0 & \dots & 0 \\ 0 & 2^{-d} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q^{-d} \end{bmatrix} \otimes I_n$$

An Alternative Representation (1)

- Borrowing from Johansen (1995), construct matrix $B_{0\perp}$ orthogonal to B_0:

$$\underbrace{B_0}_{r\times n} \cdot \underbrace{B'_{0\perp}}_{n\times(n-r)} = \mathbf{0}_{r\times(n-r)}$$

- Consider then the TV decomposition (CKM16) that divides $\mathbb{R}^n$ into the sub-spaces spanned onto the rows of $B_0 \cdot \Omega_t^{1/2}$ and $B_{0\perp}\Omega_t^{-1/2}$:

$$I_n = \Omega_t B'_0 \left(B_0 \Omega_t B'_0\right)^{-1} B_0 + B'_{0\perp} \left(B_{0\perp} \Omega_t^{-1} B'_{0\perp}\right)^{-1} B_{0\perp} \Omega_t^{-1}$$

- Using this decomposition, we can rewrite the model as a FAVAR:

$$y_t = \sum_{\ell=1}^{\max(p,q)} (\Gamma_\ell \Omega_t B'_0 \Xi_t^{-1} + A_\ell) F_{t-\ell} + \sum_{\ell=1}^{q} \Gamma_\ell B'_{0\perp} \Xi_{\perp,t}^{-1} G_{t-\ell} + u_t$$

where $F_t = B_0 \cdot y_t$, $G_t = B_{0\perp} \Omega_t^{-1} y_t$, $\Xi_t = B_0 \Omega_t B'_0$ and $\Xi_{\perp,t} = B_{0\perp} \Omega_t^{-1} B'_{0\perp}$

An Alternative Representation (2)

- Multiplying both sides of the previous representation by B_0 and $B_{0\perp}\Omega_t^{-1}$:

$$F_t = \sum_{\ell=1}^{q} B_0 \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_0(\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \omega_t$$

$$G_t = \sum_{\ell=1}^{q} B_{0\perp}\Omega_t^{-1} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_{0\perp}\Omega_t^{-1}(\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \psi_t$$

where $$\begin{bmatrix} \omega_t \\ \psi_t \end{bmatrix} = \begin{bmatrix} B_0 u_t \\ B_{0\perp}\Omega_t^{-1} u_t \end{bmatrix} \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix} \Xi_t & 0 \\ 0 & \Xi_{\perp,t} \end{bmatrix} \right)$$

- so that $\begin{bmatrix} F_t \\ G_t \end{bmatrix}$ evolves as VAR with block uncorrelated errors, since

$$\mathbb{E}(\omega_t \psi_t') = \mathbb{E}(B_0 u_t u_t' \Omega_t^{-1} B_{0\perp}') = B_0 \Omega_t \Omega_t^{-1} B_{0\perp}' = \mathbf{0}_{r\times(n-r)}$$