Invisible Market Forces with Observable Effects

This version: October 29, 2020

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ACKNOWLEDGMENTS: I had a series of motivating discussions with Raphael Abiry, Matthias Blonski, Oivind Nilsen, Paul Reimers, Uwe Walz, and Christine Zulehner. David Heller brought to my attention the topic of estimation methods for markets in disequilibrium. I have also received helpful comments from participants of the Goethe University's Management and Microeconomics Colloquium of 2020 and the Brown Bag seminar of the Leibniz Institute SAFE. I am thankful to all. The estimates and the analysis in this article that is based on Information Resources Inc. data are by the author and not by Information Resources Inc.

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Abstract

The limited availability of methodologies that evaluate departures from market-clearing discourages the application of estimation methods that allow them. Nevertheless, shortages and surpluses appear in a plethora of markets not only under exceptional but also under normal circumstances. In this article, I propose a statistical assessment of the market-clearing condition, which allows the comparison of equilibrium and disequilibrium models for which the likelihoods are known. An application of the methodology using US retail and scanner deodorant data provides evidence that, during times of distress, exogenous shocks can strengthen the price mechanism of markets that are not characterized by some innate failure. In prosperous times, market participants may become more complacent and, as a result, the price mechanism can be weakened. Analytic expressions, simulations, and computational benchmarks for five market models are also presented. The results of this article may serve in empirical justifications of deviations from market-clearing.

JEL-Classification: C01, C18, C34, C52, D50, D45

Keywords: market-clearing, disequilibrium, marginal effects, model selection, maximum likelihood, Markov switching

I. Introduction

Equilibrium concepts are analytically convenient. Their analytic convenience is typically accompanied, however, by strong behavioral and institutional assumptions. Assumptions that are likely not to be met in some settings in which market failures, price rigidities, and liquidity constraints arise, thereby rendering the study of equilibrium to the physical equivalence of studying gravity in the vacuum. Eliciting information about a market that is not characterized by a state of perpetual clearing using structural models that are based on such an assumption casts reasonable doubts on the validity of the results of the methodological approach. In response to this critique, deviations from the market-clearing condition in the context of models for markets in disequilibrium have been proposed to alleviate these misspecification problematics.

The estimation of models for markets in disequilibrium has found its way in a variety of forms in applications for a diverse set of markets, with labor and financial markets constituting the most prevalent usage cases. Nevertheless, the complexity of the estimation on the one hand, and the lack of a general statistical methodology for assessing the validity of the selected model on the other hand, prevented the methods from receiving attention in cases that their adoption could have been appropriate.

In the present article, I contribute to the literature of such estimation methods by proposing a readily applicable statistical assessment of the appropriateness of the market-clearing condition. Although the methodology is presented using an equilibrium and four prevalent disequilibrium models, the approach is based on information criteria instead of parameter testing, which does not require the compared models to be nested and allows the comparison of any number of equilibrium and disequilibrium models for which the likelihoods can be calculated. As a result, the approach is relevant even for comparing models that are not covered in this article.

I use granular retail microdata at a store, product week level containing information from both the demand and supply side of the US deodorant markets of Eau Claire and Pittsfield from 2001 until 2012 to illustrate how the proposed methodology can reveal obscure aspects relating to the efficiency of the price mechanism. Specifically, the application of the methodology provides evidence that when a crisis affects a market only through side effects, it can awaken the competition forces and improve the efficiency of the market. The awakening result is relevant when, during prosperous times, market participants become complacent, resulting in lazier market forces and weakening the effectiveness of the price adjustment mechanism.

In addition, by calculating the marginal effects of demand and supply controls on the shortage probabilities, I propose a way to obtain a state-dependent, system-wide evaluation of how changes in market factors affect the state of the market. As in binary response models a marginal effect provides an estimate of a factor's influence on the probability of observing the binary event, so in this article, a marginal effect provides an estimate of a factor's influence on the probability of observing a shortage or a surplus in a market. In contrast to the direct examination of the estimated coefficients of each side of the market separately, both sides are taken into account when calculating marginal effects, which allows analyzing the influence of factors with channels that act through both demand and supply.

Last but not least, analytic derivations for the gradients of all the models and the Hessians of the equilibrium and two of the disequilibrium models are provided. Knowledge of derivatives is central in computational optimization algorithms and the new expressions substantially contribute to the maximum likelihood estimation efficiency. Using benchmarking simulations, I collect statistical measurements and present the computational benefits that result from using the expressions in gradient-based optimization methods.

The remaining of the article is organized in the following way. Section II introduces the five econometric market models upon which the analysis of the article is based. The section does not only describe the models but also discusses applications of each model in recent literature. Section III presents the marginal effects on the shortage probabilities and proposes the methodology that assesses the statistical appropriateness of the market-clearing condition in its econometric modeling. An extensive Monte Carlo experiment is used to illustrate the statistical properties of the methodology. Section IV is indicative of how the models and the methodology of this article can be used. It provides evidence that in some markets, exogenously driven times of distress can increase the effectiveness of the free-market forces and lead to fewer market shortages. Section V presents the technical results of the article. It gives likelihood representations that are based on moments of prices and quantities. Furthermore, it contains expressions of analytically calculated gradients for the likelihoods of the five discussed models and the Hessians of the equilibrium and two of the disequilibrium models. Lastly, the section presents the calculated statistics from the benchmarking simulations. The last section concludes.

II. Models for Markets in Equilibrium and Disequilibrium

This section introduces the five market models that are used in the discussion of this article; a model for markets in equilibrium and four models for markets in disequilibrium. The models are similar in that their specifications are built on joint-normality assumptions, they describe, however, different market structures. The equilibrium model is characterized by a condition that ascribes that the market under review clears, while the disequilibrium models describe markets in which the minimum between the demanded and supplied quantity is observed, i.e. in which the short-side rule holds. Whereas the equilibrium model is a stochastic system of three linear simultaneous equations, the disequilibrium models are stochastic systems of two or three linear simultaneous equations coupled with a nonlinear observability condition.

The equilibrium model is examined under the lens of full information maximum likelihood estimation. In particular, the equilibrium system is a special case of the stochastic systems of simultaneous equations studied by Balestra and Varadharajan-Krishnakumar (1987). Models for markets in equilibrium are almost exclusively estimated using two- or three-stage least squares. It can be shown that the maximum likelihood estimation of such systems is asymptotically equivalent to the linear estimation. The maximum likelihood approach is used in the assessment of market-clearing that I propose in this article to ensure the comparability of the equilibrium estimates with those of the models for markets in disequilibrium, which are exclusively obtained by full information maximum likelihood estimations.

Estimating models for markets in disequilibrium has a long, multibranch history, with the methodological foundations of such econometric tools being established in the pioneering work of Fair and Jaffee (1972); Amemiya (1974); Maddala and Nelson (1974); Quandt and Ramsey (1978). While alternative specifications of models for markets in disequilibrium can be found in the literature, the econometric approach laid in the aforementioned articles has been by far the most commonly used one in empirical applications. To increase the coverage and relevance of the analysis in this article, I focus on the four model variations that are considered by Maddala and Nelson (1974).

The models for markets in disequilibrium that I consider belong in the general class of Markov switching models. They switch between states of excess demand and states of excess supply. Moreover, these models can be classified into two basic categories. Models with known sample separation, i.e. models in which observations are classified in either demand or supply based on an exogenous separation rule, and models with unknown sample separation, i.e models in which no such rule is imposed. Models with unknown sample separation do not use observable information to ex-ante classify the observations in one of the two states. In contrast, models with known sample separation classify observations in one of the two regimes before any estimation occurs. The classification criterion is model specific and it typically uses information on price changes.

Both the equilibrium and the disequilibrium models have as starting point the demand and supply market forces. The equilibrium model represents a paradigm of market efficiency, while the disequilibrium models are in principle relevant for markets that, due to some rigidity or market anomaly, fail to clear. In most cases, the analysis of markets is based on equilibrium models. Nevertheless, there are applications of disequilibrium models in markets of loans, agricultural goods, housing, and labor. The majority of previous work has considered such models on the basis of argumentation that there is a reason to believe that the market under review was not in equilibrium at a given time. Although in most cases argumentation can be reasonable, it remains only intuitive and not based on statistical evidence.

As structural econometric models, the disequilibrium specifications that are considered in this article are not concerned with the reasons that a market fails. Without imposing any particular market failure reasoning, disequilibrium models can be estimated when demand and supply data are available and, depending on the definition of the market of interest, either aggregated or disaggregated data can be used. Conditional on data availability, one can control for information asymmetries, bargaining power, search frictions, and any other specifics that lead to shortages or surpluses in a market. The econometric specification, however, besides estimating such effects, does not require a specific channel through which these failures arise.

In the equilibrium model, the demand and supply forces are augmented by the market-clearing condition, which prescribes that demanded and supplied quantities are equated and are both simultaneously observed. For the disequilibrium models, the starting point of the analysis ceases to be a market-clearing condition and instead, demanded and supplied quantities are explicitly treated as being distinctly observed. The observable traded quantity in the market may belong exclusively in demand or supply and, thus, only reveals partial information. The estimation methods of models for markets in disequilibrium elicit demand and supply forces from the short-side rule in such partial information specifications.

i. Equilibrium model

The model of the equilibrium case constitutes the benchmark model of this article. The market forces neutralize each other eliminating in the process all shortages and surpluses. The equilibrium model is specified by

$$\begin{aligned} D_{nt} &= X'_{d,nt}\beta_d + P_{nt}\alpha_d + u_{d,nt}, \\ S_{nt} &= X'_{s,nt}\beta_s + P_{nt}\alpha_s + u_{s,nt}, \\ Q_{nt} &= D_{nt} = S_{nt}, \end{aligned} \tag{EM}$$

where Q denotes the traded quantity, D the demanded quantity, S the supplied quantity, P the market price, and X_d and X_s are vectors of controls. The third equation of the system is the market-clearing condition. Substituting the traded quantity in the demand and supply equations gives a two by two, stochastic linear system on prices and quantities. Although, there is theoretical background and reasonable economic argumentation in favor of non-linear demand and supply equations, the econometric estimation of such models is more complicated and, hence, the majority of empirical applications relies on linear markets models.

The stochastic terms u_d and u_s form a disturbance vector that is independently and identically distributed according to

$$\begin{bmatrix} u_d \\ u_s \end{bmatrix} \sim \mathrm{N}\left(0, \Sigma\right) \quad \text{where} \quad \Sigma = \begin{bmatrix} \sigma_d^2 & \sigma_d\sigma_s\rho \\ \sigma_d\sigma_s\rho & \sigma_s^2 \end{bmatrix}.$$

Within each date, the demand and supply shocks can be correlated and the parameter ρ controls their correlation. Model (EM) can be estimated both when u_d and u_s are correlated or independent. Deterministically setting ρ to zero gives specifications in which the demand and supply shocks are independent, while allowing its estimation corresponds to specifications with correlated shocks.

The control variables of the demand and supply equations may partially coincide, but a necessary identifying condition is that there is at least one variable on each side that is not included on the other side. The separate notation of prices and their coefficients is not only due to their economic significance but also due to their special contribution to the structure of the system. The significance of prices in the market-clearing mechanism is reflected in the gradient equations that determine the estimators of the price coefficients. Prices adjust for the markets to clear and, as shown in proposition V.6, this special trait leads to a form of necessary optimization conditions for the coefficients of prices that differ from those of the rest of the controls. This is not true for all the models for markets in disequilibrium, as in cases in which price dynamics are absent, price coefficients can be treated similarly to those of other controls.

ii. Basic model

I will refer to the first disequilibrium model that I consider as the basic model. The basic model is specified by

$$\begin{aligned} D_{nt} &= X'_{d,nt}\beta_d + u_{d,nt}, \\ S_{nt} &= X'_{s,nt}\beta_s + u_{s,nt}, \\ Q_{nt} &= \min\{D_{nt}, S_{nt}\}. \end{aligned} \tag{BM}$$

The notation and the distributional assumptions are the same as in model (EM). In the absence of market-clearing and any price dynamics, there is no need to introduce a separate notation for the price coefficients in this model. In contrast to the equilibrium model, the estimators of prices are specified in a way that is identical to that of the estimators of rest of the control variables. Prices can be, and typically are, included in $X_{d,nt}$ and $X_{s,nt}$. If demand is inelastic, one can, of course, introduce an econometric specification that does not include a price variable in $X_{d,nt}$. This remark is also true for the supply equation, in which case a price variable may be excluded from $X_{s,nt}$.

An observation belongs either in an excess demand regime or an excess supply regime and these states are mutually exclusive. The equilibrium point is a zero probability state and can be included in any of the two regimes without losing generality. The convention that I am following in this article is to classify it as an excess demand state. The estimation of the model does require that the sample is separated into excess demand and excess supply observations ex-ante. Estimating the model gives an ex-post classification of the observations in demand and supply that, conditional on the model, maximizes the probability of observing the sample.

The basic model is the simplest among the four disequilibrium setups that I consider here and it is by far the most commonly used in empirical applications. Even though it has been numerously applied, most applications do not allow for correlation between demand and supply shocks. The model has been used to analyze both aggregated and disaggregated data.

Concerning aggregated data, the basic model has been used in the study of financial markets. Mody and Taylor (2013) use the independent shock specification to estimate supply and demand functions for international flows of bond, equity, and syndicated loans. The main market specification is essentially inelastic as the coefficients of country-specific interest rates were not statistically significant and were not reported. In this context, they find that asymmetric information affects the supply of capital that is available in developing countries. Felices and Orskaug (2011) also use the independent shock specification to estimate time-varying credit rationing probabilities for various emerging economies. Their approach consists of a two-step methodology, in which firstly, they estimate spreads in a two-stage least square equilibrium model and, subsequently, use the estimated values as instruments in their disequilibrium specification. By construction, the equilibrium model does not allow shortages to exist and this approach potentially skews the estimated shortages obtained from the disequilibrium estimation. Herrera et al. (2013) apply the basic model with independent shocks using macroeconomic data to study Egypt's credit market. The authors also estimate an equilibrium model to examine whether their findings remain robust under the market-clearing hypothesis. They do not calculate the likelihood of the equilibrium case and, although they use information criteria to select the disequilibrium specification that is reported, they do not apply the same approach to compare between the equilibrium and disequilibrium specifications. The econometric model of the bank of Italy uses the basic model to quantify the impact of credit constraints on investment[2]. Vouldis (2015) uses the same model with independent shocks to study the evolution of credit demand and supply in Greece and Pazarbasioglu (1996) to study the effects of the 1991-92 banking crisis to the credit market of Finland.

The basic model has also been used in a diverse set of applications using granular disaggregated bank credit market data. Heller et al. (2019) estimate the basic model allowing for correlated shocks between supply and demand. Using firm-level data of SMEs and large enterprises in Germany, the authors use the estimated shortages in a second stage regression to study the effects of the great recession on financial constraints and the recovery of the affected firms. Kremp and Sevestre (2013) estimate the basic model, conditional on observing a positive interest rate to alleviate selection bias, in the study of the credit rationing of the French SMEs. Although that the state of financial constraints that are estimated by the disequilibrium model is continuous, their analysis categorizes firms as being financially constraint in a binary fashion, by using a one-half threshold on the estimated shortage probabilities. Farinha and Félix (2015) conduct a similar analysis using Portuguese companies, while Hurlin and Kierzenkowski (2007) apply the model in the credit market of Poland using supply-side data. Carbó Valverde et al. (2009) and Carbo-Valverde et al. (2016) estimate the basic model with firm-level data for the Spanish market. The former article employs the disequilibrium estimates to examine the relationship between bank market-power and credit rationing and the latter to provide evidence that small financially constrained firms rely more on trade credit during crises. Atanasova and Wilson (2004) estimate the basic model using credit market panel data from the UK to study the propagation of monetary policy changes.

There are also applications of the model in non-financial markets. For instance, Lewis and Makepeace (1981) use it to estimate labor demand and supply using aggregated UK data. Another labor market application is found in Baird et al. (2019), in which the authors use US data from multiple sources to study the market of anesthesiologists. Besides the basic model, the authors estimate an extension of the disequilibrium model using shortage indicators and an equilibrium model. They compare the out-of-sample predictive power of the models, but neither do they provide a specification assessment for the whole sample nor do they consider the validity of market-clearing.

iii. Directional model

I will refer to the second disequilibrium model that I consider as directional model. In the directional model, the sample is ex-ante separated in observations belonging in the demand or supply-side based on the direction

[2] See Bulligan et al. (2017, p. 27).

of price movements. The inception of this form of separation stems from Fair and Jaffee (1972).

The directional model is given by

$$\begin{aligned} D_{nt} &= X'_{d,nt}\beta_d + u_{d,nt}, \\ S_{nt} &= X'_{s,nt}\beta_s + u_{s,nt}, \\ Q_{nt} &= \min\{D_{nt}, S_{nt}\}, \\ \Delta P_{nt} \geq 0 &\implies D_{nt} \geq S_{nt}. \end{aligned} \tag{DM}$$

The notation and the assumptions on the demand and supply shocks are as in model (BM). The directional model augments the basic model's system with a simple sample separation condition. If prices increase at a particular date, then the observation corresponding to this date is classified as being in an excess demand regime. Observations are classified in the excess supply regime when prices decrease.

Economic arguments based on market efficiency suggest that when an excess demand state is observed, market forces increase prices until the resulting increased supplied and reduced demanded quantities reach the equilibrium point. The separation condition of the directional method reverses the last argument and postulates that price movements can always be attributed to this mechanism and, if so, then one can simply observe through price changes the regime to which an observation belongs. When estimating the model in empirical applications, this assumption should be taken into account when assessing the validity of the model. In addition, either supply or demand has to be inelastic when estimating the directional model. The usage of prices in the separation rule identifies the traded quantity with the demanded or supplied quantity and, therefore, cannot subsequently be reused in both equations to identify the price coefficient[3].

The directional method is used by Laffont and Garcia (1977). The estimation of the model's coefficients is simple, in particular when no correlation between the shocks is allowed, in which case the estimation of demand can be conducted separately from that of supply. The model has not been applied as often as model (BM), potentially due to its restrictive stipulations. By separating the sample, one imposes more structure on the estimation procedure, but this leads to better estimates only in cases that the separation rule reflects the true underlying data generating process. If the separation condition is inaccurate for the underlying market, the additional requirement translates to a misspecification source.

[3]See also Maddala and Nelson (1974, p 1021).

iv. Deterministic adjustment model

I refer to the third disequilibrium model of this article as deterministic adjustment model. In this model, the price adjustments are analogous to the market shortages or surpluses. Specifically, the model is specified by

$$\begin{aligned} D_{nt} &= X'_{d,nt}\beta_d + P_{nt}\alpha_d + u_{d,nt}, \\ S_{nt} &= X'_{s,nt}\beta_s + P_{nt}\alpha_s + u_{s,nt}, \\ Q_{nt} &= \min\{D_{nt}, S_{nt}\}, \\ \Delta P_{nt} &= \frac{1}{\gamma}\left(D_{nt} - S_{nt}\right). \end{aligned} \tag{DA}$$

The additional, with respect to model (BM), equation of this model has a dual role. Firstly, as in the directional model, it introduces a separation rule. Secondly, it quantifies how prices respond in deviations from market-clearing. The effect of shortages and surpluses on price movements is controlled by the reciprocal of parameter γ. In contrast to the directional model, the deterministic adjustment model introduces one additional parameter to estimate, allowing thereby the estimation of price coefficients on both sides of the market. The specification of price dynamics, in this case, is in accord with the classic equalizing attributes that the demand and supply forces have in most general equilibrium settings of functioning free-markets.

The remaining notation and the assumptions on the distributions of the disturbances are as in model (EM). Prices are denoted separately from other controls similar to the equilibrium case. Their coefficients α_d and α_s are optimally determined by more convoluted gradient relations in comparison with the coefficients β_d and β_s, with the extra complexity arising from the entanglement of demand and supply through the price dynamics.

The price dynamics notation that I am using in this article is mildly different from the one in Maddala and Nelson (1974). In particular, the price dynamics in the latter article are given by $\Delta P_{nt} = \tilde{\gamma}\left(D_{nt} - S_{nt}\right)$. The only point of the parameter space that is excluded by using the reciprocal of γ is that of $\tilde{\gamma} = 0$, which does not, however, cause any loss of generality as in the zero-case the deterministic adjustment model becomes degenerate; it is a model in which prices are completely rigid. The benefit of inverting the parameter is that the expressions of section V become relatively simpler.

The deterministic adjustment model was introduced by Laffont and Garcia (1977)[4]. Mayer et al. (2015) use a disequilibrium model that resembles the deterministic adjustment model to identify the determinants of national broadband penetration. The approach departs and extends the framework laid by Maddala and Nelson (1974). Firstly, the adjustment factor is allowed to be non-homogeneous and secondly, instead of maximum likelihood, the authors use a GMM estimation approach. Loberto and Zollino (2016) use another variation of the model to study the effects of the crisis on the housing market of Italy. They also extend the model by

[4] The authors refer to it as quantitative model due to the quantitative nature of the price dynamics. I adopt the deterministic adjustment naming convention to distinguish this model from the disequilibrium model of section II.v, which also has a quantitative nature.

introducing a two-sided price adjustment condition, through which price responses to shortages are potentially different from responses to surpluses of equivalent size.

v. Stochastic adjustment model

The last disequilibrium model that I consider in this article extends the deterministic adjustment model by adding confluent factors and stochasticity in the price dynamics. The price dynamics are not only quantitative but also stochastic and, thus, I will refer to the model of this case as the stochastic adjustment model. With those additions, the stochastic adjustment model takes the form

$$\begin{aligned} D_{nt} &= X'_{d,nt}\beta_d + P_{nt}\alpha_d + u_{d,nt}, \\ S_{nt} &= X'_{s,nt}\beta_s + P_{nt}\alpha_s + u_{s,nt}, \\ Q_{nt} &= \min\{D_{nt}, S_{nt}\}, \\ \Delta P_{nt} &= \frac{1}{\gamma}\left(D_{nt} - S_{nt}\right) + X'_{p,nt}\beta_p + u_{p,nt}. \end{aligned} \tag{SA}$$

The notation of this model is as in model (DA) with the addition of X_p, which is a vector of price controls. The stochastic terms of this model are also assumed to be joint-normally distributed at each date, i.e.

$$\begin{bmatrix} u_d \\ u_s \\ u_p \end{bmatrix} \sim \mathrm{N}\left(0, \Sigma\right) \quad \text{where} \quad \Sigma = \begin{bmatrix} \sigma_d^2 & \sigma_d\sigma_s\rho_{ds} & \sigma_d\sigma_p\rho_{dp} \\ \sigma_d\sigma_s\rho_{ds} & \sigma_s^2 & \sigma_s\sigma_p\rho_{sp} \\ \sigma_d\sigma_p\rho_{dp} & \sigma_s\sigma_p\rho_{sp} & \sigma_p^2 \end{bmatrix},$$

and to be independent across dates. I abuse notation and use the same symbol for the variance-covariance matrix. The stochastic adjustment model is the only one that has three stochastic terms and, therefore, it is always clear from context whether I refer to the three-dimensional stochastic adjustment matrix or to the two-dimensional matrix of all the remaining cases.

The specification of the model is relevant when factors that are exogenous to demand and supply affect how prices are determined. For example, a government intervention that targets breaking collusive practices in an oligopoly market can potentially be captured by the factors contained in $X_{p,nt}$. The additional controls in the stochastic adjustment model give extra degrees of freedom in estimations, but also render the optimization of the likelihood more numerically intensive. In view of this, it is not always true that the extra degrees of freedom will be accompanied by better estimation results.

III. Specification Assessment and Marginal Effects

This section contains the methodological contributions of the article. Firstly, a methodology that can be used to statistically assess whether the market-clearing condition constitutes an appropriate econometric description of the market-state is introduced. Using artificial market data, the performance of the methodology is tested against the data generating processes of all five models of section II. Secondly, the marginal effects on the shortage probabilities are introduced and described.

i. Assessing market-clearing through model selection

Is market-clearing an appropriate assumption when studying a particular market? Despite that disequilibrium models have found a plethora of applications in the literature, only in a few cases, the robustness of the models against the equilibrium alternative has been examined. Providing statistical evidence of whether market-clearing is a suitable assumption for a given market has proven not to be straightforward. The assessment of disequilibrium has already been considered by Quandt (1978) in the context of the stochastic price adjustment model. In this pioneering work, the author proposes a series of methods to statistically test whether a market is better described by the stochastic price adjustment model or by an equilibrium model. The article considers the problem from the perspective of parameter testing and, among the proposed tests, there is also a likelihood ratio test considered.

The likelihood ratio can be used to test the statistical significance of one model parameter separately, or many model parameters jointly. For instance, to test whether demand is inelastic, one can calculate the likelihood of the unconstrained specification and the likelihood of the constraint model, which is obtained by setting the price coefficient of the demand equation to zero, and compare their likelihoods. In a similar manner, to test whether the supply and demand shocks are correlated, one can estimate both the model with correlated and independent shocks and calculate the ratio of their likelihoods.

However, the likelihood ratio cannot be used to test whether a disequilibrium specification is more plausible in comparison with an equilibrium one. As it is indicated from the previous examples, the test can only be used to compare models that are related in a specific way. In particular, the constrained model, i.e. the model obtained by superimposing the condition of the hypothesis that is to be tested, has to be nested into the unconstrained model. As it is also noted by Quandt (1978), the equilibrium model can only be obtained from the disequilibrium model as a limiting case and, as a result, the likelihood ratio test is only heuristic.

There are also alternative attempts to address the specification question. For example, shin Hwang (1980) uses the deterministic adjustment model to derive another test based on parameter stability. Instead, Bowden (1978) develops a methodology for testing the disequilibrium hypothesis using re-parameterizations of the deterministic and stochastic adjustment models. There are also a few applications of these approaches. The likelihood ratio test of Quandt (1978) is applied by Perez (1998) as a heuristic instrument to determine whether there was credit rationing in the US market during the years 1981 to 1991. The test of Bowden (1978) is used in Ito and Ueda (1981).

The limitation of the above approaches is that they are model specific. Besides, however, the disequilibrium models that I examine in this article, many authors use modifications of these models that are more suitable for the market that they study. As a result, the previous approaches are not always applicable in practice. In contrast, the assessment of market-clearing that I propose in this section can be used with any combination of models whose likelihoods can be calculated.

The assessment is neither heuristic nor based on parameter testing, but rather draws from information theory. Instead of condensing the market-clearing hypothesis in a set of parameters and testing their significance, the issue is approached from a model selection perspective. One or more disequilibrium models can be compared with the equilibrium model to examine which structure better describes the data. Each model alternative is perceived as a potential structure of the market that generated the data and, from this view, criteria based on information theory can be used when selecting between these alternatives.

As the likelihoods of the disequilibrium models that I examine here can be calculated, all four models are compatible with the methodology that I propose. The equilibrium model, however, is typically estimated using either a two-, as proposed in Theil (1953), or a three-stage least square methodology, as proposed in Zellner and Theil (1962). Maximum likelihood estimations are avoided as they are computationally more demanding and require the likelihood to be known.

The methodological assessment of the current article requires also that the equilibrium model is estimated using maximum likelihood. Although one can evaluate the equilibrium likelihood using the least square estimates, comparing the resulting value with the values of the disequilibrium likelihoods evaluated using the maximum likelihood estimates can potentially be numerically inaccurate. As shown in Balestra and Varadharajan-Krishnakumar (1987), there is an asymptotic equivalence between the generalized three-stage linear and the full-information maximum likelihood estimators. However, the equivalence is only limiting and practically, using the linear estimators to evaluate the likelihood of the equilibrium model, may result in values that are significantly different from the values obtained when using the maximum likelihood estimators.

The methodology can be summarized in the three simple steps that follow.

1. Estimate the considered disequilibrium model(s) and calculate its(their) likelihood(s) at the optimum.

2. Estimate the equilibrium model using the full information maximum likelihood and calculate the optimized likelihood.

3. Calculate an information criterion using the values of the likelihoods obtained from the two previous steps.

Akaike's information criterion can be a suitable choice for the third step, but one may alternatively use the corrected Akaike or the Bayesian information criteria for the assessment of market-clearing. Depending on the sample size, one selection statistic may have advantages over the other. Moreover, the proposed methodology is relevant not only for the assessment of the market-clearing hypothesis but also when one fine-tunes her identification strategy. Information criteria have already been used in this way by Herrera et al. (2013). The methodology is essentially a model selection approach and it can be applied when selecting the disequilibrium specification that is more appropriate for the given sample.

The assessment has to be interpreted with care as there a few sharp edges. Firstly, the methodology assesses the appropriateness of the econometric model. Although market-clearing is a helpful concept for economic analysis, it remains a conceptual tool that might as well almost never be reflected in actual market conditions. Nevertheless, it can provide a simple and appropriate way to econometrically study a market on many occasions, and the methodology provides a statistical suggestion of whether the examined market constitutes such a case. Secondly, the assessment is conditional on the data. If the data do not provide all the relevant information, for instance only data from one side of the market are available, then the information criteria might select a model that does not correspond to the true underlying state of the market.

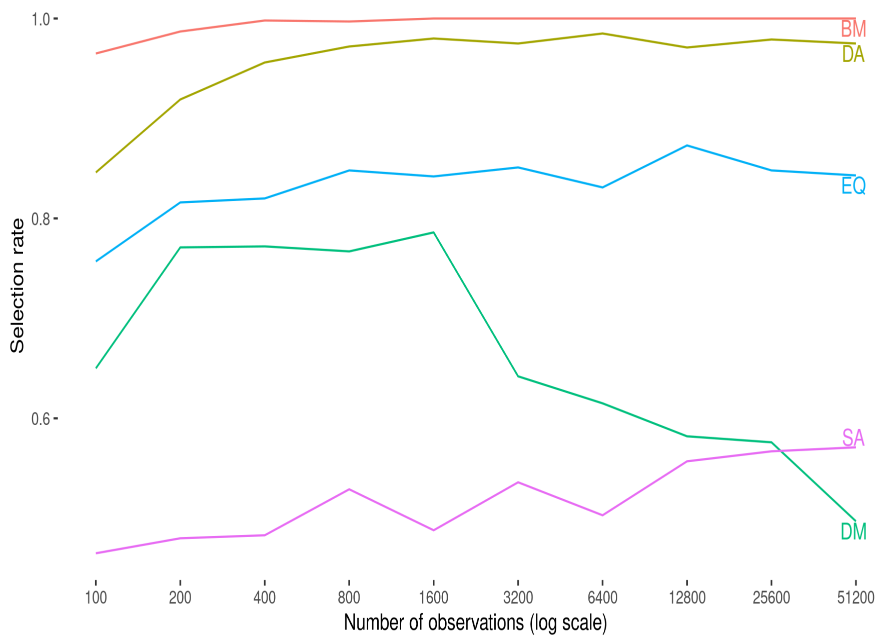
ii. Performance of the methodology

This section examines the performance of the methodology in a controlled environment. Each model of section II is simulated 1000 times for 10 different sample sizes. The sample sizes of each simulated panel consist of 5 time-points and of an exponentially increasing number of entities. In each model's simulation, both the panel data as well as the model's coefficients are randomly drawn from normal distributions. In each simulation round, it is ensured that the simulated prices and quantities are positive and that the simulated data do not lead to shortages or surpluses for more than 90% of the draws in order to avoid extreme cases that are typically not found in real market data sets. Finally, it is ensured that all models can be estimated for the drawn sample. If any of these conditions is not true, the simulation round is repeated. The market-clearing assessment methodology is applied when all the above conditions are met and the results are aggregated by calculating the success rate of each simulated model. A successful round is considered to be a round in which the selected model, among the available five alternatives, coincides with the model that generated the data. In all other cases, the round is considered unsuccessful. Appendix B.i contains the details of the simulation process.

Figure 1 presents the results of the procedure. The methodology is best performing when the data are generated using the basic model, reaching 100% success recognition of the process for more than 1600 observations. The case of the deterministic adjustment model closely follows with successful recognition that reaches 98%. Considering that the equilibrium event has zero probability, recognizing with 85% success rate that markets clear in the underlying data generating process is the most striking simulation result. The worst performance is achieved in the case of the directional model, which whereas it starts with 77% success rates, it eventually drops to 50%. The stochastic adjustment model begins with low success rates and reaches 57% successful selection as the number of observations grows.

There are a few points that the simulation exercise reveals. Firstly, the basic model offers a flexible market structure with a minimalistic set of degrees of freedom. As it is evident from fig. 1, these characteristics constitute an advantage when it comes to statistical inference. When there is a structural deficiency or a market failure present in the market of interest and the specification of price dynamics is unknown or dubious, the basic model

Figure 1: Success rates of the selection criterion.



constitutes a conducive estimation structure. Secondly, in an application in which price dynamics are important, the deterministic adjustment model offers the most flexible alternative. Furthermore, the specification of the deterministic adjustment model is very similar to that of the equilibrium model, which makes comparing these two cases easier. Thirdly, a selection of the equilibrium model by the methodology is a strong indication that market-clearing is a good approximation of the state of the market. Fourthly, the directional model is potentially more useful when applied with panel datasets for which the time dimension is prevalent. For a fixed number of time-points, the performance of the model decreases as the number of observations increases. Fifthly, the performance of the methodology is relatively hindered by the complexity of the data generating process of the stochastic adjustment model or from the numerical stability properties of its estimation. Applications of the stochastic adjustment model should involve large datasets.

iii. Marginal effects

How can one interpret the models' estimates? All models of this article are systems of stochastic linear equations and their coefficients can be interpreted separately in the usual fashion. Nevertheless, the direct interpretations of the coefficients capture effects on only one side of the market. On many occasions, prices and other variables affect both the demand and the supply sides of the market, and the effects obtained separately from each equation of the system do not reveal the overall effect on the state of the market.

In the models for markets in disequilibrium, the market state is characterized by the short-side rule $Q =$

$\min\{D, S\}$ and the estimated shortages give the models' predictions on the state of the market. Examining the effects on the shortages offers, therefore, a way to examine how changes in the control variables affect the market holistically.

The calculations of marginal effects on the shortage probabilities are similar to the corresponding calculations of binary response and truncated models. For simplicity, I introduce the marginal effects using model (BM), but the result is extendable to all other disequilibrium models. Under joint normality, the probability of an observation belonging in the supply equation is given by

$$\pi_S = \mathrm{P}(D \geq S) = \Phi\left(\frac{X_d'\beta_d - X_s'\beta_s}{\sigma}\right),$$

where $\sigma^2 = \sigma_d^2 + \sigma_s^2 - 2\sigma_d\sigma_s\rho$. I use Φ to denote the standard normal distribution function and ϕ to denote the standard normal density. Excess demand and excess supply are complementary events and, hence, $\pi_D = 1 - \pi_S$. For brevity, let

$$\xi_d = \frac{X_d'\beta_d - X_s'\beta_s}{\sigma}.$$

The random variable ξ_d gives the expected shortage normalized by the variance of the difference of the shocks. One may define the normalized surplus variable analogously. This section discusses ξ_d, but all the results have analogous counterparts if one alternatively uses ξ_s.

The normalized shortage is a unit-less variable as both the numerator and the denominator are measured in terms of the quantity units. This variable is the disequilibrium models' point estimate of the state of the market. The marginal effects of the control variables on ξ_d are constant, which means that irrespective of the state of the market, changes in the controls affect the intensity of shortages in the same way.

A more dynamic interpretation of the shortages can be obtained by examining the probabilities of their realizations. Similar to non-linear probability models, the marginal effects on the probabilities of the shortages are state-dependent and the intensity that a change in a control variable affects the probability of observing a shortage depends on the current market state. The further is a system from the market-clearing point, the less prominent are the marginal effects of the control variables. Outlying shortage states have small probabilities to occur and changes in the control variables leave these probabilities small. In contrast, the probabilities of observing states that are close to market-clearing are affected more intensely from control variable changes.

The following theorem summarizes the discussion and makes the last statements precise.

Theorem III.1 (Marginal effects on the shortage probabilities)**.** *Suppose that an explanatory variable exists only in demand equation and is indexed by k_d. Then the marginal effect of a change in the variable on the probability of excess demand is*

$$\partial\pi_S = \phi(\xi_d)\frac{\beta_{d,k_d}}{\sigma}.$$

If the variable belongs only in the supply equation and is indexed by k_s, then the effect is

$$\partial \pi_S = -\phi(\xi_d) \frac{\beta_{s,k_s}}{\sigma}.$$

Lastly, if it belongs in both equations and indexed k_d and k_s respectively, the effect is

$$\partial \pi_S = \phi(\xi_d) \frac{\beta_{d,k_d} - \beta_{s,k_s}}{\sigma}.$$

Proof. By inspection. □

Due to the complementarity of the events of excess demand and excess supply, the effects on surplus probabilities have the opposite signs of those of the shortage effects. The marginal effects of theorem III.1 are not equation specific. Instead, they account for the entire market specification. The advantage from examining $\partial \pi_S$ in comparison to $\partial \xi_d$ is that it takes into account the market state. The disadvantage is that the interpretation of the effect concerns the probability of observing a shortage and not the shortage itself, but in any case, both effects qualitatively point to the same direction as the standard normal density is positive and the signs of $\partial \pi_S$ and $\partial \xi_d$ coincide.

IV. Market-Clearing in Times of Distress

By applying the market-clearing assessment methodology, this section illustrates that exogenous shocks can improve the effectiveness of the free-market mechanism in cases of markets that are in principle competitive and they do not suffer from any innate market failure. I use granular micro-level data from the collection of IRI marketing datasets[5] containing information from both the demand and supply side of the US deodorant retail markets of Eau Claire and Pittsfield from 2001 until 2012.

Contrary to expectations one might have a priori, I provide statistical evidence that the price mechanism of the deodorant market was more efficient during the years of the crisis. The data indicate that market-clearing constitutes a statistically more plausible econometric assumption during the years of the crisis, while the short-side rule describes the deodorant market better in normal times.

i. Data description

From the supply side, I use data containing information on sale volumes and values per week, per product, and per store. From the demand side, I use scanner data from visits of households in the retail stores of the two counties. The household trip data contain purchased quantities and expenditure for each product that a household bought at a given trip to a retail store. I aggregate household purchases on a week, store, product base.

[5] A detailed description of the data is given by Bronnenberg et al. (2008).

Each household is linked with demographic data. When aggregating the purchases data, I use the demographic variables to construct measures of centrality for the demographics of the representative household purchasing a particular product in a given store and on a given date. For instance, the average of the income classes of the households that visited a store at a given week is used to proxy the income of the representative visiting household.

For each product, I extract the volume measured in ounces from the IRI stub specification of the product and calculate the average price per ounce sold by the store at a given week. Traded quantities are also transformed to be measured in ounces. The products of the IRI data are not by default linked for all the available years. Specifically, the products are linked from 2001 until 2006, from 2008 until 2011, while the years 2007 and 2012 have their own identifiers. To study the evolution of the market-clearing hypothesis during the great recession I link the observed products using a supervised semi-automated matching process that is based on the Levenshtein semi-metric, on a trigram similarity measure, and on a self-developed measure that uses the symmetric difference of the atoms of the stub specifications of the products. The details of the matching process can be found in appendix C.i.

The resulting set is linked with product information and subsequently with company and vendor data. The product data contain information on product attributes, such as packaging, scent, form, etc. Furthermore, each store observation is linked with data from the corresponding store data files. From this link, information on the type of stores, the market that a store operates, and the total weekly sales and volumes are obtained. Finally, stores are linked with their corresponding chains. I deflate prices using the quarterly, seasonally adjusted series GDP implicit price deflator from 2001 to 2012 with 2012 as base year[6].

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
No Stores	48	53	43	44	43	44	42	38	39	40	40	38
No Chains	11	12	14	14	14	11	11	13	13	13	13	13
No Products	648	776	752	690	729	685	708	651	621	511	527	510
No Companies	23	24	22	24	25	25	24	22	22	20	18	20
Eau Claire Obs	2685	3257	2622	3025	2773	2461	1772	754	618	550	767	597
Pittsfield Obs	8403	9465	7673	5668	5823	5346	4953	5233	4573	3766	3638	2986
Sample Obs	11088	12722	10295	8693	8596	7807	6725	5987	5191	4316	4405	3583

Table 1: Variation within years.

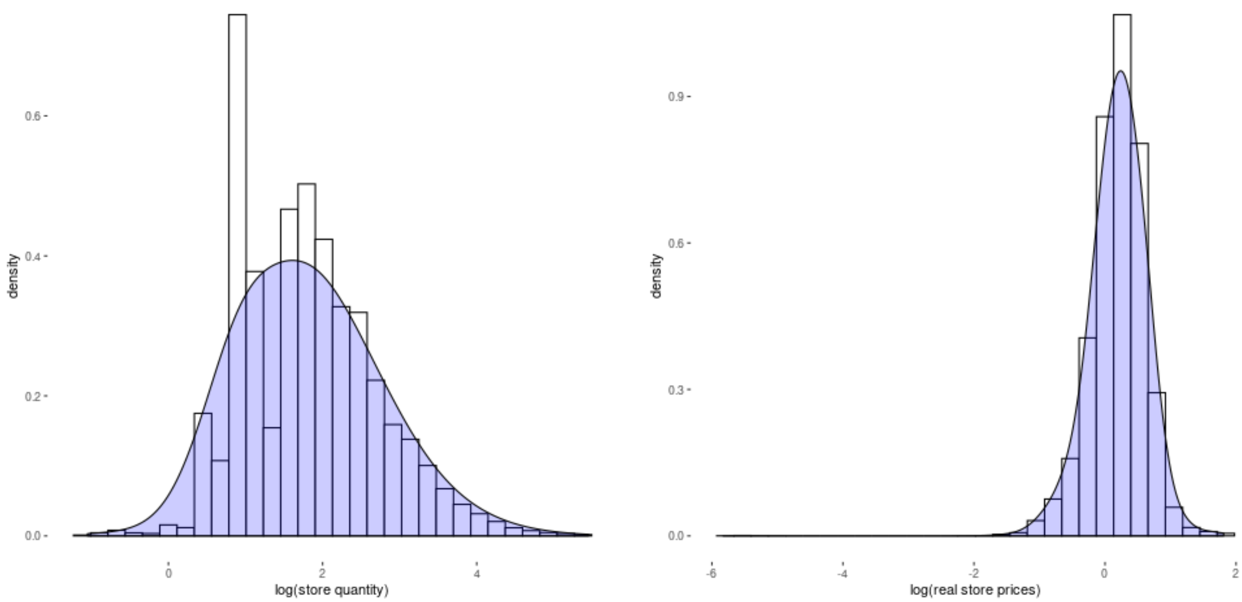
Table 1 shows the variation of the dataset within each year concerning the stores, the chains of the stores, the products, as well as the companies that produced them. Although that in all of the above dimensions

[6] U.S. Bureau of Economic Analysis, Gross Domestic Product: Implicit Price Deflator [GDPDEF], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GDPDEF, May 5, 2020

the dataset is balanced, the decreasing number of observations is potential a source of survivorship bias. I focus on two consecutive year period estimates to avoid reporting biased results. Figure 2 depicts the sample's distributions of prices and quantities for all the available years. Except for a few low price outliers that potentially correspond to special offers and bulk purchases, the distribution of prices is fairly concentrated around its mean. The distribution of quantities is more fragmented, which should come without surprise, as the volume of the packages and containers in the deodorant market is largely standardized across products.

Figure 2: Sample distributions of prices and quantities.

(a) Quantities. (b) Prices.



ii. Analysis and discussion

In this section, I am using p, s, and t for indexing products, stores and time-points respectively. Besides prices, I include store indicators to capture customer service idiosyncrasies, company indicators to capture brand loyalty effects, as well as monthly indicators to capture seasonal effects in the demand specification. Lastly, I am including the average income of the households visited each store divided by the average number of family members to capture effects relating to the purchasing power of the representative household buying a particular product at a particular store at each week. The empirical demand specification is summarized by

$$\log D_{pst} = \beta_{d,0} + \alpha_d \log P_{pst} + \beta_{d,1} \frac{\text{income class}}{\text{family size}} + \text{store} + \text{company} + \text{month} + u_{d,pst}.$$

Since not all products are sold by all stores, the supply side specification includes product indicators to capture product-specific effects. It also includes chain indicators, to capture effects that are attributed to chain-level promotion policies, and interaction terms for the month and the market of the observation. In summary, the specification is given by

$$\log S_{pst} = \beta_{s,0} + \alpha_s \log P_{pst} + \text{product} + \text{chain} + \text{market} \times \text{month} + u_{s,pst}.$$

Using the above market setup, I estimate the equilibrium and the deterministic adjustment models for 11 one-year periods for the years of the sample beginning from 2002 and apply the methodology of section III to examine how the market-clearing assumption compares to flexible deterministic price dynamics[7]. The analysis is focused on comparing the deterministic adjustment and the equilibrium models because the retail deodorant market is not characterized by structural inefficiencies or market failures which would cast reasonable doubt to the assumption of the prices being responsive to market forces.

Criterion	Model	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
AIC	DA	30910.60*	28485.14*	25248.99*	25930.73*	25432.59*	19060.84	10601.65	3585.18*	3574.95*	3163.76*	2085.74*
	EM	33693.03	29727.45	31668.23	33646.00	30474.08	17774.66*	6974.49*	5816.69	6345.59	6339.17	2740.40
BIC	DA	37675.30*	35172.29*	31503.78*	32048.19*	33664.82*	26771.62	15264.81	5399.20*	5330.57*	5123.67*	3630.67*
	EM	40980.37	36268.20	38087.74	40375.16	40206.49	25142.25*	10324.20*	8781.60	9018.50	10148.13	4356.89

Table 2: Model selection per year

Table 2 presents the results of the analysis. Each cell of the table contains the calculated values of Akaike (upper part) and Baysean's (lower part) information criteria for the model that corresponds in the row of the cell and the year that corresponds in the cell's column. The selection of each information criterion for each year is indicated by a $*$ following the value of the selected model. For the majority of the years, the deterministic adjustment model constitutes a more suitable alternative for explaining the data in comparison with the equilibrium model. The only two years for which the situation is reversed are 2007 and 2008, the year that the financial crisis began, and the year that its effects were the greatest in the US. The difference in the values of both the information criteria is more prevalent for 2008, which is ostensibly counterintuitive. One should have in mind, however, firstly that the crisis concerned primarily the financial sector and secondly that the deodorant market was not characterized by some structural inefficiency that led to a breakdown, rather it was indirectly affected by the collapse of the financial market. With these two points in mind, the result of table 2 becomes more sensible.

In normal times, the deodorant market constitutes only a small share of a household's expenditure. The prices of deodorants are low and many consumers may rely on memory when purchasing such products instead of conducting extensive market search. When households behave in such a way, their sets of considered products shrink, making the demand of the products more inelastic, and giving firms the ability to raise prices. Thus, the market forces are weakened and the market mechanism becomes lazier. In times of distress, when income

[7] One year is skipped to keep the estimated models comparable because, despite that the equilibrium model is temporal, the calculation of the price dynamics of the deterministic adjustment model requires the calculation of lagged prices.

falls and small expenditures become more important, or times preceding distress, when households perceive that even small expenditures are important for the future, they become more prudent and rely more on market search instead of memory. Such behavior enlarges consideration and restricts the ability of the sellers to increase prices. In addition, companies that are financially constraint due to the financial crisis have incentives to, by becoming more competitive, raise revenues, and use their cash flows as a short-term refinancing alternative. This intuition is in line with the result of table 2, which shows that the market mechanism was less prevalent before 2007, it strengthened in $2007-2008$, and subsequently reverted to its usual state after 2008.

Figure 3 provides further evidence from both the demand and supply sides of the market to support the economic intuition. Figure 3a plots the Herfindahl–Hirschman index accumulated for each year of the sample using market shares calculated from deflated prices. Although there is a negative correlation in the market concentration indices between Pittsfield and Eau Claire, the index for the whole sample indicates that competition has strengthened in 2008 and 2009. Specifically, the reduction in the concentration index during the years of the crisis is greater than the increase in the index from 2001 until 2007. Figure 3b plots the average household expenditure in the market normalized by the per capita household income class. The expenditure is calculated using the scanner prices recorded in household trips, adjusted for inflation using the GDP deflator. The substantial decrease of expenditures in the Pittsfield market dominates the sample's movement, an observation which is particularly strong in 2008. In accordance with the direction suggested by the market-clearing assessment methodology, the anticipation and the realization of increased competition from the supply-side, combined with reduced demand-side expenditure, increased market efficiency during the times of distress.

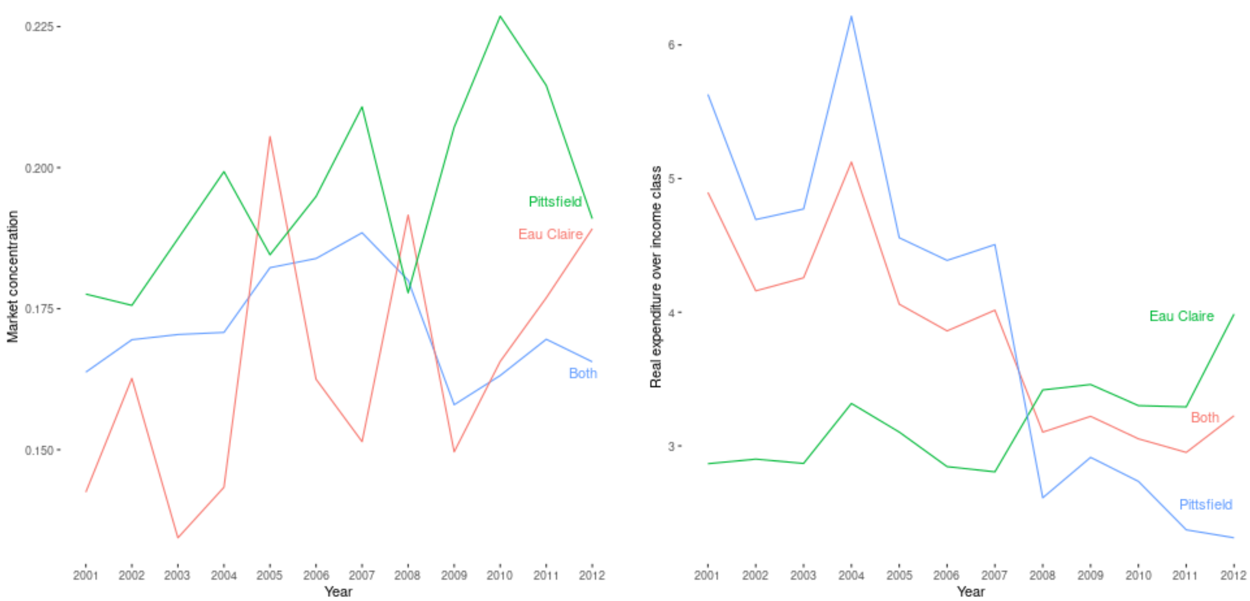
V. Likelihood Optimization

This section presents the modeling and technical contributions of the article. It contains the results concerning the likelihoods of the equilibrium and disequilibrium models, as well as their gradients and Hessians. Although analytic expressions of the likelihoods of the disequilibrium models can be found in existing literature, the alternative representations that I am presenting here are based on price and quantity moments, which are more receptive to economic interpretations. Each representation is summarized in proposition and the proofs of these statements are given in appendix A.i.

Concerning the derivatives of the likelihoods, Maddala and Nelson (1974) calculate the gradient and the Hessian of the basic model with independent demand and supply shocks. I extend the basic model's calculations to also cover the case of correlated shocks. The gradients of each of the remaining models are given for correlated shocks since the independence case is nested within the correlated one and can be obtained by performing some simplifications in the general expressions. I also calculate the Hessians for the equilibrium, the basic, and the directional models with correlated shocks. The calculations for these expressions can be found in the online appendix. Table 3 summarizes the calculations of the derivatives.

Figure 3: Expenditure and competition.

(a) Supply side competition. (b) Demand side expenditure.



		Shocks	
Market	Model	Independent	Correlated
Disequilibrium	Basic	Maddala and Nelson (1974)	This article
	Directional	This article	
	Deterministic Adjustment	This article (only gradient)	
	Stochastic Adjustment	This article (only gradient)	
Equilibrium	FIML	This article	

Table 3: Analytic expressions of gradients and Hessians.

Besides the analytic insight that they provide, which admittedly can be obscured by the complexity of their expressions, the calculation of the derivatives is beneficial in two practical ways. Firstly, they provide more information to the optimization methods, which can improve the maximum likelihood estimations' accuracy. This point becomes especially relevant when the identified parameters are in a neighborhood of a pole of the likelihood. The gradients can be numerically approximated using finite differences, but the approximation errors of such differences are greater nearby the poles of a function. In some cases, the numerical approximation of the gradient completely fails and the estimation of the model using numerically calculated derivatives becomes impossible. The analytic derivatives alleviate this problem. Secondly, the use of extra information from the expressions of the gradients significantly reduces the average computational needs of estimating the models. By conducting extensive benchmarking simulations in Hesse's high-performance computing cluster of Goethe University, I collect and present statistics on the performance of maximum likelihood estimations of the models considered in this article.

i. Expressions of likelihoods

To facilitate the presentation of the likelihoods, I introduce some simplifying notation. Let X be a random variable among $\{D, S, P\}$. A fundamental expression that repeatedly appears in the equations of the discussed models is

$$h_X = \frac{X - \mu_X}{\sigma_X}.$$

It denotes the normalized deviation of the random variable X from its mean. It is expressed in relative terms as the numerator's unit and the standard deviation's unit are canceled out. If X and Y are two distinct random variables in $\{D, S, P\}$, I use ρ_{XY} to denote the correlation between X and Y. Another important expression in this context takes the form of a weighted difference between the deviations of X and Y from their respective means. It is given by

$$z_{XY} = \frac{h_X - h_Y \rho}{\sqrt{1 - \rho_{XY}}},$$

where ρ is the correlation of the underlying disturbances of X and Y. Both the correlation (ρ_{XY}) and the deviations (h_X, h_Y) are unit-less and therefore z_{XY} is also expressed in relative terms. The variable z_{XY} collapses merely to h_X when the disturbances of X and Y are independent.

The first likelihood I present in this section is the full-information likelihood of the equilibrium model. The general full-information likelihood for systems of simultaneous equations can be found in Balestra and Varadharajan-Krishnakumar (1987). Here, I specialize the likelihood expression for a linear market system. This expression can be used when calculating the information criterion of the market-clearing assessment given in section III.i.

Proposition V.1 (Equilibrium model's likelihood)**.** *In model (EM), prices and quantities are joint-normally distributed with moments*

$$\mu_P = \frac{X'_{d,nt}\beta_d - X'_{s,nt}\beta_s}{\alpha_s - \alpha_d},$$
$$\sigma_P^2 = \frac{\sigma_d^2 + \sigma_s^2 - 2\sigma_d\sigma_s\rho}{(\alpha_s - \alpha_d)^2},$$
$$\mu_Q = \frac{X'_{d,nt}\alpha_s\beta_d - X'_{s,nt}\alpha_d\beta_s}{\alpha_s - \alpha_d},$$

$$\sigma_Q^2 = \frac{\alpha_s^2\sigma_d^2 + \alpha_d^2\sigma_s^2 - 2\alpha_d\alpha_s\sigma_d\sigma_s\rho}{(\alpha_s - \alpha_d)^2},$$

and

$$\sigma_{QP} = \frac{\alpha_s\sigma_d^2 + \alpha_d\sigma_s^2 - (\alpha_d + \alpha_s)\,\sigma_d\sigma_s\rho}{(\alpha_s - \alpha_d)^2}.$$

The log-likelihood of the equilibrium model is written in terms of these moments as

$$\ell(\alpha_d, \beta_d, \alpha_s, \beta_s, \sigma_d^2, \sigma_s^2, \rho) = -\frac{1}{2}\sum_{nt} \log\left\{\log(2\pi) + \log\left(\sigma_Q^2\sigma_P^2\left(1-\rho_{QP}^2\right)\right) + \frac{h_Q^2 - 2h_Q h_P \rho_{QP} + h_P^2}{1-\rho_{QP}^2}\right\}.$$

In the basic model, the short-side rule determines whether the observed quantity comes from the demand or the supply side. The expressions for h_D and h_S are simply formulated as

$$h_{D,nt} = \frac{Q_{nt} - X'_{d,nt}\beta_d}{\sigma_d} \quad \text{and } h_{S,nt} = \frac{Q_{nt} - X'_{s,nt}\beta_s}{\sigma_s}.$$

The variances of the supplied and demanded quantities merely coincide with the corresponding variances of the shocks of the demand and supply equations. The correlation between D and S is equal to the correlation of the shocks. The next proposition gives the likelihood of the basic model.

Proposition V.2 (Basic model's likelihood)**.** *The log-likelihood of model (BM) is given by*

$$\ell(\beta_d, \beta_s, \sigma_d^2, \sigma_s^2, \rho) = \sum_{nt} \log\left\{\frac{1}{\sigma_d}\phi(h_{D,nt})\left[1 - \Phi\left(z_{SD,nt}\right)\right] + \frac{1}{\sigma_s}\phi(h_{S,nt})\left[1 - \Phi\left(z_{DS,nt}\right)\right]\right\}.$$

In the directional model, the sample is separated based on the observed price changes. Due to this sample separation, an observation of the sample contributes exclusively either to the excess demand or to the excess supply case. The expressions for the deviations, the variances, and the correlation between demanded and supplied quantities are as of those in the basic model.

Proposition V.3 (Directional model's likelihood)**.** *The log-likelihood of model (DM) is*

$$\ell(\beta_d, \beta_s, \sigma_d^2, \sigma_s^2, \rho) = \underbrace{\sum_{\Delta P < 0} \log \frac{\phi(h_{D,nt})\left[1 - \Phi\left(z_{SD,nt}\right)\right]}{\sigma_d}}_{\ell_d} + \underbrace{\sum_{\Delta P \geq 0} \log \frac{\phi(h_{S,nt})\left[1 - \Phi\left(z_{DS,nt}\right)\right]}{\sigma_s}}_{\ell_s}.$$

For the case of independent shocks, the simple separation rule of the directional model allows to conveniently disentangle the densities of the excess demand and supply regimes in the likelihood function. The likelihood becomes additively separable with respect to demand and supply variables, which allows ℓ_d and ℓ_s to be estimated independently.

When deterministic price adjustments are considered, the likelihood of the disequilibrium model assumes a more complicated form. In contrast to the densities of the first two disequilibrium models which are univariate, the density of the deterministic adjustment model has two variables. The sample is also separated in the case of the deterministic adjustment model.

Proposition V.4 (Deterministic adjustment model's likelihood)**.** *Suppose that the market of model (DA) is characterized by excess supply. Prices and demanded quantities are joint-normally distributed with moments*

$$\mu_P = \frac{\gamma P_{n,t-1} + X'_{d,nt}\beta_d - X'_{s,nt}\beta_s}{\gamma + \alpha_s - \alpha_d},$$
$$\sigma_P^2 = \frac{\sigma_d^2 + \sigma_s^2 - 2\sigma_d\sigma_s\rho}{\left(\gamma + \alpha_s - \alpha_d\right)^2},$$
$$\mu_D = X'_{d,nt}\beta_d + \alpha_d\mu_P,$$
$$\sigma_D^2 = \alpha_d^2\sigma_P^2 + \sigma_d^2 + 2\alpha_d\frac{\sigma_d^2 - \sigma_d\sigma_s\rho}{\gamma + \alpha_s - \alpha_d},$$

and

$$\sigma_{DP} = \alpha_d\sigma_P^2 + \frac{\sigma_d^2 - \sigma_d\sigma_s\rho}{\gamma + \alpha_s - \alpha_d}.$$

Suppose instead that the market is in an excess demand state. Prices and supplied quantities are joint-normally distributed. Prices' moments are as before and supplied quantities' moments are

$$\mu_S = X'_{s,nt}\beta_s + \alpha_s\mu_P,$$
$$\sigma_S^2 = \alpha_s^2\sigma_P^2 + \sigma_s^2 - 2\alpha_s\frac{\sigma_s^2 - \sigma_d\sigma_s\rho}{\gamma + \alpha_s - \alpha_d}.$$

The covariance of prices and supplied quantities is given by

$$\sigma_{SP} = \alpha_s\sigma_P^2 - \frac{\sigma_s^2 - \sigma_d\sigma_s\rho}{\gamma + \alpha_s - \alpha_d}.$$

Finally, the log-likelihood is completely determined by these moments. Specifically

$$\ell(\alpha_d, \beta_d, \alpha_s, \beta_s, \gamma, \sigma_d^2, \sigma_s^2, \rho) = \ell_D + \ell_S$$

where

$$\ell_D = -\sum_{\Delta P<0}\left[\log\left(2\pi\right) + \frac{1}{2}\log\left(\sigma_D^2\sigma_P^2\left(1-\rho_{DP}^2\right)\right) + \frac{1}{2}\frac{h_D^2 - 2h_Dh_P\rho_{DP} + h_P^2}{1-\rho_{DP}^2}\right]$$

and ℓ_S is obtained by replacing D with S in the last equation and summing over the observations with $\Delta P \geq 0$.

The representation of the likelihood of proposition V.4 is given in terms of the moments of the model's prices and quantities. Besides contributing to giving a compact likelihood representation, these moments have estimates that have meaningful economic interpretations. For instance, if the model is used in a market where prices are given by interest rates, μ_P is the expected rate of return for an observation, conditional on the observable variables. The mean of this estimator provides an estimate for the market's mean expected return. Analogously, one can obtain an estimate of the market returns' volatility using σ_P^2.

The last likelihood of the section concerns the stochastic adjustment model. The sample is not separated in this model and the market system has three sources of stochasticity. As a result, the derived densities are three-dimensional and the likelihood expression is more convoluted.

Proposition V.5 (Stochastic adjustment model's likelihood)**.** *Prices, demanded and supplied quantities are joint-normally distributed in model (SA). Their moments are given by*

$$\mu_P = \frac{\gamma P_{n,t-1} + X'_{d,nt}\beta_d - X'_{s,nt}\beta_s + X'_{p,nt}\gamma\beta_p}{\gamma + \alpha_s - \alpha_d},$$

$$\sigma_P^2 = \frac{\sigma_d^2 + \sigma_s^2 + \gamma^2\sigma_p^2 - 2\sigma_d\sigma_s\rho_{ds} + 2\gamma\sigma_d\sigma_p\rho_{dp} - 2\gamma\sigma_s\sigma_p\rho_{sp}}{(\gamma + \alpha_s - \alpha_d)^2},$$

$$\mu_D = X'_{d,nt}\beta_d + \alpha_d\mu_P,$$

$$\sigma_D^2 = \alpha_d^2\sigma_P^2 + \sigma_d^2 + 2\alpha_d\frac{\sigma_d^2 - \sigma_d\sigma_s\rho_{ds} + \gamma\sigma_d\sigma_p\rho_{dp}}{\gamma + \alpha_s - \alpha_d},$$

$$\mu_S = X'_{s,nt}\beta_s + \alpha_s\mu_P,$$

$$\sigma_S^2 = \alpha_s^2\sigma_P^2 + \sigma_s^2 - 2\alpha_s\frac{\sigma_s^2 - \sigma_d\sigma_s\rho_{ds} - \gamma\sigma_s\sigma_p\rho_{sp}}{\gamma + \alpha_s - \alpha_d}.$$

The covariances are

$$\sigma_{DS} = \alpha_d\alpha_s\sigma_P^2 - \alpha_d\frac{\sigma_s^2 - \sigma_d\sigma_s\rho_{ds} - \gamma\sigma_s\sigma_p\rho_{sp}}{\gamma + \alpha_s - \alpha_d} + \alpha_s\frac{\sigma_d^2 - \sigma_d\sigma_s\rho_{ds} + \gamma\sigma_d\sigma_p\rho_{dp}}{\gamma + \alpha_s - \alpha_d} + \sigma_d\sigma_s\rho_{ds},$$

$$\sigma_{DP} = \alpha_d\sigma_P^2 + \frac{\sigma_d^2 - \sigma_d\sigma_s\rho_{ds} + \gamma\sigma_d\sigma_p\rho_{dp}}{\gamma + \alpha_s - \alpha_d},$$

and

$$\sigma_{SP} = \alpha_s\sigma_P^2 - \frac{\sigma_s^2 - \sigma_d\sigma_s\rho_{ds} + \gamma\sigma_s\sigma_p\rho_{sp}}{\gamma + \alpha_s - \alpha_d}.$$

Lastly, the log-likelihood is completely determined by these moments as

$$\ell(\alpha_d, \beta_d, \alpha_s, \beta_s, \gamma, \beta_p, \sigma_d^2, \sigma_s^2, \sigma_p^2, \rho_{ds}, \rho_{dp}, \rho_{sp}) = \sum_{nt} \log\left(L_D + L_S\right)$$

where

$$L_D = \frac{\exp\left(\frac{2h_D h_P \rho_{DP} - h_D^2 - h_P^2}{2(1-\rho_{DP}^2)}\right)}{\sqrt{(2\pi)^2\sigma_D^2\sigma_P^2(1-\rho_{DP}^2)}}\left(1 - \Phi\left(\frac{h_S(1-\rho_{DP}^2) - h_D(\rho_{DP}\rho_{SP} - \rho_{DS}) + h_P(\rho_{DS}\rho_{DP} - \rho_{SP})}{\sqrt{(1-\rho_{DP}^2 - \rho_{DS}^2 - \rho_{SP}^2 + 2\rho_{DP}\rho_{DS}\rho_{SP})(1-\rho_{DP}^2)}}\right)\right).$$

and L_S is obtained by swapping D and S in the last equation.

An equivalent, alternative representation of the likelihood of the stochastic adjustment model is given in Quandt (1978). The value-added from the representation in the current article, as also argued for the case of proposition V.4, is that the likelihood is expressed in economically meaningful terms; namely in terms of moments of prices, demanded, and supplied quantities.

ii. Expressions of gradients and Hessians

The following definitions simplify the presentation of the section's results. Let $i, j \in \{D, S\}$ be distinct indices and set

$$\psi_i = \phi\left(h_i\right)\phi\left(z_{ji}\right),$$

$$\Psi_i = \phi\left(h_i\right)\left(1 - \Phi\left(z_{ji}\right)\right).$$

For the models that involve sample separation, it is useful to introduce index sets for the separation rule. Let

$$I_D = \{nt\colon \Delta P < 0\} \quad \text{and} \quad I_S = \{nt\colon \Delta P \geq 0\}.$$

Lastly, for distinct $i, j \in \{D, S, P\}$ define

$$\rho_{1,ij} = \frac{1}{\sqrt{1 - \rho_{ij}}}$$

and $\rho_{2,ij} = \rho_{ij}\rho_{1,ij}$.

Proposition V.6 (Equilibrium model likelihood's derivatives)**.** *The gradient of the equilibrium model's log-likelihood is*

$$\nabla l = \sum_{nt}\left[\frac{2X_d\rho_1 z_d}{\sigma_d} \quad \frac{2X_s\rho_1 z_s}{\sigma_s} \quad \frac{h_d\rho_1 z_d - 1}{\sigma_d^2} \quad \frac{h_s\rho_1 z_s - 1}{\sigma_s^2} \quad \rho_1^2\left(2h_d h_s - 2\rho_2\left(h_d z_d + h_s z_s\right) + \rho\right)\right].$$

The Hessian is given in the online appendix.

Proposition V.7 (Basic model likelihood's derivatives)**.** *Let L be the likelihood of an observation of model (BM). Its gradient is completely specified by the equations*

$$\begin{aligned}
\partial_{\beta_i} L &= \left(\frac{\psi_j\rho_1}{\sigma_i\sigma_j} + \frac{\Psi_i h_i - \psi_i\rho_2}{\sigma_i^2}\right) X_i', \\
\partial_{\sigma_i^2} L &= \frac{h_i\psi_j\rho_1}{2\sigma_i^2\sigma_j} + \frac{\Psi_i\left(h_i^2 - 1\right) - h_i\psi_i\rho_2}{2\sigma_i^3}, \\
\partial_\rho L &= \rho_1^2\left(\frac{\psi_i z_i}{\sigma_i} + \frac{\psi_j z_j}{\sigma_j}\right),
\end{aligned}$$

where i, j are distinct indices in $\{D, S\}$. The Hessian is given in the online appendix.

In many cases, it is more convenient to work with the log-likelihood instead of the likelihood function. Proposition V.7 can be used to derive the expressions for the the log-likelihood derivatives. Specifically, for a particular observation one obtains

$$\begin{aligned}
\ell_{nt} &= \log L_{nt}, \\
\nabla\ell_{nt} &= \frac{\nabla L_{nt}}{L_{nt}},
\end{aligned}$$

and

$$\mathrm{D}^2\ell_{nt} = \frac{L_{nt}\mathrm{D}^2 L_{nt} - \nabla' L_{nt}\nabla L_{nt}}{L_{nt}^2}.$$

Finally, the expressions of the derivatives of the model's log-likelihood are obtained by summing over all the observations.

The independent shocks case can be obtained in a similar manner by relatively simpler calculations compared to those of proposition V.7. The resulting formulas are specializations of the corresponding expressions of the correlated case for $\rho = 0$. In such a case, the derivatives with respect to ρ are irrelevant and many terms in the remaining derivatives vanish due to $\rho_1 = 1$ and $\rho_2 = 0$.

Corollary V.8 (Basic model with independent shocks likelihood's derivatives)**.** *The gradient of the likelihood of an observation of model (BM) with independent shocks is specified by*

$$\partial_{\beta_i} L = \left(\frac{\psi_j}{\sigma_i \sigma_j} + \frac{\Psi_i h_i}{\sigma_i^2} \right) X_i',$$
$$\partial_{\sigma_i^2} L = \frac{h_i \psi_j}{2\sigma_i^2 \sigma_j} + \frac{\Psi_i \left(h_i^2 - 1 \right)}{2\sigma_i^3},$$

where i, j are distinct indices in $\{D, S\}$. The Hessian is given in the online appendix.

The expressions of corollary V.8 are alternative, equivalent representations of the expressions derived in Maddala and Nelson (1974). Analogous corollaries can be formulated for the remaining propositions of this section. I avoid doing so, to save some space.

Proposition V.9 (Directional model likelihood's derivatives)**.** *The gradient of the log-likelihood of model (DM) is specified by*

$$\partial_{\beta_i} \ell = \sum_{I_i} \frac{\Psi_i h_i - \psi_i \rho_2}{\Psi_i \sigma_i} X_i' + \sum_{I_j} \frac{\psi_j \rho_1}{\Psi_j \sigma_i} X_i',$$
$$\partial_{\sigma_i^2} \ell = \sum_{I_i} \frac{\Psi_i \left(h_i^2 - 1 \right) - h_i \psi_i \rho_2}{2\Psi_i \sigma_i^2} + \sum_{I_j} \frac{h_i \psi_j \rho_1}{2\Psi_j \sigma_i^2},$$
$$\partial_\rho \ell = \sum_{I_i} \frac{\psi_i \rho_1^2 z_i}{\Psi_i} + \sum_{I_j} \frac{\psi_j \rho_1^2 z_j}{\Psi_j},$$

where $i \neq j$ are indices in $\{D, S\}$. The derivatives specifying the Hessian are given in the online appendix.

Proposition V.10 (Deterministic adjustment model likelihood's gradient)**.** *The gradient of the log-likelihood of model (DA) is given by adding*

$$\nabla \ell_D = \sum_{I_D} \left[\begin{array}{l} \frac{\rho_{1,DP} z_{PD}}{\sigma_P} \nabla \mu_P + \frac{\rho_{1,DP} h_P z_{PD} - 1}{2\sigma_P^2} \nabla \sigma_P^2 + \frac{\rho_{1,DP} z_{DP}}{\sigma_D} \nabla \mu_D + \frac{\rho_{1,DP} h_D z_{DP} - 1}{2\sigma_D^2} \nabla \sigma_D^2 \\ + \left(\left(\rho_{1,DP}^2 \rho_{2,DP}^2 + \rho_{1,DP}^4 \right) h_D h_P - \rho_{1,DP}^3 \rho_{2,DP} \left(h_P^2 + h_D^2 \right) + \rho_{1,DP} \rho_{2,DP} \right) \nabla \rho_{DP} \end{array} \right].$$

and $\nabla \ell_S$, which is obtained by replacing D with S. The expressions for the derivatives of the moments and the correlations are given in the online appendix.

The likelihood of model (SA) combines elements from both model (BM) and model (DA) and the resulting derivative expressions become lengthier. These expressions, as well as their derivations can be found in the online appendix.

iii. Computation

In this section, I discuss the performance of model estimations and the computational benefits from using the expressions of the gradients given in section V.ii. A major difficulty of estimating models for markets in disequilibrium comes from their computational complexity. Dorsey and Mayer (1995) classify the estimation of disequilibrium models among the most demanding econometric estimation problems. The likelihoods of these models have poles and non-unique local maxima and attempts to estimate them using derivative-based methods with finite differences approximations of gradient fail on many occasions. Other authors propose methodologies that examine how the global maximum can be located, while this section illustrates how the estimation procedure can be ameliorated by using the expressions of the gradients.

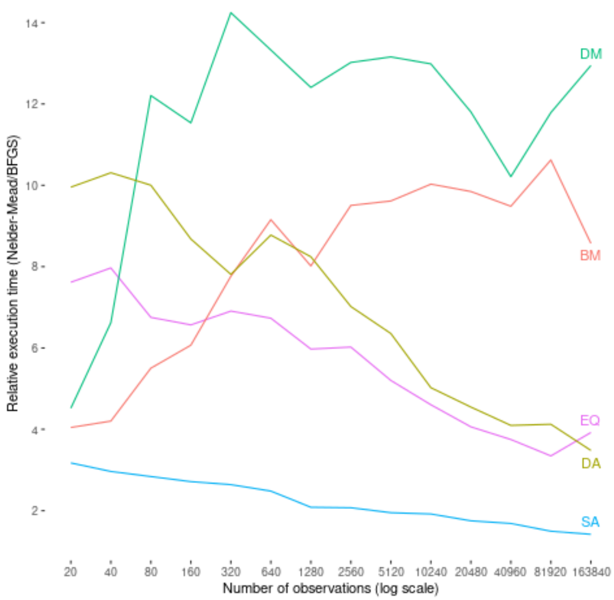
The classic estimation approach, as proposed by Maddala (1986), obtains the maximum likelihood estimates using a global, iterative Newton method. Zilinskas and Bogle (2006) propose a random interval arithmetic optimization approach and use the dataset of Fair and Jaffee (1972) and an unknown sample separation model with independent shocks to experimentally assess the performance of methodology. The results of section V.ii can be embedded in both the classic and the random arithmetic approach, and in general in any other derivative-based optimization method. Quandt and Ramsey (1978) propose an estimation methodology that uses the moment generating function of the likelihood. Bowden (1978) also considers the estimation difficulties of the deterministic and stochastic adjustment models and proposes a re-parametrization of these models that allow for simpler estimation procedures. In addition, the last article compares the application of a genetic algorithm optimization method with Nelder-Mead.

In this article, I compare the median time needed to estimate each model using BFGS, an algorithm that uses the calculated derivatives, and Nelder-Mead, an algorithm that does not. To collect statistics on the execution times I perform a series of benchmarking simulations. The model parameters are not assigned with deterministic values, but instead, they are randomly drawn from a uniform distribution. The independent and explanatory variables are drawn from normal distributions. Although the inclusion of BFGS with numerically approximated derivatives in the benchmarking exercise would have been of interest, practically this algorithm without the analytic gradients is failure-prone and its inclusion requires a significant reduction of the extent of randomization of the simulation. This would cause the collected statistics to be less informative and, therefore BFGS with numerically approximated derivatives is not considered.

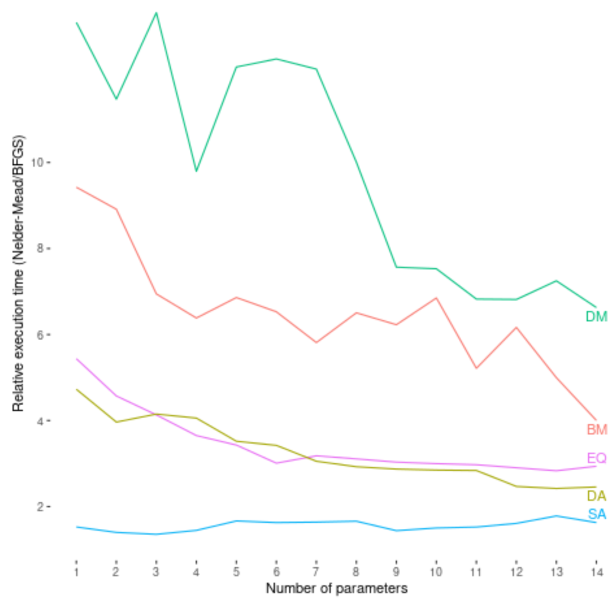
Each model is simulated 100 times and it is ensured that the simulated data are well behaved. If prices or quantities are negative, or if shortages and surpluses represent more than 90% of the sample, the simulation round is repeated. To keep things equal, each set of simulated data is used to estimate the model both with BFGS and Nelder-Mead. The execution time is saved at the end of each round. The counted time concerns only the estimation of the models and not their simulation. To spin up the processors, 10 untimed warm-up estimations are performed in the beginning. The exercise is repeated for a set of 14 different number of observations and 14 different number of model parameters.

Figure 4: Computational benefits of BFGS using gradient expressions per model.

(a) Variation over observations.



(b) Variation over model's parameters.



Figures 4a and 4b summarize the measurements. The vertical axes of both figures measure the ratio between the median estimation time of Nelder-Mead over the median estimation time of BFGS using the derived formulas. The greater is this ratio, the greater are the performance gains from using BFGS with the derived gradients. In fig. 4a demand is estimated using five parameters and supply using four parameters. The deterministic adjustment model uses one extra and the stochastic adjustment model uses four extra parameters in their corresponding price equations. The correlation between shocks is estimated for all the models. These specifications constitute the starting points the estimations of fig. 4b. The horizontal axis of fig. 4b measures the parameters that are estimated in both demand and supply in addition to the ones of the starting points.

The basic model, which is the most used among all the disequilibrium models that I consider in this article, is estimated on average at least four times faster when the expressions are used. The gains for the deterministic adjustment and equilibrium models are very similar due to the similarities that these models exhibit. In both cases, the estimation is performed at least three times faster. The least relative benefits are observed in the case of the stochastic adjustment model, in which the estimation is on average two times faster. In absolute terms, the performance benefit in the estimation of the stochastic adjustment model is the greatest because the estimation of this model is by far the most time-consuming.

VI. Conclusion

In the presence of price rigidities or other market failures, the application of estimation methods that account for deviations from the market-clearing condition becomes appealing. Although economic intuition has a primary role in determining the appropriateness of applying an econometric method, statistical evidence supporting the application of the methodology increases the credibility of the analysis.

In the present article, I provided a methodology that, conditional on the observable sample, statistically assesses the appropriateness of the market-clearing condition in the econometric description of a market. An empirical application of the methodology in the deodorant markets of Eau Claire and Pittsfield shows that, while crises may lead to greater imbalances in the price mechanism of markets that are characterized by some structural failure, the adjustment of prices can improve in times of distress in markets that are only exogenously affected. As a result, equilibrium models that entail market-clearing assumptions become more relevant than disequilibrium models during years of crises, in cases in which market participants become complacent and market forces weaken during normal times.

The presentation of the methodology was based on an equilibrium and four prevent disequilibrium market models and the performance of the assessment was illustrated using extensive simulations for data generating processes structured by these five models. The methodology, however, is not based on the specifics of the models that were used for its presentation and it can be readily applied to compare any equilibrium and disequilibrium models with known likelihoods.

Concerning the estimation of the examined models in this article, I provided representations of their likelihoods that are expressed in terms of their moments of prices and quantities. Concerning the interpretation of the effects of the included controls in these models, I introduced the calculation of the marginal effects on the shortage probabilities as a state-dependent evaluation of how the market is holistically affected by changes in the relevant variables.

Lastly, the article provides analytic expressions for the gradients of all the models and the Hessians of the equilibrium, the basic, and the directional models. By performing a series of benchmarking simulations, statistical measurements of the relative computational benefits that are associated with employing the newly derived expressions in the estimation of the five examined models were collected and presented.

A. Proofs of Propositions

All the probabilities, densities, and expectations of this section are to be understood as being conditional on the sigma algebras generated by the random variables that are observable at a given date.

i. Proof of propositions of section V.i

Proof of proposition V.1.

Combining the equations of model (EM) gives

$$P = \frac{X_d'\beta_d}{\alpha_s - \alpha_d} - \frac{X_s'\beta_s}{\alpha_s - \alpha_d} + \frac{u_d - u_s}{\alpha_s - \alpha_d},$$

from which one concludes that P is normally distributed and obtains the expressions for μ_P and σ_P^2. Substituting the closed form expression of the price to the demand equation results to

$$Q = \frac{X_d'\alpha_s\beta_d}{\alpha_s - \alpha_d} - \frac{X_s'\alpha_d\beta_s}{\alpha_s - \alpha_d} + \frac{\alpha_s u_d - \alpha_d u_s}{\alpha_s - \alpha_d},$$

from which the quantity moments are also obtained. The last two equations imply that

$$\sigma_{QP} = \mathrm{Cov}\left(\frac{u_d - u_s}{\alpha_s - \alpha_d}, \frac{\alpha_s u_d - \alpha_d u_s}{\alpha_s - \alpha_d}\right) = \frac{\alpha_s\sigma_d^2 + \alpha_d\sigma_s^2 - (\alpha_d + \alpha_s)\,\sigma_d\sigma_s\rho}{(\alpha_s - \alpha_d)^2}.$$

The joint density of prices and quantities is then given by

$$f(Q,P) = \frac{1}{2\pi\sqrt{\det(\Sigma_{QP})}}\exp\left(-\frac{1}{2}\begin{bmatrix} Q - \mu_Q & P - \mu_P\end{bmatrix}\Sigma_{QP}^{-1}\begin{bmatrix} Q - \mu_Q \\ P - \mu_P\end{bmatrix}\right),$$

where

$$\Sigma_{QP} = \begin{bmatrix} \sigma_Q^2 & \sigma_{QP} \\ \sigma_{QP} & \sigma_P^2 \end{bmatrix}.$$

The expression of the likelihood follows by calculating the quadratic form in the exponent of f. □

Proof of proposition V.2.

Following Maddala and Nelson (1974), the unconditional density of Q in model (BM) is

$$f(Q) = f(Q \mid S > Q)\,\pi_D + f(Q \mid D \geq Q)\,\pi_S.$$

The conditional density of Q in an excess demand state is given by

$$f(Q \mid S > Q) = \frac{f_D(Q)}{\pi_D},$$

where

$$f_D(Q) = \int_Q^\infty \frac{1}{2\pi\sqrt{\det(\Sigma)}}\exp\left(-\frac{1}{2}\begin{bmatrix} Q - X_d'\beta_d & S - X_s'\beta_s\end{bmatrix}\Sigma^{-1}\begin{bmatrix} Q - X_d'\beta_d \\ S - X_s\beta_s\end{bmatrix}\right)\mathrm{d}S.$$

The integral can be calcualted by a square completion and a variable change, i.e.

$$f_D(Q) = \int_Q^\infty \frac{1}{2\pi\sqrt{\det(\Sigma)}} \exp\left(-\frac{(Q - X_d'\beta_d)^2 \sigma_s^2(1-\rho^2) + ((Q - X_d'\beta_d)\sigma_s\rho - (S - X_s'\beta_s)\sigma_d)^2}{2\det(\Sigma)}\right) \mathrm{d}S$$

$$= \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left(-\frac{1}{2}h_D^2\right) \int_{\frac{1}{\sqrt{1-\rho^2}}(h_S - h_D\rho)}^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}S^2\right) \mathrm{d}S$$

$$= \frac{1}{\sigma_d}\phi(h_D)\left[1 - \Phi(z_S)\right].$$

Analogous arguments hold for the excess supply case. The likelihood then reduces to

$$L = \prod_{nt} \left(f_D(Q_{nt}) + f_S(Q_{nt})\right)$$

and the result readily follows. □

Proof of proposition V.3.

The conditional densities of Q given that the market is in an excess demand or excess supply regime are calculated as in the proof of proposition V.2. In contrast to proposition V.2, the selection condition of model (DM) is used to separate the sample and there is no need to use the total probability theorem to get the unconditional likelihoods. Instead, we need to account for the probability of observing the sample separation at hand. The likelihood is then obtained by

$$L = \left(\prod_{\Delta P<0} \frac{f_D(Q_{nt})}{\pi_D}\right)\left(\prod_{\Delta P\geq 0} \frac{f_S(Q_{nt})}{\pi_S}\right)\left(\prod_{\Delta P<0} \pi_D\right)\left(\prod_{\Delta P\geq 0} \pi_S\right)$$

$$= \left(\prod_{\Delta P<0} f_D(Q_{nt})\right)\left(\prod_{\Delta P\geq 0} f_S(Q_{nt})\right).$$

□

Proof of proposition V.4.

Combining the equations of model (DA) gives

$$(\gamma + \alpha_s - \alpha_d) P_{nt} = \gamma P_{n,t-1} + X_{d,nt}'\beta_d - X_{s,nt}'\beta_s + u_{d,nt} - u_{s,nt}.$$

Hence P_{nt} is normally distributed as an affine transformation of normally distributed variables. Its mean and variance are easily calculated from the last equation. Suppose that the market is an excess supply state. By the short-side rule, the quantity is determined by

$$Q_{nt} = D_{nt} = X_{d,nt}'\beta_d + P_{nt}\alpha_d + u_{d,nt}.$$

It follows that D_{nt} is also normally distributed. In particular, its moments are obtained by the last equation. The covariance between D_{nt} and P_{nt} is

$$\mathrm{Cov}(D, P) = \alpha_d \mathrm{Cov}(P, P) + \mathrm{Cov}(u_d, P) = \alpha_d\sigma_P^2 + \frac{\sigma_d^2 - \sigma_d\sigma_s\rho}{\gamma + \alpha_s - \alpha_d}.$$

Then, conditional on $\Delta P < 0$, the joint density of Q and P is given by

$$f_D(Q,P) = \frac{1}{2\pi\sqrt{\det \Sigma_D}} \exp\left(-\frac{1}{2} \begin{bmatrix} Q - \mu_D & P - \mu_P \end{bmatrix} \Sigma_D^{-1} \begin{bmatrix} Q - \mu_D \\ P - \mu_P \end{bmatrix} \right),$$

where

$$\Sigma_D = \begin{bmatrix} \sigma_D^2 & \sigma_{DP} \\ \sigma_{DP} & \sigma_P^2 \end{bmatrix}.$$

One can rewrite the density as

$$f_D(Q,P) = \frac{1}{2\pi\sqrt{\sigma_D^2\sigma_P^2\left(1-\rho_{DP}^2\right)}} \exp\left(-\frac{1}{2} \frac{h_{DP}^2 - 2h_{DP}h_P\rho_{DP} + h_P^2}{1-\rho_{DP}^2} \right).$$

Conditional that the market is in an excess demand state, the moments of S, its covariance with P, and the joint density of Q and P are obtained analogously. Following Amemiya (1974), the likelihood of the model is given by

$$\begin{aligned} L &= \left(\prod_{\Delta P<0} \frac{f_D(Q_{nt},P_{nt})}{\pi_D} \right) \left(\prod_{\Delta P\geq 0} \frac{f_S(Q_{nt},P_{nt})}{\pi_S} \right) \left(\prod_{\Delta P<0} \pi_D \right) \left(\prod_{\Delta P\geq 0} \pi_S \right) \\ &= \left(\prod_{\Delta P<0} f_D(Q_{nt},P_{nt}) \right) \left(\prod_{\Delta P\geq 0} f_S(Q_{nt},P_{nt}) \right). \end{aligned}$$

□

Proof of proposition V.5.

The calculations of moments in the model (SA) are extensions of the calculations in the proof of proposition V.4. In this case, the sample is not separated and we have to consider both the demanded and the supplied quantity for each observation simultaneously. The combination of the model's equations results to

$$\left(\gamma + \alpha_s - \alpha_d\right) P_{nt} = \gamma P_{n,t-1} + X'_{d,nt}\beta_d - X'_{s,nt}\beta_s + X'_{p,nt}\gamma\beta_p + u_{d,nt} - u_{s,nt} + \gamma u_{p,nt},$$

which, as in the deterministic adjustment case, proves that P_{nt} is normally distributed. The price moments of proposition V.5 are calculated from the last equation. The demand and supply system of equations, i.e.

$$\begin{aligned} D_{nt} &= X'_{d,nt}\beta_d + P_{nt}\alpha_d + u_{d,nt} \\ S_{nt} &= X'_{s,nt}\beta_s + P_{nt}\alpha_s + u_{s,nt}, \end{aligned}$$

imply that D and S are normally distributed. The calculation of their moments is also straightforward. To calculate the joint distribution of D, S and P, it remains to calculate the covariances. The covariance between D and P is calculated analogously to the deterministic adjustment proof. Similarly one obtains the covariance between S and P. The covariance between D and S is given by

$$\text{Cov}\left(D,S\right) = \alpha_d\alpha_s\text{Cov}\left(P,P\right) + \alpha_d\text{Cov}\left(P,u_s\right) + \alpha_s\text{Cov}\left(u_d,P\right) + \text{Cov}\left(u_d,u_s\right)$$

$$= \alpha_d \alpha_s \sigma_P^2 - \alpha_d \frac{\sigma_s^2 - \sigma_d \sigma_s \rho_{ds} - \gamma \sigma_s \sigma_p \rho_{sp}}{\gamma + \alpha_s - \alpha_d} + \alpha_s \frac{\sigma_d^2 - \sigma_d \sigma_s \rho_{ds} + \gamma \sigma_d \sigma_p \rho_{dp}}{\gamma + \alpha_s - \alpha_d} + \sigma_d \sigma_s \rho_{ds}.$$

The joint density of all three variables is

$$f(D, S, P) = \frac{1}{\sqrt{(2\pi)^3 \det \Sigma}} \exp \left(-\frac{1}{2} \begin{bmatrix} D - \mu_D & S - \mu_S & P - \mu_P \end{bmatrix} \Sigma^{-1} \begin{bmatrix} D - \mu_D \\ S - \mu_S \\ P - \mu_P \end{bmatrix} \right),$$

where

$$\Sigma = \begin{bmatrix} \sigma_D^2 & \sigma_D \sigma_S \rho_{DS} & \sigma_D \sigma_P \rho_{DP} \\ \sigma_D \sigma_S \rho_{DS} & \sigma_S^2 & \sigma_P \sigma_S \rho_{SP} \\ \sigma_D \sigma_P \rho_{DP} & \sigma_P \sigma_S \rho_{SP} & \sigma_P^2 \end{bmatrix}.$$

Following Quandt (1978) the conditional, on excess supply, density of Q and P is given by

$$f(Q, P \mid S > Q) = \frac{f_D(Q, P)}{\pi_S}$$

where

$$f_D(Q, P) = \int_Q^{\infty} f(Q, S, P) \, \mathrm{d}S.$$

The density f_D is obtained with a calculation that is similar to that of the proof of proposition V.2; namely by completing the square and changing variables. Introducing expressions for the determinants of the covariance matrix Σ and its minors simplifies the calculations of the model (SA). Let

$$\zeta^2 = \frac{\det(\Sigma)}{\sigma_D^2 \sigma_P^2 \sigma_S^2} = 1 - \rho_{DP}^2 - \rho_{DS}^2 - \rho_{SP}^2 + 2\rho_{DP}\rho_{DS}\rho_{SP},$$
$$\zeta_{ii} = \frac{\det(\Sigma_{i,i})}{\sigma_j^2 \sigma_k^2} = 1 - \rho_{jk}^2,$$

and

$$\zeta_{ij} = \frac{\det(\Sigma_{i,j})}{\sigma_i \sigma_j \sigma_k^2} = \rho_{ij} - \rho_{ik}\rho_{jk}$$

for i, j, k distinct indices in $\{D, S, P\}$. Furthermore, define

$$\begin{aligned} \varepsilon &= \begin{bmatrix} D - \mu_D & S - \mu_S & P - \mu_P \end{bmatrix} \Sigma^{-1} \begin{bmatrix} D - \mu_D & S - \mu_S & P - \mu_P \end{bmatrix}' \\ &= \frac{1}{\zeta^2} \left(h_D^2 \zeta_{DD} - 2h_D h_P \zeta_{DP} + h_P^2 \zeta_{PP} + \left(\frac{S - \mu_S}{\sigma_S} \right)^2 \zeta_{SS} - 2 \left(\frac{S - \mu_S}{\sigma_S} \right) (h_D \zeta_{DS} + h_P \zeta_{SP}) \right), \end{aligned}$$

and

$$\omega = \left(\frac{S - \mu_S}{\sigma_S} \right) \sqrt{\zeta_{SS}} - \frac{h_D \zeta_{DS} + h_P \zeta_{SP}}{\sqrt{\zeta_{SS}}}.$$

Using the above notation, one calculates

$$\begin{aligned} f_D(Q,P) &= \int_Q^\infty \frac{1}{\sqrt{(2\pi)^3\sigma_D^2\sigma_S^2\sigma_P^2\zeta^2}} \exp\left(-\frac{\varepsilon}{2\zeta^2}\right) \mathrm{d}S \\ &= \int_Q^\infty \frac{1}{\sqrt{(2\pi)^3\sigma_D^2\sigma_S^2\sigma_P^2\zeta^2}} \exp\left(-\frac{1}{2}\left(\frac{h_D^2 - 2h_D h_P \rho_{DP} + h_P^2}{\zeta_{SS}} + \left(\frac{\omega}{\zeta}\right)^2\right)\right) \mathrm{d}S \\ &= \frac{\exp\left(-\frac{1}{2}\frac{h_D^2 - 2h_D h_P \rho_{DP} + h_P^2}{\zeta_{SS}}\right)}{\sqrt{(2\pi)^2\sigma_D^2\sigma_P^2\zeta_{SS}}} \int_Q^\infty \frac{\sqrt{\zeta_{SS}}}{\sqrt{2\pi\sigma_S^2\zeta^2}} \exp\left(-\frac{1}{2}\left(\frac{\omega}{\zeta}\right)^2\right) \mathrm{d}S \\ &= \frac{\exp\left(-\frac{1}{2}\frac{h_D^2 - 2h_D h_P \rho_{DP} + h_P^2}{\zeta_{SS}}\right)}{\sqrt{(2\pi)^2\sigma_D^2\sigma_P^2\zeta_{SS}}} \left(1 - \Phi\left(\frac{h_S\zeta_{SS} - h_D\zeta_{DS} + h_P\zeta_{SP}}{\zeta\sqrt{\zeta_{SS}}}\right)\right). \end{aligned}$$

Analogously one calculates $f_S(Q,P)$ for the states of excess demand. The likelihood is then given by

$$L = \prod_{nt} \left(f_D(Q_{nt}, P_{nt}) + f_S(Q_{nt}, P_{nt})\right).$$

□

B. Simulation and Benchmarking

This appendix details all the simulation exercises of the article. It describes the Monte Carlo experiments of section III.ii and the benchmarking exercise of section V.iii. All the estimations and simulations of the article used the (Karapanagiotis, 2020, R package). All simulations begin with the same baseline specifications for demand and supply, which are given by

$$\begin{aligned} D_{nt} &= P_{nt}\alpha_d + X'_{d,nt}\beta_d + X'_{nt}\eta_d + u_d, \\ S_{nt} &= P_{nt}\alpha_s + X'_{s,nt}\beta_s + X'_{nt}\eta_s + u_s. \end{aligned}$$

The matrices $X_{d,nt}$ and $X_{s,nt}$ contain equation specific controls, while X contains common controls. In the equilibrium case, prices are not simulated. Instead, they are calculated so that the market clears, i.e.

$$P_{nt} = \frac{X'_{nt}(\eta_s - \eta_d) + X'_{s,nt}\beta_s - X'_{d,nt}\beta_d + u_s - u_d}{\alpha_d - \alpha_s}.$$

In the basic model case, prices are simulated as a common control, the demanded and supplied quantities are then calculated, and the observed quantity is determined by $Q_{nt} = \min\{D_{nt}, S_{nt}\}$. In the directional model, prices are also simulated as the rest of the controls and subsequently it is checked whether the condition $\Delta P_{nt} \geq 0 \implies D_{nt} \geq S_{nt}$ is true. If not, the simulation of controls and prices is repeated until a sample draw that fulfills the condition is realized. In the deterministic adjustment model, an out of sample, initial price value is drawn and the remaining prices are then generated by the rule

$$P_{nt} = \frac{X'_{nt}(\eta_s - \eta_d) + X'_{s,nt}\beta_s - X'_{d,nt}\beta_d - \gamma P_{n,t-1} + u_s - u_d}{\alpha_d - \alpha_s - \gamma}.$$

For the stochastic adjustment model, the procedure is analogous to the one with deterministic price dynamics, but the price generation rule is now given by

$$P_{nt} = \frac{X'_{nt}(\eta_s - \eta_d) + X'_{s,nt}\beta_s - X'_{d,nt}\beta_d - \gamma P_{n,t-1} + \gamma X'_{p,nt}\beta_p + u_s - u_d + \gamma u_p}{\alpha_d - \alpha_s - \gamma}.$$

i. Simulation

The Monte Carlo experiments are using a three dimensional vector for $X_{d,nt}$ and two dimensional vectors for $X_{s,nt}$ and X_{nt}. Both the coefficients and the simulated data are drawn from normal distributions. The distribution parameters for the coefficients are chosen for each model so that the simulation gives balanced data. For example, simulations that give unbalanced datasets that contain mostly excess demand, or excess supply states are avoided. The results of the Monte Carlo experiment vary when the drawn coefficients lead to extreme simulated data. Nevertheless, letting the coefficients being stochastic, instead of assigning them with constant values, increases the validity of the experiment's results.

	α_d	β_{0d}	β_{1d}	β_{2d}	η_{0d}	η_{1d}	α_s	β_{0s}	β_{1s}	η_{0s}	η_{1s}	γ	β_{0p}	β_{1p}	σ_d^2	σ_s^2	σ_p^2	ρ_{ds}^2	ρ_{dp}^2	ρ_{sp}^2
EQ	-2.7	4.90	2.10	-0.70	3.50	6.25	2.80	3.20	0.65	1.15	4.20				1.00	1.00		0.50		
BM	-0.9	8.9	0.03	-0.02	-0.03	-0.01	0.9	4.2	0.03	0.05	0.02				0.9	1.2		0.0		
DM	-0.4	3.3	0.03	0.02	0.03	0.01	0.0	0.0	3.03	0.05	0.02				1.0	0.2		-0.4		
DA	-2.7	4.9	2.1	-0.7	3.5	6.25	2.8	3.2	0.65	0.15	4.2	1.2			1.00	1.00		0.0		
SA	-0.01	9.8	0.03	-0.02	0.06	-0.01	0.01	6.1	0.09	-0.05	0.02	2.4	4.3	0.8	0.1	0.1	0.1	0.0	0.0	0.0

Table 4: Mean values of Monte Carlo parameters.

Table 4 contains the means of the normal distributions of the simulated coefficients. The variances of all the distributions are set equal to 0.1. The coefficients are redrawn for each simulation round and based on them, the data are produced according to the examined model. The simulated demand, supply, and price equation shocks are drawn from jointly normal distributions. The controls, and whenever appropriate the prices, of the models are drawn from normal distributions with mean 2.5 and variance 0.5.

Once the data are simulated, it is ensured that all the models can be estimated. When data are generated from models (EM), (BM), (DM), and (DA) they can be used to interchangeably estimate any other among these models, as the generated data have the same number of variables in all cases. In the case of model (SA) however, at least one additional control in the price equation is needed in order for the model to be estimated. Therefore, an additional dummy variable is simulated for the estimation of model (SA) when the data generated by the other models. The reverse direction is simpler. When data are generated based on model (SA), the estimation of the remaining models ignores the additional variables.

For resulting simulated data for which all the models can be estimated, the selection criterion of section III.i is applied using all five models. If the selected model coincides with the model according to which the data were generated, then the simulation round is perceived as being successful. For each model, the simulation is repeated 1000 times and the results are aggregated in fig. 1.

ii. Benchmarking

The benchmarking simulations that are used for collecting measurements of executions times are based on a similar procedure. Model coefficients are randomly drawn, but in this case from a uniform instead of a normal distribution. For measuring the execution time, the simulated data do not need to be suitable for all five models simultaneously, which allows more flexibility when drawing the coefficients. The drawn coefficients are only used in the context of a single model and the additional variability is important in execution time measurements even for improbable samples.

	α_d	β_{0d}	β_{1d}	β_{2d}	η_{0d}	η_{1d}	α_s	β_{0s}	β_{1s}	η_{0s}	η_{1s}	γ	β_{0p}	β_{1p}	σ_d^2	σ_s^2	σ_p^2	ρ_{ds}^2	ρ_{dp}^2	ρ_{sp}^2
lb	-2.9	3.9	1.1	-1.7	2.5	5.25	1.8	0.2	-0.35	0.15	3.2	0.5	0.2	0.7	0.8	0.8	0.2	0.4	0.0	0.0
ub	-0.9	5.9	3.1	0.3	4.5	7.25	3.8	2.2	1.65	2.15	5.2	0.8	0.6	0.9	1.2	1.2	0.4	0.6	0.0	0.0

Table 5: Upper and lower bounds of benchmarking parameters.

The coefficients of all models are drawn using the same uniform distributions, the details of which are shown in table 5. Prices, when appropriate, and other controls are simulated, as in the Monte Carlo case, using a normal distribution with mean 2.5 and variance 0.5.

Each set of simulated data is used to estimate the model using two different optimization algorithms. The first algorithm is BFGS and uses the expressions of the derivatives which are calculated in this article. The second algorithm is Nelder-Mead and does not use the calculated expressions. The execution times are measured for both methods. Simulation and model setup operations are not timed. The processors are warmed up using, 10 untimed, estimations that are performed at the beginning of the procedure.

Execution statistics are gathered for two simulation procedures. The first one keeps the specifications of the models constant and increases the number of observations. All the simulation rounds use 5 time-points and let the number of observations increase geometrically in the entity dimension according to 2^{n+1}, for $n = 1, \ldots, 14$. The second simulation procedure keeps the number of observations equal to 40960, which corresponds to $n = 12$ in the previous rule, and adds extra variables in the specifications of both the demand and supply equations. The coefficients of each extra variable are uniformly distributed in the interval $[-0.3, 0.3]$, while the additional simulated controls follow a normal distribution with mean 2.5 and variance 0.5.

C. Additional Empirical Analysis

i. The matching process

In the IRI dataset, stub refers to the hierarchical assignment of Unique Product Codes (henceforth UPCs) to brands, to vendors, to types, and to categories of products. The stub, however, cannot be used as an identifier for the complete IRI dataset because it is not uniquely assigned to products. Across different years, the same stub can be assigned to different products that potentially belong to different parent corporations or categories. There are three structural breaks, which occur in the years 2007, 2008, and 2012, in which the remapping of stubs occurs in the IRI dataset. For each period defined by two sequential breaks, the UPCs are uniquely assigned to the same product. Across periods, however, this is not the case.

The matching of product records across periods is performed by a semi-automatic supervised procedure of three steps. The first step is simple; all observations of two successive periods with exact matches of stub specifications are linked. The two subsequent steps start by cross-joining the products from two successive periods that were not linked in the first step. The cross join associates each unmatched product from the current period with every unmatched product of the previous period. For example, if the current period contains 10 unmatched products and the previous period 5 unmatched products, then the cross join produces 50 records. For each of these records, a series of distance measures are calculated. The first two measures, namely the Levenshtein and the trigram similarity measures, are standard. In particular, the implementations that are found in the `fuzzystrmatch` extension of `PostgeSQL` database system were used. The second and third steps use two additional measures that were implemented specifically to link the product records.

Both the additional measures are based on tokenizing a field of a dataset into atoms. An atom is considered to be an alphanumeric sequence without any white-space character. The algorithm that constructs the measures then uses the number of elements in the symmetric difference of the atoms of two records to assess whether two records coincide. In contrast to the Levenshtein and trigram similarity measures, cases in which two fields differ only by white spaces are recognized to be identical. Moreover, the symmetric difference of the atoms is empty in cases in which the two fields that are compared contain exactly the same words, but in a different order, or in cases of duplicated atoms in one of the fields.

In the second step of the matching process, the above algorithm is applied to stubs and to product attributes. For each pair of unmatched records, the symmetric difference of their corresponding sets of atoms of the stubs is calculated and all pairs for which the resulting set is empty are linked. The symmetric difference of atoms is also calculated for the product attributes that are reported in the IRI dataset. In this case, however, there is no automatic linking process that is followed because the mere matching of product attributes does not uniquely identify a single product.

While the two previous steps were automatic, the third step of the process is, instead, performed in a supervised manner. For each product in the current period that is remaining in the cross join of the unmatched products, the top three potential matches from the previous period's products are selected. These are chosen by minimizing the number of atoms in the symmetric difference of the stub specifications, maximizing the trigram similarity measure, and minimizing the Levenshtein measure. Additionally, the symmetric difference of the product attributes is reported, but it is not considered in the selection of the top potential matches because of its non-identifying nature.

ii. Elasticities and marginal effects

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
DADemand	-1.02 (0.00)	-0.96 (0.00)	-0.94 (0.00)	-0.91 (0.00)	-0.87 (0.00)	-0.89 (0.00)	-0.83 (0.00)	-1.66 (0.00)	-1.04 (NA)	-0.74 (NA)	-0.42 (NA)
DASupply	-0.32 (0.00)	-0.75 (0.00)	-0.47 (0.00)	-0.00 (1.00)	0.56 (0.00)	1.25 (0.00)	0.21 (0.35)	8.37 (0.06)	-1.01 (NA)	-0.71 (NA)	-0.37 (NA)
EQDemand	-1.01 (0.00)	-1.01 (0.00)	-0.93 (0.00)	-0.94 (0.00)	-0.84 (0.00)	-0.76 (0.00)	-0.89 (0.00)	-0.95 (0.00)	-0.92 (0.00)	-0.87 (0.00)	-0.92 (0.00)
EQSupply	-1.10 (0.00)	-1.22 (0.00)	-1.03 (0.00)	-0.17 (0.27)	1.55 (0.00)	1.86 (0.00)	1.23 (0.12)	-0.49 (0.09)	-0.47 (0.00)	-0.26 (0.62)	-0.46 (0.40)

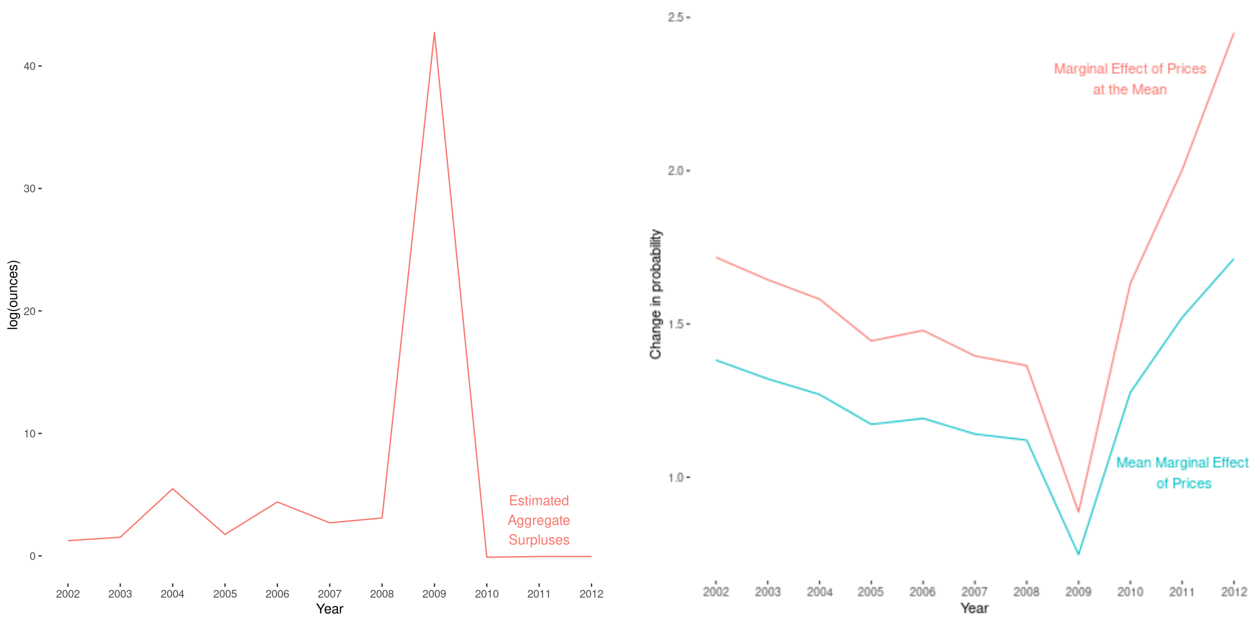
Table 6: Estimated prices elasticites

Table 6 summarizes the estimated demand and supply price elasticities for the yearly estimates of both the equilibrium and the deterministic adjustment model. The parentheses contain the P-values of the estimates. The decreased number of observations towards the end of the sample's period is reflected in higher P-values of the supply estimates for 2011 and 2012. The standard errors of the deterministic adjustment model for the last three years cannot be calculated because the Hessian is not numerically invertible. The obtained estimates for the years that surround 2008 are in alignment with economic intuition and have very low, in most cases almost zero, P-values. The estimated elasticities of the selected equilibrium model have the expected signs in both 2007 and 2008. The same is true for the signs of the estimated elasticities of the deterministic adjustment model in 2009, when the selection methodology switches from the equilibrium to the disequilibrium model.

Figure 5 depicts in parallel the yearly evolution of the market surpluses and the effects of prices on the surplus probabilities as they are estimated by the deterministic adjustment model. As shown in fig. 5a, the estimation of the deterministic adjustment model indicates that there were small surpluses in the deodorant market until 2009. In 2009, the after effects of the financial crisis become evident in the point estimate of the shortages in the form of a spiking surplus, which is the result of the combination of sustained reduced demand and increased market competition during the last two years in the deodorant market. After 2009, the market reverts to a state that is close to the previous status and exhibits small, almost zero-valued, shortages. The examination of the marginal effects of prices in fig. 5b indicates that the source of the 2009 spike of fig. 5a was exogenous to the price mechanism of the deodorant market. The marginal effects of prices on the probabilities of observing the estimated market surplus are positive for all years, as is to be expected, due to the signs of the price elasticities. The year in which prices are affecting the least the observed state of the market is 2009.

Figure 5: Deterministic adjustment model's yearly estimates

(a) Estimated surpluses. (b) Marginal effects on surplus probabilities.



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