Central Bank Digital Currency: When Price and Bank Stability Collide

Linda Schilling - École Polytechnique, CREST, and CEPR
Jesús Fernández-Villaverde - University of Pennsylvania, NBER, and CEPR
Harald Uhlig - University of Chicago, CEPR, and NBER*

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Abstract

With the introduction of a central bank digital currency, or CBDC, a central bank is forced to confront a classic issue in banking: the tension between providing a liquid means of payment and the desirable level of maturity transformation. We analyze this issue in a nominal version of a Diamond and Dybvig (1983) model, where the central bank additionally has a price stability objective. While the central bank can always deliver on its nominal obligations, runs can nonetheless occur, manifesting themselves either as an excessive real asset liquidation or as a failure to maintain price stability. Thus, we demonstrate an impossibility result that we call the CBDC trilemma: of the three goals of efficiency, financial stability (i.e., absence of runs), and price stability, the central bank can achieve at most two.

Keywords: Central bank digital currency, monetary policy, bank runs, financial intermediation, inflation targeting, CBDC trilemma.

JEL classifications: E58, G21.

*linda.schilling@polytechnique.edu, jesusfv@econ.upenn.edu, and huhlig@uchicago.edu. The contribution of Linda Schilling has been prepared under the Lamfalussy fellowship program sponsored by the ECB and was originally named "Central bank digital currency and the reorganization of the banking system." We thank Agnese Leonello and an anonymous ECB referee for insightful comments, as well as participants at numerous seminars. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the ECB.

1 Introduction

Many central banks and policymaking institutions, such as the Bank of Canada, the Bank of England, the BIS, the ECB, the IMF, the People's Bank of China, the Sveriges Riksbank, and the G30, are openly debating the implementation of a central bank digital currency, or CBDC (Barrdear and Kumhof, 2016; Bech and Garratt, 2017; Chapman et al., 2017; Lagarde, 2018; Ingves, 2018; Kahn et al., 2019; Davoodalhosseini et al., 2020; Auer and Böhme, 2020; Auer et al., 2020; Group of 30, 2020). With the introduction of a CBDC, the central bank will issue an electronic means of payment to households in the form of short-term central bank liabilities. A widespread adoption of CBDCs has the potential to be a watershed for the monetary and financial systems of advanced economies.

We argue that a CBDC will lead the central bank to lending long-term and borrowing short-term. With a CBDC, households will have access to an electronic means of payment and, thus, an attractive alternative to traditional deposit accounts, possibly triggering disintermediation in the retail banking sector.[1] This phenomenon raises the question of what the central bank should do regarding its asset side. If the central bank sticks to traditional customs and chooses to only hold liquid and safe securities such as Treasuries, this will dry up the resources for loans and projects traditionally funded via retail bank deposit funding, creating inefficiencies. Instead, if the central bank chooses to funnel the obtained CBDC funds back to the financial sector and long-term borrowers, the central bank enters the business of financial intermediation and maturity transformation.[2] In that role, the central bank will confront well-known issues, namely, preventing runs while enabling socially optimal risk-sharing allocations, in addition to its price stability mandate. As the key result of this paper, we demonstrate the central bank's inability to attain these three objectives simultaneously.

We show an impossibility result, which we term the CBDC trilemma (see Figure 1). Of the three goals of attaining the socially optimal allocation, the absence of runs, and price stability, the central bank can achieve at most two. As the main part of the CBDC trilemma, we prove that the central bank can always implement the socially optimal allocation in dominant strategies while deterring central bank runs, but only when credibly threatening high inflation, i.e., when giving up the price stability objective.

[1] As Fernández-Villaverde et al. (2020) show, a CBDC offered by the central bank may be such an attractive alternative to private bank deposits that the central bank becomes a deposit monopolist, further consolidating its role as a financial intermediator.

[2] Central banks have engaged in large-scale, long-term lending to the economy ("quantitative easing") since the financial crises in 2008-2009. The introduction of a CBDC will considerably enlarge these activities.

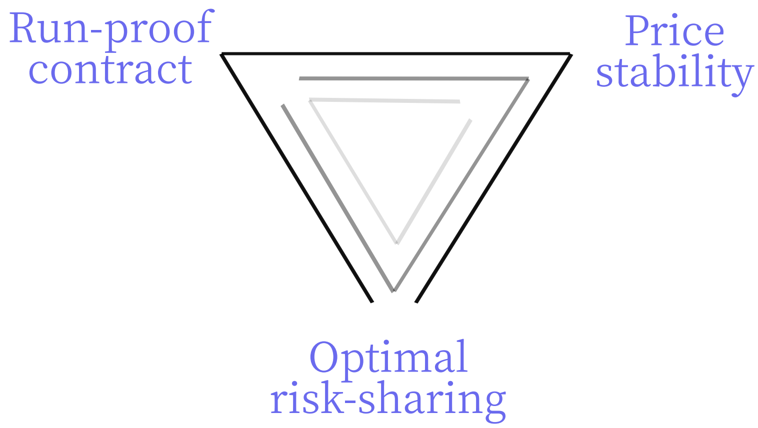


Figure 1: CBDC Trilemma: For the central bank, it is impossible to attain all three objectives at a time. When one objective is picked, at least one other objective has to be sacrificed.

To build our argument, we build on the classic model by Diamond and Dybvig (1983), which emphasizes a bank's role in maturity transformation. In the original model by Diamond and Dybvig (1983), a bank pools resources and finances real long-term projects with demand deposits that can be withdrawn at a short time horizon to meet consumption needs. In our model, households hold nominal CBDC rather than real demand deposits. The households obtain their CBDC balances in the initial period by selling their initial endowment of goods to the central bank, which invests these goods using available technologies. In turn, the central bank decides on the amount of long-term projects to be liquidated early after observing the fraction of agents seeking to spend their CBDC balances.

Importantly, in our economy, there can no longer be a "withdrawal" of deposits since the CBDC is the only asset and medium of exchange. Instead, "spending" the CBDC balances on real goods replaces deposit withdrawals.[3] As in Diamond and Dybvig (1983), some agents are subject to idiosyncratic consumption shocks, i.e., they are "impatient." These agents instantly have to spend CBDC balances on consumption, while "patient" agents can spend strategically either early or late, depending on which action yields the higher real allocation. More concretely, "patient" agents can spend early and store the goods purchased with the CBDC to consume later.

[3]Think about the "electronic dollars" that many universities and colleges issue to faculty and students in their ID cards for purchases on campus. One can spend those "electronic dollars" in different campus locations, such as vending machines and food courts, but one cannot "withdraw" the "electronic dollars" or transform them into other assets. In our model, real goods can only be traded against the CBDC, implicitly setting a form of a cash-in-advance constraint in the tradition of Svensson (1985) and Lucas and Stokey (1987). Section 9 discusses our results in the presence of a fully fledged financial system with other assets and argues that our main result remains unchanged.

What does a run on the central bank mean in our context? The central bank is the issuer of currency and can, therefore, always deliver on its nominal obligations. Thus, rationing or limited service of CBDC deposits does not arise. Instead, a run on the central bank will manifest itself as a spending spree, where more than just the fraction of impatient agents are spending their CBDC balances at a short horizon. Since the aggregate spending decision versus the available quantity of goods for purchase impacts the price level via market clearing, a central bank run can also be interpreted as a run on the price level. The price level will flexibly adjust to the level of aggregate spending and varies with the goods supply.[4]

Why would a central bank run, i.e., excessive spending, occur? In the classic bank run literature, runs happen due to high volume short-term withdrawals that enforce the large-scale liquidation of illiquid assets. But this is not the relevant mechanism here because the central bank's liabilities are nominal, while investment is real. Hence, the central bank's asset liquidation policy is independent of the agent's spending decision. Instead, the extent of asset liquidation determines the supply of real goods and, via market clearing, the price level and the purchasing power of the CBDC. If the central bank liquidates more assets today, tomorrow's goods supply declines. As agents anticipate a low supply of goods tomorrow, they understand that their CBDC balances may have a higher purchasing power today, and early spending is optimal. Since the central bank is strategic too, it anticipates the behavior of its CBDC customers. By choosing its liquidation policy wisely, the central bank can steer shoppers' incentives to spend early versus late to deter a central bank run.

To do so, the central bank needs to credibly commit to a liquidation policy that lowers the goods supply whenever many agents are spending early ("run-deterring policy"), rendering early spending *ex-post* suboptimal. While a run-deterring policy has the flavor of "suspension of convertibility," the central bank is not preventing agents from spending their account balances. Rather, the mechanism works through lowering the supply of goods by limited liquidation. A credible run-deterring policy has the drawback that the central bank has to be ready to lower the supply of goods whenever high volume spending occurs (off equilibrium), then triggering a high, spending-contingent price level and inflation.[5]

[4]Notice, more pointedly, that where there is a run on the price level, issuing additional amounts of CBDC not only does not help, but it makes the inflation worse! The ability of a central bank to issue as much CBDC as desired is a useless tool against runs on its liabilities. See Section 6 for more details.

[5]Historically, governments have limited spending through alternative policies, from calling loans to the private sector, increasing the policy interest rate to force savings, or rationing in war times, which are just forms of hidden inflation. The details of how a central bank can achieve a limited liquidation of projects are somewhat irrelevant for our main argument, but the historical evidence suggests governments have plenty of levers to do so.

The mechanism also works in the other direction. If many agents spend early, the central bank may want to provide more goods to stabilize the price level. But if the central bank willingly provides too many goods in the short run, implying a lower goods supply in the future, all agents will spend their CBDC balances early. That is, anticipated inflation tomorrow causes a central bank run and high inflation today.[6]

The obvious alternative to strategic asset liquidation is to consider the classic central bank intervention of changing the money supply in response to a run. Section 6 investigates these possibilities. We show that the central bank would need to reduce the money supply or suspend portions of the money supply as a means of payment. We argue that these are policy responses likely to wreak havoc with the trust that households place in the monetary system. Moreover, since the run-deterrence mechanism works via the aggregate supply of real goods, nominal fiscal backing, changes in the money supply, and higher nominal interest rates do not prevent agents from running on the central bank.

The assumption that real production is exogenous, depending only on the extent of liquidation, is essential for our mechanism. There is no import of goods, and monetary spending does not translate into increased real production. There are no explicit nominal rigidities. But in the second part of the trilemma, we impose exogenous price stability, which can be interpreted as an extreme form of price stickiness. There, we show that price stability imposes strong restrictions on the central bank's liquidation policy, such that the social optimum may no longer be reached, or central bank runs may reoccur.

One could argue that the introduction of a CBDC is still so far in the future that the trilemma we highlight is not empirically relevant. However, Section 9 shows that the tension among incompatible goals already exists in the current financial system, though indirectly and hidden by additional layers of economic agents. Section 9 features firms that operate long-term real production technologies, households that buy their goods from firms, private banks that finance loans to firms, and a central bank that purchases bonds from private banks to create reserves, which banks lend out, in turn, to the real economy. In such a setting, the central bank can enforce the liquidation of real production processes by not rolling over bonds purchased from banks, which in turn do not prolong loans to producing firms. Our assumption in the baseline model that the central bank engages in maturity transformation,

[6]It may be tempting to think of the runs on toilet paper and spaghetti, as happened all over the world during the start of the COVID-19 pandemic in 2020. The production technology here, however, yields risk-free real interest. Hence, the comparison to a potato field is more suitable, where potatoes are small in the short run but grow large in the long run. Harvesting small potatoes today means giving up large potatoes tomorrow.

"takes over" the production side of the economy, and liquidates real long-term projects is, therefore, a simplification for analytic convenience, not the reason behind our results.

Our results extend to a setting where the central bank directly competes with private retail banks for deposits. In Section 7, retail banks need to obtain CBDC from the central bank to service withdrawals. The central bank chooses the additional instruments of interest rates charged on bank loans as well as a market share tax in order to keep the system competitive. We show that, if the goods market is centralized, large-scale liquidation by the central bank can cause runs (excessive spending) on private banks and large-scale liquidation by the private banks can cause runs (excessive spending) on the central bank. Consequently, there is a strong coordination motive for the central bank and private banks. The central bank and private banks can deter runs by applying jointly a run-deterring liquidation policy. If coordination is not possible (e.g., by regulating private banks), the central bank can still deter runs if it controls a sufficiently large share of the deposit market.

Section 10.2 extends our results to the case of hidden competition between the central bank and private banks in the form of a synthetic CBDC. If retail banks provide the "front end" for CBDC transactions, customers may not need to hold traditional deposit accounts on top of that. The potential decline of traditional deposit accounts raises the issue of disintermediation and further strengthens the central bank's role as a financial intermediator. This decline can be avoided if the central bank engages in "pass through," funneling the funds deposited by households back to the retail banks, as Brunnermeier and Niepelt (2019) have argued. But as Section 9 outlines, the tension between the central bank's role as a financial intermediator and the objective of keeping prices stable remains with such pass-through.

Our results are interesting with regard to Diamond and Dybvig (1983), who taught us that financial intermediaries necessarily have to be prone to runs for implementing the socially optimal allocation. Moreover, if a run occurs in Diamond and Dybvig (1983), the social optimum is not attained. In our setting, the central bank always attains the social optimum since it additionally controls the price level, but only when sacrificing price stability. One may wonder how our result compares to Jacklin (1987). There, a financial intermediary can always attain the social allocation when offering equity shares to agents instead of demand deposits if trade in dividends is possible at an interim stage. Section 8 demonstrates that trade in dividends is a special case of a run-deterring policy. Moreover, we extend the result in Jacklin (1987) to a nominal dividend setting and discuss the differences from our nominal setting.

Related literature

Our paper contributes to several strands of the literature. The three papers closest to ours are Diamond and Dybvig (1983), Skeie (2008), and Allen et al. (2014). First, we contribute to the literature on financial intermediation and bank fragility. Building on the seminal Diamond and Dybvig (1983) model, we stress the central bank's role in liquidity transformation when issuing a CBDC that allows depositors to share idiosyncratic liquidity risk. Similar to Diamond and Dybvig (1983), we study the microincentives of depositors to spend from the bank. But unlike them, we employ nominal instead of real demand-deposit contracts, giving "the bank" an additional tool –the price level– to prevent runs.

Nominal demand-deposit contracts have previously been considered by Allen and Gale (1998), Diamond and Rajan (2006), Skeie (2008), Allen et al. (2014), Leiva and Mendizábal (2019), and Andolfatto et al. (2020), among others. Unlike in all these papers, in our framework, the central bank is a strategic player that observes spending and, as a response, determines the real goods supply to alter either the depositors' incentives to spend or the price level according to its objectives. Therefore, we can show that the central bank can always implement the efficient allocation in dominant strategies, and runs no longer occur. Since implementation in dominant strategies requires giving up price stability, we can also discuss the flip side of this result. We further differ from the literature above by considering a more stylized model, abstracting from private banks and firms. In our framework, the central bank takes over the activity of real investment, financial intermediation, and the management of the money supply. In Skeie (2008), large withdrawals of nominal deposits can lead to an increase in the price level, reducing the real allocation and deterring runs. In a similar model, Allen et al. (2014) show that optimal risk sharing can be achieved via nominal contracts, but their setting cannot exclude runs (in particular, compare their Section 4.4 to our Lemma 4.2). In their case, the price level reacts passively and cannot be fine-tuned to the agent's spending decisions. As we mentioned above, in both Skeie (2008) and Allen et al. (2014), the "real" side is arising from the interplay between workers and entrepreneurs (and their customers), leaving the nominal side to the banking system and the central bank. Andolfatto et al. (2020) incorporate Diamond-Dybvig financial intermediation into the new monetarist model of Lagos and Wright (2005). Di Tella and Kurlat (2021) study why banks are exposed to monetary policy. In our framework, we examine a drastically simplified model, dropping the financial intermediary sector, while these issues would arise in a richer setting.

Second, we contribute to a growing literature on the macroeconomic implications of introducing a CBDC. Berentsen (1998) is a pioneering analysis of the monetary policy implications of digital money. Chiu et al. (2019) discuss issues regarding the competition with and support of private banks. Keister and Sanches (2019) explore how the presence of a CBDC affects the liquidity premium on bank deposits and, through it, investment. Böser and Gersbach (2019a) gauge the implications of CBDCs for banking panics. Böser and Gersbach (2019b) show that the introduction of a CBDC transfers default risk to the central bank when a CBDC competes with private deposits. Fernández-Villaverde et al. (2020) demonstrate that competition for deposits between private banks and the central bank can lead to a deposit monopoly at the central bank when commercial banks cannot commit. Skeie (2019) analyzes inflation-driven digital currency runs in a nominal model where a private digital currency competes with a CBDC. In contrast to this strand of the literature, our analysis abstracts from competition between a CBDC and deposits at private banks, respectively a CBDC and private digital currency, by modeling the central bank as the monopolistic provider of demand deposits. Brunnermeier and Niepelt (2019) derive an equivalence result of allocations when introducing a CBDC if the central bank commits to redepositing CBDC funds in private banks. In comparison, we are more explicit about the micro incentives of agents to run on the central bank. Ferrari et al. (2020) discuss monetary policy transmission in a two-country DSGE model when introducing a CBDC. In our model, we focus on one country and do not feature firms, other financial agents, or assets. Instead, we focus on the depositors' microincentives to (not) run on the central bank.

Lastly, we contribute to the growing literature on cryptoeconomics that analyzes the price and exchange rate implications of crypto mining (Choi and Rocheteau, 2020; Garratt and van Oordt, 2019; Huberman et al., 2017; Prat and Walter, 2018), the micro and macroeconomics of blockchain (Amoussou-Guenou et al., 2019; Biais et al., 2019a,b; Ebrahimi et al., 2019; Leshno and Strack, 2020; Saleh, 2020) and token issuance (Cong et al., 2020; Li and Mann, 2020; Prat et al., 2019), and the macroeconomic implications of cryptocurrencies via currency competition (Benigno, 2019; Benigno et al., 2019; Fernández-Villaverde and Sanches, 2019; Schilling and Uhlig, 2019). Our paper abstracts from the existence of competing digital currencies and assumes full functionality of the CBDC account and ledger system.

2 The basic framework

Our framework builds on the classic Diamond and Dybvig (1983) model of banking. Time is discrete with three periods $t = 0, 1, 2$. There is a $[0, 1]$-continuum of agents, each endowed with 1 unit of a real consumption good in period $t = 0$. Agents are symmetric in the initial period, but can be of two types in period 1: patient and impatient. Impatient agents value consumption only in period 1. In contrast, patient agents value consumption in period $t = 2$. An agent is impatient with likelihood $\lambda \in (0, 1)$ and otherwise is patient. The agent's type is randomly drawn at the beginning of period 1 and types are private information. Since we have a continuum of agents, there is no aggregate uncertainty about the measure of patient and impatient types in the economy. Thus, λ also denotes the share of impatient agents. Preferences are represented by a strictly increasing, strictly concave, and continuously differentiable utility function over consumption $u(\cdot) \in \mathbb{R}$. We further assume a relative risk aversion, $-x \cdot u''(x)/u'(x) > 1$, for all consumption levels $x \geq 0$.

There exists a long-term production technology in the economy. For each unit of the good invested in $t = 0$, the technology yields either 1 unit at $t = 1$ or $R > 1$ units at $t = 2$. Additionally, there is a storage technology between periods 1 and 2, yielding 1 unit of the good in $t = 2$ for each unit invested in $t = 1$. All agents can access both technologies. Let $x_1 \geq 0$ denote the agent's real consumption when deciding to spend at $t = 1$, and let $x_2 \geq 0$ denote the agent's consumption when spending at $t = 2$.

2.1 Optimal risk sharing

Following Diamond and Dybvig (1983), we derive, first, the optimal allocation. The social planner collects and invests the aggregate endowment in the long technology. Given that all agents behave according to their type, the social planner maximizes *ex-ante* welfare

$$W = \lambda u(x_1) + (1 - \lambda)u(x_2) \tag{1}$$

by choosing (x_1, x_2), subject to the feasibility constraint $\lambda x_1 \leq 1$, and the resource constraint $(1 - \lambda)x_2 \leq R(1 - \lambda x_1)$. The interior first-order condition for this problem implies that the optimal allocation (x_1^*, x_2^*) satisfies:

$$u'(x_1^*) = Ru'(x_2^*). \tag{2}$$

Given our assumptions, the resource constraint binds in the optimum

$$R(1 - \lambda x_1^*) = (1 - \lambda)x_2^*. \tag{3}$$

This condition, together with equation (2), uniquely pins down (x_1^*, x_2^*) and delivers the familiar optimal deposit contract in Diamond and Dybvig (1983). Together with $R > 1$ and the concavity of $u(\cdot)$, equation (2) implies that the optimal consumption of patient agents is higher than the consumption of impatient ones: $x_1^* < x_2^*$.

Moreover, the depositors' relative risk-aversion exceeding unity and the resource constraint yield $x_1^* > 1$ and $x_2^* < R$.[7]

Diamond and Dybvig (1983) show that a demand-deposit contract can implement the efficient allocation. A key feature of their analysis is the use of a "real" demand deposit contract (i.e., a contract that promises to pay out goods in future periods). Due to a maturity mismatch between real long-term investment and real deposit liabilities, the Diamond and Dybvig (1983) environment, however, also features a bank run equilibrium under which the social optimum is not implemented. Our main contribution is to show that a nominal contract can lead to the implementation of the efficient allocation in dominant strategies. In other words, runs do not occur along the equilibrium path. The key mechanism is that the central bank can set the price level, thereby controlling the wedge between real long-term investment and nominal deposit liabilities. The implementation in dominant strategies comes at a price, requiring flexibility of the price level.

3 A nominal economy

Consider now an economy with a social planner that uses nominal contracts to implement the efficient allocation.

Nominal contracts. The planner offers contracts in a unit of account for which it is the sole issuer. Because central banks have a monopoly on currency, the planner in our analysis can be equated with the central bank or any other monetary authority with the ability to issue currency. In this paper, we refer to the unit of account as a central bank digital currency (CBDC) or digital euros. Agents who sign a contract with the central bank hand over their real goods endowment and receive CBDC balances in return. The most straightforward interpretation of our environment is to think of a CBDC as an account-based electronic currency in the sense of Barrdear and Kumhof (2016) and Bordo and Levin

[7] Following the proof in Diamond and Dybvig (1983),

$$Ru'(R) = u'(1) + \int_1^R \frac{\partial}{\partial x}(x \cdot u'(x))\, dx = u'(1) + \int_1^R (x \cdot u''(x) + u'(x))\, dx < u'(1) \tag{4}$$

by $-x \cdot u''(x)/u'(x) > 1$ for all x.

(2017), i.e., to think of a CBDC as being akin to a deposit account at the central bank. In Section 10, we show that the results of our paper largely carry over to a token-based system or hybrid systems. Agents can spend their CBDC balances by transferring them to other agents in exchange for goods. As with physical euros, we impose the constraint that agents cannot hold negative amounts of a CBDC.

Timing. At $t = 0$, the central bank creates an empty account, i.e., a zero-balance CBDC account, for each agent in the economy. Then, each agent agrees to invest her unit endowment of the good in exchange for $M > 0$ units of digital euros, credited to that agent's account. Next, the central bank invests all goods in the long-term technology.

In $t = 1$, agents learn their type and decide whether to spend their CBDC balances, M, or to roll them over. The central bank contract imposes the constraint that an agent either spends all of her balances or none at all. Because types are unobservable, the central bank cannot discriminate between patient and impatient agents to deny a patient agent access to her balances. Let $n \in [0, 1]$ denote the share and measure of agents who decide to spend in $t = 1$. The central bank observes n and then decides on the fraction $y = y(n)$ of the technology to liquidate, selling that amount in the goods market at the unit price P_1. Notice that through the resource constraint, early liquidation of the technology reduces the remaining investment and, hence, the supply of goods in $t = 2$. That is, there is a real payoff externality, and the central bank's liquidation choice in $t = 1$ determines the real supply of goods for both of the periods $t = 1$ and $t = 2$. Given n, the central bank also chooses a nominal interest rate $i = i(n)$ to be paid in period 2 on the remaining CBDC balances. Each digital euro held at the end of $t = 1$ turns into $1 + i(n)$ digital euros at the beginning of $t = 2$. Notice that $i(n) \geq -1$, given that agents cannot hold negative amounts of digital euros.

In $t = 2$, the remaining $1 - n$ depositors each have $(1 + i)M$ digital euros to spend on goods in the market at a price P_2. The remaining investment in the technology matures so that the central bank supplies $R(1 - y(n))$ units of goods in exchange for money balances. Figure 2 summarizes this timing.

Definition 1. *A central bank policy is a triple* $(M, y(\cdot), i(\cdot))$*, where* $y : [0, 1] \to [0, 1]$ *specifies the central bank's liquidation policy and* $i : [0, 1] \to [-1, \infty)$ *is the interest rate policy for every possible spending level* $n \in [0, 1]$.

Notice that M itself is not state-contingent. The logic here is that, traditionally, 1 dollar today is 1 dollar tomorrow: we maintain that tradition with that assumption here. In Section 6, we discuss an extension where we allow M to be state-contingent as well.

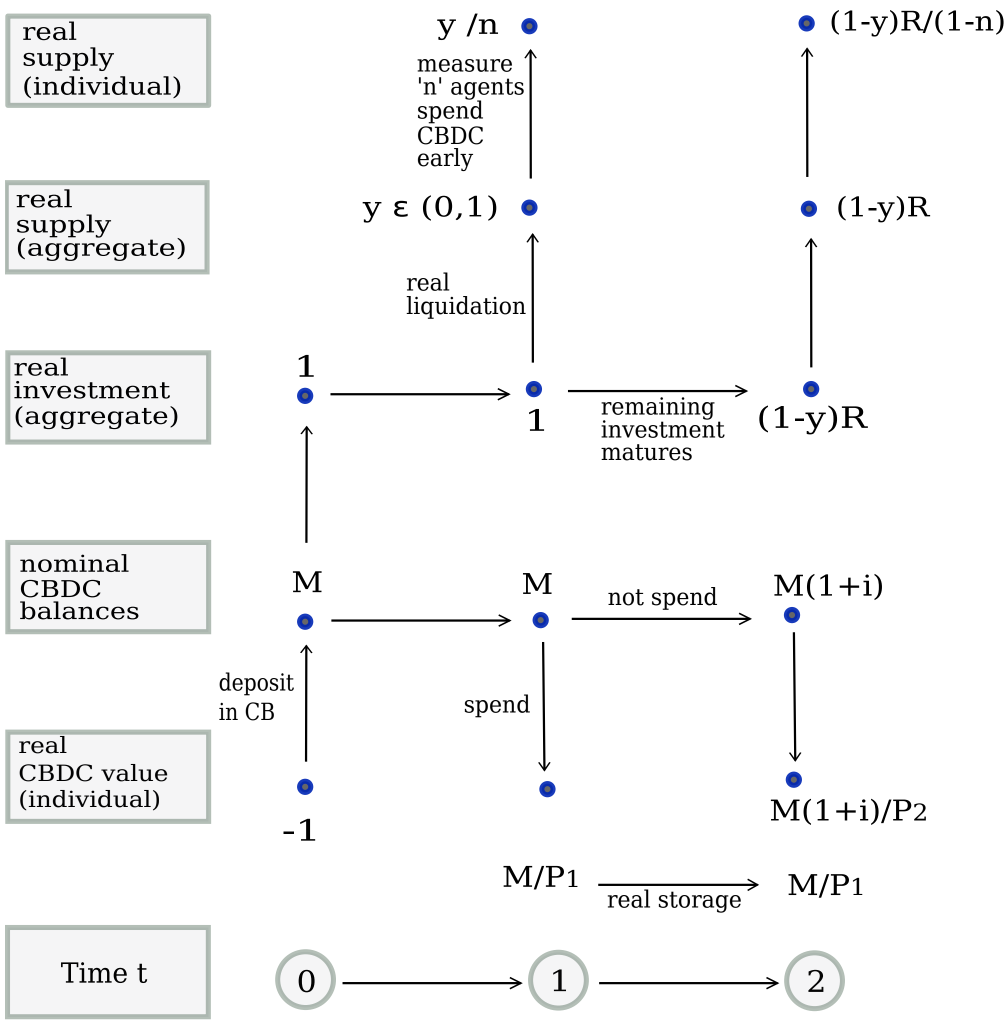


Figure 2: Nominal and real investment and contracts

Market clearing. In periods 1 and 2, agents spend their money balances for goods in a Walrasian market. The market-clearing conditions are:

$$\underbrace{nM}_{\substack{\text{nominal CBDC}\\ \text{supply in } t_1}} = P_1 \cdot \underbrace{y(n)}_{\substack{\text{real goods}\\ \text{supply in } t_1}} \tag{5}$$

$$\underbrace{(1-n)(1+i(n))M}_{\substack{\text{nominal CBDC}\\ \text{supply in } t_2}} = P_2 \underbrace{R(1-y(n))}_{\substack{\text{real goods}\\ \text{supply in } t_2}}, \tag{6}$$

which take the form of the quantity theory equation in each period. Given aggregate spending n in $t = 1$ and the central bank's policy, these conditions determine the price level, $P_1 = P_1(n)$

and $P_2 = P_2(n)$, in each period:

$$P_1(n) = \frac{nM}{y(n)} \tag{7}$$

$$P_2(n) = \frac{(1-n)(1+i(n))M}{R(1-y(n))}. \tag{8}$$

The price levels flexibly adjust in aggregate spending and the central bank's liquidation policy. The central bank chooses the initial money supply before learning the measure of spending in the intermediate period. The central bank, however, controls the supply of goods, which is chosen conditional on the measure of spending. As a result, the central bank can control the price level in period 1.[8] The nominal interest rate allows the central bank to control the price level in period 2 independently of the price level in period 1. Because the intermediary is the central bank with a monopoly on the unit of account in which contracts are denominated, the liquidation policy is flexible and becomes a monetary policy tool.

Implied real contract. For the strategic spending decision of patient agents, the individual real allocations that an agent can afford with her CBDC when spending early versus late are all that matters. The real value when spending CBDC balances in $t = 1$ equals

$$x_1 = \frac{M}{P_1}, \tag{9}$$

while the real value when spending balances in $t = 2$ equals

$$x_2 = \frac{(1+i(n))M}{P_2}. \tag{10}$$

Aggregate spending n and the liquidation policy $y(n)$ jointly determine the allocation of goods via the market-clearing conditions. The real allocations when spending in $t = 1$ versus $t = 2$ can therefore be rewritten as

$$x_1(n) = \frac{y(n)}{n} \tag{11}$$

$$x_2(n) = \frac{1-y(n)}{1-n}R. \tag{12}$$

[8] A private bank, in contrast, would need to take P_1, P_2 as given, which together with the observation n implies a unique liquidation $y(n, P_1)$. In a more detailed model, the central bank could determine the supply of goods by different instruments, such as calling loans to private banks or by moving the policy interest rate (as in New Keynesian models). The details of how that happens are not central to our argument.

Because all agents that spend CBDC in the same period have the same nominal income, the real goods supply $y(n)$ is equally distributed across all spending agents in period 1, and the supply $R(1-y(n))$ is equally allocated to all spending agents in period 2.[9]

To summarize: in $t=0$, the central bank announces and commits to a policy $(M, y(\cdot), i(\cdot))$, pinning down a spending-contingent real goods supply and an offer of a nominal contract $(M, M(1+i(\cdot)))$ in exchange for 1 unit of the good. All consumers accept the contract and the policy, meaning they have the option to spend either M digital euros in period 1 or $M(1+i(n))$ digital euros in period 2, for every possible level of aggregate spending $n \in [0,1]$. We discuss voluntary participation in contracts in Section 7.

In $t=1$, the aggregate spending level n is realized. Finally, the central bank's policy, together with the market-clearing conditions, results in the real consumption amounts $(x_1(n), x_2(n)) = (\frac{M}{P_1}, \frac{M(1+i(n))}{P_2}) = \left(\frac{y(n)}{n}, \frac{1-y(n)}{1-n}R\right)$. Notice that the central bank is fully committed to carry through with its policy (M, y, i), regardless of which n obtains and independently of the implications for the price levels (P_1, P_2). We, therefore, define

Definition 2. *A* **commitment equilibrium** *consists of a central bank policy* $(M, y(\cdot), i(\cdot))$, *aggregate spending behavior* $n \in [0,1]$ *and price levels* (P_1, P_2) *such that:*

(i) The spending decision of each individual consumer is optimal given aggregate spending decisions n, *the announced policy* $(M, y(\cdot), i(\cdot))$, *and price levels* (P_1, P_2).

(ii) Given aggregate spending n, *the central bank provides* $y(n)$ *goods and sets the nominal interest rate* $i(n)$.

(iii) Given $(n, y(n), M)$, *the price level* P_1 *clears the market in* $t=1$.
Given $(n, y(n), i(n), M)$, *the price level* P_2 *clears the market in* $t=2$.

As a particular consequence of this equilibrium concept, the price levels (P_1, P_2) flexibly adjust to the aggregate spending realization and the announced central bank policy.

4 Implementation of optimal risk sharing

In our model, the implementation of the optimal risk-sharing arrangement (x_1^*, x_2^*) is of particular interest to the central bank. Given the preferences and technology that we pos-

[9]These equations remain intuitive even if $y(n)=0$ or $y(n)=1$. Therefore, we assume that they continue to hold, despite one of the price levels being potentially ill-defined or infinite.

tulated above, only the real allocation of goods matters to the two types of agents. There is, consequently, no additional motive for the monetary authority to keep prices stable.

However, focusing only on real allocations is a narrow perspective. There is a vast literature arguing in favor of central banks keeping prices stable or setting a goal of low and stable inflation for reasons that are absent from our model. For instance, stable prices minimize the misallocations created by nominal rigidities as in Woodford (2003). Having to hold cash to accomplish transactions, such as in cash-in-advance or money-in-utility models, creates a whole range of distortions that can be minimized by deft management of the price level (think about the logic behind the Friedman rule). It certainly would therefore be reasonable to extend the social planner;s objective (1) with a term, reflecting a desire to keep prices stable. Rather than extending the model to include these considerations, which would complicate the analysis for an uncertain benefit, we shall proceed by discussing the tradeoffs between achieving the optimal real allocation of consumption and the implications of such an effort for the stability of prices.

Runs on the central bank. The first important property of the equilibrium defined above is that a nominal contract, *per se*, does not rule out the possibility of a run on the central bank. Since impatient agents only care for consumption in $t = 1$, every equilibrium will exhibit aggregate spending behavior of at least λ, implying $n \geq \lambda$.[10] Patient agents, on the other hand, spend their CBDC balances strategically in $t = 1$ or $t = 2$. They spend in $t = 1$ if they believe that the central bank policy implies $x_1 > x_2$. In that case, patient agents will use the storage technology to consume x_1 in period 2. Otherwise, patient agents will find it optimal to wait until the final period. We say,

Definition 3 (Central Bank Run)**.** *A run on the central bank occurs if patient agents also spend in* $t = 1$*,* $n > \lambda$*.*

In a bank run, the central bank is not running out of the item that it can produce freely (i.e., it is not running out of digital money). This feature will distinguish the run equilibrium here from the bank run equilibrium in Diamond and Dybvig (1983), in which a commercial bank prematurely liquidates all of its assets to satisfy the demand for withdrawals in period 1, therefore, ultimately running out of resources. If $n > \lambda$, the central bank is confronted with a run on deposits. As we will see, the real consequences of a run on the central bank with nominal contracts can be similar to its counterpart in the model with real contracts.

[10] When $y(n) = 0$, impatient agents are indifferent between spending and not spending. To break ties, we assume that they spend their CBDC balances in $t = 1$.

However, we shall demonstrate that the central bank's ability to avert a run is necessarily tied to its monopoly on currency and the implementation of a nominal contract. Importantly, by equations (11) and (12), a patient agent's optimal decision whether to run on the central bank, to spend or not, depends on the central bank's choices only through the liquidation policy $y(\cdot)$ and not via the nominal elements M and $i(n)$. By our equilibrium definition, the aggregate spending behavior n has to be consistent with optimal individual choices. These considerations imply the following lemma.

Lemma 4.1. *Given the central bank policy $(M, y(\cdot), i(\cdot))$,*

(i) The absence of a run, $n = \lambda$, is an equilibrium only if $x_1(\lambda) \leq x_2(\lambda)$.

(ii) A central bank run, $n = 1$, is an equilibrium if and only if $x_1(1) \geq x_2(1)$.

(iii) A partial run, $n \in (\lambda, 1)$, occurs in equilibrium if and only if patient agents are indifferent between either action, requiring $x_1(n) = x_2(n)$.

This lemma fully characterizes the range of equilibria, given the implied real allocation of a central bank policy. But how can policy attain the first-best allocation?

4.1 Implementation of optimal risk sharing via liquidation policy

By $(x_1^*, x_2^*) = \left(\frac{y^*}{\lambda}, \frac{R(1-y^*)}{1-\lambda}\right)$, the feasibility constraint $y \in [0,1]$, and the optimality conditions in Section 2.1, the implementation of optimal risk sharing requires a liquidation policy

$$y^*(\lambda) = x_1^* \lambda \in (\lambda, 1] \tag{13}$$

given that only impatient types spend. Similarly to Diamond and Dybvig (1983), the resource constraint $y \in [0,1]$ and $x_1^* > 1$ imply that optimal risk sharing is not feasible when all agents spend. The implied price level when n agents spend equals $P_1^*(n) = \frac{nM}{\lambda x_1^*}$. These results confirm our assertion at the start of this section that the social optimum is independent of price level stability. Combining the previous derivation with Lemma 4.1, we arrive at the following lemma.

Lemma 4.2. *The central bank policy $(M, y(\cdot), i(\cdot))$ implements optimal risk sharing (x_1^*, x_2^*) in dominant strategies if the central bank*

(i) sets $y(\lambda) = y^*$ for any $n \leq \lambda$.

(ii) sets a liquidation policy that implies $x_1(n) < x_2(n)$ for all $n > \lambda$.

The real allocation to agents and, thus, their incentives to spend or not depend on the central bank policy $(M, y(\cdot), i(\cdot))$ only through the liquidation policy $y(\cdot)$. Given that only impatient agents are spending (i.e., $n = \lambda$), then a policy choice with $y(\lambda) = y^*$ implements the social optimum. That is, there is a multiplicity of monetary policies that implement the first-best since the pair $(M, i(\cdot))$ is not uniquely pinned down. While the pair $(M, i(\cdot))$ does not affect depositors' incentives, it has an impact on prices via equations (7) and (8). Also, thanks to the existence of the storage technology, patient agents can –but do not have to– spend their CBDC balances at $t = 1$. Spending at $t = 2$ is dominant only if for every possible spending level n the real allocation at $t = 2$ exceeds the allocation at $t = 1$.

The central bank internalizes depositors' decision making. Since it observes aggregate spending behavior n *before* it liquidates any asset, the central bank is not committed to liquidating y^* if patient agents are also spending. Condition (ii) of this lemma corresponds to the classic incentive-compatibility constraint in the bank run literature: since expectations are rational, in $t = 1$, depositors correctly anticipate the central bank policy that follows spending behavior n. To deter patient agents from spending, the central bank can threaten to implement a liquidation policy $y(\cdot)$ that makes spending non-optimal *ex-post*, i.e., so that $x_1(n) < x_2(n)$ for $n \in (\lambda, 1]$. If the monetary authority can credibly threaten patient agents by setting such a liquidation policy, it deters them from spending ex-ante, and a central bank run does not occur in equilibrium. Therefore, in the unique equilibrium, only impatient agents spend, all patient agents roll over, and the social optimum is always attained.

The central bank implements "spending late" as the dominant equilibrium strategy for patient agents by fine-tuning the real goods supply via its liquidation policy, i.e., by making real asset liquidation spending-contingent.

Definition 4. *We call a liquidation policy $y(\cdot)$ "run-deterring" if it satisfies*

$$y^d(n) < \frac{nR}{1 + n(R-1)}, \qquad \text{for all } n \in (\lambda, 1]. \tag{14}$$

Such a liquidation policy implies that "roll over" is ex-post optimal $x_1(n) < x_2(n)$ even though patient agents are spending $n \in (\lambda, 1]$.

The implementation of a run-deterring policy is only possible because the contracts between the central bank and the agents are nominal. The liquidation of investments in the real technology is at the central bank's discretion, thereby controlling the real goods supply and,

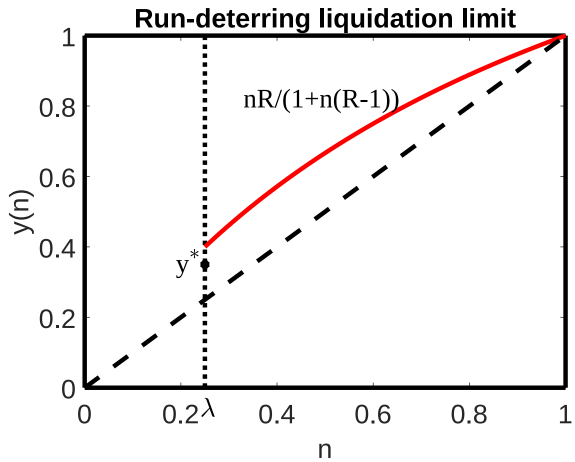


Figure 4: The upper bound of a "run-deterring" liquidation policy as a function of n is plotted in red. The bound starts at λ (for illustration purposes, here 0.25) because "impatient agents" will always spend. Note the social optimum, y^*, which is at λ in the n-axis and below the upper bound in the $y(n)$-axis and, to make interpretation easier, the 45-degree line in discontinuous segments.

for a given spending level, the real allocation in either time period. A spending-contingent liquidation policy implies a spending-contingent price level. In the case of real contracts between a private bank and depositors such as in Diamond and Dybvig (1983), in contrast, the real claims of the agents are fixed already in $t = 0$, thus pinning down a liquidation policy for every measure of aggregate spending n. In the case of high spending, rationing must occur. Similarly, in the case of nominal contracts between a private bank and depositors, the private bank has to take the price level as given, which then again pins down the liquidation policy. Alternatively, the price level adjusts via market clearing to high aggregate nominal spending (Skeie, 2008), while here it can serve as a strategic control variable.

As the main result of this paper,

Corollary 5 (Trilemma part I - No price stability)**.** *Every policy choice* $(M, y(\cdot), i(\cdot))$, $n \in [0,1]$ *with*

$$y(\lambda) = y^* \text{ and } y^d(n) < \frac{nR}{1+n(R-1)}, \qquad \text{for all } n \in (\lambda, 1], \tag{15}$$

deters central bank runs and implements the social optimum in dominant strategies. Such a deterence policy choice requires the interim price level $P_1(n)$ *to exceed the spending-dependent bound:*

$$P_1(n) > \frac{M}{R}(1 + n(R-1)), \qquad \text{for all } n \in (\lambda, 1]. \tag{16}$$

Under a credible liquidation policy (15) all agents have a dominant strategy to spend if and only if the agent is impatient; otherwise they wait. Thus, under rational behavior, runs do not occur, and by $y(\lambda) = y^*$ the social optimum always obtains. That is, a strategic real supply shock enforced by the central bank *causes* a demand shock to CBDC spending that deters runs.

The implementation, however, comes at a price. To attain feasibility of a run-deterring policy $y(\cdot)$, the central bank has to sacrifice price stability. By condition (16), the more agents spend, the larger the required price level threat to deter runs. The threat has to be credible to deter runs *ex-ante*. Agents have to believe that *ex-post* the central bank will give up price stability if realized spending behavior is excessive. Only then do runs and inflation not occur on the equilibrium path.

In Diamond and Dybvig (1983), we learned the dilemma that offering the optimal amount of risk sharing via demand-deposit contracts requires private banks to be prone to runs. Thus, a bad bank run equilibrium also exists. Our result brings this dilemma to the next level. If the bank is a central bank equipped with the power to set price levels and control the real goods supply, then optimal risk sharing can be implemented in dominant strategies such that a bank run never occurs, but only at the expense of price stability.

Observe that by the optimality conditions and the resource constraint, $y^* < \frac{\lambda R}{1+\lambda(R-1)}$ holds and that the upper bound for $y^d(n)$ is increasing in n. Therefore, the constant liquidation policy

$$y(n) \equiv y^* \tag{17}$$

implements optimal risk sharing in dominant strategies. However, there exist other liquidation policies that can accomplish the same result.

Besides its simplicity, policy (17) is equivalent to the run-proof dividend policy in Jacklin (1987), which implements the social allocation via equity shares that can be traded at the interim stage among patient and impatient depositors. Section 8 discusses the connection of this result to our model and argues that Jacklin (1987) features a special case of a run-deterring policy. The policy (17) also delivers the same result as the classic suspension-of-convertibility option, which is known to exclude bank runs in the Diamond-Dybvig world.

There is a subtle but essential difference, though, between suspension and our liquidation policy. Suspension of convertibility requires the bank to stop paying customers who arrive after the fraction λ of agents have withdrawn. By contrast, in our environment, there is no restriction on agents to spend their digital euros in period 1, and there is no suspension of accounts. Instead, it is the supply of goods offered for trade against those digital euros and the resulting change in the price level that generate the incentives for patient agents to prefer to wait. This reasoning also implies that, in our model, (nominal) deposit insurance will not deter agents from running on the central bank.

More concretely, low liquidation and thus low supply imply that the price level P_1 is pushed above an upper bound that is increasing in the aggregate spending.[11] The low liquidation policy, on the other hand, deters large spending *ex-ante*, such that the high price level (16) is a threat that is realized only off-equilibrium. But each time we have an off-equilibrium threat, we should worry about the possibility of time inconsistency. In comparison with the classic treatment of time inconsistency in Kydland and Prescott (1977), the concern here is not that the central bank will be tempted to inflate too much, but that it would be tempted to inflate too little. The central bank can avoid suboptimal allocations by committing to let inflation grow whenever necessary. A similar concern appears in models with a zero lower bound on nominal interest rates. Eggertsson and Woodford (2003) have shown that a central bank then wants to commit to keeping interest rates sufficiently low for sufficiently long, even after the economy is out of recession, to get the economy off the zero lower bound (see also Krugman, 1998, for an early version of this idea). But once the economy is away from the zero lower bound, there is an incentive to renege on the commitment to lower interest rates and avoid an increase in the price level.

In our model, we assume that the central bank fully commits such that the threat is credible. But what if the central bank is concerned with price stability and, therefore, refuses to induce a high price level?

5 The classic policy goal: Price level targeting

There are many possible reasons why central banks view the stabilization of price levels or, more generally, inflation rates as one of their prime objectives. The model here should be viewed as part of a larger macroeconomic environment, where the objective of price stability must be taken into account. That objective could arise out of concerns regarding nominal rigidities or legal mandates, and they may be socially optimal, requiring an appropriate modification of (1). The other way around, exogenous price stability can be interpreted as an extreme form of price stickiness. The task at hand, then, is to examine how price stability

[11] Our result resembles Theorem 4 in Allen and Gale (1998) and has a similar intuition. In Allen and Gale (1998), a central bank lends to a representative bank an interest-free line of credit to dilute the claims of the early consumers so that they bear a share of the low returns to the risky asset. In their environment, private bank runs are required to achieve the first-best risk allocation.

imposes constraints on central bank policy. In particular, we will document the existence of deep tensions between the three objectives of attaining the first-best outcome, deterring central bank runs, and maintaining price stability.

Addressing the time-inconsistency problem above requires the introduction of an objective function for the central bank. Given an objective function for the central bank, a time-consistent equilibrium is a commitment equilibrium such that the central bank policy $(M, y(n), i(n))$ and the resulting price levels $(P_1(n), P_2(n))$ maximize the central bank's objective function for every value $n \in [0, 1]$. A particular objective is that the central bank pursues price stability above everything else. We shall distinguish between two versions of the objective of price stability: full price stability and partial price stability. Let us start by analyzing the former.

5.1 Full price stability

Definition 6. *We call a central bank policy*

(i) P_1***-stable at level*** $\overline{P}$*, if it achieves* $P_1(n) \equiv \overline{P}$ *for the* ***price level target*** $\overline{P}$*, for all spending behavior* $n \in [\lambda, 1]$*.*

(ii) ***price-stable at level*** $\overline{P}$*, if it achieves* $P_1(n) = P_2(n) \equiv \overline{P}$ *for the* ***price level target*** $\overline{P}$*, for all spending behavior* $n \in [\lambda, 1]$*.*

In our definition, price stability here is treated as a mandate and commitment to the price level $\overline{P}$ even for off-equilibrium realizations of n. From the definition, price stability at some level $\overline{P}$ implies P_1 stability at $\overline{P}$. Hence, the second price stability criterion is stronger.

Definition 7. *Given a price goal* $\overline{P}$*, we call a commitment equilibrium a*

- P_1**-price-commitment equilibrium***, if the central bank policy is* P_1*-stable at level* $\overline{P}$
- **price-commitment equilibrium***, if the central bank policy is price-stable at level* $\overline{P}$

What constraints does the price stability objective impose on central bank policy?

Proposition 8 (Policy under Full Price Stability)**.** *A central bank policy is:*

(i) P_1*-stable at level* $\overline{P}$*, if and only if its liquidation policy satisfies:*

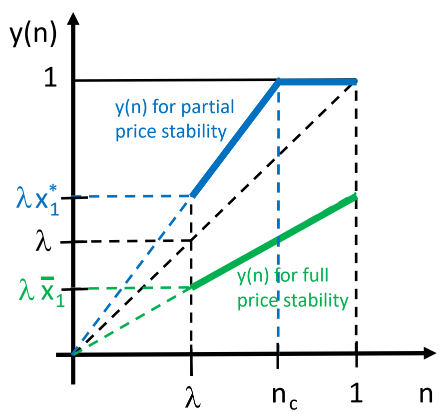
$$y(n) = \frac{M}{\overline{P}} n, \text{ for all } n \in [0, 1] \tag{18}$$

implying a real interim allocation:

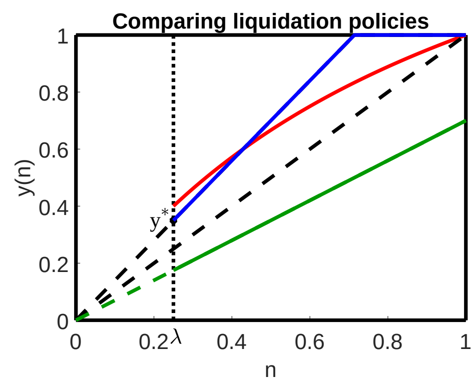
$$x_1(n) \equiv \overline{x}_1 = \frac{M}{\overline{P}} \leq 1. \tag{19}$$

(ii) A central bank policy is price-stable at level $\overline{P}$, if and only if its liquidation policy satisfies equation (18)*, its price level satisfies (19), and its interest policy satisfies:*

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1-n} R - 1. \tag{20}$$



(a) Partial vs. full price-stable liquidation policies



(b) Price-stable versus run-deterring policy

Figure 5: Fully price-stable policies are run-deterring (below the red line) but do not reach the social optimum y^*. Partially price stable policies (which are not fully price stable) are not run-deterring but can reach the social optimum. Run-deterring policies cannot be fully price stable while reaching the social optimum, since all fully price stable policies must be linear in the spending level n while having a slope below or equal to one.

A price-stable liquidation policy (18) requires asset liquidation in constant proportion to aggregate spending for all $n \in [0, 1]$; see the green line in Figure 5a, where we plot $y(n)$ for partial versus full price-stable liquidation policies. As a consequence, individual real consumption x_1 is constant regardless of aggregate spending behavior, and cuts below 1 since, due to the resource constraint, the central bank cannot liquidate more than the entire investment. Hence, a price-stable liquidation policy excludes rationing or all kinds of suspension policies. By equation (19) and again due to the resource constraint, for a given money supply M, only price levels $\bar{P} \geq M$ can be P_1- stable or price-stable. The slope of the liquidation policy is, thus, equal to or below 1. In other words, the rationing problem shows up indirectly through a lower bound on all possible price-stable central bank policies.

There is a caveat here. Should agents be able to operate the savings technology on their own, then they can always assure themselves a payoff of 1 in period $t = 1$ for every good stored in period $t = 0$. Thus, the only CBDC contract acceptable to these agents would be a "green line" coinciding with the 45-degree line and a slope of 1. Slopes below 1 are agreeable, however, if the central bank is the only entity capable of operating this technology or the only entity capable of intermediation with operators of that technology.

Recall from Section 4, that optimal risk sharing satisfies $x_1^* > 1$, while from Proposition 8 a P_1 price-stable policy requires $x_1 \leq 1$. Hence, we can show the second part of our trilemma:

Corollary 9 (Trilemma part II - No optimal risk sharing)**.** *If the central bank commits to a P_1-stable policy, then:*

(i) Optimal risk sharing is never implemented.

(ii) The no-run equilibrium is implemented in dominant strategies, i.e., there is a unique equilibrium in which only impatient agents spend, $n^* = \lambda$, and there are no central bank run equilibria.

(iii) If the central bank commits to a price-stable central bank policy, then the nominal interest rate is increasing in n and non-negative $i(n) \geq 0$ for all $n \in [\lambda, 1]$.

Intuitively, no runs take place under a P_1-stable policy since the real allocation in $t = 1$ is too low, causing all patient agents to prefer to spend late.

5.2 Partial price stability

While price stability and the absence of central bank runs may be desirable, the constraint (19), i.e., the failure to implement optimal risk sharing, is not. In particular, the implementation of the social optimum is impossible under complete price stability. Recall that optimal risk sharing at $x_1^* > 1$ triggers potential bank runs in models of the Diamond-Dybvig variety: thus, part (ii) of the proposition above should not be a surprise. Demanding price stability for all possible spending realizations of n is thus too stringent: for sufficiently high spending levels of n, equation (18) exhausts the liquidation possibilities available to a central bank, as $y(n)$ cannot exceed 1. We therefore examine a more modest goal: a central bank may still wish to ensure price stability, but may deviate from its goal in times of crises. We capture this with the following definition.

Definition 10. *A central bank policy is*

(i) ***partially*** P_1***-stable at level*** $\overline{P}$*, if for all spending behavior* $n \in [\lambda, 1]$*, either the policy achieves* $P_1(n) = \overline{P}$ *for some* ***price level target*** $\overline{P}$*, or aggregate spending satisfies* $n > \bar{P}/M$*. In the latter case, we require full liquidation,* $y(n) = 1$*.*

(ii) ***partially price-stable at level*** $\overline{P}$*, if for all spending behavior* $n \in [\lambda, 1]$*, either the policy achieves* $P_1(n) = P_2(n) = \overline{P}$ *for some* ***price level target*** $\overline{P}$*, or aggregate spending satisfies* $n > \bar{P}/M$*. In the latter case, we require* $y(n) = 1$*.*

For a graphical illustration, see the blue line in Figure 5a. Obviously, P_1-stable central bank policies are also partially P_1-stable, and price-stable central bank policies are also partially price-stable.

Definition 11. *Given a price goal* $\overline{P}$*, we call a commitment equilibrium a*

- **partial** P_1**-price-commitment equilibrium***, if the central bank policy is partially* P_1*-stable at level* $\overline{P}$
- **partial price-commitment equilibrium***, if the central bank policy is partially price-stable at level* $\overline{P}$

Recall that only price levels above the money supply $\overline{P} \geq M$ can attain full price stability. We therefore now concentrate on lower price levels $M > \overline{P}$, since attaining optimality requires $1 < x_1^* = M/\bar{P}$. Nevertheless, we also encounter a (weaker) feasibility constraint for partially price-stable policies. Since the central bank cannot liquidate more than the entire asset, $y(n) = x_1 n \in [0, 1]$ for all $n \in [\lambda, 1]$, it faces the constraint $\lambda x_1 \leq 1$. Feasibility, therefore, implies a lower bound on all possible partially stable price levels, $\overline{P} \geq \lambda M$. Furthermore, partial price stability restricts central bank policies:

Proposition 12 (Policy under Partial Price-Stability)**.** *Suppose that* $M > \overline{P} \geq \lambda M$*.*

(i) A central bank policy is partially P_1*-stable at level* $\overline{P}$*, if and only if its liquidation policy satisfies:*

$$y(n) = \min\left\{\frac{M}{\overline{P}}n, 1\right\}. \tag{21}$$

(ii) For every partially P_1*-stable central bank policy at level* $\overline{P}$*, there exists a critical aggregate spending level* $n_c \equiv \frac{\overline{P}}{M} \in (0, 1)$ *such that*

(ii.a) *For all $n \le n_c$, the price level is stable at $P_1(n) = \overline{P}$ and the real goods purchased per agent in period $t = 1$ equal $x_1(n) = \overline{x}_1 = \frac{M}{\overline{P}} > 1$ while real goods purchased per agent in period $t = 2$ equal $x_2(n) = R(1 - \overline{x}_1 n)/(1 - n)$.*

(ii.b) *For spending $n > n_c$, the real goods purchased per agent in period $t = 1$ equal $x_1(n) = 1/n$ while $x_2(n) = 0$ and the price level $P_1(n)$ proportionally increases with total spending n: $P_1(n) = Mn$*

(iii) *A central bank policy is partially price-stable at $\overline{P}$, if and only if its liquidation policy satisfies equation (21) and its interest rate policy satisfies:*

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1 - n} R - 1, \qquad \text{for all } n \le n_c. \tag{22}$$

For $n > n_c$, there is no supply of real goods in $t = 2$. Thus, P_2 and $i(n)$ are irrelevant.

(iv) *For a partially price-stable central bank policy at $\overline{P}$, there exists a spending level*

$$n_0 = \frac{R\frac{\overline{P}}{M} - 1}{R - 1} = \frac{Rn_c - 1}{R - 1} \in [0, n_c), \tag{23}$$

such that the nominal interest rate turns negative for all $n \in (n_0, n_c)$. For $R < M/\overline{P}$, the nominal interest rate is negative for all $n \in [0, n_c)$.

Proposition 12 reflects the central bank's capacity to keep the price level and the real interim allocation x_1 stable as long as spending remains below the critical level n_c. The stabilization of the price level requires the liquidation of real investment proportionally to aggregate spending by factor $M/\overline{P}$. At the critical spending level n_c, the central bank is forced to liquidate the entire asset to maintain the price level P_1. Since the central bank cannot liquidate more than its entire investment, as spending exceeds the critical level n_c, price level stabilization via the liquidation of real assets becomes impossible. For all spending behavior $n > n_c$, the real allocation to late spending agents is thus zero. The rationing of real goods implies that the price level has to rise and the real allocation declines in aggregate spending.

The spending level $n_0 < n_c$ is the level at which the real allocation to early and late spenders is just equal

$$x_1(n_0) = x_2(n_0) = \bar{x}_1. \tag{24}$$

Notice that $x_2(n)$ declines in n for $n \in [0, n_c]$. Thus, if fewer than measure n_0 of agents spend, not spending is optimal for patient agents. But for all spending realizations $n > n_0$, the allocation at $t = 2$ undercuts the allocation at $t = 1$: $x_2(n) < x_1(n)$, turning the real interest rate on the CBDC negative, and causing “spend early” to be a patient agent’s optimal response to an aggregate spending behavior in excess of n_0. Consequently, self-fulfilling runs are possible as in Diamond and Dybvig (1983), and we obtain the following result as a corollary of Proposition 12:

Corollary 13 (Trilemma part III- Runs on the Central Bank (Fragility))**.** *Under every partially P_1-stable central bank policy with $M > \overline{P} \geq \lambda M$, there is a multiplicity of equilibria:*

(i) There exists a good equilibrium in which only impatient agents spend, $n^* = \lambda$. In that case, there is no run on the central bank, the social optimum is attained and the price level is stable at level $\overline{P}$.

(ii) There also exists a bad equilibrium in which a central bank run occurs, $n^* = 1$, the social optimum is not attained, and the price level is unstable.

Proposition 12 is in marked contrast to Proposition 8. One could argue that when banking is interesting, i.e., $x_1^* > 1$, then the goal of price stability induces the possibility of runs on the central bank, the necessity for negative nominal interest rates, and the abolishment of the price stability goal, if a run indeed occurs.

6 Money supply policy or suspension of spending

It is natural to ask why the central bank cannot resort to a more classic monetary policy to resolve the trilemma and attain price stability: expansion or reduction of the money supply. In this section, let us then allow for the possibility that M is state-contingent, i.e., M is chosen as a function of aggregate spending $M = M(n)$ at $t = 1$. Therefore, a central bank policy consists of three functions $(M(\cdot), y(\cdot), i(\cdot))$.

The analysis is now straightforward and easiest to explain for the case where the liquidation policy is not state-contingent, $y(n) \equiv y^*$. To maintain price stability at some level $\overline{P}$, market clearing demands

$$nM(n) = \overline{P}y^*. \tag{25}$$

As a result, the total money balances spent in $t = 1$ stay constant in n, implying

$$n\,M(n) \equiv \lambda\,M(\lambda), \quad \text{for all } n \in [\lambda, 1]. \tag{26}$$

But spending per agent alters, as does the total money supply $M(n)$. That is, the central bank would have to commit itself to **reducing** the quantity of money in circulation in response to a demand shock encapsulated in n: the more people go shopping, the lower are individual money balances. With policy (25), $y(n) \equiv y^*$ and $i(n) \equiv i^*$ chosen so that $P_2 = \overline{P}$, the central bank can now achieve full price stability, efficiency, and financial stability. The CBDC trilemma appears to be resolved. There are several ways of thinking about this.

State-contingent money supply. A first approach is to make the amount of CBDC balances available for shopping in $t = 1$ state-contingent. Having such CBDC accounts with random balances is an intriguing possibility: it is quite impossible with paper money but fairly straightforward with electronic forms of currency. A different interpretation of this approach is to think in terms of a state-contingent nominal interest rate paid on CBDC accounts between $t = 0$ and $t = 1$. One should recognize that both of these routes are a bit odd, and contrary to how we usually treat money and interest rates. As for money, a dollar today is a dollar tomorrow: changing that amount in a state-contingent fashion probably risks severely undermining the trust in the monetary system, and trust is key for maintaining a fiat currency. As for interest rates, it is commonly understood that interest rates are agreed upon before events are realized in the future. A state-contingent interest rate turns accounts into risky and equity-like contracts, likewise undermining trust in the safety of the system (see, nonetheless, Section 8 for trade in equity).

Helicopter drops. A third way to think about the state-contingent nature of M corresponds to a classic monetary injection in the form of state-contingent lump-sum payments ("helicopter drops") $M(n) - \bar{M}$ (or taxes, if negative), compared to some original baseline $\bar{M}$. If one wishes to insist that $M(n) - \bar{M} \geq 0$, i.e., only allowing helicopter drops, then the central bank would choose $\bar{M} \leq M(1)$ as payment for goods in period $t = 0$ and thus always distribute additional helicopter money in the "normal" case $n = \lambda$ in period 1. Notice that distributional issues would arise in richer models, where agents are not coordinating on the same action, thereby distorting savings incentives.

Suspension of spending. With an account-based CBDC, there is an additional and rather drastic policy tool at the disposal of the central bank: the central bank can simply disallow agents to spend (i.e., transfer to others) more than a certain amount of their account. In other words, the bank can impose a "corralito" and suspend spending. This policy is different from the standard suspension of liquidation, as the amount of goods made available is a policy-induced choice that still exists separately from the suspension-of-spending policy. Notice also that "suspension of spending" should perhaps not be called "suspension of withdrawal." Since there are only CBDC accounts and they cannot be converted into something else, the amounts can only be transferred to another account. With the suspension-of-spending policy, the central bank could arrange matters in such a way that not more than the initially intended amount of money $\lambda M(\lambda)$ will be spent in period 1; see equation (26). In practice, the central bank would then either take all spending requests at once and, if the total spending requests exceeded the overall threshold, impose a pro-rata spending limit. Alternatively, it could arrange and work through the spending requests in some sequence (first-come-first-served), thereby possibly imposing different limits depending on the position of a request in that queue.

Monetary neutrality. Last but not least, a state-contingent money supply cannot replace the central bank's liquidation policy as the active policy variable. Not only price targeting, but also the deterrence of runs is an objective of the central bank for attaining optimal risk sharing.

A state-contingent money supply, however, does not impact the agent's spending behavior: the individual agents exclusively care for their individual real allocation at $t = 1$, y/n, versus $t = 2$, $R(1 - y)/(1 - n)$. These allocations are independent of nominal quantities (M, P_1). That is, money is neutral. Given a realization of an individual real allocation y/n, the identity:

$$\frac{y}{n} = \frac{M(n)}{P_1} \tag{27}$$

pins down a relationship that needs to hold between the money supply and the price level that clears the market. The central bank can implement all money supplies and price level pairs (M, P_1) that satisfy equation (27). And as soon as the price level goal P_1 is pinned down, contingent on the realization $\frac{y}{n}$, the money supply that solves equation (27) is unique. But in equation (27) the classic dichotomy holds, and the choice of the right-hand side (M, P_1) cannot alter the left-hand side, i.e., cannot alter incentives to run. Consequently, if the central bank wants to impact consumers' behavior to run on the central bank to implement the social optimum, it can only do so by altering the real goods supply y through adjustment of its liquidation policy.

In summary. Given the previous discussion, a state-contingent money supply strikes us as odd monetary policy. First, the usual inclination for central banks is to accommodate an increase in demand with a rise, rather than a decline in the money supply. A central bank that reacts to an increase in demand by making money scarce may undermine trust in the monetary system. In particular, and needless to say, a spending suspension might create considerable havoc; the experience in Argentina at the end of 2001 provides ample proof. Even if this was not the case, monetary neutrality implies that adjusting the money supply does not affect the run decisions of agents. Therefore, we think that this particular escape route from the CBDC trilemma needs to be treated with considerable caution.

7 Voluntary participation in CBDC and competition by private banks

The main model assumes that all consumers invest in a CBDC. It remains to clarify whether agents may be better off using the investment technology on their own, rather than relying on the central bank. This is an important question: if agents were to decide to stay in autarky and invest in the investment technology directly, they might have incentives to supply goods at the interim stage, thus, potentially undermining the central bank's liquidation policy. Similarly, if the outside option is not autarky but investing in deposits with a different, private bank, then the liquidation policy of that private bank has implications for the aggregate real goods supply at the interim stage, again impairing the effectiveness of the central bank's policy. We now discuss both.

7.1 Autarky and voluntary participation in a CBDC

Assume all but one agent invest in a CBDC. Assume that this single agent invests in the real technology at $t = 0$, yielding storage between $t = 0$ and $t = 1$, and yielding $R > 1$ when held between $t = 0$ and $t = 2$. Then, at $t = 1$, she would learn her type. If she is impatient, she will liquidate the technology, yielding 1 unit of the real good, and she would consume her good. She would not sell the good against nominal CBDC deposits, since she only cares about consumption at $t = 1$. In the case where she is impatient, she is worse off in comparison to an agent who invested in CBDCs with the central bank if the central bank offers optimal risk sharing and manages to implement a run-deterring policy. This is so, since under the latter, an individual impatient agent gets $x_1^* > 1$ real goods.

If the individual agent is patient, she will stay invested in the technology until time two. There, the technology yields $R > 1$ units of the good. The agent will, thus, be better off than under investment in a CBDC since $x_2^* < R$; see Section 2.1. But, in particular, also in the patient case, the individual agent will not offer goods for sale in the interim period, since liquidation and selling against a CBDC will only yield x_2^* in $t = 2$. Thus, in either case, patient or impatient, the agent who invests in autarky will not have an incentive to undermine the central bank's policy by increasing the goods supply in the interim period.

Does the agent prefer to remain in autarky rather than participating in the CBDC? *Ex-ante*, the risk-averse agent cannot know whether she will turn out to be patient or impatient. Diamond and Dybvig (1983) show that pooling of resources via banking can attain the social optimum under an absence of runs, while investment under autarky cannot. That is, the single agent is always better off investing in the CBDC account if the central bank offers optimal risk sharing and implements a run-deterring policy. Thus, participation in the CBDC account is individually rational.

What if the central bank runs a policy of full price stability at goal $\bar{P}$? In that case, our second main result, Corollary 9, shows that runs on the central bank do not occur but $x_1 \leq 1$. Thus, for all $x_1 < 1$, investing in a CBDC is dominated by investing in autarky. Voluntary participation thus requires $x_1 = 1$ or $M = \bar{P}$, implying $x_2 = R$. The agent is then indifferent between investing in a CBDC and staying in autarky. Yet, if she stayed in autarky, she will not undermine the central bank's liquidation policy for the reasons above.

In the case of a partial price-stable policy, the situation is as in Diamond and Dybvig (1983). *Ex-ante*, the agent cannot know whether a run occurs or not. Conditional on the no-run equilibrium, we implement the social optimum and the agent is better off investing in a CBDC. But conditional on the run equilibrium, she was better off in autarky. From within the model, it is not possible to attach likelihoods for each equilibrium.

7.2 Can private banks undermine the central bank's policy?

The question of under what circumstances consumers prefer investing in a CBDC account with the central bank rather than investing in demand deposits with private banks, with implications for how both types of banks can coexist, is addressed in Fernández-Villaverde et al. (2020). In this section, we will analyze private banks' incentives to provide goods at the interim stage, *conditional on the coexistence of private banks with the central bank.*

Goods supply. If the central bank coexists with private banks, it controls the market of goods only partially, with the remainder of the real goods being supplied by commercial banks. As before, the measure of agents is normalized to one, divided between a share $\alpha \in (0, 1)$ of agents who are CBDC customers at the central bank and a share $1 - \alpha$ who are customers at private banks. Assume that all agents invest their 1 unit endowment in their corresponding bank and that the private banks invest in the same asset as the central bank does. Then, at $t = 1$, the central bank can supply up to α goods via liquidation, while private banks can supply up to $1 - \alpha$ goods.

Assume that there is one centralized goods market to which customers and banks have access. That is, CBDC depositors can spend CBDC balances on goods supplied by private banks and private bank customers can spend their private deposit balances on goods supplied by the central bank. Let n denote the total measure of spending agents across both customer groups at the central bank and private banks, given by

$$n = \alpha\, n_{CB} + (1 - \alpha)\, n_P, \tag{28}$$

where n_{CB} is the total share of consumers at the central bank who spend, while n_P is the total share of consumers at the private bank who spend. Given total spending n in period $t = 1$, let $y_P(n)$ be the share of assets liquidated by private banks. In contrast, let $y_{CB}(n)$ be the central bank's liquidation policy, i.e., the share of assets liquidated by the central bank. The total goods supply y in the centralized market at the interim stage is then:

$$y(n) = \alpha\, y_{CB}(n) + (1 - \alpha)\, y_P(n). \tag{29}$$

Private deposit making. To collect investment in $t = 0$, the private banks offer a nominal demand-deposit account in return for 1 unit of the real good. The private nominal accounts are denominated in units of the CBDC. Due to competition with the central bank, the private contract also offers M units of the CBDC in $t = 1$ or $M(1 + i(n))$ units in $t = 2$.

To service withdrawals in terms of the CBDC, private banks first observe their customers' CBDC withdrawal needs n_P, and borrow the required amount $(1 - \alpha)n_P M$ of the CBDC from the central bank at the beginning of period $t = 1$. The central bank creates the CBDC quantity $(1 - \alpha)n_P M$ on demand for the private banks. Private banks observe CBDC spending at the central bank n_{CB}, yielding aggregate spending n. During period one, the private banks sell the share $y_P(n)$ of their real goods investment at price P_1 in the centralized market to all consumers, thus receiving proceeds of $P_1 y_P(n)(1 - \alpha)$ units of the CBDC in return, where P_1 satisfies market clearing:

$$M\Big((1 - \alpha)n_P + \alpha n_{CB}\Big) = P_1\Big(y_P(n)(1 - \alpha) + y_{CB}(n)\,\alpha\Big). \tag{30}$$

The private banks use these CBDC proceeds to (partially) repay their loan to the central bank at zero interest within period one. Since the central bank retains only partial control over the goods market, it generically no longer holds $n_{CB}M = P_1 y_{CB}(n)$. As a consequence, the private banks can hold positive or negative CBDC balances $(1-\alpha)(P_1 y_P(n) - n_P M)$ with the central bank between $t = 1$ and $t = 2$.

We seek to examine a range of possibilities for the private bank withdrawals n_P as well as liquidation choices y_P. Thus, it is useful to impose the condition that private banks make zero profits, regardless of the "circumstances" n_P or their choice for y_P. This requires some careful calculation, which we provide in Appendix 13, and only summarize here.

We assume that the central bank charges or pays the nominal interest rate $z = (RP_2/P_1) - 1$ on the excess or deficit CBDC balances of private banks, to be settled at the end of $t = 2$. Note that $z > i$, if $x_1 > 1$ and equals the internal nominal shadow interest rate regarding private bank liquidation choices. Moreover, we impose a market share tax at the end of period $t = 2$ in order to compensate for profits or losses due to circumstances n_P.

At $t = 2$, the remaining private customers spend the quantity $(1-\alpha)(1-n_P)M(1+i(n))$ of private CBDC accounts that the private banks borrow from the central bank at the beginning of $t = 2$. The private banks sell their returns on the remaining investment $R(1-y_P(n))(1-\alpha)$ at price P_2, where P_2 satisfies market clearing

$$\begin{aligned} M(1+i(n))\Big((1-\alpha)(1-n_P)+\alpha(1-n_{CB})\Big) &= \\ P_2 R\Big((1-y_P(n))(1-\alpha)+(1-y_{CB}(n))\,\alpha\Big). \quad (31) \end{aligned}$$

At the end of $t = 2$, the private banks settle their accounts with the central bank, taking into account the remaining balances at $t = 1$ adjusted for interest, the end-of-period tax compensating for circumstances n_P, the loan at the beginning of $t = 2$, and the sales proceeds at $t = 2$.

Joint liquidation policies. The actions of private banks and the central bank may not be perfectly aligned when it comes to the liquidation of assets and the supply of goods at the interim stage. Private banks can have various objectives depending on their ownership structure and may be subject to regulation of their liquidation policy, both shaping y_P. Independently of whether private banks maximize depositor welfare as in Diamond and Dybvig (1983), or pursue some other objective, the prevention of runs is key. We have seen above that runs occur if the provision of real goods at the interim stage is high. Since the market is centralized, for the spending incentives of bank customers it is irrelevant whether these goods are provided by the central bank's or the private bank's liquidation of assets.

Hence, as before, a run-deterring liquidation policy $y(\cdot)$ is a function of aggregate spending

n such that the real allocation at $t = 1$ undercuts the real allocation at $t = 2$:

$$\frac{y(n)}{n} < R\frac{(1 - y(n))}{1 - n}, \qquad \text{for all } n \in [\lambda, 1]. \tag{32}$$

Thus, again, a run-deterring policy satisfies

$$y(n) < \frac{nR}{1 + n(R - 1)}, \qquad \text{for all } n \in [\lambda, 1]. \tag{33}$$

Perfect coordination. If the central bank and the private banks coordinate perfectly, i.e., act as one entity, and have full control over the asset liquidation, then all run-deterring policies are possible, as in the case where the central bank is a monopolist. But why would they coordinate perfectly? By the market's centralization, the destiny of the central bank is intertwined with the destiny of the private banks and both types of banks have an interest in deterring runs. In particular, the private bank will, therefore, not undermine a central bank's run-deterring policy by supplying additional goods when, for instance, prices are high, since this might cause a run not only on the central bank but also on the private bank. Coordination is therefore among the equilibrium outcomes.

Runs under imperfect coordination. The following example shows how, for general liquidation policies y_P of private banks, runs can occur. Assume that the private bank, for some reason, follows a liquidation rule $y_P(n) \in [0, 1]$ where $y_P(n_b) = 1$ for all $n \geq n_b$ where $n_b \in (0, 1)$. For instance, $n_b = 1 - \alpha$, i.e., the private bank is subject to regulation and has to liquidate all assets if a fraction of its customers equal to its market share spends. In that case, as we show next, the central bank's capacity to deter runs depends on the size of the private banking sector, i.e., its market power α. Since the central bank can only control the liquidation of its own investment y_{CP}, via (32) and (29), a run-deterring policy y_{CB} needs to satisfy:

$$y_{CB}(n) < \frac{Rn - (1 - \alpha)y_P(n)(Rn + 1 - n)}{\alpha(Rn + 1 - n)}, \qquad \text{for all } n \in [\lambda, 1]. \tag{34}$$

Now assume $n > n_b$, such that $y_P(n) = 1$. If in addition the central bank has a small market share $\alpha \to 0$, then the numerator converges to $-(1-n)$, while the denominator goes to zero, $\alpha(1 + (R - 1)n) \to 0$. That is, for $n_b < n < 1$, the right-hand side in (34) goes to minus infinity such that (34) cannot hold. This implies that the run equilibrium exists.

A sufficient condition: Run-deterrence under imperfect coordination. The example above makes clear that the central bank's share in the deposit market needs to be large enough in order to prevent runs. The following proposition provides the appropriate bound under which the central bank can ensure the absence of a run, regardless of the private bank's liquidation schedule $y_P : [\lambda, 1] \to [0, 1]$.

Proposition 14. *Suppose that the central bank's share in the deposit market satisfies*

$$\alpha > \frac{1-\lambda}{(1-\lambda+R\lambda)}. \tag{35}$$

Then the central bank can always find a run-deterring liquidation policy $y_{CB} : [\lambda, 1] \to [0, 1]$, regardless of the private bank's liquidation policy $y_P : [\lambda, 1] \to [0, 1]$.

Such an $\alpha \in (0, 1)$ exists since $\frac{1-\lambda}{(1-\lambda+R\lambda)} \in (0, 1)$. Thus, the right-hand side $\frac{1-\lambda}{(1-\lambda+R\lambda)}$ of equation (35) imposes a lower bound on the balance-sheet size of the central bank as a percentage of the total demand deposit market, such that run-deterring policies remain possible despite coexisting private banks that are subject to liquidation restrictions.

Proof. [Proposition 14] We need to show that for any private bank liquidation policy $y_P : [\lambda, 1] \to [0, 1]$, there is a central bank liquidation policy $y_{CB} : [\lambda, 1] \to [0, 1]$ so that (34) is satisfied. To derive a sufficient condition on the central bank's market share α under which it can nevertheless implement a run-deterring policy, note that by $R > 1$, the right-hand side in (34) declines in the value y_p for all $\alpha \in (0, 1)$. Thus, if a central bank policy y_{CP} is run-deterring for $y_P = 1$ for all $n \in [0, 1]$, then y_{CP} is also run-deterring for a private bank policy $y_P(n) \leq 1$ for all $n \in [0, 1]$. Thus, assume $y_P = 1$ for all $n \in [0, 1]$. Then, a sufficient condition for a run-deterring policy against all private bank policies y_P is:

$$y_{CB}(n) < \frac{Rn - (1-\alpha)(Rn + (1-n))}{\alpha(1 + (R-1)n)} = 1 - \frac{1-n}{\alpha(1+(R-1)n)}, \qquad \text{for all } n \in [\lambda, 1]. \tag{36}$$

The right-hand side is increasing in n and $y_{CB}(n)$ cannot undercut zero. Thus, a sufficient condition for the existence of a policy $y_{CB} \in [0, 1]$ that satisfies (36) is an α such that:

$$0 < 1 - \frac{1-\lambda}{\alpha(1+(R-1)\lambda)}. \tag{37}$$

$\square$

8 Trade in equity shares

Diamond and Dybvig (1983) show that banks can offer the socially optimal risk-sharing allocation via demand deposits at the cost of being prone to runs. Jacklin (1987) demonstrates that a run-proof, optimal risk sharing can be implemented when banks offer shares in equity instead of demand deposits. For banks to do so, the real dividend payments $D = \lambda c_1^*$ in $t = 1$ and $R(1 - D)$ in $t = 2$ must be predetermined in $t = 0$ and there must exist a market in which to trade claims on dividends in $t = 1$. The dividends accrue to all investors, patient and impatient. When the equity market opens in $t = 1$, patient investors purchase the impatient agent's late dividend payments in return for the lower early dividend payments. This trade is incentive-compatible once types are revealed. Moreover, before learning their types in $t = 0$, all agents are willing to agree to the predetermined dividend payments. Since in $t = 1$, the equity contract does not allow impatient types to demand an additional share of their late dividend payment, runs that would enforce excess asset liquidation cannot occur.

Would the Jacklin (1987) environment also work in our nominal banking model to prevent runs on the central bank? The answer is not only yes, but in fact, that the dividend policy proposed in Jacklin (1987) is a special case of a run-deterring liquidation policy with a dividend payment equal to:

$$D = \lambda\, c_1^* = y^*, \qquad \text{for all } n \in [0, 1] \tag{38}$$

That is, the liquidation policy discussed around equation (17), which implements the social optimum in dominant strategies via CBDC demand deposits, *is* the real allocation that is implemented in Jacklin (1987) via equity shares and trade in dividends.

In both Jacklin (1987) and our special case (17), the total asset liquidation in $t = 1$ is predetermined at $t = 0$. In Jacklin (1987), per default, all agents receive a real dividend in $t = 1$ and then can trade claims on dividends in $t = 2$ for or against claims on dividends in $t = 1$. After this trade, the patient agents will have given up on their early real dividend, while the impatient types will have given up on their late real dividend. In our setting instead, agents are not allocated real goods per default in $t = 1$. Instead, the agreement is, if an agent spends her CBDC balance on goods in $t = 1$, she foregoes her right to spend her CBDC balance on goods in $t = 2$. Moreover, she will share the supply of goods with all agents that spend with her in $t = 1$, where the total share of spending agents will be unknown to her as she makes the spending decision. While in Jacklin (1987), the market-clearing price of dividends induces the optimal spending, in our model, the fixed supply of goods deters patient types from spending.

But the space of run-deterring policies that we give here is much richer than policy (17). In particular, a run-deterring policy can allow for spending-contingent liquidation $y(n)$, $n \in [\lambda, 1]$, where liquidation is not constant in n. With such a policy, liquidation is not predetermined in $t = 0$, yet, runs will not occur, and the social optimum is implemented in dominant strategies if $y(\lambda) = y^*$.

8.1 Jacklin (1987) with nominal contracts

Notice, however, that our banking model features nominal contracts, while in Jacklin (1987), dividends are denominated in real terms. By our main result (5), a run-deterring policy requires an inflation threat (16). What if the dividend payments were nominal? Does inflation necessarily arise there too for deterring runs? And what is a run on a bank under trade in equity shares?

To answer these questions, assume the extreme case where agents hand over their real goods endowment in return for nominal equity shares in the central bank.[12] The total measure of all agents remains at one. The central bank pools the real goods for investment in the real technology and commits to a central bank dividend policy (D_1, D_2). The $t = 0$ agreement is that all agents receive a nominal CBDC dividend D_1 in $t = 1$ and another nominal dividend D_2 by the central bank in $t = 2$, irrespective of their type. The central bank follows a liquidation policy $y(n)$, where –as before– $n \in [0, 1]$ denotes the measure of agents who go shopping with CBDC in $t = 1$. Since dividends are paid to all shareholders, the total nominal CBDC supply equals D_1 in $t = 1$ and equals D_2 in $t = 2$. The central bank sets a price level P_1 at time $t = 1$ and P_2 in $t = 2$ that clears the goods market. In $t = 1$, types realize and impatient types want to consume as much as possible in $t = 1$. Impatient types can can sell their claims on a nominal dividend D_2 in $t = 2$ in return for nominal dividends D_1 in $t = 1$ to purchase consumption goods provided by the central bank.

In Jacklin (1987), dividends are real and, thus, promise consumption in a one-to-one relation. With nominal dividends, this is no longer true. Crucially, the central bank sees shareholders and shoppers as two different agent groups.

Let $n \in [0, 1]$, the measure of agents who go shopping with their CBDC in $t = 1$ to spend dividends $\tilde{D}_1 \geq D_1$, after trade in nominal dividends has taken place. Assume that there is no storage technology for nominal dividends. That is, either an agent trades D_1 for

[12] Recall that, historically, many central banks sold shares to the public at large that paid dividends. Even today, one can buy shares of the central banks of Japan and Switzerland.

consumption goods with the central bank directly or trades D_1 in return for a claim on a larger nominal dividend $\tilde{D}_2 \geq D_2$ in $t = 2$. Otherwise, D_1 would expire. For example, D_1 can be considered a nominal claim on $t = 1$ consumption goods with a flexible exchange rate (food stamps with an expiration date of "1 meal," where the size of the meal decreases with demand). After trade has occurred, the aggregate nominal CBDC supply equals D_1 and is supplied by measure n agents who demand $y(n)$ goods at a market-clearing price P_1. In particular, the total nominal dividend supply D_1 is independent of trade.

We define a run on nominal equity shares as the incidence where patient types are unwilling to trade their early dividends for late dividends with impatient types, meaning $n > \lambda$. That is, patient types also go shopping for real goods early by spending their nominal dividends D_1, and the dividends trade between the agent groups partially collapses. After observing the total measure of shoppers n who jointly supply dividends D_1, the central bank supplies $y(n)$ goods according to its policy. The market-clearing price P_1 satisfies $n \cdot \frac{D_1}{n} = P_1 \, y(n)$, respectively:

$$D_1 = P_1 \, y(n) \tag{39}$$

Likewise in $t = 2$

$$D_2 = P_2 R(1 - y(n)) \tag{40}$$

The real allocations per agent equal in $t = 1$,

$$x_1 = \frac{y(n)}{n} = \frac{D_1}{P_1 n} \tag{41}$$

and in $t = 2$

$$x_2 = \frac{R(1 - y)}{1 - n} = \frac{D_2}{P_2(1 - n)} \tag{42}$$

An important difference between the nominal CBDC demand-deposit contract we discussed in previous sections and the model with nominal equity shares is that, if the liquidation policy is constant in the measure of shopping agents n, $y(n) = \textit{const}$ for all $n \in [0, 1]$, then the price level must be stable in both $t = 1$ and $t = 2$; see (39) and (40). This result holds because the total supply of nominal dividends that are traded for goods in $t = 1$ is constant at D_1. With a nominal CBDC demand-deposit contract, in contrast, the price level varies in n even if total liquidation is constant, because the nominal supply of CBDCs in $t = 1$ depends on the share of spending agents.

8.2 Runs with nominal contracts

Despite the stable price level, in this nominal version of Jacklin (1987) runs can occur: patient types might not be willing to trade their early nominal dividends for late nominal dividends. That is, the key mechanism in the "real" Jacklin (1987) is not the determination of equity shares and dividends in $t = 0$ but rather that the backing of the dividend payments, the real supply of goods, is predetermined in $t = 0$.

To see that, keep the nominal dividend payments D_1, $D_2 > D_1$ strictly positive. Set $y(n) = 1$ for all $n \in [0, 1]$. That is, the central bank liquidates all real technology at the interim stage so that the goods supply in $t = 2$ is zero, $R(1 - y(n)) = 0$. Consequently, late dividend payments D_2 have zero real value, $P_2 \to \infty$, and all agents, patient and impatient, go shopping for goods in $t = 1$, implying $n = 1$ and trade in nominal equity shares collapses.

The central bank can, however, implement the social optimum by setting a liquidation policy with $y(n) = y^*$ for $n = \lambda$ that simultaneously deters patient types from shopping early so that nominal equity shares are traded. The latter happens when the individual real allocation in $t = 1$ undercuts the real allocation in $t = 2$, $x_1 < x_2$. Via equations (41) and (42), early shopping is deterred for patient types if $\frac{y(n)}{n} < \frac{R(1-y(n))}{1-n}$ for all $n \in (\lambda, 1]$, respectively, when $y(n) < \frac{nR}{1+n(R-1)}$. This familiar constraint imposes the condition that liquidation policy be "run-deterring," as in equation (14). The requirement of run-proofness implies a particular design on the real value of the aggregate dividends via equation (39):

$$\frac{D_1}{P_1} < \frac{nR}{1 + n(R-1)}, \qquad \text{for all } n \in (\lambda, 1] \tag{43}$$

Since the nominal dividend payments are predetermined in $t = 0$, they cannot depend on the share of shoppers n. The right-hand side of (43) is increasing in n, and therefore reaches its minimum in $n = \lambda$. If the central bank wants to follow a fixed price level path, $P_1 = \bar{P}$, then the dividends have to satisfy

$$D_1 < \bar{P}\frac{\lambda R}{1 + \lambda(R-1)} \tag{44}$$

so that patient types have no incentive to shop early. By $\frac{\lambda R}{1+\lambda(R-1)} =: \hat{y} \in (0, 1)$, the constant liquidation policy $\hat{y}$ is feasible, and run-proof and implements the price level $\bar{P}$ irrespective of the measure of shoppers $n \in [\lambda, 1]$. For a spending-flexible liquidation policy $y(n)$ that varies in n, the price level will have to adjust for keeping the predetermined dividend constant at D_1, akin to the case of the nominal CBDC demand-deposit contract.

To conclude, generically, the nominal version of Jacklin (1987) is prone to runs, and a central bank faces a trade-off between implementing the social optimum in a run-proof way and keeping prices stable. When setting the specific run-deterring policy $y(n) = y^*$ for all $n \in [\lambda, 1]$, then the nominal-equity-share setting yields price stability for sure. In contrast, the nominal CBDC demand-deposit contract has varying prices, but only off the equilibrium path. Yet, the nominal equity share setting requires the existence of an interim market to trade dividends. The existence of such a market is not required under nominal CBDC contracts.

9 The financial system

Our model abstracts from many features of the financial system. In our baseline setting, we only have households and the central bank interacting with each other, dropping the financial intermediary sector entirely. This can appear as rather different from the institutional framework seen in practice and the risk-sharing framework in place. This section links the two and motivates the stripped-down setup in Section 2 and beyond.

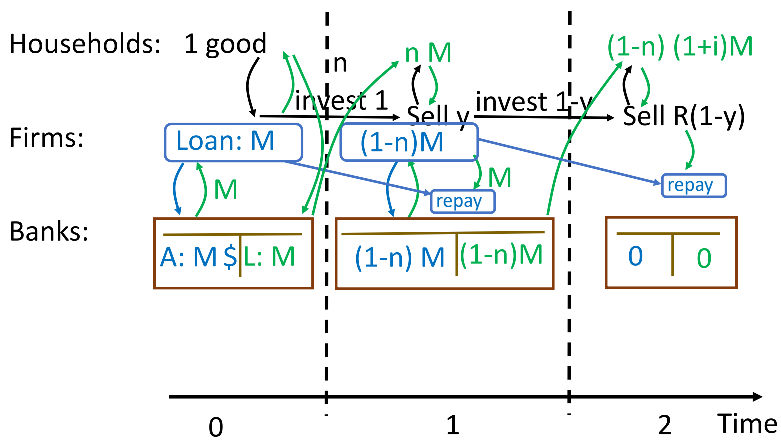


Figure 6: The Financial System 1: Households, firms, and banks.

Consider Figure 6, which shows a three-period model with households, firms, and banks. Each household starts with 1 unit of the real good in period 0. In period 0, banks provide firms with one-period loans totaling M units of money. The firms provide banks with an IOU or loan agreement, adding to the asset side of the bank. Firms use their M units of money to purchase the goods from households and invest them. Households, in turn, deposit the money received with banks, creating bank liabilities of M in the form of deposit accounts.

Total assets are equal to total liabilities of the bank, indicated by the T-account. In period 1, a fraction n of households withdraws their deposits and use them to buy goods from firms. Firms sell y units of the goods to the households, keeping $1-y$ invested for sale in period 2. The firm uses its receipts of nM units of money for the partial repayment of the one-period loan obtained in period 0. The firm thus needs to obtain a new loan, totaling $(1-n)M$, and to use that loan as well as the money received in the goods market in order to repay its period-0 loan completely. In period 2, the remaining investment of the firm generates $R(1-y)$ goods, which are sold to the household. We allow the bank to pay a nominal interest rate of i on deposits held until period 2. Households withdraw their entire deposits. With the cash receipts at hand, the firm repays the original loan. The bank finishes period 2 with a balance sheet of length zero.

A few remarks are in order. We have assumed that there is no interest between period 0 and period 1. Interest rates can be introduced but would clutter notation at this point. We have assumed that loans are one-period loans. Alternatively, we could have assumed that loans are long term, but callable in period 1. The degree to which banks are willing to roll over these loans determines the number of goods the firms have to sell in period 1, in order to be able to repay the original loan in full. The less the bank is willing to roll over the loan, the larger the amount nM the firm needs to repay in $t=1$. If the firm faces an exogenous price level P_1 in t_1, the firm has to increase the liquidation fraction y, since market clearing $nM/P_1 = y$ has to hold. Therefore, the exogenous price level and the bank's willingness to roll over the loan jointly determine the good's supply in $t=1$. In period 2 and in order for the balance sheet of the bank to end up with zero on both sides as well as with the firms selling all output against all the remaining money in the deposit accounts of households, prices and interest rates have to appropriately clear the markets. These issues will be sorted out in our model: for the purpose of the motivational description here, we shall simply assume that this is so.

Note that "money" in Figure 6 is inside money created by the banking system. In principle, all the transactions could take place per appropriate bank-to-bank and account-to-account transfers within the banking system. In practice, however, "withdrawal" of deposit accounts is understood to allow the conversion of deposit accounts into cash and then paying with cash in turn. There is no "cash" in Figure 6 and no central bank.

Figure 7 thus introduces a central bank and a role for cash transactions. Indeed, we now assume that all transactions are for cash only and that households hold cash across periods rather than using deposit accounts at banks. While it is not hard to enrich matters further

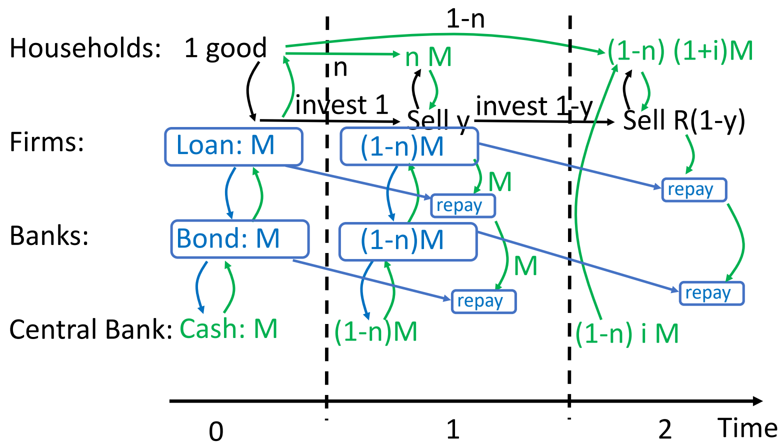


Figure 7: The Financial System 2: Introducing a central bank and open market operations.

and allow for a hybrid deposit-cash-based system, it would seem to unnecessarily complicate matters further. What is important here is that cash is the most liquid means of payment. Households no longer "withdraw" some accounts, turning their withdrawal into cash: it is cash that they have at hand.

In Figure 7, therefore, firms seek to obtain cash when obtaining a loan from a bank. Banks cannot create cash: cash is outside money. The banks, therefore, first need to obtain cash from the central bank. They do so by selling one-period bonds to the central bank for cash, as the first step in period 0. This is a standard open market operation: the central bank purchases bonds using central bank money. Note that the bonds sold by the banks in Figure 7 are bonds underwritten by the bank. In practice –and outside of financial distress episodes– central banks insist on only purchasing government bonds in open market operations. It would not be hard to introduce another layer into the structure in Figure 7, where a government issues bonds to be originally held by banks, which in turn may sell them to the central bank, but that would not create a substantive difference for our analysis. In practice, central banks typically pay for OMT bond purchases using reserves rather than cash, i.e., crediting the central bank accounts of selling banks. The system in Figure 7 could be enriched by allowing for the distinction between central bank reserves and cash. In practice, however, banks can turn these reserve accounts into cash as needed: the distinction would not make a substantive difference. We allow the central bank to pay a nominal interest on cash held by households between period 1 and period 2. While this may appear to be rather hard to do with cash, it will be easily feasible when cash is replaced with CBDC: we thus include this feature here.

In sum, Figure 7 shows the complete financial system, including banks, firms, and a central bank, and it shows a central bank interacting with banks via standard open market operations. It is now the central bank that determines via its open market operations in period 1 the degree to which banks can extend loans to firms in period 1, in turn influencing real activity per the sales y by firms. This influence of the central bank on the volume of loans extended by private banks and on real activity should be familiar from standard textbooks on money and banking and accords well with leading research on this topic (Kashyap and Stein, 2000; Kashyap et al., 1996). While matters are more complicated in practice and involve additional detail and steps, the important point is that the financial system in Figure 7 is a conventional one.

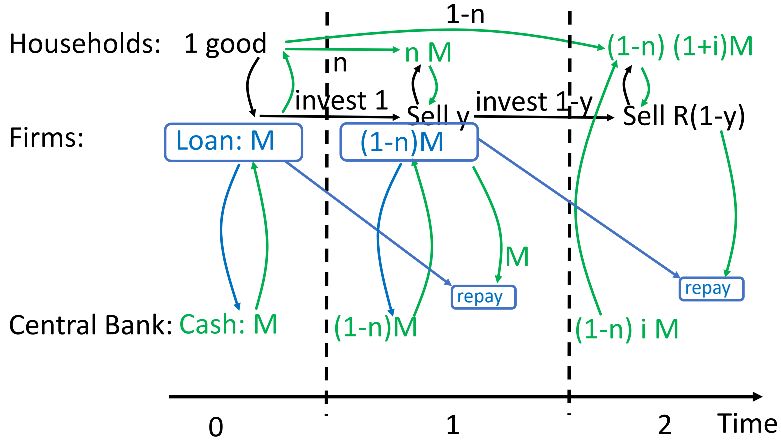


Figure 8: The Financial System 3: Stripping out banks.

For our benchmark analysis, the banks in Figure 7 turn out to be an unnecessary layer: one might as well have firms sell their loans directly to the central bank. This is the system envisioned in Figure 8. One might even see this system as compatible with some of the measures undertaken during the 2007-2008 financial crisis or pandemic episodes, where central banks indeed purchased bonds issued by the private sector rather than bonds issued by banks or the government. The key for us, however, is simply that there is little substantive distinction between the financial structure in Figures 7 and 8.

Finally, one can strip out the firms too, and simply assume that the central bank undertakes the real investment and sale of goods itself, as shown in Figure 9. This picture of the financial system surely looks odd: central banks do not engage in real production nor do they sell goods directly to households! We agree. The point is simply that the financial structure shown in Figure 9 can be understood as the conventional financial structure shown in Fig-

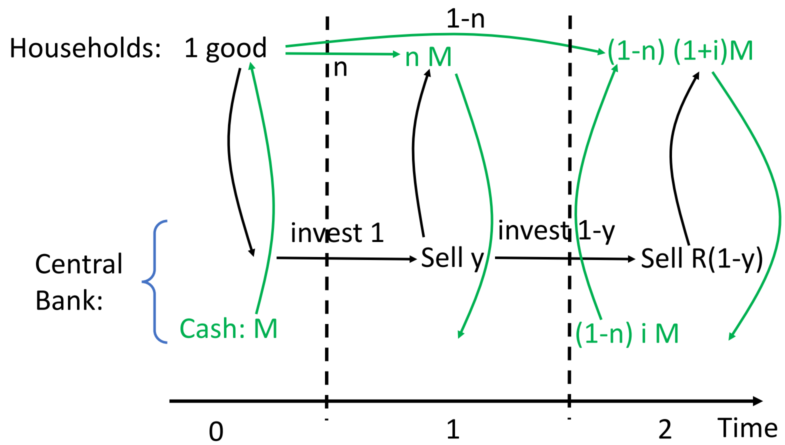


Figure 9: The Financial System 4: Only households and a central bank.

ure 7, stripping out layers of little relevance to our analysis. However, it is good to keep the system shown in Figure 7 in mind in order to understand how households interact with the central bank. Households are not "entitled" to real goods, using their cash, as may seem the case in Figure 9: rather households just spend cash, as was made clear in the description of Figure 7. Moreover, households do not "withdraw" balances from some central bank account in Figure 9. They receive cash in $t = 0$, and there is nothing more to be withdrawn: cash is the most liquid form of payment. Instead, agents spend their cash for real goods in either $t = 1$ or $t = 2$. Finally, and given that we allow an interest on unspent cash balances to be paid in period 2, we could allow for interest payments on the overall cash balance between period 0 and period 1. However, what matters in the analysis above is the amount of cash in the hands of households in $t = 1$ and not how much cash they originally received in $t = 0$.

10 Extensions

10.1 Token-based CBDC

With a token-based CBDC, a central bank issues anonymous electronic tokens to agents in period 1, rather than accounts.[13] These electronic tokens are more akin to traditional ban-

[13]This can be done with or without a blockchain. In the second case, a centralized ledger to record transactions can be kept by a third party that is separate from the central bank. That third party could also potentially pay interest or impose a suspension of spending. For the purpose of this paper, we do not need to worry about the operational details of such a third party or to specify which walls should exist between it and the central bank to guarantee the anonymity of tokens.

knotes than to deposit accounts. Trading with tokens only requires trust in the authenticity of the token rather than knowledge of the identity of the token holder. Thus, token-based transactions can be made without the knowledge of the central bank.

With appropriate software, digital tokens can be designed in such a way that each unit of a token in $t = 1$ turns into a quantity $1 + i$ of tokens in $t = 2$, with i to be determined by the central bank at the beginning of period $t = 2$: even a negative nominal interest rate is possible.[14]

With that, the analysis in the previous sections still holds, since nothing of essence depends on the identity of the spending agents other than total CBDC tokens spent in the goods market. With a token-based CBDC, agents obtain M tokens in period $t = 0$, and decide how much to spend in periods $t = 1$ and $t = 2$. Thus, the same allocations can be implemented except for those that require the suspension of spending, as discussed in Subsection 6.

For the latter, the degree of implementability depends on technical details outside the scope of this paper. Note that even with a token-based system, the transfer of tokens usually needs to be registered somewhere, e.g., on a blockchain. It is technically feasible to limit the total quantity of tokens that can be transferred on-chain in any given period. A pro-rata arrangement can be imposed by taking all the pending transactions waiting to be encoded in the blockchain, taking the sum of all the spending requests, and accordingly dividing each token into a portion that can be transferred and a portion that cannot. It may be that off-chain solutions arise circumventing some of these measures, but their availability depends on the precise technical protocol of the CBDC token-based system. In the case where the token-based CBDC is operated by a centralized third party, such an implementation is even easier.

10.2 Synthetic CBDC and retail banking

With a synthetic CBDC, agents do not hold the central bank's digital money directly. Rather, agents hold accounts at their own retail bank, which in turn holds a CBDC not much different from current central bank reserves. This may be due to tight regulation by the monetary authority. The retail banks undertake the real investments envisioned for the central bank

[14] Historically, we have examples of banknotes bearing positive interest (for instance, during the U.S. Civil War, the U.S. Treasury issued notes with coupons that could be clipped at regular intervals) and negative interest (demurrage-charged currency, such as the prosperity certificates in Alberta, Canada, during 1936). Thus, an interest-bearing electronic token is novel only in its incarnation, but not in its essence.

in our analysis above. A synthetic CBDC, therefore, corresponds to the model sketched in Section 7.2 with $\alpha = 0$.

The key difference from the current cash-and-deposit-banking system is that cash does not exist as a separate central bank currency or means of payment. That is, in a synthetic CBDC system, agents can transfer amounts from one account to another, but these transactions are always observable to the banking system and, thereby, the central bank. Likewise, agents (and banks) cannot circumvent negative nominal interest, while they could do so in a classic cash-and-deposit banking system by withdrawing cash and storing it.

For the purpose of our analysis, observability is key. Our analysis is relevant in the case of a systemic bank run, i.e., if the economy-wide fraction of spending agents exceeds the equilibrium outcome. Much then depends on the interplay between the central bank and the system of private banks. For example, if the liquidation of long-term real projects is up to the retail banks, and these retail banks decide to make the same quantity of real goods available in each period, regardless of the nominal spending requests by their depositors, then the aggregate price level will have to adjust. The central bank may seek to prevent this either by imposing a suspension of spending at retail banks or by forcing banks into higher liquidation of real projects: both would require considerable authority for the central bank. Proposition 14, for instance, says that with $\alpha = 0$, the central bank alone cannot implement a run-deterring policy when offering a synthetic CBDC. Run deterrence then requires retail banks to control liquidation in a particular way.

10.3 Cash

The key difference to a fully cash-based system is that spending decisions can only be observed in the goods market, rather than by also tracing accounts or transactions on the blockchain. In principle, the payment of nominal interest rates on cash is feasible, but is demanding in practice. Excluding nominal interest rates on cash, due to these practical considerations, implies the cash-and-deposit banking system discussed in Section 10.2 and the restrictions discussed there. The tools available to a central bank are now considerably more limited. These limitations may be a good thing, as they may impose a commitment technology and may thus lead to the prevention of an equilibrium systemic bank run in the first place, but the restricted tool set may be viewed as a burden *ex-post*, should such a bank run occur.

11 Conclusion

Diamond and Dybvig (1983) have taught us that the implementation of the social optimum via the financial intermediation of banks comes at the cost of making these banks prone to runs. This dilemma becomes a trilemma when the central bank acts as the intermediary offering a CBDC because central banks are additionally concerned about price stability. As our main result, a central bank that wishes to simultaneously achieve a socially efficient solution, price stability, and financial stability (i.e., absence of runs) will see its desires frustrated. We have shown that a central bank can only realize two of these three goals at a time.

12 Appendix A: Proofs

Proof. [Proposition 8] Proof (i): Via the market clearing condition (7), setting $P_1(n) \equiv \bar{P}$ for all n requires $y(n) = \frac{M}{\bar{P}}n$, for all $n \in [0,1]$. Thus, via (11), $x_1(n) = y(n)/n = \frac{M}{\bar{P}}$ is constant for all n. Last, since the central bank cannot liquidate more than the entire investment in the real technology, $y(n) \in [0,1]$ for all n, together with x_1 constant requires, in particular, $\frac{M}{\bar{P}} = x_1 = x_1(1) = y(1) \leq 1$. Thus, $M \leq \bar{P}$. Proof (ii): When additionally requiring price stability, $P_1(n) = P_2(n) \equiv \bar{P}$, the market clearing condition (8) together with (18) yields (20). □

Proof. [Corollary 9] Proof (i): We know that price stability demands $x_1 \leq 1$ but the social optimum satisfies $x_1^* > 1$. Proof (ii): $\overline{x}_1 \leq 1$ implies $x_2(n) = \frac{1-y(n)}{1-n}R = \frac{1-n\overline{x}_1}{1-n}R \geq R > 1 \geq \overline{x}$. Since the real value of the allocation at $t = 2$ always exceeds the real value of the time one allocation at $t = 1$, patient agents never spend at $t = 1$; thus, there are no runs. Proof (iii): By equation (19), $\frac{\overline{P}}{M} \geq 1$, implies $i(n) = \frac{\frac{\overline{P}}{M}-n}{1-n}R - 1 \geq R - 1 > 0$ for all $n \in [\lambda, 1]$ by $R > 1$. Further, $\frac{\overline{P}}{M} \geq 1$ implies that $i(n)$ increases in n. □

Proof. [Proposition 12] Proof (i): Equation (21) follows immediately from (7) and the constraint $y(n) \leq 1$. Proof (ii): In $n = n_c$, we have $\frac{M}{\bar{P}}n = 1$. Therefore, $n_c > 0$. By assumption $\bar{P} < M$, thus $n_c < 1$, with $n_c \in (0,1)$. Equation (21) implies that $x_1(n) = y(n)/n$ is constant at the level $\overline{x} = M/\overline{P}$, as long as $y(n) < 1$: this is the case for $n < n_c$. For $n \geq n_c$, $y(n) \equiv 1$. All goods are liquidated, so $x_1(n) = 1/n$. Equation $P_1(n) = Mn$ follows from equation (7). Proof (iii): Equation (22) follows from (8) combined with (21). Proof (iv): This is straightforward, when plugging in (21) into $P_2(n)$ and observing that n_0 is positive only for $R > M/\overline{P}$. □

13 Appendix B: Private bank accounting

Consider the collective of private banks with market share $(1-\alpha) \in (0,1)$. For the sake of brevity, we refer to the collective as "the private bank." A fraction n_P of the private bank's customers spend in $t = 1$, while a fraction n_{CB} of the central bank's customers do so, for a total fraction n of all agents $n = (1-\alpha)n_P + \alpha n_{CB}$. Agents are promised M units of the CBDC, when spending in $t = 1$, or $M(1+i)$ units, when spending in $t = 2$. The central bank liquidates y_{CB} goods in period $t = 1$, while the private bank liquidates y_P, for total liquidation $y = (1-\alpha)y_P + \alpha y_{CB}$. For accounting, we introduce some notation. The private bank borrows CBDC L_1 from the central bank to meet withdrawals at the beginning of each period, repaying the loan at the end of the period with the sales proceed S_1 from selling real goods. No interest is charged for the within-period loan.

The difference D_1 at the end of period $t = 1$ is kept on account at the central bank, earning or paying the nominal interest rate z, to be settled at the end of period $t = 2$. Further, the bank has to pay a tax $\tau(1 - \alpha)$ denoted in CBDC at the end of period 2 (or receive this as a subsidy, if $\tau < 0$). The interest rate z and the tax τ are chosen by the central bank (CB in the accounting below), and may depend on n_P and choices y_P of the private bank. We seek to calculate x and τ so that the private bank makes zero profits, i.e., is left with zero CBDC balances D_2 at the end of period 2, after having liquidated and sold all its remaining goods at the end of period 2. Then:

Accounting in period $t = 1$:

$$\begin{aligned} \text{Loan from CB:} \quad L_1 &= (1 - \alpha) n_P M \\ \text{Sales proceeds:} \quad S_1 &= (1 - \alpha) P_1 y_P \\ \text{Difference:} \quad D_1 &= S_1 - L_1 = (1 - \alpha)(P_1 y_P - n_P M) \end{aligned}$$

Accounting in period $t = 2$:

$$\begin{aligned} \text{Loan from CB:} \quad L_2 &= (1 - \alpha)(1 - n_P)(1 + i) M \\ \text{Sales proceeds:} \quad S_2 &= (1 - \alpha) P_2 R (1 - y_P) \\ \text{CB account:} \quad A_2 &= (1 + z) D_1 - \tau(1 - \alpha) \\ \text{Difference:} \quad D_2 &= A_2 + S_2 - L_2 \\ &= (1 - \alpha)\Big(P_2 R + ((1 + z) P_1 - P_2 R) y_P - (1 + i) M - (z - i) n_P M - \tau\Big) \end{aligned}$$

Market clearing:

$$\begin{aligned} \text{In } t = 1: \quad P_1 y &= nM \\ \text{In } t = 2: \quad P_2 R (1 - y) &= (1 - n)(1 + i) M \end{aligned}$$

Sum $(1 + i)$ times the market clearing equation for P_1 with the equation for P_2 to obtain $P_2 R + ((1 + i) P_1 - P_2 R) y = (1 + i) M$. Use the latter equation to replace $(1 + i) M$ in the last expression for D_2 to find

$$\frac{D_2}{P_1(1-\alpha)} = (i-s)(y_P - y) + (z-i)(y_P - n_P x_1) - \frac{\tau}{P_1} \tag{45}$$

where, as usual, $x_1 = \frac{M}{P_1}$ is the amount of real goods acquired by agents in period $t = 1$ and where we introduce:

$$s = \frac{P_2}{P_1} R - 1 \tag{46}$$

to denote the "shadow" nominal interest rate for private banks, equating liquidating a unit of the good in $t = 1$, selling at P_1 and investing at the shadow nominal return $1 + s$ to keeping the unit of good and thus selling R units at price P_2. Notice that $y = nx_1$ and the market clearing equations imply

$$1 + s = (1+i)\frac{1-n}{1-x_1 n} x_1 \tag{47}$$

and, thus, $s > i$, whenever $x_1 > 1$. In particular, this is the case at the efficient outcome. We note that $s = i$, if and only if $x_1 = 1$, which is the maximal full price-stable solution as well as the market allocation, when agents engage in self-storage.

Suppose now that the private bank sells exactly as many goods as purchased by its withdrawing customers, i.e., $y_P = n_P x_1$. Absent τ, equation (45) reveals that the private bank will make a loss or profit, if $x_1 \neq 1$ and if $y_P \neq y$, i.e., $n_P \neq n$. For example, if the share of private-bank customers who go shopping in $t = 1$ is larger than the average share of customers who shop economy-wide, $n_P > n$, and if the allocation achieves $x_1 > 1$ and thus $s > i$, then the private bank incurs a loss $D_2 < 0$, absent τ, as the opportunity costs for servicing agents in $t = 1$ are high. We shall use these observations to fix the tax τ to compensate for these losses or profits, and assume that

$$\tau = P_1(i-s)(n_P - n)x_1 \tag{48}$$

from here onward. This τ depends on the specifics of the bank only via the "circumstances" n_P and does not depend on the choice y_P. To take care of the case where $y_P \neq n_P x_1$, we use the central bank-account interest rate z. Solving for z per setting $D_2 = 0$ in (45) and imposing (48) yields the following result, which we formulate as a proposition.

Proposition 15. *Suppose τ satisfies (48). Then*

$$\{D_2 = 0\} \Leftrightarrow \Big(\{y_P = n_P x_1\} \text{ or } \{z = s\}\Big). \tag{49}$$

In sum, taxing the "circumstance" profits per (48) and paying an internal interest rate z on central bank balances equal to the shadow nominal interest rate s achieves the objective that private banks make zero profits, regardless of their circumstances n_P and regardless of their liquidation choice y_P.

Lemma 16. *If the private bank sets $y_P \equiv y_{CB}$, then the interest rate for which the private bank's balances with the central bank are zero equals $z = i$ and $\tau = 0$.*

That is, if the private bank liquidates the same share of assets as does the central bank, then the interest rate on CBDC balances $z = i$ sets bank profits to zero.

Proof. [Lemma 16] With $\tau = 0$, the CBDC balance at the end of $t = 2$ equals

$$\begin{aligned} D_2 &= (1-\alpha)\left(P_2 R(1-y_p) - (1-n_p)(1+i)M + (1+z)(P_1 y_p - n_p M)\right) \\ &= (1-\alpha)M * \left(\begin{array}{c} (1+i)\left(\frac{(1-y_p)(1-n)}{1-y} - (1-n_p)\right) \\ +(1+z)\left(\frac{n y_p}{y} - n_p\right) \end{array} \right) \end{aligned} \tag{50}$$

where, at the last equality, we have plugged in P_1 and P_2. Then,

$$\frac{(1-y_p)(1-n)}{1-y} - (1-n_p) = -\left(\frac{n y_p}{y} - n_p\right) \tag{51}$$

if and only if

$$\frac{y(1-y_p) - n(y-y_p)}{y(1-y)} = 1 \tag{52}$$

For $\alpha \in (0,1)$, $y_P \equiv y_{CB}$ implies $y_p = y$. If $y = y_p$, then equations (52) and (51) are true. Thus, for $y = y_p$ the choice $z = i$ puts $D_2 = 0$. $\square$

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