# Central Bank Digital Currency: Central Banking For All?

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**Abstract**

The introduction of a central bank digital currency (CBDC) allows the central bank to engage in large-scale intermediation by competing with private financial intermediaries for deposits. Yet, since a central bank is not an investment expert, it cannot invest in long-term projects itself, but relies on investment banks to do so. We derive an equivalence result that shows that absent a banking panic, the set of allocations achieved with private financial intermediation will also be achieved with a CBDC. During a panic, however, we show that the rigidity of the central bank's contract with the investment banks has the capacity to deter runs. Thus, the central bank is more stable than the commercial banking sector. Consumers internalize this feature *ex-ante*, and the central bank arises as a deposit monopolist, attracting all deposits away from the commercial banking sector. This monopoly might endanger maturity transformation.

*Keywords*: central bank digital currency, central banking, intermediation, maturity transformation, bank runs, lender of last resort.

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# 1 Introduction

We investigate the implications for the financial architecture of introducing a central bank digital currency, or CBDC. In particular, we focus on the effects of a CBDC on financial intermediation and maturity transformation. We show how the central bank must offer contracts on par with those of commercial banks to make the CBDC sufficiently attractive as an alternative to commercial bank deposits. While this achieves social efficiency, we also point out a sinister counterpart. We assume that the central bank cannot invest in long-term projects on its own, but has to rely on investment banks instead. In turn, this reliance renders central bank CBDC deposits safer and, thus, more attractive than deposit contracts at commercial banks, endangering the commercial banking sector.

Recent advances in cryptographic and distributed ledger techniques (von zur Gathen, 2015; Narayanan et al., 2016) have opened the door to the widespread use of digital currencies. While the lead in introducing these currencies came from private initiatives such as Bitcoin, Ethereum, and Libra, researchers and policymakers have explored the possibility that central banks can also issue their own digital currencies, aptly called a CBDC.[1]

The introduction of a CBDC can represent an important innovation in money and banking history. Besides its potential role in eliminating physical cash, a CBDC will allow the central bank to engage in large-scale intermediation by competing with private financial intermediaries for deposits (and, likely, engaging in some form of lending of those deposits). In other words, a CBDC amounts to giving consumers the possibility of holding a bank account with the central bank directly.

Defenders of a CBDC have been rather explicit about this implication. For instance, Barrdear and Kumhof (2016, p. 7) state: "By CBDC, we refer to a central bank granting universal, electronic, $24x7$, national-currency-denominated and interest-bearing access to its balance sheet." In this paper, we will use these authors' definition as the working concept of a CBDC.[2]

[1]See, without trying to be exhaustive, Adrian and Mancini-Griffoli (2019), Agur et al. (2019), Barontini and Holden (2019), Berentsen and Schar (2018), Brunnermeier et al. (2019), Barrdear and Kumhof (2016), Bordo and Levin (2017), Danezis and Meiklejohn (2015), Dyson and Hodgson (2016), Ketterer and Andrade (2016), Mancini-Griffoli et al. (2018), Rogoff (2014), and Wadsworth (2018). The BIS, the ECB, the Sveriges Riksbank, and the Bank of Japan —among other central banks— have issued official reports examining different aspects of CBDCs.

[2]Notice that the sense in which most economists have defined a CBDC as an account-based currency goes well beyond a basic notion of central bank-issued electronic money (which may or may not fully replace physical cash). To focus our investigation, we will avoid the discussion of the relative merits of other forms of central bank-issued electronic money, such as a token-based central bank cryptocurrency or traditional electronic reserves. Conversely, one could consider situations where the central bank opens its account facilities to all citizens, even in the absence of a CBDC. As we will see in Section 2, this opening has occurred often in history, and it was defended in the 1980s by Tobin (1987, p. 172) as "deposited currency." Nearly all of the analysis in our paper carries over to a "deposited currency" environment.

Similarly, Bordo and Levin (2017) propose that either "an account-based CBDC could be implemented via accounts held directly at the central bank" [p. 7], or that a "CBDC could be provided to the public via specially designated accounts at supervised commercial banks, which would hold the corresponding amount of funds in segregated reserve accounts at the central bank" [p. 8], an option often referred to as a synthetic CBDC. In this formulation, the differences between a CBDC held directly at the central bank or a commercial bank with segregated reserves are mostly inconsequential for our analysis, for they have the same implications for financial intermediation.

Thus, we could be at a juncture where changes in technology –namely, the introduction of digital currencies– may justify a fundamental shift in the architecture of a financial system, a central bank "open to all." As argued by Friedman and Schwartz (1986) in a classic paper, external forces –in this case, technological innovations– might shape how we think about the role of government in setting up monetary and financial institutions and make us opt for alternative arrangements.

These considerations are already relevant to monetary policy. In June 2018, Swiss voters turned down the sovereign-money (*Vollgeld*) initiative that would have given the central bank a monopoly on issuing demand deposits, an idea motivated in part by the possibility of a Swiss CBDC. Despite the *Vollgeld* initiative being soundly defeated at the ballot box, similar designs are bound to be widely discussed during the coming years.

A CBDC is frequently promoted as a vehicle for financial inclusion of a previously unbanked population as well as a safer alternative to commercial bank accounts. With that, the central bank needs to make the CBDC sufficiently attractive relative to interest-bearing and risk-sharing commercial bank deposit contracts. Inevitably, this means that the central bank will find itself confronted with traditional banking functions such as maturity transformation and financial intermediation. The purpose of this paper is to think these issues through to the very end and answer the key questions that arise.

What effects will the introduction of a CBDC and the opening of central bank facilities have on financial intermediation? Will a CBDC impair the role of the financial system in allocating funds to productive projects? Or can we reorganize the financial system in such a way that a CBDC will still allow the right flow of funds between savers and investors? Will a CBDC make bank runs disappear and stabilize the financial system?

To answer these questions, we build on the classic model by Diamond and Dybvig (1983), augmented by the presence of a central bank. We select this model because it emphasizes the role of banks in maturity transformation: banks finance long-term projects with demand deposits, which may be withdrawn at a shorter horizon. Exploring how a CBDC will interact with maturity transformation is a first-order consideration that has not been thoroughly examined by the literature, often more interested in questions such as the consequences of a CBDC for interest rates, tax evasion, or anonymity (for a review of that literature, see Beniak, 2019). Furthermore, the model by Diamond and Dybvig (1983) allows us to explore how the propensity for bank runs might change with the introduction of a CBDC and to compare those results with previous findings in the literature.

More concretely, we consider a framework in which the CBDC takes the form of demand deposit accounts of the public at the central bank. Like a commercial bank, the central bank holds assets funded by these liabilities, but in contrast to commercial banks, we assume that the central bank cannot invest in high-return long-term projects. This constraint might be due to the central bank not having a good technology to screen, monitor, and liquidate productive projects. The central bank can, instead, rely on investment banks to engage in wholesale loans. We derive an interesting equivalence result that shows that the set of allocations achieved with private financial intermediation (which, in the Diamond-Dybvig framework, can deliver an *ex-ante* first-best allocation) can also be achieved with a CBDC, provided competition with commercial banks is allowed.

This equivalence result vindicates some of the views of proponents of a CBDC: the socially optimal amount of maturity transformation can still be produced in an economy where the central bank has been opened to all. But our equivalence result has a sinister counterpart. If the competition from commercial banks is impaired (for example, through some fiscal subsidization of central bank deposits), the central bank has to be careful in its choices to avoid creating havoc with maturity transformation.

We can think about our equivalence result as belonging to the tradition of public finance Modigliani-Miller theorems (Sargent, 2009, ch. 8). These theorems highlight how the same allocation can often be implemented through very different government policies. In our case, the socially optimal allocation can be implemented with and without a CBDC. But these Modigliani-Miller theorems also emphasize how strict are the conditions under which such results work. In the presence of a CBDC, we are likely to find powerful political-economic forces that will break the equivalence conditions. For instance, we can expect directives or subsidies to provide preferences for investment in some class of projects over others that are favored by the electoral process.

The fragility of the equivalence result is even clearer when we add to the analysis the consideration of commercial and central bank runs. Under bank runs, the central bank can still offer the socially optimal deposit contract, but the conditions for runs in commercial and central banks are different. The intuition is as follows. The wholesale loan of the central bank to the investment bank is not callable. This implies that the central bank's indirect investment in the long asset is protected from early liquidation. This protection either deters runs on the central bank or makes them less likely than runs on the commercial banking sector. Consumers internalize this feature and exclusively deposit with the central bank. That is, due to the rigidity of the central bank's contract with the investment banks, the central bank becomes the monopoly provider of deposits.

Importantly, this monopoly power allows the central bank to deviate from offering the socially optimal deposit contract. If the central bank exercises this market power, it will deviate from offering the socially optimal deposit contract, and the economy will not have the first-best amount of maturity transformation.

Commercial banks do not have access to the same rigid contract because of two fundamental aspects of public law in nearly all legal systems: the seniority of the debt to the central bank and the protection it enjoys against forced liquidation. Once the central bank starts dealing with investment banks, it will displace commercial banks from this market because, thanks to its seniority, it will obtain a higher expected rate of return. Also, the protection against forced liquidation will allow the central bank to use the investment banks and deny withdrawal requests that exceed the level of its short assets. A central bank can default but not go bankrupt. These two legal features make the nature of the central bank's assets and liabilities essentially different from a commercial bank's assets and liabilities. This element of CBDCs is an overlooked but key factor behind the consequences of this new money.[3]

Our paper is related to several important branches of monetary economics. In terms of money and banking theory, we are closest to Diamond and Dybvig (1983), and all the subsequent literature. Our main equivalence result resembles some aspects of Brunnermeier and Niepelt (2019), who show the equivalence of private and public monies in quite a different environment without maturity transformation. By contrast, we highlight, in particular, the equivalence of financial arrangements in terms of maturity transformation, a novel result in the literature, and derive our results by studying the incentives of individual depositors.

There is also an emerging literature on CBDC. Beyond the many papers cited above, Andolfatto (2020), Böser and Gersbach (2019), Brunnermeier and Niepelt (2019), Chiu et al. (2019), Niepelt (2020), and Keister and Sanches (2019) discuss related issues to ours regarding the interaction between the commercial banking sector and a CBDC. A distinctive feature of our paper is that we pay much attention to allocations under bank runs and analyze how a CBDC can and cannot prevent these runs.

Finally, notice that the debate on CBDC belongs to a broader debate on narrow banking (Pennacchi, 2012).This literature has been concerned with how alternative forms of financial

[3]Of course, one could advocate the reform of public law and the elimination of these legal features, but that would fundamentally impair many of the other roles of governments in taxation, conventional monetary policy, etc. The most diverse legal systems agree on this point: to function properly, a modern government requires seniority of its debts and protection against forced liquidation.

intermediation may unravel banks' deposit markets (von Thadden, 1998). For example, Jacklin (1987), among others, shows that a profit-sharing contract between a bank and depositors would prevent runs and monopolize the market.

The rest of the paper is organized as follows. Section 2 frames the current discussion within the historical background. Section 3 introduces our model of a central bank open to all. Section 4 defines and characterizes the equilibrium of the economy. Section 5 analyzes bank runs. Section 6 presents some discussion of the robustness of our results. Section 7 concludes.

# 2 Historical Background

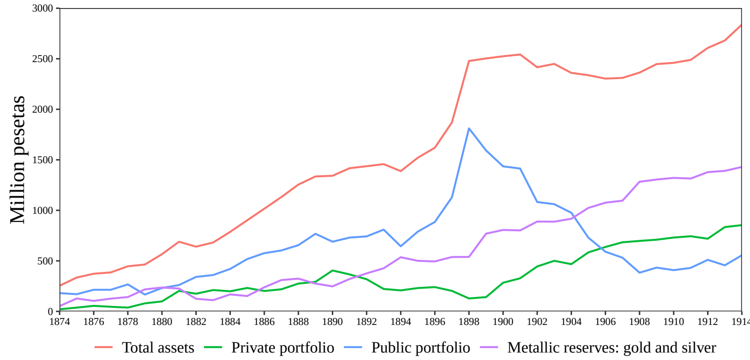
Historically, many central banks have allowed deposits by and extended loans to firms and private citizens at large. These activities were often considered more important for the central banks than monetary policy, both in terms of daily operations and the top management priorities. Indeed, many governments saw the positive impact on economic growth of a central bank's commercial activities –namely, the supply of demand deposits, the creation of credit, the integration of payment systems, etc.– as the motivation to create such institutions.[4]

Perhaps the most famous case of a central bank engaged in commercial activities is the Bank of England. This institution, established in 1694 as a privately owned limited-liability corporation, was given "the right to maximise its profits through undertaking a general banking business, including through issuing paper money, taking deposits, lending on mortgages and dealing in bills of exchange as well as gold and silver" (Kynaston, 2017, p. 5). The Bank of England vigorously pursued such a goal for over two centuries, encroaching on other commercial banks' business through direct competition and the lobbying effort with Parliament at Westminster for additional legal privileges to protect its private interests against potential competitors.

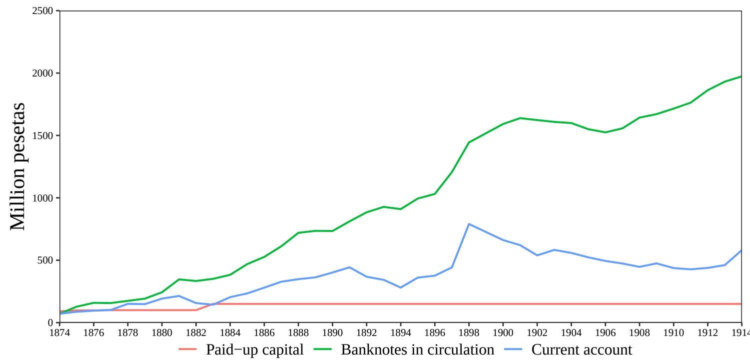
In the U.S., both the First and Second Banks of the United States participated actively in the borrowing and credit markets (Cowen, 2000, and Knodell, 2017). In fact, the bank war between Andrew Jackson and Nicholas Biddle was linked directly to the Second Bank of the United States' operations with firms and merchants, which Jackson considered favored his political rivals (Remini, 2017).

Sometimes, central banks' commercial activities were so extensive that they became the dominant players in the borrowing and lending markets. For example, in 1900, the Bank of Spain (*Banco de España*), with 58 branches that covered all major towns across the nation,

[4] Bindseil (2019) offers a rich historical narrative of how early central banks were deeply involved in lending and borrowing with private parties and the motivations behind their creation.




(a) Assets




(b) Liabilities

Figure 1: Banco de España, Assets and Liabilities, 1874-1914.

held 68% of total assets and 73% of all demand deposits in the Spanish financial sector (Martín-Aceña, 2017, 2018). In Figure 1, we plot the assets and liabilities of the Bank of Spain from 1874 to 1914. Panel 1a shows how, by the start of the 20th century, the bank's private portfolio of loans was larger than the public portfolio of treasury bonds. We can also appreciate, in panel 1b, the large size of the current accounts (i.e., demand deposits) on the liabilities side of the bank.

A related development was the existence of postal savings systems. The first such system was the Post Office Savings Bank (POSB) in the United Kingdom, which opened in 1861. In the U.S., such a system operated from 1911 to 1967, reaching by the end of World War II around 10% of assets of the commercial banking sector. These postal savings systems took advantage of the existing network of post offices to offer government-backed deposit accounts and other financial services such as easy and cheap money transfers to private citizens. If we think about a consolidated public-sector balance sheet, deposits at a postal savings system are fully equivalent to deposits at a central bank: they are deposits in two different agencies of the same public sector. Political-economic constraints, however, may make such full equivalence break down in practice (for instance, if a government treats profits and losses from a postal savings system differently from profits and losses from a central bank).[5]

The sharp distinction between a central bank operating only with primary depository institutions and commercial banks dealing with members of the public at large is, to no small extent, a post-WWII development. This move was induced, among other reasons, by the governments' desire to directly control discretionary monetary policies once the gold standard had disappeared.[6]

These new economic conditions led to the nationalization of many central banks, such as the Bank of England in 1946 and the Bank of Spain in 1962, regardless of the political inclination of governments (left-leaning in the U.K. in 1946, right-wing authoritarian in Spain in 1962). But, even today, one can trade shares of many central banks on stock exchanges, including the Swiss National Bank and the Bank of Japan. Although these shares come with severe voting rights limitations, their active trading is proof that central banks used to be engaged in a set of activities very different from the pure conducting of conventional monetary policy.

The arrival of digital money has reopened the debate about the role of central banks. First, CBDCs have become feasible. Second, the internet allows a central bank to skip building an extensive network of branches, either directly or in cooperation with existing commercial banks. Both factors suggest that we can, and very likely will, revisit the sharp separation wall between central banks and the public at large. But, for such an endeavor, we require a formal economic model.

# 3 A Model of Banking

Our benchmark economy builds on the canonical banking model by Diamond and Dybvig (1983). This framework focuses on the role of banks as maturity transformation providers. Thus, we can use our model to investigate how a CBDC, and the subsequent opening of central bank deposits to the public at large, affects financial intermediation and the bank runs that might break it down.

We depart from the standard version of the model on two points. First, less signifi-

[5] Governments across the world also often own controlling stakes in commercial banks. However, those banks keep a separate corporate structure subject to regular private law and, thus, are not equivalent to postal systems, which are usually integral parts of the public sector and subject to public law.

[6] Also, as economies grew, maintaining central bank facilities open to the public at large became increasingly challenging from an operational perspective. This was, let us remember, the time before the arrival of the internet and cheap computing. Commercial banks, in comparison, found that economic growth allowed them to gain size and offer more attractive terms to a wide range of consumers, eroding the role of central banks in providing retail services and payment facilities.

cantly, we distinguish between commercial and investment banks. This distinction clarifies our results, and nothing of substance beyond some intuition is lost by eliminating it. We could rework our main propositions by allowing a richer set of contracts between banks and consumers and interbank loans. Also, dealing separately with each type of bank opens the door to extensions of the model incorporating the observed regulatory differences between commercial and investment banks.

Second, more relevantly, we add a government-controlled central bank. Mechanically, since we are interested in CBDCs, our model must have a central bank. But there is a more subtle point at play. One primary purpose of our analysis is to verify that an efficient allocation can be implemented when a government-controlled institution competes with private banks for deposits. Such discussions can be logically separated from any consideration of digital money, even if digital money makes the debate more salient by overcoming previous logistic hurdles that opening central bank balance sheets to the public could bring. The equivalence result that emerges from our analysis can be viewed as an implementation exercise relevant for the discussions regarding the role of central banks in the provision of deposit accounts to the public at large.

Let us then introduce the main blocks of our model. There are three periods indexed by $t = 0, 1, 2$. In each period, there is a single good that can be used for either consumption or investment. The economy is populated by consumers, commercial and investment bankers, and a central bank. Let us now describe each agent in turn. Subsections 3.1 and 3.2 follow Allen and Gale (2009), except for introducing investment banks in Subsection 3.2.

## 3.1 Consumers

There is a $[0, 1]$-continuum of *ex-ante* identical consumers, each of whom has an endowment of one unit of the good in period 0. The consumer's utility function is:

$$U(c_1, c_2) = \begin{cases} u(c_1) \text{ with probability } \lambda, \\ u(c_2) \text{ with probability } 1 - \lambda, \end{cases}$$

where $c_1 \in \mathbb{R}_+$ denotes consumption in period 1 and $c_2 \in \mathbb{R}_+$ denotes consumption in period 2. The utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, strictly concave, and continuously differentiable.

This setting has an interpretation where *ex-ante* identical consumers are subject to idiosyncratic consumption shocks in $t = 1$. With probability $\lambda \in (0, 1)$, an agent is an early (impatient) consumer who values consumption in period 1; with probability $1 - \lambda$, an agent is a late (patient) consumer who values consumption in period 2. Each consumer learns her type (i.e., whether she is an early or late consumer) in period 1, which is private information. This informational asymmetry will prevent banks from discriminating among consumers. In period 1, consumers can visit a central location, which occurs sequentially in random order.

Finally, the consumer has access to a storage technology that carries one unit of the good from period 0 into one unit of the good in period 1 and, similarly, one unit of the good from period 1 into one unit of the good in period 2. This technology allows the patient consumer to withdraw her deposit from the bank in period 1 even if she prefers to consume in period 2. This feature will become relevant when we deal with bank runs.

## 3.2 Banks

There are a large number of banks that make investments on behalf of consumers. Banks have access to two types of investment technologies: a short- and a long-term technology. The short-term technology (or *short asset*) is a constant-returns-to-scale technology that takes one unit of the good at date $t = 0, 1$ and converts it into one unit of the good at date $t + 1$. We can think about this technology as storage, such as the vault in the basement of a bank.[7]

The long-term technology (or *long asset*) is a risk-free constant-returns-to-scale technology that transforms one unit of the good in period 0 into $R > 1$ units of the good in period 2. If the long-term technology is liquidated prematurely in period 1, it pays off $\kappa \in (0, 1)$ units of the good for each unit invested. We can think about this technology as a productive project that requires time to yield its fruits and subject to an early liquidation cost.

There are two types of banks: commercial banks and investment banks. Both types of banks are found in the central location. In what follows, we use the terms "banker" and "bank" interchangeably. Commercial (also called retail) banks offer demand deposit contracts (to be described momentarily) to consumers and use the proceedings to invest in short and long assets.

While also having access to the storage technology and the long assets, investment bankers only offer contracts contributing non-negative profits in period 2. To put it differently: investment banks do not provide liquidity by offering demand deposits to consumers. All future cash flows from investment banks to a counterpart are already determined in $t = 0$, and the counterpart cannot demand payments in $t = 1$, as is the case with demand deposits at commercial banks. Consequently, and since the long-asset return dominates the short-asset returns, investment bankers choose only to operate the long-term technology and maximize period-2 profits.

[7] This storage technology is the same as the one available to consumers. Banks, however, have the advantage that they can pool the risks of early and late consumers, while individual consumers cannot sign insurance contracts among themselves against their idiosyncratic liquidity risk.

We include in our interpretation of investment banks not only financial institutions that call themselves by that name, but also industrial banks (historically common in Continental Europe, Japan, and South Korea), and any other investment vehicles, such as retirement funds, whose goals are centered around long-term returns.

### 3.2.1 Deposit contracts

A commercial bank offers a deposit contract $(\hat{c}_1, \hat{c}_2) \in \mathbb{R}^2_+$ to consumers. If the consumer accepts a banker's contract, she must deposit one unit of the good with the banker in period 0. The banker invests a portion $y$ of the goods in the short-term technology, and the remaining portion $1 - y$ is invested in the long-term technology.

The banker promises to pay either $\hat{c}_1$ units of the good to a consumer on demand in period 1 or the lower of the amount $\hat{c}_2$ and the resources available to the banker, which are equally divided across all remaining consumers in period 2. The banker is committed to paying $\hat{c}_1$ units to consumers arriving in period 1 either by using the returns from the short-term investment or by liquidating long-term projects until all resources are exhausted. Any leftover resources in period 1 are invested in the short-term investment technology. The banker consumes any leftover resources in period 2.

The bankers maximize their period-2 consumption, which cannot be negative, per their choice of the deposit contract. Finally, bankers are in Bertrand competition when offering deposit contracts to consumers and, thus, make zero profits.

If we close the model here, before the introduction of a central bank, allocations would result in the same allocation as in the standard Diamond-Dybvig setup. In equilibrium, the bankers would consume nothing and the expected utility of the consumers is maximized, subject to the feasibility constraints arising from the problem above. As a result, the equilibrium allocation would be first-best efficient *ex-ante* (even if subject to possible runs *ex-post*). We refer to the contract associated with the first-best efficient allocation as $(c_1^*, c_2^*)$. Appendix A provides the details.

## 3.3 The central bank

In our model, the central bank is a government-controlled institution with access to the short-term investment technology but no access to the long asset. The central bank may, however, contract with investment banks. Furthermore, the central bank cannot rely on independent sources of taxation. Let us unpack the four components of this definition.

#### 3.3.1 The central bank is a government-controlled institution

The first component of our definition is that the central bank is a government-controlled institution. Our definition highlights effective control instead of formal ownership. Many central banks have complex ownership structures that are the product of historical accidents and political-economic bargainings. Think, for instance, about the intricate design of the Federal Reserve System in the U.S. However, in practice, all central banks in the advanced economies behave similarly due to governance rules that place them firmly under public control. This is true even when the central banks enjoy operational independence to pursue goals like price stability laid down by the legislative branch. Bartels et al. (2016) provide evidence on the irrelevance of ownership structures among 35 OECD central banks.

Government control, however, has two crucial consequences: the priority for the payment of claims due to the government (or government-controlled institutions) and protection against forced liquidation. First, a common feature of legal systems is that the government has absolute seniority when liquidating a bankrupt debtor's debts. In the U.S., this seniority is inherited from the common law and established by the federal priority statute, 31 U.S.C. §3713. Federal debt seniority has been upheld by the Supreme Court's case law since *U.S. v. State Bank of North Carolina*, 31 U.S. 29 (1832), and is interpreted broadly in favor of the U.S. This seniority holds even if other creditors have a lien on a property of the debtor or the debts are payable in the future (e.g., bonds to be paid at a future date).[8] Second, central banks cannot be forced into liquidation. A central bank can default, but it is next to impossible to use the legal system to recover any government asset to cover the creditor losses (unless these assets are outside the territory of the sovereign).

While these two consequences do not play any role in deriving our main equivalence result in Section 4, they will be important in Section 5, when we analyze bank runs. We will develop this argument when we get there.

#### 3.3.2 The central bank and the short-term asset

The first component of our definition is that we allow the central bank to access the short-term asset. All central banks have storage technologies: a simple visit to the gold vault at the Federal Reserve Bank of New York verifies this statement. Such a visit proves, as well, an important but subtle point: the vault allows for the storage of a real good, gold. The Diamond-Dybvig model deals with real goods, not nominal contracts. Scaling up storage facilities could be costly but is well within the capabilities of modern states.

[8] For more details, see https://www.justice.gov/jm/civil-resource-manual-206-priority and https://www.fiscal.treasury.gov/files/dms/debt-treatise-partII.pdf.

### 3.3.3 The central bank and the long-term asset

The third component of our definition is that we do not allow the central bank to have access to the long-term asset. This assumption captures the idea that private banks have a comparative advantage in monitoring loans to extract their full return (or, in an alternative formulation, in liquidating non-performing loans à la Diamond and Rajan, 2001). Private banks have developed over decades expertise in screening, monitoring, and liquidating productive projects. Thus, it is reasonable to assume that a central bank does not have access to the same investment opportunities as private banks.[9] The assumption also makes the economics of our paper more interesting: the central bank is not a simple clone of a commercial bank on a larger scale.

### 3.3.4 No fiscal backing

The last component of our definition is that the central bank is not fiscally backed, either directly through seigniorage, or indirectly through taxation, such as a subsidy from the general-government budget. In other words, the central bank has access only to goods provided by consumers in period 0 or the proceeds from investing these deposits. Conversely, the commercial banks are not taxed by any special levy that might make their deposits unattractive.

This fourth component will be key for our analysis and, yet, at the same time, the most fragile. If a central bank had fiscal backing, it would have an advantage with respect to commercial banks that would render the rest of the analysis trivial. At the same time, political-economic considerations are likely to be a first-order consideration in the actual running of a central bank open to all. Many political groups will lobby the bank to change its borrowing and lending policies so as to achieve the group's preferred outcomes, even if this action requires fiscal backing. Similarly, considerations about the ownership structure of central banks and dividend payments, which we argued above are mainly inconsequential at the moment, might resurface.

### 3.3.5 Deposits at the central bank

The central bank can offer the same kind of deposit contract $(d_1, d_2)$ described above to consumers, i.e., it competes for deposits with the commercial banks. Specifically, in exchange for one unit of the good at date 0, the central bank allows consumers to withdraw either $d_1 \in \mathbb{R}_+$ units in period 1 or $d_2 \in \mathbb{R}_+$ units in period 2.

[9] We could relax the assumption by allowing the central bank to have access to the long-term asset with a return $R^* < R$. Since we will show an equivalence result where, under certain circumstances, the central bank can circumvent its investment limitations, such an extension would only strengthen our argument.

From our previous discussion, the central bank is in a disadvantaged position to compete with the commercial banks. Lacking access to long-term investment opportunities, a central bank might seem less capable of engaging in liquidity transformation. We will show that, despite this disadvantage, the central bank can still compete in the retail deposit market by contracting with the investment banks to access the long-term technology, referred to as wholesale deposits. Implicit in this step is the assumption that a central bank will not face frictions in the wholesale deposit market (such as a lack of information or the expertise to operate in it). Given that central banks already participate in this market, this assumption is empirically sound. For instance, the ECB operates its refinancing operations (MROs), longer-term refinancing operations (LTROs), targeted longer-term refinancing operations (TLTROs), and pandemic emergency longer-term refinancing operations (PELTROs) with a wide variety of financial institutions.

In period 0, all bankers and the central bank play a *simultaneous* game, referred to as the demand deposit game, when offering both retail and wholesale deposit contracts. After observing the contracts posted by all bankers and the central bank, all consumers make their deposit decisions. The central bank then deposits a portion $1-x$ of each deposited unit with the investment banks, investing the remainder $x \in [0,1]$ in the short-term technology.

Let $\ell_2 \in \mathbb{R}_+$ be the contract between the central bank and the representative investment bank. In this contract, the private investment bank receives wholesale deposits from the central bank in period 0, invests them in the long asset, and returns, in period 2, $\ell_2$ to the central bank per unit deposited. The investment bank only offers this contract if it results in non-negative profits, i.e., if

$$\ell_2 \leq R. \tag{1}$$

Let $f \in [0,1]$ denote the fraction of consumers who deposit with the central bank in period 0. Because each consumer deposits one unit, the central bank receives $f$ units from all consumers. Let $\alpha \in [0,1]$ denote the fraction of consumers who decide to withdraw in period 1. The budget constraints for the central bank in periods 1 and 2 are:

$$\alpha d_1 \leq x \tag{2}$$

and

$$(1-\alpha)\, d_2 \leq \ell_2(1-x) + x - \alpha d_1, \tag{3}$$

respectively.

Notice that we do not assume a particular behavior of the central bank (such as profit or social welfare maximization) beyond the requirement that the central bank satisfies its budget constraints (i.e., no fiscal backing). In the tradition of public finance, we will postulate below an equilibrium indexed by the central bank choice of deposit contract (regardless of how it is determined) and characterize it for a class of relevant contracts.

### 3.4 The consumer's problem

Consider now the consumer's problem. An individual consumer must decide whether to deposit with the central bank, consuming either $c_1 = d_1$ upon withdrawal in period 1 or $c_2 = d_2$ in period 2, or with a commercial bank, consuming either $c_1 = \hat{c}_1$ upon withdrawal in period 1 or $c_2 = \hat{c}_2$ in period 2. Consumers choose the contract that delivers the highest *ex-ante* expected utility. If both contracts offer the same utility, then some fraction $f$ will pick the central bank, and the remaining fraction will pick the commercial banks. In that case, the fraction $f$ is indeterminate.

To complete the description of consumer behavior in period 0, we need to specify deposit decisions in the initial period and withdrawal strategies in the intermediate period. Let $h_i \in \{0,1\}$ denote the deposit decision of consumer $i \in [0,1]$, where $h_i = 0$ represents depositing with a commercial bank and $h_i = 1$ represents depositing with the central bank. Let $h = \{h_i\}_{i \in [0,1]}$ denote the profile of deposit choices in the initial period.

A withdrawal strategy for consumer $i$ is a variable $\sigma_i \in \{1,2\}$ that indicates the date at which consumer $i$ withdraws from the banking system. An early consumer always withdraws in the intermediate period with probability one. A late consumer may choose to withdraw early, depending on her beliefs about other patient consumers' actions. Let $\sigma = \{\sigma_i\}_{i \in [0,1]}$ be the complete profile of withdrawal strategies, and let $\sigma_{-i}$ denote the profile of strategies for all investors except $i$. In period 1, consumer $i$ selects a best response $\sigma_i$ in the withdrawal game, given her expectations of other agents' strategies $\sigma_{-i}$.

## 4 Equilibrium

We are ready to define an equilibrium for the economy with a central bank. We restrict our attention to symmetric equilibria, where all investment banks and all commercial banks use the same contract.

**Definition 1** *An equilibrium is a contract $\ell_2$ between the central bank and the representative investment bank, a demand-deposit contract $(\hat{c}_1, \hat{c}_2)$ for the representative commercial bank, a demand-deposit contract $(d_1, d_2)$ for the central bank, deposit decisions $h \in \{0,1\}$ in the initial period, a strategy profile $\sigma \in \{1,2\}$ for the withdrawal game in the intermediate period, a fraction $\alpha \in [0,1]$ of consumers who withdraw in period 1, and a fraction $f \in [0,1]$ of consumers depositing with the central bank such that:*

1. *In period* $0$*, given the contracts* $(\hat{c}_1, \hat{c}_2)$ *and* $(d_1, d_2)$*, each consumer* $i \in [0, 1]$ *optimally deposits one unit of the good with a financial institution by selecting the contract that offers the highest expected utility. The strategy profile* $\sigma$ *is a Nash equilibrium of the withdrawal game in period* $1$*.*

2. *Each commercial bank chooses the contract* $(\hat{c}_1, \hat{c}_2)$ *to maximize profits in period* $2$*, given* $(d_1, d_2)$*.*

3. *Each investment bank offers the contract* $\ell_2$*, provided it satisfies the non-negative profit condition (1).*

4. *The budget constraints (2) and (3) for the central bank hold with equality, given the strategy profile* $\sigma$*.*

5. *Withdrawals in period* $1$ *satisfy* $\alpha = 1 - \int_{\{i \in [0,1]: \sigma_i = 2\}} di$*.*

6. *Initial deposits* $f$ *at the central bank satisfy* $f = \int h_i di$*.*

While, in this economy, the central bank does not necessarily act as a self-interested agent, it still competes with the commercial banks for deposits from consumers. This competition could lead commercial banks to behave differently than they would do in the absence of a central bank. In particular, $0 < f < 1$ can only occur if consumers are indifferent between depositing at the central bank or a commercial bank. Nevertheless, we next show that the central bank can replicate the socially optimal contract described in Appendix A by relying on the investment banking sector.

**Lemma 4.1 (Replication of the Optimal Contract)** *The central bank replicates the socially optimal commercial bank contract by setting* $x = y^*$, $d_1 = c_1^*$, $d_2 = R(1 - y^*)/1 - \lambda$*. To offer this optimal contract, the central bank requires* $l_2 = R$*, implying zero profits to investment banks.*

**Proof.** [Lemma 4.1] The allocation $x = y^*$, $d_1 = c_1^*$, $d_2 = R(1 - y^*)/1 - \lambda$ with $l_2 = R$ is feasible by (1), (2), and (3) and with $\alpha = \lambda$ and optimal by (A.1), and (A.2). ■

Our next result shows that, in equilibrium, the socially optimal contract is always offered by some bank.

**Proposition 4.1** *In equilibrium, the socially optimal contract is offered either by the commercial banks or the central bank or both. If both the central bank and the commercial bank have customers,* $f \in (0, 1)$*, then both banks are offering the optimal contract.*

If only commercial banks offer the optimal contract, they absorb the entire deposit market ($f = 0$). Conversely, if only the central bank offers the optimal contract ($f = 1$). If both kinds of banks offer the optimal contract, by indifference, every $f \in [0, 1]$ is an equilibrium.

**Proof.** [Proposition 4.1] We first show that, in equilibrium, all commercial bankers that have deposits make zero profits and offer the socially optimal contract.

Let $\tau$ denote the commercial bank's profit. Consider the following depositor utility maximization problem, which guarantees profit $\tau$:

$$V(\tau) = \max_{(y,c_1,c_2)\in\mathbb{R}^3_+} \left[\lambda u(c_1) + (1-\lambda) u(c_2)\right]$$

subject to $0 \leq y \leq 1$, $\lambda c_1 \leq y$, and

$$(1-\lambda) c_2 \leq R(1-y) + y - \lambda c_1 - \tau.$$

Let $(y(\tau), c_1(\tau), c_2(\tau))$ denote the solution to this problem. The value function $V(\tau)$ gives the maximum expected utility for the consumer for each profit level $\tau \in \mathbb{R}_+$ for the banker. Given our assumptions on preferences and technologies, we obtain a unique interior solution characterized by:

$$u'\left(\frac{y}{\lambda}\right) = Ru'\left(\frac{R(1-y)-\tau}{1-\lambda}\right),$$

which implicitly defines $y(\tau)$. Then, we have $c_1(\tau) = \frac{y(\tau)}{\lambda}$ and $c_2(\tau) = \frac{R[1-y(\tau)]-\tau}{1-\lambda}$.

The envelope theorem implies $V'(\tau) < 0$ for any $\tau > 0$. All consumers contract only with those banks that offer the highest utility-yielding contract. All commercial banks internalize that the banks that offer the best contract absorb the entire market of deposits. If the commercial banker's profit is non-zero $\tau > 0$, then any banker who offers a contract $(y(\tau'), c_1(\tau'), c_2(\tau'))$ with $0 < \tau' < \tau$ will end up attracting all consumers and, therefore, make non-zero profits. Due to this price competition, we can only obtain an equilibrium for $\tau \to 0$. The result follows by $(y(\tau), c_1(\tau), c_2(\tau)) \to (y^*, c_1^*, c_2^*)$ pointwise: if $f < 1$, then the commercial banks with customers offer the optimal contract. Fix $V^* \equiv \lim_{\tau\to 0} V(\tau)$, the value implied by the optimal contract.

What if the central bank also attracts some, and potentially all customers, $f \in (0, 1]$? We next show that in this case, the central bank must also be offering the optimal contract.

By Lemma 4.1, the central bank is capable of offering the socially optimal commercial bank contract. Since the central bank does not have access to better technology, and by $\ell_2 \leq R$ (otherwise the investment banking sector runs losses) and $\alpha \geq \lambda$, the central bank cannot offer a contract better than the socially optimal commercial bank contract.

Suppose the central bank offers a contract $(d_1, d_2)$ subject to the feasibility constraints

(1), (2), and (3). Such a contract will result in the expected utility $\lambda u(d_1) + (1-\lambda) u(d_2)$ for the consumer. Because $V(\tau)$ is continuous, and decreasing, there exists a profit $\tau_c \geq 0$ such that the commercial bank can replicate the central bank contract $V(\tau_c) = \lambda u(d_1) + (1-\lambda) u(d_2)$. If $\tau_c = 0$, the central bank is offering the socially optimal contract. In that case, the commercial bank can make the central bank customers indifferent by also setting the optimal contract, achieving by this any $f \in [0,1]$ in equilibrium. If $\tau_c > 0$, then the central bank is not offering the socially optimal contract. Thus, the commercial bank can lure the central bank's customers away ($f = 0$) by offering a contract that implies a profit $\tau < \tau_c$. Other commercial banks can further undercut the profit and again the socially optimal contract is offered in equilibrium as $\tau \to 0$ via Bertrand competition. Further, since the central bank anticipates this competition, it may set the socially optimal contract in the first place. ■

The previous result shows how competition limits how much surplus investment banks can extract from the central bank. For any $\ell_2 < R$, the central bank fails to offer the socially optimal contract. Due to Bertrand competition with the commercial banking sector, the central bank attracts zero deposits, which leads to zero initial investment in the investment banking sector. The competition for deposits between the commercial banking sector and the central bank disciplines the investment banking sector.

Lemma 4.1 and Proposition 4.1 show that, since a central bank is capable of replicating the socially optimal contract by relying on the investment banking sector, the presence of a CBDC (or, more generally, of a central bank "open to all") can still deliver the same maturity transformation that commercial banks do in its absence. Moreover, if the socially optimal contract is offered and if all consumers behave according to their types (i.e., consumers withdraw if and only if impatient), then the socially optimal contract is indeed attained. This result holds independently of whether the contract is offered by the commercial bank or the central bank.

This equivalence result backs up some of the statements of the defenders of a CBDC: in equilibrium, we still obtain the socially optimal amount of maturity transformation. However, this equivalence result has a sinister counterpart. If the conditions behind Lemma 4.1 and Proposition 4.1 are broken, for instance, because the central bank receives fiscal backing, the competitive forces that create the right amount of maturity transformation disappear and the central bank must tread carefully in deciding how to avoid creating suboptimal levels of maturity transformation. While the central bank can do so, nothing in the model is ensuring such an outcome. In particular, there is the threat that, in the absence of the counterbalancing forces of competition, political-economic mechanisms might lead the central bank to outcomes that are suboptimal.

# 5 Runs

In the previous section, we show a fundamental equivalence result between commercial and central banks' deposit contracts. Nevertheless, the payments written in the deposit contract were feasible only if consumers behaved according to their types. But what if they do not do so?

The contract offered by the central bank is not identical in functionality to the contract offered by the commercial banks. The central bank contract is more rigid, since the central bank cannot call its loan to the investment bank. That is, the central bank can serve up to the returns of the short asset and not more. In comparison, the commercial bank has direct control over its investment: the commercial bank can serve consumers in the interim period on demand by liquidating the short and long asset. Hence, once we allow for banking panics where consumers mimic the impatient type and withdraw early to secure their deposits, the rigidity of the central bank's contract has implications for depositor incentives and equilibrium outcomes.

## 5.1 Why do commercial banks not deposit with the investment banks?

As mentioned in Subsection 3.3, commercial banks do not engage in the same contract with the investment banks for two complementary reasons deeply rooted in public law all across the world. These two reasons make central banks' assets and liabilities inherently different from the commercial banks' assets and liabilities.

First, the seniority of central bank debt means that commercial banks avoid depositing with investment banks that also receive deposits from the central bank. The simplest way to see this is to introduce, in the next paragraphs, a small probability that a long-term asset fails.

Imagine that a long-term project reaches fruitful completion and returns $R$ only with probability $\chi$. With probability $1-\chi$, the long-term project fails and, in liquidation, we can recover only $\gamma R$ where $0 < \gamma < 1$. If a commercial bank invests in a long-term project by itself, the expected return is $\chi R + (1-\chi)\gamma R$. Let us consider the case where the commercial bank has, instead, deposited in an investment bank, where the central bank has also deposited. The fraction of deposits that finance the project that comes from the central bank is $\rho > 0$. In case of the failure of the long-term project, at liquidation, the central bank is senior and obtains $min(\rho, \gamma R)$, that is, the minimum of either of all its investment ($\rho$) or the liquidation value of the project. In comparison, the commercial bank gets the residual $max(0, \gamma R - \rho)$.[10] Thus,

[10]This argument assumes that investment banks cannot discriminate *against* the central bank in terms of

Bertrand competition among the commercial banks will force them to avoid the investment bank and rely on their own long-term projects to be able to offer a better rate of return to their depositors.[11]

This problem is similar to the situation in sovereign debt lending. Since lending from multilateral institutions like the IMF and World Bank is senior to private lending, even if the latter is older in time, new multilateral programs crowd out private lending by increasing its expected loss in the event of default (see the evidence for crowding out in Krahnke, 2020).

We will skip introducing the parameters $\chi$ and $\gamma$ in our model. Except to illustrate the point above, they only complicate algebra and they can always be trivially small and still deliver the desired result.

The second reason is that the central bank cannot be forced into liquidation, but commercial banks can. A commercial bank that faces too many withdrawals in the interim period cannot argue that its assets are tied down with the investment bank (as a "commitment device" to avoid runs) to refuse payment: bankruptcy will be the consequence.[12] In comparison, a central bank can default, but it is next to impossible to use the legal system to force its liquidation. A central bank will use this protection against liquidation to prevent runs.

## 5.2 Commercial bank runs

Following Diamond and Dybvig (1983), we now show how commercial banks are prone to self-fulfilling runs. Consider a consumer who has deposited with the commercial bank. By Proposition 4.1, we know that this bank must be offering the socially efficient contract $(c_1^*, c_2^*)$. Assume that, at $t = 1$, the consumer learns that she is patient. Since her type is unobservable, she may nevertheless act as if she were impatient and decide to withdraw. When would she do this?

If only a measure $\alpha = \lambda$ of consumers withdraw, the payoffs are exactly as in her contract. If, however, patient consumers also withdraw, i.e., $\alpha \in (\lambda, 1]$, the payoffs to rolling over

interest rates. Most legal systems preclude this discrimination: governments cannot be offered worse prices than private parties for equivalent products. Notice that the investment bank cannot get around this problem by offering different tranches of returns on an underlying long-term asset. The seniority of the central bank would overrule any privately contracted seniority tranching.

[11] Interestingly, consumers do not seem to be too interested in lock-ins. For example, the well-known "closed-end funds" puzzle documents that closed-end funds that lock in capital and offer liquidity to investors through exchange-traded fund shares trade at a 10%-20% discount (Lee et al., 1991). More in general, consumers react tepidly to deposit contracts that introduce some equity components. It is not the point of our paper to analyze why. We just take these observations as given.

[12] Unless the regulatory system allows a mandatory stay in deposits, which most systems do not. Ennis and Keister (2009) highlights that, for plausibly parameter values, suspension of convertibility is not *ex-post* optimal. In fact, deposit freezes and payment rescheduling with court intervention, are usually only allowed in cases of system-wide runs.

and withdrawing deviate from the payoffs promised in the contract. This is because the commercial bank has committed to paying $c_1^*$ to every consumer who demands back her deposit in $t = 1$. To finance withdrawals above $\lambda$, the commercial bank needs to liquidate the long-term asset, which reduces payoffs to those consumers who roll over.

We say that a run on the commercial bank occurs if the commercial bank is forced to liquidate not only its investment in the short-term asset but also its investment in the long-term asset to satisfy short-term withdrawals. That is, if $\alpha c_1^* > y^* + (1-y^*)\kappa$, or equivalently if

$$\{\text{Run on Commercial Bank}\} \quad \Leftrightarrow \quad \{(\alpha - \lambda)\, c_1^* > (1 - y^*)\kappa\} \tag{4}$$

In the case of a run, consumers who roll over receive zero, and consumers who withdraw receive the payoff $c_1^*$ only with a certain likelihood due to rationing. The interpretation here is that $\alpha > \lambda$ consumers queue at the commercial bank to receive their deposits back. As explained above, the bank serves consumers by using short assets and liquidating long-term assets. If there are not enough long-term assets to liquidate, the commercial bank chooses a random subset of consumers $\frac{y^*+(1-y^*)k}{\alpha c_1^*}$ in the queue to whom it serves the payment $c_1^*$.[13] Conditional on no run, the consumers' payoffs are as in the original contract.

The payoff matrix is, then:

| Event/ Action | withdraw | rollover |
|---|---|---|
| no run | $u(c_1^*)$ | $u\left(\frac{R[(1-y^*)-(\alpha-\lambda)c_1^*/\kappa]}{1-\alpha}\right)$ |
| run | $\frac{y^*+(1-y^*)k}{\alpha c_1^*} \cdot u(c_1^*)$ | 0 |

Thus, we find a classic strategic complementarity in actions: conditional on a bank run, the payoff from withdrawing exceeds the payoff from rolling over and withdrawing deposits is optimal. Conversely, if only a few consumers withdraw, $\alpha = \lambda$, then the payoff from rolling over is larger, $c_1^* < c_2^*$ (recall Appendix A). Thus, for a patient consumer, it is optimal to roll over her deposit. We summarize this idea in the proposition below, taken from Diamond and Dybvig (1983).

**Proposition 5.1 (Commercial Bank Multiple Equilibria)** *The withdrawal game of commercial bank consumers has two pure equilibria. There is a good equilibrium in which all patient consumers roll over their deposit, $\alpha = \lambda$, and the socially optimal contract is attained. But there is also a bank run equilibrium, where all patient consumers panic and withdraw, $\alpha = 1$. In the latter case, the socially optimal contract is not attained.*

[13] Goldstein and Pauzner (2005) make this assumption about which consumers receive payments to avoid the possible complications of a sequential service constraint pointed out by Wallace (1988).

## 5.3 Central bank runs

Runs on central banks are a very different phenomenon than runs on commercial banks. During a bank panic, the central bank can only serve withdrawals up to the returns stemming from the short asset since it cannot call the loan to the investment bank.[14]

This constraint has two implications. First, the payoffs under a run on the central bank differ from payoffs under a run on the commercial bank. In the social optimum, the central bank invests $y^* = \lambda c_1^*$ in the short asset and loans the remaining $1 - y^*$ to the investment bank. Thus, the central bank cannot serve more than a measure $y^*$ of withdrawals, while the commercial bank can serve up to a measure $y^* + (1 - y^*)\kappa$ by liquidating long assets early.

Second, the incident that triggers a run on the central bank differs from the triggering event for a run on a commercial bank. A run on the central bank occurs if $\alpha c_1^* > y^*$ or, equivalently, if:

$$\{\text{Run on Central Bank}\} \quad \Leftrightarrow \quad \{\alpha > \lambda\} \tag{5}$$

In the incident of a run, the central bank can allocate only real goods of quantity $y^*$ to a measure $\alpha$ of agents. As in the commercial bank case, we assume that consumers queue and receive $c_1^*$ units with likelihood $\lambda/\alpha$.

But how do we treat the remaining consumers who were not served? Recall that the central bank cannot be forced into liquidation. Instead, a central bank can adopt two different schemes: punishment and equal treatment.

### 5.3.1 Punishment

The central bank can decide to punish consumers who contribute to the run by not paying any consumers who try to withdraw beyond the measure $\lambda$. All returns earned in $t = 2$ will go exclusively to consumers who roll over. Then, the payoff matrix becomes:

| Event/ Action | withdraw | rollover |
|---|---|---|
| no run, $\alpha = \lambda$ | $u(c_1^*)$ | $u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ |
| run, $\alpha > \lambda$ | $\frac{\lambda}{\alpha} \cdot u(c_1^*) + \left(1 - \frac{\lambda}{\alpha}\right) \cdot 0$ | $u\left(\frac{R(1-y^*)}{1-\alpha}\right)$ |

Under this scheme, a consumer who rolls over is rewarded when withdrawals are high, since she shares the total returns with fewer consumers. For $\alpha > \lambda$,

$$u\left(\frac{R(1-y^*)}{1-\alpha}\right) > u\left(\frac{R(1-y^*)}{1-\lambda}\right). \tag{6}$$

[14] The central bank does not have fiscal backing and, this being a real model, it cannot issue fiat money to meet consumers' demand. Schilling et al. (2020) relax the latter assumption by building a nominal model of CBDCs.

In addition, during a run, due to punishment and queuing, a withdrawing consumer receives $c_1^*$ only with a certain likelihood:

$$\frac{\lambda}{\alpha}u(c_1^*) < u(c_1^*). \tag{7}$$

Last, we know $c_1^* < c_2^* \equiv \frac{R(1-y^*)}{1-\lambda}$, since the utility function is concave and by $R > 1$ and equation (A.2). Altogether, for $\alpha > \lambda$:

$$\frac{\lambda}{\alpha}u(c_1^*) < u\left(\frac{R(1-y^*)}{1-\alpha}\right) \tag{8}$$

Thus, the resulting payoff matrix breaks the strategic complementarity of actions and runs do not happen. To patient consumers, rolling over is dominant, and, in equilibrium, only impatient consumers withdraw. Formally, we have our next proposition.

**Proposition 5.2 (Central Bank Equilibria: No Run under Punishment)** *If the central bank punishes consumers who contribute to a run, then the withdrawal game of central bank consumers has a unique equilibrium. All patient consumers roll over, and only impatient consumers withdraw. Runs on the central bank do not occur. The socially optimal contract is always attained when offered.*

This result is reminiscent of a mandatory stay for consumers that enforces a halt of service. Diamond and Dybvig (1983) already show that the combination of punishment and a regulatory intervention (here the deposit of the central bank in the investment bank) can deter runs.

### 5.3.2 Equal treatment

Now assume instead that the central bank forces consumers beyond measure $\lambda$ who cannot be served in the interim period to roll over their deposits. Consequently, the payoff to rolling over is independent of what other consumers do and the resulting payoff matrix becomes:

| Event/ Action | withdraw | rollover |
|---|---|---|
| no run, $\alpha = \lambda$ | $u(c_1^*)$ | $u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ |
| run, $\alpha > \lambda$ | $\frac{\lambda}{\alpha} \cdot u(c_1^*) + \left(1 - \frac{\lambda}{\alpha}\right) \cdot u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ | $u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ |

As before, the rigidity of the central bank's contract with the investment bank prevents runs from happening. We still have that $c_1^* < c_2^* \equiv \frac{R(1-y^*)}{1-\lambda}$ and rolling over is dominant if the consumer is patient. The new proposition is as follows.

**Proposition 5.3 (Central Bank Equilibria: No Run under Equal Treatment)** *Assume the central bank does not punish consumers who contribute to the run, but treats them as if they had rolled over their deposits. Then, the withdrawal game of central bank consumers has a unique equilibrium. All patient consumers roll over and only impatient consumers withdraw. Runs do not occur. The socially optimal contract is always attained when offered.*

## 5.4 Deposit monopoly

Given that consumers anticipate outcomes before depositing with the commercial or the central bank, we obtain our next result when runs occur at the commercial bank with a strictly positive probability along the equilibrium path:

**Proposition 5.4 (Central Bank Deposit Monopoly under Bank Runs)** *If the central bank offers the socially optimal contract, then via either a punishment or an equal treatment scheme, it will attract all deposits in the market away from the commercial banking sector.*

We omit the proof in the interest of space, but it is easy to see what is happening here. Even if the commercial banking sector offers the socially optimal contract to compete with the central bank, the commercial bank cannot prevent the bad bank run equilibrium from occurring. Thus, the consumers are better off depositing at the central bank. If consumers believe that commercial bank runs can happen with positive probability, they will be strictly better off by depositing at the central bank. Only in the case where consumers believe that commercial bank runs do not happen, will they be indifferent between depositing at the central bank and at the commercial bank. Since even a small tremble in beliefs will break this indifference, we do not consider this limit case further.

Since consumers internalize that the central bank contract is run-proof, the central bank enjoys market power. In particular, the central bank can offer a different deposit contract from the socially optimal one and still obtain a monopoly of all deposits. We can see this through a variational argument with respect to the result in Proposition 5.4.

Let $U_1$ be the expected utility from the run-prone commercial bank deposit contract and $U_2$ the expected utility from the run-proof central bank deposit contract. If runs occur with a strictly positive probability, $U_1 < U_2$ even if both contracts are socially optimal *ex-ante*. Thus, the central bank can find a $c_1 < c_1^*$ such that $U_1 < U(c_1) < U_2$. That is, $c_1$ maintains run-proofness but i) lowers the utility of the consumer and generates positive profits for the central bank, and ii) departs from the socially optimal allocation.

## 5.5 The risks of deposit monopoly

Why is there a risk that a central bank that enjoys deposit monopoly will use it and depart from the socially optimal allocation? Because of political-economic forces. Let us provide a concrete example. On November 27, 2019, a group of academics and various social groups signed an open letter to Christine Lagarde asking the ECB to act on climate change through a commitment "to gradually eliminating carbon-intensive assets from its portfolios." Indeed, if the central bank has market power, it can divert investment toward environmental-friendly technologies. Operationally, the central bank can use its positive profits from the previous paragraph and distribute them, through lower interest rates, to firms with a lower $CO_2$ imprint. The political process, which in Europe has recently witnessed large electoral gains by green parties, is likely to look at this policy most favorably.

The political-economic pressures are endless: diverting investment toward firms that lead in gender and racial equality, diverting investment toward firms to bridge the rural-urban divide, diverting investment toward poorer regions, diverting investment toward firms that offer a better balance of family and work life, diverting investment toward "strategic" sectors of high added value, diverting investment toward "national champions," and so forth. We can even think of less benign reasons, such as diverting investment to firms owned by the supporters of the political party in power.

In fact, the TRLOs III of the ECB already conditions the interest rate applied to the participating banks' lending patterns.[15] In the U.S., the Community Reinvestment Act of 1977 redirects loans by commercial banks toward local communities, and it is easy to see how such a program could be extended to a Federal Reserve System "open to all."

Admittedly, many of the policy goals enumerated in the previous paragraph exist because the market allocation is not optimal. For example, climate change is caused by market failures. Is it a good idea to use the central bank's monopoly power to fight the market failures behind climate change? Decades of disappointing experiences with government-owned banks subsidizing investment in otherwise laudable goals such as fighting regional disparities in Europe suggests that exploiting the central bank's market power and departing from the competitive-market maturity transformation might not be the best tool to improve social welfare. For the poor performance of these efforts, see Fernández-Villaverde et al. (2013). Pigouvian taxes, for example, seem a much cleaner alternative to fight climate change.

The central bank's resilience to runs is a double-edged sword: it avoids financial panics, but it destroys the competitive forces that discipline central banks that are "open to all."

[15] See `https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32019D0021(01)`. Monnet (2018) describes some of the long history of central banks in Europe redirecting credit toward social outcomes preferred by the political process.

# 6 Robustness

Our benchmark model can be extended in many different directions to better reflect the possible advantages and disadvantages of a CBDC. In the interest of space, we focus on the robustness of our results to fundamental runs due to asset risk and the presence of central bank regulation.

## 6.1 Fundamental runs due to asset risk

In our benchmark model, the long-term asset has no aggregate risk: it returns $R$ for sure in $t = 2$. Consider now, instead, the case where bank loans are subject to the risk that the borrower (i.e., the long-term project) fails to repay for some exogenous reason (credit risk). If information reaches markets that a borrower may not repay (i.e., the bank has issued a bad loan), we might have a run on the bank driven by information, and not, as before, by panic (Chari and Jagannathan, 1988, and Goldstein and Pauzner, 2005, hereafter GP).

To model these information-driven runs, we assume that the asset return $R$ is only paid with probability $p(\theta)$. With probability $1 - p(\theta)$, it pays zero. The function $p(\cdot)$ is strictly increasing and differentiable, with $p(0) = 0$, and $\theta \sim U[0, 1]$ is the random state of the economy. Bank consumers observe noisy, private, and correlated signals about the return probabilities before making their decision $\theta_i = \theta + \varepsilon_i$, $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$, where the noise terms $\varepsilon_i$ are i.i.d. and independent of the state distribution.

The payoffs at the commercial bank then become:

| Event/ Action | withdraw | rollover |
|---|---|---|
| no run | $u(c_1^*)$ | $p(\theta)\, u\left(\frac{R[(1-y^*)-(\alpha-\lambda)c_1^*/\kappa]}{1-\alpha}\right)$ |
| run | $\frac{(1-y^*)\kappa}{(\alpha-\lambda)c_1^*} \cdot u(c_1^*)$ | 0 |

In a global games environment, GP show that a run on the commercial bank continues to exist under these circumstances either due to miscoordination or bad fundamentals. More relevant for us, the central bank will also be subject to runs. Consider the case of equal treatment under aggregate risk:

| Event/ Action | withdraw | rollover |
|---|---|---|
| no run, $\alpha = \lambda$ | $u(c_1^*)$ | $p(\theta)\, u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ |
| run, $\alpha > \lambda$ | $\frac{\lambda}{\alpha} \cdot u(c_1^*) + \left(1 - \frac{\lambda}{\alpha}\right) \cdot p(\theta)\, u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ | $p(\theta)\, u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ |

Then, for sufficiently low state realizations, i.e., $\theta \in [0, \underline{\theta}]$, with $\underline{\theta}$ such that:

$$u(c_1^*) = p(\underline{\theta}) u \left( \frac{R(1 - y^*)}{1 - \lambda} \right), \tag{9}$$

"withdraw" is a dominant action and we have a central bank run.

Unlike the case for the commercial bank described in GP, depositors at the central bank do not play a coordination game. In equilibrium, all impatient consumers withdraw, and, thus, $\alpha$ cannot be below $\lambda$. Thus, every patient consumer is pivotal: if she withdraws, she triggers a run on the central bank. But since the investment in the long asset is protected from liquidation by the rigidity of the contract, for every $\theta > \underline{\theta}$, rolling over the deposit is dominant.

The lower dominance region for commercial bank runs is determined by the same threshold $\underline{\theta}$ given by condition (9). However, due to miscoordination, the range of state realizations for which commercial bank runs occur is larger. There exists a critical state $\theta_b \in (\underline{\theta}, 1]$ such that commercial bank runs occur for all states in $[0, \theta_b)$.[16]

Consequently, there exists an interval of state realizations $(\underline{\theta}, \theta_b)$ for which runs occur on the commercial bank, but not on the central bank under equal treatment. Consumers internalize that the central bank is safer *ex-ante*, and solely deposit with the central bank. Thus, as before, the central bank has a monopoly power that it can exploit to deviate from the socially optimal deposit contract.

Our next proposition characterizes central bank runs under the assumption that the central bank offers the socially optimal deposit contract, a natural benchmark for our investigation, generating a deposit monopoly.

**Proposition 6.1 (Deposit Monopoly under Asset Risk and Equal Treatment)** *If the central bank offers the socially optimal deposit contract, under asset risk and equal treatment, central bank runs occur for state realizations in* $[0, \underline{\theta}]$*, but do not arise for states* $[\underline{\theta}, 1]$*. Ex-ante, runs on the central bank occur less often than runs on commercial banks and, therefore, the central bank attracts all deposits.*

In the case of a run on the central bank, we do not get the optimal allocation if the asset fails to pay or if an impatient consumer is forced to wait until the long asset matures. Also, as we mentioned above, while the proposition is driven by the rigidity of the contract, the main result resembles a setting where a commercial bank invests in the risky asset directly

[16] Using a global games approach (Carlsson and Van Damme, 1993), GP show the existence of such a critical state $\theta_b > \underline{\theta}$ when depositors observe noisy private and correlated signals $\theta_i = \theta + \varepsilon_i$ about the state of the world, where $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$ are noise terms that are independent of the state $\theta$.

and a regulator has the authority to stop runs to protect asset liquidation once a critical measure of consumers has withdrawn. For further details, see Schilling (2018).

Now let us return to the case where the central bank punishes consumers who contribute to the run under asset risk. The payoff matrix becomes:

| Event/ Action | withdraw | rollover |
|---|---|---|
| no run, $\alpha = \lambda$ | $u(c_1^*)$ | $p(\theta)\ u\left(\frac{R(1-y^*)}{1-\lambda}\right)$ |
| run, $\alpha > \lambda$ | $\frac{\lambda}{\alpha} \cdot u(c_1^*) + \left(1 - \frac{\lambda}{\alpha}\right) \cdot 0$ | $p(\theta)\ u\left(\frac{R(1-y^*)}{1-\alpha}\right)$ |

Consider the state $\underline{\theta}$ defined by condition (9). For all $[\underline{\theta}, 1]$, rolling over remains dominant because, in equilibrium, all impatient types withdraw and $\alpha \geq \lambda$. Therefore:

$$\frac{\lambda}{\alpha} u(c_1^*) \leq u(c_1^*) \leq p(\theta)\ u\left(\frac{R(1-y^*)}{1-\lambda}\right) \leq p(\theta)\ u\left(\frac{R(1-y^*)}{1-\alpha}\right). \tag{10}$$

We look now at the range $[0, \underline{\theta})$. Under equal treatment, consumers' withdrawal is dominant for state realizations in $[0, \underline{\theta})$. Under punishment, however, this is not true. To see this, assume the state is realized in $\theta \in (0, \underline{\theta})$. Then, $p(\theta) > 0$. Because the pro rata share $u\left(\frac{R(1-y^*)}{1-\alpha}\right)$ goes to infinity for $\alpha \to 1$ and because $\frac{\lambda}{\alpha} \cdot u(c_1^*) \in [\lambda u(c_1^*), u(c_1^*)]$ is bounded, there exists a measure of withdrawals $\alpha(\theta) > \lambda$ such that:

$$\frac{\lambda}{\alpha} u(c_1^*) < p(\theta)\ u\left(\frac{R(1-y^*)}{1-\alpha}\right) \tag{11}$$

for all $\alpha > \alpha(\theta)$. That is, for every low state realization $(0, \underline{\theta})$, rolling over is optimal if withdrawals are sufficiently high.

We can formalize this argument in the next proposition, again under the assumption that the central bank offers the socially optimal deposit contract.

**Proposition 6.2 (Deposit Monopoly under Asset Risk and Punishment)** *If the central bank offers the socially optimal deposit contract, under asset risk, a central bank that punishes consumers for contributing to runs is at least as stable as the central bank that applies equal treatment. Therefore, under asset risk, the central bank is always more stable than the commercial banking sector and attracts all deposits away from the commercial banks.*

The intuition for this result is simple. The punishment of consumers who contribute to a run implies a reward for consumers who roll over. Moreover, the rigidity of the contract protects consumers who roll over from receiving nothing (recall that $\theta = 0$ has zero probability and, thus, the asset always pays with some probability). Hence, as soon as enough consumers withdraw, the reward for rolling over is large enough to compensate for a bad outlook of the asset. The rigidity of the contract paired with the punishment-reward scheme self-regulates withdrawals and deters runs.

The deterrence of runs is always optimal and efficient if the long asset is free of risk. Under aggregate risk, however, early asset liquidation is efficient if the asset's continuation value $p(\theta)R$ falls short of the liquidation value $\kappa$ (Allen and Gale, 1998). In this situation, the rigidity of the central bank's contract can cause harm and inefficient investment during a recession (i.e., a time of low $p(\theta)R$).

## 6.2 Central bank regulation

Our analysis has focused on the setting where the central bank can invest in both the storage technology and a loan to the investment bank. In principle, however, nothing prevents the central bank from investing in a demand-deposit contract with a commercial bank to conduct maturity transformation.

Unlike the rigid contract with the investment bank, the commercial bank serves deposit withdrawals in the interim period on demand. As consumers start withdrawing from the central bank, the central bank withdraws from the commercial bank to finance those demands from its deposits. Since the commercial bank may liquidate the entire long asset in the interim period, consumers who roll over at the central bank may receive a zero payment. As consumers internalize this feature, central bank runs become more likely *ex-ante* in comparison to the case where the central bank abstains from investing in the commercial bank and only uses the investment bank. Moreover, a run on the central bank may trigger a run on the corresponding commercial bank and vice versa. Our analysis suggests the usefulness of prohibiting the central bank from investing in maturity-transforming institutions. In that way, we can prevent a bank run cascade (contagion). For related ideas, check Dasgupta (2004).

Recall that the central bank's investment in the long asset through investment banks exposes the central bank to runs if the long asset is risky. If, by regulation, the central bank only invests in the short asset (i.e., highly liquid assets), central bank runs do not occur, but the socially optimal contract is only offered by the commercial banking sector. As we found before, central banks face a trade-off between offering the socially optimal contract and being run-proof.

## 6.3 Deposit insurance

Notice that most of our analysis regarding the effects of a CBDC would remain unchanged if we introduce deposit insurance. First, an actuarially fair deposit insurance (for instance, against fundamental asset risk) does not violate the condition of no fiscal backing of the central bank since it does not change the deposits' expected value at a commercial bank in the absence of runs.

Second, deposit insurance does not wholly eliminate bank runs. Large deposits are not covered by deposit insurance (especially when we go to the "shadow banking sector," which plays the same role as commercial banks in our models). As shown during the 2007-2008 financial crisis, we can have an even quicker bank run among large depositors than with small consumers. Thus, a CBDC may favor large depositors more than smaller ones, even if the motivation for a CBDC has often been to help the latter, not the former. Nevertheless, even small consumers may run in the presence of deposit insurance to avoid the delays and inconveniences of having to deal with the insurance corporation to recover their deposits. Therefore, a CBDC that eliminates bank runs might attract them, which confers market power on the central bank.

# 7 Conclusions

In this paper, we have investigated the implications of an account-based central bank digital currency (CBDC), focusing on its potential competition with the traditional maturity-transforming role of commercial banks, as in Diamond and Dybvig (1983). The central bank cannot invest in long-term projects itself, but instead has to rely on the expert knowledge of investment banks to do so. We have derived an equivalence result that shows that the set of allocations achieved with private financial intermediation will also be achieved with a CBDC, provided competition with commercial banks is allowed and if consumers do not panic.

Nevertheless, our equivalence result has a sinister counterpart. If the competition from commercial banks is impaired (for example, through some fiscal subsidization of central bank deposits), the central bank has to be careful in its choices to avoid creating havoc with maturity transformation. Furthermore, we have shown that the rigidity of the central bank's contract with investment banks deters runs. In equilibrium, consumers internalize this feature and exclusively deposit with the central bank such that the central bank arises as a deposit monopolist, attracting deposits away from the commercial banking sector. But this monopoly power eliminates the forces that induce the central bank to deliver the socially optimal amount of maturity transformation.

In closing, recall that the analysis in this paper has been entirely real: all contracts are denominated in real quantities. We pursue the implications of a CBDC in a nominal setting and its inter-relations with more traditional monetary policy in Schilling et al. (2020).

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# A Appendix: The commercial bank deposit contract

In this appendix, we derive the banking solution to the liquidity insurance problem, provided there are only commercial banks and consumers behave according to their type. The optimal contract takes the form of a standard demand deposit contract (i.e., a depositor can choose to withdraw from the bank either in period 1 or 2). Bertrand competition among the banks forces them to maximize the expected utility of consumers subject to feasibility conditions. Formally, the private banking arrangement solves:

$$\max_{(y,c_1,c_2)\in\mathbb{R}^3_+} \lambda u\left(c_1\right) + \left(1-\lambda\right) u\left(c_2\right)$$

subject to $0 \le y \le 1$, $\lambda c_1 \le y$, and $\left(1-\lambda\right) c_2 \le R\left(1-y\right) + y - \lambda c_1$. where $y$ is the fraction of deposits invested in the short-term technology and the remainder $1-y$ is invested in the long-term technology. With this contract, it is optimal for all patient consumers to withdraw only in period 2, provided all other patient consumers do so, i.e., $\alpha = \lambda$ is an equilibrium. This withdrawal behavior is also the first-best solution to the social planning problem where the planner knows the type of each agent. Because of the properties of the utility function, the banking allocation when patient consumers withdraw only in period 2 is given by:

$$c_1^* = \frac{y^*}{\lambda}, \quad c_2^* = \frac{R\left(1-y^*\right)}{1-\lambda}, \tag{A.1}$$

$$u'\left(\frac{y^*}{\lambda}\right) = Ru'\left(\frac{R\left(1-y^*\right)}{1-\lambda}\right). \tag{A.2}$$

This allocation coincides with the efficient allocation derived in the planner's problem. Thus, the private banking arrangement provides an efficient way to insure against the idiosyncratic liquidity risk in the economy (i.e., the event of being an early consumer).

It is well known in the literature that, since types are unobservable and unverifiable and consumers have access to the storage technology, the previously described banking arrangement also features a bank run equilibrium, where $\alpha = 1$. If we assume that the bank is required to liquidate all of its assets to satisfy the demand of consumers, and if all consumers withdraw, then, for $c_1^* > 1$, the bank cannot serve all consumers. Since consumers know the potential of a liquidity squeeze *ex-ante*, a self-fulfilling run can occur in equilibrium with late consumers withdrawing early and storing the good because they believe others will withdraw early. Ennis and Keister (2009) refer to an economy that admits a bank run equilibrium such as this one as an economy with a *fragile* banking system. Obviously, the bank run equilibrium is suboptimal compared to the first-best solution. While it is interesting to consider second-best planning problems for comparison, this is not the purpose of this paper.