

Bank of England  
Chief Economists Workshop

12 July 2021

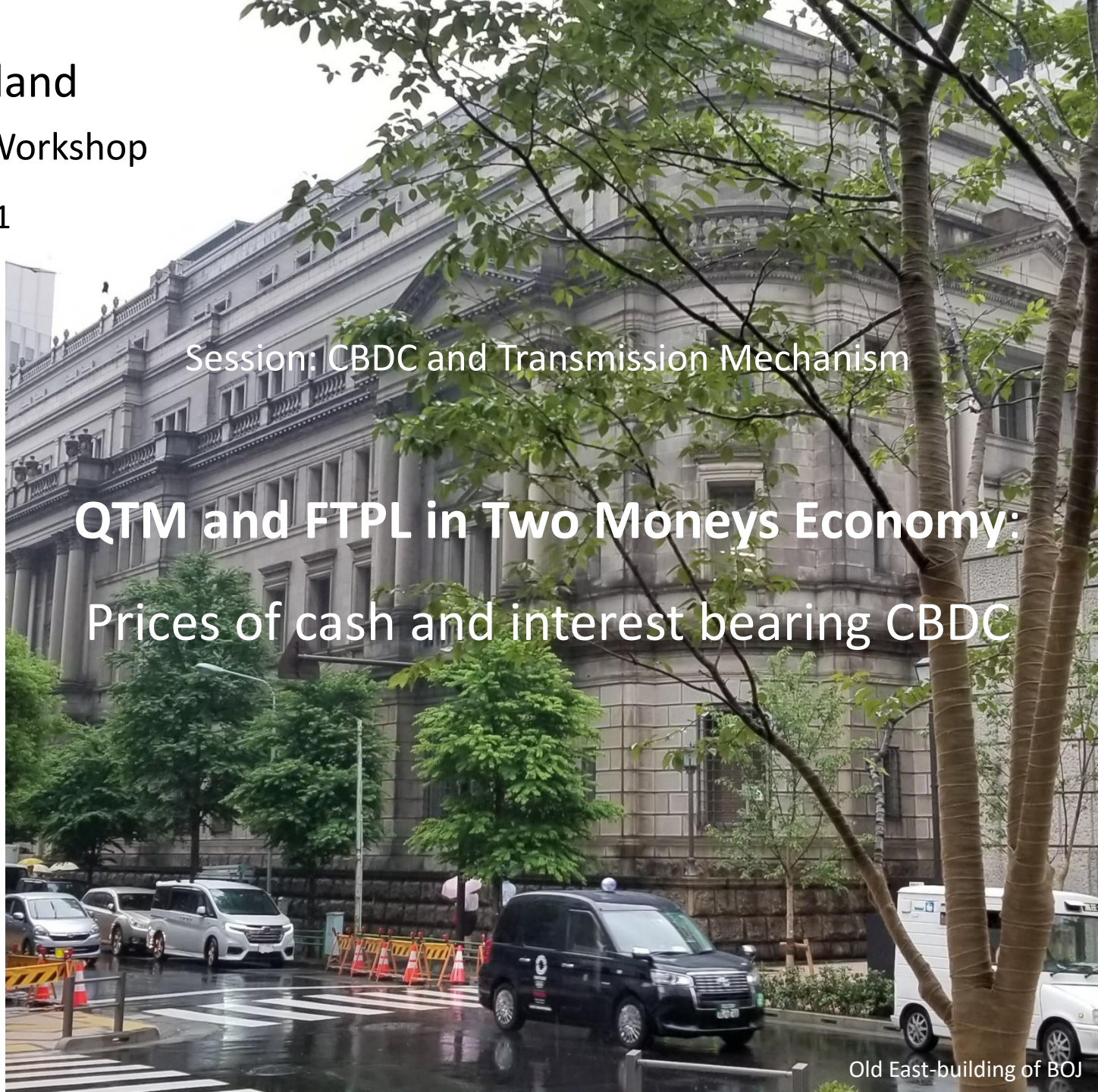
Session: CBDC and Transmission Mechanism

**QTM and FTPL in Two Moneys Economy:  
Prices of cash and interest bearing CBDC**

Head of FinTech center  
Yutaka SOEJIMA



BANK OF JAPAN



Old East-building of BOJ

# Disclaimer

1. Presentation and slide do not represent BoJ's view and thought.
2. Ideological experimentation of Macroeconomic approach to CBDC and e-money

# Three motivations

1. Prices may differ by choice of payment measures.
  - Due to *merchant fee* and *cash handling cost* at merchants
  - While consumers face the same price, merchants sales differ.
  - Gap of payment costs introduce two prices in representative agent model.
2. Exchange rate among cash and CBDC
  - The rate may *divert from unity* due to *conversion costs*.  
e.g. ATM fee for deposit money, Prepaid/Withdrawal fee for CBDC
3. Interest-bearing CBDC
  - Context of policy tools; *Gesell* type money transfer to stimulate consumption, c.f. Limit in withdraw period of old banknote; effective date of money
  - CBDC with *gift certificates convertible to CBDC* has *near* interest-bearing feature.

# Policy implications

1. Nominal interest rates differ among cash/CBDC/bond
  - Intertemporal substitution of consumption can be triggered by CBDC as well as bond.
  - Cash/CBDC dependent goods would be endogenously determined. But out of scope in this study
2. Relative real interest rate also work
  - Due to two prices and conversion cost between cash and CBDC
  - Even in zero nominal rate of CBDC, relative real rate can cause the substitution.
3. Many variants of FTPL and QTM
4. What happens in Seigniorage?

# Assumption of Model

Model is based on Makoto Saito (2021)

1.  $M_0$  : cash with zero interest rate
2.  $M_1$  : CBDC with interest rate  $i_1$
3. Exchange rate among  $M_0$  and  $M_1$ :  $e$  (per unit of M1)
4.  $P_0$  : price of cash dependent goods
5.  $P_1$  : price of CBDC dependent goods
6.  $i_1$  : interest rate of  $M_1$
7.  $i_B$  : interest rate of bond B
8. Utility function:  $u(c) + v_1(M_1/P_1) + v_0(M_0/P_0)$

$\left\{ \begin{array}{l} e > 1: \text{conversion cost} \\ e < 1: \text{conversion gift} \end{array} \right.$   
Alternative: additive term of the cost in both conversion directions

# Budget constraint for representative household

Measured by  $P_1$  ( $M_1$  numeraire)

$$B_t + M_{1t} + M_{0t}/e_t = P_{1t}(y_t - c_t - \text{tax}_t) \quad \rightarrow \text{Fiscal surplus:}$$
$$+ (1+i_{Bt})B_{t-1} \quad \text{fs} = \{\text{tax} - (y - c)\}, \text{ real term}$$
$$+ (1+i_{1t})M_{1t-1}$$
$$+(e_t/e_{t-1})M_{0t-1}/e_{t-1}$$

$M_0$  measured by  $P_1$  varies with change in  $e$

$B$  also measured by  $P_1$

No conversion cost among  $B$  and  $M_1$

i.e. No friction in digital world, but friction exists across digital and physical

# Consolidated government's budget constraint

Real consolidated gov. debt:  $rcgd_t = (B_t + M_{1t} + M_{0t}/e_t) / P_{1t}$

$$rcgd_{t-1} = rcdg_t / (1+r_t)$$

$$+ fs_t / (1+r_t)$$

Fiscal surplus

$$+ \frac{(i_{Bt} - i_{1t}) m_{1t-1}}{(1+i_{Bt})}$$

Seigniorage by CBDC

$$+ \frac{(i_{Bt} + de_t/e_{t-1}) m_{0t-1}}{(1+i_{Bt})}$$

Seigniorage by cash

= CBDC/cash holding costs

where

$$m_{1t} = M_{1t}/P_{1t}, \quad m_{0t} = M_{0t}/(e_{1t}P_{1t})$$

$$r_t = (1+i_{Bt}) P_{1t-1}/P_{1t}$$

# Consolidated government's budget constraint

$$\begin{aligned} \text{Real value of gov. debt} = & \text{PV of fiscal surplus} \\ & + \text{PV of seigniorage by CBDC} \\ & + \text{PV of seigniorage by cash} \\ & + \text{terminal value of gov. debt} \end{aligned}$$



## (Reference) Optimality conditions

Maximize  $\sum [1/(1+\rho)^{\tau-t+1} \{u(c_\tau) + v_1(m_{1\tau}) + v_0(m_{0\tau})\}]$  s.t const.

$$\text{Euler eq : } 1/(1+\rho) * u'(c_{t+1})/u'(c_t) * (1+r_t) = 1$$

$$\begin{aligned} \text{e-money mkt eqbm: } v_1'(m_{1t}) &= (i_{Bt} - i_{1t}) u'(c_t) \\ \text{cash mkt eqbm: } v_0'(m_{0t}) &= (i_{Bt} + de_t/e_{t-1}) u'(c_t) \end{aligned} \quad \begin{array}{l} \text{MU = MC} \\ \text{determine} \\ \text{money demand} \end{array}$$

$$\text{interest parity: } i_{1t} v_1'(m_{1t}) = - de_t/e_{t-1} v_0'(m_{0t})$$

$$\text{i.e. } i_{1t} = - de_t/e_{t-1} (v_0'/v_1')$$

# Focal cases

1.  $i_B > i_1 = 0$  Standard case of full seigniorage gain
2.  $i_B > i_1 > 0$  Original seigniorage used for CBDC issuance
3.  $i_B > 0 > i_1$  Seigniorage from CBDC negative rate
4.  $i_B = i_1 > 0$  Infinite CBDC demand (Another liquidity trap)

Exchange rate  $e_t$  provides points of discussion.

1.  $-de_t/e_{t-1} < 0$  Increase in  $e_t$  introduces new seigniorage
2.  $e_t > 1$  Conversion cost makes relative price of cash goods.  
c.f.  $m_{0t} = M_{0t}/(e_{1t}P_{1t})$ , see budget const.  $(e_t/e_{t-1})M_{0t-1}/e_{t-1}$   
No substitution between two goods introduces new tax.
3.  $e_t < 1$  Conversion **gift** from gov. to household

# Implications for price theory: QTM

Steady state eqbm (constant  $c_t$ )

given  $i_{Bt} > i_1 \geq 0$  and

that CB controls constant  $i_1$  and constant money growth  $\mu_1, \mu_0$ .

## QTM world

Real money demands  $m_1$  &  $m_0$  are stable.

$$dP_1/P_1 = \mu_1, \quad dP_0/P_0 = \mu_0,$$

$$P_{1t} = (M_{1t} + M_{0t}/e_t)/(m_1 + m_0) \quad \text{and} \quad P_{0t} = (e_t M_{1t} + M_{0t})/(m_1 + m_0).$$

Moneys determine price levels.

# Implications for price theory: FTPL(1)

Given  $i_{Bt} = i_1$  (or  $i_{Bt} = i_1 = de_t/e_{t-1}$  (=0 at SS): New Friedman rule)

Opportunity cost to hold CBDC is zero.

CBDC demand may exceed saturation level of money utility.

Case1: Suppose CBDC supply goes beyond the level,

## **FTPL world**

Current real balance of bond + current excess supply of  $M_1$

= PV of fiscal surplus in future

Undetermined  $M_1$  is financed by a part of the fiscal surplus.

The fiscal surplus determine  $P_t$ .

# Implications for price theory: FTPL(2)

Case2: Given  $M_{1t}$  supply which just saturates money utility, excess demand for bond may arise.

## **FTPL world**

Current real balance of bond + current excess supply of B  
= PV of fiscal surplus in future

Undetermined B is financed by a part of the fiscal surplus.

The fiscal surplus determine  $P_t$ .

There is an interim case among the case 1 and 2 with excess supplies of band and CBDC.

# Reference

Saito, Makoto, Saito M. (2021) Central Bank Cryptocurrencies in a Competitive Equilibrium Environment: Can Strong Money Demand Survive in the Digital Age?. In: Strong Money Demand in Financing War and Peace. Advances in Japanese Business and Economics, vol. 28, Springer.