

Bank of England
Chief Economists Workshop

12 July 2021

Session: CBDC and Transmission Mechanism

**QTM and FTPL in Two Moneys Economy:
Prices of cash and interest bearing CBDC**

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Disclaimer

1. Presentation and slide do not represent BoJ's view and thought.
2. Ideological experimentation of Macroeconomic approach to CBDC and e-money

Three motivations

1. Prices may differ by choice of payment measures.
 - Due to *merchant fee* and *cash handling cost* at merchants
 - While consumers face the same price, merchants sales differ.
 - Gap of payment costs introduce two prices in representative agent model.
2. Exchange rate among cash and CBDC
 - The rate may *divert from unity* due to *conversion costs*.
e.g. ATM fee for deposit money, Prepaid/Withdrawal fee for CBDC
3. Interest-bearing CBDC
 - Context of policy tools; *Gesell* type money transfer to stimulate consumption, c.f. Limit in withdraw period of old banknote; effective date of money
 - CBDC with *gift certificates convertible to CBDC* has *near* interest-bearing feature.

Policy implications

1. Nominal interest rates differ among cash/CBDC/bond
 - Intertemporal substitution of consumption can be triggered by CBDC as well as bond.
 - Cash/CBDC dependent goods would be endogenously determined. But out of scope in this study
2. Relative real interest rate also work
 - Due to two prices and conversion cost between cash and CBDC
 - Even in zero nominal rate of CBDC, relative real rate can cause the substitution.
3. Many variants of FTPL and QTM
4. What happens in Seigniorage?

Assumption of Model

Model is based on Makoto Saito (2021)

1. M_0 : cash with zero interest rate
2. M_1 : CBDC with interest rate i_1
3. Exchange rate among M_0 and M_1 : e (per unit of M1)
4. P_0 : price of cash dependent goods
5. P_1 : price of CBDC dependent goods
6. i_1 : interest rate of M_1
7. i_B : interest rate of bond B
8. Utility function: $u(c) + v_1(M_1/P_1) + v_0(M_0/P_0)$

$\left\{ \begin{array}{l} e > 1: \text{conversion cost} \\ e < 1: \text{conversion gift} \end{array} \right.$
Alternative: additive term of the cost in both conversion directions

Budget constraint for representative household

Measured by P_1 (M_1 numeraire)

$$\begin{aligned} B_t + M_{1t} + M_{0t}/e_t &= P_{1t}(y_t - c_t - \text{tax}_t) && \rightarrow \text{Fiscal surplus:} \\ &+ (1+i_{Bt})B_{t-1} && \text{fs} = \{\text{tax} - (y - c)\}, \text{ real term} \\ &+ (1+i_{1t})M_{1t-1} \\ &+(e_t/e_{t-1})M_{0t-1}/e_{t-1} \end{aligned}$$

M_0 measured by P_1 varies with change in e

B also measured by P_1

No conversion cost among B and M_1

i.e. No friction in digital world, but friction exists across digital and physical

Consolidated government's budget constraint

Real consolidated gov. debt: $rcgd_t = (B_t + M_{1t} + M_{0t}/e_t) / P_{1t}$

$$rcgd_{t-1} = rcdg_t / (1+r_t)$$

$$+ fs_t / (1+r_t)$$

Fiscal surplus

$$+ \frac{(i_{Bt} - i_{1t}) m_{1t-1}}{(1+i_{Bt})}$$

Seigniorage by CBDC

$$+ \frac{(i_{Bt} + de_t/e_{t-1}) m_{0t-1}}{(1+i_{Bt})}$$

Seigniorage by cash

= CBDC/cash holding costs

where

$$m_{1t} = M_{1t}/P_{1t}, \quad m_{0t} = M_{0t}/(e_{1t}P_{1t})$$

$$r_t = (1+i_{Bt}) P_{1t-1}/P_{1t}$$

Consolidated government's budget constraint

$$\begin{aligned} \text{Real value of gov. debt} = & \text{PV of fiscal surplus} \\ & + \text{PV of seigniorage by CBDC} \\ & + \text{PV of seigniorage by cash} \\ & + \text{terminal value of gov. debt} \end{aligned}$$

(Reference) Optimality conditions

Maximize $\sum [1/(1+\rho)^{\tau-t+1} \{u(c_\tau) + v_1(m_{1\tau}) + v_0(m_{0\tau})\}]$ s.t const.

$$\text{Euler eq : } 1/(1+\rho) * u'(c_{t+1})/u'(c_t) * (1+r_t) = 1$$

$$\begin{aligned} \text{e-money mkt eqbm: } v_1'(m_{1t}) &= (i_{Bt} - i_{1t}) u'(c_t) \\ \text{cash mkt eqbm: } v_0'(m_{0t}) &= (i_{Bt} + de_t/e_{t-1}) u'(c_t) \end{aligned} \quad \begin{array}{l} \text{MU = MC} \\ \text{determine} \\ \text{money demand} \end{array}$$

$$\text{interest parity: } i_{1t} v_1'(m_{1t}) = - de_t/e_{t-1} v_0'(m_{0t})$$

$$\text{i.e. } i_{1t} = - de_t/e_{t-1} (v_0'/v_1')$$

Focal cases

1. $i_B > i_1 = 0$ Standard case of full seigniorage gain
2. $i_B > i_1 > 0$ Original seigniorage used for CBDC issuance
3. $i_B > 0 > i_1$ Seigniorage from CBDC negative rate
4. $i_B = i_1 > 0$ Infinite CBDC demand (Another liquidity trap)

Exchange rate e_t provides points of discussion.

1. $-de_t/e_{t-1} < 0$ Increase in e_t introduces new seigniorage
2. $e_t > 1$ Conversion cost makes relative price of cash goods.
c.f. $m_{0t} = M_{0t}/(e_{1t}P_{1t})$, see budget const. $(e_t/e_{t-1})M_{0t-1}/e_{t-1}$
No substitution between two goods introduces new tax.
3. $e_t < 1$ Conversion **gift** from gov. to household

Implications for price theory: QTM

Steady state eqbm (constant c_t)

given $i_{Bt} > i_1 \geq 0$ and

that CB controls constant i_1 and constant money growth μ_1, μ_0 .

QTM world

Real money demands m_1 & m_0 are stable.

$$dP_1/P_1 = \mu_1, \quad dP_0/P_0 = \mu_0,$$

$$P_{1t} = (M_{1t} + M_{0t}/e_t)/(m_1 + m_0) \quad \text{and} \quad P_{0t} = (e_t M_{1t} + M_{0t})/(m_1 + m_0).$$

Moneys determine price levels.

Implications for price theory: FTPL(1)

Given $i_{Bt} = i_1$ (or $i_{Bt} = i_1 = de_t/e_{t-1}$ (=0 at SS): New Friedman rule)

Opportunity cost to hold CBDC is zero.

CBDC demand may exceed saturation level of money utility.

Case1: Suppose CBDC supply goes beyond the level,

FTPL world

Current real balance of bond + current excess supply of M_1

= PV of fiscal surplus in future

Undetermined M_1 is financed by a part of the fiscal surplus.

The fiscal surplus determine P_t .

Implications for price theory: FTPL(2)

Case2: Given M_{1t} supply which just saturates money utility, excess demand for bond may arise.

FTPL world

Current real balance of bond + current excess supply of B
= PV of fiscal surplus in future

Undetermined B is financed by a part of the fiscal surplus.

The fiscal surplus determine P_t .

There is an interim case among the case 1 and 2 with excess supplies of bond and CBDC.

Reference

Saito, Makoto, Saito M. (2021) Central Bank Cryptocurrencies in a Competitive Equilibrium Environment: Can Strong Money Demand Survive in the Digital Age?. In: Strong Money Demand in Financing War and Peace. Advances in Japanese Business and Economics, vol. 28, Springer.