

# Cross-Subsidies in Household Finance

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# Overview

- Ordinary people struggle to make good financial decisions.  
(Gomes, Haliassos, Ramadorai, 2021, Badarinza, Campbell, Ramadorai, 2016, Campbell, 2006.)
  - ▶ Behavioral biases/limited attention.
  - ▶ Search frictions.
  - ▶ Quality of financial advice.
  - ▶ **Complex contracts and product design.**
- Considerable heterogeneity in financial sophistication, prices paid for financial products.
  - ▶ Less sophisticated consumers make mistakes, get worse deals.
  - ▶ Cross-subsidize more sophisticated consumers who are “cut in on the deal.”  
(Miles, 2004, Gabaix and Laibson, 2006, Armstrong and Vickers, 2012.)
- Sophistication is correlated with wealth and income.  
(Campbell, Ramadorai, Ranish, 2019, Greenwald, Leombroni, Lustig, Van Nieuwerburgh, 2021).
  - ▶ **Design of financial products and contracts can materially amplify inequality.**

# A Simple Model of Cross-Subsidization

- Consumers:
  - ▶ Endowed with base good at price  $p_l$ , defaulted into add-on good at price  $p_h$ .
  - ▶ Can substitute to base good at price  $p_l$  by paying  $k$  (same utility).
  - ▶ Unit demand, aggregate normalized to 1.
- Firm:
  - ▶ Sells base good ( $p_l$ ) and add-on good ( $p_h > p_l$ ), both prices positive.
- Costs and choice:
  - ▶ Assume household costs distributed uniformly  $k \sim U(0, \bar{k})$ .
    - ▶ Define threshold  $k^* \equiv p_h - p_l$ .
    - ▶ Households with  $k > k^*$  pay add-on price  $p_h$ .
    - ▶ Households with  $k \leq k^*$  pay base price  $p_l$  in addition to cost.

## Cross-Subsidization From High to Low Cost Consumers

- Expected firm revenues:

$$\frac{k^*}{\bar{k}}p_l + \left(1 - \frac{k^*}{\bar{k}}\right)p_h \quad (1)$$

- Consider single price  $p^*$  (no add-on pricing) under expected revenue equivalence:

$$\underbrace{p^*}_{\text{Revenues under single price}} = \underbrace{\frac{k^*}{\bar{k}}p_l + \left(1 - \frac{k^*}{\bar{k}}\right)p_h}_{\text{Revenues exp. under dual price}} \quad (2)$$

- Which implies:

$$p_l < p^* < p_h \quad (3)$$

- Cross-subsidy (dual- vs. single-price) is transfer from high  $k$  to low  $k$  households:
  - ▶ When moving to single rate world, low  $k$  consumers lose  $\frac{k^*}{\bar{k}}p^* - \frac{k^*}{\bar{k}}p_l$ , equivalent to...
  - ▶ ...high  $k$  consumers' gain:  $\left(1 - \frac{k^*}{\bar{k}}\right)p_h - \left(1 - \frac{k^*}{\bar{k}}\right)p^*$

## Moving Forward

- Differential sophistication can create cross-subsidies in household finance markets. (US, Danish FRMs (Campbell, 2006, Keys et al. 2018, Andersen, Campbell, Ramadorai, Ranish, 2020).)
  - ▶ How big are these transfers in different markets (ARMs, insurance, credit)?
  - ▶ Who pays/receives them? ( $k$  across regions, income, wealth, race, gender).

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  - ▶ Who wins and loses in counterfactual alternative pricing design (here  $p^*$ )?
  - ▶ What will happen to aggregates like product take-up, average prices?
  - ▶ Broader applications such as effects of new technology (e.g., Fuster, Goldsmith-Pinkham, Ramadorai, Walther, 2021).

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- Next: application to UK ARM mortgage setting.  
ICHF: Badarinza, Campbell, Ramadorai, 2016, Badarinza, Balasubramaniam, Ramadorai, 2019.



# Refinancing Cross-Subsidies in the Mortgage Market

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Disclaimer: Any views expressed here are not meant to represent those of the Bank of England or members of its policy committees.

## This Paper

- Studies mortgage refinancing using rich and granular administrative data in the U.K. on the total outstanding stock of mortgages.
  - ▶ U.K. has an ARM system with initial teaser rates fixed for 2-5 years.
  - ▶ Initial discounted teaser rates automatically adjust to high variable revert rate after fixation period unless refinanced into another teaser rate.
  - ▶ Prompt refiners and sluggish refiners suggests presence of cross-subsidies.
- Builds a partial equilibrium model of the UK mortgage market with heterogeneity in refinancing costs and heterogeneous valuations for housing.
- Structurally estimates model parameters to match moments in the data.
- Uses parameters to assess size of cross-subsidy by comparing to a counterfactual single-rate market design.
- Shows how cross-subsidies vary across income groups and areas of the U.K.; provides evidence that they are regressive.

## Compared to Simple Model: Richer Model of Household Refinancing

- Fixed household parameter  $k$  unrealistically implies “terminal refinancing date” given loan amortization.
  - ▶ Model both a persistent component of  $k$  as well as a random shock to “refinancing attention” (as in Andersen, Campbell, Nielsen, Ramadorai, 2020).
  - ▶ Captures empirical transitions between discounted and revert rates.
- In the data, households pick different loan sizes.
  - ▶ Model the intensive margin decision assuming a distribution of value for housing; households trade off housing utility against increased mortgage cost.
  - ▶ Match outstanding mortgage stock, not just numbers on different tariffs.
- Households refinance multiple times in the data, not just once.
  - ▶ Model is dynamic, describing refinancing over the life of the mortgage.
  - ▶ Assume that model is in steady state to simplify structural estimation and moment-matching.

## Institutional Framework and Data

# The UK Mortgage Market

- Mortgages pay “teaser-rate” for initial fixation period (2-5 years), which reverts to high standard variable rate (SVR) unless refinanced after fixation period.
  - ▶ Similar to credit cards, cellphone/electricity plans (Armstrong and Vickers, 2012).
  - ▶ Significant refinancing incentives at the end of fixed period (Cloyne et al., 2019).
  - ▶ High prepayment penalties deter early refinancing.
- Pricing based on product characteristics: lender, rate type, fixation period, loan-to-value.
- Prices homogenous across borrowers conditional on product (different from US).
- 2019 FCA Mortgage Market Study notes that remortgaging is easy, and most often with initial lender.
  - ▶ Filter ~40K of 2M on reset rate that cannot refinance (“mortgage prisoners”).
  - ▶ Filter potentially constrained borrowers (high LTV, payment shortfalls etc).

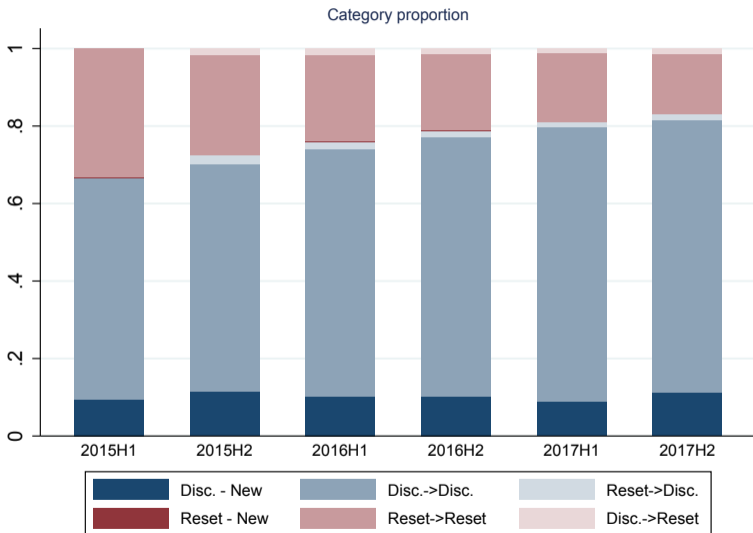
# Example

Mortgages available	Maximum loan to value	Initial rate	Differential to Bank of England base rate (currently 0.25%)	Then changing to Santander's Standard Variable Rate	The overall cost for comparison is (APR)	Product fee	Additional benefits	Early repayment charge (ERC)	Monthly cost	Compare up to three rates
2 year fixed rate	80%	1.64%	n/a	4.49%	4.1%	£999	Free valuation and £250 cashback	3% + Repay £250 cashback	£813	<input type="checkbox"/>
2 year fixed rate	85%	1.74%	n/a	4.49%	4.1%	£999	Free valuation and £250 cashback	3% + Repay £250 cashback	£823	<input checked="" type="checkbox"/>
2 year fixed rate	85%	2.14%	n/a	4.49%	4.2%	£0	Free valuation and £250 cashback	3% + Repay £250 cashback	£861	<input checked="" type="checkbox"/>
2 year fixed rate First Time Buyer Exclusive	90%	2.24%	n/a	4.49%	4.2%	£999	Free valuation and £250 cashback	3% + Repay £250 cashback	£871	<input type="checkbox"/>
5 year fixed rate	80%	2.44%	n/a	4.49%	4.0%	£999	Free valuation and £250 cashback	5% + Repay £250 cashback	£891	<input type="checkbox"/>

## Data

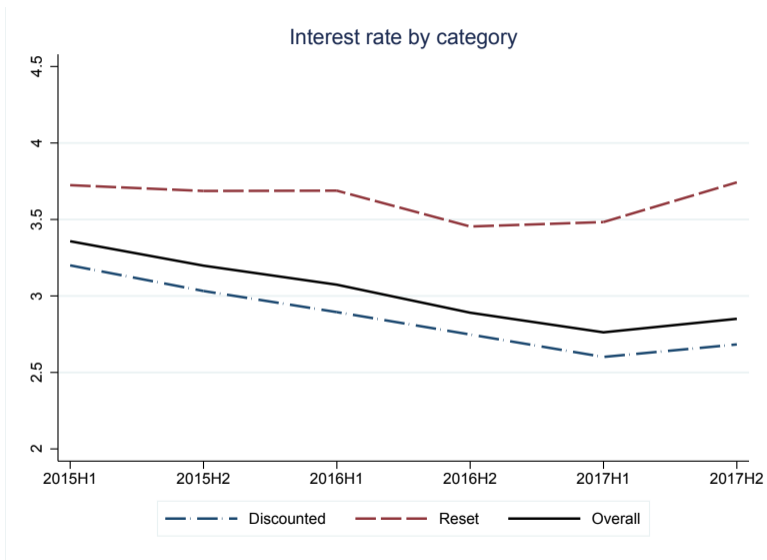
- Data sourced from the Financial Conduct Authority (FCA) (Dataset PSD: 007).
- Tracks stock of all outstanding loans issued by regulated financial institutions in the U.K. at a semi-annual frequency.
- Data from June 2015–December 2017, we mainly utilize stock at June 2015 (2015H1) in this draft.
- Eliminate buy-to-let and tracker mortgages, focus on discounted and revert rate mortgages.
- 3.59M mortgages, £470B aggregate debt in 2015H1 (filtering refinancing-constrained borrowers).
- Granular mortgage details, tracked over time, limited borrower characteristics (age, income, location).
- Used in a range of studies (Cloyne et al., 2019; Robles-Garcia, 2019; Benetton, 2021).

# Fraction of Mortgage Stock on Discounted and Reset Rates





# Interest Rates in Different Categories



## An Outline of the Model

## Model: Assumptions

- Households:
  - ▶ Pay a fixed cost  $k_{i,t} = k_i \varepsilon_{i,t}$  at the point of refinancing:
    - ▶  $k_i$  is persistent cost for household  $i$ .
    - ▶  $\varepsilon_{i,t}$  household-specific multiplicative shock. Non-negative, iid with  $f(\varepsilon_{i,t})$ .
  - ▶ Household per-period housing value  $v_i$ ; valuations, costs described by joint cdf  $G(v_i, k_i)$ , pdf  $g(v_i, k_i)$ .
- Mortgages:
  - ▶ Last for  $T$  periods.
  - ▶ Discounted rate  $r_d$  for an initial  $T_d$  periods.
  - ▶ Reset rate  $R > r_d$  after  $T_d$  periods, if the household does not refinance.
- Choices:
  - ▶ Maximize flow utility:  $v_i h_i^\alpha - m(l_i, r, T)$ .  $0 < \alpha < 1$  parameter governs housing utility. At each  $T_d$  households decide on which  $r$  (i.e.,  $R$  or  $r_d$ ).
  - ▶ At  $t = 0$ , households choose loan size  $l_{i,0}$  to finance a property priced at  $h_i$ , where  $h_i = \omega l_{i,0}$ . which implies a fixed LTV =  $\frac{1}{\omega}$ .

## Optimal Refinancing

- Household refinancing follows a threshold rule. In the last period refinance if and only if  $k_{i,T} < k_i^*(T) = m(l_{i,T-1}, R, 1) - m(l_{i,T-1}, r_d, 1)$ .
- Defines Bellman Equation which we solve using backward induction.
- For a given borrower and refinancing cost shock, larger loans provide greater incentives to refinance, and over time, the appeal of refinancing decreases.

### Value functions

- Households choose an initial loan size that maximizes their discounted utility.
  - ▶ Optimal loan size  $l_{i,0}^*(v_i, k_i)$  depends directly on households' housing valuations  $v_i$  and indirectly on their refinancing costs  $k_i$  through anticipated interest rates.
  - ▶ Consumers participate in mortgage market if utility with a mortgage greater than alternative of renting (extensive margin condition).

# Structural Estimation

## Outline of Structural Estimation

- The model allows for a convenient aggregation of outstanding mortgages.
- There is a nice mapping back to the mortgage stock data, so we can match moments from these data under a steady state assumption.
- We estimate key parameters that capture the distributions of housing valuations and refinancing costs, and the variance of refinancing cost shocks.
- Focusing on the stock rather than flows offers several advantages:
  1. Facilitates computing aggregate lender revenues.
  2. Estimated parameters are not influenced by changes over short periods of time.
  3. Captures behavior across the maturity spectrum.
  4. Cost of focusing on stock is the steady-state assumption.

## Model Fit

	DATA	MODEL
MEAN LOAN BALANCE, DISCOUNTED RATE	140,647	143,697
STANDARD DEVIATION LOAN BALANCE, DISCOUNTED RATE	105,062	106,551
MEAN LOAN BALANCE, RESET RATE	112,692	113,741
STANDARD DEVIATION LOAN BALANCE, RESET RATE	79,684	76,546
MEAN REMAINING YEARS, DISCOUNTED RATE	20.57	18.63
STANDARD DEVIATION REMAINING YEARS, DISCOUNTED RATE	7.73	7.91
MEAN REMAINING YEARS, RESET RATE	16.84	15.56
STANDARD DEVIATION REMAINING YEARS, RESET RATE	6.95	7.40
SHARE OF MORTGAGES ON DISCOUNTED RATE, 0-5 PERCENTILE	52.72	52.82
SHARE OF MORTGAGES ON DISCOUNTED RATE, 5-25 PERCENTILE	56.36	58.03
SHARE OF MORTGAGES ON DISCOUNTED RATE, 25-50 PERCENTILE	61.48	60.12
SHARE OF MORTGAGES ON DISCOUNTED RATE, 50-75 PERCENTILE	67.76	63.73
SHARE OF MORTGAGES ON DISCOUNTED RATE, 75-95 PERCENTILE	73.77	72.10
SHARE OF MORTGAGES ON DISCOUNTED RATE, 95-100 PERCENTILE	81.19	83.66
TRANSITION FROM RESET RATE TO DISCOUNTED RATE	16.52	16.42
SHARE OF OWNERS	63.13	64.50

## Cross-Subsidies and How They are Distributed



## Model: Computing Cross-Subsidies

- To compute cross-subsidies, we consider a counterfactual in which all households pay a single constant interest rate  $r_f$  (we consider different values of  $r_f$ , below).
- Optimal loan size  $l_{i,0}^{**}(v_i, k_i)$  in this case maximizes the value function at origination evaluated at  $k_i = 0$ .
- We can compute the aggregate number and balance of mortgages in this scenario.
- We also apply the model to groups  $j = 1, \dots, J$  of households, i.e.:

$$r_f \sum_{j=1}^J Q_j(r_f) = \sum_{j=1}^J (r(Q_{0j}(r) + Q_{1j}(r)) + RQ_{2j}(R)),$$

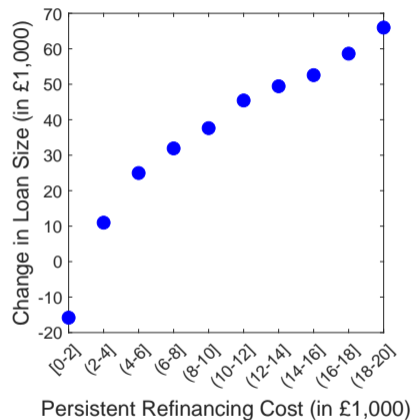
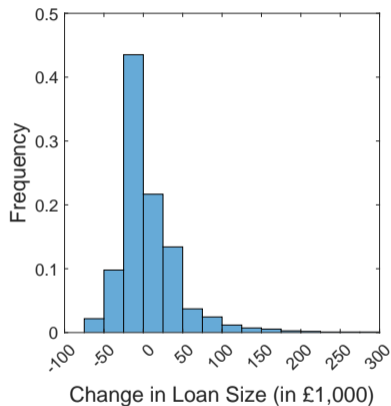
which we can use to calculate group-specific (e.g., income, geographic regions) cross-subsidies.

## Single Interest Rate Scenarios

Several different values considered for single interest rate  $r_f$ :

1. The average discounted rate, i.e.,  $r_f = 333$  bps.
2. The loan-weighted average interest rate observed in the data.
3. The rate that yields the same revenue as the composite of the populations on the discounted rate and the reset rate (constant revenue assumption, requires model to compute).
4. The average reset rate, i.e.,  $r_f = 383$  bps.

# Differences in Mortgage Size, Dual-Rate to Single-Rate



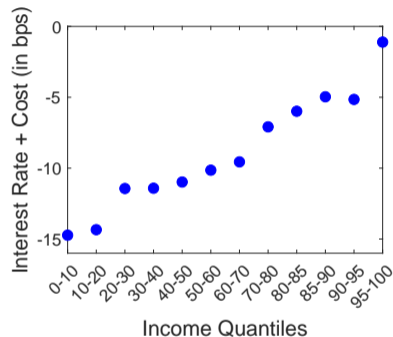
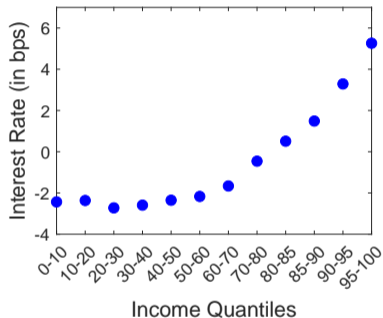
Notes: Left panel shows distribution of changes in loan sizes at origination between single rate counterfactual and baseline dual rate market. Right panel shows average change in loan sizes for households with different  $k_i$ 's (in bins of £1,000) using revenue-equivalence (Panel B), UK-wide.

## Cross-Subsidies Across Income and Regional Groups

- Next, we re-estimate the model for a set of subgroups of the data:
  - ▶ 12 income groups (10 income deciles, top decile further subdivided into two groups).
  - ▶ 12 U.K. regions and devolved administrations.
- Using group-specific parameters, calculate:
  - ▶ Average interest rate difference (under single- vs dual-rate) for each group.
  - ▶ Average loan balance difference.
  - ▶ Average annual payment difference...
- There is considerable *within-group* variation in the data, but in this exercise, focus on *across-group* distribution of cross-subsidies.

# Differences in Outcomes, Dual-Rate to Single-Rate, Income Groups

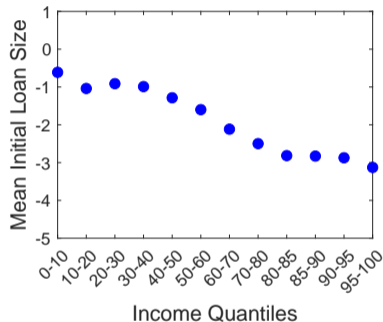
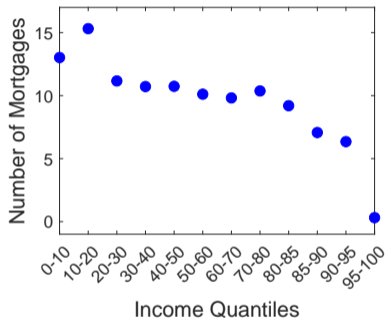
Higher average rates, seemingly small differences in net rates.



- Revenue equivalence implies that interest rates increase in aggregate since balances fall more for high-loan borrowers (next).
- Difference between raw changes in interest rates and net including  $k$ .

# Differences in Outcomes, Dual-Rate to Single-Rate, Income Groups

Significant adjustments to mortgage debt respond to changes in net interest rates.



- Larger regressive effect for mortgage debt (6-8pp) than net rate (10bp).
- Driven by extensive margin changes for low-income groups, intensive margin changes for high-income groups.

## Descriptive Statistics, Income Groups

	INC. LEVEL	PROP. (DISC.)	DISC. RATE	RESET RATE	BAL.
0-10	24,604	0.66	3.45	3.98	60,144
10-20	29,483	0.64	3.45	3.90	73,839
20-30	34,564	0.64	3.44	3.86	84,721
30-40	39,581	0.64	3.41	3.82	94,547
40-50	44,986	0.64	3.37	3.77	104,950
50-60	51,327	0.64	3.34	3.73	116,473
60-70	59,412	0.64	3.30	3.69	130,123
70-80	71,261	0.66	3.25	3.65	149,041
80-85	80,290	0.66	3.19	3.62	169,791
85-90	94,142	0.67	3.13	3.61	190,849
90-95	122,708	0.68	3.04	3.59	227,788
95-100	214,886	0.69	2.88	3.52	345,904

## Cross-Subsidy Mechanisms

- Cross-subsidy calculation compares outcomes in single- and dual-rate worlds
- Whether households benefit under the single-rate counterfactual depends on both their refinancing cost  $k$  and valuation for housing  $v$ .
- The single-rate world unambiguously benefits those with high  $k$  because these households spent most of their time on the high reset rate in the dual-rate world.
- High  $v$  households—typically higher income—benefit from the status quo since  $\uparrow v \rightarrow \uparrow l_0 \rightarrow \uparrow k^*(\bullet) \implies$  more time spent on the discounted rate.
- Large effects on extensive margin for low-income households who enter the single-rate market but are deterred from dual-rate market.
- Large effects on intensive margin for high-income households who take smaller loans in single-rate market.



## Summary

- Structurally estimate refinancing cross-subsidies in the U.K. mortgage market.
- Match broad features of the data, with realistic parameters that highlight significant cross-household variation in refinancing costs.
- Under counterfactual single-rate system:
  - ▶ High-refinancing cost borrowers benefit; their loan balances increase significantly relative to dual-rate status quo.
  - ▶ Higher income groups and wealthier regions of the U.K. see bigger increases in rates than poorer groups/regions.
- Counterfactual comparisons also show that loan sizes (and takeup) grow more for poorer groups/regions, and shrink for richer groups/regions.  
“Democratization” of mortgage takeup in single-rate world.

## Where Next?

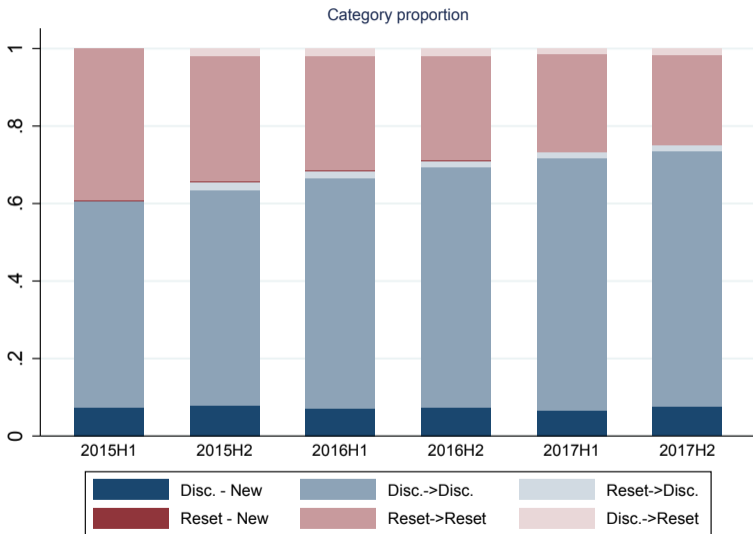
- Broader point: Design of household financial system can affect income and wealth inequality. This contributes to unpopularity of finance; political consequences.
- Calls for more careful study of:
  - ▶ The preferences, beliefs, and constraints of households.
  - ▶ Financial product and contract design.
  - ▶ Frictions impeding efficient use of these products and contracts.
- Some reflections:
  - ▶ Cross-subsidies are ubiquitous (FRMs, ARMs, insurance (Gottlieb–Smetters, 2021)).
  - ▶ Regressive outcomes can show up in less obvious places (this paper).
  - ▶ Seemingly too-complex contracts can also have unintended positive consequences given non-standard household preferences (e.g., Calvet et al. 2021).
- We need more empirical and theoretical work on these topics.

# Appendix

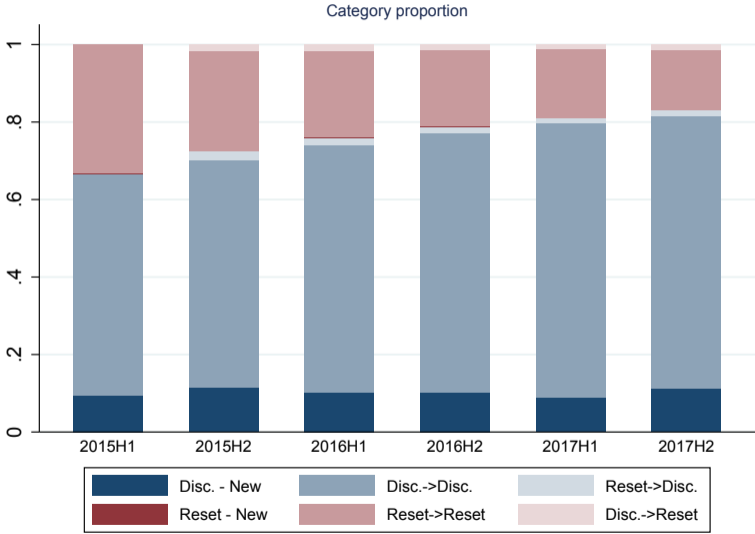
## Data Filters to Remove Refinancing-Constrained Households

	2015H1	2015H2	2016H1	2016H2	2017H1	2017H2
All	100%	100%	100%	100%	100%	100%
(1) LTV $\geq$ 100	1.3%	1.2%	1.6%	2.0%	1.9%	3.2%
(2) LTV $\geq$ 95	2.3%	1.9%	2.2%	2.4%	2.4%	3.6%
(3) Balance $\leq$ 10000	1.3%	1.2%	1.3%	1.3%	1.4%	1.5%
(4) Balance $\leq$ 30000	6.5%	6.5%	6.7%	6.7%	6.9%	6.9%
(5) Short-term arrears	2.0%	1.7%	1.7%	1.6%	1.5%	1.5%
(6) Non-performing	5.5%	5.0%	3.9%	3.9%	3.8%	3.6%
All excl. (2),(4),(6)	86.4%	87.2%	87.7%	87.4%	87.4%	86.3%

# Fraction of Mortgages on Discounted and Reset Rates



# Fraction of Mortgage Stock on Discounted and Reset Rates



## Descriptive Statistics, U.K. Regions and Devolved Administrations

	PROP. (DISC.)	DISC. RATE	RESET RATE	BAL.
NORTHERN IRELAND	0.59	3.42	4.00	88,790
NORTH EAST (ENGLAND)	0.60	3.48	3.77	93,488
SCOTLAND	0.61	3.40	3.83	102,084
WEST MIDLANDS (ENGLAND)	0.61	3.39	3.67	110,089
WALES	0.62	3.42	3.78	100,026
NORTH WEST (ENGLAND)	0.63	3.44	3.82	103,406
YORKSHIRE AND THE HUMBER	0.64	3.44	3.85	100,650
EAST MIDLANDS (ENGLAND)	0.64	3.41	3.71	106,786
SOUTH WEST (ENGLAND)	0.67	3.31	3.61	128,260
EAST OF ENGLAND	0.69	3.24	3.72	146,888
SOUTH EAST (ENGLAND)	0.69	3.19	3.66	165,072
LONDON	0.69	3.00	3.83	207,592

## Value Functions

- Value function at time  $T$  with refinancing threshold  $k_i^*(T)$ :

$$\begin{aligned}V_T(k_i, l_{i,T-1}) &= \mathbb{E}_{\varepsilon_{i,T}} \left[ \max \left\{ -m(l_{i,T-1}, R, 1), -m(l_{i,T-1}, r_d, 1) - k_i \cdot \varepsilon_{i,T} \right\} \right] \\ &= \mathbb{P}(k_i \cdot \varepsilon_{i,T} \leq k_i^*(T)) \cdot (-k_i \cdot \mathbb{E}[\varepsilon_{i,T} | k_i \cdot \varepsilon_{i,T} \leq k_i^*(T)] - m(l_{i,T-1}, r_d, 1)) \dots \\ &\quad \dots + (1 - \mathbb{P}(k_i \cdot \varepsilon_{i,T} > k_i^*(T))) \cdot (-m(l_{i,T-1}, R, 1))\end{aligned}$$

- Similarly, define  $V_t(k_i, l_{i,t-1})$  for a generic period:

$$\begin{aligned}V_t(k_i, l_{i,t-1}) &= \dots \\ &\dots \mathbb{E}_{\varepsilon_{i,t}} \left[ \max \left\{ -m(l_{i,t-1}, R, T-t+1) + \beta \cdot V_{t+1}(k_i, l_{i,t-1} \cdot (1+R) - m(l_{i,t-1}, R, T-t+1)), \dots \right. \right. \\ &\quad \left. \left. \dots - m(l_{i,t-1}, r_d, T-t+1) - k_i \cdot \varepsilon_{i,t} + \beta \cdot V_{t+1}(k_i, l_{i,t-1} \cdot (1+r_d) - m(l_{i,t-1}, r_d, T-t+1)) \right\} \right]\end{aligned}$$



## Model: Aggregation and the Stock of Mortgages

- Define three groups ( $g$ ) of mortgages, and derive the aggregate number  $N_g(\cdot)$  and aggregate balance  $Q_g(\cdot)$  of mortgages in each group.
  - ▶ Expressions can be directly mapped to observed stock of mortgages in each category, under the assumption that the market is in steady-state.
- First, recursively define the endogenous distribution  $H_t(\cdot)$  of loan balances after  $t$  periods from their origination, given evolution of loan balances and refinancing policy:

$$H_0(z) = \iint_{\{(v_i, k_i): v_i \geq v_i^*(k_i) \cap l_{i,0}^*(v_i, k_i) \leq z\}} dG(v_i, k_i),$$

$$H_t(z) = \int_{\{l_{i,t-1}: l_{i,t}(r, l_{i,t-1}) \leq z\}} dH_{t-1}(l_{i,t-1}).$$

## Model: Aggregation and the Stock of Mortgages - Group 0

- Group 0: households with mortgage of initial size  $l_{i,0}^*(v_i, k_i)$ , on initial discount period.

$$N_0(r_d) = M \int_{-\infty}^{+\infty} \int_{v_i^*(k_i)}^{+\infty} dG(v_i, k_i),$$

$$Q_0(r_d) = N_0(r_d) \int_0^{+\infty} z dH_0(z) = M \int_{-\infty}^{+\infty} \int_{v_i^*(k_i)}^{+\infty} l_{i,0}^*(v_i, k_i) dG(v_i, k_i).$$

- ▶ Intuition: recall mass  $M$  of households entering the market in each time period. The fraction of them getting (discounted-rate) mortgages equals those of them satisfying the extensive margin condition  $v_i > v_i^*(k_i)$ , with the outer integral integrating across the  $k_i$  distribution.

## Model: Aggregation and the Stock of Mortgages - Group 1

- Group 1: Mortgages of households who refinance into paying the discounted rate.
- In each period  $t \in \{1, \dots, T - 1\}$ , the number  $N_{1,t}(r_d)$  of mortgages is:

$$N_{1,t}(r_d) = N_0(r_d) \int_{\{l_{i,t}: r(l_{i,t}, k_{i,t}) = r_d\}} dH_t(l_{i,t})$$

- ▶ Intuition: combines all borrowers who have a refinancing cost  $k_{i,t}$  below the cutoff point  $k_i^*(t + 1)$ , and thus have policy functions  $r(l_{i,t}, k_{i,t}) = r_d$ .
  - ▶ Thus, the aggregate number  $N_1(r_d)$  of mortgages is  $N_1(r_d) = \sum_{t=1}^{T-1} N_{1,t}(r_d)$
- The aggregate balance of this group is the sum of balances on  $r_d$ :

$$Q_{1,t}(r_d) = N_0(r_d) \int_{\{l_{i,t}: r(l_{i,t}, k_{i,t}) = r_d\}} l_{i,t} dH_t(l_{i,t}).$$

- ▶ Thus, the aggregate balance equals  $Q_1(r_d) = \sum_{t=1}^{T-1} Q_{1,t}(r_d)$ .

## Model: Aggregation and the Stock of Mortgages - Group 2

- Group 2: Mortgages of households who did not refinance, and pay the reset rate.
- In each period  $t \in \{1, \dots, T - 1\}$ , the number  $N_{2,t}(R)$  of mortgages is:

$$N_{2,t}(R) = N_0(r_d) \int_{\{l_{i,t}: r(l_{i,t}, k_{i,t})=R\}} dH_t(l_{i,t}),$$

- ▶ Intuition: set of borrowers who have refinancing cost above cutoff point  $k_i^*(t + 1)$ , and thus have policy functions  $r(l_{i,t}, k_{i,t}) = R$ .
- ▶ Thus, the aggregate number of households who pay the reset rate equals  $N_2(R) = \sum_{t=1}^{T-1} N_{2,t}(R)$ .
- The aggregate balance of this group is the sum of balances on  $R$ :

$$Q_{2,t}(R) = N_0(r_d) \int_{\{l_{i,t}: r(l_{i,t}, k_{i,t})=R\}} l_{i,t} dH_t(l_{i,t}).$$

- ▶ Thus, the aggregate balance equals  $Q_2(R) = \sum_{t=2}^T Q_{2,t}(R)$ .

## Computing Cross-Subsidies: Single Interest Rate

- To compute cross-subsidies, we consider a counterfactual in which all households pay a single constant interest rate  $r_f$  (we consider different values of  $r_f$ ).
- Optimal loan size  $l_{i,0}^{**}(v_i, k_i)$  maximizes the value function at origination evaluated at  $k_i = 0$ . We get aggregate number and balance of mortgages:

$$N(r_f) = MT \int_{-\infty}^{+\infty} \int_{v_i^{**}(r_f)}^{+\infty} dG(v_i, k_i),$$
$$Q(r_f) = M \sum_{t=1}^T \gamma_{r_f}(t-1) \int_{-\infty}^{+\infty} \int_{v_i^{**}(r_f)}^{+\infty} l_{i,0}^{**}(v_i, k_i = 0) dG(v_i, k_i),$$

where

$$\gamma_{r_f}(t-1) = \frac{l_{i,t}(r_f, l_{i,0})}{l_{i,0}} = \frac{(1+r_f)^T - (1+r_f)^t}{(1+r_f)^T - 1},$$

is the beginning-of-period- $t$  share of the initial loan still to be repaid, and  $v_i^{**}(r_f)$  is the household that is indifferent between getting a mortgage or not in this constant rate scenario, i.e.:  $W_0(v_i^{**}, k=0, l_{i,0}^{**}(v_i^{**}, k_i=0)) = \frac{\bar{u}}{1-\beta}$ .

## Descriptive Statistics, Income Groups (2017H1)

	INC. LEVEL	PROP. (DISC.)	DISC. RATE	RESET RATE	BAL.
0-10	25,435	0.75	2.90	3.78	61,726
10-20	30,470	0.74	2.87	3.68	76,792
20-30	35,737	0.75	2.85	3.62	88,696
30-40	40,962	0.75	2.82	3.57	99,790
40-50	46,597	0.76	2.77	3.52	111,548
50-60	53,167	0.76	2.73	3.48	124,665
60-70	61,536	0.77	2.68	3.43	140,210
70-80	73,712	0.78	2.63	3.38	161,470
80-85	82,981	0.78	2.57	3.35	183,833
85-90	97,194	0.78	2.53	3.34	206,593
90-95	126,414	0.79	2.47	3.33	246,039
95-100	216,018	0.79	2.39	3.28	370,173

## Descriptive Statistics, U.K. Regions/Devolved Administrations (2017H1)

	PROP. (DISC.)	DISC. RATE	RESET RATE	BAL.
NORTHERN IRELAND	0.71	2.82	3.71	92,513
NORTH EAST (ENGLAND)	0.71	2.87	3.56	97,234
SCOTLAND	0.72	2.80	3.59	105,329
WEST MIDLANDS (ENGLAND)	0.74	2.79	3.43	116,606
WALES	0.73	2.85	3.55	104,046
NORTH WEST (ENGLAND)	0.74	2.85	3.59	108,855
YORKSHIRE AND THE HUMBER	0.74	2.85	3.62	105,504
EAST MIDLANDS (ENGLAND)	0.76	2.79	3.45	113,622
SOUTH WEST (ENGLAND)	0.79	2.72	3.35	136,328
SOUTH EAST (ENGLAND)	0.80	2.59	3.39	178,564
EAST OF ENGLAND	0.80	2.62	3.44	160,469
LONDON	0.79	2.46	3.58	227,780

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