FTPL Puzzle Redux with Market Segmentation

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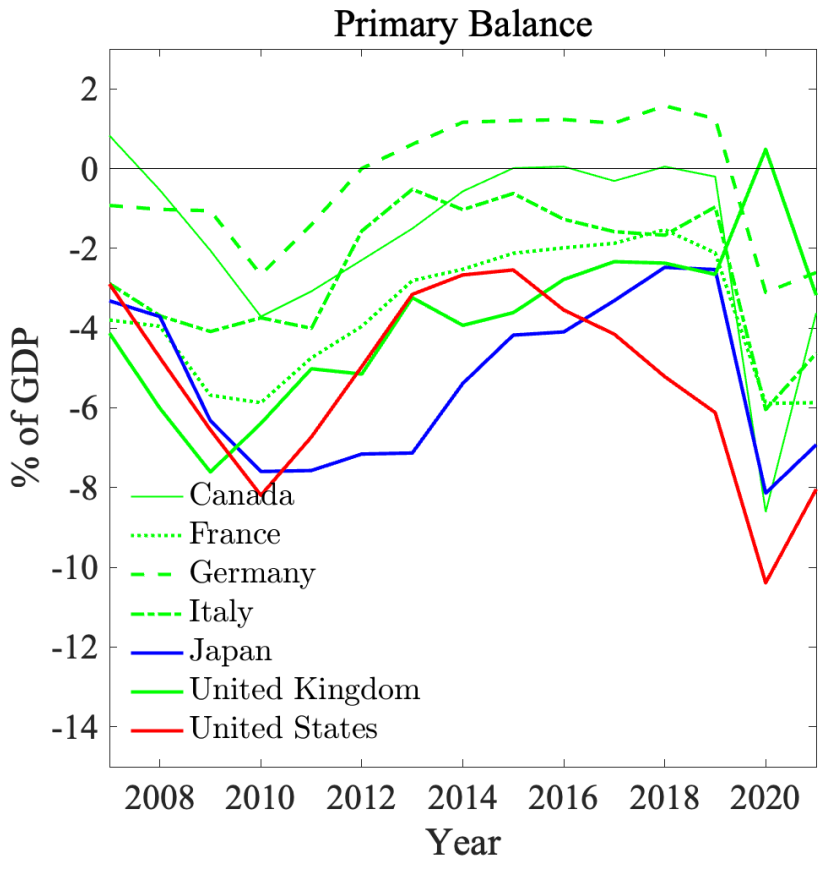
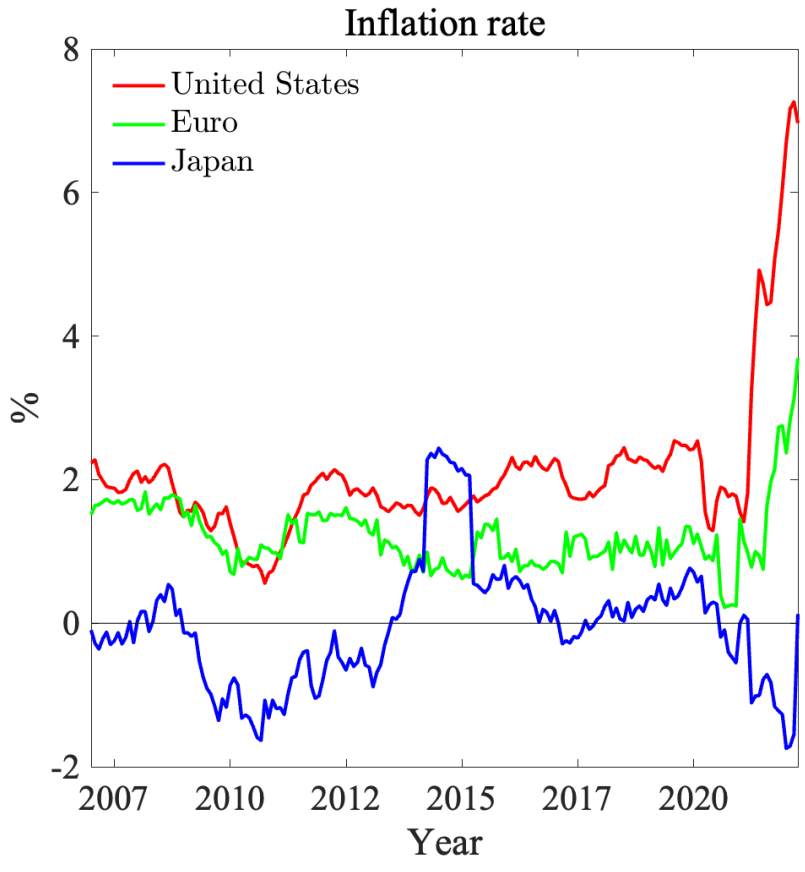
[1]Disclaimer: The views expressed in this paper are those of the authors. They do not necessarily reflect the official views of the Bank of Japan. Any remaining errors are mine.

Motivation: Low inflation with large fiscal deficit





- FTPL equation: real debt equals the present value of future real surplus

$$\frac{B}{P} = \mathbb{E}\left[\sum_{t=1}^{\infty} \beta^t s_t\right] ? \Rightarrow \textit{FTPL puzzle}$$

(Brunnermeier Merkel & Sannikov 2020, Cochrane 2022)

What we do

- **Research Question**:
  - What is a potential solution for the puzzle of FTPL?
  - How the government fiscal deficit affect inflation and debt?
- **What this paper does**:
  - Propose NK-FTPL model with financial friction (market segmentation)
- **Findings**:
  1. Deflationary regime emerges resulting from a failure of risk sharing of government budget constraint
  2. There is a threshold which determines inflationary and deflationary equilibrium selection
  3. The threshold depends on
     1. the interest rate differential of long-term and short-term bond
     2. the growth rate
     3. the size of debt

Why Market Segmentation?

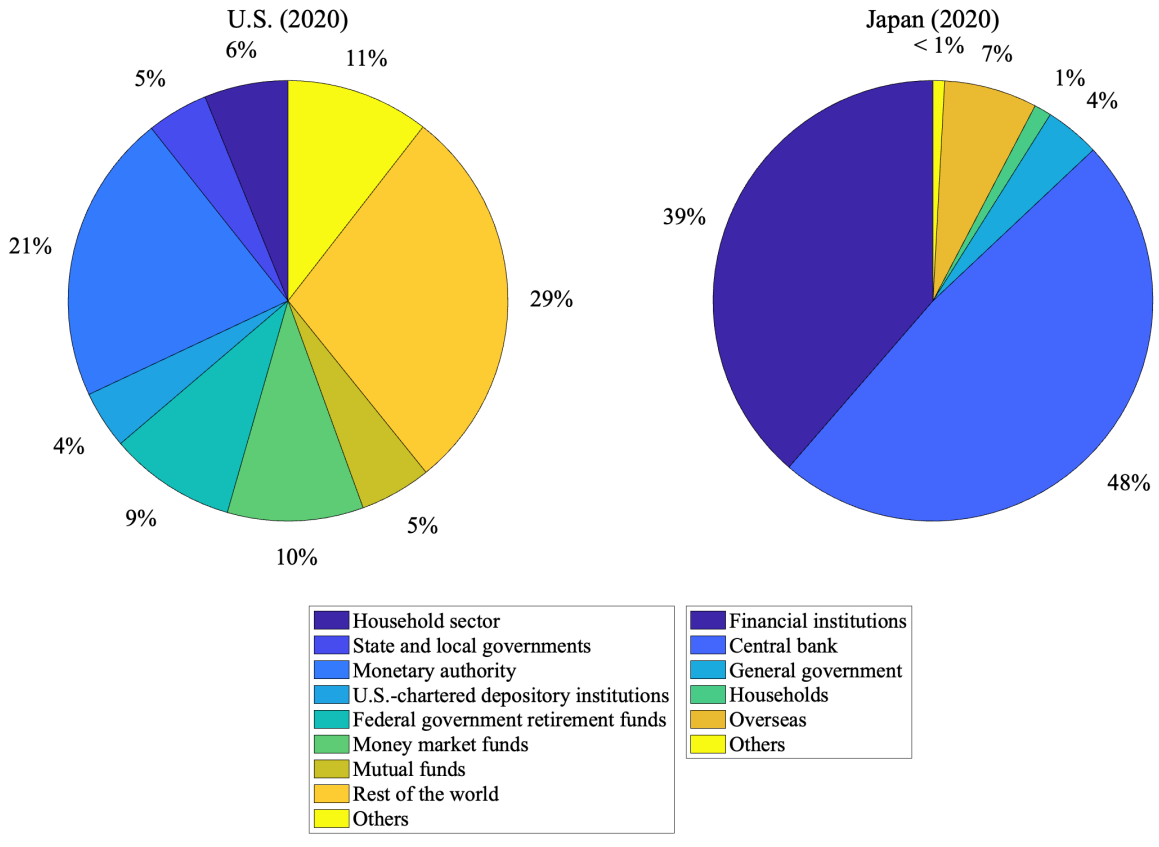


Figure: Owners of the government bonds

NOTE.—The data source for the US is 'Z.1 Financial Accounts of the United States' (Board of Governors of the Federal Reserve System). For Japan, we use 'Flow of Funds Accounts' (Bank of Japan).

Model Setup and Basics

- Model setup
  - Base model (NK-FTPL literature): Leeper 1991, Woodford 2003, Bianchi & Ilut 2017, Bianchi & Melosi 2017
  - With market segmentation à la Alvarez Atkeson & Kehoe 2007, Gabaix & Maggiori 2015, Itskhoki & Mukhin 2021
    - The public debt has risk premium and held only by financial intermediaries
  - Investigate fiscal transfer shock

HH and Firm ⇒ FI ⇒ Gov ⇒ MC

- Representative HHs maximize their expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi} \right)$$

subject to the budget constraint:

$$P_t C_t + \frac{F_t^S}{R_t^S} + M_t \leq W_t L_t + F_{t-1}^S + M_{t-1} + \Pi_t^{PF} + \Pi_t^{FI} - T_t + TR_t$$

- Firm's production technology (Cobb-Douglas):

$$Y_t(i) = A_t L_t(i), \quad A_t \equiv \gamma^t$$

Marginal cost of production:

$$MC_t = \frac{W_t L_t(i)}{Y_t(i)} = \frac{W_t}{\gamma^t}, \quad \Pi_t(i)^{PF} = \left(P_t(i) - MC_t\right) Y_t(i)$$

Price setting à la Calvo:

$$\pi_t = k_p \left(mc_t - p_t\right) + \beta \mathbb{E}_t \pi_{t+1}$$

where $k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}$

HH and Firm ⇒ **FI** ⇒ Gov ⇒ MC

- Portfolio choice of financial intermediaries:

$$P_t^m H_{t+1}^L = \frac{\mathbb{E}_t \left[R_{t+1}^L \right] - R_t^S}{\omega \Sigma}$$

where ω is the CARA parameter, and Σ is the variance-covariance matrix. The return on long-term bond is:

$$\mathbb{E}_t \left[R_{t+1}^L \right] \equiv \mathbb{E}_t \frac{1 + \delta P_{t+1}^m}{P_t^m}$$

- Budget constraint of financial intermediary:

$$P_t^m H_t^L + \frac{H_t^S}{R_t^S} = 0$$

- Profit transfer to HHs:

$$\Pi_t^{\mathsf{FI}} = \left(1 + \delta P_t^m\right) H_{t-1}^L + H_{t-1}^S$$

HH and Firm ⇒ FI ⇒ **Gov** ⇒ MC

- The fiscal authority:

$$\underbrace{B_{t-1}^{S}+\left(1+\delta P_{t}^{m}\right)B_{t-1}^{L}}_{\text{Gov. should pay at date } t}-T_{t}+TR_{t}=\frac{B_{t}^{S}}{R_{t}^{S}}+P_{t}^{m}B_{t}^{L}$$

Log-linearizing around the steady state after de-trending along the BGP:

$$\tilde{b}_{t}=\frac{\overline{R^{L}}}{\gamma}\tilde{b}_{t-1}+b_{ss}\frac{\overline{R^{L}}}{\gamma}\left(r_{t}^{L}-\pi_{t}-\Delta y_{t}\right)-\tilde{\tau}_{t}+\sigma_{tr}\varepsilon_{t}^{tr},\quad \varepsilon_{t}^{tr}\sim iid(0,1)$$

The response of the fiscal authority to debt is:

$$\tilde{\tau}_{t}=\phi_{b}\tilde{b}_{t-1}$$

- The central bank controls CB_t^S to determine the short-term rate:

$$\frac{R_{t}^{S}}{\overline{R^{S}}}=\left[\frac{R_{t-1}^{S}}{\overline{R^{S}}}\right]^{\rho_{m}}\left[\left(\frac{\Pi_{t}}{\overline{\Pi}}\right)^{\phi_{\pi}}\right]^{(1-\rho_{m})}e^{\sigma_{m}\varepsilon_{t}^{m}},\quad \frac{CB_{t}^{S}}{R_{t}^{S}}-CB_{t-1}^{S}=M_{t}-M_{t-1}$$

HH and Firm ⇒ FI ⇒ Gov ⇒ **MC**

- Good market clearing:

$$C_t = Y_t$$

- Short-term bond market clearing:

$$\underbrace{B_t^S}_{=0} = F_t^S + H_t^S + \underbrace{CB_t^S}_{\text{MP instrument}}$$

- Long-term bond market clearing:

$$B_t^L = H_t^L$$

Definition (Equilibrium system)

Given state variables at time t, $\{F_t^S, CB_t^S, B_t^L, R_{t-1}^S\}_t$*, and prices,*
a competitive equilibrium consists of stochastic process $\{C_t, L_t, Y_t, F_{t+1}^S, H_{t+1}^S, H_{t+1}^L, B_{t+1}^L, \Pi_t^P, \Pi_t^{FI}, P_t, W_t, R_t^S, R_t^L\}_t$ *such that:*
(i) $\{C_t, L_t, F_{t+1}^S\}$ *maximize the infinite horizon utility subject to the budget constraint;*
(ii) goods producing firms choose $\{P_t, Y_t, L_t\}$ *to maximize the profit;*
(iii) financial intermediaries's position solves mean-variance problem $\{H_{t+1}^S, H_{t+1}^L\}$*;*
(iv) government follows the tax rule to determine $\{PB_t\}$*;*
(v) CB follows the Taylor rule through open market operation to choose $\{R_t^S\}$*; and*
(vi) market clearing conditions are satisfied.

5 equations governing the economy

$$\text{Households' IE equation:} \quad \sigma c_t = \sigma \mathbb{E}_t c_{t+1} - \phi_\pi \pi_t + \mathbb{E}_t \pi_{t+1},$$

$$\text{NK Phillips curve:} \quad \pi_t = \lambda c_t + \beta \mathbb{E}_t \pi_{t+1},$$

$$\text{Gov. budget constraint:} \quad \tilde{b}_t = \left(\frac{\overline{R^L}}{\gamma} - \phi_b \right) \tilde{b}_{t-1} + b_{ss} \frac{\overline{R^L}}{\gamma} \left(r_t^L - \pi_t - \Delta c_t \right) + \sigma_s \varepsilon_t^s,$$

$$\text{Debt to GDP ratio:} \quad \tilde{b}_t = \kappa \mathbb{E}_t r_{t+1}^L + (1 - \kappa) \phi_\pi \pi_t - p_t - c_t,$$

$$\text{Return on the LT bond:} \quad r_t^L = \frac{\delta}{\overline{R^L}} p_t^m - p_{t-1}^m,$$

where $\lambda = \kappa_p (\sigma + \phi)$ and $\kappa = \frac{\overline{R^L}}{\overline{R^L} - \overline{R^S}}$.

New Regime in Fiscal-led Regime

Proposition

In the case of full nominal rigidity, $\lambda(\equiv \kappa_p(\sigma+\phi)) \to 0$*:*

i. equilibrium is deflationary when $\{\phi_\pi < 1, \phi_b^{inf} < \phi_b < \phi_b^{def}\}$*;*

ii. equilibrium is inflationary when $\{\phi_\pi < 1, \phi_b < \phi_b^{inf}\}$*,*

$$\text{where} \quad \phi_b^{def} \equiv \frac{\overline{R^L}}{\gamma} - 1 + \frac{b_{ss}}{\gamma}\left(\overline{R^L} - \overline{R^S}\right),$$

$$\phi_b^{inf} \equiv \frac{\overline{R^L}}{\gamma} - 1 + \Theta\left[\frac{b_{ss}}{\gamma}\left(\overline{R^L} - \overline{R^S}\right)\right] + (1-\Theta)\left[1 - \frac{\overline{R^L}}{\delta}\right], \quad \Theta \equiv \frac{\overline{R^L} - \delta}{\overline{R^L} - \delta\frac{\overline{R^S}}{\overline{R^L}}}.$$

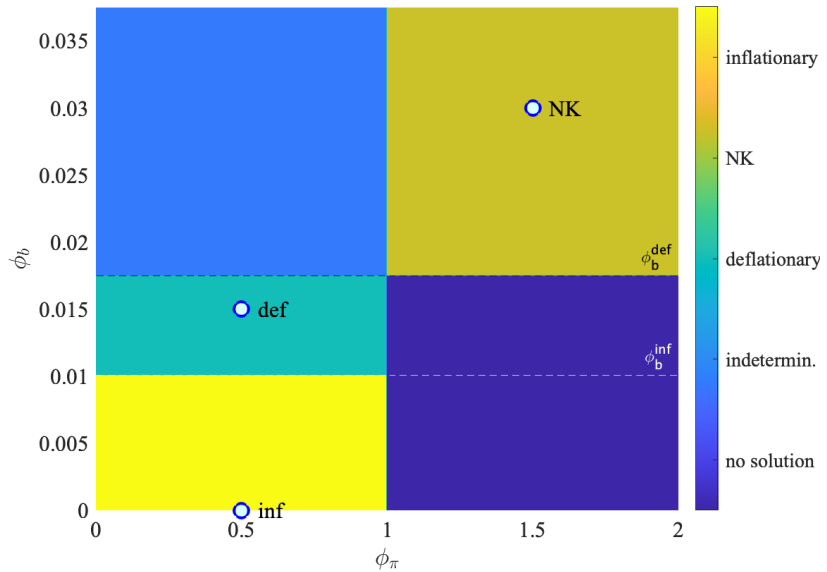


Figure: Determinacy regions with fiscal-led inflationary region and deflationary region

Two equilibrium (Deflationary and Inflationary) Mechanism

1. Deflationary
   - Financial intermediaries lever up to earn returns from long-term bonds.
   - Current primary deficit is financed not by inflation but by future primary surplus

2. Inflationary
   - Financial intermediaries leverage down because they would not get a return on long-term bonds.
   - Current primary deficit is financed not by future primary surplus but by inflation

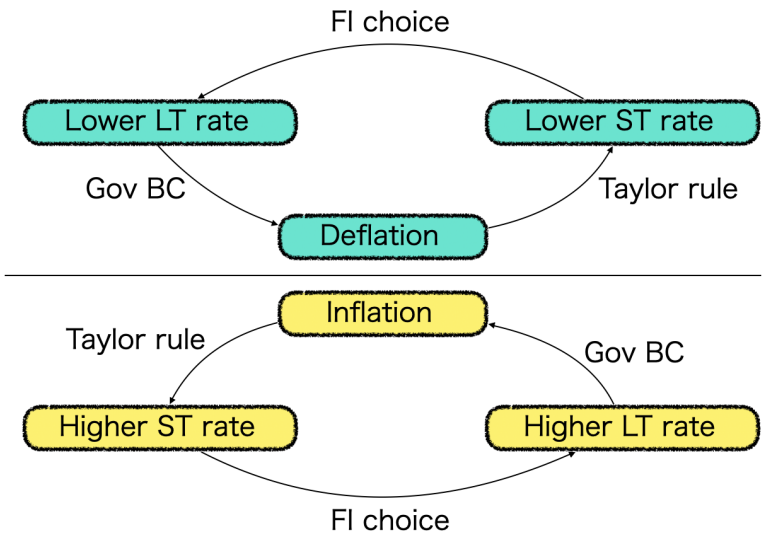


Figure: Main machanism

IRF to Fiscal Transfer Shock

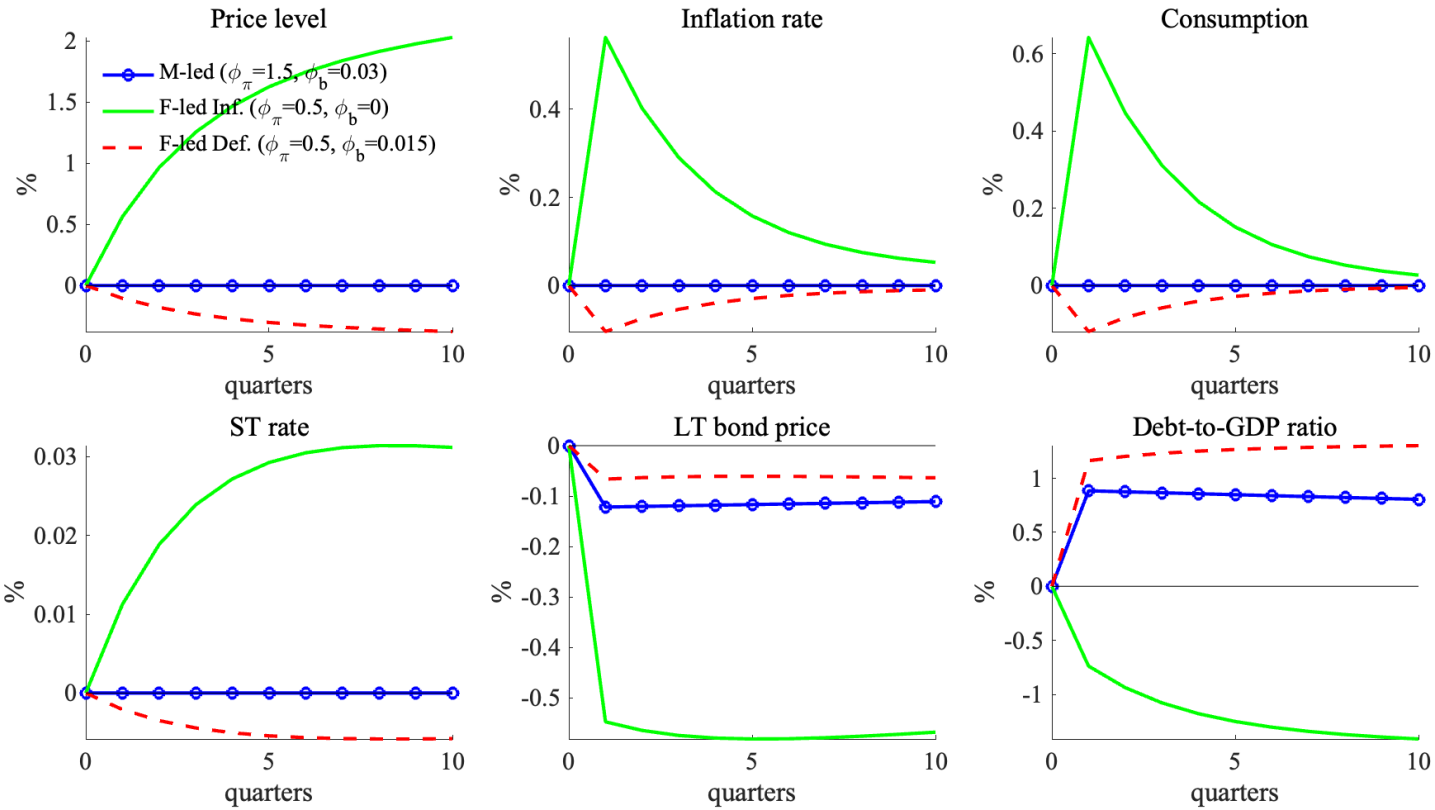


Figure: IRFs to 1% negative fiscal deficit shock

NOTE.—'M-led' is the Monetary-led regime (blue circle line), 'F-led Inf.' is the Fiscally-led Inflationary regime (green solid line), and 'F-led Def.' is the Fiscally-led Deflationary regime (dashed red line), respectively.

Argument on R Minus G

- Literature for $r - g$ argument:
  - Blanchard 2019, Sims 2019, Brunnermeier Merkel & Sannikov 2020, Cochrane 2022, Reis 2022, Jiang Lustig Van-Nieuwerburgh & Xiaolan 2022, Mian Straub & Sufi 2022, Angeletos Lian & Wolf 2023, etc.

- Blanchard 2021, The Mayekawa Lecture at the 2021 BOJ-IMES Conference:
  - ... Even if there is no change in fundamentals, if investors start becoming anxious and anticipate some probability of default, they will ask for a higher spread. ... Emerging market economies are particularly prone to this danger, but it is not irrelevant even for advanced economies.
  - Central banks have a role to play here. If investors start worrying about debt sustainability without any change in fundamentals, central banks with deep pockets can play the role of a large stable investor to prevent self-fulfilling non-fundamental runs on government debt.

- This paper's contribution to the argument on $r - g$:
  - Even if $r < g$ $\left(\overline{R^L} < \gamma, \text{ in the paper}\right)$, the risk premia held in the market $\left(\overline{R^L} - \overline{R^S}\right)$, the growth rate (γ), and the size of debt (b_{ss}), do matter for equilibrium selection.
  - Credibility on both monetary and fiscal policy, ϕ_π and ϕ_b, is important to understand the dynamics of inflation and debt conditional on exogenous shocks.

Conclusion

- **Research Question**:
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  - How the government fiscal deficit affect inflation and debt?
- **What this paper does**:
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     - the interest rate differential of long-term and short-term bond
     - the growth rate
     - the size of debt

New Regime in Fiscal-led Regime (Partial Nominal Price Rigidity)

Proposition

(i) Equilibrium is deflationary when $\{\phi_\pi < 1, \phi_b < \phi_b^{def}, F(\phi_\pi, \phi_b) < 0\}$

(ii) Equilibrium is inflationary when $\{\phi_\pi < 1, \phi_b < \phi_b^{def}, F(\phi_\pi, \phi_b) > 0\}$

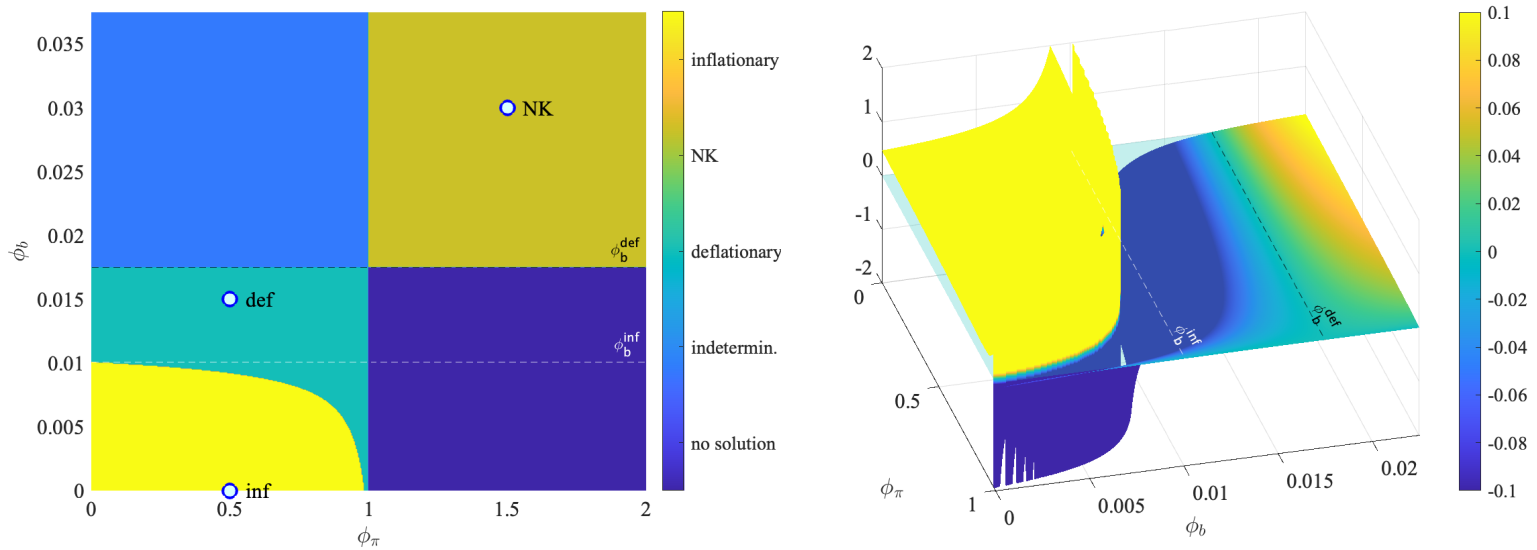


Figure: Determinacy regions with fiscal-led inflationary region and deflationary region with nominal price rigidity

$F(\phi_\pi, \phi_b)$

$$F(\phi_\pi, \phi_b) = -\frac{\sigma \frac{1-\beta e_6}{1-\beta \phi_\pi} \frac{1}{b_{ss} \frac{\overline{R^L}}{\gamma \overline{\Pi}}}}{(q_{55} q_{66} - q_{56}) q_{77} - (q_{75} q_{66} - q_{76})},$$

where

$$q_{55} q_{66} - q_{56} = \frac{\left[\sigma \frac{1-\beta e_6}{1-\beta \phi_\pi} (1 - e_5) - \frac{\lambda}{\beta} \left(1 - \frac{1-\beta e_6}{1-\beta \phi_\pi}\right)\right] A_5 + (e_5 - e_6) B_5}{(e_5 - e_6)(e_5 - e_4)},$$

$$q_{75} q_{66} - q_{76} = \frac{\left[\sigma \frac{1-\beta e_6}{1-\beta \phi_\pi} (1 - e_7) - \frac{\lambda}{\beta} \left(1 - \frac{1-\beta e_6}{1-\beta \phi_\pi}\right)\right] (A_7 q_{77} - e_7) + (e_7 - e_6)(B_7 q_{77} - e_7 + 1)}{(e_7 - e_6)(e_7 - e_4)},$$

$$q_{77} = -\frac{\frac{\overline{R^L}}{\gamma \overline{\Pi}} - \phi_b - e_7}{e_7 - e_5},$$

$$e_{4,6} = \frac{1}{2\beta} \left(\beta + \frac{\lambda}{\sigma} + 1 \mp \sqrt{\left(\beta + \frac{\lambda}{\sigma} + 1\right)^2 - 4\beta \left(1 + \frac{\lambda}{\sigma} \phi_\pi\right)} \right),$$

$$e_5 = \frac{\overline{R^L}}{\gamma \overline{\Pi}} - \phi_b + \frac{b_{ss}}{\gamma \overline{\Pi}} \left(\overline{R^L} - \overline{R^S}\right), \quad e_7 = \frac{\overline{R^L}}{\delta},$$

$$A_i = e_i + \frac{e_i}{(1 - e_i)} \frac{1}{\kappa} - \left(1 - \frac{1}{\kappa}\right) \phi_\pi, \quad B_i = e_i - \frac{1}{\kappa} - 1, \quad \text{for } i = 5, 7,$$

$$\lambda = \kappa_p(\sigma + \phi) \text{ and } \kappa = \frac{\overline{R^L}}{\overline{R^L} - \overline{R^S}}.$$

Deflationary Fiscally-led Region

Deflationary fiscally-led region.

$$\Delta\phi_b \equiv \phi_b^{def} - \phi_b^{inf} = (1-\Theta)\left[\frac{b_{ss}}{\gamma}\left(\overline{R^L} - \overline{R^S}\right) + \left(\frac{\overline{R^L}}{\delta} - 1\right)\right], \quad \Theta \equiv \frac{\overline{R^L} - \delta}{\overline{R^L} - \frac{\overline{R^S}}{\overline{R^L}}\delta}.$$

Assumption:

$$\overline{R^L} = \overline{R^S} + \delta\left(R^* - \overline{R^S}\right) = (1-\delta)\overline{R^S} + \delta R^*$$

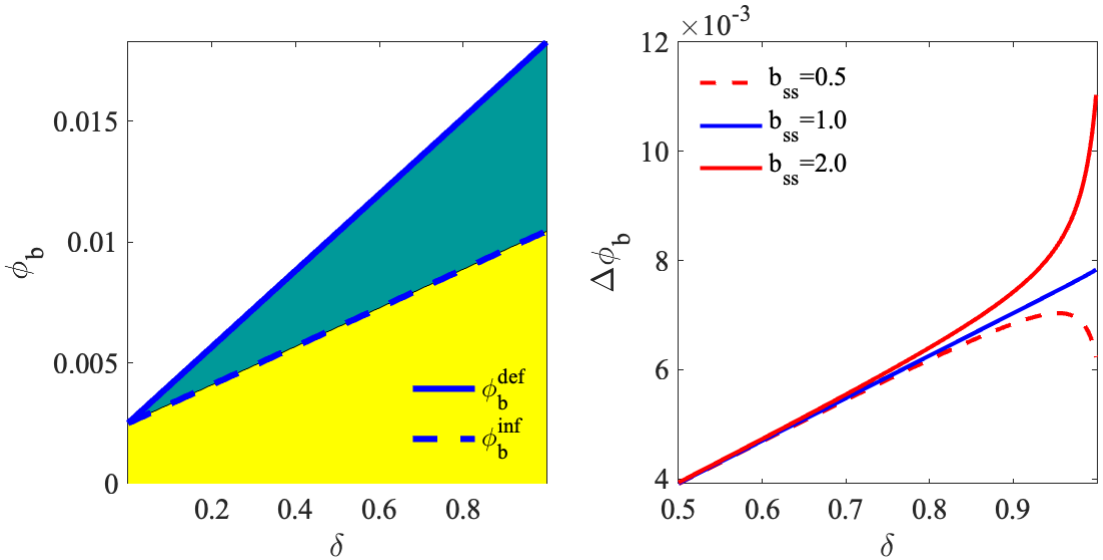
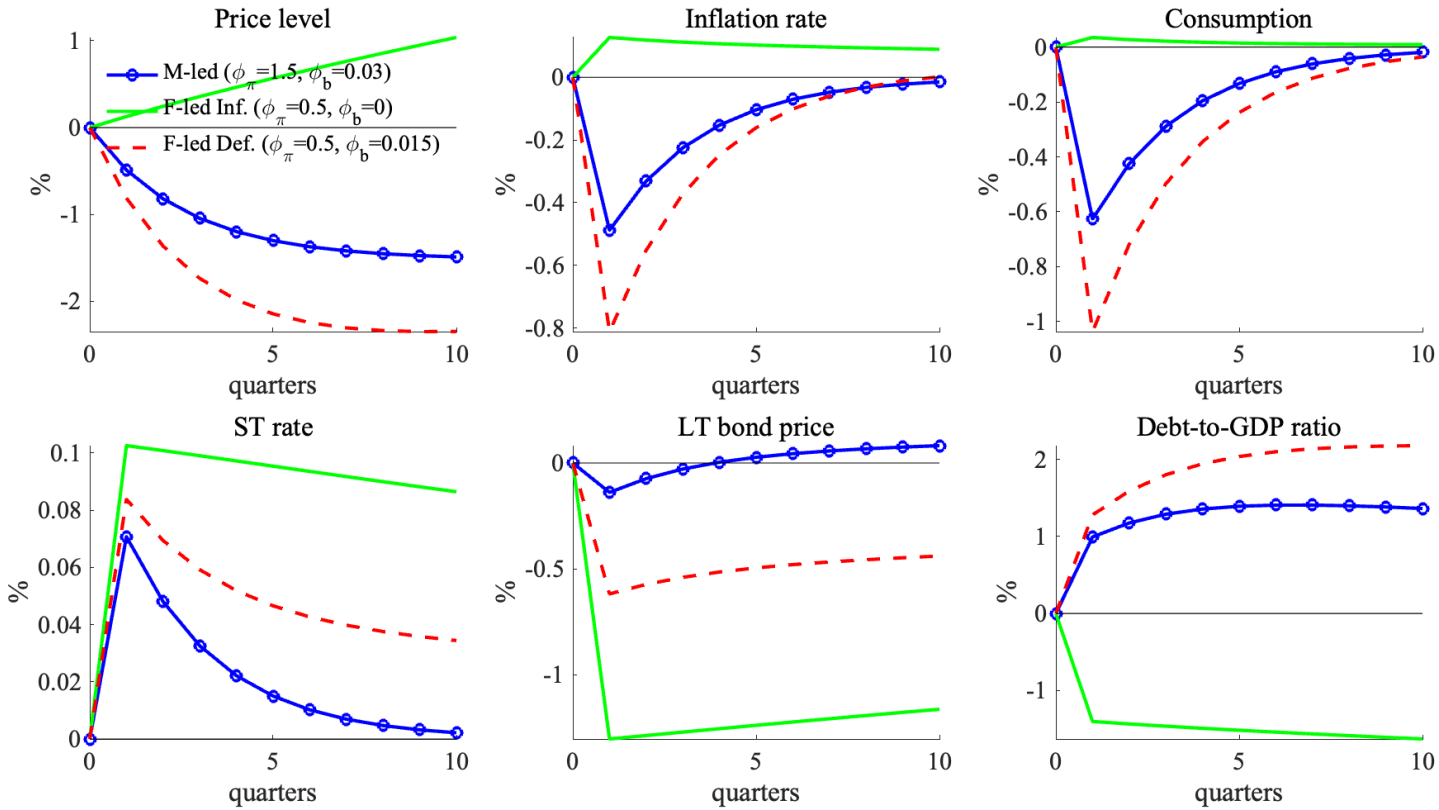


Figure: The measure of the size of the deflationary region in terms of the maturity measure and the size of the debt

IRF to 10 bps Interest Hike Monetary Policy Shock



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Mapping to the Data

- Inflation Data during Covid-19 Crisis
  - The more strict tax rule is, the lower inflation rate is.

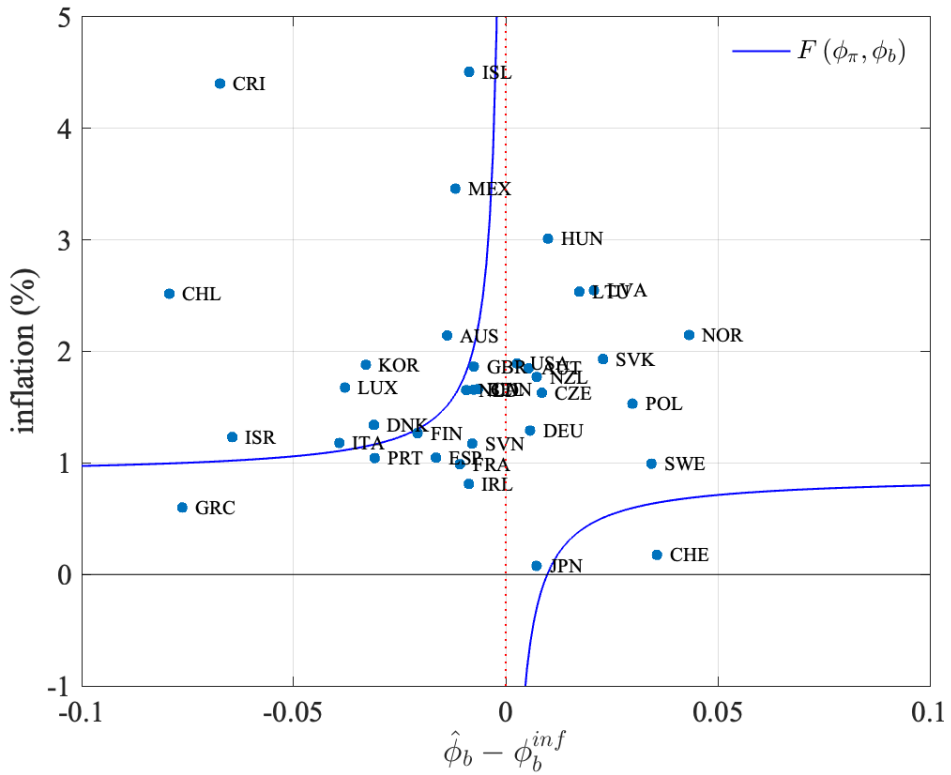

Figure: $\hat{\phi}_b - \phi_b^{inf}$ in OECD countries

NOTE.—The panel shows a plot for $\hat{\phi}_b - \phi_b^{inf}$ and average inflation rates after the Covid-19 crisis in OECD countries.