Optimal Monetary Policy during a Cost-of-Living Crisis

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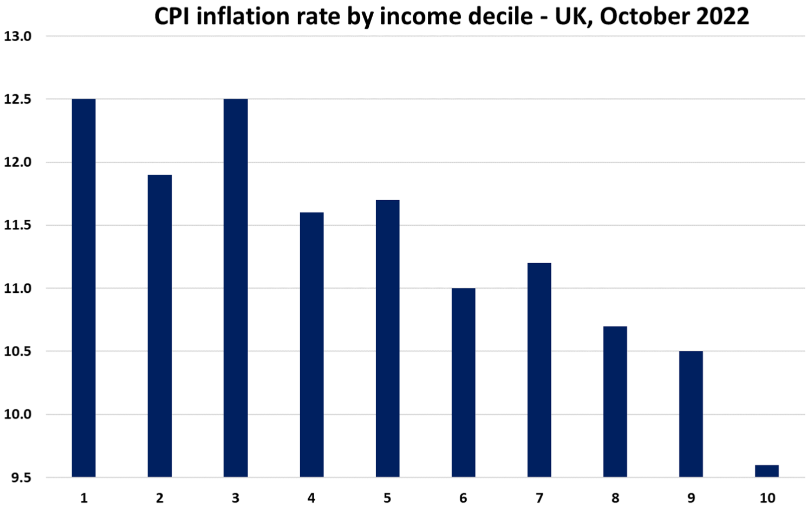
Bank of Englad - New Evidence on the Monetary Transmission Mechanism

May 21, 2024

Cost-of-Living Crisis



Source: ONS.

How should monetary policy respond?

This paper

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2) Analytical characterisation (“sufficient statistics”)
   - NKPC wedges
   - additional price index: *Marginal* CPI
   - transmission of sectoral shocks (necessity vs luxury)

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3) Optimal policy
   - Main finding: delayed tightening is optimal

Literature

Households

Unit mass of households, indexed by i. Die with probability δ. Idiosyncratic productivity level $\theta(i)$. Born with some initial level of wealth, $b(i)$.

K goods sectors, indexed by $k = 1, 2, ...$ Continuum of symmetric varieties within each sector, indexed by j.

Utility:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta(1-\delta))^{t+s} \left(U(\mathbf{c}_{t+s}(i)) - \chi(n_{t+s}(i)/\theta(i)) \right)$$

where

$$U(\mathbf{c}) = U(\mathcal{U}_1(\mathbf{c^1}), ..., \mathcal{U}_K(\mathbf{c^K}))$$

- *Outer* utility function U is weakly separable in products produced in different sectors, and twice differentiable.
- *Inner* utility function $\mathcal{U}_k$ is concave, symmetric and twice Fréchet differentiable.

Households

- Households decide on consumption, labour supply and bond holdings.
- Budget constraint household i:

$$e_t(i) + b_t(i) = R_{t-1} b_{t-1}(i) + n_t(i) W_t + \sum_k \varsigma(i) div_{k,t},$$

where $e_t(i) = \sum_k e_{k,t}(i) = \sum_k \int_0^1 p_{k,t}(j) c_{k,t}(i,j) di$.

- Within each household type, a fraction of households lives hand-to-mouth, setting $b_t(i) = b_{t-1}(i)$.

Firms

- Monopolistically competitive. Maximize expected PV of profits.
- Can adjust their price only with probability $1 - \theta_k$.
- Production function, allowing for I-O linkages:

$$y_{k,t}(j) = A_{k,t} F_k(n_{k,t}(j), \tilde{Y}_{1,k,t}(j), .., \tilde{Y}_{K,k,t}(j))$$

  - aggregate + sectoral productivity shocks
- Demand constraint:

$$y_{k,t}(j) = \int_0^1 d_k \left(p_{k,t}(j), \mathbf{p}_{k,t}, e_{k,t}(i) \right) di + \tilde{d}_k \left(p_{k,t}(j), \mathbf{p}_{k,t}, \right) \tilde{Y}_{k,t}.$$

Government & Market clearing

- Fiscal policy eliminates steady-state markups.
- Monetary policy rule:
$$\hat{R}_t = \phi \pi_{cpi,t} + u_t^R.$$
alternatively: optimal policy
- Markets for goods, bonds and labor clear.
- Deceased households are replaced by their steady-state versions

New Keynesian Phillips Curve

sector k, no I-O linkages

$$\pi_{k,t} = \kappa_k \tilde{\mathcal{Y}}_t + \lambda_k \left(\mathcal{NH}_t + \mathcal{M}_{k,t} - \mathcal{P}_{k,t} \right) + \beta \mathbb{E}_t \pi_{k,t+1},$$

$$\tilde{\mathcal{Y}}_t = \hat{\mathcal{Y}}_t - \hat{\mathcal{Y}}_t^*, \qquad \text{(Output gap)}$$

$$\mathcal{NH}_t = \sum\nolimits_l (\overline{\partial_e e_l} - \bar{s}_l)(\hat{P}_{l,t} - \hat{P}_{l,t}^*), \qquad \text{(Non-homotheticity wedge)}$$

$$\mathcal{M}_{k,t} = \int \gamma_{e,k}(i) \frac{c_k(i)}{C_k} \hat{c}_{k,t}(i) di - \Gamma_k \tilde{\mathcal{Y}}_t, \qquad \text{(Endogenous markup wedge)}$$

$$\mathcal{P}_{k,t} = (\hat{P}_{k,t} - \hat{P}_{cpi,t}) - (\hat{P}_{k,t}^* - \hat{P}_{cpi,t}^*), \qquad \text{(Relative price wedge)}$$

Homotheticity: $\overline{\partial_e e_l} - \bar{s}_l = \mathcal{NH}_t = 0$

CES: $\gamma_{e,k}(j) = \Gamma_k = \mathcal{M}_{k,t} = 0$

Output gap

Case without HtM

Output gap:

$$\tilde{\mathcal{Y}}_t = \mathbb{E}_t \tilde{\mathcal{Y}}_{t+1} - \sigma \mathbb{E}_t \left(\hat{R}_t - \pi_{mcpi,t+1} - \hat{r}_t^* \right)$$

where $\pi_{mpci,t} \equiv \sum_k \overline{\partial_e e_k} \pi_{k,t}$ is the *Marginal* CPI index.

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Natural interest rate:

$$\hat{r}_t^* = \frac{1}{\sigma + \psi} \sum_{l=1}^{K} \left(\psi \overline{\partial_e e_l} + \bar{s}_l \right) \left(\hat{A}_{l,t+1} - \hat{A}_{l,t} \right),$$

Natural level of demand:

$$\hat{\mathcal{Y}}_t^* = \sum_{l=1}^{K} \frac{\psi \overline{\partial_e e_l} + \bar{s}_l}{1 + \psi / \sigma} \hat{A}_{l,t}$$

Policy insights: analytical results

Two simplifying assumptions:

A1. The NKPC slope κ is common across sectors.

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Result 2 (divine coincidence under CES preferences)
When $\mathcal{M}_{k,t} = 0$, fluctuations in the output gap can be eliminated by stabilising the Marginal CPI index $\pi_{mpci,t} \equiv \sum_k \overline{\partial_e e_k} \pi_{k,t}$.

$$\pi_{mcpi,t} = \kappa \tilde{\mathcal{Y}}_t + \beta \mathbb{E}_t \pi_{mcpi,t+1}.$$

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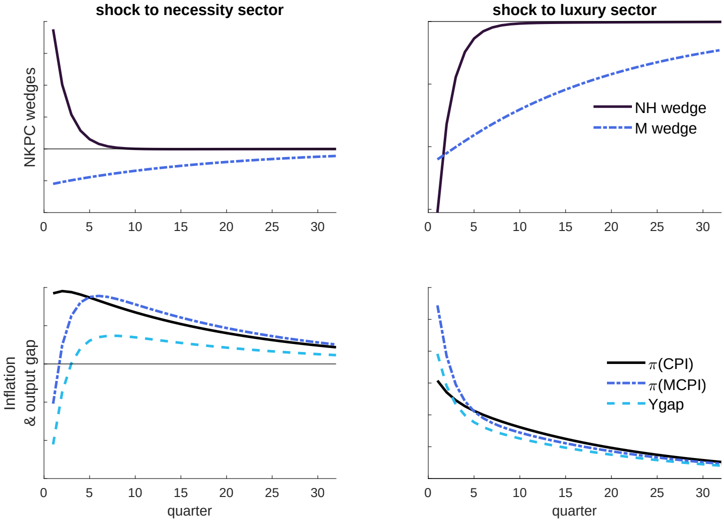
Result 3 (breakdown coincidence under non-CES preferences)
When $\mathcal{M}_{k,t} \neq 0$ there does not exist any inflation index which can be fully stabilised together with the output gap.

NKPC shifts up during a Cost-of-Living Crisis

Illustration

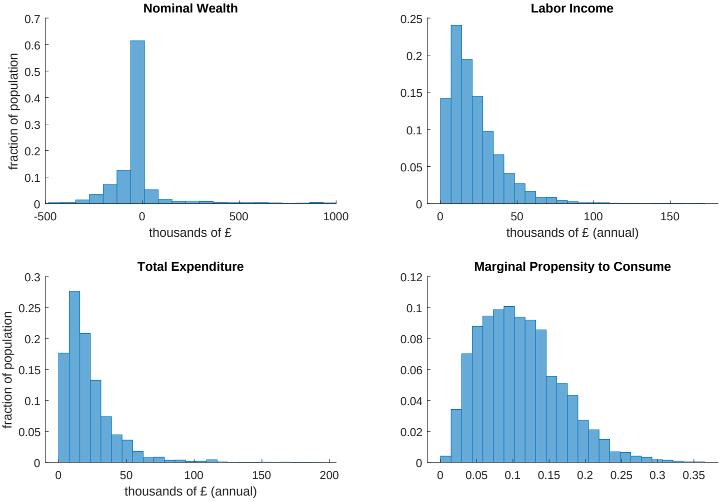


Model solution

The full model has a block-recursive structure:

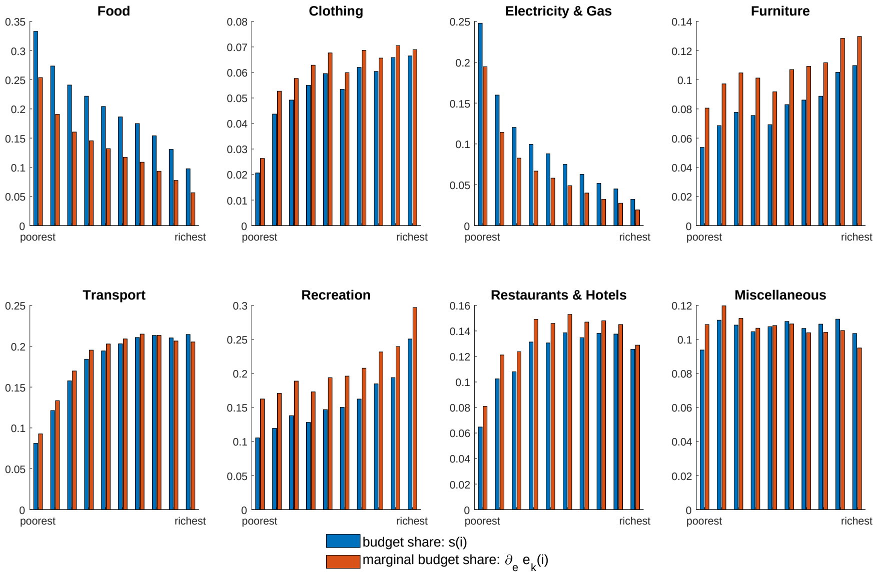
- Can write as block of $5K + 3$ core equations
  - keeps track of relevant distributional objects.
- Straightforward to solve for dynamics distributions and aggregates.
- Quantitative implementation: discipline with data from the Living Costs and Food (LCF) survey.

Full model - distributions



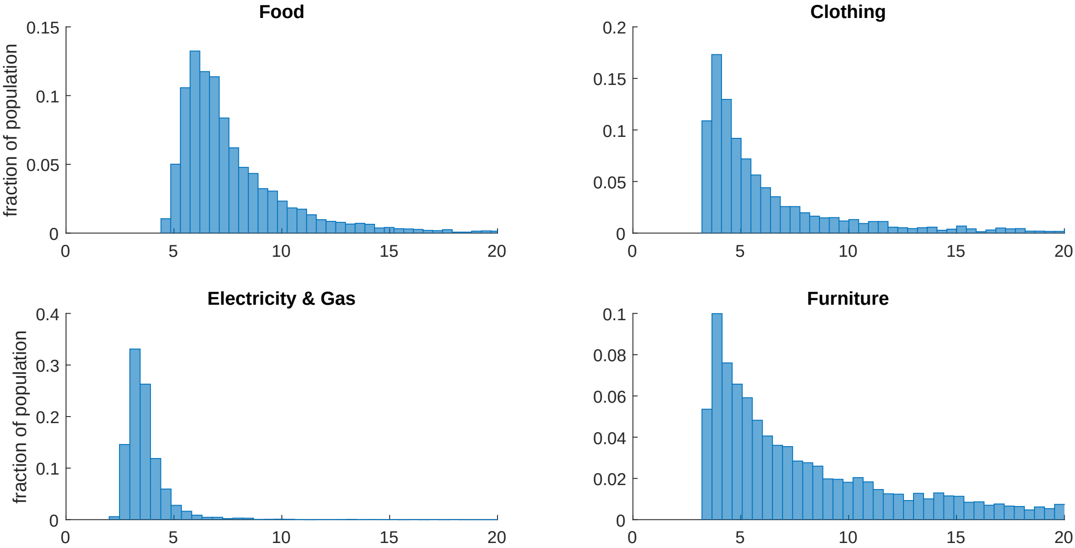
Source: LCF. MPC's calibration based on UK evidence from Albuquerque and Green (2022).

Budget shares



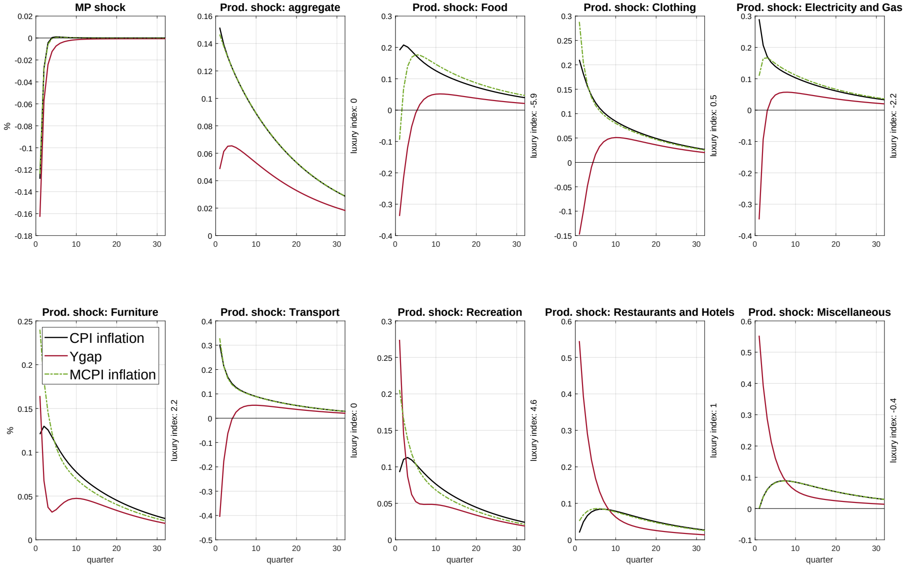
Distribution of demand elasticities

histograms



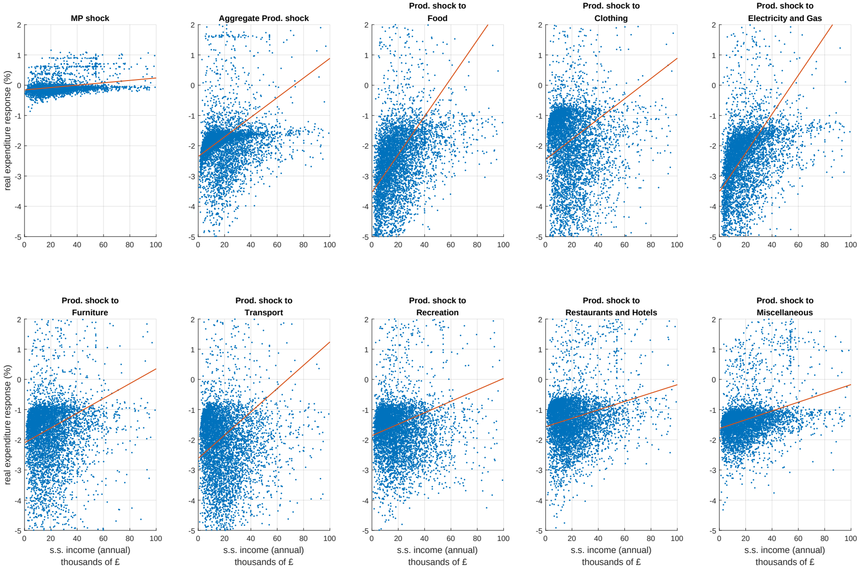
Responses to aggregate and sectoral shocks

Full model with IO linkages and HtM households



Effects across the distribution

Response of consumption in first year following the shock



Optimal monetary policy

Replace the rule for R_t by an optimizing CB who maximizes:

$$\mathcal{W} = \mathbb{E}\left(1-\delta\right)\int G(V^0(i), i)di + \delta \sum_{t_0=0}^{\infty} \beta^{t_0} \int G(V^{t_0}(i), i)di,$$

subject to all remaining model equations, where

$$V^{t_0}(i) = \sum_{s=0}^{\infty}\left(\left(1-\delta\right)\beta\right)^s \left(v\left(e^{t_0}_{t_0+s}(j), P_{1,t_0+s}, \ldots, P_{K,t_0+s}\right) - \chi\left(\frac{n^{t_0}_{t_0+s}(i)}{\theta(i)}\right)\right)$$

Social welfare function

Two assumptions:

i) CB treats steady-state inequality as efficient:

$$G'\left(V^{t_0}(i), i\right)\partial_e v\left(e(i), \hat{P}_1, ..., \hat{P}_K\right) = 1.$$

ii) CB weighs households' utility fluctuations equally:

$$G''\left(V_{ss}^{t_0}(i), i\right) = 0.$$

$\Rightarrow$ Pareto weight: $g(i) = \frac{E}{\psi\theta(j)Wn(i)+\sigma e(i)}$

Optimal policy: analytical results

under assumptions A.1-A.2

Result 7 (comparison to basic NK model):
If θ_k , $\bar{\epsilon}_k$ and $\bar{\epsilon}^s_k$ are equal across sectors, then the optimal policy problem can be expressed as:

$$\min_{\{\tilde{\mathcal{Y}}_t, \pi_{cpi,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\sigma+\psi}{\sigma\psi} \tilde{\mathcal{Y}}_t^2 + \tilde{\vartheta} \pi_{cpi,t}^2 \right)$$
$$s.t.\ \pi_{cpi,t} = \kappa \tilde{\mathcal{Y}}_t + \beta(1-\delta)\mathbb{E}_t \pi_{cpi,t+1} + \lambda(\mathcal{M}_t + \mathcal{NH}_t),$$

where $\tilde{\vartheta} = \frac{\bar{\epsilon}\theta}{(1-\theta)(1-\beta\theta)}$, and where the wedges $\mathcal{M}_t \equiv \sum_{k=1}^{K} \bar{s}_k \mathcal{M}_{k,t}$ and $\mathcal{NH}_t$ evolve independently of monetary policy (Result 1).

$\rightarrow$ heterogeneity matters for optimal policy!

Optimal policy

under assumptions A.1-A.2

Result 8 (dynamics under optimal policy)
The responses of the output gap and inflation to necessity and luxury shocks have the opposite sign under optimal policy, both in the short and in the medium run. The signs of the responses are summarised in the following table:

	Y gap	CPI	MCPI	NH wedge
Necessity shock (short run)	-	+	-	+
Necessity shock (medium run)	+	-	+	-
Luxury shock (short run)	+	-	+	-
Luxury shock (medium run)	-	+	-	+

Optimal policy

under assumptions A.1-A.2

Result 9 (comparison to strict CPI targeting)
Compared to a strict CPI targeting policy, the optimal policy is initially relatively loose (tight) following a negative necessity (luxury) shock, and relatively tight (loose) later on.

→ Delayed tightening during a cost-of-living crisis

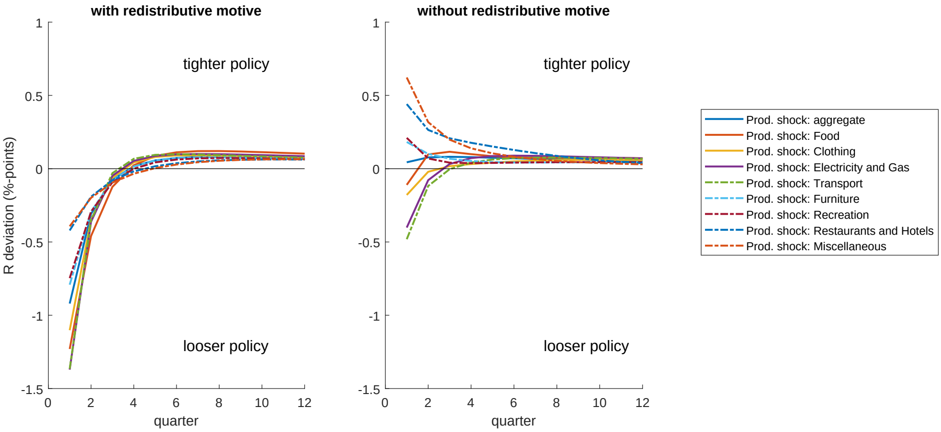
Optimal policy - full model

Including I-O linkages and HtM households

- Q. Is Optimal Policy looser or tighter than a rule $\hat{R}_t = \phi \pi_{cpi,t}$, in particular following necessity shocks?
- Idea: can implement optimal policy as an interest rule + "guidance" (=announced deviations from rule).
- Solve numerically for "guidance".

Optimal policy - full model

Including I-O linkages and HtM households



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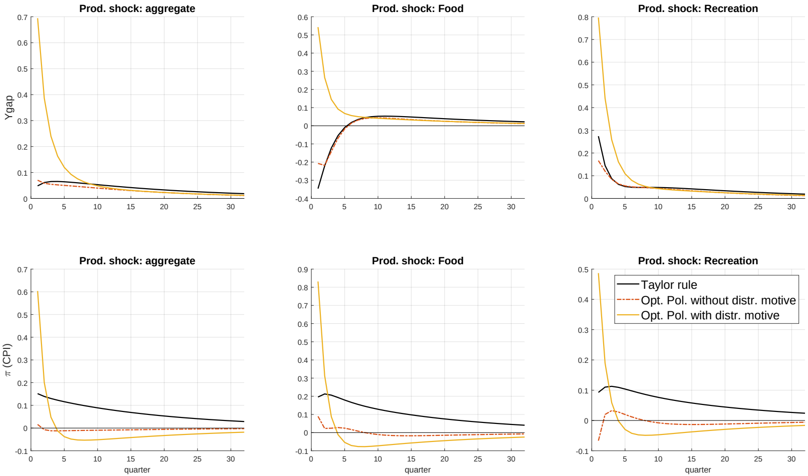

Conclusion

- Tractable multi-sector NK model with inequality and generalized preferences
  - *realistic heterogeneity in income, wealth and expenditures*

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- Productivity shocks turn into markup shocks
  - *but with rich dynamics governed by inequality*
  - *transmission highly dependent on sectoral source of the shock (necessity vs luxury)*

Conclusion

- Tractable multi-sector NK model with inequality and generalized preferences
  - *realistic heterogeneity in income, wealth and expenditures*
- Productivity shocks turn into markup shocks
  - *but with rich dynamics governed by inequality*
  - *transmission highly dependent on sectoral source of the shock (necessity vs luxury)*
- Emergence of *marginal* CPI as complementary metric for policy
- Optimal policy is initially accommodative during cost-of-living crisis; tighten with a delay.

References I

Acharya, S., E. Challe, and K. Dogra (2023) "Optimal monetary policy according to HANK," *American Economic Review*, 113 (7), 1741–1782.

Almås, Ingvild (2012) "International Income Inequality: Measuring PPP Bias by Estimating Engel Curves for Food," *American Economic Review*, 102 (3), 1093–1117.

Argente, David and Munseob Lee (2021) "Cost of Living Inequality During the Great Recession," *Journal of the European Economic Association*, 19 (2), 913–952.

Auclert, Adrien (2019) "Monetary Policy and the Redistribution Channel," *American Economic Review*, 6, Working Paper.

Baqaee, D., E. Farhi, and K Sangani (2021) "The Supply-Side Effects of Monetary Policy," Working paper.

Bayer, Christian, Ralph Luetticke, Lien Pham-Dao, and Volker Tjaden (2019) "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," *Econometrica*, 87 (1), 255–290.

References II

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas Sargent (2021) "Inequality, Business Cycles and Monetary-Fiscal Policy," *Econometrica*, 6, 2559–2599.

Blanco, C. and S. Diz (2021) "Optimal monetary policy with non-homothetic preferences," mimeo.

Boppart, Timo (2014) "Structural Change and the Kaldor Facts in a Growth Model With Relative Price Effects and Non-Gorman Preferences," *Econometrica*, 82, 2167–2196.

Challe, E. (2020) "Uninsured unemployment risk and optimal monetary policy in a zero-liquidity economy," *American Economic Journal: Macroeconomics*, 12 (10), 241–283.

Challe, E. and X. Ragot (2016) "Inflation at the Household Level," *Economic Journal*, 126, 135–164.

Comin, D., D. Laskhari, and M. Mestieri (2021) "Structural Change with Long-run Income and Price Effects," *Econometrica*, 89 (1), 311–374.

References III

Dávilla, E. and A. Schaab (2022) "Optimal Monetary Policy with Heterogeneous Agents: Discretion, Commitment, and Timeless Policy," Working paper.

Debortoli, Davide and Jordi Galí (2017) "Monetary Policy with Heterogeneous Agents: Insights from TANK models," mimeo.

Engel, Ernst (1857) "Die Productions- und Consumtionsverhältnisse des Königreichs Sachsen," *Zeitschrift des Statistischen Bureaus des Koniglich Sachsischen Ministerium des Inneren*, 8-9.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima (2016) "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy," Working Paper 12-21, Federal Reserve Bank of Philadelphia.

Guerrieri, V., G. Lorenzoni, L. Straub, and I. Werning (2022) "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?" *American Economic Review*, 112 (5).

Hamilton, Bruce (2001) "Using Engel's Law to Estimate CPI Bias," *American Economic Review*, 91 (3), 619–630.

References IV

Herrendorf, B., R. Rogerson, and A. Valentinyi (2014) "Growth and Structural Transformation," *Handbook of Economic Growth*, 2, 855–941.

Houthakker, H.S. (1957) "An International Comparison of Household Expenditure Patterns, Commemorating the Centenary of Engel's Law," *Econometrica*, 25, 532–551.

Jaravel, X. and A. Olivi (2021) "Prices, Non-homotheticities, and Optimal Taxation The Amplification Channel of Redistribution," Working paper.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante (2017) "Monetary Policy According to HANK," *American Economic Review*, 108 (3), 697–743.

LaO, J. and A. Tahbaz-Salehi (2019) "Optimal Monetary Policy in Production Networks," Working paper.

Le Grand, F., A. Martin-Baillon, and X. Ragot (2021) "Should Monetary Policy Care About Redistribution? Optimal Fiscal and Monetary Policy with Heterogeneous Agents," Working paper.

References V

McKay, A. and C. Wolf (2023) "Optimal Policy Rules in HANK," Working paper.

McKay, Alisdair, Emi Nakamura, and Jon Steinsson (2016) "The Power of Forward Guidance Revisited," *American Economic Review*, 106 (10), 3133–3158.

Melcangi, D. and V. Sterk (2019) "Stock Market Participation, Inequality and Monetary Policy," Working paper.

Nuno, G. and C. Thomas (2022) "Optimal Redistributive Inflation," *Annals of Economics and Statistics*, 146, 3–64.

Pasten, E., Schoenle R., and M. Weber (2020) "The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy," *Journal of Monetary Economics*, 116, 1–22.

Portillo, Rafael, Luis-Felipe Zanna, Stephen O'Connel, and Richard Peck (2016) "Implications of Food Subsistence for Monetary Policy and Inflation," *Oxford Economic Papers*, 68 (3), 782–810.

References VI

Rubbo, E. (2023) "Networks, Phillips Curves and Monetary Policy," *Econometrica*, 91 (4), 1417–1455.

Werning, Iván (2015) "Incomplete markets and aggregate demand,"Technical report, National Bureau of Economic Research.

Xhani, D. (2021) "Correcting Market Power with Taxation: a Sufficient Statistic Approach," Working paper.

Literature

New Keynesian +

- **Multiple Sectors:** Pasten, R. and Weber (2020); Rubbo (2023); LaO and Tahbaz-Salehi (2019); Baqaee, Farhi and Sangani (2021); Guerrieri, Lorenzoni, Straub and Werning (2022), etc.

- **Heterogeneous households:** McKay, Nakamura and Steinsson (2016); Gornemann, Kuester and Nakajima (2016); Challe and Ragot (2016); Auclert (2019); Werning (2015); Kaplan, Moll and Violante (2017); Debortoli and Galí (2017); Bayer, Luetticke, Pham-Dao and Tjaden (2019), etc.

- **Non-homothetic preferences:** Portillo, Zanna, O'Connel and Peck (2016); Melcangi and Sterk (2019); Blanco and Diz (2021), etc.

back

Literature

- **Non-homothetic preferences +**
  - **Growth:** Herrendorf, Rogerson and Valentinyi (2014); Boppart (2014); Comin, Laskhari and Mestieri (2021), etc.
  - **Inequality:** Engel (1857); Houthakker (1957); Hamilton (2001); Almås (2012); Argente and Lee (2021), etc.
  - **Taxation:** Jaravel and Olivi (2021); Xhani (2021), etc.

- **Optimal Policy in HANK**: Challe (2020); Bhandari, Evans, Golosov and Sargent (2021); Le Grand, Martin-Baillon and Ragot (2021); Nuno and Thomas (2022); Dávilla and Schaab (2022); Acharya, Challe and Dogra (2023); McKay and Wolf (2023).

back

Definitions

$$\lambda_k = \frac{(1-\theta_k)(1-\beta\theta_k)}{\theta_k}\frac{\bar{\epsilon}_k-1}{\bar{\epsilon}_k-1+\bar{\eta}_k}$$

$$\gamma_{e,k}(j) = \left(1 - \frac{\epsilon_k(j)}{\bar{\epsilon}_k}\left(1+\epsilon_k^s(j)\right)\right)\frac{1}{\bar{\epsilon}_k-1}$$

$$\bar{\epsilon}_k = \int \frac{e_k(j)}{E_k}\epsilon_k(j)dj$$

$$\bar{\eta}_k = \left(-\int\left(\epsilon_k(j)-\bar{\epsilon}_k\right)^2\frac{e_k(j)}{E_k}dj + \int\frac{\epsilon_k^s(j)}{\epsilon_k(j)}\frac{e_k(j)}{E_k}dj\right)/\bar{\epsilon}_k$$

$$\bar{s}_k = E_k/E$$

$$\bar{\xi}_k = \int_j \frac{\vartheta(j)Wn(j)}{\int_j\vartheta(j)Wn(j)}\xi_k(j)dj$$

$$\Gamma = \sum_k \bar{s}_k \int \gamma_{e,k}(j)\xi_k(j)\frac{e(j)}{E_k}dj$$

$$\mathcal{M}_t^D = \bar{s}_k\sum_k \mathcal{M}_{k,t}^D$$

$$\mathcal{M}_t^P = \sum_k \bar{s}_k \sum_l \int_j \frac{e_k(j)}{E_k}\gamma_{e,k}(j)\rho_{k,l}(j)dj\cdot\left(\hat{P}_{l,t}-\hat{P}_{k,t}\right)$$

Back

Endogenous markup wedge

Tractable distributional dynamics

$$\mathcal{M}_{k,t} = \mathcal{M}^P_{k,t} + \mathcal{M}^E_{k,t}$$
$$\mathcal{M}^E_{k,t} = \Gamma\hat{\mathcal{Y}}_t + \mathcal{M}^D_{k,t}$$
$$\mathcal{M}^P_{k,t} = \sum_l \mathcal{S}_{k,l}\left(\hat{P}_{l,t} - \hat{P}_{k,t}\right)$$

$$\mathcal{M}^E_{k,t} = \mathbb{E}_t\mathcal{M}^E_{k,t+1} - \bar{\gamma}_{e,k}\bar{\sigma}^{\mathcal{M}}_k\hat{R}_t + \sum_l \bar{\gamma}_{e,k}\bar{\sigma}^{\mathcal{M}}_{k,l}\mathbb{E}_t\pi_{l,t+1} - \frac{\delta}{1-\delta}\mathbb{E}_t\mathcal{M}^0_{k,t}$$

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$$\frac{1}{(1-\delta)R}\hat{\mathcal{M}}^0_{k,t} = \hat{\mathcal{M}}^0_{k,t-1} - \int \gamma_{b,k}(j)\frac{b(j)}{RE}dj\left(\hat{R}_t - \sum_l \bar{s}_l\pi_{l,t+1}\right)$$
$$- \left(1 + \frac{\bar{\psi}}{\bar{\sigma}}\right)\int \gamma_{b,k}(j)\frac{wn(j)}{WL}dj\hat{\mathcal{Y}}_t + \frac{R-1}{R}\hat{\mathcal{M}}^E_{k,t}$$
$$- \sum_l \int \gamma_{b,k}(j)\left(\frac{e(j)}{E}\left(\bar{s}_l - s_l(j)\right) + \frac{wn(j)}{WL}\left(\bar{\psi}_l - \psi_l(j)\right)\right)dj\hat{P}_{l,t}$$

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Output gap

Output gap:

$$\tilde{\mathcal{Y}}_t = (\frac{1}{\bar{\sigma}} + \frac{1}{\psi})(\hat{\mathcal{Y}}_t - \hat{\mathcal{Y}}_t^*)$$

Aggregate demand index:

$$\hat{\mathcal{Y}}_t = \mathbb{E}_t \hat{\mathcal{Y}}_{t+1} - \bar{\sigma} \left(\hat{R}_t - \mathbb{E}_t \pi_{cpi,t+1} - \mathbb{E}_t \tilde{\pi}_{\mathcal{NH},t+1} \right),$$

where

$$\tilde{\pi}_{\mathcal{NH},t} = \sum_{k=1}^{K} \left(\frac{\bar{\sigma}_k + \psi \bar{\bar{\xi}}_k}{\bar{\sigma} + \psi} - \bar{s}_k \right) \pi_{k,t}$$

Flex-price agg. demand index:

$$\hat{\mathcal{Y}}_t^* = \sum_k \frac{\psi \bar{\bar{\xi}}_k + \bar{s}_k}{1 + \bar{\sigma}} \hat{A}_{k,t}$$

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Welfare loss

Assumptions A1-A2 and $\mathcal{M} = 0$

$$\mathcal{L}_s^{\tilde{\mathcal{Y}}} = \frac{\bar{\psi}}{1 + \frac{\bar{\psi}}{\bar{\sigma}}} \left\{ \tilde{\mathcal{Y}}_s^2 - \mathcal{C}_s^{\tilde{\mathcal{Y}}} \right\}$$

$$\mathcal{L}_s^{\pi} = \sum_k \vartheta \bar{s}_k \cdot \pi_{k,s}^2$$

$$\mathcal{L}_s^{d} = \mathbb{E}_\delta \int g(j) \left(\tau_{t_0}(j) + \sum_k \sum_{s \geq t_0} \frac{R-1}{R^{s+1-t_0}} \frac{e(j)}{E} s_k(j) A_{k,s} \right)^2 dj$$

$$- 2\mathbb{E}_\delta \int \frac{\xi(j)}{1 + \frac{\theta(j) W n(j) \psi}{e(j)\sigma}} \tau_{t_0}(j) \sum_k \sum_{s \geq t_0} \frac{R-1}{R^{s+1-t_0}} A_{k,s} dj)$$

Welfare loss

Assumptions A1-A2 and $\mathcal{M} = 0$

$$
\begin{aligned}
\mathcal{L}_s^{\tilde{\mathcal{Y}}} &= \frac{\bar{\psi}}{1 + \frac{\bar{\psi}}{\bar{\sigma}}} \left\{ \tilde{\mathcal{Y}}_s^2 - \mathcal{C}_s^{\tilde{\mathcal{Y}}} \right\} \\
\mathcal{L}_s^{\pi} &= \sum_k \vartheta \bar{s}_k \cdot \pi_{k,s}^2 \\
\mathcal{L}_s^{d} &= \mathbb{E}_\delta \int g(j) \left(\tau_{t_0}(j) + \sum_k \sum_{s \geq t_0} \frac{R-1}{R^{s+1-t_0}} \frac{e(j)}{E} s_k(j) A_{k,s} \right)^2 dj \\
&\quad - 2\mathbb{E}_\delta \int \frac{\xi(j)}{1 + \frac{\theta(j) W n(j) \psi}{e(j)\sigma}} \tau_{t_0}(j) \sum_k \sum_{s \geq t_0} \frac{R-1}{R^{s+1-t_0}} A_{k,s} dj)
\end{aligned}
$$

where

$$
\tau_{t_0}(j) = \left(1 - \frac{1}{R}\right) \sum_{s \geq t_0} \frac{1}{R^{s-t_0}} \left(\frac{b(j)}{RE} \left(R_s - \pi_{cpi,s+1}\right) - \sum_k \frac{e(j)}{E} \left(s_k(j) - \bar{s}_k\right) \right)
$$

$$
\mathcal{C}_s^{\tilde{\mathcal{Y}}} = \mathbb{E}_\delta \int \frac{\left(1 - \frac{1}{R}\right) \frac{b(j)}{E}}{\cdots} \left(\sum \frac{R-1}{\cdots} \left(\tilde{\mathcal{Y}}_s - \sum \left(\xi(j) - \bar{\xi}\right) \left(P_{k,s} - P^*{}_{k,s}\right) \right) \right)^2 dj
$$

Welfare loss

general

$$\mathcal{L}_s^{\tilde{\mathcal{Y}}} = \frac{\bar{\psi}}{1+\frac{\bar{\psi}}{\bar{\sigma}}} \int \frac{e(j)}{E} \left(\hat{W}_s - \sum_k \xi_k(j) \left(\hat{P}_{k,s} + \hat{A}_{k,s} \right) \right)^2 dj + \mathcal{C}_s^{\tilde{\mathcal{Y}}}$$

$$\mathcal{L}_s^{\pi} = \sum_k \bar{s}_k \vartheta_k \pi_{k,s}^2$$

$$\mathcal{L}_s^{s} = -\sum_k \bar{s}_k \sum_l \mathcal{S}_{k,l} \left(\hat{P}_{l,s} + \hat{A}_{l,s} \right) \left(\hat{P}_{k,s} + \hat{A}_{k,s} \right)$$

$$\mathcal{L}_s^{r} = \frac{\bar{\sigma}}{1+\frac{\bar{\psi}}{\bar{\sigma}}} \sum_{k,l} \mathcal{E}_{k,l} \left\{ \left(\hat{P}_{k,s} + \hat{A}_{k,s} \right) \left(\hat{P}_{l,s} + \hat{A}_{l,s} \right) - \mathcal{C}_{k,l}^{r} \right\}$$

$$\mathcal{L}^{d} = \mathbb{E}_{\delta} \int g(j)\, \hat{\tau}_{t_0}(j)^2 dj + \mathcal{C}^{d}$$

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