

A Model of Monetary Singleness ^{*}

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September 21, 2026

Abstract

This paper develops a three-period banking model in which profit-maximising banks choose the unit of account of their liabilities and whether those liabilities circulate as a transferable medium of exchange. When banks denominate liabilities differently, cross-bank settlement requires honouring claims whose values diverge across states, creating a settlement friction tied to asset heterogeneity. This friction raises transfer fees, reduces debt capacity, and, beyond a threshold level of divergence, renders interbank payments infeasible. Coordination on a shared unit of account can extend the range over which exchange remains feasible, though it need not be efficient when asset divergence is high. The framework clarifies why digital monies backed by assets that differ from those of incumbent banks are more prone to fragmentation. It also shows how public instruments operate differently. Central bank reserves reduce settlement frictions through a common settlement asset, while cash provides an alternative channel through central bank money.

Keywords: Banking, Money, Singleness, Unit of account

JEL codes: E41, E42, E58

^{*}The views expressed in this paper are those of the author and do not necessarily reflect the position of the Bank of England or its committees. I am grateful to My Hedlin, Nick McLaren, Cyril Monnet, Dirk Niepelt, Marcus Opp, David Skeie, and Kathy Yuan for their valuable comments and suggestions. I also thank the participants at the BoE/WBS Innovation in Money and Payments Conference, the London FIT Workshop, the BoE Macro Brownbag, and the Bocconi/CEPR/ECB The Future of Payments Conference for their helpful feedback and discussions.

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1 Introduction

Private digital monies, including stablecoins, tokenised deposits, and platform currencies, increase heterogeneity in the payment system. Each instrument may be backed by a different asset portfolio, denominated in a distinct unit of account, and subject to issuer-specific constraints on transferability. When two issuers denominate liabilities differently, cross-bank settlement requires honouring claims whose value differs across states. This creates a cross-bank settlement friction determined by the promise-keeping constraint that must hold in every state. Settlement becomes more costly, transfer fees rise, debt issuance falls, and, above a threshold, interoperability breaks down.

Monetary singleness is the property that different forms of money exchange at par and therefore have a single uniform value across a currency area, regardless of issuer or where they are used. In the model, this corresponds to par trading across bank liabilities and frictionless interoperability in settlement. The focus is singleness within a monetary system, not global singleness across currencies. Departures from par at the global level typically reflect monetary sovereignty and exchange-rate adjustment, rather than a failure of the payment system. In the national setting studied here, par trading and interoperability across issuers are policy objectives, and whether singleness is preserved depends on the degree of asset-base divergence and on the policy instruments that shape the settlement environment.

Departures from par are not hypothetical. Stablecoins have traded at discounts during periods of stress, cross-platform transfers incur fees, and new entrants need not adopt the same unit of account as incumbent bank deposits. The policy question is whether the entry of digital monies backed by heterogeneous assets generates persistent deviations from par that reduce the gains from monetary exchange. Garratt and Shin (2023) argue that even small departures from par can propagate through the payment system and erode money’s coordinating function. Chiu and Monnet (2025), by contrast, suggest that modest departures may be tolerable if digital money delivers offsetting benefits.

A related question is how policy instruments affect singleness once private incentives create pressures toward fragmentation. Gorton (2020) emphasises that regulation and supervision support bank liabilities as a medium of exchange by reducing default risk. Central bank reserves provide a common settlement asset that aligns effective asset values across issuers. Cash, as physical central bank money, provides a state-invariant settlement claim and an alternative settlement channel, which can sustain interoperability when direct interbank transfers are costly, as discussed by Armelius et al. (2020) and in BIS (2003) and Rivadeneyra et al. (2024).

This paper develops a three-period banking model in which profit-maximising banks choose the unit of account of their liabilities and whether those liabilities circulate as a transferable medium of exchange. It extends Doepke and Schneider (2017) by making unit-of-account choice endogenous and linking it to interbank settlement. The central result is that heterogeneous asset backing alone, without default risk or asymmetric information, can make monetary fragmentation a Nash equilibrium outcome. The settlement friction induced by cross-denominated claims tightens promise-keeping constraints, reduces debt capacity, and makes interbank transfer fees endogenous. As asset-base divergence increases, fee-free interoperability can give way to costly transfers and, eventually, to autarky. The model then shows how public instruments such as reserves, cash, and common-unit-of-account mandates reshape the boundary between singleness and fragmentation.

Two channels generate inefficiency. First, cross-denominated settlement obligations diverge from the receiving bank's output in adverse states, which tightens promise-keeping constraints and reduces borrowing capacity. Second, interbank transfer fees compensate banks for that settlement friction and reduce the amount that must be settled across banks, but they also narrow the range over which liabilities circulate and therefore reduce their value as a medium of exchange. These two channels organise the paper's equilibrium and policy results.

A simple two-state version of the model makes these mechanisms transparent. Under the natural unit of account, low relative-price variance supports efficient exchange. As variance rises, the natural-unit-of-account equilibrium first becomes inefficient while transfers remain fee-free, then features positive interbank transfer fees, and eventually gives way to autarky. Common-unit-of-account equilibria remain available over a wider range. They are efficient at low levels of variance and can persist even after the natural-unit-of-account benchmark has become inefficient. At higher levels of variance, however, no common unit of account can be sustained in equilibrium. The same framework also shows how reserves, acting as a common settlement asset, and cash, acting as an alternative settlement claim, shift these boundaries in different ways. An extension then shows that when debt is information-sensitive, the value of singleness is higher because fragmentation reduces both borrowing capacity and acceptance in exchange.

The question of how the unit of account affects welfare is developed in Doepke and Schneider (2017), who argue that a shared unit reduces relative price risk along credit chains. Drenik et al. (2022) relate unit-of-account preferences to the co-movement of currency values and consumption needs, while Zúñiga (2023) compare the transactional efficiency of alternative monetary regimes. Here, unit-of-account choice is endogenous and linked to interbank settlement. The key mechanism is the settlement friction that arises

when liabilities are denominated in different units of account. That friction determines whether heterogeneous monies can coexist in equilibrium.

The paper also relates to work on short-term bank debt as a medium of exchange. Gorton and Pennacchi (1990) analyse when demandable debt can circulate because its information-insensitive design suppresses adverse selection. Gorton (2020) and Dang et al. (2020) emphasise that stable secondary-market prices are needed for the no-questions-asked property, that is, for debt to circulate without counterparties needing to investigate its quality. Lagos and Wright (2005) develop a search-theoretic framework in which monetary design shapes welfare under decentralised trade. In those settings, fragmentation arises from informational asymmetries, default risk, or liquidity shortfalls. Here, it arises when banks use different units of account, even under symmetric information and without default. Section 6 adds an informational friction at the point of exchange. In that extension, information sensitivity arises endogenously from unit-of-account choice rather than from opaque asset pools, and reserves remove both frictions at once.

The paper also relates closely to recent work on digital money and payment-system fragmentation. Brunnermeier et al. (2021) study how platform currencies can displace national monies through digital dollarisation. Garratt and Shin (2023) argue that bearer digital instruments are prone to departures from par when settlement is costly, whereas tokenised deposits settled in central bank money preserve singleness. Chiu and Monnet (2025) analyse how programmable currencies introduce heterogeneity in liquidity and transferability, creating a trade-off between commitment and informational frictions. In their model, departures from monetary singleness arise from currency design. Here, they arise from differences in the asset portfolios that back otherwise standard liabilities. This distinction matters for policy. In Chiu and Monnet, singleness is restored through currency design. Here, it requires compressing the settlement friction through a common settlement asset or a shared unit of account. Ozdenoren et al. (2025) examine how platform issuers use proprietary currencies to extract seigniorage at the cost of market fragmentation. The paper complements this literature by tracing fragmentation to the settlement friction created by asset-portfolio divergence. It also characterises when that friction makes cross-bank transferability a Nash equilibrium outcome.

Section 2 presents the model. Section 3 characterises equilibrium and derives conditions for efficient interbank trade. Section 4 studies the two-state case and illustrates the equilibrium structure with numerical examples. Section 5 analyses three policy instruments, namely a common-unit-of-account mandate, central bank reserves, and cash, and characterises their interaction. Section 6 shows how informational frictions at the point of exchange reinforce the baseline mechanism. Section 7 concludes.

2 The Model

The model has two banks, each producing a distinct basket of goods. They issue debt to labourers, who may use it as a means of payment when trading with artisans. Each bank chooses the size of its debt, its unit of account, and the conditions under which the debt can be transferred across regions. When banks use different units of account, cross-bank transfers create a state-contingent settlement friction. That friction tightens feasibility constraints, reduces borrowing capacity, and can lead banks to charge transfer fees or restrict interoperability.

2.1 Environment

There are two goods, $i \in \{A, B\}$, each of which serves as both a production output and a consumption good. Time is discrete, with three dates, 0, 1, and 2.

There are two regions. Each region contains three agents: a bank, a labourer, and an artisan. At date 0, the bank and labourer in the same region enter into a bilateral contract. The labourer provides labour input n to the bank in exchange for a debt contract, which is described below. At date 1, the labourer may meet an artisan. If a meeting occurs, the artisan can expend x units of labour effort to produce x units of a customised good. This good can be consumed only by the labourer for whom it is produced at date 2. It cannot be traded on the open market and has no value to other agents. At date 2, the bank's output is realised and a spot market opens in which goods A and B are traded. As in Doepke and Schneider (2017), the price vector $\mathbf{p}$ is exogenous and random, with convex and compact support $\mathbf{P} \subset \mathbb{R}_{>0}^2$.

All agents receive instantaneous utility $v(\mathbf{c})$ from consuming a vector $\mathbf{c}$ of goods A and B at date 2, where $v(\cdot)$ is homogeneous of degree 1. The bank derives utility only from consumption of tradable goods at date 2.

With probability $\alpha \geq 0$, the labourer meets an artisan in the same region. With probability $\beta \geq 0$, the labourer meets an artisan in the other region. With probability $1 - \alpha - \beta \geq 0$, no meeting occurs. These probabilities are symmetric across regions. An artisan who expends x units of labour to produce a customised good and then consumes a vector $\mathbf{c}$ of tradable goods has utility

$$v(\mathbf{c}) - x.$$

A labourer who provides n units of labour at date 0, receives x units of the customised good, and consumes a vector $\mathbf{c}$ of tradable goods at date 2 has utility

$$v(\mathbf{c}) + (1 + \gamma)x - n,$$

where $\gamma > 0$ implies gains from trade between the labourer and the artisan. Labourers cannot issue debt of their own. Trade between labourers and artisans can therefore occur only if bank debt can be transferred and used as a means of payment.

Each bank produces a positive vector of goods denoted by $\mathbf{y}_j$ according to the following production function

$$\mathbf{y}_j = n_j^\varphi \hat{\mathbf{y}}_j, \tag{1}$$

where n_j is the scalar quantity of labour supplied to Bank j by the labourer located in the same region as Bank j . The parameter $\varphi \in (0, 1)$ is such that the bank's production function has diminishing returns to the labour input. Each Bank produces a specific ratio of goods A and B captured by the unit vector $\hat{\mathbf{y}}_j$, which I refer to as the unit of production of Bank j . Each element of $\hat{\mathbf{y}}_j$ is weakly positive, denoting that production of each good is non-negative.

A key assumption is that the relative price risk between the two banks' asset portfolios cannot be insured at date 0. If complete insurance were available, banks could fully hedge this risk and the choice of unit of account would be irrelevant. The absence of complete markets reflects frictions that exist in reality. Contingent claims on future relative prices are often incomplete or missing, particularly for new or illiquid assets. These gaps are even more likely for digital money and intermediaries with unconventional business models.

Three frictions determine whether a common unit of account is required for efficient exchange. First, insurance against relative price risk is unavailable at date 0, so unit-of-account choice has real consequences for feasibility. Second, labourers cannot issue debt of their own, which makes bank debt the unique means of payment and gives the bank all contract-setting power. Third, banks must meet their debt obligations under every price realisation, not only in expectation. A unit of account that differs from the bank's production basket therefore tightens the promise-keeping constraints that limit borrowing capacity.

Throughout, each bank uses a single unit of account for its retail liabilities within its own region. Interbank claims can be tailored only for settlement across banks. Together with the absence of date-0 insurance, this contracting environment is what gives unit-of-account differences bite in the model.

As in Doepke and Schneider (2017), I normalise prices in the following way. First, prices

are normalised so that the price vector $\mathbf{p}$ satisfies $\tilde{P}(\mathbf{p}) = 1$ for all $\mathbf{p} \in P$ where $\tilde{P}(\mathbf{p})$ is the expenditure function at a utility level of 1 and

$$\tilde{P}(\mathbf{p}) = \min_{\mathbf{c}} \{\mathbf{p}'\mathbf{c}\}$$

subject to $v(\mathbf{c}) \geq 1$. This normalisation along with the assumption that v is homogeneous of degree 1 simplifies the analysis, since the indirect utility of consuming a bundle $\mathbf{y}$ of goods A and B at date 2 for a given price realisation is $\mathbf{p}'\mathbf{y}$. Second, the expected price of each good $i \in \{A, B\}$ is equal to 1: $E(p^i) = 1$.

2.2 Bank Debt

At date 0, the bank and labourer in region j negotiate a bilateral contract. The labourer provides n_j units of labour in exchange for bank debt $\mathbf{b}_j$, which is a promise to deliver a bundle of goods at date 2. The debt can be decomposed into a scale, $B_j > 0$, and a unit of account, $\hat{\mathbf{b}}_j$, such that $\mathbf{b}_j = B_j \hat{\mathbf{b}}_j$, where

$$\hat{\mathbf{b}}_j = \frac{\mathbf{b}_j}{\|\mathbf{b}_j\|_1}. \quad (2)$$

The contract also specifies whether the debt can be transferred to another agent. If it is transferable, it can be used as a means of payment and can therefore facilitate trade between labourers and artisans. Transferability may be restricted to artisans in the same region, or extended across regions. The bank may also charge a transfer fee. The fee $\chi_{j,k} \geq 0$ is the fraction of debt paid to bank j when the labourer transfers debt to an agent in region k . The fee may differ between transfers within the same region and transfers across regions.

The bank makes a take-it-or-leave-it offer to the labourer, subject to the feasibility constraints set out below. Banks are assumed to commit fully to the terms agreed at date 0.

At date 2, the bank must have sufficient resources to make the promised payment to the labourer for all possible price realisations $\mathbf{p}$. A debt that is not transferred to another agent is feasible if it satisfies the following condition

$$\mathbf{p}'(n_j^\varphi \hat{\mathbf{y}}_j - B_j \hat{\mathbf{b}}_j) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \quad (3)$$

At date 1, a labourer may meet an artisan and attempt to trade. If the debt issued by bank j is transferable, the labourer can request that bank j transfer a fraction $s_{j,k}$ of

the debt to an artisan in region k . All debt that bank j issues to artisans in region j is denominated in the same unit of account as the debt issued to its own labourer. Bank j can therefore allow intrabank transfers at no additional cost. Since $\gamma > 0$, such transfers are valuable whenever trade occurs. Although bank j may set an intrabank fee $\chi_{j,j}$, doing so does not relax the feasibility constraint in equation (3), since that constraint must also hold when no meeting takes place.

If the labourer requests bank j to transfer a fraction of its debt $s_{j,k} \in [0, 1]$ to an artisan located in region $k \neq j$ then the bank in region k must agree to take on this debt from bank j , effectively requiring a functioning interbank market. This interbank market is structured as follows. First, it is assumed that the bank that initiates the trade makes a take-it-or-leave-it offer to the receiving bank consisting of an interbank debt issued to the receiving bank $\mathbf{t}_{j,k}$ and the amount to be transferred between agents $s_{j,k} B_j \hat{\mathbf{b}}_j$. The interbank debt consists of a bundle of tradable goods and can be decomposed into a scale, T_j , and a unit of account, $\hat{\mathbf{t}}_{j,k}$. The unit of account $\hat{\mathbf{t}}_{j,k}$ is a unit vector and the decomposition is such that $\mathbf{t}_{j,k} = T_j \hat{\mathbf{t}}_{j,k}$.

If the receiving bank, k , accepts the terms of the transfer, it receives interbank debt $T_j \hat{\mathbf{t}}_{j,k}$ from the transferring bank, j . Bank k then takes on a fraction $s_{j,k}$ of bank j 's debt net of the transfer cost $\chi_{j,k}$. As it is assumed that bank k uses the same unit of account for all debt issued to labourers and artisans in region k , bank j ' debt is exchanged for bank k debt at the expected exchange rate at date 1 which is $\frac{E[\mathbf{p}' \hat{\mathbf{b}}_j]}{E[\mathbf{p}' \hat{\mathbf{b}}_k]}$. Given the assumption that $E[p_i] = 1$ for all $i \in \{A, B\}$ and $\hat{\mathbf{b}}_j$ and $\hat{\mathbf{b}}_k$ are unit vectors and $\frac{E[\mathbf{p}' \hat{\mathbf{b}}_j]}{E[\mathbf{p}' \hat{\mathbf{b}}_k]} = 1$.

It must also be feasible for bank k to accept the debt transfer and to pay its obligation to the artisan in region k given all possible price realisations $\mathbf{p}$. To simplify the analysis, I assume that the interbank debt received by bank k must be sufficient to cover its liabilities to the artisan in region k . Thus, it is feasible for the bank k to accept a bank transfer if the following equation holds

$$\mathbf{p}' \left(T_j \hat{\mathbf{t}}_{j,k} - s_{j,k} (1 - \chi_{j,k}) B_j \hat{\mathbf{b}}_k \right) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \quad (4)$$

A transfer of debt between bank j and bank k also changes the financial obligations of bank j . A fraction $s_{j,k}$ of bank j 's debt to the labourer in region j is cancelled, but a new interbank debt obligation to bank k is created. For the interbank transfer to be feasible, bank j must be able to repay its obligations to both the labourer in region j and the bank in region j from its output given all possible price realisations. Thus, the following must hold

$$\mathbf{p}' \left(n_j^\varphi \hat{\mathbf{y}}_j - (1 - s_{j,k}) B_j \hat{\mathbf{b}}_j - T_j \hat{\mathbf{t}}_{j,k} \right) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \quad (5)$$

As the bank initiating the transfer, in this case bank j , has all the bargaining power, it follows from equation (4) that it is optimal for bank j to ensure bank k makes zero profit from the transfer in every possible price realisation. This occurs if the following holds

$$T_j \mathbf{p}' \hat{\mathbf{t}}_{j,k} = s_{j,k} (1 - \chi_{j,k}) B_j \mathbf{p}' \hat{\mathbf{b}}_k \quad \forall \mathbf{p} \in \mathbf{P}. \quad (6)$$

For this to hold in every possible price realisation, it must be the case that the interbank loan has the same unit of account as the debt bank k issues to the region k artisan. That is, it must be the case that $\hat{\mathbf{t}}_{j,k} = \hat{\mathbf{b}}_k$. The size of the interbank transfer is thus set to ensure that equation (4) binds for all price realisations.

Should the units of account differ between banks j and k , $\hat{\mathbf{b}}_j \neq \hat{\mathbf{b}}_k$, allowing for debt to be transferred between the two banks creates additional uncertainty relating to the realisation of the value of the liabilities created in the transfer. If the receiving bank k takes on some of this risk, then the feasibility constraint, equation (4), must be slack in some situations resulting in bank k earning strictly positive profit in expectation. From the perspective of bank j it is optimal for bank j to fully insure bank k from the uncertainty that results from the mismatch of the unit of account. Here equation (4) will always hold. Allowing debt to be fully transferable for any $s_{j,k} \in [0, 1]$ and substituting in the optimal interbank contract offered by bank j , equation (5) can be rewritten as

$$\mathbf{p}' \left(n_j^\varphi \hat{\mathbf{y}}_j - (1 - \chi_{j,k}) \hat{\mathbf{b}}_k B_j \right) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \quad (7)$$

A debt contract denominated in unit of account $\hat{\mathbf{b}}_j$ is a state-contingent claim on bank j 's output: its value in state $\mathbf{p}$ is $\mathbf{p}' \hat{\mathbf{b}}_j$, which varies across states whenever $\hat{\mathbf{b}}_j$ differs from the production basket $\hat{\mathbf{y}}_j$. When bank j permits its liabilities to be transferred to artisans in region k , the interbank obligation is denominated in bank k 's unit of account $\hat{\mathbf{b}}_k$. That obligation is therefore a state-contingent claim on bank j 's output valued at $\mathbf{p}' \hat{\mathbf{b}}_k$, not at $\mathbf{p}' \hat{\mathbf{b}}_j$. When $\hat{\mathbf{b}}_k \neq \hat{\mathbf{b}}_j$, the value of what bank j owes to bank k varies differently across states from the value of bank j 's production, tightening the feasibility constraints on bank j 's debt and reducing its borrowing capacity. The transfer fee $\chi_{j,k}$ can offset this tightening to the extent allowed by the labourer's participation constraint, but cannot eliminate it entirely when cross-bank differences in units of account are large.

The settlement requirement created by cross-denominated claims is state-contingent and binds most tightly in the state that maximises the value of the promised payment relative to bank j 's output. The associated settlement requirement is

$$\epsilon_{j,k} \equiv \max_{\mathbf{p} \in \mathbf{P}} \frac{\mathbf{p}' \hat{\mathbf{b}}_k}{\mathbf{p}' \hat{\mathbf{y}}_j}. \quad (8)$$

This is the largest number of units of bank j 's output value needed across states to settle one unit of debt denominated in bank k 's unit of account. It equals 1 when $\hat{\mathbf{b}}_k = \hat{\mathbf{y}}_j$. It exceeds 1 when the two units of account differ. The excess, $\epsilon_{j,k} - 1$, captures the cross-bank settlement friction. That friction raises transfer fees, tightens the debt constraint, reduces investment, and, when sufficiently large, makes cross-bank transferability unprofitable.

Substituting the optimal interbank contract into the feasibility conditions yields two constraints. If the debt remains within bank j , feasibility requires

$$n_j^\varphi \frac{1}{\epsilon_{j,j}} - B_j \geq 0, \quad (9)$$

where $\epsilon_{j,j}$ is the state-contingent settlement requirement for debt denominated in bank j 's own unit of account, relative to its production. If the debt is transferred across banks, the corresponding feasibility condition is

$$n_j^\varphi \frac{1}{\epsilon_{j,k}} - (1 - \chi_{j,k}) B_j \geq 0, \quad (10)$$

where $\epsilon_{j,k} > 1$ implies that the cross-bank constraint is tighter than the own-bank constraint. The transfer fee $\chi_{j,k}$ partially relaxes this constraint by reducing the amount of debt that must be settled across banks.

2.3 Decentralised Trade

Allowing debt to be transferred across banks imposes an additional feasibility constraint when banks use different units of account. At the same time, transferability allows bank debt to circulate as a means of payment and therefore creates gains from trade between labourers and artisans. For debt to be transferable in equilibrium, these gains must outweigh the associated settlement friction.

Consider a labourer in region j who provides n_j units of labour to bank j in exchange for debt $\mathbf{b}_j = B_j \hat{\mathbf{b}}_j$. If the labourer does not meet an artisan at date 1, the bank delivers the promised bundle of tradable goods at date 2. Given the price normalisation and the homogeneity of $v(\cdot)$, the labourer's utility is

$$E[\mathbf{p}'\mathbf{b}_j] - n_j = B_j - n_j. \quad (11)$$

This is also the expected utility of a labourer who holds non-transferable debt and is therefore unable to trade with artisans at date 1.

Now consider the case in which the labourer in region j meets an artisan in region k . The labourer can then use bank debt as a means of payment, subject to the transfer fee $\chi_{j,k}$

agreed at date 0. The labourer makes a take-it-or-leave-it offer to the artisan, offering a fraction $s_{j,k} \in [0, 1]$ of the debt in exchange for x_k units of the customised good. The labourer therefore chooses $s_{j,k}$ and x_k to maximise

$$\max_{s_{j,k}, x_k} (1 - s_{j,k}) E[\mathbf{p}'\mathbf{b}_j] + (1 + \gamma) x_k - n_j, \quad (12)$$

subject to the artisan's participation constraint

$$s_{j,k} (1 - \chi_{j,k}) E[\mathbf{p}'\mathbf{b}_j] - x_k \geq 0. \quad (13)$$

The labourer's problem has a corner solution. The labourer either sets $s_{j,k} = 1$ and chooses x_k so that the artisan's participation constraint binds, or sets $s_{j,k} = x_k = 0$ and does not trade.

Since $\gamma > 0$, trade occurs whenever the transfer fee is sufficiently low. In particular, the fee must satisfy

$$(1 + \gamma) (1 - \chi_{j,k}) \geq 1. \quad (14)$$

Assuming the transfer fee is sufficiently low, the labourer's expected utility from tradable debt is

$$\begin{aligned} U_j = & \alpha (1 + \gamma) (1 - \chi_{j,j}) E[\mathbf{p}'\mathbf{b}_j] \\ & + \beta (1 + \gamma) (1 - \chi_{j,k}) E[\mathbf{p}'\mathbf{b}_j] \\ & + (1 - \alpha - \beta) E[\mathbf{p}'\mathbf{b}_j] - n_j, \end{aligned} \quad (15)$$

where α is the probability that the labourer from region j meets an artisan also from region j while with probability β , the labourer meets a foreign artisan from region $k \neq j$.

Using the price normalisation the above can be rewritten as

$$U_j = [(1 + \gamma) (\alpha (1 - \chi_{j,j}) + \beta (1 - \chi_{j,k})) + (1 - \alpha - \beta)] B_j - n_j, \quad (16)$$

where B_j is the size of the debt the labourer receives from bank j . The above equation will enter as a constraint in the Bank's optimisation problem which will be detailed later. The quantity of labour that a labourer is willing to supply will be larger in cases where the debt is transferable and where the transfer fees are sufficiently low. Also, since labourers are risk neutral, the unit of account of bank j 's debt does not directly enter the utility function of the labourer.

Transferability raises the liquidity value of bank debt. Labourers who can use their debt as a means of payment accept a lower wage, which expands the bank's borrowing capacity and investment. Cross-bank transferability creates an additional gain when a labourer

meets a foreign artisan, but it also introduces a cost. Because the interbank obligation is denominated in bank k 's unit of account, it is a state-contingent claim on bank j 's output priced in another unit of account. When the two units of account differ, honouring that claim is more expensive in the states in which bank j 's output is least valuable. This tightens the promise-keeping constraint and reduces debt capacity. The bank therefore faces a trade-off. Cross-bank transferability increases the liquidity value of the bank's liabilities, but it also makes them more expensive to honour across states when claims are priced in another unit of account. This trade-off determines the equilibrium transfer fee and the level of investment characterised in Section 3.

2.4 Timeline

First, at date 0 the bank in region j makes a take-it-or-leave-it offer of a promise to pay the labourer in region j a bundle of goods $\mathbf{b}_j$ in exchange for labour input n_j . The bank specifies under which conditions the debt can be transferred to another agent and specifies a transfer fee that is payable if the debt is transferred. If the labourer accepts, the bank receives labour input to its production function and the labourer becomes a debtholder.

Second, at date 1 the labourers may randomly meet an artisan. A labourer who meets an artisan and holds transferable bank debt makes a take-it-or-leave-it offer to the artisan, specifying the amount of bank debt to transfer in exchange for a quantity x of a custom good. If bank j receives a request to transfer its debt to a bank located in region k , bank j makes a take-it-or-leave-it offer to bank k specifying the size of the debt to be issued by bank k in exchange for an interbank debt issued by bank j .

Finally, at date 2, the bank's production is realised, and the banks make good on their promised payments. A competitive spot market for goods A and B opens and the exogenously given prices of the two tradable goods $\mathbf{p}$ are realised.

2.5 Constrained Optimal Allocation

The constrained optimum is the relevant benchmark because it isolates the inefficiency created by strategic behaviour from the inefficiency created by the model's underlying frictions. Strategic behaviour here takes the form of transfer fees and restricted interoperability. A planner who faces the same production technology and uninsurable price risk, but directs investment and ensures full trade in every meeting, implements this allocation.

With the price normalisation, the marginal benefit of one unit of bank output is 1 when the labourer does not meet an artisan and $1 + \gamma$ when a meeting occurs. The social planner's problem is therefore

$$\mathcal{W} = \max_{\{n_j\}_j} \sum_j [(1 + \gamma(\alpha + \beta)) E[\mathbf{p}'\hat{\mathbf{y}}_j] n_j^\varphi - n_j], \quad (17)$$

where the price normalisation implies $E[\mathbf{p}'\hat{\mathbf{y}}_j] = 1$.

The planner chooses the scale of each bank's project so that the marginal benefit of an additional unit of labour equals its marginal cost. The resulting optimal scale for bank j is

$$n_j^* = [\varphi(1 + \gamma(\alpha + \beta))]^{\frac{1}{1-\varphi}}, \quad (18)$$

and optimal welfare is

$$\mathcal{W} = \sum_j (1 - \varphi) [\varphi(1 + \gamma(\alpha + \beta))]^{\frac{\varphi}{1-\varphi}}. \quad (19)$$

Aggregate welfare is increasing in the meeting probabilities α and β , as well as in the gains from trade γ .

3 Equilibrium

This section characterises equilibrium in three steps. It first studies the contract a bank offers when liabilities circulate only within its own region. It then analyses the contract when cross-bank transfers are allowed and shows how banks decide whether to permit them. Finally, it characterises strategic interaction between banks over units of account, transfer fees, and interoperability.

Within a given contract, bank j chooses the labour input n_j , the size of debt B_j , the unit of account in which that debt is denominated, and the conditions under which the debt can be transferred. Because transfers within the same bank do not create an additional settlement friction, the central contractual margin is whether debt is allowed to circulate across banks and, if so, the interbank transfer fee $\chi_{j,k}$ that applies. The contracting environment therefore fixes the scope of private risk-sharing and unit-of-account choice at the outset. These institutional features are consequential. They make unit-of-account design economically meaningful and create a role for public interventions, such as reserves and cash, that relax settlement and risk-sharing constraints private issuers take as given.

3.1 Contracting under Autarky

First, consider the contract bank j offers when it does not allow interbank transfers. In that case, its liabilities circulate only within its own region. Bank j chooses labour n_j , debt size B_j , its unit of account, and the intrabank transfer fee $\chi_{j,j}$ to maximise expected profit conditional on not allowing interbank transfers.

The bank's optimisation problem is to maximise its profit

$$V_j = \max_{\{n_j, B_j, \mathbf{b}_j, \chi_{j,j}\}} n_j^\varphi - (1 - \alpha\chi_{j,j}) B_j, \quad (20)$$

subject to the participation constraint of the labourer, the promise-keeping constraint on its debt, and a maximum intrabank transfer fee that would be paid by the labourer.

The participation constraint requires the labourer's expected utility to be non-negative, so $U_j \geq 0$. Restricting interbank transfers eliminates gains from trade with foreign artisans. This yields

$$[\alpha(1 + \gamma)(1 - \chi_{j,j}) + 1 - \alpha] B_j \geq n_j. \quad (21)$$

The bank must ensure that the promise-keeping constraint, equation (9), holds in all states. This constraint also covers the case where the labourer transfers the debt to an artisan within the same region, regardless of whether an intrabank fee is imposed.

There is an upper limit on the intrabank transfer fee. Since the gain from trade between labourers and artisans is $(1 + \gamma)$, the fee must not be large enough to eliminate this benefit entirely. The upper bound on $\chi_{j,j}$ is

$$(1 + \gamma)(1 - \chi_{j,j}) \geq 1. \quad (22)$$

The following proposition characterises the optimal contract when bank j does not allow interbank transfers.

Proposition 1 (Autarky Contract). *Suppose bank j does not allow interbank transfers. Then the contract under autarky features zero intrabank fees ($\chi_{j,j} = 0$), investment*

$$n_j^{(0)} = [\varphi(1 + \gamma\alpha)]^{\frac{1}{1-\varphi}},$$

debt size $B_j^{(0)} = \varphi^{\frac{1}{1-\varphi}}(1 + \gamma\alpha)^{\frac{\varphi}{1-\varphi}}$, profit $V_j^{(0)} = \varphi^{\frac{\varphi}{1-\varphi}}(1 + \gamma\alpha)^{\frac{\varphi}{1-\varphi}}(1 - \varphi)$, and any unit of account satisfying $\epsilon_{j,j} \leq 1/\varphi$.

Proof. See Appendix A.1. □

It is optimal to set the intrabank fee to zero, so $\chi_{j,j} = 0$. Raising the fee increases fee revenue, but it also reduces the amount of debt the labourer can transfer to the artisan by the same amount. Since $\gamma > 0$, the gains from trade are positive. As a result, a higher fee raises the bank's cost of debt by more than it raises fee revenue.

Both the labour input chosen by the bank, $n_j^{(0)}$, and the profit it earns, $V_j^{(0)}$, are below the constrained optimum whenever $\beta > 0$. When interbank transfers are not allowed, labourers cannot use bank debt to trade with foreign artisans. They therefore value the debt less, and the bank's borrowing capacity is lower than it otherwise would be.

In equilibrium, only the labourer's participation constraint, equation (21), binds. Bank j 's unit of account $\epsilon_{j,j}$ enters the bank's problem only through the promise-keeping constraint, equation (9). It is always optimal for bank j to choose a unit of account such that this constraint never binds. The natural choice is $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$, giving $\epsilon_{j,j} = 1$, under which the promise-keeping constraint becomes $n_j^\varphi \geq B_j$ and is satisfied whenever the bank earns non-negative profit. More generally, any unit of account satisfying $\epsilon_{j,j} \leq 1/\varphi$ leaves the constraint slack; since $\varphi < 1$, there is some flexibility around which corresponds to the natural unit of account ($\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$).

3.2 Contracting with Transferable Debt

Bank j maximises profit by choosing labour n_j , debt size B_j , its unit of account $\hat{\mathbf{b}}_j$, an intrabank fee $\chi_{j,j} \geq 0$, and an interbank fee $\chi_{j,k} \geq 0$.

$$W_j = \max_{\{\chi_{j,j}, \chi_{j,k}, \hat{\mathbf{b}}_j, B_j, n_j\}} n_j^\varphi - (1 - \alpha\chi_{j,j} - \beta\chi_{j,k}) B_j, \quad (23)$$

subject to the participation constraint of the labourer, the promise-keeping constraint on the debt issued to the labourer in region j , the maximum interbank transfer fees that would be paid by the labourer and an additional promise-keeping constraint should the debt be transferred to region k .

The participation constraint of the labourer adds the gains from trade with foreign artisans to the autarky case:

$$[(1 + \gamma)(\alpha(1 - \chi_{j,j}) + \beta(1 - \chi_{j,k})) + (1 - \alpha - \beta)] B_j \geq n_j. \quad (24)$$

The promise-keeping constraint on the original debt contract is the same as in the autarky case and is given by equation (9). There is also a second promise-keeping constraint on the

interbank obligation. When bank k chooses a different unit of account from bank j , cross-region transfers create the state-contingent settlement friction captured by equation (10).

Finally, there are upper bounds on the transfer fees. The upper bound on the fee for the intrabank transfer is identical to that in the autarky case and is given by equation (22). In addition, there is an analogous upper-bound on interbank transfers given by the equation below

$$(1 + \gamma)(1 - \chi_{j,k}) \geq 1. \quad (25)$$

Proposition 2 (Transferable Debt). *Suppose bank j allows interbank transfers and bank k 's unit of account induces state-contingent settlement requirement $\epsilon_{j,k}$. Then $\chi_{j,j} = 0$, $\epsilon_{j,j} \leq 1/\varphi$, and:*

- (i) *If $\epsilon_{j,k} \leq 1/\varphi$, the interbank constraint does not bind. Bank j sets $\chi_{j,k} = 0$ and achieves the constrained optimum with investment $n_j^{(1)} = n_j^*$, debt size $B_j^{(1)} = \varphi^{\frac{1}{1-\varphi}}(1 + \gamma(\alpha + \beta))^{\frac{\varphi}{1-\varphi}}$, and profit*

$$W_j^{(1)} = \varphi^{\frac{\varphi}{1-\varphi}}(1 + \gamma(\alpha + \beta))^{\frac{\varphi}{1-\varphi}}(1 - \varphi).$$

- (ii) *If $\epsilon_{j,k} > 1/\varphi$, the interbank constraint binds. Bank j sets $\chi_{j,k} = \chi_{j,k}^*(\epsilon_{j,k}) \geq 0$ (characterised in Appendix C), invests below the optimum ($n_j^{(2)} < n_j^{(1)} = n_j^*$), and earns profit*

$$W_j^{(2)} = \left[\frac{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k}^*)}{\epsilon_{j,k}(1 - \chi_{j,k}^*)} \right]^{\frac{\varphi}{1-\varphi}} \left(1 - \frac{1 - \beta\chi_{j,k}^*}{\epsilon_{j,k}(1 - \chi_{j,k}^*)} \right),$$

which is strictly decreasing in $\epsilon_{j,k}$. Explicit expressions for $n_j^{(2)}$ and $B_j^{(2)}$ are given in Appendix B.

Proof. See Appendix B. □

Case (i) describes the benchmark in which the interbank promise-keeping constraint does not bind. In that case, bank k 's unit of account is sufficiently close to bank j 's production basket that cross-bank settlement is not costly in the relevant states. In this case, the interbank promise-keeping constraint is relatively easy to satisfy and equation (10) does not bind in equilibrium. Bank j is then able to hire the optimal amount of labour $n_j = n_j^*$ and will choose to allow its debt to be costlessly transferable to region k by setting $\chi_{j,k} = 0$.

In case (ii), bank k 's unit of account induces a large state-contingent settlement requirement $\epsilon_{j,k}$. In the worst-case state for bank j , honouring a claim denominated in $\hat{\mathbf{b}}_k$

requires more output than honouring an equally sized claim in bank j 's own production basket. The interbank promise-keeping constraint, equation (10), therefore binds and it is no longer feasible to implement the constrained optimal debt contract.

Bank j responds in two ways. It first reduces investment, so that $n_j < n_j^*$, to loosen the interbank constraint. As $\epsilon_{j,k}$ rises further, it also raises the interbank transfer fee $\chi_{j,k} > 0$. The fee is the endogenous price of interoperability. It allows bank j to recover part of the cost of honouring claims denominated in another unit of account when those claims are expensive to settle in adverse states. A higher fee tightens the labourer's participation constraint, but it also relaxes the interbank promise-keeping constraint by reducing the amount that must be transferred across banks. Bank j therefore sets the fee at the point where these effects balance.

As in the autarkic contract, it is always optimal for bank j to set $\chi_{j,j} = 0$. The only constraint that $\chi_{j,j}$ enters is the labourer's participation constraint. Raising the intrabank fee tightens that constraint more than it increases fee revenue, so it is always more efficient to hold the fee to zero and expand investment instead.

Bank j can always choose its own unit of account to ensure that the initial promise-keeping constraint, equation (9), does not bind, by setting $\hat{\mathbf{b}}_j$ sufficiently close to $\hat{\mathbf{y}}_j$. Setting $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$ implies $\epsilon_{j,j} = 1$ so the own-bank constraint is weaker than the interbank promise-keeping constraint and does not bind in equilibrium. The feasible range for bank j 's unit of account depends on the equilibrium contract, which in turn depends on bank k 's choice.

Proposition 2 implies three distinct fee regimes, separated by two cutoffs $\bar{\epsilon}_{j,k}^{(0)}$ and $\bar{\epsilon}_{j,k}^{(1)}$ with $1/\varphi < \bar{\epsilon}^{(0)} < \bar{\epsilon}^{(1)}$. Bank j charges no fee when $\epsilon_{j,k} \leq \bar{\epsilon}^{(0)}$, an interior fee when $\bar{\epsilon}^{(0)} < \epsilon_{j,k} < \bar{\epsilon}^{(1)}$, and the maximum feasible fee $\gamma/(1 + \gamma)$ when $\epsilon_{j,k} \geq \bar{\epsilon}^{(1)}$. The explicit expressions for these cutoffs are derived in Appendix C. Once the interbank constraint binds, bank profit is strictly decreasing in $\epsilon_{j,k}$. By the envelope theorem,

$$\frac{dW_j}{d\epsilon_{j,k}} = -\lambda_k \frac{n_j^\varphi}{\epsilon_{j,k}^2} < 0 \quad \text{for } \lambda_k > 0.$$

In the intermediate case $\bar{\epsilon}_{j,k}^{(0)} < \epsilon_{j,k} < \bar{\epsilon}_{j,k}^{(1)}$, the optimal transfer fee $\chi_{j,k}^*$ is characterised by a quadratic equation whose closed-form solution depends on the specific structure of the price set P and production vectors. A closed-form expression is available in the simple two-state case studied later.

3.3 Interoperability Choice

The next step is to determine whether bank j chooses to allow its liabilities to circulate beyond its own region. This decision compares the profit from transferable debt with the profit from autarky. Bank j 's decision can be formalised as

$$\pi_j(\epsilon_{j,k}) = \max \left\{ W_j(\epsilon_{j,k}), V_j^{(0)} \right\}, \quad (26)$$

where $W_j(\epsilon_{j,k})$ denotes bank j 's profit if it allows its liabilities to be transferred to region k with

$$W_j(\epsilon_{j,k}) = \begin{cases} \varphi^{\frac{\varphi}{1-\varphi}} (1 + \gamma(\alpha + \beta))^{\frac{\varphi}{1-\varphi}} (1 - \varphi) & \text{if } \epsilon_{j,k} \leq \frac{1}{\varphi} \\ \left[\frac{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k}^*(\epsilon_{j,k}))}{\epsilon_{j,k}(1 - \chi_{j,k}^*(\epsilon_{j,k}))} \right]^{\frac{\varphi}{1-\varphi}} \left(1 - \frac{1 - \beta\chi_{j,k}^*(\epsilon_{j,k})}{\epsilon_{j,k}(1 - \chi_{j,k}^*(\epsilon_{j,k}))} \right) & \text{if } \epsilon_{j,k} > \frac{1}{\varphi} \end{cases} \quad (27)$$

and where $V_j^{(0)}$ is bank j 's profit if it chooses autarky as defined in Proposition 1.

The choice bank k makes regarding its unit of account will determine to what extent the interbank promise-keeping constraint binds. The best response of bank j to bank k 's decision will be to vary both the terms of the debt contract and to vary whether or not to allow the debt to be transferable to region k or not.

In cases where $\epsilon_{j,k} \leq \frac{1}{\varphi}$, bank j will always choose to allow interbank transfers as their profit is strictly higher than in the autarkic case. However, as $\epsilon_{j,k}$ increases, bank j 's profit when allowing interbank transfers decreases. At the limit bank profit goes to zero, $\lim_{\epsilon_{j,k} \rightarrow \infty} W_j(\epsilon_{j,k}) = 0$, and thus bank j earns a higher profit in autarky.

The following proposition formalises this trade-off by establishing a unique threshold above which autarky dominates.

Proposition 3 (Autarky Threshold). *Suppose $\beta > 0$ and $\gamma > 0$. There exists a unique threshold $\bar{\epsilon}_{j,k}^{(2)} > \frac{1}{\varphi}$, implicitly defined by $W_j(\bar{\epsilon}_{j,k}^{(2)}) = V_j^{(0)}$, such that:*

- (i) bank j strictly prefers allowing interbank transfers when $\epsilon_{j,k} < \bar{\epsilon}_{j,k}^{(2)}$;
- (ii) bank j strictly prefers autarky when $\epsilon_{j,k} > \bar{\epsilon}_{j,k}^{(2)}$.

The threshold satisfies

$$\bar{\epsilon}_{j,k}^{(2)} = \bar{\epsilon}^{(2)}(\gamma, \alpha, \beta, \varphi)$$

and is strictly increasing in β and in γ .

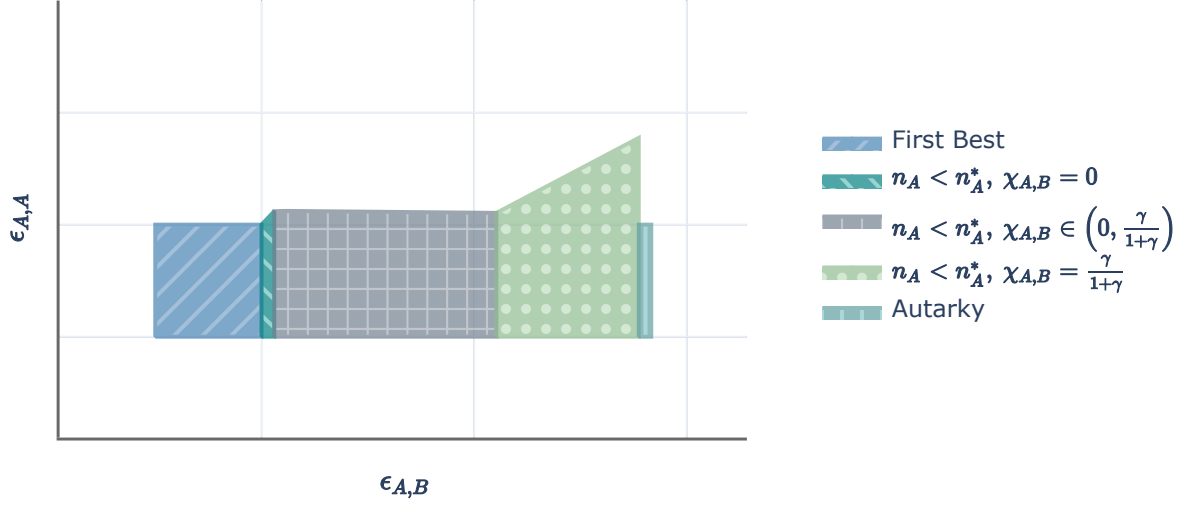


Figure 1: Bank j 's Best Responses Given Bank k 's Choice of Unit of Account

Proof. See Appendix B. □

The threshold $\bar{\epsilon}_{j,k}^{(2)}$ identifies the point at which the gains from interoperability no longer outweigh the settlement friction created by cross-bank transfers. A higher probability of meeting a foreign artisan or a larger gain from trade raises the value of allowing interbank transfers, so the threshold increases with both β and γ . Figure 1 illustrates how bank j 's best response varies with bank k 's unit of account. Bank j chooses its own unit of account so that the initial borrowing constraint in equation (9) does not bind. The figure therefore isolates how the profitability of interoperability changes as bank k 's unit of account moves away from bank j 's production basket.

Figure 1 illustrates the conditions under which a bank restricts interoperability as the other bank's unit of account diverges from its own production basket.

Consider bank k as an incumbent with an established unit of account, and bank j as an entrant that takes that choice as given. When bank k 's unit of account is close to bank j 's production basket, $\epsilon_{j,k}$ is small and the interbank promise-keeping constraint is slack. In that case, the entrant sets $\chi_{j,k} = 0$. As bank k 's unit of account moves further away from bank j 's production basket, $\epsilon_{j,k}$ rises. Claims denominated in another unit of account then become more expensive for bank j to honour in adverse states. The entrant first responds by raising the interbank transfer fee. Once $\epsilon_{j,k}$ exceeds $\bar{\epsilon}_{j,k}^{(2)}$, it withdraws from interbank transfers altogether. This is not an arbitrary restriction on transferability. It reflects the fact that settlement becomes unprofitable when claims are priced in another unit of account.

3.4 Strategic Interaction of Banks

I now characterise the strategic interaction between banks over their choice of unit of account. I consider two equilibrium concepts. The first is a staggered-entry benchmark in which bank k has committed to a unit of account before bank j enters and bank j best-responds to bank k 's choice. The second is a simultaneous-move Nash equilibrium in which both banks choose their units of account, transfer fees, and interoperability decisions at the same time. In both cases, each bank retains its choice over transfer fees and interoperability.

Consider first the staggered entry case, in which bank k has already committed to its unit of account before bank j enters. Bank j then takes $\hat{\mathbf{b}}_k$ as given and chooses its contract as in Propositions 1–3. The relevant object is the settlement requirement

$$\epsilon_{j,k} = \max_{\mathbf{p} \in \mathbf{P}} \frac{\mathbf{p}' \hat{\mathbf{b}}_k}{\mathbf{p}' \hat{\mathbf{y}}_j}.$$

If $\epsilon_{j,k} \leq 1/\varphi$, the entrant decentralises the constrained optimum by setting $\chi_{j,k} = 0$.

I now turn to the Nash equilibrium. Efficiency requires that each bank's unit of account be sufficiently close to the other's unit of production, so $\epsilon_{j,k} \leq 1/\varphi$ and $\epsilon_{k,j} \leq 1/\varphi$. When feasible units of account satisfy these inequalities for both banks, an efficient fee-free equilibrium exists. The following proposition records the optimal contract regimes induced by a given pair $(\epsilon_{j,k}, \epsilon_{k,j})$ and the corresponding equilibrium existence results.

Proposition 4 (Nash Equilibrium). *Consider the simultaneous-move game in which each bank $i \in \{j, k\}$ chooses $\hat{\mathbf{b}}_i$ and optimises its debt contract as in Propositions 1–3, yielding payoff $\pi_i(\epsilon_{i,\ell}) = \max\{W_i(\epsilon_{i,\ell}), V_i^{(0)}\}$. Autarky is always a Nash equilibrium. Given the other bank's unit of account, bank i 's optimal contract depends on $\epsilon_{i,\ell}$ and falls into one of four regimes. For a given $\epsilon_{i,\ell}$, the optimal fee $\chi_{i,\ell}^*$ is uniquely determined.*

- Zero-fee: $\epsilon_{i,\ell} \leq \bar{\epsilon}_{i,\ell}^{(0)}$, $\chi_{i,\ell}^* = 0$. Investment is at the constrained optimum if $\epsilon_{i,\ell} \leq 1/\varphi$; the interbank constraint binds and investment falls below the optimum for $\epsilon_{i,\ell} \in (1/\varphi, \bar{\epsilon}_{i,\ell}^{(0)})$.
- Positive-fee: $\bar{\epsilon}_{i,\ell}^{(0)} < \epsilon_{i,\ell} < \bar{\epsilon}_{i,\ell}^{(1)}$, $\chi_{i,\ell}^* \in (0, \gamma/(1+\gamma))$.
- Maximum-fee: $\bar{\epsilon}_{i,\ell}^{(1)} \leq \epsilon_{i,\ell} < \bar{\epsilon}_{i,\ell}^{(2)}$, $\chi_{i,\ell}^* = \gamma/(1+\gamma)$.
- Autarky: $\epsilon_{i,\ell} \geq \bar{\epsilon}_{i,\ell}^{(2)}$, interbank transfers prohibited.

If both banks choose a common unit of account $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k = \hat{\mathbf{b}}$ such that $\max_{\mathbf{p} \in \mathbf{P}} \mathbf{p}' \hat{\mathbf{b}} / \mathbf{p}' \hat{\mathbf{y}}_i \leq 1/\varphi$ for both banks, the resulting profile is an efficient fee-free Nash equilibrium.

Proof. See Appendix B. □

Efficient zero-fee equilibria, when they exist, Pareto-dominate the other contract outcomes described above. A common unit of account is sufficient for efficiency, but it is not necessary. Distinct units of account can also implement the constrained optimum if both cross-price constraints satisfy $\epsilon_{j,k} \leq 1/\varphi$ and $\epsilon_{k,j} \leq 1/\varphi$. Zero fees are not enough on their own. When $\epsilon_{i,\ell} \in (1/\varphi, \bar{\epsilon}_{i,\ell}^{(0)}]$, the interbank promise-keeping constraint binds and bank profit falls below the optimum even though no fee is charged.

Equilibrium multiplicity can arise in the baseline environment because any unit of account that satisfies the promise-keeping constraint yields the same debt capacity and the same profit. Banks are therefore indifferent across a range of feasible unit-of-account choices. Section 6 considers an extension in which this indifference no longer holds. When acceptance depends on the realised value of debt, issuers prefer units of account that minimise information sensitivity.

3.5 Common Unit of Account and Deviations from Par

Three concepts are related but distinct: efficiency, par trading, and the state-contingent value of bank liabilities.

A liability denominated in unit of account $\hat{\mathbf{b}}_j$ is a state-contingent claim. Its value in state $\mathbf{p}$ is $\mathbf{p}'\hat{\mathbf{b}}_j$. When two banks issue liabilities denominated in different units of account, those liabilities are different state-contingent claims, and their values diverge in at least one state. When both banks adopt a common unit of account, $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k$, their liabilities have the same value in every state. This removes the cross-bank settlement friction. Bank j 's interbank obligation then reduces from $\epsilon_{j,k}$ to $\epsilon_{j,j}$, so the interbank constraint is no tighter than the promise-keeping constraint. If the common basket differs from bank j 's production basket, the promise-keeping constraint may still bind at $\epsilon_{j,j} > 1$, but there is no additional friction from differences in units of account.

Par trading follows from common denomination in the absence of default risk. If $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k$, the two liabilities have identical values in every state and therefore exchange one for one. If banks choose different units of account, their liabilities differ in value in at least one state and do not trade at par. The size of this deviation increases with $\epsilon_{j,k}$ and $\epsilon_{k,j}$.

Efficiency and par are related, but they are not the same. Efficiency requires the interbank promise-keeping constraints to be slack and investment to reach n_j^* . This occurs when $\epsilon_{j,k} \leq 1/\varphi$ and $\epsilon_{k,j} \leq 1/\varphi$. These conditions can hold even when banks use different units

of account. In that case, the constrained optimum is attainable even though liabilities are different state-contingent claims and do not trade at par. A common unit of account is therefore sufficient for efficiency, but not necessary. Par trading is also not sufficient for efficiency. Liabilities may trade at par in expectation even when promise-keeping constraints bind in adverse states and investment falls below the optimum.

When production vectors diverge, all feasible equilibria involve positive transfer fees or autarky. In those cases, lower investment and reduced trade directly lower welfare relative to the constrained optimum. Section 5 studies the policy interventions that can restore coordination on an efficient equilibrium.

4 Two-State Analytical Example

This section studies a two-state, two-good version of the model in which all equilibrium objects can be expressed as functions of a single parameter ρ indexing price variance. The two-state case makes the equilibrium structure transparent. It distinguishes the natural-unit-of-account benchmark from equilibria with a common unit of account, while also allowing for equilibria that lie between these two cases.

4.1 Setup

To fix ideas, I assume that bank j produces only good A and bank k produces only good B . For convenience, I index banks by the good they produce, replacing the subscript j with A and k with B . The units of production for the two banks are

$$\hat{\mathbf{y}}_A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \hat{\mathbf{y}}_B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (28)$$

I simplify the price vector so that there are only two states that occur with equal probability. In state A , $p_A = 1 - \rho_A$ and $p_B = 1 + \rho_B$; in state B , $p_A = 1 + \rho_A$ and $p_B = 1 - \rho_B$, where ρ_A and ρ_B are normalised to be positive and capture the variance in the two prices.

In the symmetric case $\rho_A = \rho_B = \rho$, specialising the settlement requirement $\epsilon_{j,k}$ from equation (8) to the natural unit of account ($\hat{\mathbf{b}}_k = \hat{\mathbf{y}}_k$, $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$) and the two-state price process yields

$$\epsilon_{A,B} = \epsilon_{B,A} = \frac{1 + \rho}{1 - \rho}, \quad \rho \in [0, 1). \quad (29)$$

The mapping in equation (29) is strictly increasing in ρ . It therefore allows the fee and autarky cutoffs derived in ϵ -space to be translated directly into cutoffs in ρ -space.

4.2 Nash Equilibrium Characterisation

Under the natural-unit-of-account benchmark, where $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$, the mapping in equation (29) reduces the general Nash equilibrium characterisation in Proposition 4 and the fee cutoffs derived in Appendix C to functions of ρ alone. Define

$$\Gamma \equiv \frac{\gamma(1-\varphi)\beta}{1+\gamma\alpha-\beta}. \quad (30)$$

Let $\rho^{(0)}$, $\rho^{(1)}$, and $\rho^{(2)}$ denote the values of ρ obtained by mapping the corresponding cutoffs $\bar{\epsilon}^{(0)}$, $\bar{\epsilon}^{(1)}$, and $\bar{\epsilon}^{(2)}$ through equation (29). Thus

$$\rho^{(0)} = \frac{\bar{\epsilon}^{(0)} - 1}{\bar{\epsilon}^{(0)} + 1}, \quad \bar{\epsilon}^{(0)} = \frac{1}{\varphi}(1 + \Gamma(1 - \alpha - \beta)), \quad (31)$$

$$\rho^{(1)} = \frac{\bar{\epsilon}^{(1)} - 1}{\bar{\epsilon}^{(1)} + 1}, \quad (32)$$

$$\rho^{(2)} = \frac{\bar{\epsilon}^{(2)} - 1}{\bar{\epsilon}^{(2)} + 1}. \quad (33)$$

Proposition 3 defines $\bar{\epsilon}^{(2)}$ implicitly through $W_j(\bar{\epsilon}^{(2)}) = V_j^{(0)}$, so $\rho^{(2)}$ has no closed form. Under the model's parameter restrictions,

$$0 < \rho^{(0)} < \rho^{(1)} < \rho^{(2)} < 1.$$

Finally, let $\rho_0 \equiv (1 - \varphi)/(1 + \varphi)$ denote the value at which $\epsilon(\rho) = 1/\varphi$. The symbol ρ_0 refers to the promise-keeping threshold. By contrast, $\rho^{(0)}$ denotes the zero-fee cutoff. Since the zero-fee region extends beyond the point at which the interbank constraint first binds, $\rho_0 < \rho^{(0)}$. The following proposition characterises equilibrium outcomes under the natural unit of account.

Proposition 5 (Two-State Natural Unit of Account). *In the symmetric two-state model under the natural unit of account, with $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$, let $\rho^{(0)}$, $\rho^{(1)}$, and $\rho^{(2)}$ be defined by equations (31)–(33).*

- (a) ($\rho \leq \rho_0$): Both banks set $\chi^* = 0$, investment is at the constrained optimum, and all equilibria are efficient regardless of the unit of account.
- (b) ($\rho_0 < \rho \leq \rho^{(0)}$): Both banks set $\chi^* = 0$, but investment is below the constrained optimum and continuously declining in ρ .
- (c) ($\rho^{(0)} < \rho \leq \rho^{(2)}$): Both banks charge $\chi^*(\rho) > 0$. For $\rho^{(0)} < \rho < \rho^{(1)}$, the fee is interior, $\chi^*(\rho) \in (0, \gamma/(1 + \gamma))$, strictly increasing in ρ . For $\rho^{(1)} \leq \rho \leq \rho^{(2)}$, the fee is at its maximum, $\chi^* = \gamma/(1 + \gamma)$.

- (d) ($\rho > \rho^{(2)}$): Both banks choose autarky; no equilibrium with interbank transfers exists under the natural unit of account.

Proof. See Appendix D. □

As ρ increases through cases (b)–(d), the settlement cost rises and banks respond first with lower investment, then with transfer fees, and finally with autarky. Case (a) establishes a stronger result: when $\rho \leq \rho_0$, the natural unit of account maximises the interbank settlement cost over all feasible unit-of-account choices, so all equilibria are efficient regardless of the unit of account.

4.3 Common Unit of Account Equilibria

The natural-unit-of-account benchmark fixes each bank's unit of account at $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$. When banks instead coordinate on a common unit of account, $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k = [\omega, 1 - \omega]'$. In that case, the interbank settlement requirement $\epsilon_{j,k}$ is replaced by $\epsilon_{j,j}$, the settlement requirement associated with each bank's own unit of account relative to its production. By symmetry of the two banks' production baskets, $\omega = 1/2$ minimises $\max\{\epsilon_{A,A}(\omega), \epsilon_{B,B}(\omega)\}$. At this optimal common basket, both banks face $\epsilon_{j,j} = 1/(1 - \rho)$, which is strictly less than $(1 + \rho)/(1 - \rho)$ for all $\rho > 0$. Define

$$\rho_c \equiv 1 - \frac{1}{\bar{\epsilon}^{(0)}} = \frac{\bar{\epsilon}^{(0)} - 1}{\bar{\epsilon}^{(0)}}, \quad (34)$$

which is the value at which the common-unit-of-account settlement requirement $1/(1 - \rho)$ equals the zero-fee cutoff $\bar{\epsilon}^{(0)}$. The ordering $\rho^{(0)} < \rho_c$ follows from $\bar{\epsilon}^{(0)} < \bar{\epsilon}^{(0)} + 1$.

Proposition 6 (Common Unit of Account Equilibria). *In the symmetric two-state model:*

- (a) ($\rho \leq 1 - \varphi$): There exist Nash equilibria in which both banks adopt a common unit of account. An efficient such equilibrium exists.
- (b) ($1 - \varphi < \rho \leq \rho_c$): There exist Nash equilibria in which both banks adopt a common unit of account, but all such equilibria are inefficient.
- (c) ($\rho > \rho_c$): No Nash equilibrium with a common unit of account exists.

Proof. See Appendix D. □

Proposition 6 separates the existence question from the efficiency question. For $\rho \leq 1 - \varphi$, the common basket at $\omega = 1/2$ yields an efficient common-unit-of-account equilibrium. Moving away from $\omega = 1/2$ raises $\epsilon_{j,j}$. For $1 - \varphi < \rho \leq \rho_c$, common-unit-of-account equilibria still exist, but none implements the constrained optimum. In that range, even the best common basket satisfies $\epsilon_{j,j} = 1/(1 - \rho) > 1/\varphi$, so investment is distorted. For $\rho > \rho_c$, deviation to the natural unit of account is strictly profitable for each bank, so no common-unit-of-account equilibrium survives.

4.4 Welfare Analysis

Welfare comparisons in this subsection are conducted under the natural-unit-of-account benchmark, $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$. In the symmetric two-state model, this benchmark yields the largest cross-bank settlement friction among feasible unit-of-account choices. It therefore gives the lowest-welfare Nash equilibrium with interbank transfers, so welfare losses under any other equilibrium are weakly smaller. This ordering follows from symmetry and Proposition 4. The welfare loss is measured relative to the constrained optimum defined in Section 2, that is, the allocation a planner would choose given the model's production structure and uninsurable price risk. An efficient equilibrium attains this benchmark when $\chi^* = 0$ and investment reaches the constrained-optimal level n^* . Denoting per-bank welfare at the constrained optimum by W^* , the welfare loss is $\Delta W(\rho) \equiv W^* - W(\rho)$.

Proposition 7 (Welfare Loss under the Natural Unit of Account). *In the symmetric two-state model under the natural unit of account ($\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$), the per-bank welfare loss $\Delta W(\rho) \equiv W^* - W(\rho)$ relative to the constrained optimum satisfies:*

- (a) $\Delta W(\rho) = 0$ for $\rho \leq \rho_0$ (efficient equilibria).
- (b) $\Delta W(\rho) > 0$ and strictly increasing for $\rho_0 < \rho \leq \rho^{(2)}$ (zero-fee suboptimal and fee-charging cases).
- (c) $\Delta W(\rho) = W^* - V_j^{(0)}$ (constant) for $\rho > \rho^{(2)}$ (autarky).

Moreover, $\Delta W(\rho)$ is continuous on $[0, \rho^{(2)}]$ and constant for $\rho > \rho^{(2)}$. Explicit welfare expressions are provided in Appendix D.

Proof. See Appendix D. □

The proposition shows how the natural-unit-of-account benchmark deteriorates as price variance rises. At low variance, exchange is efficient. As variance increases, the benchmark first becomes inefficient while transfers remain fee-free, then supports fee-charging interoperability, and eventually gives way to autarky.

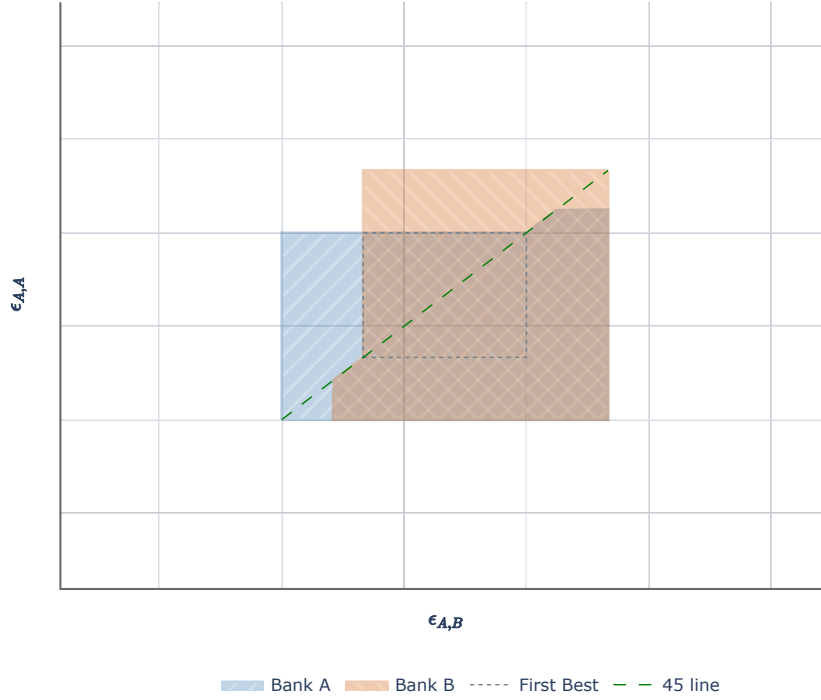


Figure 2: Nash Equilibrium Sets at $\rho_A = \rho_B = 0.4$

4.5 Illustrations

Figures 2 and 3 illustrate the equilibrium sets in the two-state case for two values of ρ . The natural unit of account and the common unit of account are useful benchmark cases within these figures, but the equilibrium set also includes intermediate unit-of-account choices.

The figures in this subsection use the parameter values $\varphi = 0.5$, $\gamma = 1.0$, $\alpha = 0.3$, and $\beta = 0.3$. These values are chosen to illustrate the transition from efficient exchange to fee-charging interoperability and, at sufficiently high price variance, to autarky.

Figure 2 illustrates the case $\rho = 0.4$. By Proposition 5(c), this value lies in the positive-fee case under the natural unit of account, so both banks charge positive transfer fees and investment is below the constrained optimum. The grey dotted boundary marks the efficient subset. Proposition 6(a) shows that efficient equilibria still exist when banks coordinate on a common unit of account, shown by the green dashed 45-degree line. Under a common unit of account, cross-denominated claims do not arise, so transfer fees fall to zero.

Figure 3 illustrates the case $\rho = 0.7$. Proposition 5(d) implies that, under the natural unit of account, both banks choose autarky because $\rho > \rho^{(2)}$. Proposition 6(c) shows that no common-unit-of-account equilibrium exists at this value of ρ .

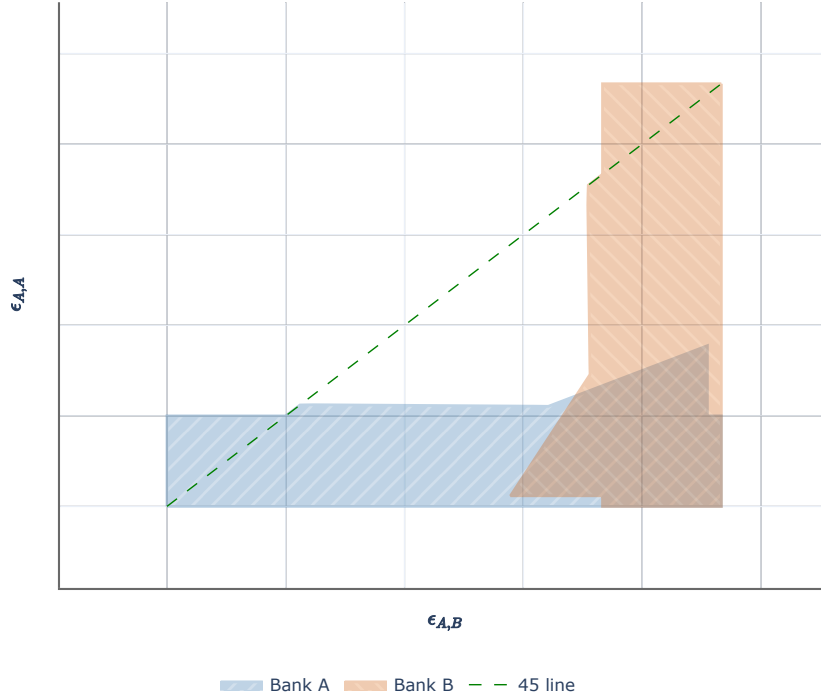


Figure 3: Nash Equilibrium Sets at $\rho_A = \rho_B = 0.7$

5 Policy Analysis

This section studies three policy instruments and their effects on equilibrium outcomes. It considers a mandate that requires issuers to adopt a common unit of account, the use of central bank reserves as a common settlement asset, and cash as an alternative settlement channel when direct interbank transfer is costly.

The model's objects have natural policy interpretations. Differences in banks' production baskets $\hat{\mathbf{y}}_j$ capture asset heterogeneity across intermediaries. The settlement requirement $\epsilon_{j,k}$ measures how costly it is for one bank to honour claims denominated in another bank's unit of account in adverse states, while the fee $\chi_{j,k}$ is the endogenous price of interoperability. The reserve parameter τ captures the extent to which reserve backing reduces variation in effective asset returns by pooling production risk through the monetary unit $\hat{\mathbf{y}}_m$. Cash denominated in $\hat{\mathbf{y}}_m$ provides an alternative settlement channel when direct interbank transfer is costly.

The analysis focuses on singleness within a monetary system. The model does not aim to explain global singleness, which is generally absent and often tolerated because it reflects additional frictions related to monetary sovereignty, exchange-rate adjustment, and policy autonomy. Foreign-currency or offshore issuers instead enter the framework through the asset-divergence channel. This distinction clarifies why deviations from singleness raise

particular concerns at the national level, where par trading and interoperability are policy objectives.

The policy tools considered here matter not because they impose outcomes that private issuers would otherwise choose, but because they relax settlement and risk-sharing constraints that are structural to the contracting environment. Reserves and cash operate on dimensions that private bilateral contracts cannot replicate within the model.

5.1 Mandating a Common Unit of Account

A common-unit-of-account mandate removes differences in the units of account used across banks. When both banks set $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_a$, their liabilities have the same state-contingent value, so the settlement friction created by differences in units of account disappears and interbank transfer fees fall to zero. The mandated unit $\hat{\mathbf{y}}_a$ need not coincide with the monetary unit $\hat{\mathbf{y}}_m$. The former is a regulatory choice of common unit of account, whereas the latter is the unit in which reserves and cash are denominated. If $\hat{\mathbf{y}}_a$ is poorly aligned with bank j 's production basket $\hat{\mathbf{y}}_j$, the mandate tightens the promise-keeping constraint.

A common-unit-of-account mandate therefore requires banks to set

$$\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_a. \quad (35)$$

Substituting this restriction into the promise-keeping constraints (9) and (10) shows that there is no gain from setting $\chi_{j,k} > 0$, and bank k faces the symmetric problem. When a common-unit-of-account Nash equilibrium already exists at $\hat{\mathbf{y}}_a$, the mandate simply selects that equilibrium.

The mandate nevertheless creates two policy risks. The first is that the choice of the common unit of account may matter. Figure 2 illustrates a case in which an efficient common-unit-of-account equilibrium exists. A mandate that selects a poorly aligned common basket can still impose an inefficient outcome.

The second risk is that no common-unit-of-account Nash equilibrium may exist without intervention. Figure 3 illustrates one such case. Here the mandated common unit of account imposes an additional constraint on banks' debt contracts and guarantees a suboptimal outcome. Banks must still satisfy the promise-keeping constraints in (9) and (10). If the mandated unit is too poorly aligned with production, the only way to satisfy those constraints is to reduce borrowing below the optimum. Appendix E provides the formal welfare characterisation.

Proposition 6 organises these cases. For $\rho \leq \rho_c$, a common-unit-of-account Nash equilibrium exists, so a mandate can operate as an equilibrium selection device. For $\rho \leq 1 - \varphi$, the symmetric basket at $\omega = 1/2$ is efficient. For $\rho > \rho_c$, no common-unit-of-account Nash equilibrium exists, so the mandate imposes a constrained outcome. By symmetry, the welfare-maximising mandated weight is $\omega = 1/2$.

The model does not explain how a common unit of account would emerge through competition. It does, however, clarify when a mandate improves coordination and when it imposes a constrained outcome instead. When $\rho \leq 1 - \varphi$, a mandate can select the efficient common basket. When $\rho > \rho_c$, it forces banks into an outcome that no common-unit-of-account Nash equilibrium would sustain.

5.2 Central Bank Reserves

Central bank reserves change the state-pricing problem by reducing the dispersion of effective asset values across states. A tax-transfer system that mutualises a fraction τ of each bank's production through the monetary unit of account $\hat{\mathbf{y}}_m$ aligns intermediaries' effective asset bases and narrows the range of state-contingent values at which cross-bank claims must be honoured. As a result, the settlement requirement associated with cross-denominated claims becomes less sensitive to adverse states. Formally, consider a government that taxes a fraction $\tau \in [0, 1]$ of each bank's output. The tax rate is announced at date 0 and collected at date 2. The government does not consume any of the tax revenues itself. Instead, it rebates tax proceeds to banks in proportion to their tax payments. Implementation is enforced by issuing each bank at date 0 a tax obligation and a corresponding transfer claim payable at date 2.

This intervention is ex ante neutral, as each bank receives a transfer equal in expected value to the tax payment they are required to make. However, taxes and transfers will differ in their units of account. Tax payments are assumed to be made in real terms, so that the unit of account of the tax payment bank j has a unit of account $\hat{\mathbf{y}}_j$. The transfer, on the other hand, in this simple example is denoted by $\hat{\mathbf{y}}_m$ where

$$\hat{\mathbf{y}}_m = \frac{1}{2} (\hat{\mathbf{y}}_j + \hat{\mathbf{y}}_k). \quad (36)$$

In this way, the government introduces fiat money and a monetary unit of account $\hat{\mathbf{y}}_m$ through a simple system of taxes and transfers. Equivalently, it provides partial insurance against relative price shocks by pooling part of each bank's output through a common settlement asset.

Now consider what this simple policy implies for the set of competitive equilibria. Bank j 's initial endowment at the beginning of date 2 now consists of their production output net of taxes and transfers. The unit of production for bank j after taxes and transfers is thus

$$\tilde{\mathbf{y}}_j = (1 - \tau) \hat{\mathbf{y}}_j + \tau \hat{\mathbf{y}}_m. \quad (37)$$

Central bank reserves reduce each bank's effective price variance from ρ to $\tilde{\rho}(\tau) = \rho(1 - \tau)$. This shifts the cutoffs in Propositions 5 and 6, expanding both the efficient-equilibrium existence region and the natural-unit-of-account zero-fee region. The resulting comparative statics are summarised in Proposition 8.

Proposition 8 (Central Bank Reserves). *In the symmetric two-state model with reserve requirement $\tau \in [0, 1]$, the effective price variance parameter is $\tilde{\rho}(\tau) = \rho(1 - \tau)$ and the state-contingent settlement cost becomes*

$$\epsilon(\rho, \tau) = \frac{1 + \rho(1 - \tau)}{1 - \rho(1 - \tau)}. \quad (38)$$

- (a) *An efficient equilibrium exists if and only if $\tilde{\rho}(\tau) \leq 1 - \varphi$, equivalently $\tau \geq \tau^{eff}$ where*

$$\tau^{eff} = \max\left\{0, 1 - \frac{1 - \varphi}{\rho}\right\}. \quad (39)$$

Under the natural unit of account, zero fees obtain if and only if $\tilde{\rho}(\tau) \leq \rho^{(0)}$, equivalently $\tau \geq \tau^*$ where

$$\tau^* = \max\left\{0, 1 - \frac{\rho^{(0)}}{\rho}\right\}. \quad (40)$$

- (b) *Under the natural unit of account, autarky is avoided if and only if $\tilde{\rho}(\tau) \leq \rho^{(2)}$, equivalently $\tau \geq \tau^A \equiv \max\{0, 1 - \rho^{(2)}/\rho\}$.*

Proof. Substituting $\rho(1 - \tau)$ for ρ in (29) gives $\epsilon(\rho, \tau) = (1 + \rho(1 - \tau))/(1 - \rho(1 - \tau))$. Under the optimal common basket $\omega = 1/2$, both banks face $\epsilon_{j,j} = 1/(1 - \tilde{\rho})$. By Proposition 4, efficient equilibria exist if and only if $1/(1 - \tilde{\rho}) \leq 1/\varphi$, i.e. $\tilde{\rho}(\tau) \leq 1 - \varphi$, yielding $\tau \geq 1 - (1 - \varphi)/\rho \equiv \tau^{eff}$. Under the natural unit of account, $\epsilon \leq \bar{\epsilon}^{(0)}$ yields $\tau \geq 1 - \rho^{(0)}/\rho \equiv \tau^*$. $\square$

Setting $\tau = 1$ guarantees efficiency for all $\rho \in [0, 1]$. This limiting case abstracts from the costs of a 100% reserve requirement, but it captures the logic of narrow banking. When all production risk is mutualised through reserves, the state-contingent settlement requirement falls to its lowest level.

5.3 Cash

Cash changes the state-pricing problem by providing an alternative claim whose value is less dispersed across states than direct interbank claims. Since $\hat{\mathbf{y}}_m$ lies between the two banks' production baskets, it reduces the effective friction associated with cross-bank transfer. As a consequence, $\epsilon_{j,m} < \epsilon_{j,k}$ for all $\rho > 0$. At date 1, the labourer can request that the bank provide cash. The bank obtains it from the central bank in exchange for a claim on its future output denominated in $\hat{\mathbf{y}}_m$ rather than in another bank's unit of account. As a universally accepted means of payment, cash is accepted by artisans in all regions.

This gives the bank an alternative way to support trade between labourers and artisans when direct interbank transfer is costly. If it allows either interbank transfers or cash withdrawals, bank j 's optimisation problem becomes

$$n_j^\varphi \geq B_j \min \{ (1 - \chi_{j,k}) \epsilon_{j,k}, \epsilon_{j,m} \}, \quad (41)$$

where $\epsilon_{j,m}$ is the settlement requirement for claims denominated in $\hat{\mathbf{y}}_m$, defined as $\hat{\mathbf{y}}_m \equiv \max_{\mathbf{p} \in \mathbf{P}} \mathbf{p}' \hat{\mathbf{y}}_m / \mathbf{p}' \hat{\mathbf{y}}_j$.

Proposition 9 (Cash as Settlement Channel). *In the symmetric two-state model with cash denominated in $\hat{\mathbf{y}}_m = \frac{1}{2}(\hat{\mathbf{y}}_A + \hat{\mathbf{y}}_B)$:*

(a) *Cash caps the effective interbank settlement cost at*

$$\epsilon_{j,m} = \frac{1}{1 - \rho},$$

which is strictly less than $\epsilon_{j,k} = (1 + \rho)/(1 - \rho)$ for all $\rho > 0$.

(b) *Under the natural unit of account, cash expands the zero-fee region from $\rho \leq \rho^{(0)}$ to $\rho \leq \rho_c$, where ρ_c is defined in equation (34).*

(c) *Under the natural unit of account, cash raises the autarky threshold from $\rho^{(2)}$ to*

$$\rho_c^{(2)} = 1 - \frac{1}{\bar{\epsilon}^{(2)}} > \rho^{(2)}.$$

(d) *If $\hat{\mathbf{b}}_k = \hat{\mathbf{y}}_m$, then $\epsilon_{j,k} = \epsilon_{j,m}$ and bank j gains no additional benefit from the cash channel. Bank k can settle using cash at $\epsilon_{k,m}$ rather than direct interbank transfer at $\epsilon_{k,j}$.*

Proof. See Appendix E. □

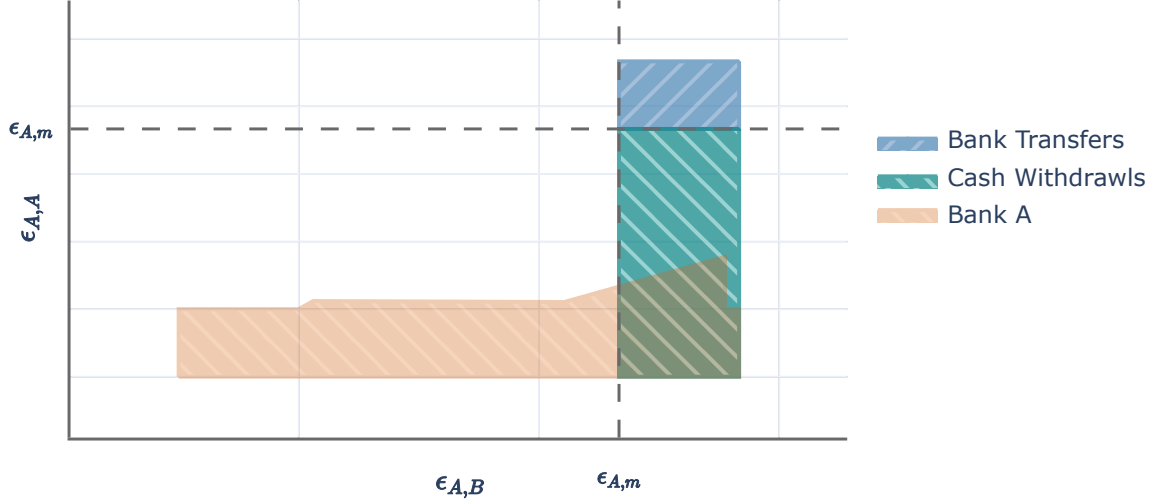


Figure 4: Bank Best Response Functions with Cash ($\epsilon_{j,m} = 1/\varphi$)

Cash affects the natural-unit-of-account benchmark by relaxing the relevant settlement friction when direct interbank transfer is costly. Under that benchmark, it expands the zero-fee region from $\rho \leq \rho^{(0)}$ to $\rho \leq \rho_c$ and raises the autarky threshold accordingly. It does not change the efficient-equilibrium existence boundary. When banks coordinate on a common unit of account, the relevant promise-keeping condition depends on $\epsilon_{j,j} = 1/(1 - \rho)$ and therefore does not depend on the cash channel.

This matters especially when a new entrant chooses a different unit of account from the incumbent. Consider the case in which bank k adopts the monetary unit of account, so that $\hat{\mathbf{b}}_k = \hat{\mathbf{y}}_m$, while bank j enters with a different unit of account such that $\epsilon_{k,j} \geq 1/\varphi$. Without cash, bank k would restrict interoperability with bank j by charging interbank fees or banning transfers altogether. This is inefficient because bank k 's depositors cannot fully benefit from trade with bank j 's depositors. If cash is available as a universally accepted means of payment, bank k can instead facilitate trade through cash withdrawals. In that case, cash insulates the incumbent from the entrant's unit-of-account choice and allows bank k to operate at the efficient scale. In this case, the main benefit of cash accrues to the incumbent bank. Bank j is then indifferent between allowing partial interoperability through cash and allowing it directly through bank k .

Figure 4 illustrates the best response of the two banks, as the policy maker introduces cash with a unit of account $\mathbf{p}_j' \hat{\mathbf{b}}_m$. In this case, bank k will always be able to achieve the optimal investment level regardless of bank j 's choice of unit of account. On the other hand, bank j will limit interoperability with both cash and bank k and choose a unit of account sufficiently far from bank k 's.

5.4 Interaction of Policy Tools

Reserves and cash affect different margins of the problem. Reserves reduce effective price variance by pooling part of each bank's production through the monetary unit. Cash reduces the settlement friction faced under the natural unit of account by providing an alternative settlement claim. The two instruments therefore matter in different ways when price variance is high.

Only reserves expand the range of price variance for which efficient equilibria exist. Without reserves, efficient equilibria require $\rho \leq 1 - \varphi$. With reserve requirement τ , that condition becomes $\rho(1 - \tau) \leq 1 - \varphi$, or equivalently $\tau \geq \tau^{eff}$. Higher reserve backing therefore allows efficient equilibria to survive at higher levels of price variance.

The two instruments are partial substitutes under the natural-unit-of-account benchmark. Without cash, a zero-fee outcome requires $\tau \geq \tau^*$. With cash, the relevant settlement condition uses $\epsilon_{j,m}(\tau) = 1/(1 - \tilde{\rho}(\tau))$ rather than $\epsilon_{j,k}(\tau)$, so the zero-fee condition becomes $\tilde{\rho}(\tau) \leq \rho_c$, equivalently

$$\tau \geq \tau_c^* \equiv \max\left\{0, 1 - \frac{\rho_c}{\rho}\right\}. \quad (42)$$

Since $\rho_c > \rho^{(0)}$, it follows that $\tau_c^* < \tau^*$ whenever both are interior. Cash therefore reduces the amount of reserve backing needed to sustain a zero-fee outcome under the natural unit of account.

This distinction matters most when price variance is high. Cash alone supports the zero-fee outcome only when $\rho \leq \rho_c$. For $\rho > \rho_c$, some positive reserve requirement remains necessary even if cash is available. At the same time, cash lowers the reserve requirement from τ^* to τ_c^* , so it still improves outcomes in high-volatility cases. When $\rho^{(0)} < \rho \leq \rho_c$, cash alone is sufficient to sustain the zero-fee outcome. When $\rho > \rho_c$, reserves and cash work together. Reserves are still needed, but cash reduces how much reserve backing is required.

5.5 New Payment Intermediaries

The model also clarifies how entry by a new payment intermediary can weaken alignment on a common unit of account when its asset portfolio differs from those of incumbent banks.

This effect operates through asset-portfolio divergence. The cutoffs $\rho^{(0)}$, $\rho^{(1)}$, and $\rho^{(2)}$ depend on the variance in relative prices across banks' asset portfolios, not on the aggregate price level, consistent with Doepke and Schneider (2017). Higher values of ρ

correspond to greater dispersion in relative prices and move the economy from fee-free exchange to fee-charging interoperability and eventually to autarky. A payment intermediary whose collateral differs systematically from that of incumbent banks raises ρ and moves the economy toward the autarky threshold. One example is a stablecoin backed by foreign-currency assets operating alongside domestic deposit banks. Banks within the same monetary system tend to hold similar asset portfolios and therefore tend to coordinate on a common unit of account. Entry by an intermediary with a more divergent portfolio can weaken that coordination. The model does not characterise entry dynamics, adoption paths, or network effects. It does identify a clear mechanism through which new payment instruments can weaken coordination. Asset-portfolio divergence raises settlement friction and makes monetary fragmentation more likely.

6 Information Sensitivity and Singleness

The policy instruments in Section 5 operate through the state-contingent pricing of cross-denominated claims under information-insensitive debt. Labourers and artisans are symmetrically informed, so debt circulates at its expected value regardless of the realised state and acceptance is unconditional whenever there are gains from trade. This section adds an informational friction at the point of exchange. Acceptance now depends on realised payoffs, which reinforces the role of the unit of account and interoperability in the baseline model. The formal analysis is developed within the symmetric two-state setting of Section 4.

The extension imposes one additional requirement. The labourer must set the terms of trade so that the artisan accepts the offered debt regardless of its realised payoff. This worst-state acceptance requirement captures the idea, developed by Gorton and Pennacchi (1990), Gorton (2020), and Dang et al. (2020), that debt circulates without valuation costs only when it is information-insensitive. In the baseline environment, labourers and artisans are symmetrically informed about the distribution of $\mathbf{p}$, so trade occurs whenever a labourer meets an artisan, provided $\gamma > 0$. The unit of account therefore matters through borrowing capacity and interoperability, but not through acceptance in exchange. Once artisans can condition acceptance on the realised payoff, however, the offered debt must be acceptable in the worst state. The labourer must therefore price debt at its minimum value rather than at its expected value.

Under this requirement, the relevant object is the worst-case value of the unit of account. I summarise this with the *information sensitivity* of bank j 's debt:

$$\sigma_j(\hat{\mathbf{b}}_j) = 1 - \min_{\mathbf{p} \in \mathbf{P}} \mathbf{p}'\hat{\mathbf{b}}_j. \quad (43)$$

Given the price normalisation $E[\mathbf{p}'\hat{\mathbf{b}}_j] = 1$, it follows that $\sigma_j \in [0, 1)$. When $\sigma_j = 0$, the debt has the same value in every state. When $\sigma_j > 0$, its worst-case value is $1 - \sigma_j < 1$, so an informed artisan can condition acceptance on the realised payoff.

In the symmetric two-state model of Section 4, bank A 's unit of account can be parameterised as $\hat{\mathbf{b}}_A = [\omega, 1 - \omega]'$ with $\omega \in [0, 1]$. Substituting this parameterisation into equation (43) gives

$$\sigma_A(\omega) = \rho |1 - 2\omega|, \quad (44)$$

which is minimised at $\omega = \frac{1}{2}$, the risk-free diversified bundle, and maximised at the corner solutions $\omega \in \{0, 1\}$.

Consider a labourer in region j who meets an artisan in region k and offers a fraction $s_{j,k} \in [0, 1]$ of their debt in exchange for x_k units of the customised good. Under this acceptance requirement, the offer must remain acceptable even in the lowest-payoff state. The relevant constraint is therefore

$$x_k \leq s_{j,k}(1 - \chi_{j,k})(1 - \sigma_j) B_j.$$

Because the labourer's utility is increasing in x_k , the labourer chooses to transfer as much debt as possible. The constraint therefore binds, so the optimum sets $s_{j,k} = 1$ and

$$x_k = (1 - \chi_{j,k})(1 - \sigma_j) B_j.$$

Under this strategy, the labourer's expected utility takes the same form as in the baseline model in equation (16), except that the gains-from-trade parameter γ is replaced by the information-adjusted parameter

$$\tilde{\gamma}_j \equiv \gamma - (1 + \gamma)\sigma_j. \quad (45)$$

There are gains from trade if and only if $\tilde{\gamma}_j > 0$, which requires $\sigma_j < \gamma/(1 + \gamma)$. The bank's choice of unit of account therefore affects whether trade can occur at all. If information sensitivity is too high, the gains from trade disappear.

Replacing γ with $\tilde{\gamma}_j$ carries the baseline contracting problem into the information-sensitive environment. The bank's profit function is unchanged, but the labourer's participation constraint becomes tighter because effective gains from trade are lower. The proofs of Propositions 1–3 therefore extend directly once γ is replaced by $\tilde{\gamma}_j$, provided $\tilde{\gamma}_j > 0$. The next proposition states the resulting equilibrium in the symmetric two-state case.

Proposition 10 (Information-Sensitive Equilibrium). *Consider the symmetric two-state model with $\rho_A = \rho_B = \rho$, and suppose the worst-state acceptance requirement holds.*

(i) If $\rho \leq 1 - \varphi$, the risk-free unit of account $\omega^* = \frac{1}{2}$ satisfies the feasibility constraint $\epsilon_{j,j} \leq 1/\varphi$ and achieves $\sigma_j^* = 0$. In that case, both banks recover the baseline efficient equilibrium with $\tilde{\gamma}_j = \gamma$.

(ii) If $\rho > 1 - \varphi$, the risk-free unit of account violates the feasibility constraint. Each bank then chooses the feasible unit of account that minimises information sensitivity, yielding

$$\omega^* = \frac{1}{2} + \frac{\rho + \varphi - 1}{2\rho\varphi}, \quad (46)$$

with minimum information sensitivity

$$\sigma_j^* = \frac{\rho + \varphi - 1}{\varphi} > 0$$

and adjusted gains from trade

$$\tilde{\gamma}_j^* = \gamma - (1 + \gamma) \frac{\rho + \varphi - 1}{\varphi}.$$

Trade remains beneficial if and only if

$$\gamma > \frac{\rho + \varphi - 1}{1 - \rho}.$$

Proof. See Appendix F. □

The threshold $1 - \varphi$ is the point at which the risk-free diversified unit of account first becomes infeasible. Below that threshold, the basket $\omega = \frac{1}{2}$ satisfies the promise-keeping constraint and delivers the baseline efficient equilibrium. Above it, the same basket is too far from each bank's production basket to remain feasible, so each bank must move away from $\omega = \frac{1}{2}$ to satisfy $\epsilon_{j,j} \leq 1/\varphi$. In the baseline two-state model, the same threshold also marks the boundary below which efficient common-unit-of-account equilibria exist, as established in Proposition 6(a).

The cutoff ρ_c plays a different role. In the baseline two-state model, it is the point beyond which no common-unit-of-account equilibrium exists. The extension leaves that logic intact, but it adds a further discipline on unit-of-account choice. In the baseline model, any feasible unit of account yields the same profit. Here, by contrast, information sensitivity matters because it reduces effective gains from trade. Once $\omega = \frac{1}{2}$ becomes infeasible, each bank therefore chooses the feasible unit of account that minimises information sensitivity. This is why ω^* is now pinned down.

In the baseline model, bank profit is independent of the unit of account once the feasibility constraint is satisfied. Any $\hat{\mathbf{b}}_j$ with $\epsilon_{j,j} \leq 1/\varphi$ delivers the same debt capacity and the

same profit. Information sensitivity removes that indifference. Bank profit rises with $\tilde{\gamma}_j$, and $\tilde{\gamma}_j$ falls as σ_j increases. Each bank therefore prefers the feasible unit of account that minimises information sensitivity. Appendix F provides the formal argument.

Information sensitivity also strengthens the role of central bank reserves. When the reserve requirement satisfies

$$\tau \geq \frac{\rho + \varphi - 1}{\rho},$$

effective price variance falls to $1 - \varphi$. At that point, the risk-free diversified unit of account becomes feasible and $\sigma_j^* = 0$. The same condition places the economy at the efficient-equilibrium boundary identified in Proposition 8(a). Reserves therefore do two things at once. They restore efficient equilibria and remove the information-sensitivity tax on gains from trade. Appendix F formalises this result.

The bank now faces a three-way trade-off in its choice of unit of account. Borrowing capacity is highest when $\epsilon_{j,j}$ is low, which favours a unit of account close to the bank's own output. Interoperability is greatest when $\epsilon_{j,k}$ is low, which favours a unit of account close to the other bank's output. Information sensitivity is minimised when $\sigma_j = 0$, which favours the risk-free diversified bundle. No feasible unit of account achieves all three objectives at once.

Replacing γ with $\tilde{\gamma}_j$ tightens the gains-from-trade condition. As a result, the transfer-fee cutoffs $\bar{\epsilon}^{(0)}$ and $\bar{\epsilon}^{(1)}$ are lower in the information-sensitive environment than in the baseline model. The welfare cost of non-singleness is therefore higher. Whenever $\sigma_j^* > 0$, both autarky welfare and efficient-equilibrium welfare fall relative to the baseline.

Cash denominated in the monetary unit of account $\hat{\mathbf{y}}_m = \frac{1}{2}(\hat{\mathbf{y}}_A + \hat{\mathbf{y}}_B)$ has information sensitivity $\sigma_m = 0$, since $\mathbf{p}'\hat{\mathbf{y}}_m = 1$ in every state. When bank debt is information-sensitive, labourers face lower effective gains from trade because $\tilde{\gamma}_j < \gamma$. Cash restores trade at full value because its payoff is state-invariant and acceptance does not depend on the realised state.

7 Conclusion

Fragmentation arises when liabilities issued by different banks are tied to different underlying assets and therefore settle at different values across states. When each bank issues debt in its own unit of account tied to its production, cross-bank settlement becomes costly. Transfer fees rise, investment falls, and beyond a critical level of asset divergence, interbank payments cease to be viable, leaving money effectively non-interoperable across

issuers. Coordination on a shared unit of account extends the range over which exchange can occur. Such coordination supports efficient exchange when banks' asset bases are sufficiently similar. As asset divergence increases, coordination remains feasible but becomes inefficient. Efficient exchange therefore does not require a shared unit of account in general. It requires that differences in underlying assets remain small enough that settlement across issuers is not prohibitively costly.

The policy analysis shows that public instruments differ in how they affect that trade-off. Central bank reserves, by acting as a common settlement asset, reduce how much effective asset values vary across states and can restore efficiency even when banks' asset bases differ substantially. Cash provides an alternative settlement channel through central bank money and limits settlement costs in environments where banks continue to issue debt against their own assets.

A common-unit-of-account mandate removes the settlement friction created by differences in units of account across issuers. If the mandated unit is poorly aligned with banks' underlying assets, however, it can still tighten balance-sheet constraints and reduce investment.

The broader implication of the analysis is that preserving singleness in modern payment systems requires more than a shared unit of account alone. The paper focuses on singleness within a monetary system, where par trading and interoperability are policy objectives. Global singleness is typically absent and often tolerated because of additional frictions related to monetary sovereignty and exchange-rate adjustment. What matters for national singleness is how private liabilities are backed, how they settle across issuers, and whether central bank money is available to anchor that process. Informational frictions in exchange environments reinforce these considerations by penalising liabilities whose value varies across states, but they do not overturn the core mechanisms of the model. In payment systems with stablecoins, tokenised deposits, or other privately issued monies backed by heterogeneous assets, interoperability depends not only on the units of account that issuers adopt, but also on settlement design and the public infrastructure that supports it.

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Appendix

A Foundational Bank Optimisation

A.1 Bank j 's problem without interbank transfers

The Lagrangian is

$$\begin{aligned}\mathcal{L}_j^{(0)} = & n_j^\varphi - (1 - \alpha\chi_{j,j})B_j \\ & + \lambda_N [((1 + \gamma)\alpha(1 - \chi_{j,j}) + 1 - \alpha)B_j - n_j] \\ & + \mu_j^+ [(1 + \gamma)(1 - \chi_{j,j}) - 1] \\ & + \mu_j^- \chi_{j,j}\end{aligned}\tag{A.1}$$

I take the first-order conditions with respect to $n_j, B_j, \chi_{j,j}$ which are as follows:

$$\frac{\partial \mathcal{L}_j^{(0)}}{\partial n_j} : \quad \varphi n_j^{\varphi-1} - \lambda_N = 0\tag{A.2}$$

$$\frac{\partial \mathcal{L}_j^{(0)}}{\partial B_j} : \quad \lambda_N [(1 + \gamma)\alpha(1 - \chi_{j,j}) + 1 - \alpha] - (1 - \alpha\chi_{j,j}) = 0\tag{A.3}$$

$$\frac{\partial \mathcal{L}_j^{(0)}}{\partial \chi_{j,j}} : \quad \alpha B_j - \lambda_N (1 + \gamma)\alpha B_j - \mu_j^+ (1 + \gamma) + \mu_j^- = 0\tag{A.4}$$

It follows from the latter equation that

$$\chi_{j,j} = 0 \iff \lambda_N > \frac{1}{1 + \gamma}\tag{A.5}$$

Rearranging the second equation yields

$$\lambda_N = \frac{1 - \alpha\chi_{j,j}}{(1 + \gamma)\alpha(1 - \chi_{j,j}) + 1 - \alpha}\tag{A.6}$$

where I note that this implies $\lambda_N > \frac{1}{1+\gamma}$ for any value of $\chi_{j,j} \geq 0$.

Thus I note that it is optimal in autarky for firms not to charge for intrabank transfers.

Substituting out λ_N in the first equation yields an equation for the quantity of labour hired in autarky

$$n_j^{(0)} = [\varphi(1 + \gamma\alpha)]^{\frac{1}{1-\varphi}}. \quad (\text{A.7})$$

The loan size follows from the budget constraint

$$B_j^{(0)} = \varphi^{\frac{1}{1-\varphi}} (1 + \gamma\alpha)^{\frac{\varphi}{1-\varphi}}, \quad (\text{A.8})$$

and the profit to bank j is

$$V_j^{(0)} = \varphi^{\frac{\varphi}{1-\varphi}} (1 + \gamma\alpha)^{\frac{\varphi}{1-\varphi}} (1 - \varphi). \quad (\text{A.9})$$

Finally, note that the bank's unit of account can be set to any value that ensures the promise keeping condition described by equation (3) holds. This is satisfied so long as

$$\epsilon_{j,j} \leq \frac{1}{\varphi}. \quad (\text{A.10})$$

A.2 Bank j 's problem with interbank transfers

Note I drop the promise-keeping constraint as Bank A can simply choose its own unit of account to ensure it does not bind.

The Lagrangian then becomes

$$\begin{aligned} \mathcal{L}_j = & n_j^\varphi - (1 - \alpha\chi_{j,j} - \beta\chi_{j,k})B_j \\ & + \lambda_N [((1 + \gamma)(\alpha(1 - \chi_{j,j}) + \beta(1 - \chi_{j,k})) + 1 - \alpha - \beta)B_j - n_j] \\ & + \lambda_k \left[\frac{n_j^\varphi}{\epsilon_{j,k}} - (1 - \chi_{j,k})B_j \right] \\ & + \mu_j^+ [(1 + \gamma)(1 - \chi_{j,j}) - 1] \\ & + \mu_k^+ [(1 + \gamma)(1 - \chi_{j,k}) - 1] \\ & + \mu_j^- \chi_{j,j} \\ & + \mu_k^- \chi_{j,k} \end{aligned} \quad (\text{A.11})$$

I take the first order conditions with respect to $n_j, B_j, \chi_{j,j}, \chi_{j,k}$ which are as follows

$$\frac{\partial \mathcal{L}_j}{\partial n_j} : \quad \varphi n_j^{\varphi-1} \left(1 + \frac{1}{\epsilon_{j,k}} \lambda_k \right) - \lambda_N = 0 \quad (\text{A.12})$$

$$\begin{aligned} \frac{\partial \mathcal{L}_j}{\partial B_j} : \quad & \lambda_N [(1 + \gamma)(\alpha(1 - \chi_{j,j}) + \beta(1 - \chi_{j,k})) + 1 - \alpha - \beta] \\ & - (1 - \alpha\chi_{j,j} - \beta\chi_{j,k}) - \lambda_k(1 - \chi_{j,k}) = 0 \end{aligned} \quad (\text{A.13})$$

$$\frac{\partial \mathcal{L}_j}{\partial \chi_{j,j}} : \quad \alpha B_j - \lambda_N(1 + \gamma)\alpha B_j - \mu_j^+(1 + \gamma) + \mu_j^- = 0 \quad (\text{A.14})$$

$$\frac{\partial \mathcal{L}_j}{\partial \chi_{j,k}} : \quad \beta B_j - \lambda_N(1 + \gamma)\beta B_j + \lambda_k B_j - \mu_k^+(1 + \gamma) + \mu_k^- = 0 \quad (\text{A.15})$$

B Proofs of Main Equilibrium Results

Proof of Proposition 2

Case (i): $\epsilon_{j,k} \leq 1/\varphi$

First assume that $\lambda_k = 0$ and later verify under which conditions this holds.

From rearranging equation (A.14) it follows that

$$\chi_{j,j} = 0 \iff \lambda_N > \frac{1}{1 + \gamma}. \quad (\text{B.1})$$

Similarly, rearranging equation (A.15) yields

$$\chi_{j,k} = 0 \iff \lambda_N > \frac{\beta + \lambda_k}{(1 + \gamma)\beta}. \quad (\text{B.2})$$

Note that in the case where $\lambda_k = 0$ equations (B.1) and (B.2) are equivalent.

Rearranging equation (A.13) yields the following equation for λ_N

$$\lambda_N = \frac{1 - \alpha\chi_{j,j} - \beta\chi_{j,k} + \lambda_k(1 - \chi_{j,k})}{1 + \gamma(\alpha + \beta) - (1 + \gamma)(\alpha\chi_{j,j} + \beta\chi_{j,k})} \quad (\text{B.3})$$

and note that in the case where $\lambda_k = 0$, equations (B.1) and (B.2) imply that $\chi_{j,j} = \chi_{j,k} = 0$.

Now substituting out λ_N from equation (A.12) and rearranging yields

$$n_j = [\varphi(1 + \gamma(\alpha + \beta))]^{\frac{1}{1-\varphi}}. \quad (\text{B.4})$$

In equilibrium, the labourer's participation constraint, equation (24) holds with strict equality and the debt size immediately follows

$$B_j = \varphi^{\frac{1}{1-\varphi}} (1 + \gamma(\alpha + \beta))^{\frac{\varphi}{1-\varphi}}. \quad (\text{B.5})$$

Substituting the equations for n_j and B_j into bank j 's profit function yields the profit function

$$W_j = \varphi^{\frac{\varphi}{1-\varphi}} (1 + \gamma(\alpha + \beta))^{\frac{\varphi}{1-\varphi}} (1 - \varphi). \quad (\text{B.6})$$

Next, note that in order for the bank's initial promise-keeping constraint, equation (9) to be satisfied, it is required that

$$\epsilon_{j,j} \leq \frac{1}{\varphi}. \quad (\text{B.7})$$

Finally, it remains to find the conditions under which $\lambda_k = 0$. For this to hold it must be the case that the interbank promise-keeping constraint, equation (10) is slack. Substituting the equations for n_j and B_j into equation (10) yields the following inequality

$$\epsilon_{j,k} \leq \frac{1}{\varphi}. \quad (\text{B.8})$$

Case (ii): $\epsilon_{j,k} > 1/\varphi$

Consider now the case where $\epsilon_{j,k} > \frac{1}{\varphi}$ and thus $\lambda_k > 0$. Here, equation (B.1) is still satisfied and $\chi_{j,j} = 0$.

Rearranging the first order conditions, and binding constraints yields the following system of equations

$$\lambda_N = \frac{1 + (1 - \chi_{j,k})\lambda_k}{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k})} \quad (\text{B.9})$$

$$n_j^{\varphi-1} \frac{1}{\epsilon_{j,k}} (1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k})) = (1 - \chi_{j,k}) \quad (\text{B.10})$$

and

$$\lambda_N = \varphi n_j^{\varphi-1} \left(1 + \lambda_k \frac{1}{\epsilon_{j,k}} \right). \quad (\text{B.11})$$

Solving this system of equations for n_j yields

$$n_j = \left[\frac{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k})}{\epsilon_{j,k} (1 - \chi_{j,k})} \right]^{\frac{1}{1-\varphi}} \quad (\text{B.12})$$

The debt level follows from rearranging equation (24)

$$B_j = \frac{n_j}{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k})} \quad (\text{B.13})$$

which yields

$$B_j = \left(\frac{1}{\epsilon_{j,k}(1 - \chi_{j,k})} \right)^{\frac{1}{1-\varphi}} [1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k})]^{\frac{\varphi}{1-\varphi}}. \quad (\text{B.14})$$

Solving for bank j 's profit yields

$$W_j = \left[\frac{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi_{j,k})}{\epsilon_{j,k}(1 - \chi_{j,k})} \right]^{\frac{\varphi}{1-\varphi}} \left(1 - \frac{1 - \beta\chi_{j,k}}{\epsilon_{j,k}(1 - \chi_{j,k})} \right) \quad (\text{B.15})$$

Proof of Proposition 3

When $\epsilon_{j,k} = 1/\varphi$, case (i) of Proposition 2 gives $W_j^{(1)} > V_j^{(0)}$ since $\beta > 0$ and $\gamma > 0$ imply that the gains from interbank trade are strictly positive. Since $\lim_{\epsilon_{j,k} \rightarrow \infty} W_j(\epsilon_{j,k}) = 0 < V_j^{(0)}$ and W_j is continuous and strictly decreasing for $\epsilon_{j,k} > 1/\varphi$ (as $dW_j/d\epsilon_{j,k} = -\lambda_k n_j^\varphi / \epsilon_{j,k}^2 < 0$ for $\lambda_k > 0$ by the envelope theorem), the intermediate value theorem yields a unique crossing point $\bar{\epsilon}_{j,k}^{(2)}$. The comparative statics follow from the observation that increasing β or γ raises $W_j^{(1)}$ relative to $V_j^{(0)}$, expanding the set of ϵ values in which transferable debt dominates. Moreover, for $\epsilon > 1/\varphi$, the constrained-case profit $W_j^{(2)}(\epsilon, \beta, \gamma) = [A/(\epsilon(1 - \chi^*))]^{\varphi/(1-\varphi)}(1 - (1 - \beta\chi^*)/(\epsilon(1 - \chi^*)))$ where $A = 1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi^*)$ is increasing in β (which raises A) and in γ (which raises both A and the parenthetical term). Since $V_j^{(0)}$ is independent of β and increases in γ at a slower rate than $W_j^{(2)}$, the threshold $\bar{\epsilon}_{j,k}^{(2)}$ defined by $W_j(\bar{\epsilon}^{(2)}) = V_j^{(0)}$ shifts upward in both parameters.

Proof of Proposition 4

Autarky. Autarky is always a feasible strategy for each bank and yields payoff $V_i^{(0)} > 0$. The strategy profile in which both banks choose autarky is therefore a Nash equilibrium for all parameter values.

Conditional contract regimes. Fix a feasible pair of units of account, equivalently a feasible pair $(\epsilon_{j,k}, \epsilon_{k,j})$. Bank i 's contract problem depends on bank ℓ 's unit of account only through $\epsilon_{i,\ell}$. Combining Propositions 2–3 and Lemma C.1 yields the four regimes listed in the proposition. The intervals are mutually exclusive and exhaustive conditional on the settlement requirement faced by bank i .

Efficient fee-free equilibrium. If $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k = \hat{\mathbf{b}}$ with $\max_{\mathbf{p} \in \mathbf{P}} \mathbf{p}'\hat{\mathbf{b}}/\mathbf{p}'\hat{\mathbf{y}}_i \leq 1/\varphi$ for both banks, then the conditions for case (i) of Proposition 2 are met simultaneously. Both banks set zero transfer fees, investment is at the constrained optimum, and the resulting profile is a Nash equilibrium.

C Fee Cutoffs

Lemma C.1 (Fee Cutoffs). *Suppose that bank j allows interbank transfers and bank k 's unit of account $\epsilon_{j,k} > \frac{1}{\varphi}$. Then there exist two cutoffs*

$$\bar{\epsilon}_{j,k}^{(0)} = \frac{1}{\varphi} \left(1 + \beta \left(\frac{\gamma(1-\varphi)(1-\alpha-\beta)}{1+\gamma\alpha-\beta} \right) \right)$$

and

$$\bar{\epsilon}_{j,k}^{(1)} = \frac{1}{\varphi} \left(1 + \gamma + \beta \left(\frac{\gamma(1-\varphi)(1-\alpha)}{1+\gamma\alpha-\beta} \right) \right)$$

where $\frac{1}{\varphi} < \bar{\epsilon}_{j,k}^{(0)} < \bar{\epsilon}_{j,k}^{(1)}$ such that

$$\chi_{j,k}^*(\epsilon_{j,k}) = 0 \iff \epsilon_{j,k} \leq \bar{\epsilon}_{j,k}^{(0)}$$

and

$$\chi_{j,k}^*(\epsilon_{j,k}) = \frac{\gamma}{1+\gamma} \iff \epsilon_{j,k} \geq \bar{\epsilon}_{j,k}^{(1)}$$

Proof. Consider equation (A.15) in the case where $\lambda_k > 0$.

There are three possible scenarios to consider.

First, consider the case where $\chi_{j,k} = 0$. This occurs if

$$\frac{1 + \lambda_k}{1 + \gamma(\alpha + \beta)} > \frac{\beta + \lambda_k}{(1 + \gamma)\beta} \quad (\text{C.1})$$

which simplifies to

$$\lambda_k < \frac{\gamma\beta(1-\alpha-\beta)}{1-\beta+\gamma\alpha}. \quad (\text{C.2})$$

Solving for λ_k in the case where $\chi_{j,k} = 0$ yields

$$\lambda_k = \frac{\varphi\epsilon_{j,k} - 1}{(1 - \varphi)} \quad (\text{C.3})$$

and combining these last two equations and rearranging yields the result that

$$\chi_{j,k} = 0 \iff \epsilon_{j,k} \leq \frac{1}{\varphi} \left(1 + \beta \left(\frac{\gamma(1-\varphi)(1-\alpha-\beta)}{1+\gamma\alpha-\beta} \right) \right) \equiv \bar{\epsilon}_{j,k}^{(0)}. \quad (\text{C.4})$$

Similarly, consider the case where the maximum interbank transfer fee is charged, that is $\chi_{j,k} = \frac{\gamma}{1+\gamma}$. This occurs if

$$\frac{1 + \frac{1}{1+\gamma}\lambda_k}{1 + \gamma\alpha} < \frac{\beta + \lambda_k}{(1 + \gamma)\beta} \quad (\text{C.5})$$

Solving for λ_k in the case where $\chi_{j,k} = \frac{\gamma}{1+\gamma}$ yields

$$\lambda_k = \frac{\varphi\epsilon_{j,k} - (1 + \gamma)}{1 - \varphi} \quad (\text{C.6})$$

and combining these last two equations and rearranging yields the result that

$$\chi_{j,k} = \frac{\gamma}{1 + \gamma} \iff \epsilon_{j,k} \geq \frac{1}{\varphi} \left(1 + \gamma + \beta \left(\frac{\gamma(1-\varphi)(1-\alpha)}{1 + \gamma\alpha - \beta} \right) \right) \equiv \bar{\epsilon}_{j,k}^{(1)}. \quad (\text{C.7})$$

To verify the ordering $1/\varphi < \bar{\epsilon}_{j,k}^{(0)} < \bar{\epsilon}_{j,k}^{(1)}$: the first inequality follows from $\bar{\epsilon}_{j,k}^{(0)} = (1/\varphi)(1 + \Gamma(1 - \alpha - \beta))$ where $\Gamma = \gamma(1 - \varphi)\beta/(1 + \gamma\alpha - \beta) > 0$ under the model's parameter restrictions ($\gamma, \beta > 0$, $\varphi < 1$, $1 + \gamma\alpha > \beta$) and $1 - \alpha - \beta > 0$. The second inequality holds since $\bar{\epsilon}_{j,k}^{(1)} - \bar{\epsilon}_{j,k}^{(0)} = (1/\varphi)(\gamma + \gamma(1 - \varphi)\beta^2/(1 + \gamma\alpha - \beta)) > 0$.

If $\bar{\epsilon}_{j,k}^{(0)} < \epsilon_{j,k} < \bar{\epsilon}_{j,k}^{(1)}$ then $\chi_{j,k} \in \left(0, \frac{\gamma}{1+\gamma}\right)$ can be found from the solution to the following quadratic equation

$$\begin{aligned} & (1 + \gamma\alpha - \beta) + (1 - \varphi)(1 - \chi_{j,k})(\beta^2 + \gamma\beta(1 - \alpha)) \\ & = (1 - \varphi)(1 - \chi_{j,k})^2\beta^2(1 + \gamma) + \varphi(1 - \chi_{j,k})(1 + \gamma\alpha - \beta)\epsilon_{j,k}. \end{aligned} \quad (\text{C.8})$$

□

D Proofs for Two-State Results

Proof of Proposition 5

Proof. The mapping $\epsilon(\rho) = (1 + \rho)/(1 - \rho)$ is strictly increasing. The ordering $\bar{\epsilon}^{(0)} < \bar{\epsilon}^{(1)}$ is verified in the proof of Lemma C.1. The ordering $\bar{\epsilon}^{(1)} < \bar{\epsilon}^{(2)}$ follows because at $\epsilon = \bar{\epsilon}^{(1)}$ the maximum fee $\chi^* = \gamma/(1 + \gamma)$ is charged and bank welfare $W_j(\bar{\epsilon}^{(1)}) > V_j^{(0)}$ (since the

bank earns positive interbank gains from trade), while $\bar{\epsilon}^{(2)}$ is defined by $W_j(\bar{\epsilon}^{(2)}) = V_j^{(0)}$ and W_j is strictly decreasing in ϵ for $\epsilon > 1/\varphi$. The cutoff ordering $\bar{\epsilon}^{(0)} < \bar{\epsilon}^{(1)} < \bar{\epsilon}^{(2)}$ therefore maps to $\rho^{(0)} < \rho^{(1)} < \rho^{(2)}$. Symmetry ($\epsilon_{A,B} = \epsilon_{B,A}$) implies both banks face identical optimisation problems, so $\chi_{A,B}^* = \chi_{B,A}^*$.

For case (a): when $\rho \leq \rho_0$, $\epsilon(\rho) \leq 1/\varphi$. Under the natural unit of account, $\chi^* = 0$ and investment is at the constrained optimum by Proposition 2(i). For any other equilibrium, bank k 's unit of account satisfies $\epsilon_{j,k} = \max_s \{\mathbf{p}'_s \hat{\mathbf{b}}_k / \mathbf{p}'_s \hat{\mathbf{y}}_j\} \leq \max_s \{\mathbf{p}'_s \hat{\mathbf{y}}_k / \mathbf{p}'_s \hat{\mathbf{y}}_j\} = \epsilon(\rho) \leq 1/\varphi$, where the first inequality follows because, in the two-state model with specialised production, $\hat{\mathbf{y}}_k$ maximises $\max_s \{\mathbf{p}'_s \hat{\mathbf{b}} / \mathbf{p}'_s \hat{\mathbf{y}}_j\}$ over all normalised baskets $\hat{\mathbf{b}}$ with $\mathbf{p}'_s \hat{\mathbf{b}} \geq 0$. Since $\epsilon_{j,k} \leq 1/\varphi$ for both banks in every equilibrium, all equilibria are efficient.

For case (b): when $\rho_0 < \rho \leq \rho^{(0)}$, $\epsilon(\rho) \leq \bar{\epsilon}^{(0)}$ ensures $\chi^* = 0$ under the natural unit of account by Lemma C.1, but $\epsilon(\rho) > 1/\varphi$ so investment is below the constrained optimum.

For case (c): when $\rho^{(0)} < \rho \leq \rho^{(2)}$, $\epsilon(\rho) > \bar{\epsilon}^{(0)}$ and bank j charges $\chi^*(\rho) > 0$ by Lemma C.1.

For case (d): when $\rho > \rho^{(2)}$, $\epsilon(\rho) > \bar{\epsilon}^{(2)}$ implies $W_j(\epsilon) < V_j^{(0)}$ by Proposition 3, so banks prefer autarky. $\square$

Proof of Proposition 6

Proof. Suppose both banks adopt a common unit of account $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k = [\omega, 1 - \omega]'$. Under a common unit of account, $\epsilon_{j,k} = \epsilon_{j,j}$ for each bank, so setting $\chi_{j,k} > 0$ tightens the interbank constraint without relaxing the promise-keeping constraint; hence $\chi_{j,k}^* = 0$. If bank j deviates to $\hat{\mathbf{b}}_j = \hat{\mathbf{y}}_j$, its promise-keeping constraint relaxes to $\epsilon_{j,j} = 1$ while its interbank settlement cost $\epsilon_{j,k} = \max_s \{\mathbf{p}'_s [\omega, 1 - \omega]' / \mathbf{p}'_s \hat{\mathbf{y}}_j\}$ is unchanged (bank k 's basket is held fixed). The deviating bank can then set $\chi_{j,k} = \chi^*(\epsilon_{j,k}) > 0$ whenever $\epsilon_{j,k} > \bar{\epsilon}^{(0)}$, strictly increasing profit relative to the common-unit-of-account outcome. By symmetry, the most favourable common basket is $\omega = 1/2$, giving $\epsilon_{j,k} = 1/(1 - \rho)$ for both banks. The condition $1/(1 - \rho) > \bar{\epsilon}^{(0)}$ is equivalent to $\rho > \rho_c$. For $\rho \leq \rho_c$, deviation does not increase profit ($\chi^*(\epsilon_{j,k}) = 0$ under deviation as well), so banks are indifferent and the common-unit-of-account profile is a Nash equilibrium. For $\rho > \rho_c$, deviation is strictly profitable at any common basket, so no common-unit-of-account Nash equilibrium exists. This establishes parts (b) and (c).

For part (a): under the common basket at $\omega = 1/2$, both banks face $\epsilon_{j,j} = 1/(1 - \rho)$. By Proposition 4, efficient equilibria exist when $1/(1 - \rho) \leq 1/\varphi$, i.e. $\rho \leq 1 - \varphi$. Since $1 - \varphi < \rho_c$ (as $\bar{\epsilon}^{(0)} > 1/\varphi$), the common-unit-of-account equilibrium exists and is efficient for $\rho \leq 1 - \varphi$. $\square$

Proof of Proposition 7

Proof. Part (a) follows immediately since zero-fee equilibria with $\epsilon(\rho) \leq 1/\varphi$ implement the constrained optimum.

For part (b), the constrained-optimum welfare is

$$W^* = \varphi^{\frac{\varphi}{1-\varphi}} (1 + \gamma(\alpha + \beta))^{\frac{\varphi}{1-\varphi}} (1 - \varphi). \quad (\text{D.1})$$

Substituting the equilibrium fee $\chi^*(\rho)$ into the bank's payoff function yields

$$\Delta W(\rho) = W^* - \left[\frac{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi^*(\rho))}{\epsilon(\rho)(1 - \chi^*(\rho))} \right]^{\frac{\varphi}{1-\varphi}} \left(1 - \frac{1 - \beta\chi^*(\rho)}{\epsilon(\rho)(1 - \chi^*(\rho))} \right), \quad (\text{D.2})$$

where $\chi^*(\rho) \equiv \chi^*(\epsilon(\rho))$ is the optimal fee from Lemma C.1 and $\epsilon(\rho) = (1 + \rho)/(1 - \rho)$. The equilibrium investment level is

$$n(\rho) = \left[\frac{1 + \gamma\alpha + \beta(\gamma - (1 + \gamma)\chi^*(\rho))}{\epsilon(\rho)(1 - \chi^*(\rho))} \right]^{\frac{1}{1-\varphi}}. \quad (\text{D.3})$$

For the maximum-fee sub-case $\rho^{(1)} \leq \rho \leq \rho^{(2)}$, substituting $\chi^* = \gamma/(1 + \gamma)$ yields

$$\Delta W(\rho) = W^* - \left[\frac{(1 + \gamma\alpha)(1 + \gamma)}{\epsilon(\rho)} \right]^{\frac{\varphi}{1-\varphi}} \left(1 - \frac{1 - \beta\gamma/(1 + \gamma)}{\epsilon(\rho)/(1 + \gamma)} \right), \quad (\text{D.4})$$

where $n_M = [(1 + \gamma\alpha)(1 + \gamma)/\epsilon(\rho)]^{1/(1-\varphi)}$. For part (c), the welfare loss in autarky is

$$\Delta W(\rho) = W^* - V_j^{(0)}, \quad (\text{D.5})$$

where $V_j^{(0)} = \varphi^{\frac{\varphi}{1-\varphi}} (1 + \gamma\alpha)^{\frac{\varphi}{1-\varphi}} (1 - \varphi)$ is the autarky payoff from Proposition 1.

Monotonicity in ρ follows from the envelope theorem: W_j is strictly decreasing in $\epsilon_{j,k}$ for $\epsilon_{j,k} > 1/\varphi$, and $\epsilon(\rho)$ is strictly increasing in ρ . In the maximum-fee sub-region, although $\chi^* = \gamma/(1 + \gamma)$ is invariant to ρ , investment n_M remains strictly decreasing in ρ via its dependence on $\epsilon(\rho) = (1 + \rho)/(1 - \rho)$. Welfare loss is therefore strictly increasing on the entire interval $(\rho_0, \rho^{(2)})$. The autarky payoff $V_j^{(0)}$ does not depend on ρ , hence ΔW is constant for $\rho > \rho^{(2)}$. Continuity across the boundary at $\rho^{(1)}$ follows from continuity of $\chi^*(\rho)$. At $\rho^{(2)}$, $W_j(\bar{\epsilon}^{(2)}) = V_j^{(0)}$ by definition, so ΔW is continuous at $\rho^{(2)}$. $\square$

Closed-Form Transfer Fee

In the symmetric two-state model, for $\rho^{(0)} < \rho < \rho^{(1)}$, substituting $\epsilon = (1 + \rho)/(1 - \rho)$ into the quadratic from the proof of Lemma C.1 and letting $u = 1 - \chi$ yields $Au^2 + B(\rho)u + C = 0$ with

$$A = (1 - \varphi)\beta^2(1 + \gamma), \quad (\text{D.6})$$

$$B(\rho) = \varphi(1 + \gamma\alpha - \beta) \frac{1 + \rho}{1 - \rho} - (1 - \varphi)(\beta^2 + \gamma\beta(1 - \alpha)), \quad (\text{D.7})$$

$$C = -(1 + \gamma\alpha - \beta). \quad (\text{D.8})$$

Since $A > 0$ and $C < 0$, the product of roots is $C/A < 0$, so there is exactly one positive root. The optimal fee is therefore

$$\chi^*(\rho) = 1 - \frac{-B(\rho) + \sqrt{B(\rho)^2 - 4AC}}{2A}. \quad (\text{D.9})$$

The fee $\chi^*(\rho)$ is strictly increasing in ρ on $(\rho^{(0)}, \rho^{(1)})$, with $\chi^*(\rho^{(0)}) = 0$ and $\chi^*(\rho^{(1)}) = \gamma/(1 + \gamma)$. Monotonicity and boundary values follow from the envelope theorem and the definitions of $\rho^{(0)}$ and $\rho^{(1)}$.

E Policy Proofs and Results

Proposition E.1 (Common Unit of Account Mandate). *In the symmetric two-state model:*

- (a) *Under any mandate $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k = \hat{\mathbf{y}}_a$, both banks optimally set $\chi_{j,k}^* = 0$.*
- (b) *For $\rho \leq \rho_c$, common-unit-of-account Nash equilibria exist without the mandate, so the mandate can operate as an equilibrium selection device. For $\rho \leq 1 - \varphi$, the symmetric mandate with $\hat{\mathbf{y}}_a = \hat{\mathbf{y}}_m$ selects an efficient outcome.*
- (c) *For $\rho > \rho_c$, no common-unit-of-account Nash equilibrium exists without the mandate. Under the mandate with $\hat{\mathbf{y}}_a = \hat{\mathbf{y}}_m$, both banks face $\epsilon_{j,m} = 1/(1 - \rho)$, and investment under the mandate falls strictly below n^* whenever $\epsilon_{j,m} > 1/\varphi$, i.e. $\rho > 1 - \varphi$.*
- (d) *Among mandates $\hat{\mathbf{y}}_a = \omega\hat{\mathbf{y}}_A + (1 - \omega)\hat{\mathbf{y}}_B$, $\omega \in [0, 1]$, social welfare is maximised at $\omega = 1/2$.*

Proof. Part (a): with $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k$, both constraints (9) and (10) carry the same unit of account. Setting $\chi_{j,k} > 0$ tightens (10) without relaxing (9), so $\chi^* = 0$. Part (b) follows from Proposition 6: common-unit-of-account equilibria exist for $\rho \leq \rho_c$, and the symmetric common basket is efficient for $\rho \leq 1 - \varphi$ since $1/(1 - \rho) \leq 1/\varphi$. Part (c): Proposition 6 establishes that no common-unit-of-account equilibrium exists for $\rho > \rho_c$. Under the mandate with $\hat{\mathbf{y}}_a = \hat{\mathbf{y}}_m$, both banks face $\epsilon_{j,m} = \max_s \{\mathbf{p}'_s \hat{\mathbf{y}}_m / \mathbf{p}'_s \hat{\mathbf{y}}_j\} = 1/(1 - \rho)$. When $\epsilon_{j,m} > 1/\varphi$ (i.e. $\rho > 1 - \varphi$), the promise-keeping constraint binds and investment under the mandate falls below n^* . Part (d): by symmetry in the two-state model, $\max\{\epsilon_{A,a}(\omega), \epsilon_{B,a}(\omega)\}$ is minimised at $\omega = 1/2$. $\square$

Proof of Proposition 9

Proof. Part (a): in the two-state model, $\epsilon_{j,m} = \max_s \{\mathbf{p}'_s \hat{\mathbf{y}}_m / \mathbf{p}'_s \hat{\mathbf{y}}_j\} = 1/(1 - \rho)$. Since $(1 + \rho)/(1 - \rho) > 1/(1 - \rho)$ for $\rho > 0$, cash is strictly cheaper than direct interbank settlement. Part (b): under the natural unit of account, cash replaces bank j 's binding interbank constraint with $\epsilon_{j,m}$ in place of $\epsilon_{j,k}$. Zero fees require $\epsilon_{j,m} \leq \bar{\epsilon}^{(0)}$, i.e. $1/(1 - \rho) \leq \bar{\epsilon}^{(0)}$, giving $\rho \leq \rho_c$. Since $\rho_c = (\bar{\epsilon}^{(0)} - 1)/\bar{\epsilon}^{(0)}$ and $\rho^{(0)} = (\bar{\epsilon}^{(0)} - 1)/(\bar{\epsilon}^{(0)} + 1)$, and $\bar{\epsilon}^{(0)} < \bar{\epsilon}^{(0)} + 1$, it follows that $\rho_c > \rho^{(0)}$. The efficient-equilibrium existence boundary $1 - \varphi$ is unchanged: efficient equilibria require $1/(1 - \rho) \leq 1/\varphi$, and cash does not relax this condition. Under a common unit of account, $\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_k$ implies $\epsilon_{j,k} = \epsilon_{j,j}$, so the interbank constraint is no tighter than the promise-keeping constraint; cash can relax the former but not the latter, leaving the binding constraint unchanged. Part (c) follows identically under the natural unit of account, with $\bar{\epsilon}^{(2)}$ replacing $\bar{\epsilon}^{(0)}$. Part (d): when $\hat{\mathbf{b}}_k = \hat{\mathbf{y}}_m$, bank j 's interbank parameter is $\epsilon_{j,k} = \max_s \{\mathbf{p}'_s \hat{\mathbf{y}}_m / \mathbf{p}'_s \hat{\mathbf{y}}_j\} = \epsilon_{j,m}$, so cash offers no additional saving. Bank k faces $\epsilon_{k,j} = \max_s \{\mathbf{p}'_s \hat{\mathbf{b}}_j / \mathbf{p}'_s \hat{\mathbf{y}}_k\}$, which may exceed $\epsilon_{k,m}$. Cash allows bank k to settle at $\min\{\epsilon_{k,j}, \epsilon_{k,m}\} = \epsilon_{k,m}$, strictly reducing its effective cost whenever $\hat{\mathbf{b}}_j \neq \hat{\mathbf{y}}_m$. $\square$

F Proofs for Section 6

Proof of Proposition 10

Proof. The substitution $\gamma \rightarrow \tilde{\gamma}_j$ preserves the structure of the baseline equilibrium analysis: bank j 's profit takes the same functional form as in Propositions 1–3 with γ replaced by $\tilde{\gamma}_j$, provided $\tilde{\gamma}_j > 0$.

For case (i): when $\rho \leq 1 - \varphi$, choosing $\omega = \frac{1}{2}$ gives $\hat{\mathbf{b}}_A = \hat{\mathbf{y}}_m$. In the two-state model, $\epsilon_{A,A}(\omega) = \max\{(1 + \rho - 2\rho\omega)/(1 - \rho), (1 - \rho + 2\rho\omega)/(1 + \rho)\}$, which at $\omega = \frac{1}{2}$ equals

$1/(1-\rho)$. The promise-keeping constraint $1/(1-\rho) \leq 1/\varphi$ holds if and only if $\rho \leq 1-\varphi$. At $\omega = \frac{1}{2}$, $\sigma_A = \rho|1 - 2 \cdot \frac{1}{2}| = 0$, so $\tilde{\gamma}_A = \gamma$ and the baseline efficient equilibrium is recovered.

For case (ii): when $\rho > 1-\varphi$, $\omega = \frac{1}{2}$ violates the promise-keeping constraint. For $\omega \geq \frac{1}{2}$, the binding component of $\epsilon_{A,A}(\omega)$ is $(1 + \rho - 2\rho\omega)/(1-\rho)$. Setting this equal to $1/\varphi$ and solving yields $\omega^* = \frac{1}{2} + (\rho + \varphi - 1)/(2\rho\varphi)$. Information sensitivity at ω^* is $\sigma_A^* = \rho(2\omega^* - 1) = (\rho + \varphi - 1)/\varphi > 0$, and $\tilde{\gamma}_A^* = \gamma - (1 + \gamma)(\rho + \varphi - 1)/\varphi$. Trade requires $\tilde{\gamma}_A^* > 0$, i.e. $\gamma > (\rho + \varphi - 1)/(1-\rho)$.

By symmetry of the two-state model, bank B 's optimum is $1 - \omega^*$. Since $\omega^* > \frac{1}{2}$ in case (ii), $1 - \omega^* < \frac{1}{2}$. A common unit of account ω_c would require $\omega_c \geq \omega^*$ (for bank A 's promise-keeping constraint) and $1 - \omega_c \geq \omega^*$ (for bank B 's, by symmetry), i.e. $\omega_c \leq 1 - \omega^* < \frac{1}{2}$. These two inequalities are contradictory, so no common unit of account satisfies both banks' promise-keeping constraints. $\square$

Corollary: Indeterminacy Breaking

Corollary F.1 (Indeterminacy Breaking). *Under the worst-state acceptance requirement, among all units of account satisfying $\epsilon_{j,j} \leq 1/\varphi$, each bank strictly prefers the one that minimises $\sigma_j(\hat{\mathbf{b}}_j)$. In the two-state model, this uniquely determines the optimal ω^* as the feasible value closest to $\frac{1}{2}$.*

Proof. Profit is increasing in $\tilde{\gamma}_j$, $\tilde{\gamma}_j$ is decreasing in σ_j , and $\sigma_j = \rho|1 - 2\omega|$ is minimised at the feasible ω closest to $1/2$. $\square$

Reserves and Information-Insensitive Debt

Proposition F.1 (Reserves and Information-Insensitive Debt). *Consider the two-state model with $\rho > 1-\varphi$ and a reserve requirement τ as in Proposition 8. If $\tau \geq (\rho + \varphi - 1)/\rho$, the effective asset-return variance $\rho(1-\tau)$ satisfies $\rho(1-\tau) \leq 1-\varphi$, so the risk-free unit of account becomes feasible and $\sigma_j^* = 0$. Reserves thus provide a benefit beyond the borrowing-capacity channel identified in Proposition 8: they eliminate the information-sensitivity tax on gains from trade, restoring $\tilde{\gamma}_j = \gamma$.*

Proof. Under reserve rate $\tau \geq (\rho + \varphi - 1)/\rho$, effective variance $\rho(1-\tau) \leq 1-\varphi$, placing the economy in case (i) of Proposition 10 with $\sigma^* = 0$ and $\tilde{\gamma} = \gamma$. $\square$