



Discussion Paper No. 2

"Inflation Dynamics and the Labour Share in the UK"

by Nicoletta Batini, Brian Jackson and Stephen Nickell

External MPC Unit Discussion Paper No. 2*

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Inflation Dynamics and the Labour Share in the UK¹

By Nicoletta Batini*, Brian Jackson[†] and Stephen Nickell[‡]

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Abstract

In recent years UK real wages have been growing faster than labour factor productivity, implying that the labour share has been rising. This paper looks at various definitions of the labour share and derives a measure for the UK, which appears positively correlated with the growth rate of the UK gross value added price deflator. Following Layard, Nickell and Jackman (1991), we investigate the relationship between inflation and the share more formally by estimating a pricing equation or “new Phillips curve” obtained from a structural dynamic model of price setting based on Rotemberg (1982) and extended to capture employment adjustment costs and the openness of the UK economy. This model nests the Sbordone (1998) and Gali, Gertler and Lopez-Salido (2000) relationship between inflation and marginal costs in the limiting case of a constant equilibrium mark-up and no employment adjustment costs. Our findings suggest that there is a stable *ceteris paribus* relationship between inflation and the labour share over the last 30 years in the UK and so the share contains information that helps to predict inflation.

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Abstract

In recent years UK real wages have been growing faster than labour factor productivity, implying that the labour share has been rising. This paper looks at various definitions of the labour share and derives a measure for the UK, which appears positively correlated with the growth rate of the UK gross value added price deflator. Following Layard, Nickell and Jackman (1991), we investigate the relationship between inflation and the share more formally by estimating a pricing equation or “new Phillips curve” obtained from a structural dynamic model of price setting based on Rotemberg (1982) and extended to capture employment adjustment costs and the openness of the UK economy. This model nests the Sbordone (1998) and Gali, Gertler and Lopez-Salido (2000) relationship between inflation and marginal costs in the limiting case of a constant equilibrium mark-up and no employment adjustment costs. Our findings suggest that there is a stable *ceteris paribus* relationship between inflation and the labour share over the last 30 years in the UK and so the share contains information that helps to predict inflation.

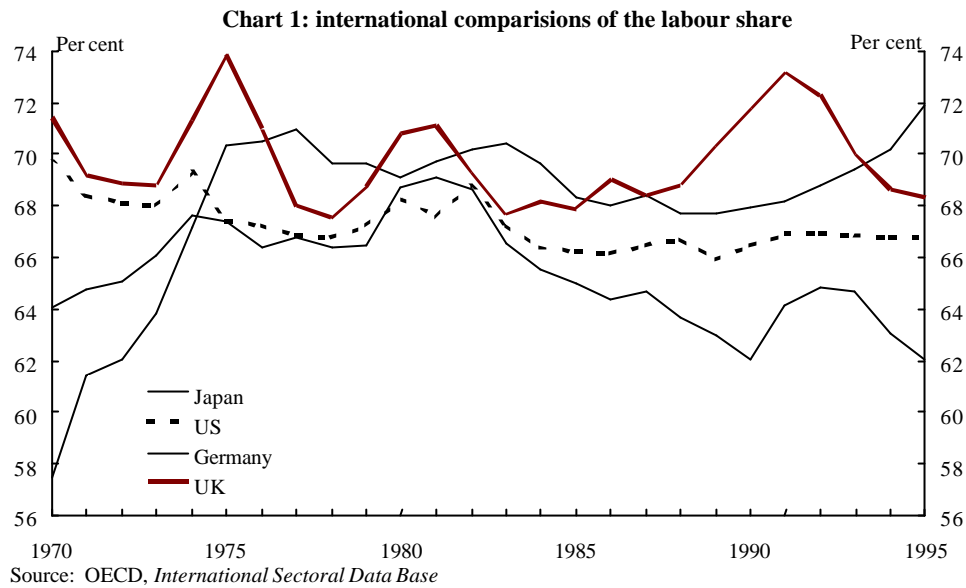
Introduction

Since 1996, real product wages in Britain have tended to rise faster than labour productivity. Intuitively, this implies that workers are getting more than they deserve. It also implies that workers' share in the 'national cake' is growing. Our purpose in what follows is to investigate whether this has implications for inflation and, thus, whether it matters for monetary policy in the UK.

The share of value added (net of indirect taxes) that accrues to workers from their supply of labour to firms is usually referred to as the "labour share". The concept of labour share goes back a long time. Ricardo originally introduced the term in *The Principles of Taxation and Political Economy* in 1821. Subsequently, the meaning and importance of the share as a measure of the division of output between workers and firms for macroeconomics has been investigated at length by both classical and neoclassical economists. Among these, Henley (1987), Sherman (1990), Praschnik and Costello (1992), Modesto and Monteiro (1993) and Kang *et al.* (1998), all looked at the determinants of the labour share and analysed its cyclical patterns. Recently, Blanchard (1997, 1998), Caballero and Hammour (1998), and Bentolila and Saint-Paul (1999), have also looked at the labour share, examining its role in the determination of unemployment or patterns in the distribution of income.¹

In real business cycle models, the labour share is often assumed to be constant, so not much importance is attached to it in explaining economic fluctuations and growth. As Bentolila and Saint-Paul (1999) point out, in the UK the labour share does indeed appear to be stationary, even if it displays large short-run fluctuations around its mean. However, this is not true in other countries, such as Japan and Germany, where the share has been following either upward or downward-sloping trends (see Chart 1).

¹ See also Artus (1984).



Importantly, as Chart 2 below illustrates, in the UK spikes in the labour share over the past have often been associated with rises in inflation, supporting the idea of a relationship between the two. This is perhaps less evident since 1995, when the labour share has increased — as real wages have been growing faster than labour factor productivity in the UK — but inflation has remained subdued.



In this paper we explore the theoretical and empirical relationship between the share of labour and inflation in the UK. We examine whether the labour share can be regarded as a good indicator of UK inflationary pressures, and, more broadly, we speculate about the future implications for inflation of the rise in the share observed since 1995.

The plan of the paper is as follows. Section 1 looks at various definitions of the labour share and derives a sensible measure of the share for the UK. Dynamic correlations indicate that this is positively correlated with inflation (measured in terms of changes in the gross value added price deflator). In addition, simple Granger causality tests suggest that the share Granger-causes inflation. Hence, section 2 examines the relationship between inflation and the share more formally, by deriving a pricing equation or a “new Keynesian Phillips curve” obtained from a structural dynamic model of pricing behaviour *à la* Layard *et al.* (1991) based on Rotemberg (1982), and extended to capture the openness of the UK economy.² This model has the Sbordone (1998) and Gali, Gertler and Lopez-Salido (2000) relationship between inflation and marginal costs as a special case, extending it to account for variations in the equilibrium price mark-up and for labour adjustment costs. In section 3 we estimate our theoretically derived pricing model by using a generalised method of moments estimator. We find a stable relationship, which implies that the measure of labour share that we employ contains information that is helpful to predict inflation. Concluding remarks follow. At the end of the paper we include a Data Appendix which describes the data that we have used in section 3, and a Technical Appendix.

1. Measuring the labour share with UK data

1.1 Definition and measurement

Algebraically, the labour share can be expressed as:

² See Roberts (1995) for an analysis of the contributions of new Keynesian economics to the study of the Phillips curve.

$$s_L \equiv WN / PY \tag{1}$$

where W is labour cost per employee, N is employment, P is the GDP deflator at factor cost, and Y is national income.

So, on the face of it, the labour share appears easy to compute: take the total compensation of employees in the economy and divide it by the national income. In practice, there are three issues to bear in mind when computing the labour share.

First, the share must be derived relative to a measure of value added that is *net of indirect taxes*. Conceptually, firms and workers can only lay claims on revenue (in terms of output per head) that actually accrues to the firm. By definition, this will be net of taxes on value added, as these are paid to the government and are not received by the firm. For this reason, we have defined the basic measure of the labour share, which we denote as 's', as the ratio of the compensation of employees to total gross value added, measured at basic prices.³

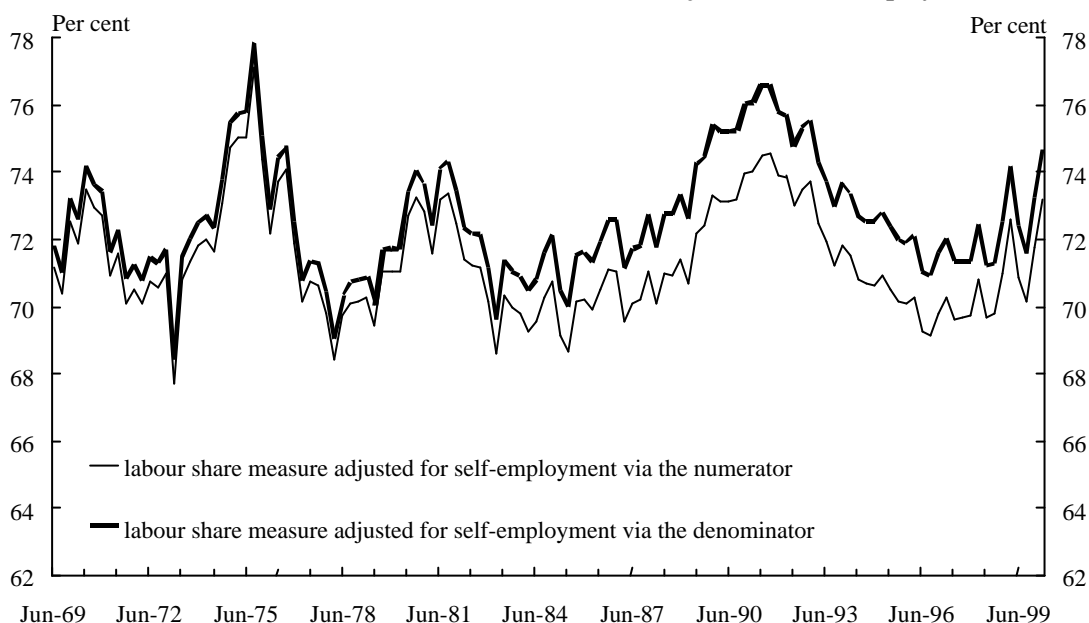
Second, as Bentolila and Saint-Paul (1999) emphasise, because the share in equation (1) represents the remuneration of employees in value added, it ignores that part of remuneration of the self-employed that constitutes a return to labour rather than to capital. There are two ways of adjusting for this. We can either augment the numerator of equation (1) to include the fraction of total compensation of self-employed that relates to labour; or we can subtract the amount of value added generated by the self-employed from its denominator.

Chart 3 below plots the self-employment adjusted labour share under these alternative approaches for the period 1969 Q2 – 2000 Q1. In particular, the thin line corresponds to the labour share including the remuneration of self-employed [numerator adjustment in equation (1)], whereas the thick line gives the employees-only measure of labour share

³ For further details on how variables are derived, refer to the Data Appendix.

[denominator adjustment in equation (1)]. Both measures assume that the average return to labour of the self-employed is equal to the average remuneration of employees.

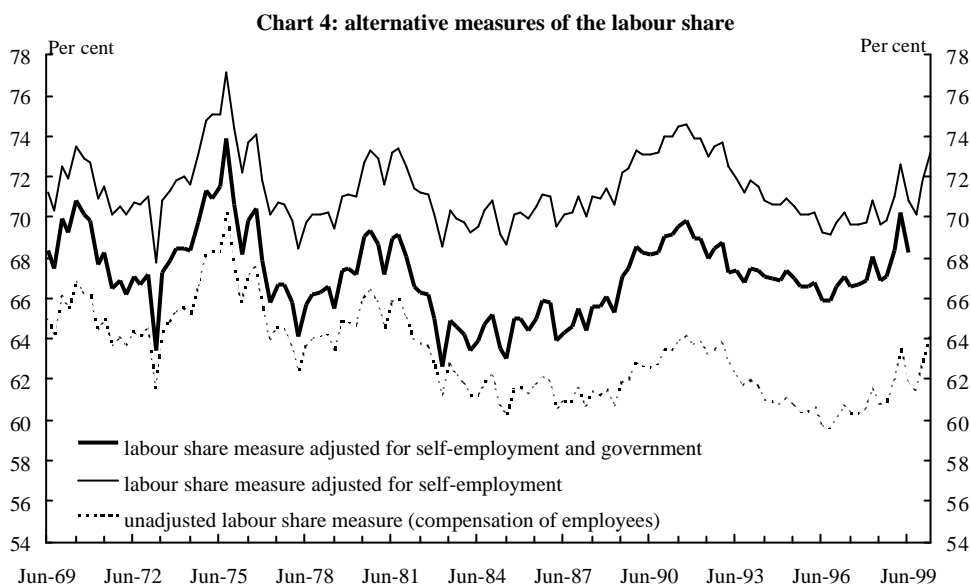
Chart 3: alternative measures of the labour share adjusted for self-employment



As the chart shows, these adjusted measures moved closely together throughout the period, and so give a broadly consistent picture of the path followed by the share over the past three decades. In what follows we will use the measure of the labour share that augments the numerator of equation (1) to incorporate the remuneration of the self-employed. This measure we refer to as the “labour share” and denote as ‘ s_L ’.

A final consideration when deriving a measure of the labour share relates to the contribution of the public sector. It might be argued that the concepts of labour and capital shares only really make sense with regard to the market sector of the economy. In this spirit, we may adjust equation (1) to remove the public sector’s inputs from the numerator and the denominator of that expression. We do so by subtracting from the numerator of the self-employed adjusted share, the compensation of employees by the general government; and by removing from the denominator of that share the general government’s total resources, essentially a measure of general government gross value added. We refer to this measure of labour share adjusted for both self-employment and

public sector considerations as the “adjusted labour share” and denote it as ‘ s_L^* ’. Chart 4 below offers a plot of the various measures of the labour share.



A casual inspection of Chart 4 suggests that the unadjusted measure of the labour share, which does not account either for self-employment or public sector considerations, seems to have a downward trend over recent decades, whereas the other two measures, s_L and s_L^* , appear to have fluctuated around a constant mean. To test the time series properties of these series more formally, we have conducted augmented Dickey-Fuller tests for the period 1972 Q3 – 1999 Q2, the results of which are shown in Table A of the Data Appendix. These tests indicate that the unadjusted measure contains a unit root, whereas both adjusted measures do not.

1.2 Some crude evidence

Chart 2 in the previous sub-section seems to suggest, at least at medium frequencies, that there is a relationship between the labour share and inflation, something we intend to investigate more formally in later sections. Crude empirical tests confirm this view.

Table 1 below lists contemporaneous and dynamic correlations computed over the period 1972 Q3 – 1999 Q2 between s_L and inflation, measured both in terms of one-quarter changes ($\mathbf{p}_t = p_t - p_{t-1}$) and four-quarter changes ($\mathbf{p}_{4t} = p_t - p_{t-4}$) in the natural log of the gross value added price deflator (henceforth ‘GVAD’). Table 2 lists analogous correlations derived over that same period using the adjusted labour share (s_L^*).

Table 1: Contemporaneous and dynamic correlations ($s_L, \mathbf{p}_t, \mathbf{p}_{4t}$)

<i>Lags</i>	0	1	2	3	4
$\text{Corr}(\mathbf{p}_t, s_{Lt-k})$	0.34	0.32	0.24	0.16	0.10
$\text{Corr}(\mathbf{p}_{4t}, s_{Lt-k})$	0.46	0.43	0.37	0.31	0.23
<i>Lags</i>	5	6	7	8	9
$\text{Corr}(\mathbf{p}_t, s_{Lt-k})$	0.05	-0.002	-0.09	-0.11	-0.10
$\text{Corr}(\mathbf{p}_{4t}, s_{Lt-k})$	0.15	0.08	-0.02	-0.08	-0.11

Table 2: Contemporaneous and dynamic correlations ($s_L^*, \mathbf{p}_t, \mathbf{p}_{4t}$)

<i>Lags</i>	0	1	2	3	4
$\text{Corr}(\mathbf{p}_t, s_{Lt-k}^*)$	0.35	0.36	0.30	0.24	0.20
$\text{Corr}(\mathbf{p}_{4t}, s_{Lt-k}^*)$	0.46	0.45	0.43	0.38	0.33
<i>Lags</i>	5	6	7	8	9
$\text{Corr}(\mathbf{p}_t, s_{Lt-k}^*)$	0.15	0.12	0.02	0.02	0.03
$\text{Corr}(\mathbf{p}_{4t}, s_{Lt-k}^*)$	0.27	0.21	0.12	0.07	0.04

In line with the informal visual evidence from Chart 2, both tables indicate that inflation is contemporaneously and dynamically positively correlated with inflation at shorter lags. As one would expect, the correlation is stronger when inflation is measured in terms of four-quarter changes in the GVAD.

These results are also supported by findings from five-lag Granger causality tests calculated over the period 1972 Q3 – 1999 Q2. We present these in Table 3 (for s_L) and

in Table 4 (for s_L^*) below. In general the hypothesis that the labour share does not Granger cause inflation is rejected at standard levels of significance.⁴ This tentative evidence seems to favor the view that the share of labour may contain corroborative or incremental information for predicting inflation.

Table 3: Pairwise Granger causality tests [(s_L, p_t) and (s_L, p_{4t})]

<i>Null Hypothesis</i>	<i>F-Statistic</i>	<i>Probability</i>
s_L does not GC p_t	2.58250	0.03082
p_t does not GC s_L	1.97776	0.08870
s_L does not GC p_{4t}	3.41162	0.00698
p_{4t} does not GC s_L	2.97902	0.01519

Table 4: Pairwise Granger causality tests [(s_L^*, p_t) and (s_L^*, p_{4t})]

<i>Null Hypothesis</i>	<i>F-Statistic</i>	<i>Probability</i>
s_L^* does not GC p_t	2.72585	0.02388
p_t does not GC s_L^*	1.85382	0.10956
s_L^* does not GC p_{4t}	3.51067	0.00584
p_{4t} does not GC s_L^*	2.90249	0.01742

Perhaps a more powerful test of the information content of the labour share is whether it is informative, at given levels of the output gap, about *changes* in inflation — since disequilibrium is associated with rising or falling inflation, rather than with the level of inflation *per se*. To pursue this final test, we considered autoregressions of the form:

$$\Delta p_t = A(L)\Delta p_{t-1} + B(L)s_{L,t-1} + C(L)(y_{t-s} - y_{t-s}^*)$$

⁴ There is also some evidence that Granger causality runs the other way, although this is not as strong in the case of the one-quarter change in the GVAD where the null of no Granger causality is not rejected at the 5 per cent significance level when s_L is considered and is not rejected at the 10 per cent level when s_L^* is considered.

where $\Delta \mathbf{p}_t = \mathbf{p}_t - \mathbf{p}_{t-1} = (p_t - p_{t-1}) - (p_{t-1} - p_{t-2})$ is the change in the one-quarter growth rate of the GVAD and $(y_t - y_t^*)$ is the output gap, and investigated the joint significance of the $B(L)$ and $C(L)$ coefficients. Similar results are also reported using the change in the four-quarter growth rate of the GVAD, $\Delta_4 \mathbf{p}_{4t} = \mathbf{p}_{4t} - \mathbf{p}_{4t-4} = (p_t - p_{t-4}) - (p_{t-4} - p_{t-8})$.

When estimating the model it became clear that we could restrict the labour share effects to be in first differences because the sum of the level effects was almost exactly zero. So below we report results of a series of F-tests examining the joint significance of four lags of changes in the labour share (Δs_L , Table 5) and the adjusted labour share (Δs_L^* , Table 6) in a regression of inflation (one-quarter and four-quarter change in the GVAD, respectively) on four lags of itself and five lags of the output gap, which we measure in terms of deviations of gross value added from its Hodrick-Prescott trend. Throughout, the sample period over which we estimate the regression is 1972 Q3 – 1999 Q2.

Table 5: Joint significance of lags ($t - 1$ to $t - 4$) of Δs_L

Dependent variable: $\Delta \mathbf{p}_t$			
Explanatory variables: $\Delta \mathbf{p}_{t-k}$, Δs_{L-t-k} , $k = 1, \dots, 4$; $(y_{t-m} - y_{t-m}^*)$, $m = 1, \dots, 5$.			
<i>F</i> -statistic	2.76	Probability	0.032
Coefficients ^(a) on lags of Δs_{L-t-m} : Δs_{L-t-1} : 0.19 (2.2); Δs_{L-t-2} : 0.20 (2.3); Δs_{L-t-3} : -0.03 (0.4); Δs_{L-t-4} : 0.12 (1.3)			
Dependent variable: $\Delta_4 \mathbf{p}_{4t}$			
Explanatory variables: $\Delta_4 \mathbf{p}_{4t-k}$, Δs_{L-t-k} , $k = 1, \dots, 4$; $(y_{t-m} - y_{t-m}^*)$, $m = 1, \dots, 5$.			
<i>F</i> -statistic	4.12	Probability	0.004
Coefficients ^(a) on lags of Δs_{L-t-m} : Δs_{L-t-1} : 0.46 (2.9); Δs_{L-t-2} : 0.38 (2.3); Δs_{L-t-3} : 0.42 (2.6); Δs_{L-t-4} : 0.56 (3.6)			

Notes: (a) *t*-statistics in parenthesis.

Table 6: Joint significance of lags ($t - 1$ to $t - 4$) of Δs_L^*

Dependent variable: Δp_t			
Explanatory variables: Δp_{t-k} , Δs_{L-t-k}^* , $k = 1, \dots, 4$; $(y_{t-m} - y_{t-m}^*)$, $m = 1, \dots, 5$.			
<i>F</i> -statistic	3.37	Probability	0.013
Coefficients ^(a) on lags of Δs_{L-t-m}^* : Δs_{L-t-1}^* : 0.16 (2.33); Δs_{L-t-2}^* : 0.19 (2.83); Δs_{L-t-3}^* : -0.006 (0.081); Δs_{L-t-4}^* : 0.12 (1.74)			
Dependent variable: $\Delta_4 p_{4t}$			
Explanatory variables: $\Delta_4 p_{4t-k}$, Δs_{L-t-k}^* , $k = 1, \dots, 4$; $(y_{t-m} - y_{t-m}^*)$, $m = 1, \dots, 5$.			
<i>F</i> -statistic	4.64	Probability	0.002
Coefficients ^(a) on lags of Δs_{L-t-m}^* : Δs_{L-t-1}^* : 0.38 (3.01); Δs_{L-t-2}^* : 0.34 (2.6); Δs_{L-t-3}^* : 0.38 (2.9); Δs_{L-t-4}^* : 0.48 (3.83)			

Notes: (a) *t*-statistics in parenthesis.

Taken together, the tables reveal that for those regressions involving both the one-quarter and four-quarter change in inflation, we can reject the hypothesis that lags of changes in the labour share (either Δs_L or Δs_L^*) have no impact.⁵ This is not just a consequence of the broad correlations generated by the two oil shocks: we obtain similar results even if we carry out the regressions using a sample period that starts in 1982.

2. A theoretical relationship between the labour share and inflation in an open economy

In this section we examine the relationship between inflation and the share more formally by deriving a pricing equation or a “new Keynesian Phillips curve” from a structural dynamic pricing model with prices and employment adjustment costs *à la* Layard, Nickell and Jackman (1991) (‘henceforth ‘LNJ’). This, in turn, is based on Rotemberg (1982), and is extended here to capture the openness of the UK economy. For simplicity, we start by deriving a theoretical relationship between inflation and the labour share for an economy that is closed and with no adjustment costs (sub-section 2.1). We then modify the model to allow for dynamic adjustment costs (sub-section 2.2). And finally,

we extend the model to account for the fact that the economy is open (sub-section 2.3). As anticipated in the Introduction, this model has the Sbordone (1998) and Gali, Gertler and Lopez-Salido (2000) “new” Phillips curve relationship between inflation and marginal costs as a special case, extending it to account for variations in the price mark-up and for labour adjustment costs.⁶ In section 3 we estimate the pricing model that we derive in section 2 to explore whether the share of labour contains information that is relevant for predicting inflation as preliminary evidence suggests.

2.1 A static closed-economy pricing model

To unveil the relationship linking inflation and the share of labour, we need a model of the pricing behaviour of firms. This pins down the linkage between prices, inflation and marginal costs. For this purpose, we start by considering the static equilibrium level of prices, that is, the price that would prevail in the absence of adjustment costs. Thus we assume that the economy is inhabited by F identical firms, labeled i , and that technology is Cobb-Douglas and can be written as:

$$Y_{it} = A_{it} N_{it}^a \quad (2)$$

where $a > 0$, Y_{it} is value added output, N_{it} is employment and A_{it} represents an exogenous productivity index capturing shifts in labour productivity. This includes the impact of both capital and total factor productivity.⁷ Following LNJ, we postulate that each firm faces a constant elasticity demand function, i.e.:

$$Y_{it} = (P_{it} / P_t)^{-h_i} Y_{dit} \quad (3)$$

⁵ In these regressions involving the one-quarter change in inflation we can also reject the hypothesis that the output gap has zero impact.

⁶ It is worth emphasizing that the “new Keynesian Phillips curve” is essentially a model of pricing behaviour, thus telling us about prices *given* wages. It is, therefore, a completely different animal from the “old Phillips curve”, which was generated by eliminating wages from the pricing model using some wage equation, leaving a relationship between price inflation and some measure of economic activity. Of course, Phillips (1958) original curve was concerned with wage inflation and excess demand in the labour market.

⁷ Capital is assumed fixed with regard to short-run variations in output.

where $h > 1$, P_{it} is the price of value added of firm i , P_t is the aggregate price of value added (i.e. the GDP deflator), and Y_{dit} is an exogenous demand index.

We then define the cost of producing output as:

$$C_t = W_{it}N_{it} + cK_i \quad (4)$$

where W_{it} represents the labour cost per employee (consisting of wages plus non-wage labour costs) and cK_i is a predetermined capital cost, which is fixed with regard to short-run variations in output.

Using (2), we can re-express cost as:

$$C_t = W_{it}Y_{it}^{1/a}A_{it}^{-1/a} + cK_i \quad (4a)$$

so that marginal cost is equal to:

$$MC_t = (1/a)(W_{it}Y_{it}^{(1/a-1)}A_{it}^{-1/a}) = (1/a)(W_{it}N_{it}/Y_{it}) \quad (\text{from (2)}) \quad (5)$$

The static equilibrium price P_{it}^* is hence given by:

$$P_{it}^* = \mathbf{m}_{it}^* MC_{it} \quad (6)$$

where \mathbf{m}_{it}^* is the equilibrium mark-up of prices on marginal cost, i.e. $\mathbf{m}_{it}^* = (1 - 1/h_{it})^{-1}$, which is decreasing in the demand elasticity.

2.2 Dynamic model based on quadratic adjustment costs

Following LNJ,⁸ we now modify the basic pricing model to encompass quadratic adjustment costs of changing both prices and employment — a specification based on Rotemberg (1982). This gives a model that is preferable to Calvo (1983) because it enables us to incorporate employment adjustment costs more easily. As we will discuss shortly, these are a crucial source of the inertia usually observed in the UK and hence should not be ignored. Throughout, lower-case letters denote natural logarithms of the corresponding upper-case variables.

To simplify the analytical solution of the dynamic optimisation problem faced by firms, and ensure linear first-order conditions, we begin by approximating the firm's real profit objective ($j(p_i)$), by a Taylor expansion around p_i^* [$p_t, p_i^* = \ln P_t, \ln P_i^*$] based on (6) or (6a). Thus:

$$j(p_i) \approx j(p_i^*) - (q/2)(p_i - p_i^*)^2 \quad (7)$$

where $j'(p_i^*) = 0$ (since p_i^* is the equilibrium price) and $q = -j''(p_i^*) > 0$. We assume that the firm wishes to maximise an objective like (7), but that it faces additional quadratic employment adjustment costs. When these are included, the firm's problem consists in deriving, at the start of period t , a price and employment path that solve:

$$\min E_{t-1} \sum_{s=0}^{\infty} f^s \left[(p_{i,t+s} - p_{i,t+s}^*)^2 + b_p / 2 (p_{i,t+s} - p_{i,t+s-1})^2 + b_n / 2 (n_{i,t+s} - n_{i,t+s-1})^2 \right] \quad (8)$$

where f is a discount factor, and E_{t-1} denotes expectations formed on the basis of information available *at the end* of period $t-1$. Objective (8) is subject to the constraint that demand is met in each period, that is:

⁸ See Layard *et al* (1991), pp. 346 and ff.

$$a_{it+s} + \mathbf{a}n_{it+s} = -\mathbf{h}(p_{it+s} - p_{t+s}) + y_{dit+s} \quad (\text{all } s \geq 0) \quad (9)$$

which is based on equations (2) and (3).

We set the demand elasticity equal to a constant, imagining that, while it may fluctuate over the cycle, the firm treats it as constant when solving this problem.⁹ Thus, using the constraint to eliminate employment, the problem reduces to:

$$\min E_{t-1} \sum_{s=0}^{\infty} \mathbf{f}^s \left\{ \begin{aligned} & \left(p_{it+s} - p_{it+s}^* \right)^2 + 1/2 (b_p + b_n \mathbf{h}^2 / \mathbf{a}^2) (p_{it+s} - p_{it+s-1})^2 - \\ & b_n \mathbf{h}^2 / \mathbf{a}^2 \left[(p_{it+s} - p_{it+s-1})(p_{t+s} - p_{t+s-1}) + \right. \\ & \left. 1/\mathbf{h} (p_{it+s} - p_{it+s-1})(y_{dit+s} - a_{it+s} - y_{dit+s-1} + a_{it+s-1}) \right] \end{aligned} \right\} \quad (10)$$

Since this is a quadratic problem, we invoke first order certainty equivalence and replace all future random variables by their expectations which, hereafter, we denote with the superscript “*e*”.

To obtain first-order conditions for this problem, we differentiate (10) with respect to the price of the individual firm, p_{it+s} . Before doing so, for notational convenience, we re-express some sets of variables in the following way: $p_{it+s} - p_{t+s}^e \equiv \tilde{p}_{it+s}$; $b_p + b_n \mathbf{h}^2 / \mathbf{a}^2 \equiv \mathbf{a}_1$; and:

$$\hat{p}_{it+s} \equiv p_{it+s}^* - p_{t+s}^e + b_p / \mathbf{q} \left(\mathbf{f} \Delta p_{t+s+1}^e - \Delta p_{t+s}^e \right) - b_n \mathbf{h} / \mathbf{q} \mathbf{a}^2 \left(\mathbf{f} \Delta (y_{dit+s+1}^e - a_{it+s+1}^e) - \Delta (y_{dit+s}^e - a_{it+s}^e) \right) \quad (11)$$

Then, the first order condition for the firm’s profit maximisation problem is:

⁹ The fact is that it may change systematically over the years may, therefore, lead to shifts in the model’s parameters.

$$\mathbf{f}\mathbf{a}_1\tilde{p}_{it+s+1} - [\mathbf{q} + \mathbf{a}_1(\mathbf{f}+1)]\tilde{p}_{it+s} + \mathbf{a}_1\tilde{p}_{it+s-1} = -\mathbf{q}\hat{p}_{it+s} \quad (s > 0) \quad (11a)$$

The standard (first period) solution to this second order difference equation (or Euler equation) is:

$$\tilde{p}_{it} = \mathbf{I}\tilde{p}_{it-1} + (1-\mathbf{I})(1-\mathbf{f}\mathbf{l}) \sum_{j=0}^{\infty} (\mathbf{f}\mathbf{l})^j \hat{p}_{it+j} \quad (12)$$

where \mathbf{I} is the unique stable root of:

$$\mathbf{f}\mathbf{a}_1\mathbf{I}^2 - [\mathbf{q} + \mathbf{a}_1(\mathbf{f}+1)]\mathbf{I} + \mathbf{a}_1 = 0 \quad (13)$$

We now make the expectations in (12) more explicit, i.e.:

$$p_{it} - E_{t-1}p_t = \mathbf{I}(p_{it-1} - p_{t-1}) + (1-\mathbf{I})(1-\mathbf{f}\mathbf{l}) \sum_{j=0}^{\infty} (\mathbf{f}\mathbf{l})^j E_{t-1}\hat{p}_{it+j} \quad (14)$$

and shift (14) one period forward. By taking expectations dated $t-1$, multiplying by $(\mathbf{f}\mathbf{l})$ and subtracting from (14), we obtain:

$$(p_{it} - E_{t-1}p_t) = \mathbf{f}\mathbf{l}E_{t-1}(p_{it+1} - p_{t+1}) - \mathbf{f}\mathbf{l}^2E_{t-1}(p_{it} - p_t) + \mathbf{I}(p_{it-1} - p_{t-1}) + (1-\mathbf{I})(1-\mathbf{f}\mathbf{l})E_{t-1}\hat{p}_{it} \quad (15)$$

We can use the assumption that firms are identical to note that $p_{it} = p_t$. Hence, from (3), $Y_{it}/Y_{dit} = Y_t/Y_{dt} = 1$. Using (9) then ensures that the aggregate version of (11) is:

$$\hat{p}_{t+s} = p_{t+s}^* - E_{t-1}p_{t+s} + b_p/\mathbf{q}(\mathbf{f}E_{t-1}\Delta p_{t+s+1} - E_{t-1}\Delta p_{t+s}) - b_r\mathbf{h}/\mathbf{q}\mathbf{a}(\mathbf{f}E_{t-1}\Delta n_{t+s+1} - E_{t-1}\Delta n_{t+s}) \quad (16)$$

where $p_{t+s}^* = E_{t-1} \ln \mathbf{m}_{t+s} + E_{t-1} mc_{t+s}$ (with $mc_t \equiv \ln MC_t$).

So the aggregate version of (15) becomes:

$$\begin{aligned} (p_t - E_{t-1} p_t) = & (1-I)(1-fI) E_{t-1} p_t^* - (1-I)(1-fI) E_{t-1} p_t + \\ & (1-I)(1-fI) b_p / q (fE_{t-1} p_{t+1} - E_{t-1} p_t) - \\ & (1-I)(1-fI) (b_n h / q a) (fE_{t-1} \Delta n_{t+1} - E_{t-1} \Delta n_t) \end{aligned} \quad (17)$$

where $p_t = p_t - p_{t-1}$, the rate of inflation. From the definition of p^* [see (6)], this becomes:

$$\begin{aligned} (p_t - E_{t-1} p_t) = & (1-I)(1-fI) E_{t-1} \ln \mathbf{m}_t^* + (1-I)(1-fI) E_{t-1} (mc_t - p_t) + \\ & (1-I)(1-fI) b_p / q (fE_{t-1} p_{t+1} - E_{t-1} p_t) - \\ & (1-I)(1-fI) (b_n h / q a) (fE_{t-1} \Delta n_{t+1} - E_{t-1} \Delta n_t) \end{aligned} \quad (17a)$$

Setting $(1-I)(1-fI) = w$, we can re-write the above as:

$$\begin{aligned} (wb_p / q) p_t = & (wb_p / q) fE_{t-1} p_{t+1} + wE_{t-1} \ln \mathbf{m}_t^* + wE_{t-1} (mc_t - p_t) - \\ & w (b_n h / q a) (fE_{t-1} \Delta n_{t+1} - E_{t-1} \Delta n_t) + \mathbf{n}_t' \end{aligned} \quad (18)$$

where $\mathbf{n}_t' = -(1-wb_p / q) (p_t - E_{t-1} p_t)$, an innovation, or:

$$\begin{aligned} p_t = & fE_{t-1} p_{t+1} + q / b_p E_{t-1} \ln \mathbf{m}_t^* + q / b_p E_{t-1} (mc_t - p_t) - \\ & (b_n h f / b_p a) E_{t-1} \Delta n_{t+1} + (b_n h / b_p a) E_{t-1} \Delta n_t + \mathbf{n}_t \end{aligned} \quad (19)$$

where $\mathbf{n}_t = -\left[(1-wb_p / q) / (wb_p / q) \right] (p_t - E_{t-1} p_t)$.

Equation (19) is the final structural equation governing the aggregate price level in our model. If we note that $(p_t - mc_t)$ is the actual mark-up of price on marginal cost, $\ln \mathbf{m}_t$, say, we can rewrite (19) as:

$$\mathbf{p}_t = \mathbf{f}E_{t-1}\mathbf{p}_{t+1} + \frac{\mathbf{q}}{b_p}E_{t-1}(\ln \mathbf{m}_t^* - \ln \mathbf{m}_t) - \left(\frac{b_n \mathbf{h}}{b_p \mathbf{a}} \right) E_{t-1}(\mathbf{f}\Delta \mathbf{h}_{t+1} - \Delta \mathbf{h}_t) + \mathbf{n}_t \quad (19')$$

This reveals that inflation depends positively on the gap between the equilibrium mark-up and the actual mark-up, this gap arising because of the stickiness of prices. So one interpretation of (19) is that inflation will be higher, *ceteris paribus*, if the equilibrium mark-up exceeds the actual mark-up, because, in this case firms will be raising prices in an attempt to restore their profitability.

Equation (19) has a similar structure to that proposed by Gali *et al.* (2000) which, *mutatis mutandis*, has the form:¹⁰

$$\mathbf{p}_t = \mathbf{f}E_{t-1}\mathbf{p}_{t+1} + \mathbf{I}'(mc_t - p_t) + const + error$$

The differences between this and our equation (19) are that we allow for a variable mark-up and also account for employment adjustment costs. However, both ours and the Gali *et al.* (2000) model imply non-neutrality, i.e. a non-zero relationship between inflation and real variables in the long run.¹¹ This follows from the fact that the process generating price rigidity in the two models is unaffected by the general level of inflation.

Thus in the Calvo (1983) model, which underlies Gali *et al.* (2000), each firm resets its price with probability $(1-\mathbf{q})$ in each period, irrespective of the general level of inflation.

This seems unlikely. As Sims (1988, p.77) remarks:

“[I]f there is such a thing as an economy with a rock-solid inflation rate of 40 per cent, plus or minus 2 per cent, per year, institutions would surely adapt, so that prices would be announced in catalogs and wage contracts with smooth growth paths paralleling the

¹⁰ The notation in Gali *et al.* (2000) is slightly different (their \mathbf{b} is equal to our \mathbf{f} , and their mc_t is real marginal cost which corresponds to our $(mc_t - p_t)$). In their model, \mathbf{I}' is decreasing in the degree of price rigidity as is our coefficient on $(mc_t - p_t)$, which is decreasing in b_p .

¹¹ Note, in the long-run steady state, $\mathbf{p}_t = \mathbf{p}_{t+1} = E_{t-1}\mathbf{p}_{t+1}$, and we find that $(1-\mathbf{f})\mathbf{p}_t = \text{real factors}$.

smooth aggregate price path. Nominal rigidity would set in about this price path in much the same form as we see around the zero inflation rate in low-inflation economies.”

It is, in fact, more plausible that the probability of each firm resetting its price will rise with the general level of inflation, since the costs of not doing so will most certainly rise with this general level. In this case, firms will reset their prices far more frequently when general inflation is 10 percent per month than when general inflation is 1 percent per month [see Ball *et al.* (1988) for a formal analysis of such a mechanism].¹² Similarly, in the quadratic adjustment cost model of Rotemberg (1982), the cost of adjusting prices depends only on the absolute change in prices $|p_{it} - p_{it-1}|$, independently of the general level of inflation.

An alternative way of specifying price adjustment costs which ensures long-run neutrality is to suppose that the costs depend on the deviation of price changes from the general level of inflation, i.e.:

$$\text{adjustment costs} = (b_p / 2) [(p_{it} - p_{it-1}) - (p_{t-1} - p_{t-2})]^2 \quad (20)$$

If we use adjustment costs as described in (20), equation (15) is unchanged, but the definition of \hat{p}_{it} is now given by:

$$\hat{p}_{it} = p_{it}^* - p_t^e + b_p / q (f \Delta p_{t+1}^e - \Delta p_t^e) - b_n h / q a^2 (f \Delta (y_{dit+1}^e - a_{it+1}^e) - \Delta (y_{dit}^e - a_{it}^e)) \quad (21)$$

Then the aggregate equation (19) becomes:

$$\begin{aligned} \Delta p_t = & f E_{t-1} \Delta p_{t+1} + q / b_p E_{t-1} \ln m_t^* + q / b_p E_{t-1} (mc_t - p_t) - (b_n h f / b_p a) E_{t-1} \Delta n_{t+1} \\ & + (b_n h / b_p a) E_{t-1} \Delta n_t + n_t \end{aligned} \quad (19a)$$

¹² Dotsey, King and Wolman (1999) show how to endogenise the probability of price changes.

This has exactly the same form as (19), except that it has the property that in the long run, inflation is independent of real factors, although they influence the change in inflation.

2.3 An open-economy dynamic pricing model

In order to estimate equations (19) and (19a), we must first specify the equilibrium price mark-up on marginal cost, \mathbf{m}^* . The equilibrium mark-up depends on the demand elasticity facing the firm [see (6) and the following discussion] and this will in turn depend on the extent of competition faced by the firm in the product market. There are three factors which are worth noting in this context. First, the extent of product market competition may be subject to long-term secular shifts arising, for example, from changes in the rigor of anti-trust regulation or the extent of trade barriers. These we denote by z_{pt} . Second, the extent of product market competition may, in an open economy, depend on the weakness of foreign competition, which we measure by $(p_t^w - p_t)$, where p_t^w is the world price of domestic GDP. So, for example, if the world price of domestic GDP decreases because of an appreciation of the domestic currency, then domestic firms may face increased competition and will tend to lower their equilibrium mark-up. Finally, the equilibrium mark-up may be affected by the overall state of the domestic economy. For example, in the model of oligopolistic collusion developed by Rotemberg and Saloner (1986), price wars tend to break out in booms leading to a reduced mark-up at such times. By contrast, the model of secret price-cutting due to Green and Porter (1984), has price wars breaking out in slumps.¹³ Either way, this suggests that we should allow the equilibrium mark-up to be influenced by the state of the business cycle which we capture by $(y_t - y_t^*)$, the deviation of output from trend (i.e. the output gap). Note that this term is to capture cyclical effects on the equilibrium mark-up which are over and above the cyclical effects on the actual mark-up that we have derived explicitly in our model and arise from price stickiness. Overall, therefore, we may specify the equilibrium mark-up by:

¹³ A comprehensive overview of these models may be found in Chapter 6 of Tirole (1988).

$$\ln \mathbf{m}_t^* = \mathbf{m}_0 + z_{pt} + \mathbf{m}_1 (y_t - y_t^*) + \mathbf{m}_2 (p_t^w - p_t) \quad (22)$$

Turning to the real marginal cost $(mc_t - p_t)$, from equation (5), this is given by:

$$mc_t - p_t = -\ln \mathbf{a} + w_t + n_t - y_t - p_t = -\ln \mathbf{a} + s_{Lt} \quad (23)$$

where s_{Lt} is the (log) share of labour. This remains true even when we take account of material inputs, so long as we remain strictly within the Cobb-Douglas framework. However, as Bentolila and Saint-Paul (1999) explain, this may have to be adjusted for the real price of imports, $(p_{mt} - p_t)$, if we assume more general technologies. For example, if the import requirement of gross output is rising at the margin, marginal costs will be increasing in the real price of imports (see the Technical Appendix). So in general we have:

$$mc_t - p_t = -\ln \mathbf{a} + s_{Lt} + \mathbf{m}_3 (p_{mt} - p_t) \quad (24)$$

The operational equations (22) and (24) maybe substituted into (19) and (19a) to generate the basis of estimated models. These are as follows. Equation (19) generates:

$$\mathbf{p}_t = \mathbf{a}_0 + \mathbf{f}E_{t-1}\mathbf{p}_{t+1} + \mathbf{q}/b_p E_{t-1}z_{pt} + \mathbf{q}\mathbf{m}_1/b_p E_{t-1}(y_t - y_t^*) + \mathbf{q}\mathbf{m}_2/b_p E_{t-1}(p_t^w - p_t) + \mathbf{q}/b_p E_{t-1}s_{Lt} + \mathbf{q}\mathbf{m}_3/b_p E_{t-1}(p_{mt} - p_t) - (b_r \mathbf{h} \mathbf{f} / b_p \mathbf{a}) E_{t-1} \Delta n_{t+1} + (b_r \mathbf{h} / b_p \mathbf{a}) E_{t-1} \Delta n_t + n_t \quad (25)$$

whereas equation (19) generates:

$$\Delta \mathbf{p}_t = \mathbf{a}_0 + \mathbf{f}E_{t-1}\Delta \mathbf{p}_{t+1} + \mathbf{q}/b_p E_{t-1}z_{pt} + \mathbf{q}\mathbf{m}_1/b_p E_{t-1}(y_t - y_t^*) + \mathbf{q}\mathbf{m}_2/b_p E_{t-1}(p_t^w - p_t) + \mathbf{q}/b_p E_{t-1}s_{Lt} + \mathbf{q}\mathbf{m}_3/b_p E_{t-1}(p_{mt} - p_t) - (b_r \mathbf{h} \mathbf{f} / b_p \mathbf{a}) E_{t-1} \Delta n_{t+1} + (b_r \mathbf{h} / b_p \mathbf{a}) E_{t-1} \Delta n_t + n_t \quad (26)$$

3. An empirical investigation

We now confront the models of inflation described in the previous section with UK data. For this purpose we estimate equations like (25) and (26) or slight modifications of those equations. To deal with the expectation terms in equations (25) and (26), we estimate the models using the generalised method of moments (GMM). We prefer this method to alternative instrumental variable regression methods because, by exploiting orthogonality conditions between some function of the parameters in the model and a set of instrumental variables, it is typically more efficient and robust. For comparison, however, we also report some estimates obtained using the standard instrumental variables (IV) estimator.

All data are quarterly time series over the period 1972 Q3 – 1999 Q2.¹⁴ Inflation, the dependent variable in our regressions that we indicate by $\mathbf{p}_t = p_t - p_{t-1}$, is measured by the first difference of the GVAD.

We do not model z_{pt} since we do not possess a time series measure of average product market competition. However, we consider two alternative measures of the output gap. The first, $(y_t - y_t^*)$, is derived by calculating deviations of the log of real output from its Hodrick-Prescott (HP) trend (with the smoothing parameter set at 1,600). The second measure, $(y_t - \hat{y}_t)$, is obtained as a residual from an estimate of potential output based on a Cobb-Douglas production function. n_t is our measure of employment, given by the log of subordinated employment plus employment of the self-employed. w_t represents *per capita* wages, and is calculated as the difference between the log of the total compensation of employees, adjusted to account for the labour remuneration of the self-employed, and n_t . We have two marginal cost adjustment variables: (i) Δoil_t denoting the change in the real price of oil, where oil_t is the log of the oil spot price in US dollars first adjusted for the log of the sterling/US dollar bilateral exchange rate, and then deflated via the GVAD; and (ii) $rpm_t = pm_t - p_t$, a measure of the relative price of imports, where pm_t

¹⁴ The Data Appendix provides more details about the series we use.

is equal to the log of the ratio of total imports at current prices to total imports at constant prices. We give oil a special status in imports because its price is subject to such enormous fluctuations.

We measure the weakness of foreign competition with com_t . This is given by the log of the ratio of an index of M6 (that is, the G7 countries, excluding the UK) export prices (goods and services), which uses the weights employed to construct the effective exchange rate, and the GVAD, p_t . Finally, s_{Lt} and s_{Lt}^* indicate the labour share adjusted for self-employment contributions with and without adjustment for public-sector considerations, respectively, as described in section 1.

We estimate equation (25) as it is. However, to estimate equation (26) we note that it may be rewritten as follows.

$$\mathbf{p}_t = \frac{1}{1+\mathbf{f}} \left[\mathbf{a}_0 + \mathbf{f}E_{t-1}\mathbf{p}_{t+1} + \mathbf{p}_{t-1} + \mathbf{q}/b_p E_{t-1} z_{pt} + \mathbf{q}\mathbf{m}_1/b_p E_{t-1} (y_t - y_t^*) + \mathbf{q}\mathbf{m}_2/b_p E_{t-1} (p_t^w - p_t) + \right. \\ \left. \mathbf{q}/b_p E_{t-1} s_{Lt} + \mathbf{q}\mathbf{m}_3/b_p E_{t-1} (p_{mt} - p_t) - (\mathbf{b}_n \mathbf{h} \mathbf{f} \mathbf{b}_p \mathbf{a} \mathbf{f})_{t-1} \Delta_{t+1} - \psi (\mathbf{b}_n \mathbf{h} \mathbf{b}_p \mathbf{a} \mathbf{f})_{t-1} \Delta_t + \hat{\mathbf{n}}_t \right] \quad (26a)$$

where $\hat{v}_t = v_t + \mathbf{f}(\mathbf{p}_t - E_{t-1}\mathbf{p}_t)$.

Equation (26a) has the same structure as (25), with the exception that \mathbf{p}_{t-1} is included and the coefficients on $E_{t-1}\mathbf{p}_{t+1}$ and \mathbf{p}_{t-1} now sum to unity.

3.1 Estimation results

We now present results of the estimation of equations (25) and (26a). To obtain GMM estimates of these equations, we have written down the moment conditions as orthogonality conditions between an expression including the parameters in the equations and a set of instrumental variables x_t . More specifically, under rational expectations, equation (25) defines the set of orthogonality conditions:

$$E_t \left\{ \left[\mathbf{p}_t - \begin{bmatrix} \mathbf{a}_0 + \mathbf{f}\mathbf{p}_{t+1} + \mathbf{q}/b_p z_{pt} + \mathbf{q}\mathbf{m}_1/b_p (y_t - y_t^*) + \mathbf{q}\mathbf{m}_2/b_p (p_t^w - p_t) + \mathbf{q}/b_p s_{Lt} + \\ \mathbf{q}\mathbf{m}_3/b_p (p_{mt} - p) - (b_h \mathbf{f}/b_p \mathbf{a}) \Delta n_{t+1} + (b_n \mathbf{h}/b_p \mathbf{a}) \Delta n_t \end{bmatrix} x_t \right] \right\} = 0$$

Similarly, the set of orthogonality conditions associated with equation (26a) is given by:

$$E_t \left\{ \left[\mathbf{p}_t - \frac{1}{1+\mathbf{f}} \begin{bmatrix} \mathbf{a}_0 + \mathbf{f}\mathbf{p}_{t+1} + \mathbf{p}_{t-1} + \mathbf{q}/b_p z_{pt} + \mathbf{q}\mathbf{m}_1/b_p (y_t - y_t^*) + \mathbf{q}\mathbf{m}_2/b_p (p_t^w - p_t) + \\ \mathbf{q}/b_p s_{Lt} + \mathbf{q}\mathbf{m}_3/b_p (p_{mt} - p) - (b_h \mathbf{f}/b_p \mathbf{a}) \Delta n_{t+1} + (b_n \mathbf{h}/b_p \mathbf{a}) \Delta n_t \end{bmatrix} x_t \right] \right\} = 0$$

Throughout, we use the same vector of instruments x_t . This includes five lags of inflation, the labour share (either s_{Lt} or s_{Lt}^* depending on which one appears in the regression), the output gap [either $(y_t - y_t^*)$ or $(y_t - \hat{y}_t)$, again depending on which of these two measures features in the regression], employment, wages, our measure of the weakness of foreign competition, the relative price of imports, and changes in the real price of oil.

Table 7a below presents estimates of the coefficients in the inflation equations (25) and (26a). The table is organised as follows. Column (a) reports estimation results for equation (25) using the adjusted labour share, s_{Lt}^* , and the output gap series based on the HP filter, $(y_t - y_t^*)$. Column (b) presents results for the unrestricted version of (26a) where the coefficients on \mathbf{p}_{t+1} and \mathbf{p}_{t-1} do not sum to unity, while column (c) reports the restricted version. Both columns (b) and (c) have the same variables for the share and the gap. Columns (d) to (f) report similar results for regressions that employ the alternative output gap series, based on a production function measure of potential output, $(y_t - \hat{y}_t)$.

In general, the parameter estimates in the table are consistent with the theoretical framework set out in section 2. In the baseline case estimates, column (a), inflation appears to be highly forward-looking, with a coefficient on expected inflation equal to 0.69 (t -value = 8.3). As we expected, the labour share term is strongly significant in this

equation entering with a coefficient of 0.16 (t -value = 4.9), which implies that a 1 per cent rise in the share of labour gives rise to a 0.16 percentage point increase in inflation. Additional cost elements, namely real import prices and the change in oil prices, are also important. The employment terms Δn_{t+1} and Δn_t are correctly signed and significant in this equation, suggesting that employment adjustment costs are relevant for pricing decisions and hence for inflation in the UK. On the other hand, in this baseline regression, both variables that capture variations in the equilibrium mark-up, i.e. the output gap and the term capturing the impact of foreign competition, are not significant.

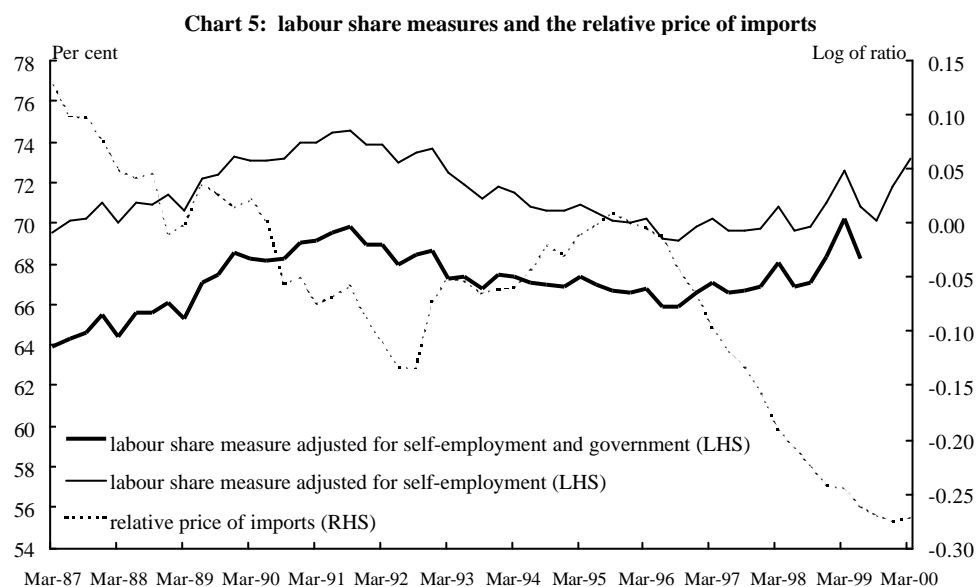
When we model adjustment costs so as to envisage long-run neutrality between real and nominal variables in the inflation equation, results are not greatly altered. In this respect, column (b) in Table 7a shows estimates of the unrestricted version of equation (26a), which introduces a lag term in inflation on the right hand side, but does not restrict the coefficients on the inflation terms to sum to unity. The coefficient on the lagged inflation term is significant but small (0.15, t -value = 4.3). Furthermore, when we impose the restriction that the weights on p_{t+1} and p_{t-1} sum to unity in column (c), the restriction is decisively rejected by the data (F -statistic = 34.8, p -value = 0.00), a result that is not new for estimates of UK inflation equations. Taken together, this implies that the persistence in inflation relative to output that is usually observed in UK data may be ultimately related to the sluggish movement in the share of labour as well as in other relevant variables, particularly contemporaneous and expected changes in employment, capturing employment adjustment costs. In general, introducing a lagged inflation term has the effect of making other regressors in the equation — notably the degree of foreign competition, the relative price of imports and the output gap — more significant. When the restriction is actually imposed [column (c)], the coefficients on expected and current employment, and foreign competition become even more significant, although the sign of the latter effect changes. The coefficient on the labour share remains positive when this restriction is imposed, but it is no longer significant. However, since the restriction is decisively rejected, significant weight should not be attached to the results in column (c).

We now turn to the estimation results obtained when we use the output gap measure that we derive as a residual of an estimate of a Cobb-Douglas production function, $(y_t - \hat{y}_t)$ [see columns (d), (e), (f)]. Three things emerge looking at these results. First, in line with results from the regressions using $(y_t - y_t^*)$, in these regressions the coefficient on labour share is always ‘correctly’ signed (positive) and strongly significant. In some instances, the coefficient on the share is even larger than in the former regressions [notably, in column (d) this coefficient is 0.26 compared with 0.16 in the similar regression of column (a) using $(y_t - y_t^*)$]. Second, as in the previous case, the restriction that the weights on expected and lagged inflation terms sum to one is rejected (F -statistic = 42.5, p -value = 0.00). Imposing this restriction [(column (f))] makes the foreign competition variable more significant relative to the case where this restriction is not imposed. Finally, the role of the output gap in all the regressions is not a strong one, although it is generally negative, indicating a degree of counter-cyclicity in the equilibrium mark-up.

Taken together, the results in Table 7a suggest that a theory-based pricing model that accounts for the impact of employment adjustment costs and openness of the economy in the pricing decisions of firms offers a good portrait of UK inflation over the period 1972 Q3 – 1999 Q2. Importantly, in line with the crude empirical evidence in section 2, and with the intuition behind our theory-based pricing model, the results in Table 7a suggest unanimously that the share of labour may contain information that is useful in predicting inflation in the UK.

In this sense, for instance, these regressions can help us explain why the labour share and inflation seem to have taken different paths since 1995 (see Chart 2). Focussing on our baseline equation [equation (a) in table 7a], we find no evidence of parameter instability

in this equation, so a structural break cannot explain this finding.¹⁵ In the framework of equation (a), a more plausible explanation for the divergent paths in the share and inflation seems instead to be the sharp fall of the relative price of imports (rpm_t), one of the regressors in equation (a), after 1995. As Chart 5 below shows, this paralleled the post-1995 surge in the labour share and thereby counterbalanced the inflationary effects of a rising labour share (rpm_t also enters with a positive coefficient in the equation). It is possible that if in the future the price of relative imports reverts to its pre-1995 level, other things being equal, the current level of the share may unleash inflationary pressures as it did in the past for comparable levels.



In Table 7b we present some further variants of the basic model (25). In column (g) we replace the adjusted share of labour, $s_{L_t}^*$, with s_{L_t} , the measure that refers to the whole economy rather than just the market sector. The equation appears to be well specified with the coefficient on the labour share remaining significant, although it is only around two-thirds the size of that in Table 7a, column (a). In column (h) of Table 7b, we simply

¹⁵ Results from a rolling regression of our baseline equation (column (a) in Table 7a) indicate that the coefficient on the labour share has been stable over the sample of estimation around a level of 0.15. This is confirmed by an analysis of residuals from this equation, which reveals that the fit has improved, rather than deteriorated, since 1995 Q2 (the standard deviation of the residuals over the 1972 Q3 –1995 Q2 period is actually higher than that computed over the whole sample period (0.0117 compared with 0.0108)).

check that the pre-whitening procedure we use in GMM is not generating major changes in the results. So here we remove it, and, as can be seen, the basic structure remains broadly unchanged. However, it is worth noting that in column (h), the coefficients on com_t and rpm_t are almost exactly the opposite of those in column (a). This is perhaps due to the broad similarity between these two variables (see the Data Appendix) which obviously makes it difficult separately to identify their individual coefficients.

Turning to columns (i) and (j) of Table 7b, we report standard IV estimates using both measures of the output gap. While the overall structure of the estimates is similar to the corresponding GMM equations [columns (a) and (d) of table 7a], the coefficients are generally less well determined, as might be expected given that the GMM estimator should be more efficient. Nevertheless, the labour share coefficient remains significant and is of the same order of magnitude.

The impact of the Single European Act on UK inflation

The remaining columns of Table 7b consider the longer term stability of the equilibrium mark-up of prices on marginal cost. With the signing of the Single European Act in the UK in 1986, a procedure for progressively removing internal barriers within the European Single Market was put in train. This steadily increased the degree of product market competition facing UK firms, so one potentially interesting issue is whether or not there is any evidence of a fall in the equilibrium mark-up over this period. In column (k) we address this issue by including a dummy variable (D_t) which takes the value of 1 after 1990 Q1. In the estimation, this dummy appears significantly negative, indicating that the mark-up may have been consistently smaller after this point. In fact, a similarly-constructed dummy starting at any point between mid-1987 and mid-1991 is significant, but not outside that interval. So there is no evidence, for example, of a shift in the constant term following the introduction of inflation targeting in 1992. However, there is weak evidence of an increase in the labour share coefficient around this time for, in column (l), we allow this coefficient to change (using a linear spline) and the increase in the coefficient has a t -value of 1.5.

Table 7a – Explaining UK Inflation [based on (25) and (26a)]

Dependent variable: $p_t = \Delta p_t$

	(a)	(b)	(c)	(d)	(e)	(f)
<i>Constant</i>	-0.66 (4.9)	-0.70 (6.8)	-0.08 (0.7)	-1.07 (8.0)	-0.95 (7.8)	-0.34 (2.8)
p_{t+1}	0.69 (8.3)	0.48 (9.3)	0.73 (18.9)	0.44 (7.7)	0.46 (8.1)	0.68 (16.1)
p_{t-1}	-	0.15 (4.3)	0.27 (7.2)	-	0.10 (2.3)	0.32 (7.7)
$(y_t - y_t^*)$	0.022 (0.4)	0.08 (1.9)	-0.05 (1.2)	-	-	-
$(y_t - \hat{y}_t)$	-	-	-	-0.031 (1.0)	-0.035 (1.4)	-0.059 (2.0)
<i>com_t</i>	-0.001 (0.1)	-0.012 (1.5)	0.032 (3.6)	-0.021 (1.5)	-0.019 (1.7)	0.047 (3.5)
s_{Lt}^*	0.16 (4.9)	0.17 (6.8)	0.020 (0.7)	0.26 (8.1)	0.23 (7.8)	0.080 (2.8)
<i>rpm_t</i>	0.017 (1.5)	0.036 (3.2)	-0.042 (3.8)	0.061 (3.6)	0.054 (3.8)	-0.050 (3.3)
Δoil_t	-0.024 (8.0)	-0.022 (8.2)	-0.024 (10.4)	-0.023 (7.6)	-0.024 (8.5)	-0.029 (10.4)
Δn_{t+1}	-1.04 (3.5)	-0.78 (3.2)	-1.04 (3.7)	-1.12 (3.2)	-0.89 (2.6)	-1.42 (4.0)
Δn_t	1.27 (4.2)	0.89 (3.2)	1.18 (4.4)	1.80 (4.5)	1.54 (4.2)	1.89 (4.7)
Estimation method	GMM	GMM	GMM	GMM	GMM	GMM
χ^2 test of over-identifying restrictions	$\chi^2(31) = 18.6$	$\chi^2(30) = 18.2$	$\chi^2(30) = 18.6$	$\chi^2(31) = 15.4$	$\chi^2(30) = 15.6$	$\chi^2(30) = 17.3$
Adjusted R^2	0.484	0.583	0.523	0.470	0.515	0.435
Test of the restriction	-	-	F(1,98)=34.8	-	-	F(1,98)=42.5

Table 7b – Explaining UK Inflation [variants of (25)]

Dependent variable: $p_t = \Delta p_t$

	(g)	(h)	(i)	(j)	(k)	(l)
<i>Constant</i>	-0.47 (3.0)	-0.41 (2.0)	-0.69 (2.2)	-0.62 (2.1)	-0.40 (2.8)	-0.28 (2.5)
p_{t+1}	0.79 (10.0)	0.65 (5.8)	0.41 (2.2)	0.35 (1.9)	0.70 (9.5)	0.78 (18.3)
$(y_t - y_t^*)$	-0.021 (0.4)	0.016 (0.3)	0.16 (1.4)	-	-0.001 (0.0)	0.08 (2.0)
$(y_t - \hat{y}_t)$	-	-	-	0.086 (1.3)	-	-
com_t	-0.001 (0.2)	0.024 (2.2)	-0.003 (0.2)	-0.017 (0.7)	0.016 (1.8)	0.014 (1.7)
s_{Lt}^*	-	0.10 (2.0)	0.17 (2.2)	0.15 (2.2)	0.10 (2.8)	0.067 (2.5)
s_{Lt}	0.11 (3.0)	-	-	-	-	-
Rpm_t	0.011 (1.1)	-0.008 (0.5)	0.041 (1.4)	0.056 (1.7)	-0.017 (1.2)	-0.013 (1.2)
Δoil_t	-0.015 (5.7)	-0.010 (2.4)	-0.021 (2.9)	-0.023 (3.3)	-0.019 (7.0)	-0.016 (6.3)
Δn_{t+1}	-0.84 (2.4)	-1.21 (4.0)	-0.48 (0.7)	-0.66 (1.0)	-0.99 (3.6)	0.38 (1.6)
Δn_t	1.07 (3.2)	1.31 (4.2)	0.30 (0.5)	0.45 (0.6)	0.99 (3.4)	-0.23 (0.9)
D_t	-	-	-	-	-0.004 (2.6)	-
$s_{Lt}^* (post 1992)$	-	-	-	-	-	0.057 (1.5)
Estimation method	GMM	GMM*	IV	IV	GMM	GMM
χ^2 test of over-identifying restrictions	$\chi^2(31) = 17.7$	$\chi^2(31) = 16.9$	na	na	$\chi^2(30) = 19.0$	$\chi^2(30) = 18.6$
Adjusted R^2	0.514	0.514	0.598	0.599	0.532	0.497

Notes to Tables (7a) and (7b):

- (i) Absolute t -values in parentheses. Sample period 1972 Q3 – 1999 Q2.
- (ii) Instruments include lags $t-1, t-2, \dots, t-5$ for the variables $\mathbf{p}_t, (y_t - y_t^*)$ [or $(y_t - \hat{y}_t)$], $s_{L_t}^*$ [or s_{L_t}], $com_t, rpm_t, \Delta oil_t, n_t,$ and w_t .
- (iii) The GMM method uses pre-whitening, whereby a preliminary VAR(1) is estimated to “soak up” the correlation in the moment conditions. Column (h) reports an equation where pre-whitening is not used (denoted GMM*).
- (iv) $(y_t - y_t^*)$ is the output gap based on deviations from a Hodrick-Prescott trend. $(y_t - \hat{y}_t)$ is based on deviations from a Cobb-Douglas based measure of potential output. n_t is employment, w_t is the *per capita* wage, $s_{L_t}^*$ and s_{L_t} are variants of the labour share (see section 1), rpm_t is the real price of imports, com_t is real world prices, and oil_t is the real price of oil. \mathbf{p}_t is inflation, the first difference of the natural log of the gross value added price deflator. $D_t = 1$ for $t > 1990$ Q1, zero otherwise.
- (v) In column (l), the variable $s_{L_t}^* (post\ 1992)$ takes the value $(s_{L_t}^* - s_{L_t}^* (1992Q3))$ for $t \geq 1992$ Q3 and zero otherwise.

4. Conclusions

We have examined the relationship between labour's share and the rate of inflation in the context of a simple dynamic model of firms pricing behaviour (the "new Keynesian Phillips curve"). This is based on an adjustment cost model, which includes costs of adjusting both prices and employment. Our main conclusions are as follows.

(i) The share of labour in the UK (corrected for self-employment) has been stationary over the last 30 years.

(ii) The share of labour has a significant impact on inflation, given the output gap. This is true both within a VAR framework and in the context of a forward-looking Phillips curve. A one per cent increase in labour's share generates a 0.16 percentage point rise in inflation, *ceteris paribus*.

(iii) Our theoretical and empirical findings on the UK dynamic pricing relationship also help us understand why the surge in the labour share observed since 1995 did not have significant effects on inflation. This is due to the 'compensating' impact of falling relative import prices over that period, which also matter for that relationship. It follows that if the relative price of imports reverts to its pre-1995 levels, a high level of the share as the one currently observed may eventually drive up in inflation, other things being equal.

(iv) Finally, there is some evidence that the equilibrium price mark-up on marginal cost fell at the end of the 1980s as the European Single Market was being completed.

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Data Appendix

The following variables are used in the regressions estimated in this paper.¹⁶

$s_t = \ln[(\text{HAEA}_t/\text{ABML}_t)*100]$ where HAEA_t is the compensation of employees, including the value of social contributions payable by the employer, and ABML_t is gross value added (current prices) measured at basic prices, excluding taxes less subsidies on products.

$s_{L_t} = \ln[\{(\text{HAEA}_t * A_t)/\text{ABML}_t\} * 100]$, where $A_t = (\text{E}_t + \text{SE}_t)/\text{E}_t$. From June 1978, E_t is given by BCAJ_t , the number of employee workforce jobs (seasonally adjusted), while SE_t is given by DYZN_t , the number of self-employment workforce jobs (seasonally adjusted). Prior to June 1978, data for these two series are only available for the second quarter of each year: these observations have been linearly interpolated in order to obtain observations for E_t and SE_t for the other quarters.

$s_{L_t}^* = \ln[\{((\text{HAEA}_t - \text{NMXS}_t) * A_t)/(\text{ABML}_t - \text{GGGVA}_t)\} * 100]$, where NMXS_t is compensation of employees paid by the general government. The series GGGVA_t is a measure of the part of gross value added attributable to the general government. From 1987 Q1 to 1997 Q4, this is given by NMXN_t , which is general government gross value added. This series is not available for other quarters, so to derive the series GGGVA_t for these quarters, it is assumed that NMXN_t grows at the same rate as the sum of its two largest components, NMXS_t and NMXV_t , the general government gross operating surplus. Together these two components account for around 98 per cent of NMXN_t throughout the sample period.

$p_t = \ln[(\text{ABML}_t/\text{ABMM}_t)*100]$, where ABMM_t is gross value added (constant prices) measured at basic prices, excluding taxes less subsidies on products. Two measures of inflation are considered: $p_t = p_t - p_{t-1}$ and $p_{4t} = p_t - p_{t-4}$. Two corresponding measures

¹⁶ Four letter codes refer to series produced by the UK Office for National Statistics.

of the change in inflation are also considered: $\Delta \mathbf{p}_t = \mathbf{p}_t - \mathbf{p}_{t-1} = (p_t - p_{t-1}) - (p_{t-1} - p_{t-2})$
and $\Delta_4 \mathbf{p}_{4t} = \mathbf{p}_{4t} - \mathbf{p}_{t-4} = (p_t - p_{t-4}) - (p_{t-4} - p_{t-8})$.

$y_t = \ln(\text{ABMM}_t)$. Applying a Hodrick-Prescott filter to this series provides a statistical measure of trend output, y_t^* , and of the output gap, $(y_t - y_t^*)$. An alternative measure of the output gap, $(y_t - \hat{y}_t)$, is obtained as a residual from an estimate of potential output based on a Cobb-Douglas production function, as described in Bank of England (1999, p.27).

$n_t = \ln(E_t + SE_t)$. $\Delta n_t = n_t - n_{t-1}$. $w_t = \ln[((\text{HAEA}_t * A_t) / (E_t + SE_t)) * 100]$.

$oil_t = \ln[(\text{PETSPOT}_t / \text{AJFA}_t) * 100] - p_t$, where PETSPOT_t is the oil spot price (average of Brent Crude, West Texas and Dubai Light) in \$US, with data obtained from Bloomberg. The series AJFA_t is the sterling - \$US exchange rate. $\Delta oil_t = oil_t - oil_{t-1}$.

$com_t = \ln[(\text{WPX}_t / \text{EER}_t) * 100] - p_t$, where WPX_t is the weighted average of export prices for the G7 countries (excluding UK), with effective exchange rate weights applied. Export price data have been obtained from the IMF's *International Financial Statistics* data base and from Datastream. The series EER_t is the UK's nominal effective exchange rate and is also obtained from the *International Financial Statistics* data base.

$rpm_t = \ln[(\text{IKBI}_t / \text{IKBL}_t) * 100] - p_t$, where IKBI_t is total imports (current prices), and IKBL_t is total imports (constant prices).

Chart A: Dependent and explanatory variables used in regressions

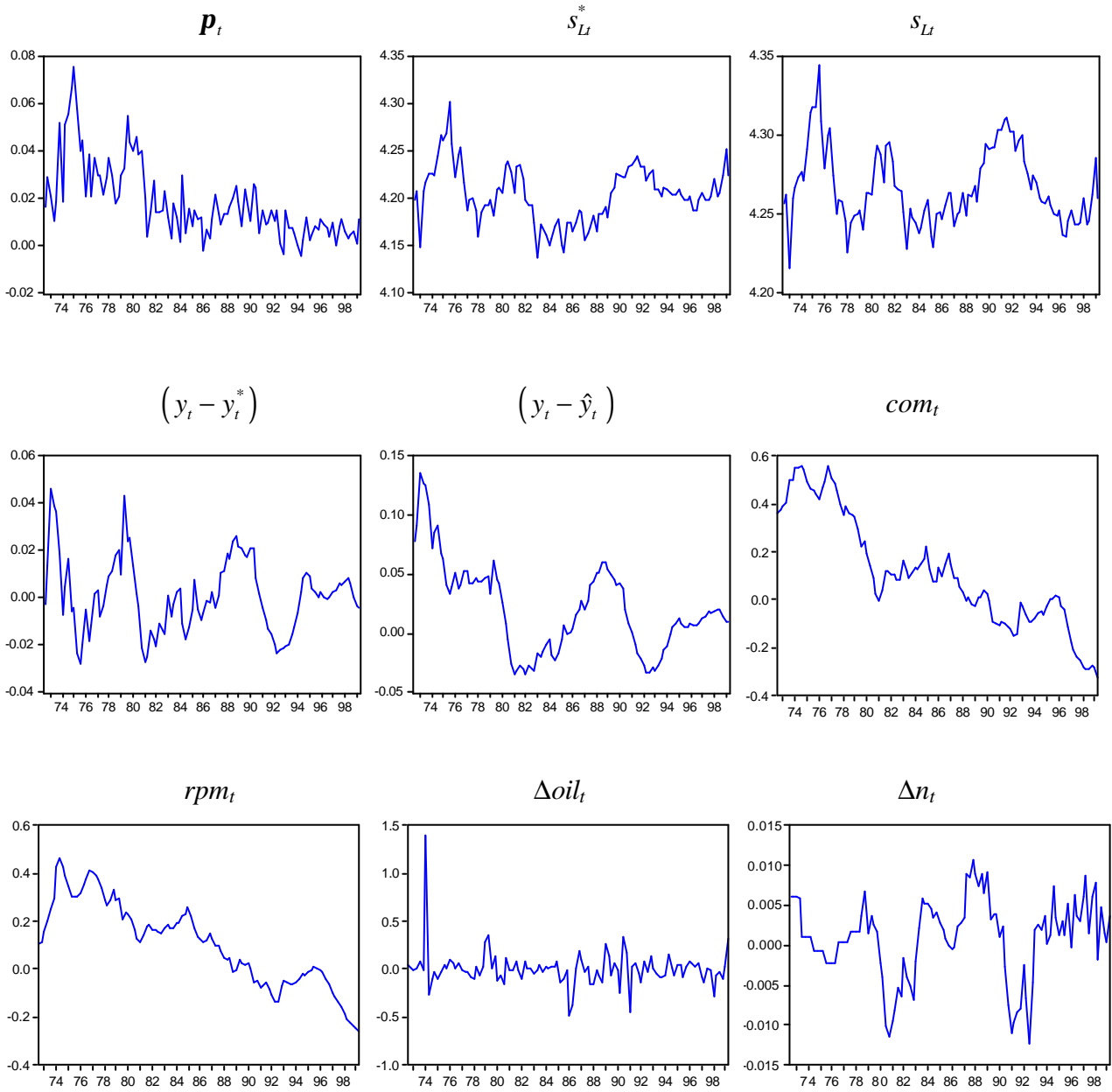


Table A: ADF tests for model variables			
Variable	ADF statistic	Specification	H₀: Unit root
s	-3.066105	Trend, intercept	Accept
Δs	-3.443830***	No trend, no intercept	Reject
s_{Lt}	-3.129561**	No trend, intercept	Reject
Δs_{Lt}	-3.534816***	No trend, no intercept	Reject
s_{Lt}^*	-2.683510*	No trend, intercept	Reject
Δs_{Lt}^*	-3.480881***	No trend, no intercept	Reject
p_t	-1.930334	Trend, intercept	Accept
\mathbf{p}_t	-3.827583**	Trend, intercept	Reject
$\Delta \mathbf{p}_t$	-5.782957***	No trend, no intercept	Reject
\mathbf{p}_{4t}	-2.888511	Trend, intercept	Accept
$\Delta_4 \mathbf{p}_{4t}$	-3.220575***	No trend, no intercept	Reject
y_t	-2.087793	Trend, intercept	Accept
Δy_t	-2.719457***	No trend, no intercept	Reject
$(y_t - y_t^*)$	-3.484631***	No trend, no intercept	Reject
$\Delta(y_t - y_t^*)$	-5.021502***	No trend, no intercept	Reject
$(y_t - \hat{y}_t)_t$	-2.107960**	Trend, intercept	Reject
$\Delta(y_t - \hat{y}_t)$	-4.034647***	No trend, no intercept	Reject
com_t	-3.277054*	Trend, intercept	Reject
Δcom_t	-5.069141***	Trend, intercept	Reject
rpm_t	-4.111657***	Trend, intercept	Reject
Δrpm_t	-4.991285***	No trend, no intercept	Reject
oil_t	-2.858666	Trend, intercept	Accept
Δoil_t	-5.893210***	No trend, no intercept	Reject
n_t	-2.737882	Trend, intercept	Accept

Δn_t	-3.069719***	No trend, no intercept	Reject
w_t	-2.480260	Trend, intercept	Accept
Δw_t	-3.637233**	Trend, intercept	Reject
<p>The sample period for each regression is 1972 Q3 – 1999 Q2, except for $\Delta(y_t - y_t^*)$ where the sample period is 1972 Q4 – 1999 Q2. A lag length of four is used for each regression. One, two and three asterisks are used to indicate that the null hypothesis of a unit root is rejected at the ten, five and one per cent significance levels respectively.</p>			

Technical Appendix

Role of the price of materials in marginal cost

Suppose technology has the form:

$$Y_G = AK^{1-a}N^a \quad (\text{A1})$$

and the material input requirement depends only on Y_G and is given by $m(Y_G)Y_G$, $m' > 0$.

The idea here is that to produce a given amount of gross output, Y_G , we require quantities of capital and labour satisfying (A1) plus material inputs, M , satisfying:

$$M = m(Y_G)Y_G \quad (\text{A2})$$

$m'(Y_G)$ could well be positive because as output rises for given K , less and less efficient machines are used so the per unit material input per unit of output goes up.

Now consider short-run variations (i.e. K fixed). The cost of producing gross output, C_G , is given by:

$$C_G(Y_G) = WN + P_m m(Y_G)Y_G \quad (\text{A3})$$

where W is the wage and P_m is the price of materials. So the corresponding marginal cost, MC_G , is:

$$MC_G = W \partial N / \partial Y_G + P_m m'(Y_G)Y_G + P_m m(Y_G)$$

or

$$MC_G = \frac{WN}{\mathbf{a}Y_G} + P_m m(Y_G) \mathbf{e}_m + P_m m(Y_G) \quad (\text{A4})$$

using (A1) and noting that $\mathbf{e}_m = m'(Y_G) Y_G / m(Y_G)$, the elasticity of unit material input with respect to gross output.

Now consider the connection between gross output and value added. If Y is value added at price P , then from (A1), the numerical quantity of value added is the same as the numerical quantity of gross output, so:

$$Y = Y_G \quad (\text{A5})$$

Then, by the definition of value added:

$$PY = P_G Y_G - P_m M \quad (\text{A6})$$

where P_G is the price of gross output. Then (A4), (A2) imply:

$$PY_G = P_G Y_G - P_m m(Y_G) Y_G$$

and so:

$$P = P_G - P_m m(Y_G) \quad (\text{A7})$$

Now define the marginal cost of producing value added as MC . The super-normal profit generated by producing an extra unit of output is $P_G - MC_G = P - MC$, since it must be the same either way because $Y = Y_G$. (Note, under competition, both are zero).

Consequently:

$$\begin{aligned}
MC &= P - P_G + MC_G \\
&= -P_m m(Y_G) + \frac{WN}{\mathbf{a}Y_G} + P_m m(Y_G) \mathbf{e}_m + P_m m(Y_G) && \text{(from A7, A4)} \\
&= \frac{WN}{\mathbf{a}Y} + P_m m(Y_G) \mathbf{e}_m && \text{(from A5)}
\end{aligned}$$

So:

$$\begin{aligned}
\frac{MC}{P} &= \frac{WN}{\mathbf{a}PY} + m(Y_G) \mathbf{e}_m \left(\frac{P_m}{P} \right) \\
&= \frac{s_L}{\mathbf{a}} + m(Y_G) \mathbf{e}_m \left(\frac{P_m}{P} \right) && \text{(A8)}
\end{aligned}$$

Derivation of equation (26a):

Starting from (26),

$$\begin{aligned}
\Delta \mathbf{p}_t &= \mathbf{a}_0 + \mathbf{f}E_{t-1} \Delta \mathbf{p}_{t+1} + \mathbf{q} / b_p E_{t-1} z_{pt} + \mathbf{q}\mathbf{m}_1 / b_p E_{t-1} (y_t - y_t^*) + \mathbf{q}\mathbf{m}_2 / b_p E_{t-1} (p_t^w - p_t) + \\
&\quad \mathbf{q} / b_p E_{t-1} s_{Lt} + \mathbf{q}\mathbf{m}_3 / b_p E_{t-1} (p_m - p) - (b_n \mathbf{h}\mathbf{f} / b_p \mathbf{a}) E_{t-1} \Delta n_{t+1} + (b_n \mathbf{h} / b_p \mathbf{a}) E_t \Delta n_t + \mathbf{n}_t
\end{aligned}$$

$$\begin{aligned}
\mathbf{p}_t - \mathbf{p}_{t-1} &= \mathbf{a}_0 + \mathbf{f}E_{t-1} \mathbf{p}_{t+1} - \mathbf{f}E_{t-1} \mathbf{p}_t + \mathbf{q} / b_p E_{t-1} z_{pt} + \mathbf{q}\mathbf{m}_1 / b_p E_{t-1} (y_t - y_t^*) + \mathbf{q}\mathbf{m}_2 / b_p E_{t-1} (p_t^w - p_t) + \\
&\quad \mathbf{q} / b_p E_{t-1} s_{Lt} + \mathbf{q}\mathbf{m}_3 / b_p E_{t-1} (p_m - p) - (b_n \mathbf{h}\mathbf{f} / b_p \mathbf{a}) E_{t-1} \Delta n_{t+1} + (b_n \mathbf{h} / b_p \mathbf{a}) E_t \Delta n_t + \mathbf{n}_t
\end{aligned}$$

$$\begin{aligned}
\mathbf{p}_t &= \mathbf{a}_0 + \mathbf{f}E_{t-1} \mathbf{p}_{t+1} - \mathbf{f}E_{t-1} \mathbf{p}_t + \mathbf{p}_{t-1} + \mathbf{q} / b_p E_{t-1} z_{pt} + \mathbf{q}\mathbf{m}_1 / b_p E_{t-1} (y_t - y_t^*) + \mathbf{q}\mathbf{m}_2 / b_p E_{t-1} (p_t^w - p_t) + \\
&\quad \mathbf{q} / b_p E_{t-1} s_{Lt} + \mathbf{q}\mathbf{m}_3 / b_p E_{t-1} (p_m - p) - (b_n \mathbf{h}\mathbf{f} / b_p \mathbf{a}) E_{t-1} \Delta n_{t+1} + (b_n \mathbf{h} / b_p \mathbf{a}) E_t \Delta n_t + \mathbf{n}_t
\end{aligned}$$

$$\begin{aligned}
\mathbf{p}_t + \mathbf{f}\mathbf{p}_t &= \mathbf{a}_0 + \mathbf{f}E_{t-1} \mathbf{p}_{t+1} + \mathbf{f}(\mathbf{p}_t - E_{t-1} \mathbf{p}_t) + \mathbf{p}_{t-1} + \mathbf{q} / b_p E_{t-1} z_{pt} + \mathbf{q}\mathbf{m}_1 / b_p E_{t-1} (y_t - y_t^*) + \mathbf{q}\mathbf{m}_2 / b_p E_{t-1} (p_t^w - p_t) + \\
&\quad \mathbf{q} / b_p E_{t-1} s_{Lt} + \mathbf{q}\mathbf{m}_3 / b_p E_{t-1} (p_m - p) - (b_n \mathbf{h}\mathbf{f} / b_p \mathbf{a}) E_{t-1} \Delta n_{t+1} + (b_n \mathbf{h} / b_p \mathbf{a}) E_t \Delta n_t + \mathbf{n}_t
\end{aligned}$$

$$\mathbf{p}_t = \frac{\mathbf{a}_0}{1+\mathbf{f}} + \frac{\mathbf{f}}{1+\mathbf{f}} E_{t-1} \mathbf{p}_{t+1} + \frac{1}{1+\mathbf{f}} \mathbf{p}_{t-1} + \frac{1}{1+\mathbf{f}} \left[\mathbf{q} / b_p E_{t-1} z_{pt} + \mathbf{q} \mathbf{m}_1 / b_p E_{t-1} (y_t - y_t^*) + \mathbf{q} \mathbf{m}_2 / b_p E_{t-1} (p_t^w - p_t) + \right. \\ \left. \mathbf{q} / b_p E_{t-1} s_{Lt} + \mathbf{q} \mathbf{m}_3 / b_p E_{t-1} (p_m - p_t) - (b_r \mathbf{h} \mathbf{f} / b_p \mathbf{a}^2 \mathbf{f}) E_{t-1} \Delta n_{t+1} + (b_r \mathbf{h} / b_p \mathbf{a}^2) E_t \Delta n_t \right] + \hat{\mathbf{n}}_t$$

where $\hat{v}_t = \frac{v_t + \mathbf{f}(\mathbf{p}_t - E_{t-1} \mathbf{p}_t)}{1+\mathbf{f}}$.