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**Discussion Paper** No.14

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# National accounts revisions and output gap estimates in a model of monetary policy with data uncertainty

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**Abstract**

This paper looks at some implications of data uncertainty for monetary policy. We combine national accounts data revisions with optimal control and filtering experiments on a calibrated model to discuss policy implications of price-versus-volume data uncertainty in GDP data for the United Kingdom. We find some degree of negative correlation between revisions to real GDP and GDP deflator data. We develop a methodology for estimating the output gap which takes account of the benefit of hindsight and decreasing measurement errors through time. Our optimal control experiments reveal that monetary policy makers would be led to place greater weight on nominal GDP data and correspondingly less weight on separate, uncertain estimates of prices and volume growth. However, estimates of real growth and also the output gap matter even when there is much uncertainty of this type. Our results also suggest that estimates of the level of inflationary pressure and nominal GDP data become more important when the economy is prone to inflationary overreactions to shifts in technological progress.

Key words: Data uncertainty, monetary policy, Kalman filter, optimal control.

JEL classification: E01, E52.

**Summary**

It is often the case that information available to policymakers, expressed as economic data, is implicitly or explicitly considered to be unchangeable. In practice, policymakers need to take account of data uncertainty and data revisions in their decision processes (this has been recently underlined in speeches by members of the Monetary Policy Committee in the United Kingdom; see Bell 2004, Lomax 2004, and Bean 2005). This paper looks at some implications of data uncertainty for monetary policy.

We combine national accounts data revisions with optimal control and filtering experiments on a calibrated model to discuss policy implications of price-versus-volume data uncertainty in GDP data for the UK. We show first of all that there is a degree of negative correlation between revisions to real GDP growth and revisions to changes in the GDP deflator. We then analyse how revisions in national accounts data can affect estimates of the output gap. We show how the benefit of hindsight means that past errors in the real growth rate and potential output growth do not fully cumulate onto the best estimate of the current output gap. We draw from this insight to develop a 'hindsight' or discounted output gap measurement model in which past errors decay. The point is that the discounted output gap estimate gives different (and possibly better) information than a mechanical estimate. This may be a more realistic view of how monetary policy makers measure the level of inflationary pressure.

Our optimal control experiments find that monetary policy makers, when adjusting their policy instrument to take account of data revisions, would be led to place greater weight on nominal GDP data and correspondingly less weight on separate, uncertain estimates of prices and volume growth as data uncertainty increases. However, we also find that estimates of the level of real activity cannot be ignored by the central bank and matter even when there is much uncertainty of this type. If demand shocks are more dominant, the policymaker will put more emphasis on inflation data. Larger supply shocks in our model do not have powerful second-round effects, so the weights of the indicators, including nominal GDP, do not change that much. When technological progress shocks are large, real GDP growth data are more important, and output gap data less so. This could be interpreted as a situation in which the policymaker uses real output growth data to assess what is happening to potential output growth.

We then modify the baseline model to take account of technological progress shocks that push potential output beyond a 'speed limit' and create inflation (or, on the contrary, that tend to make the output gap more negative and create temporary disinflations). Our simulations confirm that the output gap estimates and, as price/volume mismeasurement builds up, nominal GDP growth data become more important when the economy is prone to overreact to shifts in technological progress in this way.

## 1 Introduction

Monetary policy models are often characterised by a monetary authority setting a policy instrument (typically a short-term interest rate) in order to achieve certain objectives expressed in terms of macroeconomic variables, subject to available information on these and other variables. It is often the case that the information available to policymakers, expressed as economic data, is implicitly or explicitly considered to be unchangeable and perfectly measured. This is not a very realistic assumption. In practice policymakers need to take account of data uncertainty and data revisions in their decision processes.

In this paper we focus on a particular, possible case of data uncertainty — *price/volume uncertainty* — and discuss its implications for monetary policy. The general idea behind this type of uncertainty is that, given an estimate for economic activity in money or value terms, errors in the price component and the real output component of said estimate of nominal activity might cancel out, or cancel out even if the errors compound one another. Friedman and Schwartz (1982), for example, acknowledge that there are statistical as well as economic reasons to expect a negative correlation between real income and prices[(1)]. Recent national accounts revisions for the United Kingdom in Autumn 2003 showed an altered picture for real growth and prices, leaving money spending and output broadly unchanged (see King 2003).

We look at revisions to vintages of national accounts data for the UK on a quarterly basis from 1989 Q3 onwards. We find that revisions to quarterly real GDP growth and the quarterly change in the GDP deflator are, to some extent, negatively correlated. This implies that revisions in the allocation of an estimate of activity to its price and volume components are one source, among many, of data uncertainty and revisions. In the rest of the paper we use a calibrated model of monetary policy to derive some implications about this type of data uncertainty that we hope are reasonably robust.

A negative correlation between revisions to price and volume measures may arise because of the difficulty that statistical offices face in deriving, from aggregate output or expenditure national accounts data, separate estimates for real activity and prices or price deflators[(2)]. The separation of

(1) Friedman and Schwartz (1982) observe that in general two out of the three magnitudes (nominal income, real income and prices) are independently calculated, so, as a result, errors of estimates of real income and prices are negatively correlated.
(2) This does not usually apply to some sectors of the economy where output is measured directly in volume terms.

estimates of real activity from prices can be much more difficult when the relative prices of various goods and services are shifting. One can imagine a situation in which frequent changes in relative prices may lead to changes in deflator estimates and hence changes in volume estimates of activity without significant changes to the overall value (or current price) estimate of economic activity. This means that periods where relative prices change frequently, or where there are shifts in the long-run trends of relative prices, may be periods of particularly high price/volume data uncertainty.

The importance of taking changes in relative prices into consideration is certainly one of the main reasons behind the move towards annually chain-linked national accounts in many countries, including recently the United Kingdom (see, for example, Lynch 1996, Tuke and Reed 2001)(3). The introduction of annual chain-linking in the national accounts should, through the more frequent updating of weights and price deflators used in the compilation of the national accounts, to a certain extent address the problem of uncertainty about the split between (aggregate) real activity and the price level, although it is conceivable that particularly rapid changes in relative prices that occur, for example, between the period chosen as a price base and the present, could still have an effect.

In this paper we focus on two dimensions of the problem that data uncertainty of this type may pose for monetary policymakers: the first is how much to rely on nominal GDP data compared to real GDP data if the former are consistently less prone to revisions than the latter; the second is how much emphasis should be placed on data for real GDP *growth* as opposed to the *level* of real activity(4).

The first question loosely relates to a debate about whether nominal GDP can provide an intermediate monetary policy target when there are supply-side shocks with no second-round effects. Supply-side shocks with no second-round effects are rather like our price/volume 'measurement error shocks', in that they need no policy reaction and are likely to raise real volumes whilst lowering prices, but are offset in the nominal data.

(3) Before September 2003, the national accounts in the UK would be rebased every five years.
(4) Revisions also pose a problem for *ex-post* evaluation. Orphanides (1998, 2001) has observed that the practice of using estimates of monetary policy rules to evaluate monetary policy *ex-post* is complicated by the fact that these rules are generally estimated on the basis of the most recent available data. However, the most recent available data rarely, if ever, coincide with the information set available to the policymaker at the time his decisions are made. Consequently, there is a case for trying to evaluate policy based on a real-time data set, *i.e.* with the data, or at least an estimate of the data that were available to the policymaker at the time a certain policy was set.

This is *not* to say that we are testing the relevance of nominal GDP targeting. Our focus is very much on an inflation-targeting regime where nominal GDP data are but one in a set of indicators. In addition to this, we assume that data of high quality for the target measure of inflation (interpreted as a consumer price index) are available, and that nominal GDP data (when deflated by consumer price inflation) are considered as an alternative measure of real activity to real growth data or output gap level estimates(5).

The second question that we pose, when considering real GDP growth versus real GDP levels, is linked to the concept of the output gap, as a measure for gauging the amount of inflationary pressure in the economy. For example, Federal Reserve Board Governor Edward Gramlich suggested that policymakers should concentrate more on the *rate of change* of real variables than the *levels* of real variables under measurement error(6). We analyse how (price/volume) data uncertainty may affect estimates of the *level* of the output gap, and whether any mismeasurement is serious enough to lead policymakers to ignore output gap estimates but instead take into consideration growth rates of macroeconomic variables when implementing policy.

Our approach is to understand the implications of this type of data uncertainty within a structural dynamic model of the transmission mechanism. The mix of economic shocks expected to be hitting inflation and output determines which data source is more relevant. We focus on three types of economic shocks: price adjustment curve shocks, shifts in technological progress that raise potential output, and demand shocks. We also consider scenarios in which the shifts in technological progress are over-predicted by agents, and so can have inflationary consequences.

We find the following result: as (price versus volume) data uncertainty increases, monetary policymakers would place greater emphasis on nominal GDP data and, correspondingly, less emphasis on the separate, possibly very uncertain estimates of prices and volumes in interest rate setting. But our calibrations indicate that very uncertain estimates of real activity cannot be entirely disregarded: indeed, these real activity estimates matter even when data uncertainty is very high. We find, furthermore, that the more agents in the economy are prone to overanticipate shifts in potential output growth — a scenario which, as we will see, can be characterised with the presence of a 'speed limit' effect in the transmission mechanism — the more the output gap data

(5) Svensson (2001), for example, adopts this approach in discussing the role of money as an indicator in an inflation targeting regime.
(6) See Gramlich (1999).

or nominal GDP growth data become important.

The next section, section 2, begins by explaining how price/volume uncertainty can arise and explores what it can mean for monetary policy. It then goes on to provide some evidence of the scale of data revisions, by looking at a real-time data set for the UK for real and nominal GDP and estimates of output gap obtained using the aforementioned real-time data set. Section 3 presents our formal model of monetary policy, introduces data uncertainty into the model, and discusses some issues regarding monetary policy models and certainty equivalence. Section 4 presents results of experiments when our model is calibrated to fit to some extent the UK economy. Section 5 discusses the introduction of data uncertainty, and section 6 concludes.

## 2 Price/volume data uncertainty

We now look at revisions to national accounts data from our real-time data set. A negative correlation between revisions to real GDP and the GDP deflator would show that, to some extent, these revisions compensate one another. We then use the real GDP data from our data set to obtain real-time estimates of potential output and the output gap, considering how revisions to national accounts data relate to estimates of the output gap.

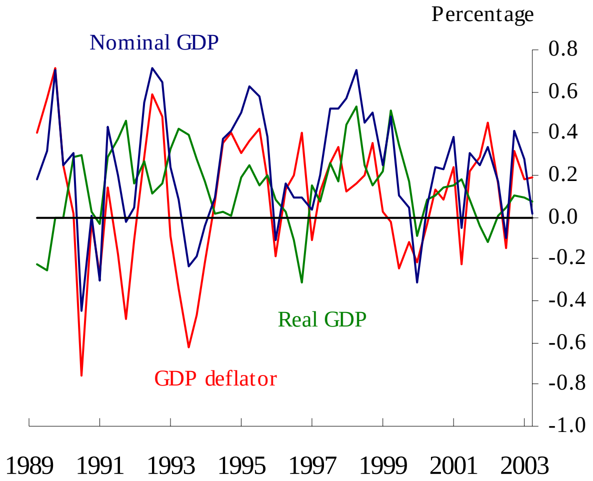
### *2.1 Evidence on price/volume mismeasurement and its effects on growth data*

Are effects of price/volume data uncertainty visible in any way? We should see, to some degree, a negative correlation between revisions to real GDP growth and revisions to changes in the overall price level, in this case the GDP deflator.

Chart 1 plots the overall revisions (defined as the revision between the first estimate for a given quarter, starting in 1989 Q3, and the estimate for that quarter in the latest vintage of data in our data set, where the data are available up to 2004 Q1) to the quarterly growth rates of GDP at constant prices and current prices[7]. Also included are the revisions to growth in the GDP deflator, calculated as the ratio of current price and constant price GDP. Table A shows summary statistics for the revisions.

(7) Since the introduction of annual chain-linking, in the UK ‘GDP at constant prices’ is referred to as ‘chained-volume measure’. We will often refer to the old definition.

Chart 1. Cumulative revisions to quarterly growth rates




Note: Centred 3 quarter moving average

| **Quarterly change in:** | | |
|---|---|---|
| Real GDP (1) | std dev | 0.35 |
| | abs dev | 0.27 |
| | mean | 0.13 |
| GDP deflator (2) | std dev | 0.70 |
| | abs dev | 0.54 |
| | mean | 0.11 |
| Nominal GDP growth | std dev | 0.67 |
| | abs dev | 0.54 |
| | mean | 0.24 |
| **Correlation between (1) and (2)** | | -0.35 |

**Table A: Summary statistics on cumulative revisions to the quarterly growth rates**

Certain patterns stand out. First of all, the mean revision to nominal GDP growth is larger than the mean revisions for real GDP and GDP deflator growth. Revisions to national accounts data, over a long period of time, will be due, among other things, to both updated source data and to methodological changes. It is not straightforward to separately identify the expected impact of these revisions on the three measures of nominal income, real income and prices. Another, more interesting, point to note from table A is that revisions to real GDP growth and to quarterly deflator inflation have to some extent been offsetting – the correlation between overall revisions to the quarterly change in the GDP deflator and quarterly real GDP growth is minus 0.35. This finding is consistent with some degree of price/volume uncertainty.

From chart 1 it also seems that deflator and nominal growth revisions are quite strongly correlated:

the correlation coefficient is in fact 0.875. A strong positive correlation between deflator and nominal growth revisions does not rule out a significant negative correlation between real growth and deflator inflation revisions. In fact, in appendix C we show in a simple decomposition how a positive (but still smaller than one) covariance between errors in nominal growth values and errors in deflator inflation is consistent with a negative correlation between measurement errors in deflation and real volume growth[(8)].

Here we have looked only at cumulative or total revisions, that is, the revision between the first estimate of the data and the latest vintage. Different results could be obtained by looking at revisions to growth rates between vintages that are a fixed distance apart, rather than just the revision between the first estimate and the latest vintage. For example, Akritidis (2003) shows that over a sample ranging from 1993 Q1 to 2000 Q4, the largest revisions to real GDP growth rates occur in the later stages of the national accounts compilation process, that is, after the publication of two Blue Books[(9)] following the first estimate of the data point under consideration.

We checked to see if the correlation coefficient between revisions to real GDP growth and the change in the GDP deflator changed when we used the data in our data set that had been through at least two Blue Books (meaning that we considered data up to 2001 Q4) and found that the correlation between quarterly GDP deflator inflation revisions and quarterly GDP growth revisions became only slightly more negative. We then calculated the same correlation coefficient over different revision windows, rather than between the first estimate and an end-point. The revisions were calculated over the following intervals: one, four, eight, 16, 20 and 24 quarters after the first release. Table B describes the results.

It is comforting that the correlation coefficients found are all negative except in one case.

### *2.2 Data revisions and the output gap*

We have discussed data uncertainty while looking at data revisions in a real-time data set with real GDP, nominal GDP and the GDP deflator. We can now look at the implications of this type of data uncertainty for the output gap, an economic concept which should indicate how much inflationary

(8) We are grateful to Charlie Bean for pointing this out to us.
(9) In the UK, the Blue Book, published by the Office for National Statistics (ONS), is the key annual publication for national accounts statistics. The Blue Book provides detailed estimates of national product, income and expenditure for the UK.

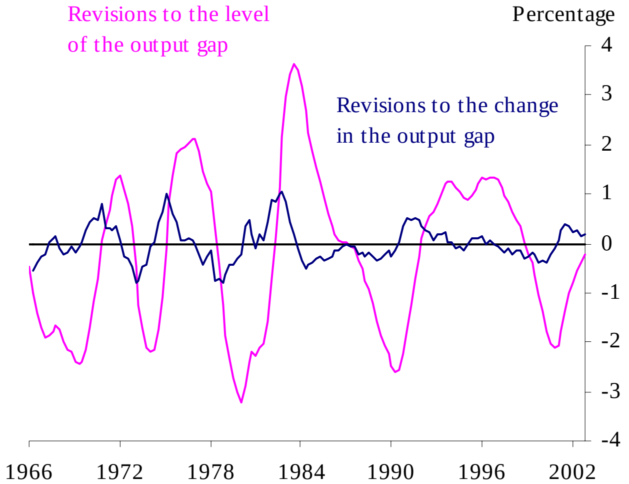
| No of quarter after first release | Correlation | sample (no of obs) |
|---|---|---|
| 1 | -0.142 | 1989Q3:2003Q3 (57) |
| 4 | -0.157 | 1989Q3:2002Q4 (54) |
| 8 | 0.118 | 1989Q3:2002Q1 (51) |
| 16 | -0.297 | 1989Q3:2001Q1 (43) |
| 20 | -0.222 | 1989Q3:1999Q1 (39) |
| 24 | -0.167 | 1989Q3:1989Q1 (35) |

**Table B: Correlation between fixed window revisions to the quarterly growth rates and deflator inflation**

pressure there is in the economy[(10)]. In what follows, the output gap is defined by the percentage difference between the level of real GDP and the level of potential output.

Some intriguing evidence about what we would expect on the relative extent of output gap uncertainty is provided by chart 2 below, taken from Walsh (2003a)[(11)]. According to his calculations for the United States, data revisions have affected official estimates of the output gap much more than they have influenced estimates of *changes* in the output gap. We can see this in the figure below. If we were to take these calculations at face value, we would conclude that estimates of the level of the output gap in the US are more uncertain than the growth rate of potential output.

Chart 2. Cumulative revisions to the output gap and the change in the output gap (US)

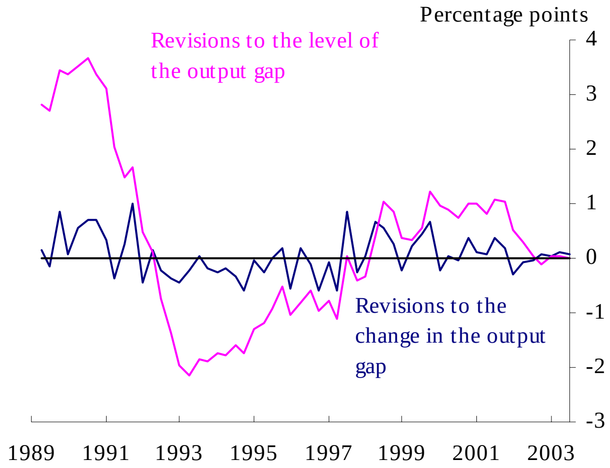



Source: Walsh (2003a)

(10) We do not address the separate issue of the choice between alternative indicators of the level of inflationary pressure in the economy, such as, for example, real marginal costs or the real rate gap versus the output gap.
(11) We are grateful to Carl Walsh for supplying us with the data for chart 2.

To derive comparable estimates for the UK we used our real-time data set, using each vintage of real GDP data in levels to calculate different vintages of estimates of potential output and hence output gaps (using a Hodrick-Prescott filter with a smoothing parameter of 1600). Chart 3 is our version of Walsh's chart for recent UK data, which confirms that on this side of the Atlantic too the revision in the output gap estimate tends to be much larger than revisions to the trend growth rate estimate, at least when judged from this method.

Chart 3. Cumulative revisions to the output gap and the change in the output gap (UK)




Source: Own calculations

Summary statistics are shown in table C. We can see that the unconditional standard deviation of the revision to the *level* of the output gap is more than five times that of the *change* in the output gap or to the potential output growth rate.

| Variable | | |
|---|---|---|
| Output gap (level) | std dev | 1.55 |
| | abs dev | 1.21 |
| | mean | 0.31 |
| Change in output gap | std dev | 0.38 |
| | abs dev | 0.29 |
| | mean | 0.05 |
| Trend growth | std dev | 0.27 |
| | abs dev | 0.20 |
| | mean | 0.08 |

**Table C: Summary statistics on cumulative revisions to the output gap**

Judged from these estimates, data revisions seem to affect the estimate of the *level* of the output gap much more than it affects estimates of *growth rates* (of real GDP, or of potential output, or the difference between the two). The main reason why mechanical estimates of the output gap shift so much with revisions is that they are derived by assuming that potential output is a smooth process. Therefore, by construction, potential output is unlikely to shift as much as real GDP.

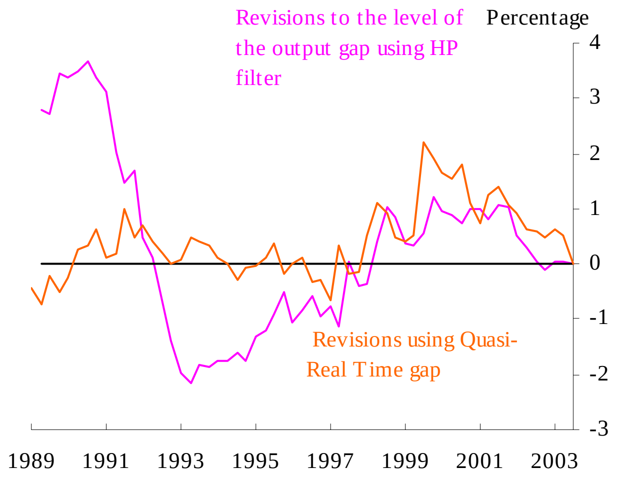
There are, however, three related reasons why mechanical estimates of the output gap level shift so much more than estimates of the change in trend growth. First, as McCallum and Nelson (2004) argue, the mechanical method may be incorrectly assuming that potential output is a smooth process. Data revisions which reflect news about potential output are not allowed to affect the estimation.

A second reason is that we are not allowing for the possibility that we may know more about potential output as time goes on. One reason is that, as Kapetanios and Yates (2004) assume, there may be some decay in the measurement error associated with data, as better information gets incorporated into later vintages of data.

Even abstracting from revisions, simply knowing how real GDP growth evolves in subsequent quarters helps to pin down the output gap more accurately. For example, in the literature on potential output estimation, methods such as the Hodrick-Prescott filter are often criticised for placing too much weight on observations at the end of the sample. The first estimate we produce using this method is therefore vulnerable to an 'end-of-sample' estimation error.

Orphanides and Van Norden (2001), using data for the US from 1965 Q1 to 2000 Q1, find that end-of-sample estimation errors are a major cause of real-time output gap uncertainty. They carried out this test by estimating what they call a *quasi real-time output gap*, where potential output for time $t$ is estimated using data only up to and including time $t$ on the *final vintage*. Subtracting the estimated gap in real time from this quasi real-time gap for each vintage gives us an estimate of what proportion of the cumulative revision is due to data revisions only. We carried out the same experiment on our UK data and found that the standard deviation of the cumulative revision was reduced by more than a half: it fell to 0.64 percent from 1.55 percent, confirming that the end-point problem is an important (but not the only) reason why mechanical filters fail to capture the true gap. Chart 4 plots the quasi-revision in the gap against the total revision in the gap and shows that the former is less variable. Table D shows some summary statistics.

Chart 4. Revisions to the UK output gap estimates




| Variable | | |
|---|---|---|
| Output gap (level) | std dev | 1.55 |
| | abs dev | 1.21 |
| | mean | 0.31 |
| Quasi real-time output gap (level) | std dev | 0.64 |
| | abs dev | 0.48 |
| | mean | 0.42 |

**Table D: Summary statistics on cumulative revisions to the output gap**

What does this tell us about our model of output gap measurement error? This confirms that the benefit of hindsight matters in output gap estimation; that knowing what will happen to real GDP in subsequent quarters matters in picking out the correct output gap today. Together, allowing for revisions and for hindsight argues for a model of output gap mismeasurement that should allow for some discounting.

A third problem with the method could be that it makes the assumption that only aggregate data are relevant. The mechanical approach takes no account of the influence disaggregate information would have on any view of the supply side and hence potential output. New information on components of labour inputs, for example, could help change our views about underlying productivity trends; new information on relative price movements could help change our view about values and volumes of goods.

Building on these intuitions, in what follows we assume that the process of estimating the level of potential output following a revision and a rebasing can be more sophisticated than subtracting a smoothed growth rate from real GDP growth data and then cumulating those changes. We call this a 'hindsight' or discounted output gap.

## 3 A monetary policy model

### *3.1 A monetary policy problem without data mismeasurement*

In this section we discuss how we can combine these stylised facts on data uncertainty with a more complex and realistic analysis of macroeconomic variables. We begin first by laying out a monetary policy problem in a world without measurement error, and then adapt the model to allow for data uncertainty.

Let us consider a two-equation model similar to the one in, for example, Lippi and Neri (2004). The first equation is a version of a Phillips curve in which inflation depends on expected future inflation $\left(E\left[\pi_{t+1} \mid I_t\right]\right)$ , past inflation $(\pi_{t-1})$, the lagged output gap $\left(y_{t-1} - y^*_{t-1}\right)$ and Phillips curve shocks $(\varepsilon_t)$. The output gap is written as the difference between actual and flexible-price output. We can write this Phillips curve as:

$$\pi_t = \alpha_0 E\left[\pi_{t+1} \mid I_t\right] + (1 - \alpha_0)\,\pi_{t-1} + \alpha_1\left(y_{t-1} - y^*_{t-1}\right) + \varepsilon_{t+1}. \qquad \textbf{(1)}$$

The second equation is an IS curve where output is an increasing function of future output, the previous period's output and a negative function of a measure of a real *ex-ante* interest rate:

$$y_t = \beta_0 E\left[y_{t+1} \mid I_t\right] + \left(1 - \beta_0\right) y_{t-1} - \beta_1 (i_t - E\left[\pi_{t+1} \mid I_t\right]) + \eta_{t+1} + \chi\omega_{t+1} \qquad \textbf{(2)}$$

where $\pi_t = p_t - p_{t-1}$ is the rate of inflation with $p_t$ being the log of the aggregate price level, $y_t$ is the log of the level of output and $i_t$ is the nominal interest rate, set by the monetary authority. The coefficient $\alpha_0$ lies between zero and unity and $\alpha_1$ and $\beta_1$ are positive. For reasons we discuss below, $\beta_0$ lies between 0.5 and unity. The terms $\varepsilon_t$ and $\eta_t$ are stochastic processes that represent supply and demand shocks. The term $\omega_t$ in **(2)** is a stochastic process that represents a technology shock (below we explain the reason for the presence of this technology shock). These processes are assumed to be autoregressive such that

$$\omega_t = \rho_\omega \omega_{t-1} + e_{\omega t};$$

$$\varepsilon_t = \rho_\epsilon \varepsilon_{t-1} + e_{\varepsilon t}$$

and

$$\eta_t = \rho_\eta \eta_{t-1} + e_{\eta t}$$

where $e_{\omega t}$, $e_{\varepsilon t}$ and $e_{\eta t}$ are all zero-mean, normally distributed shocks with variances $\sigma_{\omega}^{2}$, $\sigma_{\epsilon}^{2}$ and $\sigma_{\eta}^{2}$ and the parameters $\rho_{\omega}$, $\rho_{\varepsilon}$, $\rho_{\eta}$ all lying between zero and unity.

The final equation in our description of the transmission mechanism defines potential output as the flexible-price level of output. We begin by relating the flexible-price real interest rate to the technical progress growth term, $\omega_{t+1}$:

$$r_t^* = \left(\frac{\omega_{t+1}}{\beta_1}\right). \qquad \textbf{(3)}$$

We now justify **(3)** by showing it implies a familiar linear relation between the real interest rate and the rate of output growth in the steady state.

A first step is to obtain an expression for potential output that is consistent with the flexible-price concept. We eliminate all parameters associated with nominal rigidities from the IS curve, equation **(2)**. Setting $\chi = 0$ and $\sigma_{\eta}^{2} = 0$ we have[(12)]

$$y_t^* = \beta_0 E\left[y_{t+1}^* \mid I_t\right] + \left(1-\beta_0\right) y_{t-1}^* - \beta_1 r_t^*. \qquad \textbf{(4)}$$

Substituting **(3)** into **(4)** we have

$$y_t^* = \beta_0 E\left[y_{t+1}^* \mid I_t\right] + \left(1-\beta_0\right) y_{t-1}^* - \omega_{t+1}. \qquad \textbf{(5)}$$

We can now rewrite equation **(5)** as

$$\left(E\left[y_{t+1}^* \mid I_t\right] - y_t^*\right) = \frac{1-\beta_0}{\beta_0}\left(y_t^* - y_{t-1}^*\right) + \frac{\omega_{t+1}}{\beta_0} \qquad \textbf{(6)}$$

with $\omega_t = \rho_{\omega}\omega_{t-1} + e_{\omega t}$. To get the steady-state values, take $y^{bg} - y_{t-1}^{bg} \equiv y_t^* - y_{t-1}^* = \left(E\left[y_{t+1}^* \mid I_t\right] - y_t^*\right) = c$ and $\omega^{bg} = \omega_{t+1}$ to be constant in equation **(6)**, so that

$$y^{bg} - y_{t-1}^{bg} = c = \frac{\omega^{bg}}{\left(2\beta_0 - 1\right)} = \frac{\beta_1}{\left(2\beta_0 - 1\right)} r_{bg}^*. \qquad \textbf{(7)}$$

This then gives the linear relation between output growth and the real rate in the steady state, as one would expect. Note however that in order for this equation to describe output growth as a positive function of the steady-state real rate, it must be that $\beta_0 > 0.5$.

Now we can derive an IS curve in terms of the output gap. As equation **(5)** implies $\omega_{t+1} = -\left(y_t^* - \beta_0 E\left[y_{t+1}^* \mid I_t\right] - \left(1-\beta_0\right) y_{t-1}^*\right)$, we can substitute into **(2)** to give such an

(12) Our assumption is that the parameter $\chi$ is associated with a nominal rigidity.

expression

$$
\begin{aligned}
y_t - y_t^* \quad = \quad & \beta_0 \left( E \left[ y_{t+1} \mid I_t \right] - E \left[ y_{t+1}^* \mid I_t \right] \right) + \left( 1 - \beta_0 \right) \left( y_{t-1} - y_{t-1}^* \right) \\
& - \beta_1 ( i_t - E \left[ \pi_{t+1} \mid I_t \right] ) \\
& + \eta_{t+1} + (1 + \chi) \, \omega_{t+1}. \qquad \textbf{(8)}
\end{aligned}
$$

Using our definition of the real rate we can rewrite **(8)** as

$$
\begin{aligned}
y_t - y_t^* \quad = \quad & \beta_0 \left( E \left[ y_{t+1} \mid I_t \right] - E \left[ y_{t+1}^* \mid I_t \right] \right) + \left( 1 - \beta_0 \right) \left( y_{t-1} - y_{t-1}^* \right) \\
& - \beta_1 \left( ( i_t - E \left[ \pi_{t+1} \mid I_t \right] ) - r_t^* \right) \\
& + \eta_{t+1} + \chi \omega_{t+1}. \qquad \textbf{(9)}
\end{aligned}
$$

Looking at equation **(9)**, we can see that the technology growth shocks $\omega$ affect the output gap if and only if $\chi \neq 0$. A negative value of $\chi$ can be interpreted as the deflationary impact of a supply side improvement, such as a rise in labour productivity, or a fall in the natural rate of unemployment. This type of shock will lower the output gap, and through the output gap will have a downward effect on (future) inflation.

How do we interpret the case where $\chi > 0$? We can interpret this as an extra demand shock, over and above the $\eta$-shocks, that has a positive impact on the output gap, *i.e.* it captures an increase in overall demand, not (completely) matched by an increase in overall supply before triggering extra inflationary pressure. One interpretation of a positive $\chi$ is that it captures the inflationary impact of the output gap rising above a certain 'speed limit' (given by its constant balanced growth rate, or trend growth)[13]. Note that there is an important conceptual difference with the $\eta$-shocks, which affect actual output, rather than potential output. In our baseline case we assume that $\chi = 0$, while in section 5.2 we will relax this assumption.

(13) This is a different and additional motivation to Walsh's (2003b, 2003d) explanation of a 'speed limit' policy. See section 4.

The transmission mechanism is now summarised by equation **(1)** and equation **(8)**. The two equations express current inflation and output in terms of expected future inflation and the expected future output gap, lagged inflation and output gap, current interest rates and unobserved shocks only. To complete the set-up we need to describe the objectives of the monetary authority. We will assume that the policymaker seeks to minimise squared deviations of inflation from zero, and of output from its flexible-price level. This means that the instantaneous (expected) loss function will have the following form:

$$E\left[L_s \mid I_t\right] = \frac{1}{2}[E\left[(\pi_s)^2 \mid I_t\right] + \lambda E\left[\left((y_s - y_s^*\right)^2 \mid I_t\right]] \quad \textbf{(10)}$$

with $\lambda > 0$, and $s \geqslant t$.

This instantaneous loss function specified above implies the following intertemporal problem for the central bank:

$$\min_{i_t,..,\infty} \sum_{s=t}^{\infty} \kappa^{s-t} E\left[L_s \mid I_t\right], \text{ with } 0 < \kappa < 1 \quad \textbf{(11)}$$

subject to **(1)** and **(8)**[14].

In the next section we consider the situation in which equations **(1)** and **(8)** represent the true state of the economy, but that the policymakers operate under the constraint of only having imperfect information about the true values of real output, inflation and the output gap[15].

### *3.2 Allowing for mismeasured data*

In our set-up the policymaker has four *independent* sources of data estimates to learn about the state of the economy. These are (consumer price) inflation, real GDP data, the output gap and nominal GDP data. In this section we describe the measurement error in these sources. In what follows we will generally distinguish between the observed and true values of data; with the superscript ~ indicating the (mismeasured) observed data.

The first source of data is consumer price inflation data. We will assume that consumer price inflation is the measure of inflation targeted by the central bank, *i.e.* the inflation measure that

(14) If we multiply **(10)** by $1 - \kappa$, this formulation of the loss function is very similar to that used in, *e.g.*, Rudebusch and Svensson (1999), except for the absence of a term in the loss function that depends on the change in nominal interest rates. As $\kappa \longrightarrow 1$, the instantaneous loss function will approximate the weighted sum of the unconditional variances of inflation and the output gap.
(15) There is no informational asymmetry between the monetary authority and other agents in the economy.

appears in the loss function **(10)**. We will also assume that inflation defined by this consumer price measure (henceforth referred to as CPI inflation) is perfectly measured (or at least is never revised), so that the value observed by the policymaker and the true value coincide:

$$\widetilde{\pi}_t = \pi_t. \tag{12}$$

Despite the fact that CPI inflation is perfectly measured, in order to assess the amount of inflationary pressure in the economy, the policymaker needs estimates of other variables that define the transmission mechanism. But in our set-up, the data estimates of these variables are measured inaccurately.

Real GDP is measured imperfectly, and we interpret this as a price/volume error, as will be more clear below. We express real GDP in the following way:

$$\widetilde{y}_t - \widetilde{y}_{t-1} = y_t - y_{t-1} + \psi_t, \tag{13}$$

where $\psi_t$ is assumed to be a normally distributed variable with zero mean and variance $\sigma_{\psi}^2$.

Nominal GDP can be expressed as (approximately) real GDP plus the price level, measured by the GDP deflator. We assume that GDP deflator inflation deviates from CPI inflation because of two types of errors. First, there are deflator errors which only distort the relationship between deflator inflation data and true inflation. And second, there is the converse of the price/volume errors seen above. We express the difference between observed deflator inflation $\left(\widetilde{def}_t\right)$and true CPI inflation $(\pi_t)$ as:

$$\widetilde{def}_t - \pi_t = -\psi_t + \upsilon_t \tag{14}$$

with the deflator measurement error $\upsilon_t$ assumed to be a normally distributed variable with zero mean and variance of $\sigma_{\nu}^2$.

If we look at the sign of $\psi$, in the previous two expressions it is clear that we assume that revisions to real activity and the price level (as expressed by the GDP deflator) compensate one another. This implies that in current price terms, GDP growth is not affected by price/volume errors but only by the deflator errors $\nu$. Nominal GDP growth data in period $t$ is therefore:

$$\widetilde{n}_t = \pi_t + y_t - y_{t-1} + \upsilon_t. \tag{15}$$

The policymaker will use data estimates available to him to assess the amount of inflationary pressure in the economy. Real GDP growth data can be used as a 'speed-limit' indicator, with high rates of growth indicating a widening of the output gap, for example. In the context of our model, nominal GDP data deflated by CPI inflation can also serve as an indicator for movements in real activity. Finally, the policymaker can use estimates of the output gap level $(\widetilde{y}_{t+1} - \widetilde{y}^*_{t+1})$ to infer where the true output gap is. Given our assumptions about data measurement, how should we model output gap (level) uncertainty? How does data uncertainty in this context distort the policymaker's reading of the level of the output gap?

In contrast to the way in which potential output was estimated in the previous section — *i.e.* the filtering of real GDP data — we assume that policymakers do not just use real GDP data to formulate their view of potential output, but use information about a plethora of macroeconomic variables (for example, investment and the capital stock, labour market data, import prices and world demand). Some judgement (using a wider array of data and models) has to be made about whether the measurement error in real GDP carries over to potential output in such a way that it drops out of the output gap calculation, or if it is the case that it compounds the error in estimating the true level of inflationary pressure.

We can formalise these arguments into a model of output gap (mis)measurement with hindsight as follows. First we will describe how errors are introduced in the measurement of the output gap through the process by which potential output growth is estimated. Our model of potential growth measurement combines two popular approximations. A first approximation is that potential output growth is constant, as in the expression below:

Assumption 1. $\widetilde{y}^*_t - \widetilde{y}^*_{t-1} = c$

where $c$ is the constant steady-state growth rate. This means that the estimates for potential output are insensitive to price/volume measurement errors, but also do not allow for technological progress shocks to affect potential growth. The opposite alternative is that potential output growth follows actual output growth quite closely. This implies taking into account, for example, technological progress shocks — which would be a desirable feature — but also data uncertainty errors as well as monetary shocks. We can express this second assumption as

Assumption 2. $\widetilde{y}^*_t - \widetilde{y}^*_{t-1} = \widetilde{y}_t - \widetilde{y}_{t-1}$.

In what follows we assume that the approach chosen is a linear combination of the two procedures outlined above, with a parameter $\rho_1$ determining the weight. Ignoring the constant, we have:

$$\widetilde{y}_t^* - \widetilde{y}_{t-1}^* = (1-\rho_1)\left(\widetilde{y}_t - \widetilde{y}_{t-1}\right). \qquad \textbf{(16)}$$

This is in keeping with the spirit of the Hodrick-Prescott filter, where the smoothing parameter plays the role of $\rho_1$, in determining the trade-off between assuming the trend is linear and tracking the actual data. It seems plausible that $\rho_1$ tends to be greater than 0.5 as most potential output measurement techniques in practice rely more heavily on smoothing.

The measurement error in potential output growth is given by subtracting true potential output growth $\left(y_t^* - y_{t-1}^*\right)$ from this estimate of potential growth described in equation **(16)**:

$$\begin{aligned} & \widetilde{y}_t^* - \widetilde{y}_{t-1}^* - \left(y_t^* - y_{t-1}^*\right) \\ = \; & (1-\rho_1)\,\psi_t + (1-\rho_1)\left(y_t - y_{t-1} - y_t^* - y_{t-1}^*\right) - \rho_1\left(y_t^* - y_{t-1}^*\right). \end{aligned} \qquad \textbf{(17)}$$

Given that the measurement error in real output growth is equal to the price/volume error, that is, $\widetilde{y}_t - \widetilde{y}_{t-1} - (y_t - y_{t-1}) = \psi_t$, we can now write the measurement error in the change in the output gap as

$$\begin{aligned} & \widetilde{y}_t - \widetilde{y}_{t-1} - \left(\widetilde{y}_t^* - \widetilde{y}_{t-1}^*\right) - (y_t - y_{t-1}) + \left(y_t^* - y_{t-1}^*\right) \\ = \; & \rho_1\psi_t - (1-\rho_1)\left(y_t - y_{t-1} - y_t^* - y_{t-1}^*\right) + \rho_1\left(y_t^* - y_{t-1}^*\right). \end{aligned} \qquad \textbf{(18)}$$

We have specified how the *change* in the output gap is mismeasured. A final step is to derive a model of mismeasurement in the *level* of the output gap. If we were to take our stylised facts on output gap revisions from section 2 at face value, we would assume that predicting the current output gap is more difficult than predicting current potential output growth, as errors in measuring past growth rates cumulate onto the level. That would imply that the measurement error in the current output gap level would rise without bound. Rather than allow these errors to cumulate undiscounted onto the estimate of the output gap level, we find it more plausible to assume that the influence of an error in real GDP growth rates wanes with successive vintages. So past price/volume errors matter less for the current output gap than current errors. Formally we relate the mismeasurement in our discounted output gap to current and past price/volume errors by

$$\begin{aligned} \widetilde{y}_t - \widetilde{y}_t^* &= y_t - y_t^* \\ &+ \left( \sum_{s=0}^{s=t} \rho_0^s \left( \rho_1 \psi_{t-s} - (1-\rho_1)\left(y_t - y_{t-1} - y_t^* - y_{t-1}^*\right) + \rho_1 \left(y_t^* - y_{t-1}^*\right)\right)\right) \end{aligned} \quad \textbf{(19)}$$

with $1 > \rho_0 > 0$.

We can re-write equation **(19)** in autoregressive form as

$$\widetilde{y}_t - \widetilde{y}_t^* - y_t - y_t^* = \rho_0 \left(\widetilde{y}_{t-1} - \widetilde{y}_{t-1}^* - y_{t-1} - y_{t-1}^*\right) - (1-\rho_1)\left(y_t - y_{t-1} - y_t^* - y_{t-1}^*\right) + \rho_1 \psi_t + \rho_1 \left(y_t^* - y_{t-1}^*\right). \quad \textbf{(20)}$$

To check that the autoregression is a reasonable description of the output gap measurement error, we carried out some estimates on our real-time data set. We regress the cumulative revision in the mechanical output gap estimate on its past value with the sample from 1989Q3 to 2004Q1. This gives us an estimate of

$$\widetilde{y}_t - \widetilde{y}_t^* - \left(y_t - y_t^*\right) = \underset{(25.9)}{0.93} \left[\left(\widetilde{y}_{t-1} - \widetilde{y}_{t-1}^*\right) - \left(y_{t-1} - y_{t-1}^*\right)\right] + \underset{(-0.52)}{(-3 * 10^{-4})} \quad \textbf{(21)}$$

with t-statistics in brackets, an $R^2$ of 0.93, a standard error of 0.43%, and an LM test statistic for fourth order serial correlation at 4.8 (insignificant at 90%).

To complete our understanding of the discounted output gap mismeasurement, we need an expression for potential output to substitute in equation **(20)**. From the derivations following equation **(3)**, remember that we can rewrite equation **(5)** as

$$\left(E\left[y_{t+1}^* \mid I_t\right] - y_t^*\right) = \frac{1-\beta_0}{\beta_0}\left(y_t^* - y_{t-1}^*\right) + \frac{\omega_{t+1}}{\beta_0} \quad \textbf{(22)}$$

with $\omega_t = \rho_\omega \omega_{t-1} + e_{\omega t}$, but that in order for this equation to describe a constant growth rate where output growth is a positive function of the steady-state real rate, it must be that $\beta_0 > 0.5$.

Given this restriction we describe potential output by adopting the backward solution to the following rational expectation equation:

$$y_t^* = y_{t-1}^* + \frac{(1-\beta_0)}{\beta_0}\left(y_{t-1}^* - y_{t-2}^*\right) + \frac{1}{\beta_0}\omega_t. \quad \textbf{(23)}$$

This is a valid solution but not the only stationary solution to equation **(22)**, when $\beta_0 > 0.5$. In appendix D we show that other (more complicated) stationary growth rate solutions are possible. But we also show that the backward solution is similar to the only unique solution that would hold under a model with discounting, and on these grounds we retain equation **(23)** as our model of potential output growth.

To conclude, we can now lay out the common information set of policymakers and agents at time $t$:

$$I_t = \left\{ \begin{array}{c} \widetilde{\pi}_t, ..., \widetilde{\pi}_0,\ \widetilde{y}_t - \widetilde{y}_{t-1}, ..., \widetilde{y}_0 - \widetilde{y}_{-1}, \widetilde{n}_t, ...\widetilde{n}_0, \widetilde{y}_t - \widetilde{y}_t^*, ..., \widetilde{y}_0 - \widetilde{y}_0^* \\ \lambda, \beta_0, \beta_1, \alpha_0, \alpha_1, \chi, \rho_\omega, \rho_\varepsilon, \rho_\eta, \rho_0, \rho_1, \sigma_\omega^2, \sigma_\epsilon^2, \sigma_\eta^2, \sigma_\nu^2, \sigma_\psi^2. \end{array} \right\}.$$

Note that the only data the policymaker has are the current value and history of the four indicators. However the policymaker also has information on all the parameters of the model, including second-order information and also knowledge on the measurement error processes. In the next section we show how that information can be used to derive an optimal link between data and policy.

### *3.2.1 Some intuition on the nominal growth indicator*

We can interpret both real and nominal GDP data as indicators for the true output gap. Equation **(18)** shows that the effect that errors in real output growth have on the policymaker's estimate of the output gap is composed of both price/volume errors and technological progress shocks:

$$\begin{aligned} & \widetilde{y}_t - \widetilde{y}_{t-1} - \left(\widetilde{y}_t^* - \widetilde{y}_{t-1}^*\right) - y_t - y_{t-1} + \left(y_t^* - y_{t-1}^*\right) \\ = \ & \rho_1 \psi_t \\ & - (1-\rho_1)\left(y_t - y_{t-1} - y_t^* - y_{t-1}^*\right) \\ & + \rho_1 \left(y_t^* - y_{t-1}^*\right). \end{aligned} \quad \textbf{(24)}$$

Thus, the variance of prediction errors in using real growth data to predict the change in the output gap is

$$\begin{aligned}
&\rho_1^2 \left(\sigma_\psi^2 + E\left[\left(y_t^* - y_{t-1}^*\right)^2\right]\right) \\
&+ (1-\rho_1)^2 E\left[(y_t - y_{t-1} - y_t^* - y_{t-1}^*)^2\right] \\
&- (1-\rho_1)\,\rho_1 E\left[(y_t - y_{t-1} - y_t^* - y_{t-1}^*), \left(y_t^* - y_{t-1}^*\right)\right]. \qquad \textbf{(25)}
\end{aligned}$$

Similarly, by inserting equation **(19)** in equation **(15)** we can see that errors in nominal GDP growth data lead to errors in the estimates of the change in the true output gap because of GDP deflator measurement error and technological progress shocks, and also due to the price/volume error:

$$\begin{aligned}
\widetilde{n}_t - \left(\widetilde{y}_t^* - \widetilde{y}_{t-1}^*\right) - \widetilde{\pi}_t &= \upsilon_t - \left(y_t^* - y_{t-1}^*\right) + (1-\rho_1)\left(\widetilde{y}_t - \widetilde{y}_{t-1}\right) \\
&= (1-\rho_1)\left(y_t - y_{t-1} - y_t^* + y_{t-1}^*\right) \\
&\quad + (1-\rho_1)\left(y_t^* - y_{t-1}^*\right) \\
&\quad + (1-\rho_1)\,\psi + \upsilon_t. \qquad \textbf{(26)}
\end{aligned}$$

From equation **(26)**, the variance of prediction errors in the nominal GDP growth indicator is

$$\begin{aligned}
&(1-\rho_1)^2 E\left[\left(y_t - y_{t-1} - y_t^* + y_{t-1}^*\right)^2\right] \\
&+ (1-\rho_1)^2 \left(\sigma_\psi^2 + E\left[\left(y_t^* - y_{t-1}^*\right)^2\right]\right) \\
&+ \sigma_\nu^2 \\
&+ (1-\rho_1)^2 E\left[\left(y_t^* - y_{t-1}^*\right)\left(y_t - y_{t-1} - y_t^* + y_{t-1}^*\right)\right]. \qquad \textbf{(27)}
\end{aligned}$$

Subtracting equation **(25)** from equation **(27)**, we have

$$(1-2\rho_1)\left(\sigma_\psi^2 + E\left[\left(y_t^* - y_{t-1}^*\right)^2\right]\right) + \sigma_\nu^2 - (1-3\rho_1)\,E\left[(y_t - y_{t-1} - y_t^* - y_{t-1}^*), \left(y_t^* - y_{t-1}^*\right)\right].$$

When expressed in terms of the variance of the prediction of the true change in the output gap, the trade-off between nominal GDP growth and real GDP growth data depends therefore on the variances of deflator mismeasurement, of price/volume mismeasurement and of the growth in technological progress. (There is a term capturing the covariance between potential output growth and the change in the true output gap, but this matters only when overreaction leads to a ‘speed limit’ effect).

The more deflator mismeasurement there is, the more real growth data would be favoured. And the smoother it is assumed that potential output growth is, and hence the closer $\rho_1$ is to 1, the worse real GDP growth data would perform relative to nominal GDP growth data as an indicator of the change in the true output gap. If we can assume $\rho_1 > 0.5$ (as seems a fair reflection of popular practice) then we can also say that a greater price/volume error and a more volatile technological shock process would both imply that real GDP growth performs relatively worse as an indicator of the output gap.

### *3.3 Our other calibrations*

In order to carry out our experiments we need to calibrate the model for some values that broadly describe the UK economy. The parameter values we used are summarised in the following tables E and F, in the second column of each table.

The third column of each table contains calibrations and estimations from other authors, which refer not only to the UK, but also the euro area. It is clear that it is difficult to pin down values for even some of the most important parameters. A wide range of estimates are available, and estimates based on other countries may not be relevant for the UK. Nevertheless we hope that our parameters are not too far out of line with what others find plausible.

Note that we take the rate of discounting in our estimate of output gap measurement, $\rho_0$, to be 0.93, as this is the number we get from our econometric estimates of the mechanical output gap error in the previous section. Our calibration of $\rho_1$ cannot be estimated, and so we set it at 0.7 which we think gives a plausible trade-off between output gap measurement error and the real growth data measurement error; *i.e.* one which characterises smoothing as the more common practice.

| Description | Baseline value | Other calibrations/ estimations |
|---|---|---|
| Forward vs backward weight in Phillips curve ($\alpha_0$) | 0.4 | $(0.4)^b$<br>$(0.50)^c$, $(0.3)^d$, $(0.5)^f$ |
| Slope of Phillips curve ($\alpha_1$) | 0.3 | $(0.002)^d$, $(0.18)^f$ |
| Forward vs backward weight in IS curve ($\beta_0$) | 0.625 | $(0.6)^a$, $(0.25)^d$, $(0.6)^f$<br>$(0.25)^d$, $(0.56)^f$ |
| Interest rate multiplier ($\beta_1$) | 0.6 | $(0.7)^a$, $(0.08)^e$, $(0.06)^f$ |
| Technological progress shocks autoregression $\rho_\omega$ | 0.95 | $(0.95)^c$ |
| Price supply shocks autoregression $\rho_\varepsilon$ | 0 | |
| Demand shocks autoregression $\rho_\eta$ | 0 | |
| Standard deviation of price supply shocks ($\sigma_{e\varepsilon}$) | 0.31% | $(0.03)^d$<br>$(0.16)^f$ |
| Standard deviation of demand shocks ($\sigma_{e\eta}$) | 0.41% | $(0.41)^a$<br>$(0.39)^d$, $(0.18)^f$ |
| Standard deviation of technological progress shocks ($\sigma_{e\omega}$) | 0.9% | $(0.925)^e$, $(0.81)^f$ |
| Overreaction to technological progress shocks ($\chi$) | 0 | |
| [a] Banerjee and Batini (2003, table 3), [b] Batini, Jackson and Nickell (2000, table 7b, col i). | | |
| [c] Paez-Farrell (2003); [d] Lippi and Neri (2004); [e] Ellison and Scott (2000); [f] Smets (2003). | | |

**Table E: Table of Parameter Values (quarterly)**

| Description | Baseline value | Other calibrations/ estimations |
|---|---|---|
| Standard deviation of price/volume measurement error ($\sigma_\psi$) | 0.50% | $(0.24)^d$ |
| Standard deviation of deflator measurement error ($\sigma_\upsilon$) | 0.25% | $(0.26)^f$ |
| Elasticity of current price/volume error in determining current output gap error ($\rho_1$) | 0.7 | |
| Discounting of past errors in affecting current output gap error ($\rho_0$) | 0.93 | |
| Weight on output costs in society loss function ($\lambda$) | 0.8 | |
| Discount on welfare losses ($\kappa$) | 0.99 | |

**Table F: Table of Parameter Values 2 (quarterly)**

As a guide to interpreting our calculation, we can show that the unconditional variance of output gap errors is

$$E\left[\left(\left(\widetilde{y}_t-\widetilde{y}_t^*\right)-\left(y_t-y_t^*\right)\right)^2\right]=\frac{1}{1-\left(\rho_0\right)^2}E\left[\left(\widetilde{y}_t-\widetilde{y}_{t-1}-\widetilde{y}_t^*+\widetilde{y}_{t-1}^*-y_t+y_{t-1}+y_t^*-y_{t-1}^*\right)^2\right];$$

**(28)**

that is $\frac{1}{1-\left(\rho_0\right)^2}=7.4$ times greater than the variance of real growth mismeasurement from equation **(25)**[16].

### *3.4 The interaction of data uncertainty and economic uncertainty*

When looking at the monetary policy implications of data uncertainty it is important to distinguish between two different kinds of uncertainty in this model. One concerns the economic shocks in **(1)** and **(8)**, that is, $\varepsilon_t$, $\eta_t$ and $\omega_t$. This type of uncertainty is often referred to in the literature as *additive* uncertainty. These are unobservable at the time the policymakers set policy, their expected values are often zero and the optimal policy decision does not depend on the variance of these shocks. In fact, if the expected values of the shocks were zero, the coefficients of the policy rule would be the same as if the economy described by equations **(1)** and **(8)** were completely non-stochastic[17].

However, data uncertainty about the values of the *state* variables creates a form of non-additive uncertainty in the model. Aoki (2003), Swanson (2004) and Svensson and Woodford (2003) show that we can split our understanding of the non-additive data uncertainty problem into two stages. The first stage is a linear relation between the policy instrument (the nominal interest rate) and the expected values of unobserved targets. That first stage is certain-equivalent (and so independent of any information about the variances of the shocks). The second stage tells us how those expected values of the unobserved targets relate to the observed data. This second stage is *not* a certain-equivalent relation, in the sense that policymakers (and agents) use their beliefs about the noisiness of observed data when they choose how much emphasis to place on each indicator.

These two stages can, however, be calculated separately and then combined[18]. We use the MatLab programs developed by Gerali and Lippi (2003) and in appendix B below we show how

(16) Strictly speaking we should not be comparing the measurement error in the output gap estimate in predicting the level of the true output gap to the measurement error in real growth in predicting the change in the true output gap.
(17) See, for example, the discussion in Ljungqvist and Sargent (2000, ch.4).
(18) This is the case providing policymakers and agents share the same information set, the transmission mechanism in linear and that objectives are quadratic. See Svensson (2003) and Svensson and Woodford (2004).

we set up the model in a form consistent with their software and the model seen in Svensson and Woodford (2003): the first stage is derived as a solution to an optimal linear regulator problem and the second stage that links the observed data to expectations of inflation and output, is derived as a Kalman filter. We are interested in deriving measures of how the nominal interest rate is linked to the observed data. In this way, this framework can be used to inform us about the emphasis that should be placed on CPI inflation, real growth data, nominal GDP growth data and the estimated output gap in setting interest rates, when there is substantial uncertainty about the price/volume split.

That rule is however conditional on beliefs about variances, meaning that as policymakers' views about variance evolve, so will the rule change. To bring this out, we test the sensitivity of our rule to different assumptions about variances (the mix of shocks).

## 4 Results

### *4.1 Monetary policy under no data uncertainty*

As we discussed above, the results of our exercise can be split into two stages. The output from the first stage is an instrument rule linking nominal interest rates to the expected but unobserved values of variables. These coefficients are presented for two different types of policy setting. The first is related to discretionary monetary policy, whereby the central bank minimises the loss function subject to the constraints presented by the state of the economy. The resultant policy will be time-consistent. However, the presence of forward-looking elements in the framework we are analysing brings about a wedge between the optimal discretionary policy, and a policy that would be obtained if the central bank could credibly commit to a certain path for policy, because of the effect this commitment would have on private sector expectations (for a discussion, see, for example, Woodford 1999, Woodford 2000, Walsh 2003c, McCallum and Nelson 2004). One thing to note straight away is that this wedge between discretionary policy and policy with commitment is different from the one that underpinned the 'inflationary bias' in policymaking analysed in many papers, starting from Kydland and Prescott (1977) and Barro and Gordon (1983).

In the case of 'inflationary bias' monetary policy models, suboptimal policy outcomes were the result of the policymaker's attempts to expand real output levels above its potential level (or pushing unemployment below its natural rate)[19], and the private sector's realisation of this in a rational expectations framework. In the type of monetary policy we are analysing, the suboptimality of discretionary policy is brought about by the fact that the policymaker re-optimises in every period, without taking into account the effect of his actions on private sector expectations[20]. Another way of expressing this is (from Woodford 2000), is that it is the failure of making the conduct of monetary policy *history-dependent* that makes discretionary policy suboptimal[21]. It may be interesting to assess whether the dominance of commitment carries over to our case of price/volume data uncertainty.

Walsh (2003b, 2003d) discusses why indicators of rates of change might be important under these circumstances. He explains the role of 'speed limit' policies, demonstrating that a central bank concerned with stabilising inflation and the change of the output gap (instead of the level) can usefully impart some inertia to eliminate some of the relative losses of a discretionary (compared to a commitment) policy. However we should bear in mind that one circumstance when this matters is when there are persistent price adjustment curve shocks, which is not a feature of our calibrations.

In appendix B, we show the algebraic form of the first-stage instrument rule[22] as equation **(B-13)** in the case of discretion and as equation **(B-5)** in the case of commitment. Our simulation (in the baseline case) gives numerical values for the coefficients in the first stage as shown below first for discretion

$$\begin{aligned} i_t &= 0.93E\left[\pi_{t-1} \mid I_t\right] + 1.09E\left[\left(y_{t-1} - y^*_{t-1}\right) \mid I_t\right] + 1.55E\left[\varepsilon_t \mid I_t\right] \\ &\quad +1.67E\left[\eta_t \mid I_t\right] + 3.25E\left[\omega_t \mid I_t\right] \end{aligned} \quad \textbf{(29)}$$

(19) The reason for this being that the level of potential output (natural rate of unemployment) was too low (too high) from a social welfare perspective.
(20) Lippi and Neri (2004) note that, technically, under discretion, policy is a function of the state variables alone, while under commitment, policy also includes the Lagrange mulitipliers of the forward-looking variables (costate variables).
(21) A further motivation for the existence of an inflationary bias may be the existence of asymmetric preferences in the policymaker's (or in society's) objective function (see, for example, Cukierman and Gerlach 2003).
(22) See, for example, Svensson (2003) for a discussion on the difference between *instrument* rules, which express the policy instrument as a function of current and lagged predetermined variables, and *targeting* rules, which are equilibrium conditions involving the target variables only.

and then under commitment

$$
\begin{aligned}
i_t \quad = \quad & -6.78E\left[\pi_{t-1} \mid I_t\right] - 2.76E\left[\left(y_{t-1} - y^*_{t-1}\right) \mid I_t\right] - 11.29E\left[\varepsilon_t \mid I_t\right] \\
& +1.67E\left[\eta_t \mid I_t\right] + 3.25E\left[\omega_t \mid I_t\right] + 8.20E\left[\mu_{xt} \mid I_t\right] - 3.21E\left[\mu_{yt} \mid I_t\right] \qquad \textbf{(30)}
\end{aligned}
$$

where $\mu_{xt}$ and $\mu_{yt}$ are the costate variables associated with expected inflation and expected output respectively, and are defined in appendix B, equation **(B-5)** . The costate variables, or Lagrange multipliers, capture the marginal loss from having committed to react to past values of inflation and output gap (in expectation only: these values are unobserved). These factors are absent from the instrument rule under discretion, where no such commitment is possible.

### *4.2 Impulse responses of the model*

We can illustrate how well the model describes the responses of inflation, output and interest rates to each of three economic fundamental shocks: shocks to technological progress, the price shock and the real demand shock. We produce responses both under commitment and discretion.

The responses of inflation, the output gap and interest rates to a temporary one percentage point rise in technological progress are shown in the first column of chart 7. Interest rates track the natural rate, rising and then very slowly falling back to their former value, taking about five years. The slow return of interest rates is therefore consistent with our assumption that technological progress features a strong autocorrelation. Having said that, interest rates are at first slow to react and because of that the growth rate of output rises by slightly more on impact than does potential growth. A small and short-lived output gap opens up and leads to higher inflation. The charts do not show that the shock raises the potential growth rate, which reaches a peak of about 3% (on a quarterly rate) and then very slowly dies out. On the whole though, the inflationary effects of a rise in technological progress are small relative to the rise in output.

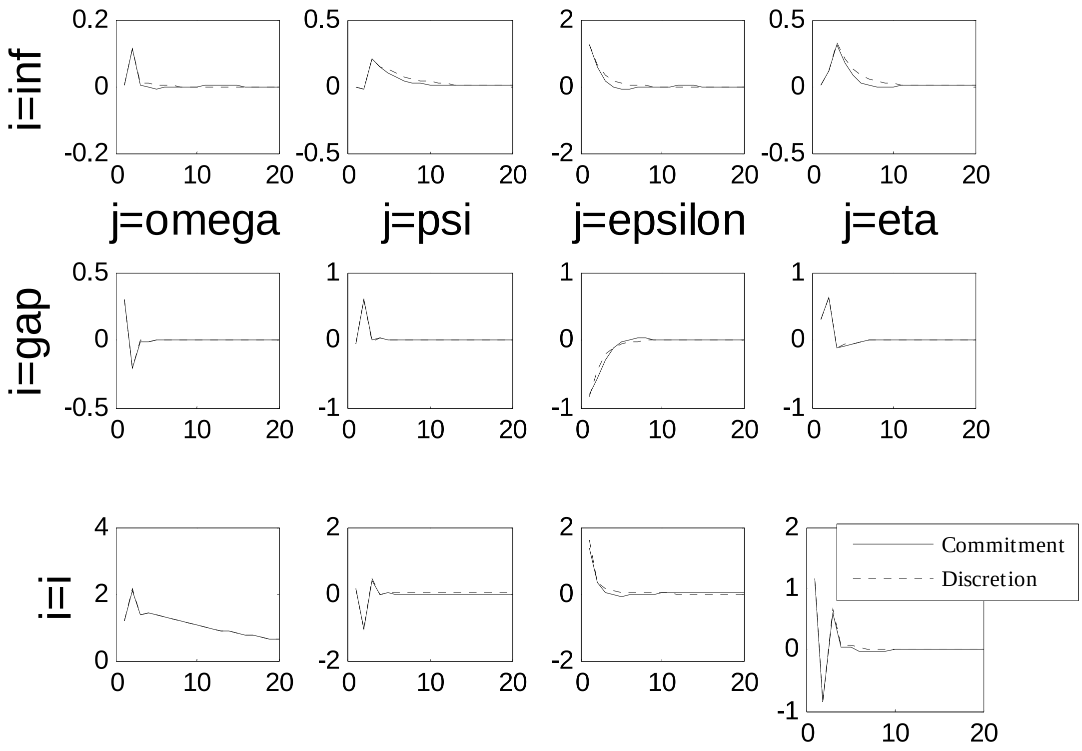
It should be remembered, however, that the simulations in chart 7 were carried out under the assumption that there is no overreaction to the technological progress, *i.e.* that $\chi = 0$. We relax this assumption in section 5.2 to show how technological progress can lead to higher inflation, and a worse trade-off, when there are bottlenecks in supply.

The price supply shock ($\varepsilon$, in the third column) is assumed to have no autocorrelation and dies out quickly. It raises inflation by about 1% (on a quarterly rate) immediately. The nominal interest rate rises too because policy does not accommodate the shock but rather reacts to it. The net effect is a higher *ex-ante* real rate and a fall in output (below potential, as potential output is invariant to this shock) of about 0.8%.

The responses to the real demand shock ($\eta$, in the fourth column) are also not dissimilar to those seen in other models. The shock raises the output gap by about 1% immediately and this feeds onto inflation a year later (by about 0.4%). Nominal interest rates are immediately raised to counter this inflationary impulse but then lowered as the inflationary pressure diminishes.

Column 2 in chart 7 describes the effect of a non-economic shock: a 1% rise in the measurement error $\psi$. It is interesting to note that qualitatively the responses are very similar to those that would result from a 'policy shock', *i.e.* interest rates rise, this subsequently drives down the output gap and lowers inflation. The policymaker here is being misled by a data measurement shock which initially has no impact on economic fundamentals, so perhaps it is not that surprising that the results obtained are similar to what other models display in the case of a shock to the policy rule.

Chart 7. Impulse response of variable i to 1pp temporary shocks in j under commitment and discretion




i=inflation, the output gap and nominal interest rate (quarterly)

We should remember that the responses to all shocks depend on the policymaker's priors with regard to the variances of disturbances. As a stark illustration, charts 8 and 9 plot the impulse responses of inflation and interest rates to a price/volume measurement shock under two very different cases[(23)]. The solid lines indicate cases where the model was solved with the policymaker believing there is no data measurement error at all, *i.e.* when the price/volume error comes as a complete surprise. The dashed lines show instead what happens if the central bank was prepared for very significant measurement error. They do not react at all to the out-turns of real GDP and the output gap estimates, in the knowledge that these data do not reveal any useful information.

(23) Under both cases, a discretionary policy is used.

Chart 8. Response of quarterly inflation rates to a 1pp price/volume error shock

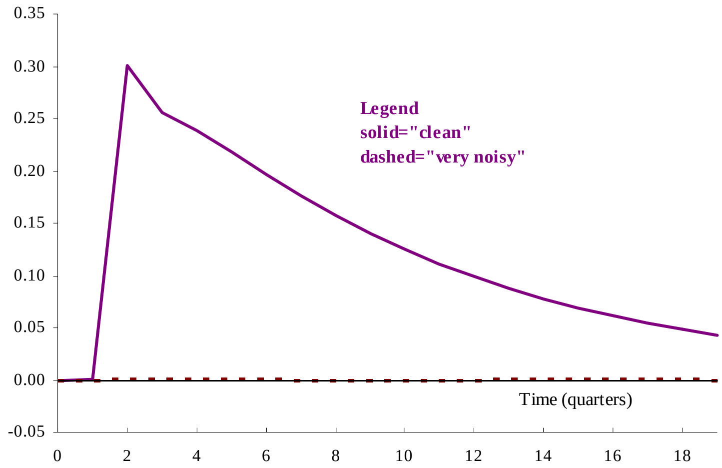
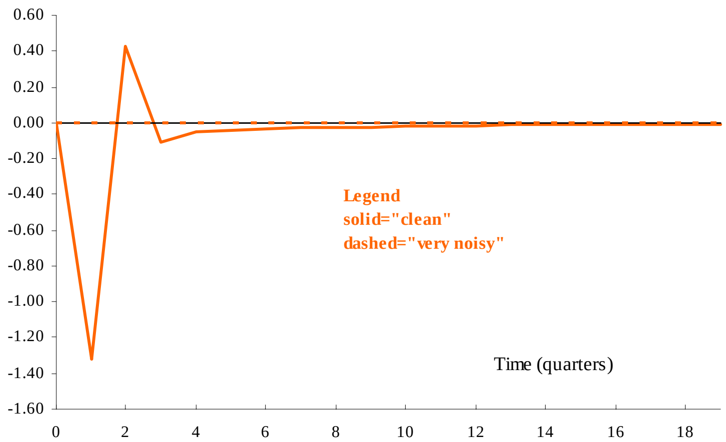



Chart 9. Response of quarterly interest rates to a 1pp price/volume error shock




## 5 Monetary policy with data uncertainty

Having described the properties of the model, we now turn to explore the effects of data uncertainty. More specifically, we want to explain what effect price/volume uncertainty has on policy setting. There are two ways we can gauge the influence of each data series: first as the effect of expectational data errors on revisions to interest rates (without knowing why those errors arise) and second by the size of impulse responses of interest rates to specific data-noise shocks.

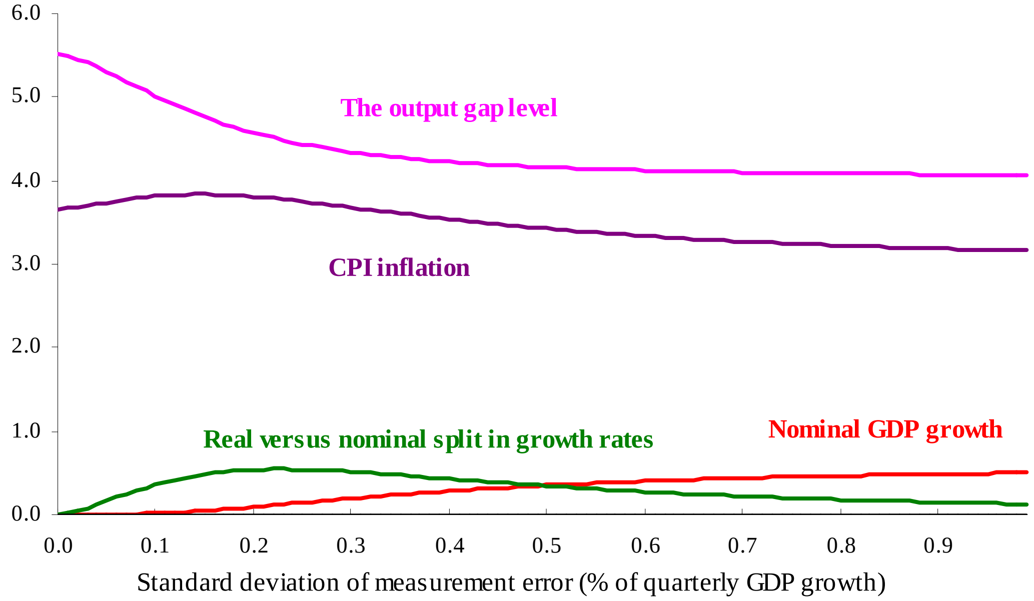
To explain the first measure, recall that policymakers form expectations of next period's data according to a Kalman filter. So when the data outturns are different to these expectations they will be led to revise both their policy decisions and their view of the underlying shocks. The principle behind the first measure of data relevance is that we can look at the revisions in each data series, and see how these revisions, through the coefficients on the (expected) pre-determined variables, affect interest rates. These coefficients are determined in our model when solved under discretion as the vector $\mathbf{F}_d\mathbf{U}$ in the expression

$$E\left[i_t \mid I_t\right] - E\left[i_t \mid I_{t-1}\right] = \mathbf{F}_d\mathbf{U}\left(E\left[\mathbf{z}_t \mid I_t\right] - E\left[\mathbf{z}_t \mid I_{t-1}\right]\right). \tag{31}$$

Appendix B shows how this equation is determined in more detail, but to summarise, $\left(E\left[i_t \mid I_t\right] - E\left[i_t \mid I_{t-1}\right]\right)$ is the revision in interest rates; $\left(E\left[\mathbf{z}_t \mid I_t\right] - E\left[\mathbf{z}_t \mid I_{t-1}\right]\right)$ are the surprises in the data that cause the revisions in interest rates; the vector $\mathbf{F}_d$ contains the coefficients placed on the expectations of the unobserved pre-determined variables in setting interest rates and are given in equation **(29)**; and the matrix $\mathbf{U}$ links the expectations of the pre-determined variables to the surprises in the four data variables. (We omit a measure of this under commitment because the relationship is complicated by the fact that the history of past surprises in data also matter for the current interest rate surprise. See appendix B.).

Chart 10 plots the results for different degrees of price/volume measurement error. It is worth noting that the coefficients in chart 10 are not to be interpreted as coefficients of a policy rule, but the extent to which expectational errors in the four sources of data affect interest rates.

Chart 10. The coefficients of expectational errors in each series in affecting interest rates (discretion)




Three facts stand out.

1. Chart 10 shows how data measurement errors cause real output growth data to be less reliable. The coefficient on real GDP growth (or, as labelled in the chart, the coefficient on the real versus nominal split in growth rates) moves closer to zero while those on nominal GDP growth and CPI inflation move away from zero as data measurement errors increase.
2. Note, though, that this only happens for very high levels of noise. This suggests that there is still much to be gained from obtaining information separately from price and volume estimates even when the standard deviation of the measurement error is as large as that of supply-side price shocks (at 0.31%).
3. Although it decreases as the data become more noisy, the coefficient placed on the output gap converges to a positive number. The implication of this is that the output gap has some role even with substantial mismeasurement of prices and volumes.

A different way of assessing the importance of each observed variable is to capture how sensitive interest rates are to data noise errors in those variables. The principle is that if a series is important then interest rates will erroneously react to an unexpected noise movement in that data series. Appendix B shows how this measure is derived as the maximum impulse of interest rates to a data noise shock in each of our four data series.

Chart 11 plots the calculations for our calibrations, showing how this evolves as the standard deviation of measurement error increases from zero to the very high value of 1% (in units of quarterly GDP growth rate) under commitment. Chart 12 replicates this chart with discretionary policy.

Chart 11. Commitment: Maximum* response of quarterly interest rates to a 1pp measurement error shock in:

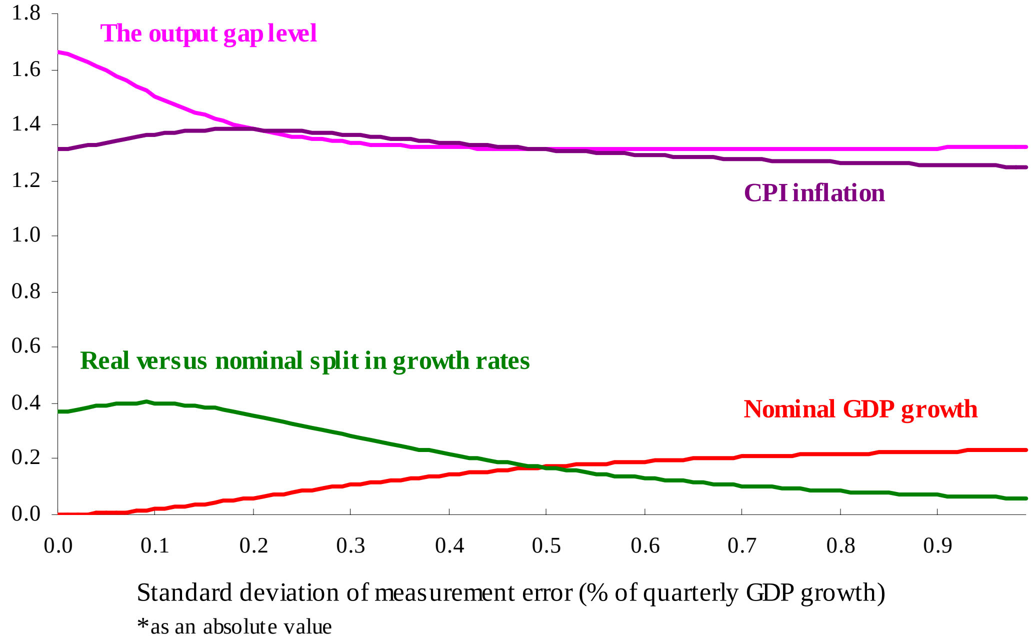
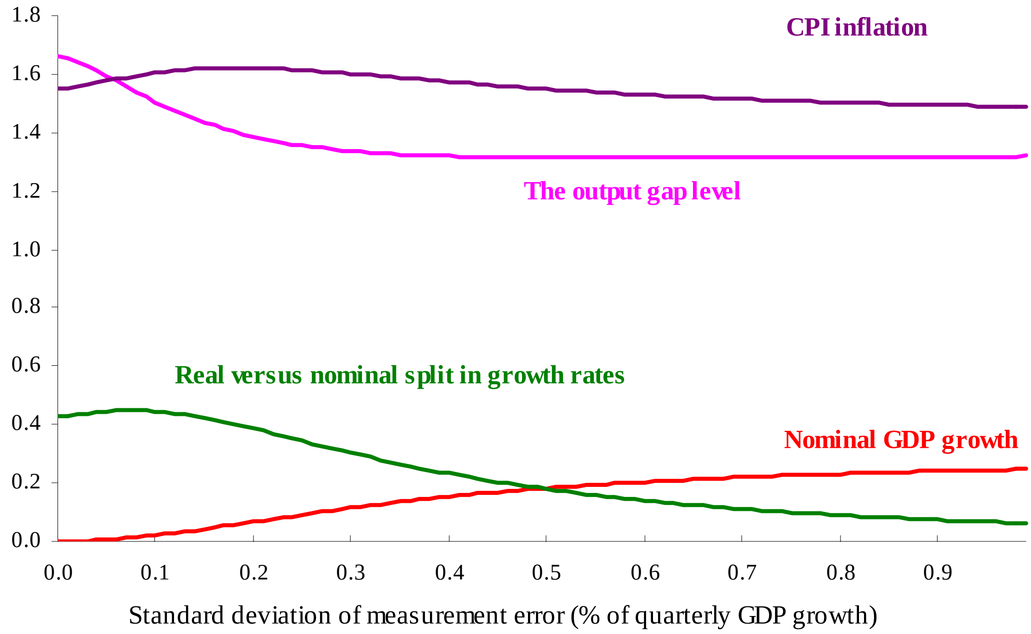



Chart 12. Discretion: Maximum* response of quarterly interest rates to a 1pp measurement error shock in:




We can see that the results seen above are to some extent preserved even when viewed from this different perspective. The sensitivity of interest rates to nominal GDP and CPI inflation data rises, whilst the sensitivity to real GDP growth and the output gap is lower. However this happens only for high levels of noise, and there still remains some weight attached to the output gap estimate even with a large degree of mismeasurement.

It might be worth explaining this last result. It should be recalled that we are experimenting with a particular type of measurement error — price/volume uncertainty. A key feature is that this error affects both real growth data and the output gap indicator, but because the output gap estimate also depends on hindsight, it affects real output growth data and the output gap indicator in different ways. Because the noise is common to both indicators the policymaker is able to compare the behaviour of both and, from that comparison, extract useful information about the underlying economic shocks. It is important to emphasise that this result is specific to our type of data mismeasurement. If we were, for example, to increase the variance of a measurement error in the output gap completely unrelated to any other indicator, the weight on that output gap estimate would be driven down to zero.

Finally, we can confirm that even though our policymaker responds as best as he can by shifting his emphasis to the data in which he has more confidence, data measurement errors do bring about some welfare losses. Chart 13 describes what data noise means for inflation, interest rate and output volatility. The volatility of the output gap increases, but less than that of inflation.

These results stand somewhat in contrast to the findings of McCallum (2003), who finds that potential output mismeasurement raises inflation volatility the most[(24)]. However, unlike McCallum (2003), we are assuming that the policymaker has good inflation data and also that he uses second-order information in forming expectations. This would make it easier for the policymaker to control inflation.

Finally, in chart 14, we compare the welfare losses under discretion and under commitment. As we would expect both increase with the measurement error. The losses under discretion increase by more, so that the relative cost of discretionary policy rises with poor data. The chart does not tell us how much more. One clue might be that from our impulse responses in chart 7 we detected very little difference between the discretionary and commitment policies. On this basis it does not

(24) McCallum and Nelson (2004) use a different model of potential output mismeasurement to us. They embed that in a forward-looking framework to show how output gap mismeasurement implies serious welfare losses.

seem that the additional welfare loss resulting from following a discretionary policy with data uncertainty rather than a policy with commitment is not quantitatively large. If we added a degree of persistence to the Phillips curve shock (*i.e.* $\rho_\varepsilon \neq 0$), then we would find a greater welfare cost of adopting a discretionary policy, and some greater role for rates of volume change. However, this does not depend on the degree of price/volume error.

Chart 13 Volatility under greater price/volume uncertainty

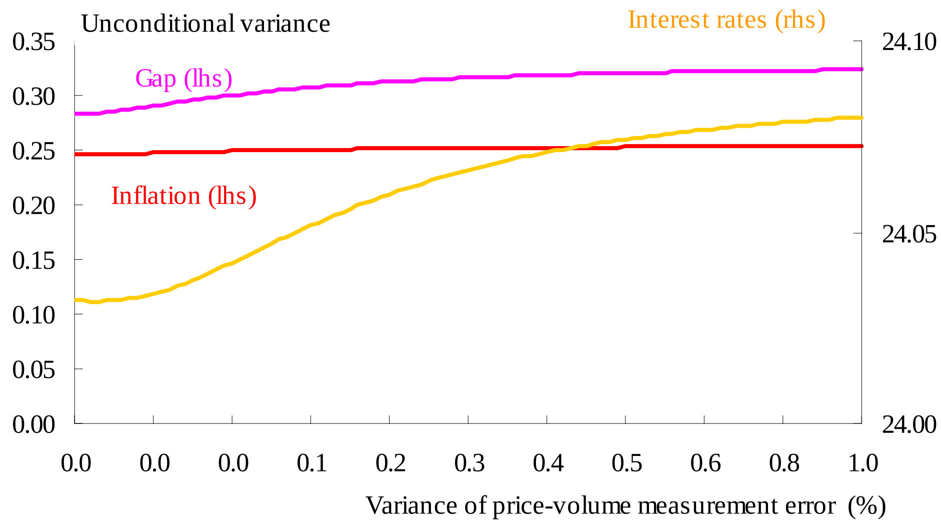
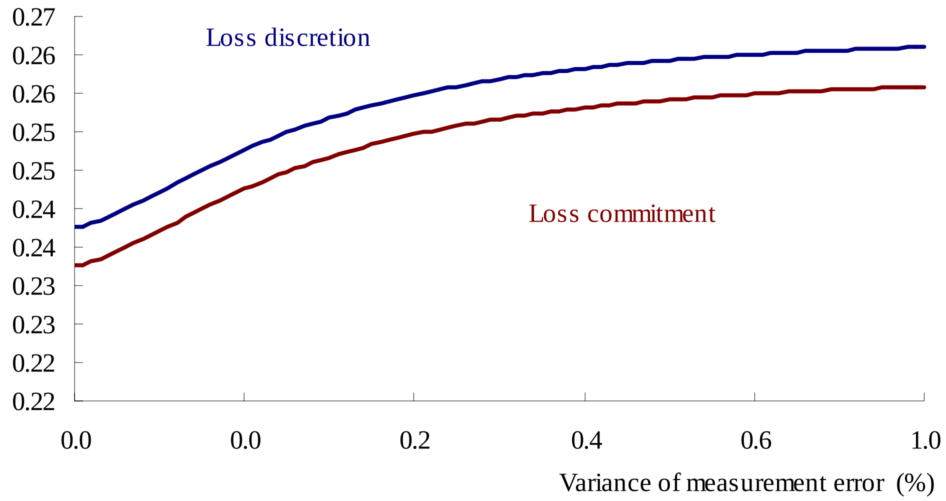



Chart 14. Losses under discretion and commitment



### *5.1 Varying the economic shocks*

In our set-up, the policymaker knows the distribution of the shocks that hit the economy and uses that information in distributing weight among different information sources. Our results will therefore be sensitive to different assumptions about the variances of the shocks to these information sources. In this section we explore how our results vary with the mix of shocks. We revisit our calibrations with different assumptions for the shocks that hit the economy.

Chart 15 reproduces our calculations on the sensitivity of interest rates to data noise but with five times as large real demand shocks in the case of policy with commitment. Comparing that to chart 12 we can see that for a given amount of price/volume mismeasurement, the policymakers will place *more* emphasis on inflation data if demand shocks were dominant. If the mismeasurement in the GDP deflator were of a small scale, nominal GDP data might also become more important under these shocks. In the absence of price/volume mismeasurement, the output gap estimate would dominate as an indicator of inflationary pressure.

Chart 16 reproduces the calculations for the base case with five times as large price supply shocks. When compared to chart 12, the coefficient on nominal GDP is little changed for a given measurement error. Price supply-side shocks in this forward-looking model do not have powerful second-round effects. It is not surprising, then, that nominal GDP growth data do not become a more prominent indicator in the presence of price/volume mismeasurement and Phillips curve shocks.

Chart 17 shows simulations with the standard deviation of technological progress shocks multiplied by five compared to the baseline case. When technological progress growth is a major source of shocks, real growth data matter more, and, correspondingly, output gap data matter less. Perhaps one can think of this as a situation in which the policymaker uses real output growth data to gauge what is happening to potential output growth and the natural real rate of interest, and rely on real GDP growth data as a guide to policy decisions. But this happens when there is a small amount of data mismeasurement. As the data error increases, the coefficient associated with real GDP growth falls and that on nominal GDP growth data rises. The combination of an uncertain potential output process and noisy real output data leads to both the real GDP growth data series and the output gap estimates becoming less relevant as indicators. CPI inflation data become more important as it is used in conjunction with nominal growth data to understand where real activity is, compared to its potential level.

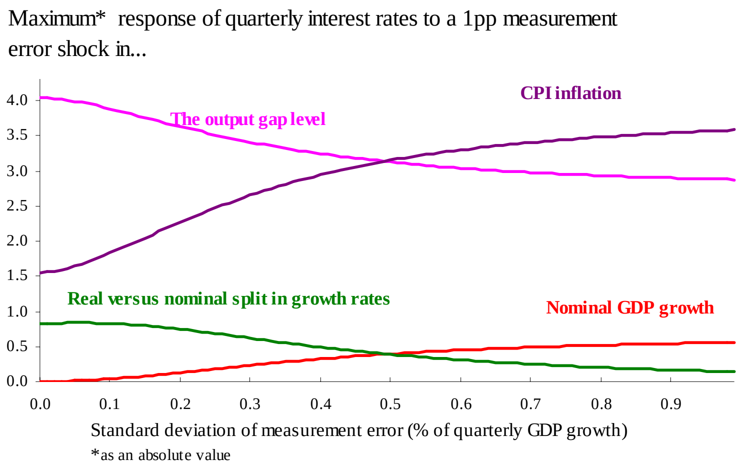



Chart 15. Large demand shocks

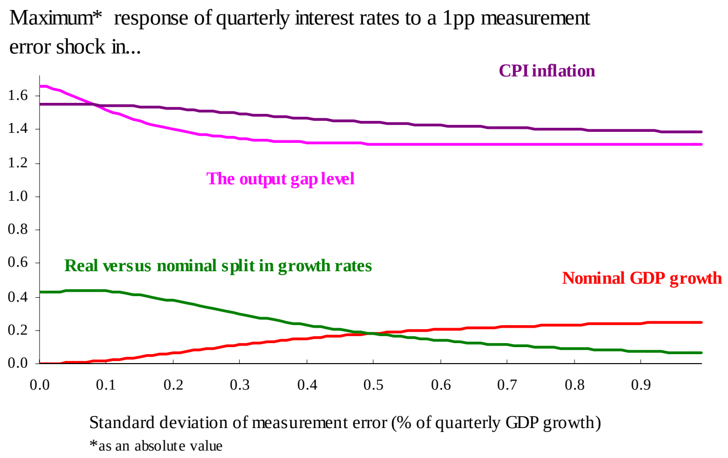



Chart 16. Large price-supply shocks

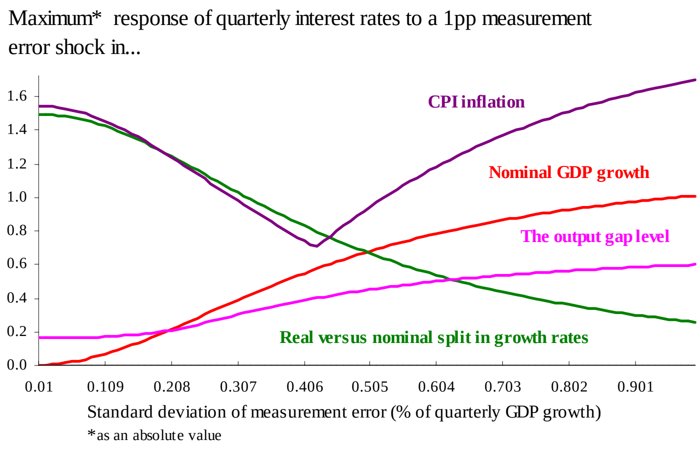



Chart 17. Large technology shocks

### 5.2 *Allowing for technological progress shocks with inflationary consequences*

In our simulations so far we have only allowed for technical progress shocks with no consequences for the output gap. Rewriting our original output gap equation:

$$\left(y_{t+1} - y^*_{t+1}\right) = \left(y_t - y^*_t\right) - \beta_2(i_t - E_t\pi_{t+1}) + \eta_{t+1} + (1+\chi)\,\omega_{t+1}, \qquad \textbf{(32)}$$

we see that so far we have assumed that $\chi = 0$, *i.e.* shifts in technological progress growth have an equal impact on actual and potential output.

Charts 18 and 19 show what happens to the impulse responses of inflation and the output gap when we relax that assumption in two opposite ways. Chart 18 features a technological progress shock that pushes the output gap above a 'speed limit' (see discussion above in section 4), *i.e.* $\chi = 1$. Chart 19, on the other hand, features a technological progress shock which tends to reduce the output gap, or to make it more negative ($\chi = -1$). Now actual output is lagging developments in potential capacity. The key message is that in the first case, technology shocks have an inflationary effect but in the second that effect is disinflationary.

Chart 18. Response of quarterly inflation and the output gap to a temporary 1pp technological progress shock under ($\chi = 1$)

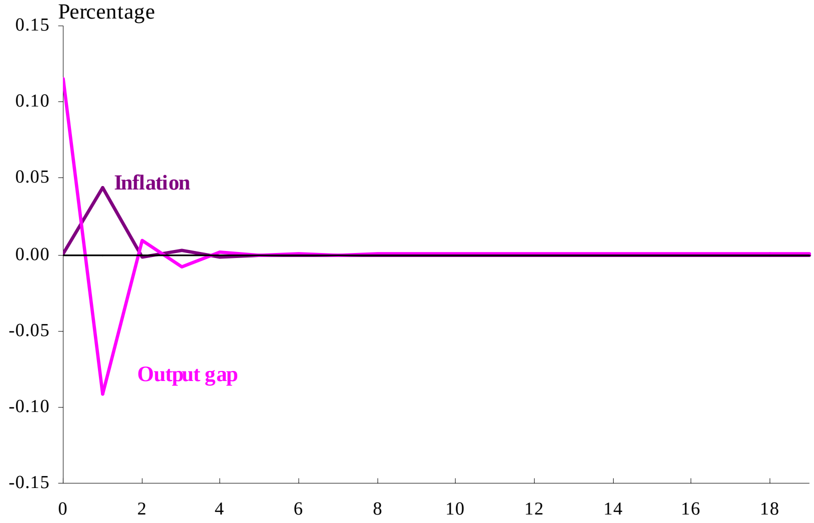
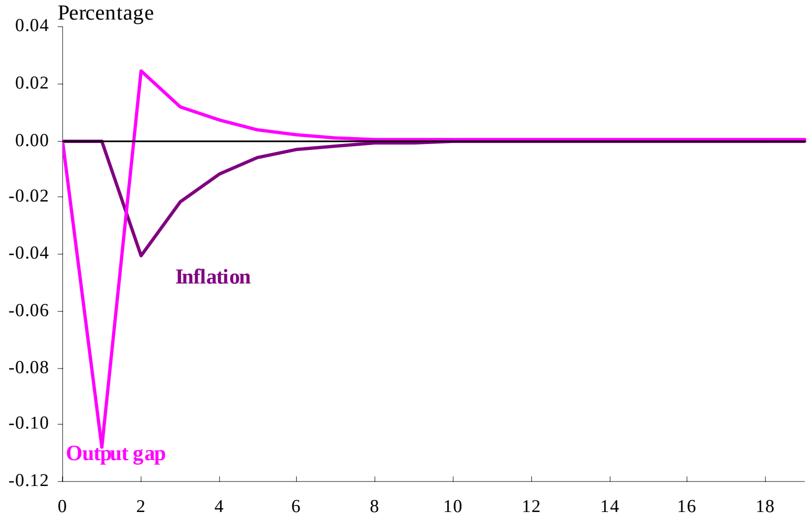



Chart 19. Response of quarterly inflation and the output gap to a temporary 1pp technological progress shock under ($\chi = -1$)




We now repeat, and show in charts 20, 21 and 22, our baseline simulations but allow for: first, an 'overheating' reaction to technological progress shocks ($\chi = 1$), and second, a supply-side improvement following the technological progress shocks ($\chi = -1$).

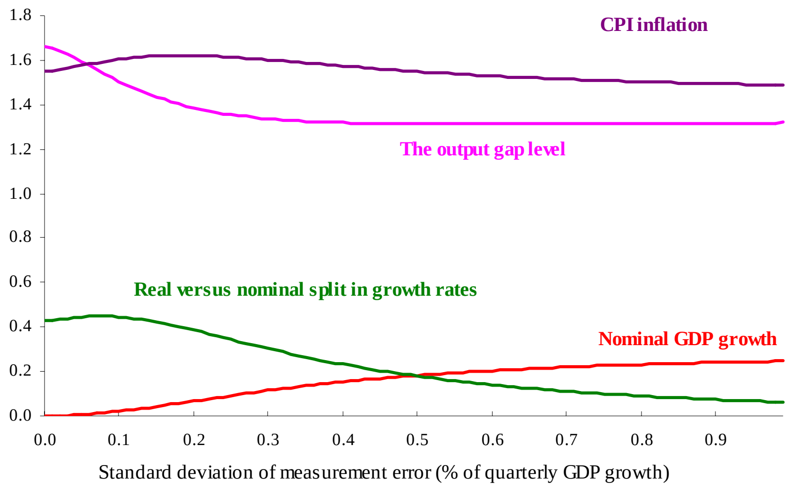



Chart 20. Baseline case (discretion) ($\chi = 0$)

Maximum* response of quarterly interest rates to a 1pp measurement error shock in...

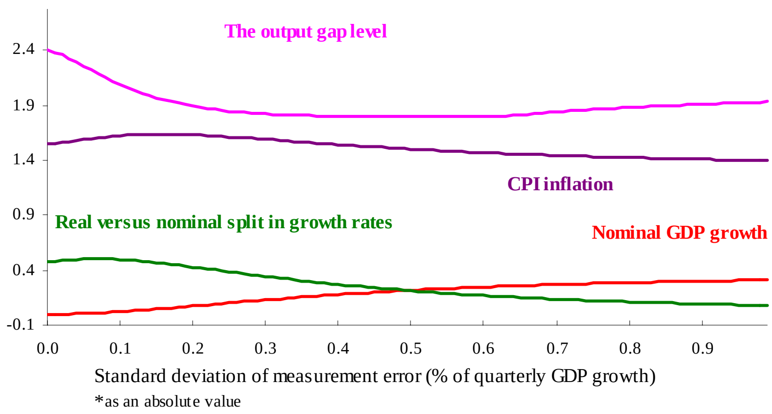



Chart 21. 'Overheating ' following technological progress shock ($\chi = 1$)

Maximum* response of quarterly interest rates to a 1pp measurement error shock in...

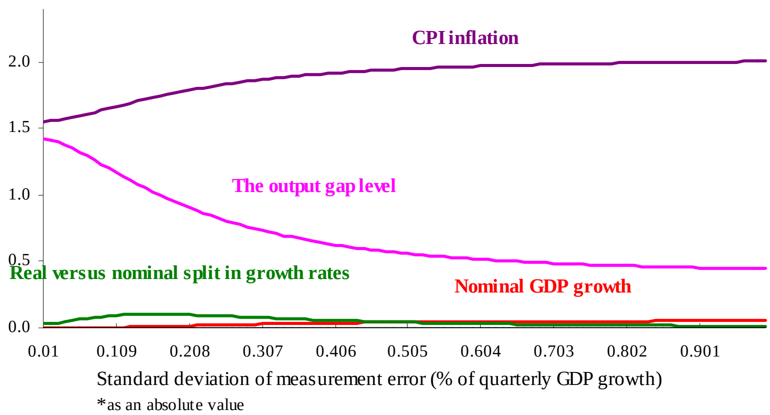



Chart 22. Supply-side improvement after technological progress shock ($\chi = -1$)

The responses to real output and nominal GDP growth data are sensitive to this ‘overreaction’ of demand above supply given by a positive value of $\chi$. If the output gap widens following $\chi$-shocks, then we can see that interest rates react more to real output growth data, or, if those are mismeasured, nominal GDP data. This is because inflationary responses to technological progress, perhaps arising through bottlenecks, are picked up by real output growing faster. Having said that, we had to allow for a substantial widening of the output gap to obtain greater responses, and again it seems sensible to argue that, based on our calibrations, the presence of ‘speed-limit’ effects[25] would not by itself lead us to ignore levels data in favour of rate of volume change indicators.

## 6 Conclusion

We have found that the presence of price/volume uncertainty in data would lead monetary policy makers to place more emphasis on nominal GDP data and less on real GDP data and the output gap. But, at least in our calibrations, a policymaker would not completely ignore indicators such as real GDP or the output gap even with a very high degree of uncertainty. In part this is because we allow for the benefit of hindsight in our model of how the output gap is measured so that past errors in estimating growth rates of real GDP and potential output do not just cumulate undiscounted onto the estimate of the current level of inflationary pressure. We also show that if technological progress shocks lead to a widening output gap, and an upward effect on inflation, then ‘hindsight’ or discounted output gap estimates and nominal GDP data become even more important.

It seems that such data measurement problems have to be severe to lead us to ignore real GDP data altogether, a result which echoes the comments of Meltzer (2001). This is particularly true when economies are hit by disturbances that require policymakers to get a firm grasp of the level of inflationary pressure that is currently building up in the economy.

(25) This ‘speed-limit’ effect lacks microfoundations. We choose to do it as a link between the output gap (in the IS curve) and the rate of change of technological progress but one can think of different ways of modelling this.

## Appendix A: A formal description of the model

Consider first equation **(1)**

$$\pi_t = \alpha_0 E\left[\pi_{t+1} \mid I_{pt}\right] + (1-\alpha_0)\,\pi_{t-1} + \alpha_1\left(y_{t-1} - y^*_{t-1}\right) + \varepsilon_{t+1} \qquad \textbf{(A-1)}$$

which we can rearrange as

$$E\left[\pi_{t+1} \mid I_{pt}\right] = \frac{\pi_t}{\alpha_0} - \frac{(1-\alpha_0)}{\alpha_0}\pi_{t-1} - \frac{\alpha_1}{\alpha_0}\left(y_{t-1} - y^*_{t-1}\right) - \frac{1}{\alpha_0}\varepsilon_{t+1}. \qquad \textbf{(A-2)}$$

Now turning to equation **(8)**,

$$\begin{aligned} y_t - y^*_t &= \beta_0\left(E\left[y_{t+1} \mid I_{pt}\right] - E\left[y^*_{t+1} \mid I_{pt}\right]\right) + \left(1-\beta_0\right)\left(y_{t-1} - y^*_{t-1}\right) \\ &\quad -\beta_1(i_t - E\left[\pi_{t+1} \mid I_{pt}\right]) \\ &\quad +\eta_{t+1} + (1+\chi)\,\omega_{t+1}, \end{aligned} \qquad \textbf{(A-3)}$$

we can also rearrange this as

$$\begin{aligned} \left(E\left[y_{t+1} \mid I_t\right] - E\left[y^*_{t+1} \mid I_t\right]\right) &= \frac{\left(y_t - y^*_t\right)}{\beta_0} - \frac{\left(1-\beta_0\right)}{\beta_0}\left(y_{t-1} - y^*_{t-1}\right) \\ &\quad +\frac{\beta_1}{\beta_0}i_t - \frac{\beta_1}{\beta_0}E\left[\pi_{t+1} \mid I_t\right] \\ &\quad -\frac{1}{\beta_0}\eta_{t+1} - \frac{(1+\chi)}{\beta_0}\omega_{t+1}. \end{aligned} \qquad \textbf{(A-4)}$$

Substituting out for expected inflation from equation **(A-3)**

$$\begin{aligned} \left(E\left[y_{t+1} \mid I_t\right] - E\left[y^*_{t+1} \mid I_t\right]\right) &= \frac{\left(y_t - y^*_t\right)}{\beta_0} - \frac{\left(1-\beta_0\right)}{\beta_0}\left(y_{t-1} - y^*_{t-1}\right) \\ &\quad -\frac{\beta_1}{\beta_0}\left(\frac{\pi_t}{\alpha_0} - \frac{(1-\alpha_0)}{\alpha_0}\pi_{t-1} - \frac{\alpha_1}{\alpha_0}\left(y_{t-1} - y^*_{t-1}\right) - \frac{1}{\alpha_0}\varepsilon_{t+1}\right) \\ +\frac{\beta_1}{\beta_0}i_t &\quad -\frac{1}{\beta_0}\eta_{t+1} - \frac{(1+\chi)}{\beta_0}\omega_{t+1}. \end{aligned} \qquad \textbf{(A-5)}$$

Rearranging gives

$$\begin{aligned}\left(E\left[y_{t+1} \mid I_t\right] - E\left[y^*_{t+1} \mid I_t\right]\right) &= \frac{1}{\beta_0}\left(y_t - y^*_t\right) + \left(\frac{\alpha_1\beta_1}{\alpha_0\beta_0} - \frac{\left(1-\beta_0\right)}{\beta_0}\right)\left(y_{t-1} - y^*_{t-1}\right) \\ &\quad + \frac{\beta_1}{\beta_0} i_t - \frac{\beta_1}{\beta_0}\frac{1}{\alpha_0}\pi_t + \frac{\beta_1}{\beta_0}\frac{(1-\alpha_0)}{\alpha_0}\pi_{t-1} \\ &\quad + \frac{\beta_1}{\alpha_0\beta_0}\varepsilon_{t+1} - \frac{1}{\beta_0}\eta_{t+1} - \frac{(1+\chi)}{\beta_0}\omega_{t+1}. \qquad \textbf{(A-6)}\end{aligned}$$

To convert to first-order form we write

$$E\left[\pi_{t+1} \mid I_t\right] = \frac{\pi_t}{\alpha_0} - \frac{(1-\alpha_0)}{\alpha_0} q_{1t} - \frac{\alpha_1}{\alpha_0} q_{2t} - \frac{1}{\alpha_0}\varepsilon_{t+1} \qquad \textbf{(A-7)}$$

and

$$\begin{aligned}\left(E\left[y_{t+1} \mid I_t\right] - E\left[y^*_{t+1} \mid I_t\right]\right) &= \frac{1}{\beta_0}\left(y_t - y^*_t\right) + \left(\frac{\alpha_1\beta_1}{\alpha_0\beta_0} - \frac{\left(1-\beta_0\right)}{\beta_0}\right) q_{2t} \\ &\quad + \frac{\beta_2}{\beta_0} i_t - \frac{\beta_1}{\beta_0}\frac{1}{\alpha_0}\pi_t + \frac{\beta_1}{\beta_0}\frac{(1-\alpha_0)}{\alpha_0} q_{1t} \\ &\quad + \frac{\beta_1}{\alpha_0\beta_0}\varepsilon_{t+1} - \frac{1}{\beta_0}\eta_{t+1} - \frac{(1+\chi)}{\beta_0}\omega_{t+1} \qquad \textbf{(A-8)}\end{aligned}$$

where

$$q_{1t+1} = \pi_t \qquad \textbf{(A-9)}$$

and

$$q_{2t+1} = \left(y_t - y^*_t\right). \qquad \textbf{(A-10)}$$

Finally to eliminate the shocks from the equations for future variables we need to define

$$e1_{\omega t+1} \equiv e_{\omega t+2},$$

$$e1_{\eta t+1} \equiv e_{\eta t+2},$$

and

$$e1_{\varepsilon t+1} \equiv e_{\varepsilon t+2}.$$

Then we can write

$$\begin{aligned}\left(E\left[y_{t+1} \mid I_t\right] - E\left[y^*_{t+1} \mid I_t\right]\right) &= \frac{1}{\beta_0}\left(y_t - y^*_t\right) + \left(\frac{\alpha_1\beta_1}{\alpha_0\beta_0} - \frac{\left(1-\beta_0\right)}{\beta_0}\right)q_{2t} \\ &+ \frac{\beta_2}{\beta_0}i_t - \frac{\beta_1}{\beta_0}\frac{1}{\alpha_0}\pi_t + \frac{\beta_1}{\beta_0}\frac{(1-\alpha_0)}{\alpha_0}q_{1t} \\ &+ \frac{\beta_1}{\alpha_0\beta_0}\rho_\varepsilon\varepsilon_t - \frac{1}{\beta_0}\rho_\eta\eta_t - \frac{(1+\chi)}{\beta_0}\rho_\omega\omega_t \\ &+ \frac{\beta_1}{\alpha_0\beta_0}e1_{\varepsilon t} - \frac{1}{\beta_0}e1_{\eta t} - \frac{(1+\chi)}{\beta_0}e1_{\omega t} \qquad \textbf{(A-11)}\end{aligned}$$

and

$$E\left[\pi_{t+1} \mid I_t\right] = \frac{\pi_t}{\alpha_0} - \frac{(1-\alpha_0)}{\alpha_0}q_{1t} - \frac{\alpha_1}{\alpha_0}q_{2t} - \frac{1}{\alpha_0}\rho_\varepsilon\varepsilon_t - \frac{1}{\alpha_0}e_{1\varepsilon t}. \qquad \textbf{(A-12)}$$

Our shocks are given by

$$\omega_{t+1} = \rho_\omega\omega_t + e1_{\omega t},$$

$$\varepsilon_{t+1} = \rho_\epsilon\varepsilon_t + e1_{\varepsilon t},$$

and

$$\eta_{t+1} = \rho_\eta\eta_t + e1_{\eta t}.$$

Defining

$$g_{t+1} = \left(y^*_{t+1} - y^*_t\right), \qquad \textbf{(A-13)}$$

we can write

$$g_{t+1} = \frac{\left(1-\beta_0\right)}{\beta_0}g_t + \frac{1}{\beta_0}\left(\rho_\omega\omega_t + e1_{\omega t}\right) \qquad \textbf{(A-14)}$$

and

$$\delta_{t+1} = \frac{\left(1-\beta_0\right)}{\beta_0}g_t + \frac{1}{\beta_0}\left(\rho_\omega\omega_t + e1_{\omega t}\right) + \rho_0\delta_t. \qquad \textbf{(A-15)}$$

We also have

$$h_{t+1} = \psi_{t+1}; \qquad \textbf{(A-16)}$$

$$\gamma_{t+1} = \psi_{t+1} + \rho_0\gamma_t; \qquad \textbf{(A-17)}$$

and

$$\varsigma_{t+1} = y_t - y_{t-1} - y^*_t + y^*_{t-1} + \rho_0\varsigma_t. \qquad \textbf{(A-18)}$$

Equations **(A-7) - (A-18)** can now be expressed in state-space form

$$E_t \mathbf{x}_{t+1} = \mathbf{A}_1 \mathbf{x}_t + \mathbf{b} i_t + \mathbf{C} \mathbf{e}_{t+1} \qquad \textbf{(A-19)}$$

where:

$$\mathbf{x}_{1t+1} = \begin{bmatrix} q_{1t+1} \\ q_{2t+1} \\ g_{t+1} \\ h_{t+1} \\ \delta_{t+1} \\ \gamma_{t+1} \\ \varsigma_{t+1} \\ \varepsilon_{t+1} \\ \eta_{t+1} \\ \omega_{t+1} \\ e_{1\varepsilon t+2} \\ e_{1\eta t+2} \\ e_{1\omega t+2} \\ \pi_{t+1} \\ \left(y_{t+1} - y^*_{t+1}\right) \end{bmatrix};$$

$$\mathbf{A}_2 = [\mathbf{0}];$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix};$$

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\beta_1}{\beta_0} \end{bmatrix}^T;$$

and

$$\mathbf{e}_{t+1} = \begin{bmatrix} e_{\omega t+2} \\ e_{\psi t+1} \\ e_{\varepsilon t+2} \\ e_{\eta t+2} \end{bmatrix}.$$

$$
\mathbf{A}_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & \frac{\left(1-\beta_0\right)}{\beta_0} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta_0}\rho_\omega & 0 & 0 & \frac{1}{\beta_0} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\left(1-\beta_0\right)}{\beta_0} & 0 & \rho_0 & 0 & 0 & 0 & 0 & \frac{1}{\beta_0}\rho_\omega & 0 & 0 & \frac{1}{\beta_0} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & \rho_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\epsilon & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\eta & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\omega & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{(1-\alpha_0)}{\alpha_0} & -\frac{\alpha_1}{\alpha_0} & 0 & 0 & 0 & 0 & 0 & -\frac{\rho_\epsilon}{\alpha_0} & 0 & 0 & -\frac{1}{\alpha_0} & 0 & 0 & \frac{1}{\alpha_0} & 0 \\
\varpi_1 & \varpi_2 & 0 & 0 & 0 & 0 & 0 & \varpi_3 & \varpi_4 & \varpi_5 & \varpi_6 & \varpi_7 & \varpi_8 & \varpi_9 & \varpi_{10}
\end{bmatrix},
$$

where

$$\varpi_1 = \frac{(1-\alpha_0)\,\beta_1}{\alpha_0\beta_0};$$

$$\varpi_2 = \frac{\alpha_1\beta_1 - \alpha_0\left(1-\beta_0\right)}{\alpha_0\beta_0};$$

$$\varpi_3 = \frac{\rho_\epsilon\beta_1}{\alpha_0\beta_0};$$

$$\varpi_4 = -\frac{\rho_\eta}{\beta_0};$$

$$\varpi_5 = -\frac{\rho_\omega\,(1+\chi)}{\beta_0};$$

$$\varpi_6 = \frac{\beta_1}{\alpha_0\beta_0};$$

$$\varpi_7 = -\frac{1}{\beta_0};$$

$$\varpi_8 = -\frac{(1+\chi)}{\beta_0};$$

$$\varpi_9 = -\frac{\beta_1}{\alpha_0\beta_0};$$

and

$$\varpi_{10} = \frac{1}{\beta_0}.$$

We can also re-write equation **(19)** as

$$\widetilde{y}_t - \widetilde{y}_t^* = y_t - y_t^* + \rho_1 \gamma_t + \rho_1 \delta_t - (1-\rho_1)\left(y_t - y_{t-1} - y_t^* + y_{t-1}^* + \rho_0 \varsigma_t\right) \qquad \textbf{(A-20)}$$

with

$$\delta_t = y_t^* - y_{t-1}^* + \rho_0 \delta_{t-1}; \qquad \textbf{(A-21)}$$

$$\varsigma_t = y_{t-1} - y_{t-2} - y_{t-1}^* + y_{t-2}^* + \rho_0 \varsigma_{t-1}; \qquad \textbf{(A-22)}$$

and

$$\gamma_t = \psi_t + \rho_0 \gamma_{t-1} \qquad \textbf{(A-23)}$$

as in equations **(A-13)** - **(A-18)** above. The link between observables and the unobserved variables that matter for welfare is given by

$$\mathbf{z}_t = \mathbf{D}_1 \mathbf{x}_t + \mathbf{v}_t, \qquad \textbf{(A-24)}$$

with

$$\mathbf{z}_t = \begin{bmatrix} \widetilde{\pi}_t \\ \widetilde{y}_t - \widetilde{y}_{t-1} \\ \widetilde{y}_t - \widetilde{y}_t^* \\ \widetilde{n}_t \end{bmatrix},$$

$$\mathbf{D}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & (1-\rho_1) & 0 & 0 & \rho_1 & \rho_1 & -(1-\rho_1)\rho_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

and

$$\mathbf{v}_t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \nu_t \end{bmatrix}.$$

The variances of the shocks are given by

$$E\mathbf{e}_{t+1}\left(\mathbf{e}_{t+1}\right)' = \Sigma_{ee} = \begin{bmatrix} \sigma_\omega^2 & 0 & 0 & 0 \\ 0 & \sigma_\psi^2 & 0 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & 0 & \sigma_\eta^2 \end{bmatrix},$$

and

$$E\mathbf{v}_{t+1}\left(\mathbf{v}_{t+1}\right)' = \Sigma_{vv} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\nu}^{2} \end{bmatrix}$$

assuming that the covariances between the measurement error uncertainty and the fundamental shocks are zero. The policymaker's period loss function can be re-expressed in the following way:

$$L_t = \mathbf{x}_t'\mathbf{Q}\mathbf{x}_t \qquad \textbf{(A-25)}$$

where

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & & \cdots & 0 \\ 0 & \lambda & 0 & 0 & \cdots & & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & & \cdots & 0 \\ \vdots & \vdots & \vdots & & \ddots & & & \vdots \\ & & & & & & & \\ & & & & & & \ddots & \vdots \\ \vdots & \vdots & \vdots & & & & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \end{bmatrix}.$$

## Appendix B: Summary measures of the relative importance of different indicators

In this section we show formally how we derive our two summary measures of the relative importance of different data indicators to policy setting. To summarise, from appendix A, the problem that monetary policymakers solve is to minimise

$$E\left[\sum \delta^s L_{t+s} \mid I_t\right]$$

where

$$L_t = \mathbf{x}_t' \mathbf{Q} \mathbf{x}_t \qquad \textbf{(B-1)}$$

by choice of $i_t$ given the state-space system

$$E\left[\mathbf{x}_{t+1} \mid I_t\right] = \mathbf{A}_1 \mathbf{x}_t + \mathbf{b} i_t + \mathbf{C} \mathbf{e}_{t+1} \qquad \textbf{(B-2)}$$

with $\mathbf{e}_{t+1}$ being jointly normally distributed with zero means and a variance-covariance matrix $\Sigma_{ee}$; and the signal system written as

$$\mathbf{z}_t = \mathbf{D}_1 \mathbf{x}_t + \mathbf{v}_t, \qquad \textbf{(B-3)}$$

with $\mathbf{v}_t$ being jointly normally distributed with zero means and a variance-covariance matrix $\Sigma_{\upsilon\upsilon}$. The information set is

$$I_t = \left\{\mathbf{z}_t, \lambda, \beta_0, \beta_1, \alpha_0, \alpha_1, \chi, \rho_\omega, \rho_\varepsilon, \rho_\eta, \rho_0, \rho_1, \sigma_\omega^2, \sigma_\epsilon^2, \sigma_\eta^2, \sigma_\nu^2, \sigma_\psi^2\right\}.$$

Let us divide the unobserved variables into the pre-determined variables ($\mathbf{x}_{1t+1}$) and forward-looking variables ($\mathbf{x}_{2t+1}$) such that

$$\mathbf{x}_{1t+1} = \begin{bmatrix} q_{1t+1} \\ q_{2t+1} \\ g_{t+1} \\ h_{t+1} \\ \delta_{t+1} \\ \gamma_{t+1} \\ \varsigma_{t+1} \\ \varepsilon_{t+1} \\ \eta_{t+1} \\ \omega_{t+1} \\ e_{1\varepsilon t+2} \\ e_{1\eta t+2} \\ e_{1\omega t+2} \end{bmatrix}$$

and

$$\mathbf{x}_{2t+1} = \begin{bmatrix} \pi_{t+1} \\ \left(y_{t+1} - y^*_{t+1}\right) \end{bmatrix}.$$

We first describe the model solution under commitment, and then under discretion.

***Commitment***

Svensson and Woodford (2003) show that under commitment the solution can be described as a function of current and past states

$$i_t = \mathbf{F}_{c0} E\left[\mathbf{x}_{1t} \mid I_t\right] + \Phi \Xi_{t-1}; \qquad \textbf{(B-4)}$$

$$\Xi_t = \mathbf{S} E\left[\mathbf{x}_{1t} \mid I_t\right] + \Sigma \Xi_{t-1}; \qquad \textbf{(B-5)}$$

and

$$\mathbf{x}_{2t} = \mathbf{G}_c E\left[\mathbf{x}_{1t} \mid I_t\right] + \Gamma \Xi_{t-1} \qquad \textbf{(B-6)}$$

starting from $\Xi_{-1} = 0$.

The solution to the noise-signal problem can be written as

$$E\left[\mathbf{x}_{1t} \mid I_t\right] = E\left[\mathbf{x}_{1t} \mid I_{t-1}\right] + \mathbf{K}\mathbf{z}_t,$$

or, in terms of expectational errors,

$$E\left[\mathbf{x}_{1t} \mid I_t\right] = E\left[\mathbf{x}_{1t} \mid I_{t-1}\right] + \mathbf{U}\left(\mathbf{E}\left[\mathbf{z_t} \mid \mathbf{I_t}\right] - \mathbf{E}\left[\mathbf{z_t} \mid \mathbf{I_{t-1}}\right]\right),$$

where the Kalman gain matrix $\mathbf{K}$ is given by

$$\mathbf{K} = \mathbf{PL}^T \left(\mathbf{LPL}^T + \Sigma_{vv}\right)^{-1};$$

and $\mathbf{U}$ is

$$\mathbf{U} = \mathbf{K}\left(\mathbf{I} + \mathbf{N}_1\mathbf{K}\right)^{-1}.$$

$$\mathbf{P} = \mathbf{H}\left[\mathbf{P} - \mathbf{PL}^T \left(\mathbf{LPL}^T + \Sigma_{vv}\right)^{-1} \mathbf{LP}\right]\mathbf{H}^T + \Sigma_{ee}$$

and

$$\mathbf{L} = \mathbf{D_1^1} + \mathbf{D}_2^1\mathbf{G}^1$$

where $\mathbf{D}_1^1$ and $\mathbf{D}_2^1$ is the partition of $\mathbf{D}$ along the lines of the pre-determined variables ($\mathbf{x}_{1t+1}$) and forward-looking variables ($\mathbf{x}_{2t+1}$) and so in our case $\mathbf{D}_2^1 = \mathbf{0}$.

To complete our description we define the matrices $\mathbf{N}_1$ and $\mathbf{H}$ below:

$$\mathbf{N}_1 = -\mathbf{D}_2^1\mathbf{G}^1 + \mathbf{G}^1;$$

$$\mathbf{H} = \mathbf{A}_{11}^1 + \mathbf{A}_{12}^1\mathbf{G}^1;$$

with

$$\mathbf{G}^1 = -\left(\mathbf{A}_{22}^1\right)^{-1}\mathbf{A}_{21}^1$$

and the matrices $\mathbf{A}_{11}^1$, $\mathbf{A}_{21}^1$, $\mathbf{A}_{12}^1$ and $\mathbf{A}_{22}^1$ being the partitions of $\mathbf{A}_1$ along $\mathbf{x}_{1t}$ and $\mathbf{x}_{2t}$. Note that the matrices that define the solution to the filtering problem do not depend on the matrices that define the solution to the optimal control problem. That is because of the separation property.

Svensson and Woodford (2003) show that the dynamics of the model under commitment can be summarised as combining **(B-4)**, **(B-5)** and **(B-6)** with

$$\mathbf{x}_{1t+1} = \left(\mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{G} + \mathbf{B}_1\mathbf{F}_c\right)\mathbf{x}_{1t} + \left(\mathbf{A}_{12}\Gamma + \mathbf{B}_1\Phi\right)\Xi_{t-1} + \mathbf{e}_{t+1} \qquad \textbf{(B-7)}$$

$$\mathbf{z}_t = \mathbf{D}_1\mathbf{x}_t + \mathbf{v}_t, \qquad \textbf{(B-8)}$$

and

$$E\left[\mathbf{x}_{1t+1} \mid I_t\right] = \left(\mathbf{H} + \mathbf{J}_c\right)E\left[\mathbf{x}_{1t} \mid I_t\right] + \Psi\,\Xi_{t-1}, \qquad \textbf{(B-9)}$$

starting from $\mathbf{x}_{1t+1}$ and $\Xi_{-1} = 0$, where

$$\Psi = \mathbf{A}_{11}^1\Gamma + \mathbf{A}_{12}^1\Phi; \qquad \textbf{(B-10)}$$

$$\mathbf{G}_c^2 = \mathbf{G}_c - \mathbf{G}^1; \qquad \textbf{(B-11)}$$

and

$$\mathbf{J}_c = \mathbf{B}_1\mathbf{F}_c + \mathbf{A}_{11}^1 + \mathbf{A}_{12}^1\mathbf{G}_c^2. \qquad \textbf{(B-12)}$$

Our second summary measure captures the maximum absolute impulse response of interest rates to a 'data-noise' shock to each data source shown in Charts 11,12 and 15-20. The data-noise shocks enter as elements of vector $\mathbf{v}_t$ , for this experiment redefined as $\mathbf{v}_t = \left[\tau - \inf, \tau - g\,\text{row}\,th, \tau - gap, \tau - nomg\,\text{row}\,th\right]^T$ and the impulse responses would be derived from solving **(B-4)**, **(B-5)**, **(B-7)**, **(B-8)** and **(B-9)**.

***Discretion***

The solution to the optimal policy problem under discretion can be expressed as a function of the pre-determined unobservable variables only,

$$i_t = \mathbf{F}_d E\left[\mathbf{x}_{1t} \mid I_t\right]; \qquad \textbf{(B-13)}$$

and

$$\mathbf{x}_{2t} = \mathbf{G}_d E\left[\mathbf{x}_{1t} \mid I_t\right] \qquad \textbf{(B-14)}$$

starting from $\Xi_{-1} = 0$ as before. The solution to the filtering problem is as we described it under commitment. Taking expectational errors of equation **(B-13)**, we can write our first measure of policy sensitivity to data releases — the interest rate response in terms of expectational errors — as

$$\begin{aligned} E\left[i_t \mid I_t\right] - E\left[i_t \mid I_{t-1}\right] &= \mathbf{F}_d\left(E\left[\mathbf{x}_{1t} \mid I_t\right] - E\left[\mathbf{x}_{1t} \mid I_{t-1}\right]\right); \\ \Rightarrow \quad E\left[i_t \mid I_t\right] - E\left[i_t \mid I_{t-1}\right] &= \mathbf{F}_d\mathbf{U}\left(E\left[\mathbf{z}_t \mid I_t\right] - E\left[\mathbf{z}_t \mid I_{t-1}\right]\right) \end{aligned} \qquad \textbf{(B-15)}$$

Chart 10 in Section 5 shows these coefficients in equation **(B-15)** as our first measure of the importance of data (at least under discretion). Svensson and Woodford (2003) show that the dynamics of the model under discretion can be summarised as combining **(B-14)** and **(B-13)** with

$$\mathbf{x}_{1t+1} = \left(\mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{G} + \mathbf{B}_1\mathbf{F}_d\right)\mathbf{x}_{1t} + \mathbf{e}_{t+1} \qquad \textbf{(B-16)}$$

$$\mathbf{z}_t = \mathbf{D}_1\mathbf{x}_t + \mathbf{v}_t, \qquad \textbf{(B-17)}$$

and

$$E\left[\mathbf{x}_{1t+1} \mid I_t\right] = \left(\mathbf{H} + \mathbf{J}_d\right) E\left[\mathbf{x}_{1t} \mid I_t\right], \qquad \textbf{(B-18)}$$

starting from $\mathbf{x}_{1t+1}$ and $\Xi_{-1} = 0$, where

$$\mathbf{G}_d^2 = \mathbf{G}_d - \mathbf{G}^1; \qquad \textbf{(B-19)}$$

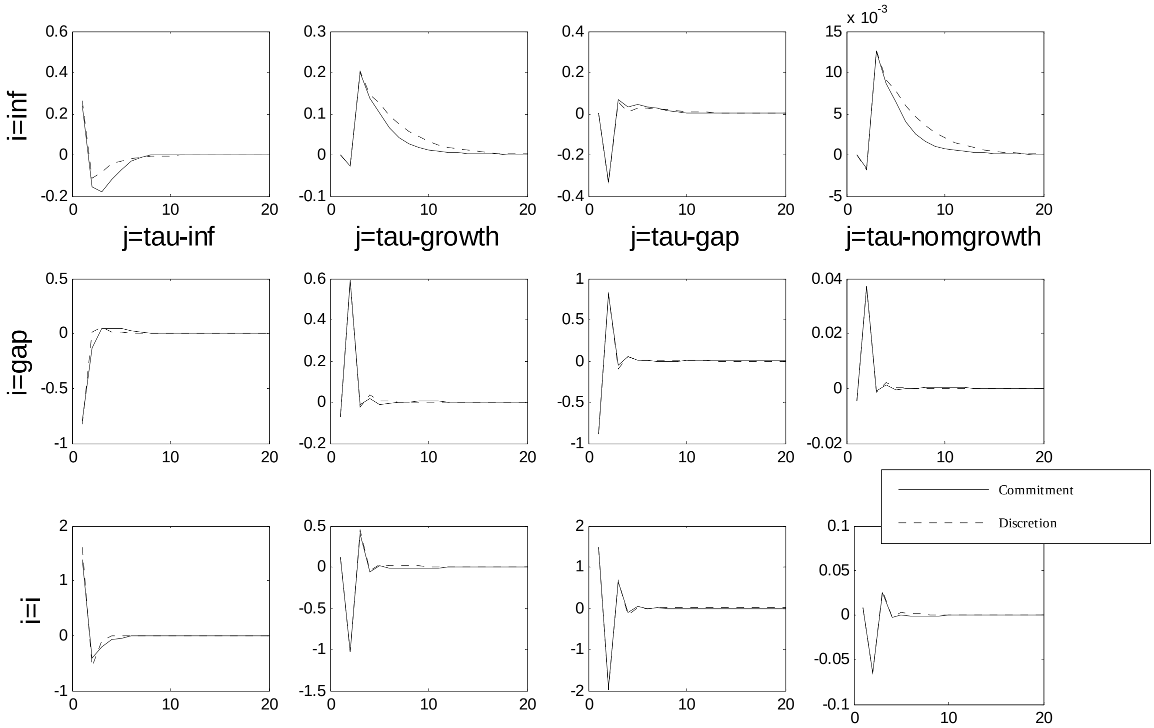
and

$$\mathbf{J}_d = \mathbf{B}_1\mathbf{F}_d + \mathbf{A}_{11}^1 + \mathbf{A}_{12}^1\mathbf{G}_d^2. \qquad \textbf{(B-20)}$$

Our second summary measure captures the maximum absolute impulse response of interest rates to a 'data-noise' shock to each data source shown in Charts 11,12 and 15-20. The data-noise shocks enter as elements of vector $\mathbf{v}_t$ which for this experiment are redefined as $\mathbf{v}_t = \left[\tau - \text{inf}, \tau - \text{growth}, \tau - \text{gap}, \tau - \text{nomgrowth.}\right]^T$ and the impulse responses would be derived from solving **(B-13)**, **(B-14)**, **(B-16)**, **(B-17)** and **(B-18)**.

For completeness below we show the responses of inflation, the output gap and interest rates to a 1% unexpected temporary rise in the data-noise shocks called $\tau -$ inf, $\tau -$ growth, $\tau -$gap and $\tau -$ nomgrowth, under both commitment and discretion for baseline parameter values.

Effect of 1% shocks on j on variable i under commitment and discretion



## Appendix C: The negative correlation between growth and deflator revisions

Let $\widetilde{z}$ denote the observed data on the true value $z$. Let us define the errors in measuring nominal growth $n$ and deflator inflation $def$ respectively as:

$$n = \widetilde{n} + e$$

and

$$def = \widetilde{def} + u$$

where $e$ and $u$ are jointly normally distributed errors, with mean zero, variances given by $\sigma_e^2$ and $\sigma_u^2$ respectively, and covariance $E(e, u)$. Assume that the variance of nominal growth is $a^2$ times larger than that of price growth, *i.e.* $\sigma_e^2 = a^2\sigma_u^2$. The covariance and the correlation coefficient $\rho_{eu}$ are therefore related by the following expression:

$$E(e, u) = \rho_{eu}\sigma_e\sigma_u = \rho_{eu}a\sigma_u^2 \quad \textbf{(C-1)}$$

Given these assumptions, real or volume growth will be given by:

$$y_t - y_{t-1} = n - \pi^{def} = \widetilde{n} + e - \widetilde{def} - u = \widetilde{n} - \widetilde{def} + e - u = \widetilde{n} - \widetilde{def} + \nu$$

where $\nu = e - u$.

Given that the error in real growth is equal to the error in nominal growth minus the error in price growth, the variance of $\nu$ will be given by:

$$E(\nu^2) = E(e^2) + E(u^2) - 2E(e, u) = \sigma_e^2 + \sigma_u^2 - 2\rho_{eu}\sigma_e\sigma_u.$$

Substituting **(C-1)** into the expression above we obtain:

$$E(\nu^2) = (1 + a^2)\sigma_u^2 - 2\rho_{eu}a\sigma_u^2 = \sigma_u^2((a - \rho_{eu})^2 + (1 - \rho_{eu}^2))$$

after some simple algebraic manipulations. The correlation of $\nu$ and $u$, which corresponds to the negative relationship between revisions to inflation and real growth described in section 3, will be given by:

$$\rho_{\nu u} = \frac{E(e - u, u)}{\sqrt{E(\nu^2)E(u^2)}} = \frac{E(e, u)}{\sqrt{E(\nu^2)E(u^2)}} - \frac{E(u^2)}{\sqrt{E(\nu^2)E(u^2)}}$$

This will be equal to:

$$\rho_{\nu u} = \left( \frac{a\rho_{eu} - 1}{\sqrt{\sigma_u^2((a-\rho_{eu})^2 + (1-\rho_{eu}^2)}} \right) \sigma_u^2 = -\left( \frac{1 - a\rho_{eu}}{\sqrt{(a-\rho_{eu})^2 + (1-\rho_{eu}^2)}} \right) \qquad \textbf{(C-2)}$$

Looking at **(C-2)**, we can see that $\rho_{\nu u}$ is decreasing in $a$. This means that, for a given correlation between price and nominal measurement errors ($\rho_{eu}$), larger measurement errors for prices will imply a larger *negative* correlation between price and volume measurement errors.

If we consider the case in which $a = 1$, we will have:

$$\rho_{\nu u} = -\left( \sqrt{\frac{(1-\rho_{eu})}{2}} \right). \qquad \textbf{(C-3)}$$

The case in which $a = 1$ corresponds to one in which the variance of the error in nominal growth is equal to the error in price growth. From table 1, we can see that in our data set, the values for $\sigma_e$ and $\sigma_\nu$ are similar. For $a = 1$, and $\rho_{eu} = 7/8$ (the value in table 1), $\rho_{\nu u}$ is equal to $-1/4$, which is not that different to the estimate reported in table 1.

In general, for $a > 1/\rho_{eu}$, the above expression will be negative, and the correlation between errors in changes in prices and changes in volumes will be negative. Given that $a \equiv \sigma_e/\sigma_u$, this implies that $a > 1/\rho_{eu}$ implies $\sigma_{eu} > 1$, *i.e.* that errors in nominal values and prices covary positively.

## Appendix D: An explicit expression for potential output

Our model for potential output is given by:

$$y_t^* = \beta_0 E\left[y_{t+1}^* \mid I_t\right] + \left(1-\beta_0\right) y_{t-1}^* - \omega_{t+1} \qquad \textbf{(D-1)}$$

and

$$\omega_t = \rho_\omega \omega_{t-1} + e_{\omega t}. \qquad \textbf{(D-2)}$$

Defining

$$z_t = \left(y_t^* - y_{t-1}^*\right),$$
$$\upsilon_t = -\frac{\omega_{t+1}}{\left(1-\beta_0\right)},$$
$$\varepsilon_t = -\frac{e_{\omega t+1}}{\left(1-\beta_0\right)}$$

and

$$a = \left(\frac{\beta_0}{1-\beta_0}\right)$$

we can write this model in terms of equations **(D-3)** and **(D-4)** below:

$$z_t = aE\left[z_{t+1} \mid I_t\right] + \upsilon_t \qquad \textbf{(D-3)}$$

and

$$\upsilon_t = \rho_\omega \upsilon_{t-1} + \varepsilon_t \qquad \textbf{(D-4)}$$

with $\varepsilon_{t-1}$ not part of the information set at time t.

The 'backward' solution to equations **(D-3)** and **(D-4)** is given by

$$z_t^b = \frac{z_{t-1}^b - \upsilon_{t-1}}{a} \qquad \textbf{(D-5)}$$

or equivalently

$$y_t^{b*} = y_{t-1}^{b*} + \frac{\left(1-\beta_0\right)}{\beta_0}\left(y_{t-1}^{b*} - y_{t-2}^{b*}\right) + \frac{1}{\beta_0}\omega_t. \qquad \textbf{(D-6)}$$

We have shown in the text that $\beta_0 > 0.5$. But then there are an infinite number of stationary solutions[26]. So as we need to assume $\beta_0 > 0.5$, we need to choose some criteria to restrict the

(26) See, for example, Gourieroux and Montfort (1997) (pages 468-470). The forward solution

$$z_t^F = \sum_{t=0}^{\infty} a^i \rho_\omega^i \upsilon_t$$

would converge if $\left|a\rho_\omega\right| = \left|\frac{\rho_\omega\beta_0}{1-\beta_0}\right| < 1$ but would be the only unique solution if $|a| = \left|\frac{\beta_0}{1-\beta_0}\right| < 1$.

solutions.

If we were to allow for a discount rate then the model for potential output would instead be written as

$$y_t^* = \beta_f E\left[y_{t+1}^* \mid I_t\right] + \beta_d y_{t-1}^* + \omega_{t+1} \quad \textbf{(D-7)}$$

with $\beta_f + \beta_d < 1$. If we were to define stability as requiring that the level of output should be stationary, then the unique stable solution to this new model would be(27)

$$y_t^* = \kappa_1 y_{t-1}^* + \kappa_2 \omega_{t+1} \quad \textbf{(D-8)}$$

with

$$\kappa_1 = \frac{1 - \sqrt{1 - 4\beta_f}\beta_d}{2\beta_f}$$

and

$$\kappa_2 = \frac{1}{1 - \beta_f\left(\kappa_1 + \rho_\omega\right)}.$$

For standard values for discounting, $\beta_f + \beta_d$ can be very close to 1, and in this case $\frac{1-\sqrt{1-4\beta_f}\beta_d}{2\beta_f} \simeq \frac{\left(1-\beta_0\right)}{\beta_0}$ so that this unique solution to the problem with discounting is very close to the backward solution in the problem without discounting. On these grounds it seems sensible to adopt the backward solution even though it is not strictly speaking unique.

(27) To derive this solution, we find the roots of the characteristic equation and single out the only root that is less than one in absolute value.

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