External MPC Unit

Discussion Paper No.23

Time-Varying Yield Curve Dynamics and Monetary Policy

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March 2008

ISSN: 1748 – 6203

The authors wish to thank Boragan Aruoba, Francis Diebold and Glenn Rudebusch for kindly sharing their yield data, and Iryna Kaminska, Christopher Schleicher, Iain De Weymarn, James Proudman, Peter Westaway, Mike Joyce, Steffen Sorensen and an anonymous referee for comments.

This paper has been externally refereed.

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Abstract

The dynamics of the US economy are modelled using a time-varying structural vector autoregression that incorporates information from the yield curve. We find important changes in the dynamics of macroeconomic variables such as inflation and the federal funds rate. In addition our results suggest a change in the relationship between the yield curve and macroeconomic variables. The monetary policy shocks of the early 1980s explain a large portion of the persistence of inflation and the level of the yield curve. Shocks to the level of the yield curve account for the persistence of the federal funds rate. We use our time-varying model provides to revisit the evidence on the expectations hypothesis.

JEL classification: E44, E52, C15.

Keywords: Nelson-Siegel, time variation, inflation expectations, credibility building, evidence on expectations hypothesis.

Summary

Since the mid-1980's, the US economy has experienced low inflation and stable output growth. A number of recent papers have analysed the dynamics of this 'great-moderation' using systems of equations known as Vector Autoregressions (VARs): a set of equations where the explanatory variables in each equation are the complete set of lagged variables in the system. GDP growth, inflation and the nominal interest rate are the typical variables included in VARs that describe the transmission mechanism of monetary policy. These empirical models are subject to the criticism that they include a limited amount of information. If, in reality, the central bank examines a wider set of variables when setting policy, estimates of the monetary policy shock derived from these small empirical models may be biased–ie not completely disentangled from non-policy shocks. As a consequence an accurate assessment of structural shifts may be hampered.

The aim of this paper is to use a VAR model that is less susceptible to this criticism. In particular, we augment the standard three variable VAR with variables that describe the level, slope and curvature of the yield curve. These additional yield curve variables contain information about private sector expectations. This additional information may alleviate the biases referred to above by ensuring that the forward looking aspect of monetary policy is accounted for in our empirical model. In addition, we allow the relationship between the yield curve and the macroeconomy (embodied in our VAR) to change over time. We use this model to investigate how the dynamics of US macroeconomic variables have changed over time and how these changes are related to changing properties of the yield curve.

The main results can be summarised as follows. The level of the yield curve is highly correlated with the one-year ahead inflation forecasts of the Fed Greenbook and the Survey of Professional Forecasters. Monetary policy shocks account for most of the persistence in inflation around the mid-1970s

and the beginning of the 1980s. The persistence of the federal funds rate is driven by shocks to the level of the yield curve, whereas the variance is explained by monetary policy shocks. Our model fits the data well with forecasts of long-term yields close to actual out turns over most of the sample period.

1 Introduction

Since the mid-1980s, the United States has experienced low inflation and stable output growth. This phenomenon has been documented in many recent studies. For example, Cogley and Sargent (2002) and Cogley and Sargent (2005) report a significant fall in the volatility of US output and inflation after the mid-1980s. Cogley and Sargent (n.d.) show that the persistence of inflation was also significantly lower in the subsequent period.

The possible role played by monetary policy in bringing about this 'great moderation' has been analysed in a series of papers. For example, Cogley and Sargent (2002) report a significant change in the degree of 'activism' of US monetary policy. As in Clarida *et al.* (2000) the authors argue that the fall in the level and persistence of US inflation in the 1980s and the 1990s coincided with an increase in the degree of activism. Some of the subsequent literature has been less favourable to this "good policy" hypothesis. For example, the evidence on US policy activism reported in Cogley and Sargent (2005) and based on an extended model is less clear cut than the authors' earlier work. Primiceri (2005) suggests that 'planting Greenspan in the 1970s' would have had little impact on inflation during that period. Similarly Sims and Zha (2006) show that a model that allows for variation in the volatility of shocks fits US data better than a model that allows for a change in the monetary policy rule.[1]

However, the arguments in Castelnuovo and Surico (2005) and Benati and Surico (2007) suggest that these results may be the outcome of model mis-specification. In particular, these studies argue that the amount of information incorporated in these VAR models is relatively limited. Typically, the VAR models used in these studies (e.g. Cogley and Sargent (2005)) consist of three or four variables – usually a short term interest rate, output

[1] Note that this evidence is mostly based on time varying VAR models. Based on a New Keynesian DSGE model Lubik and Schorfheide (2004) provide evidence in favour of a policy shift in the United States.

growth and inflation. This feature has two potential consequences. Firstly, missing variables could lead to biases in the reduced form VAR coefficients. Secondly, the omission of some variables could hinder the correct identification of structural shocks. For example, Lubik and Schorfheide (2004), show that when the Taylor principle is not satisfied (i.e. the monetary authority accommodates inflationary pressure), the dynamics of the economy in a DSGE model are characterised by a latent variable. Lubik and Schorfheide (2004), Castelnuovo and Surico (2005) and Benati and Surico (2007) show that this latent variable is a function of inflation expectations and that the *interpretation* of structural VAR estimates may be misleading if expectations are not taken into account directly.

The aim of this paper is to use a time-varying VAR model that is less susceptible to this problem. In particular, this paper examines the changing dynamics of the US economy using a time-varying VAR model that incorporates information extracted from the term structure of interest rates. We augment a standard time-varying VAR model with factors extracted from the term structure. These factors summarise information about the level and shape of the yield curve and, as our results show, the level of the yield curve is strongly correlated with measures of inflation expectations. By using this augmented VAR model, our aim is to minimise the possible omitted variable bias referred to above.

The basic premise of our paper is in line with a number of recent studies that have used similar models to highlight the link between the yield curve and the macroeconomy. Recent examples include, Diebold, Rudebusch and Aruoba (2006) and Diebold and Li (2006) for the US and Lindholdt *et al.* (2006) for the UK. In addition, Cogley (2004), Lindholdt *et al.* (2006), Rudebusch and Wu (2006) and Diebold, Li and Yue (2006) show that the dynamics of the yield curve (in the US and the UK) may have changed over time.

The contribution of this paper is twofold. Firstly, the analysis in this paper brings together the latest developments in the macro-finance literature on the bidirectional feedback between the yield curve and the economy, and the observation that both sides of this relationship have been historically characterized by substantial instabilities. We specify the link between macro and finance as in the Nelson-Siegel generalization by Diebold, Rudebusch and Aruoba (2006), and model both the interactions and the evolution of the factors using time-varying coefficients and stochastic volatilities. Secondly, to our knowledge, this is the first paper that provides systematic investigation into shifts in the link between the economy and the yield curve for the US. In addition, this paper is one of the first to use information from the yield curve in an analysis of the 'great moderation'.

The main results from our analysis can be summarized as follows. The level factor is highly correlated with the one-year ahead inflation forecasts of the Fed Greenbook and the Survey of Professional Forecasters and hence can be thought of as proxying for inflation expectations. Monetary policy shocks account for most of the persistence in inflation and the level factor around 1974 and the beginning of the 1980s. The persistence of the federal funds rate is driven by changes in shocks to the level of the yield curve, whereas the variance is explained by monetary policy shocks. Deviations from the expectations hypothesis are rare and coincided with two well-known episodes of US monetary policy history: the credibility building of the new Fed policy regime initiated with Paul Volcker's appointment, and the sequence of 7 consecutive 50 basis points rate cuts in the early 1990s.

The paper has four sections. Section 2 describes a generalization of the Nelson-Siegel model using a FAVAR with time-varying coefficients and stochastic volatilities. The empirical results are presented in Section 3. The evidence on the expectation hypothesis is revisited in Section 4. Section 5 concludes. Details on the estimation procedure are provided in the Appendix A.

2 Modelling yield curve and macro dynamics

Recent work by Cogley and Sargent (2005) and Sims and Zha (2006) has shown that the dynamics of key macroeconomic variables have evolved significantly over time. In addition, several studies (e.g. Lindholdt *et al.* (2006) and Rudebusch and Wu (2006)) have shown that the dynamics of the yield curve may be time-varying. While the recent macro-finance literature has convincingly advocated the case for the existence of a bidirectional link between the term structure and the rest of the economy, to the best of our knowledge no studies have yet tried to model time variations in the yield curve and the economy simultaneously. To this end, we design a generalization of Nelson-Siegel interpolation in the context of a FAVAR model with time-varying coefficients and stochastic volatilities. It is worth emphasizing that we also allow for time variation in the cross correlations between macro and financial factors.

2.1 A generalisation of Nelson-Siegel model

Our model is a generalisation of the latent dynamic factor model used in-Diebold, Rudebusch and Aruoba (2006). The observation equation of the state space system is based on the yield curve model developed by Nelson and Siegel (1987):

$$y(\tau) = L_t + \frac{1-e^{-\tau\lambda}}{\tau\lambda} S_t + \left(\frac{1-e^{-\tau\lambda}}{\tau\lambda} - e^{-\tau\lambda}\right) C_t + e\left(\tau\right)_t \tag{1}$$

where $y(\tau)$ denotes yields with maturity τ and L_t, S_t and C_t denote the (unobserved) level, slope and curvature factors.

Factor dynamics are given by the following time varying VAR

$$Z_t = \alpha_t + \sum_{p=1}^{P} \beta_{t,p} Z_{t-p} + v_t \tag{2}$$

where $Z_t = \{L_t, S_t, C_t, Y_t, \pi_t, R_t\}$ denotes the data matrix and $v_t = \Omega_t^{1/2}\omega_t$ with $\omega_t \sim N\left(0, I_6\right)$. Note that along with the unobserved factors, Z_t contains three macroeconomic variables: the growth rate of industrial production (Y_t), annualized inflation (π_t) and the federal funds rate (R_t).

Following Cogley and Sargent (2005) amongst others, we postulate a random walk for the evolution of the VAR coefficients:

$$\Phi_t = \Phi_{t-1} + \eta_t \tag{3}$$

where $\Phi_t = \left[\alpha_t, \beta_{t,p}\right]$.

The covariance matrix of the VAR innovations, v_t, is factored as

$$VAR\,(v_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})' \tag{4}$$

The time-varying matrices H_t and A_t are defined as:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 & 0 & 0 \\ 0 & 0 & 0 & h_{4,t} & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{5,t} & 0 \\ 0 & 0 & 0 & 0 & 0 & h_{6,t} \end{bmatrix} \tag{5}$$

$$A_t \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 & 0 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 & 0 & 0 \\ \alpha_{51,t} & \alpha_{52,t} & \alpha_{53,t} & \alpha_{54,t} & 1 & 0 \\ \alpha_{61,t} & \alpha_{62,t} & \alpha_{63,t} & \alpha_{64,t} & \alpha_{65,t} & 1 \end{bmatrix} \tag{6}$$

with the $h_{i,t}$ evolving as geometric random walks,

$$\ln h_{i,t} = \ln h_{i,t-1} + u_t$$

Following Primiceri (2005), we postulate that the non-zero and non-unit elements of the matrix A_t evolve as driftless random walks,

$$\alpha_t = \alpha_{t-1} + \varepsilon_t \ , \tag{7}$$

and we assume that the vector $[e(\tau)_t', v_t', \eta_t', \varepsilon_t', u_t']'$ is distributed as

$$\begin{bmatrix} e(\tau)_t \\ v_t \\ \eta_t \\ \varepsilon_t \\ u_t \end{bmatrix} \sim N(0, V), \tag{8}$$

$$V = \begin{bmatrix} R & 0 & 0 & 0 & 0 \\ 0 & \Omega_t & 0 & 0 & 0 \\ 0 & 0 & Q & 0 & 0 \\ 0 & 0 & 0 & S & 0 \\ 0 & 0 & 0 & 0 & G \end{bmatrix} \text{ and } G = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_6^2 \end{bmatrix} \tag{9}$$

Note that by ordering the federal funds rate last and imposing the normalization (6) we are also identifying the monetary policy shock as the only shock that does not have a contemporaneous effect on the other variables in the system. As noted by Primiceri (2005), such ordering is also consistent with the fact that the yields are dated at the beginning of each month. Ordering the level factor first implies that no other shock in the system has a contemporaneous effect on the determinants of the level of the yield curve.

The model in equations (1) to (9) provides a flexible framework for analysing the interaction between the yield curve and macroeconomy. In particular, the model allows us to investigate how this interaction has evolved over time while simultaneously accounting for changes in the volatility of the shocks. In addition, the Nelson–Siegel framework imposes some restrictions on the yield curve that may help to improve the fit of the model[2]– it guarantees positive forward rates at all horizons and a discount factor that approaches zero as maturity increases. Note, however, that our model does not incorporate some of the additional structure seen in recent macro-finance models (e.g. Ang and Piazzesi (2003)). In particular, our model does not incorporate no-arbitrage restrictions. This is primarily because of technical constraints–imposing these restrictions in a time-varying framework is still

[2]Relative to a model which includes *unrestricted* factors from the yield curve.

a task in progress. A drawback of this simplification is that we cannot estimate the term premium directly. To the extent that our yield-macro model with time-varying parameters and stochastic volatility is correctly specified, however, the residuals of the observation equations can be interpreted as estimates of the term-premia.[3]

2.2 Estimation

The model in equations 1 to 9 is estimated using the Bayesian methods described by Kim and Nelson (2000).[4] In particular, we employ a Gibbs sampling algorithm that approximates the posterior distribution. The algorithm exploits the fact that given observations on Z_t the model is a standard time-varying parameter model.

A detailed description of the prior distributions and the sampling method is given in the Appendix. Here we summarise the basic algorithm which involves the following steps:

1. Given initial values for the factors, simulate the VAR parameters and hyperparameters

    - The VAR coefficients ϕ_t and the off-diagonal elements of the covariance matrix α_t are simulated using the methods described by Carter and Kohn (2004)
    - The volatilities of the reduced form shocks H_t are drawn using the date by date blocking scheme introduced by Jacquier *et al.* (2004).
    - The hyperparameters Q and S are drawn from an inverse wishart distribution while the elements of G are simulated from an inverse gamma distribution.

[3] Note also that the model is silent about the role of the real term structure, an aspect that is potentially important in terms of the great moderation.

[4] Ang, Dong and Piazzesi (2005) use Bayesian methods to estimate a time-invariant, no-arbitrage model.

2. Given initial values for the factors, draw the covariance matrix R.

    - Note that we calibrate the parameter λ in the observation equation 1 to the value used in Diebold and Li (2006). This is primarily because estimating λ involves estimating a non-linear system of equations which complicates our algorithm considerably. It is precisely for this reason that Diebold and Li (2006) set $\lambda = 0.0609$. Note that the value of this parameter determines the maturity at which the loading on the curvature factor achieves it maximum. As two or three year maturities are commonly used in this regard, Diebold and Li (2006) set $\lambda = 0.0609$ which is the value that maximizes the loading on the curvature factor at 30 months. Given data on Z_t and $y(\tau)$ and a value for λ, the variances are then simulated from an inverse gamma distribution.

3. Simulate the factors conditional on all the other parameters

    - This is done by employing the methods described by Kim and Nelson (1999b).

4. Go to step 1.

We use 60000 Gibbs sampling replications and discard the first 56000 as burn-in. The moments of the retained draws show little fluctuation providing evidence in favour of convergence of the Gibbs sampling algorithm. Results are available upon request.

3 Results

This section describes the empirical results of the generalized Nelson-Siegel model developed in Section 2. We report estimates of the factors and their stochastic volatilities, and decompose the variance of the variables in our FAVAR.

3.1 Factors

We consider U.S. Treasury yields with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months. The yields are derived from bid/ask average price quotes, from January 1970 through December 2000, using the unsmoothed Fama and Bliss (1987) approach.[5] To initialize the factors and the autoregressive parameters, we use data from McCulloch and Kwon (n.d.) for yields with maturities of 3, 6, 12, 24, 60 and 120 months over the period January 1959 to December 1969.[6] Inflation is measured as monthly changes in the consumer price index, the policy instrument is the federal funds rate and, following Evans and Marshall (2001), the measure of real activity is industrial production which, unlike the capacity utilization rate, is available since 1959.

Figure 1 presents the estimates of the factors together with the central 68% posterior bands. In addition, we also show 'empirical counterparts' of the factors. These 'empirical counterparts' of the factors can be thought of as proxies for the level, slope and curvature of the yield curve and are calculated as simple functions of the yields at different maturities:

$$\begin{array}{cc} Level: & y_t(3) + y_t(24) + y_t(120)/3 \\ Slope: & y_t(3) - y_t(120) \\ Curvature: & 2y_t(24) - y_t(3) - y_t(120) \end{array}$$

These proxies or counterparts are regularly used by finance practitioners and provide a good cross-check on the Bayesian estimates of the yield curve factors.

The top left panel shows the level factor (dark line), the bands (red lines) and the counterpart (blue line). The correlation between the level factor and

[5] This is the data-set employed by Diebold, Rudebusch and Aruoba (2006). We use this data as it is comprehensive in its time-series and maturity coverage. We require the latter for accurate estimation of the yield curve factors. Note that an investigation by Bliss (1996) concludes that the unsmoothed Fama and Bliss (1987) method of yield curve estimation performs well in comparison with other existing techniques.

[6] The data are available at http://www.econ.ohio-state.edu/jhm/ts/mcckwon/mccull.htm . Note that we obtain very similar results using the initial sample of alternative lengths.

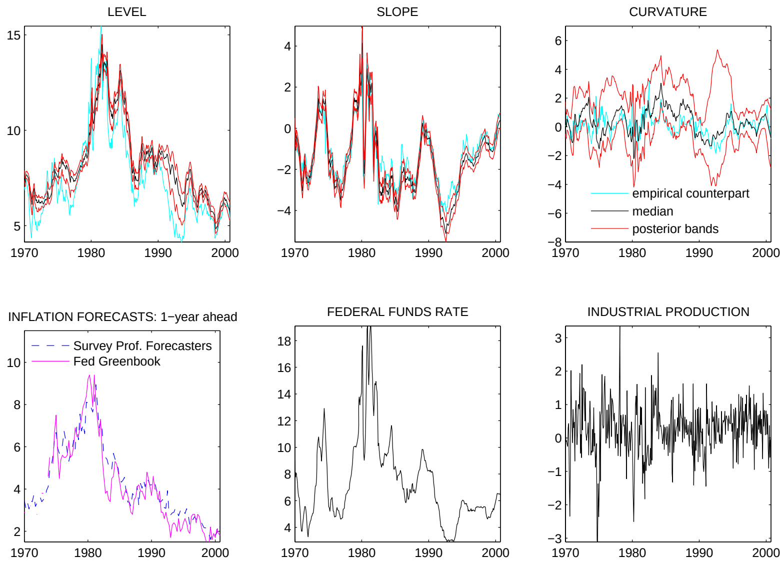


Figure 1: Factors and empirical counterparts

its counterpart is remarkable: 0.91, which is 14% higher than the number obtained by Diebold, Rudebusch and Aruoba (2006) using a time-invariant yield-macro model.

The bottom left panel reports two measures of inflation expectations: the Survey of Professional Forecasters (SPF) and the Fed Greenbook forecasts one-year ahead.[7] The correlation between our estimated level factor and the forecasts of the SPF, which is available at quarterly frequency over the full sample, is 0.69. The comovements with the Greenbook forecasts are apparent too, thereby confirming a strong association between the level of the yield curve and inflation expectations (see Kozicki and Tinsley (2001) and Hordahl *et al.* (2006)).

3.2 Volatilities

Homoskedasticity is a recurrent assumption in the macro-finance literature. In this section, we show that, in fact, significant time variation also characterizes the evolution of volatilities of the observed and unobserved factors. The first row of Figure 2 displays the square root of the stochastic volatilities of the yield curve components. The standard deviations of the level and slope innovations show a stable path until 1979, then a rapid increase up to 1981, and finally a smooth decline back to the pre-1979 values by the second-half of the 1980s.

The stochastic volatilities of inflation and output in the left and right panels of the bottom row reach their highest values around 1974. Since 1985, both series have been more stable fluctuating around considerably lower values: this pattern has become known as the Great Moderation and is extensively discussed in Bernanke (20 Feb 2004). Finally, the volatility of the monetary policy shock in the middle panel of the bottom row is characterized by two peaks over a downward sloping trend. The largest peak

[7] These forecasts are available on the web site of the Federal Reserve Bank of Philadelphia, respectively at http://www.phil.frb.org/econ/spf/spfmed.html (SPF), and http://www.phil.frb.org/econ/forecast/croushoresdatasets.html (Greenbook).

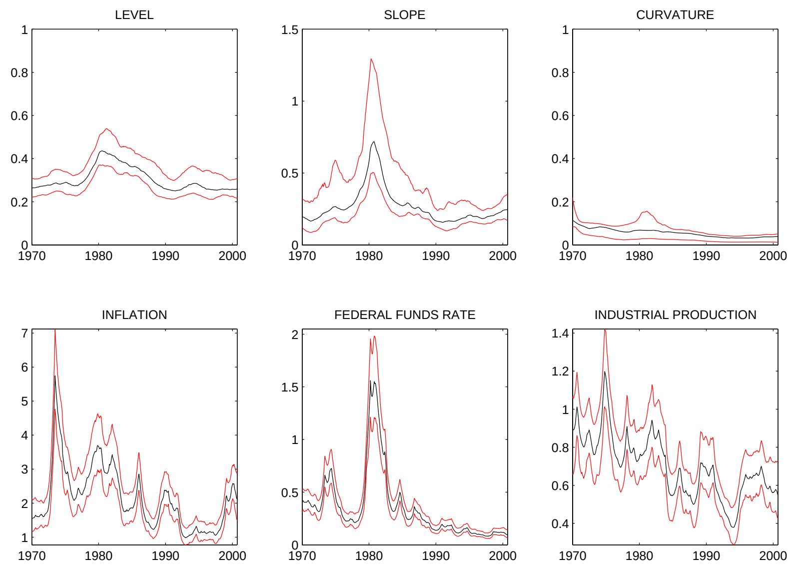


Figure 2: Standard deviations of the residuals

in the mid 1980s coincided with the onset of Paul Volcker's chairmanship of the Federal Reserve when targets for non-borrowed reserves were introduced.

3.3 Variance decomposition

In this section we decompose the unconditional variance of each endogenous variable in the FAVAR into contributions from the monetary policy shock and shocks to the level of the yield curve at each point in time and at different frequencies. This decomposition allows us to examine how the contribution of these shocks has evolved over time and whether these shocks play a role in determining the long-run variation in the endogenous variables. Chart 3 presents the contribution of the monetary policy. Note that the height of the surface in each panel represents the contribution. The values on the Y-axis denote the frequencies with values close to zero representing long run movements.

The results on the contributions of the monetary policy shock can be summarized as follows. First, at the beginning of the 1980s the monetary policy shock explained more than 50% of the variances of the level factor, inflation and industrial production at low frequency. This is consistent with the findings of Canova and Gambetti (2006) that most of the high inflation persistence of those years was attributable to the monetary policy shock. Second, during the same period, the monetary policy shock was also an important source of persistence in the level factor. Third, the monetary policy shock has affected the variance of the policy rate mainly at business cycle and high frequencies.

The contributions of the level factor shock to the normalized spectra are shown in Chart 4. As the level factor moves closely with inflation expectations, this shock could be capturing a change in inflation expectations, possibly induced by an unanticipated or imperfectly credible shift in the central bank implicit inflation target. Interestingly, the shock to the level of the yield curve accounts for most of the persistence of inflation and in-

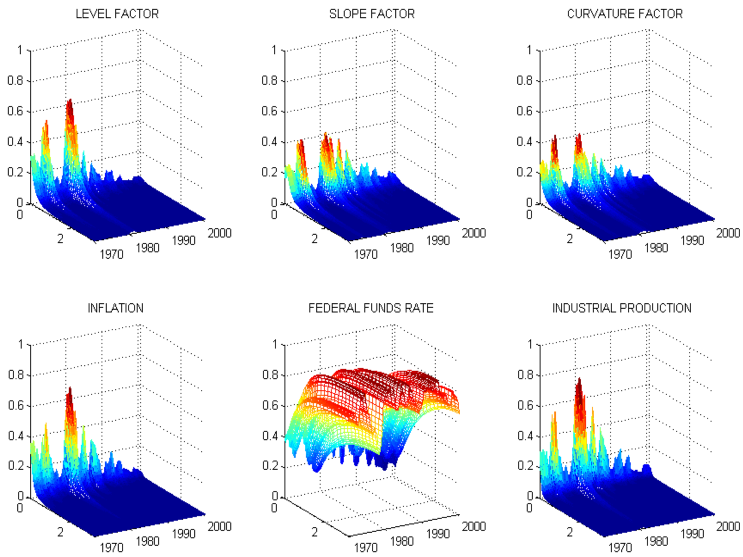


Figure 3: Variance decomposition - contribution of the monetary policy shock

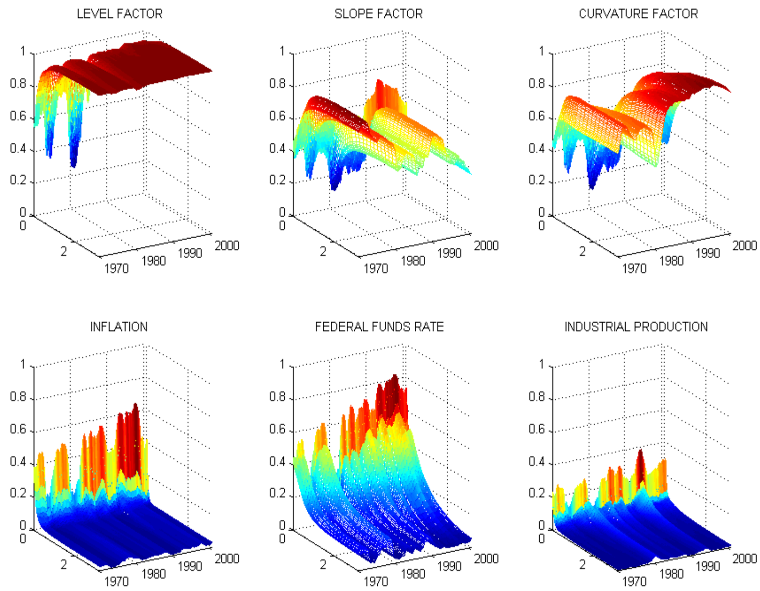


Figure 4: Variance decomposition - contribution of the level factor shock

flation expectations in periods when, as in the most recent past, inflation persistence has been low.

4 Another look at the expectations hypothesis

Expectations theory predicts that movements in the long rates are due to movements in expected future short rates. Any differences between actual long rates and expected short rates reflect a term premium, which is typically assumed to vary across maturities and over time.

A substantial body of work has concentrated on testing the expectations hypothesis, with evidence in favour of the theory hard to find (see for instance Campbell, Lo and MacKinley, 1997). Our framework allows us to revisit this problem using a time-varying generalization of Nelson-Siegel model. In particular, our framework allows us to assess whether (the lack of) time-variation in the dynamics of both yield curve and macroeconomic variables can account for the failure of the expectations hypothesis documented in earlier contributions: apparent deviations from the expectations theory may reflect neglected parameter instability.

4.1 Model with time-varying coefficients

The Bayesian approach taken in this paper provides us with a very natural way of accounting for parameter uncertainty when constructing bands around the central predictions of the expectations hypothesis.[8] In a similar vein, Cogley (2004)estimates a bivariate VAR in the tradition of Campbell and Shiller (1991) allowing for drifting parameters.

The Expectations Hypothesis (EH) consistent (pure discount) bond yield is:

[8] In a classical framework, a time-varying parameter model imposes a so heavy computantional burden as to make unfeasible considering parameter uncertainty (see Carriero, Favero and Kaminska, 2006 for an alternative procedure based on recursive estimations).

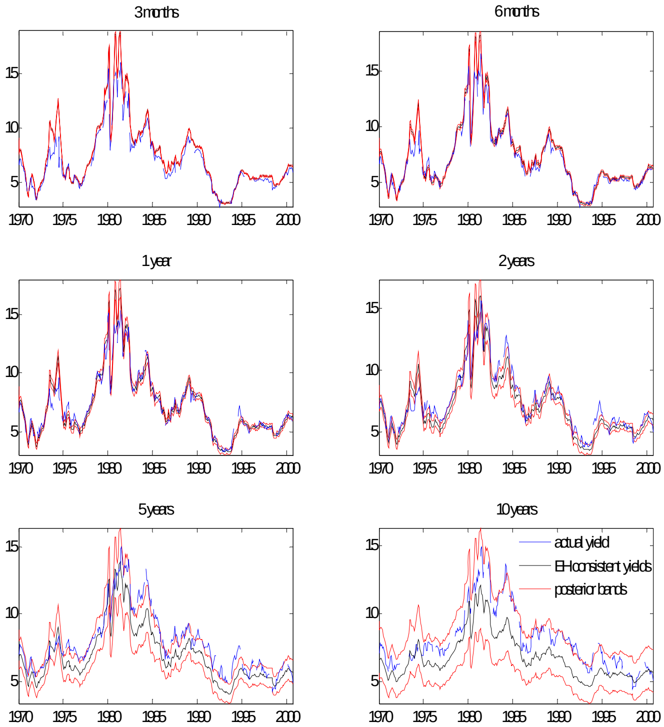


Figure 5: Actual vs. Expectations Hypothesis consistent yields: model with time-varying coefficients

$$y_t\left(\tau\right)^{EH} \equiv \left(\frac{1}{\tau}\right)\sum_{i=0}^{\tau-1} E_t y_{t+i}\left(1\right) + c_\tau \tag{10}$$

where τ and c_τ represent the maturity and the term premium.

In Figure 5, we compare actual yields with the theoretical yields constructed using (10) with $c_\tau = 0$. At each point in time, and conditional on the information available at time t, we compute the h-months ahead forecasts of the one-month yield for $h = 1, ..120$ using the time-varying model (1)-(3), (8)-(9). Note that although this exercise does not amount to a formal test of

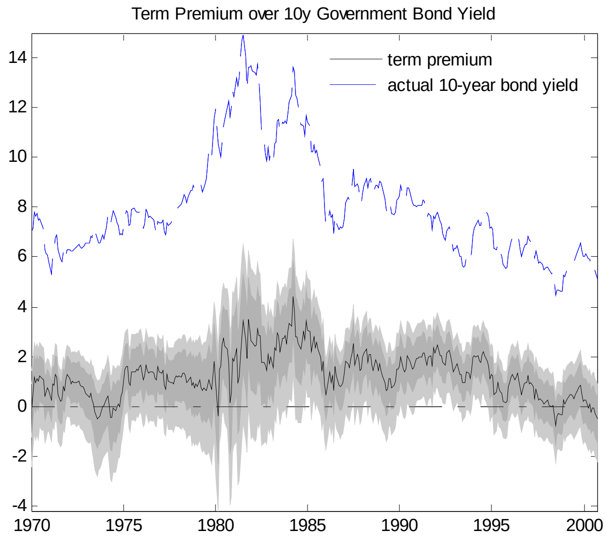


Figure 6: Term premium over the 10-year government bond yield

the expectations hypothesis, it does allow us to assess if the results from our time-varying FAVAR are consistent with the predictions of the hypothesis. In addition, we can carry out the same exercise for a time-invariant model to infer the relative performance of our extended model.

Figure 5 shows that the theoretical yields track actual yields remarkably well, especially at short maturities. The actual 5- and 10-year rates rarely fall outside the 90% posterior bands, with the largest deviations associated with the first half of the 1980s. A comparison of our results with those from the fixed-coefficient model in Diebold, Rudebusch and Aruoba (2006)suggests that time variations in the yield curve dynamics and monetary policy are, indeed, important for improving the accuracy of the forecasts based on the expectations hypothesis.

Figure 6 provides a closer inspection of the results for the 10-year bond yields by plotting the term premium, defined as the difference between actual and EH consistent yields, together with the central 68% and 90% posterior bands (dark and light grey areas). The term premium is positive for most of the sample and the zero is outside the bands in only two episodes. The first episode took place in the first half of the 1980s and coincided with the credibility building of the new policy regime initiated with the appointment of Paul Volcker as Fed Chairman. The second episode began with the early 1990s recession and continued until 1994 when the Fed reversed the policy path after 7 consecutive cuts which halved the federal funds rate from 7% to 3.5% in a few months. Excluding the early 1980s episode, which stands out for magnitude in Figure 6, movements in the term premium are modest.

4.2 Model with fixed coefficients

A direct way of assessing the significance of time variation in the parameters of the model is to compare the results in Figure 5 with the Expectations Hypothesis consistent yields generated by a model featuring fixed coefficients[9].

As stochastic volatility enters the forecasts of the endogenous variables in neither specifications, the difference in the projections of the two models will provide us with a metric for evaluating whether the coefficients do change significantly over time.

Chart 7 is the time-invariant counterpart of Chart 5. It should be noted that, similarly to the results on variance decomposition, the findings in this section focuses on the evolution of the coefficients, and therefore any change in the dynamics of the yield curve can only come from time-variation in the parameters of the model.

A comparison of the Expectations Hypothesis consistent yields implied by the two FAVARs lead us to two conclusions. First, the coefficients are characterized by a significant amount of time-variation as the theoretical

[9] Appendix B shows the estimated yield factors from this model. The estimates of the level, slope and curvature factors implied by the FAVAR with fixed coefficients are less precise than the estimates implied by the time-varying model. The correlation between the level factor and its empirical counterpart in the fixed-coefficient model is 0.80, as opposed to 0.91 in the time-varying model.

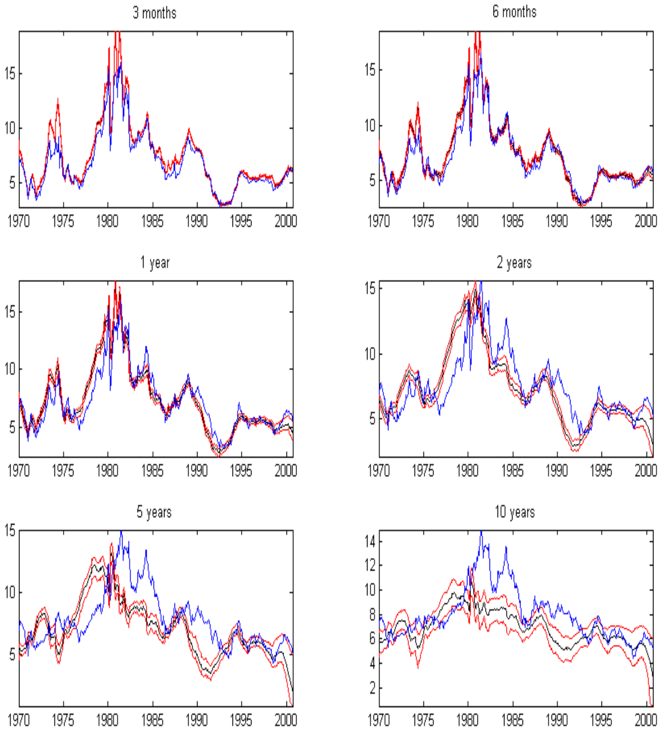


Figure 7: Actual vs. Expectations Hypothesis consistent yields: model with fixed coefficients

yields obtained under the time-invariant model are significantly different from actual yields in far more occasions than under the time-varying parameter specification in Chart 5. Second, the residual c_τ in equation (10) is far more volatile for the fixed-coefficient model than for the FAVAR with drifting coefficients, thereby suggesting that time-varying dynamics matter for understanding the evolution of the US yield curve.

5 Conclusions

This paper has studied the evolution of the link between the yield curve and the U.S. economy. We have developed a macro-finance FAVAR model with time-varying coefficients and stochastic volatilities based on the Nelson-Siegel generalization by Diebold, Rudebusch and Aruoba (2006).

The monetary policy shocks of the early 1980s were the main determinants of the persistence of inflation and the level of the term structure during those years. The tendency of the Fed to smooth movements in the federal funds rate reflects changes in the yield-curve-embodied inflation expectations. The only two significant failures of the expectations hypothesis are associated with the credibility building of Volcker's Fed Chairmanship and the sharp reduction of the policy rate at the beginning of the 1990s.

In a stimulating contribution, Diebold, Li and Yue (2006) develop a model of global yield curve dynamics and report sub-sample estimates which suggest that the properties of the global factors have changed remarkably over the last twenty years. A promising avenue for future research will be to explore the temporal evolution of the link between the yield curves of several countries and the global economy.

Appendix A: Priors and Estimation

Consider the time-varying VAR model given by equations (1) and (2).

Prior Distributions and starting values

Factors

We center our prior on the factors (and obtain starting values) by using the least squares estimator employed by Diebold and Li (2006). The prior covariance of the states ($P_{0/0}$) is set equal to an identity matrix.

The prior on the diagonal elements of R is assumed to be inverse gamma:

$$R_{ii} \sim IG(R_{ii0}, 1)$$

where $R_{ii0} = 1$.

VAR coefficients

The prior for the VAR coefficients is obtained via a fixed coefficients VAR model estimated over the sample 1959:01 to 1969:12 using data for yields at $\tau = 3, 6, 12, 24, 60, 120$ along with the macroeconomic variables. Estimates based on initial samples of alternative length yield very similar results. Φ_0 is therefore set equal to

$$\Phi_0 \sim N(\hat{\phi}^{OLS}, V^{OLS})$$

Elements of H_t

Let $\hat{v}^{ols}$ denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data described above. The prior for the diagonal elements of the VAR covariance matrix (5) is as follows:

$$\ln h_0 \sim N(\ln \mu_0, I_6 \times 10)$$

where μ_0 are the diagonal elements of $\hat{v}^{ols}$.

Elements of A_t

The prior for the off diagonal elements A_t is

$$A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$$

where $\hat{a}^{ols}$ are the off diagonal elements of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$.

Hyperparameters

The prior on Q is assumed to be inverse Wishart

$$Q_0 \sim IW\left(\bar{Q}_0, T_0\right)$$

where $\bar{Q}_0$ is assumed to be $var(\hat{\phi}^{OLS}) \times 10^{-5}$ and T_0 is the length of the sample used for calibration.

The prior distribution for the blocks of S is inverse Wishart:

$$S_{i,0} \sim IW(\bar{S}_i, K_i)$$

where $i = 1..6$ indexes the blocks of S. $\bar{S}_i$ is calibrated using $\hat{a}^{ols}$. Specifically, $\bar{S}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by 10^{-3}.

Following Cogley and Sargent (2005), we postulate an inverse-Gamma distribution for the elements of G,

$$\sigma_i^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right)$$

Simulating the Posterior Distributions

Factors and Factor Loadings

This closely follows Bernanke *et al.* (2005).

Factors Conditional on a value for λ and draws for the remaining parameters, the factors are drawn using the methods of Carter and Kohn (2004). For details see Kim and Nelson (1999a).

Elements of R As in Bernanke *et al.* (2005) R is a diagonal matrix. The diagonal elements R_{ii} are drawn from the following inverse gamma distribution:

$$R_{ii} \sim IG\left(\bar{R}_{ii}, T+1\right)$$

where

$$\bar{R}_{ii} = \hat{e}\left(\tau\right)' \hat{e}\left(\tau\right) + R_{ii0}$$

and $\hat{e}\left(\tau\right) = y(\tau) - \left(\hat{L}_t + \frac{1-e^{-\tau\lambda}}{\tau\lambda}\hat{S}_t + \left(\frac{1-e^{-\tau\lambda}}{\tau\lambda} - e^{-\tau\lambda}\right)\hat{C}_t\right)$ with $\hat{L}_t, \hat{S}_t, \hat{C}_t$ denoting a draw of the three factors. $\lambda = 0.0609$

Time Varying VAR

Given an estimate for the factors, the model becomes a VAR model with drifting coefficients and covariances. This model has become fairly standard in the literature and details on the posterior distributions can be found in a number of papers including Cogley and Sargent (2005), and Primiceri (2005). Here, we describe the algorithm briefly.

VAR coefficients Φ_t As in the case of the unobserved factors, the time-varying VAR coefficients are drawn using the methods described by Carter and Kohn (2004).

Elements of H_t Following Cogley and Sargent (2005), the diagonal elements of the VAR covariance matrix are sampled using the methods described by Jacquier *et al.* (2004).

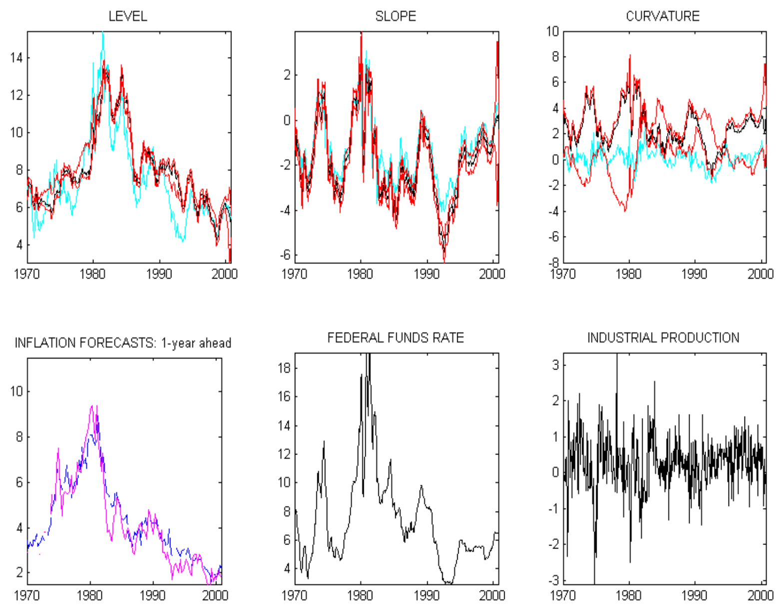
Element of A_t Given a draw for Φ_t the VAR model can be written as

$$A_t'\left(\tilde{Z}_t\right) = u_t$$

where $\tilde{Z}_t = Z_t - \alpha_t - \sum_{p=1}^{P} \beta_{t,p} Z_{t-p} = v_t$ and $VAR(u_t) = H_t$. This is a system of equations with time-varying coefficients and given a block diagonal form for $Var(\tau_t)$ the standard methods for state space models described by Carter and Kohn (2004)can be applied.

VAR hyperparameters Conditional on Z_t, $\phi_{l,t}$, H_t, and A_t, the innovations to $\Phi_{l,t}$, H_t, and A_t are observable, which allows us to draw the hyperparameters—the elements of Q, S, and the σ_i^2—from their respective distributions.

Appendix B: Estimated factors from a time-invariant model



Factors and empirical counterparts in the fixed-coefficients model

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