External MPC Unit

Discussion Paper No. 36
Did output gap measurement improve over time?

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Adrian Chiu[1] and Tomasz Wieladek[2]

Abstract

We study whether the accuracy of real-time estimates of the output gap produced by the OECD has improved over time by examining a panel dataset on real-time output gap revisions for 15 countries from 1991 Q1 — 2005 Q4. We use a simple panel data regression and a state space model, with common and country-specific factors, to assess whether the mean of the absolute value of output gap revisions has changed. We also apply a Mincer-Zarnowitz regression to determine whether the composition of measurement errors has shifted. Surprisingly, we do not find evidence that the size of output gap revisions has decreased over time; on the contrary, they seem to have increased. But the fraction of measurement errors due to omitted contemporaneous information has declined. This suggests that while the OECD's accuracy in output gap measurement has failed to improve, its estimates are becoming less noisy.

Keywords: Output gap, real-time data, dynamic common factor model.

JEL classification: E01, E32.

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1. Introduction

As is well known, real-time estimates of the output gap are often subject to considerable revision over time.[1] This unreliability of real-time output gap measures has been widely documented for the US (Orphanides and Van Norden, 2002), the UK (Nelson and Nikolov, 2003), Canada (Coyen and Van Norden, 2005), Norway (Bernhardsen, Eitrheim, Jore and Roisland, 2004), as well as several other OECD countries (Tosetto, 2008). This is a serious problem for monetary policy makers; for example, as Orphanides (2001) documents, an output gap measurement mistake led the Federal Reserve to pursue policies that eventually led to the Great Inflation of the 1970s. Similarly, Nelson and Nikolov (2003) document that output gap measurement mistakes led to interest rate settings that deviated about 500 basis points from what would have been consistent with the actual output gap during the 'Lawson boom' in the UK.[2]

At the same time, there are reasons to believe that the accuracy of real-time output gap estimates may have improved over time. In particular, a number of research papers raised awareness of the scope and implications of these measurement problems in the 1990s. This may have led to an increase in resources to solve them. Modern computing and communication technology also allowed analysts to incorporate a much greater amount of contemporaneous information into their initial output gap estimates, probably helping them to minimise measurement errors associated with data noise. Finally, several OECD countries introduced various business surveys reporting on spare capacity in various parts of the economy; these are likely to improve initial output gap estimates. In other words, as a result of these methodological changes, the size of past output gap mistakes may not be a good guide to the present.

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[1] Orphanides and Van Norden (2002) document three main reasons for output gap measurement errors. First and most obviously, real GDP data is often revised – sometimes substantially – after the initial estimates are published. Second, there is considerable uncertainty in the level of potential output at any point in time. Finally, researchers might also change their methodology for estimating potential GDP, or their assumptions within chosen methods, over time

[2] The Lawson boom was an unsustainable period of fast real GDP growth and rising prices in the UK in the late 1980s, as a result of low interest rates. This time period was named after Nigel Lawson, the UK's Chancellor of the Exchequer at the time.

To assess whether this is the case, we examine changes in the absolute values of output gap revisions (hereafter 'absolute output gap revisions') in a panel data set of 15 OECD countries from 1991Q1 to 2005Q4. Initially, we use a simple panel data model to test whether the mean of absolute output gap revisions has changed over time; we do this by estimating the model over two different time subsamples and a rolling window of 16 quarters. We then use a more flexible dynamic common factor model, with both a common factor and country-specific factors, to verify these results. Finally, we examine whether the decomposition of output gap revisions between 'news' and 'noise' has also changed over time[3]. We argue that improvement in information processing associated with modern communication and computing technology should probably reduce the size of measurement errors due to delayed inclusion of information available at the time. To test this hypothesis, we estimate a Mincer-Zarnowitz (1969) forecast efficiency regression, as before, across two different sub-samples and on a rolling window of 16 quarters.

Our results fail to find evidence of a decline in the size of absolute output gap revisions over time; on the contrary, these revisions seem to have increased. But we do find that the fraction of output gap revisions due to the delayed inclusion of contemporaneous information has indeed declined. This suggests that, while statisticians did become better at processing contemporaneous information into real-time estimates of the output gap, these estimates did not improve in accuracy, perhaps due to the difficulty of disentangling the trend and the cycle.

The rest of this paper is structured as follows. Section 2 describes the data and section 3 the empirical methodology. Section 4 presents the results and section 5 concludes.

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[3] Revisions can occur either as a result of delayed inclusion of information available at the time initial output gap estimates were made (the 'noise' view), or new information which was not available at the time (the 'news' view).

2. Data

Our data are taken from the OECD's real-time output gap database, first published in August 2008.[4] This database contains real-time output gap estimates for 15 OECD countries for the period from 1991 Q1 onwards. We treat the output gap figures published in the 2010 Q4 OECD Economic Outlook as our 'final' vintage of output gap data. Output gap revisions are constructed by subtracting the real-time estimate of the output gap (hereafter, 'real-time vintage') at each point in time from its 2010Q Q4 estimate ('final vintage'). We judge that real-time vintage output gap estimates towards the end of our sample could still be significantly revised in the future, which may introduce bias in our estimates. To mitigate this problem we drop any observation past 2005Q4.

The OECD defines the output gap as actual real GDP minus potential real GDP, as a percentage of potential real GDP (Tosetto, 2008). Potential real GDP is defined as the level of output that an economy can produce at a constant rate of inflation. However, unlike actual GDP, it cannot be directly observed from economic data. The OECD therefore indirectly infers the output gap from economic data using a 'production function' approach, rather than relying on mechanical time-series filters to measure potential output. This could mean that output gap measurements include the judgement of OECD country analysts, similar to the process followed in many central banks today.[5] Another advantage of using OECD data for this investigation is that the similarity in methodology makes output gap estimates comparable across countries.

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[4] http://www.oecd.org/document/1/0,3343,en_2649_33715_41054465_1_1_1_1,00.html
[5] For details on this method, see Giorno, Richardson and Roseveare and Van Den Noord (1995).

Figure 1: Distribution of output gap revisions

Mean = 0.36
Std. Dev. = 1.25
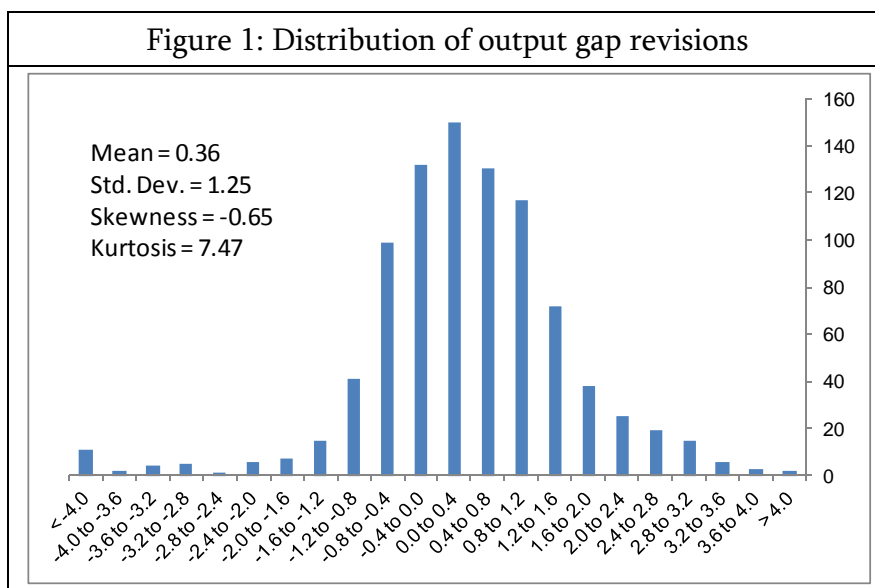Skewness = -0.65
Kurtosis = 7.47

Figure 1 shows the distribution of output gap revisions in the OECD database for all 15 countries, for the period 1991 Q1 to 2005 Q4. 64% of all revisions have a positive value, suggesting that a positive revision is more likely than a negative one. It is centred around a mean of 0.36, which is roughly 43% of the average size of real-time output gap estimates in our sample.

3. Methodology

In this study we aim to assess whether the accuracy of output gap estimates made in real time has improved over time. We measure accuracy as the absolute value of output gap revisions, since we are interested in the size rather than the sign of any error. First we test whether the size of the mean of absolute output gap revisions has changed between the 1990s and 2000s with a simple panel data model. To provide greater detail on how the mean of absolute value output gap revisions has changed over time, we re-estimate the same model over a rolling window of 16 quarters. We also estimate rolling regression coefficients for the same model to assess how the mean of the absolute output gap revisions has changed over time. We then use a more sophisticated dynamic common

factor model, with both a common factor and individual country-specific factors, to verify these results.

In the final section, we examine whether the composition of revisions has changed over time with a Mincer-Zarnowitz regression approach. In particular, one would have expected that with the introduction of modern communication and computing technology, and the associated improvements in information processing, the fraction of measurement error due to the delayed inclusion of contemporaneous information should have declined over time. As before, we first estimate the Mincer-Zarnowitz regression model over two subsamples and then on a rolling window of 16 quarters.

3.1 Panel data model

We begin to explore our data set with a simple panel data model. We start by estimating the following regression equation[6]:

$$Y_{i,t} = \alpha_i + \delta D_{i,t} + e_{i,t} \,, \tag{1}$$

where $Y_{i,t}$ is the absolute output gap revision, α_i a country-specific fixed effect and $D_{i,t}$ a dummy variable, which takes the value of one for any time period after 2000 Q1. δ is therefore the coefficient of the greatest interest to us, as its sign and the degree of statistical significance will be informative of whether the size of revisions did change over time. The choice of this particular break point may be seen as arbitrary. To address this, we estimate equation (1a) below for overlapping rolling window of 16 quarters.

$$Y_{i,t} = \alpha_i + e_{i,t} \,, \tag{1a}$$

We then plot the resulting mean of absolute output gap revisions and its 95th confidence band to assess how it has evolved over time.

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[6] There are alternative approaches to testing for structural breaks; for example, see Zivo t and Andrews (1992).

3.2 Dynamic Common Factor model

Model (1a) might seem restrictive once it is considered that there are at least two different ways in which the mean of absolute output gap revisions can change over time. Some improvements in technology, in particular in information processing due the availability of faster computers, would probably lead to changes in revisions which are common to all the 15 OECD countries in our sample. At the same time, the availability of business surveys of spare capacity could lead to country-specific changes in output gap revisions. Similarly, errors in judgement about the cycle and the trend could occur either at the global or country-specific level. The common and country-specific component could therefore conceivably be moving in opposite directions. In this case, the simple panel data model presented in 3.1 would suggest the absence of a change when it is actually present.

To verify the results from the simple panel data model, we therefore use a Bayesian dynamic common factor model with one common factor and individual country-specific factors. In previous applications, such methods have been employed to study and identify the international business cycle from domestic investment, consumption and output data across a range of countries, and to quantify the contribution of the world 'factor' (representing the world business cycle) to their variation. Applied to the question at hand, the common factor may reflect changes in absolute output gap revisions which are in common to all of the countries. Similarly, the country-specific components will reveal whether the size of absolute revisions has changed at the country level.

We implement the following model:

$$Y_{i,t} = \mu_{i,t} + \gamma_i w_t + e_{i,t} \quad e_{i,t} \sim N(0, R_i) \tag{2}$$

$$\mu_{i,t} = \mu_{i,t-1} + v_{i,t} \quad v_{i,t} \sim N(0, Q_i) \tag{3}$$

$$w_t = \varphi w_{t-1} + n_t \quad n_t \sim N(0, N) \tag{4}$$

$$E[e_{i,t}e_{j,t}] = 0, \quad E[v_{i,t}v_{j,t}] = 0, \quad \forall \, i \neq j \tag{5}$$

where $Y_{i,t}$ is the absolute output gap revision in country i at time t, $\mu_{i,t}$ is a country-specific factor that evolves according to a random walk, w_t is a common factor which drives time series in all of the countries and γ_i is the country-specific factor loading relating the factor to the individual country time series. R_i is the covariance matrix of the idiosyncratic component, Q_i the covariance matrix governing the shocks to time-variation and N the variance of the common factor. Both $e_{i,t}$ and $\mu_{i,t}$ are country-specific and would, in any conventional state space model, be observationally equivalent. To ensure that we can separate the two, we model $\mu_{i,t}$ as a time-varying mean, which only evolves slowly over time. This is achieved via a fairly tight prior on Q_i, the matrix governing the size of time-varying shocks, to embody our belief that changes in output gap measurement technology only occur slowly over time.[7]

For simplicity of notation we will refer to this model in the following state space form for the rest of the paper:

$$Y_t = HW_t + e_t, \quad e_t \sim N(0, R) \tag{6}$$

$$W_t = \Phi W_{t-1} + g_t \quad\quad g_t \sim N(0, G) \tag{7}$$

where $W_t = [w_t; \mu_{i,t} \dots \mu_{k,t}]'$ and $\Phi = [\varphi; 1 \dots 1]'$ where k is the number of countries. The matrix H contains the corresponding factor-loadings as well as an identity matrix to account for the fact that the $\mu_{i,t}$'s enter the measurement equation in levels directly. G is a square matrix with N and the Q_i's on its diagonal. $e_t = [e_{i,t} \dots e_{k,t}]'$ and R is a square matrix with R_i's on the diagonal

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[7] This specification follows the approach presented in Cogley and Sargent (2001) and Del Negro and Otrok (2008) who also model time variation as smoothly changing coefficients across periods to capture permanent structural change.

3.2.1 Dynamic Common Factor model – Identification

From a purely statistical point of view, the above model is subject to two distinct identification problems: neither the scales nor the signs of the factor and the factor loadings are identified.

Like most dynamic common factor models, our model is subject to the problem that the relative scale of the model is indeterminate. One can multiply the vector of factor loadings, Γ, by a constant d for all i, which gives $\hat{\Gamma} = d\Gamma$. We can also divide the factor by d, which yields $\widehat{w_t} = \frac{w_t}{d}$. The scale of the model $\hat{\Gamma}\widehat{w_t}$ is thus observationally equivalent to the scale of the model Γw_t. In order to solve this problem, we take the approach that is widely applied elsewhere and set N, the variance matrix of the error term of the factor to 1.

In addition, the model is subject to the rotational indeterminacy problem (Harvey (1993)). For any $k \: x \: k$ orthogonal matrix F there exists an equivalent specification such that the rotations $\Gamma^* = F\Gamma$ and $w_t^* = Fw_t$ produce the same distribution for Y_t as in the original model. This implies that the signs of the factor loadings and the common factor are not separately identified. This can be easily seen when setting F=-1, as in this case $\Gamma^* w_t^*$ and Γw_t are observationally equivalent. In order to solve this problem we follow Del Negro and Otrok (2008) and impose one of the factor loadings to be positive, as this permits the identification of the sign of the factor and thus the rest of the model.

From an economic point of view, the common factor w_t is likely to capture unobserved factors that are associated with improvements in methodology that all countries experienced simultaneously. This could be, for instance, an improvement in information processing as a result of better computer technology. The country-specific factors are likely to capture changes in country-specific measurement, such as the availability of various spare capacity business surveys.

3.2.2 Dynamic Common Factor model - Implementation

Dynamic factor models can be estimated with maximum likelihood methods (Gregory, Head and Raynauld (1997)). Nevertheless, with a large number of series in the cross-section, the resulting likelihood functions may have odd shapes (Bernanke, Boivin and Eliasz (2005)). To address this problem, Kose, Otrok, and Whiteman (2003) use the methods introduced in Carter and Kohn (1994) to develop a Bayesian Kalman filter procedure, which permits them to estimate large dynamic common factor models easily. The model is estimated with a Bayesian technique, Gibbs sampling.

Typically, researchers 'de-mean' the data by subtracting them from their own averages, and standardize the variances to unity prior to estimating a dynamic common factor model. We follow previous work in standardizing the variances to unity, since this will prevent the most volatile series in the data from dominating the evolution of the common factor. But since we plan to model the mean of each series explicitly via the country-specific factors, we do not 'de-mean' the data.

One crucial decision is the prior on Q_i, the variance-covariance matrix of the disturbance term of each country-specific factor, which governs the amount of time-variation. We follow the approach set out in Cogley and Sargent (2001) and set the prior on Q_i proportional to the variance-covariance matrix of the country-specific time series. We thus set $Q_i = \alpha^2 K$ where α is a proportionality constant. Intuitively, K can be described as the uncertainty surrounding the time-varying means. Given that we standardised the data to have a standard deviation of 1, $K = 1$. Setting $\alpha = .035$ for instance would impose a prior that time variation cannot explain more than 3.5% of the uncertainty around K. Cogley and Sargent (2001) suggest that this value for α is conservative, but realistic to explain permanent structural change in quarterly data. We follow their suggestion, but a higher (.1) or lower (.01) value of α does not change our results qualitatively. Please see appendix A for greater details about the estimation procedure.

Testing for convergence

We replicate the above algorithm 100,000 times with Gibbs sampling and discard the first 50,000 replications as burn-in, keeping only every 10th draw in order to reduce auto-correlation among the draws. We then obtain the parameter estimates of the posterior distribution from the last 5,000 replications by taking the median and constructing 95% quantiles around it. We follow previous work and try various length of the iterative process. The results do not change, whether we replicate the model 100,000 times and retain 10,000 draws or replicate it 10,000 times and retain the final 1,000 draws for inference.

Two possible test for convergence in our case are the two-sided Kolmogorov-Smirnov and Cramer-von-Mises test. These tests permit the statistical assessment of whether the underlying distribution behind two random samples is the same. We split the retained 5,000 draws in two samples and test whether they differ. Since we could never reject the null hypothesis that both samples come from the same distribution, we conclude that our procedure always seems to converge.

3.3 Mincer-Zarnowitz regression model

Apart from the size, the composition of output gap measurement errors could have changed over time as well. Previous work by Boschen and Grossman (1982), Mankiw, Runkle and Shapiro (1984), Mankiw and Shapiro (1986), Maravall and Pierce (1986), Mork (1987, 1990), Patterson and Heravi (1992), Croushore and Stark (2001, 2003), Faust, Rogers and Wright (2005), Swanson and Van Dijk (2006), and Aruoba (2008) argues that revisions to real-time output gap estimates invariably fall into two categories, labelled 'news' and 'noise'. We follow this classification here. Under the 'noise' category, measurement errors are uncorrelated with the true values. They pollute the preliminary output gap data and would need to be filtered out in order for the preliminary number to become an optimal estimate. If revisions are mostly due to 'noise', real-time output gap measurement errors – thus revisions to output gap estimates – might be reduced by

increasing the resources dedicated to incorporating omitted contemporaneous information. On the other hand, under the 'news' category, revisions are the result of the arrival of new information about the state of the economy, which is not available at the time of measurement. In this case, the estimate can only be improved after the inclusion of such additional information.

To measure the relative importance of 'noise' and 'news', we follow Faust, Rogers and Wright (2005) and express the preliminary data X_t^p as the sum of final data X_t^f and an error term ε_t, i.e. $X_t^p = X_t^f + \varepsilon_t$. Under the 'news' view, the real-time output gap reflects all information available at the time of the data release. In this case ε_t will only reflect information available after the release, implying that X_t^p and ε_t are uncorrelated. On the other hand, if all of the revision were due to subsequent inclusion of information available at the time of release, ε_t would be uncorrelated with X_t^f. In intermediate case ε_t will, of course, be correlated with both. As in Faust *et al*, we run what is essentially a forecast efficiency test, sometimes known as the Mincer-Zarnowitz (1969) test, to distinguish between these two views. If news explain all of the measurement error, then the revision, i.e. the difference between X_t^p and X_t^f, should be uncorrelated with any information available at time t. But if revisions at least in part reflect noise, ε_t and X_t^p would be correlated. In particular, X_t^p would predict the revisions. Formally, the regression is:

$$R_{i,t} = \alpha_i + \beta X_{i,t}^p + u_t \tag{8}$$

where α_i is a fixed effect and $R_{i,t} = X_t^f - X_t^p$. The Mincer-Zarnowitz (1969) procedure is in essence an F-test of the null-hypothesis that $\alpha_i = \beta = 0$, which would imply that output gap revisions are unpredictable. To assess whether statistical authorities have improved upon their ability to incorporate contemporaneous information into preliminary data releases, we estimate this regression for the 1990s and 2000s separately. As before, we also report rolling regression estimates obtained with a 16 quarter window for verification.

4. Results

4.1 Panel-data model

Estimates of equation (1) are presented in Table 1. Our interest is to assess whether there is a statistically significant difference in the mean of the absolute output gap revisions between the 1990s and 2000s. According to the results in Table 1, we can reject the null-hypothesis that $\delta = 0$. Indeed, the fact that δ is highly statistically significant and positive suggests that the mean absolute output gap revision has become larger over time.

Table 1

	Panel data model
Constant	0.86
(std. error)	*(0.03)*
δ	0.18
(std. error)	*(0.05)*
Adjusted R^2	0.30

Figure 2 shows the estimates of the mean of the output gap revisions obtained with a rolling panel fixed effects regression, together with its 95% confidence band. The first observation reflects the estimate for the time between 1991Q1 and 1994Q4. As one can clearly see, the mean of absolute output gap errors seems to have increased towards the end of our sample, which is consistent with the results presented in Table 1.

Figure 2: Five-year moving average revision and its 95% confidence band
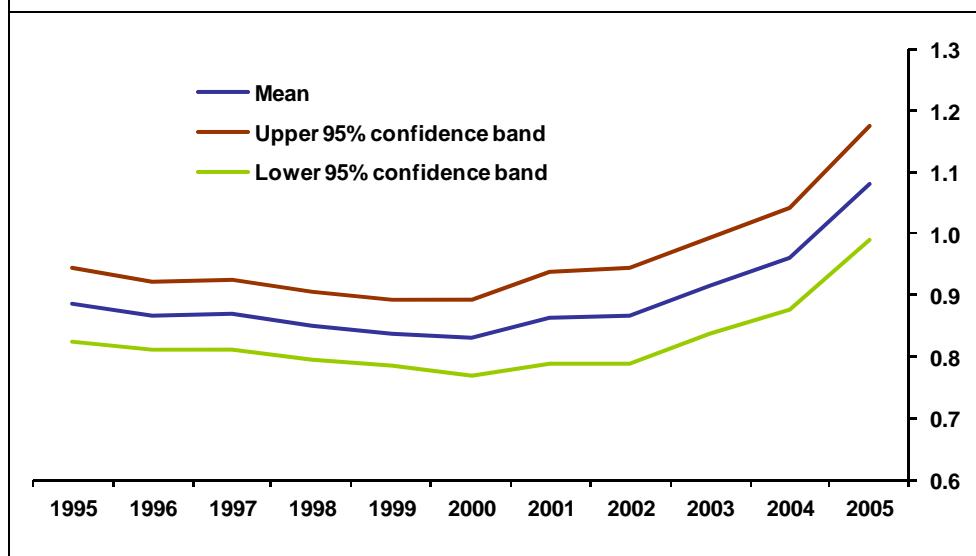
4.2 Dynamic common factor model

We present all of the results from the dynamic common factor model below. Figure 3 depicts the common factor together with 95% confidence bands.[8] The common factor does not seem to move much during the 1990s, but increases in the 2000s. Although this result is not statistically significant, interestingly, the dynamic common factor evolves in a way similar to the estimate from the rolling panel data model obtained over a window of 16 quarters. It also confirms the result that the mean of the absolute output gap error seems to have increased over time.

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[8] Note that if the distribution of the common factor were to be normal, then these would be equivalent to standard 95th percentile confidence bands.
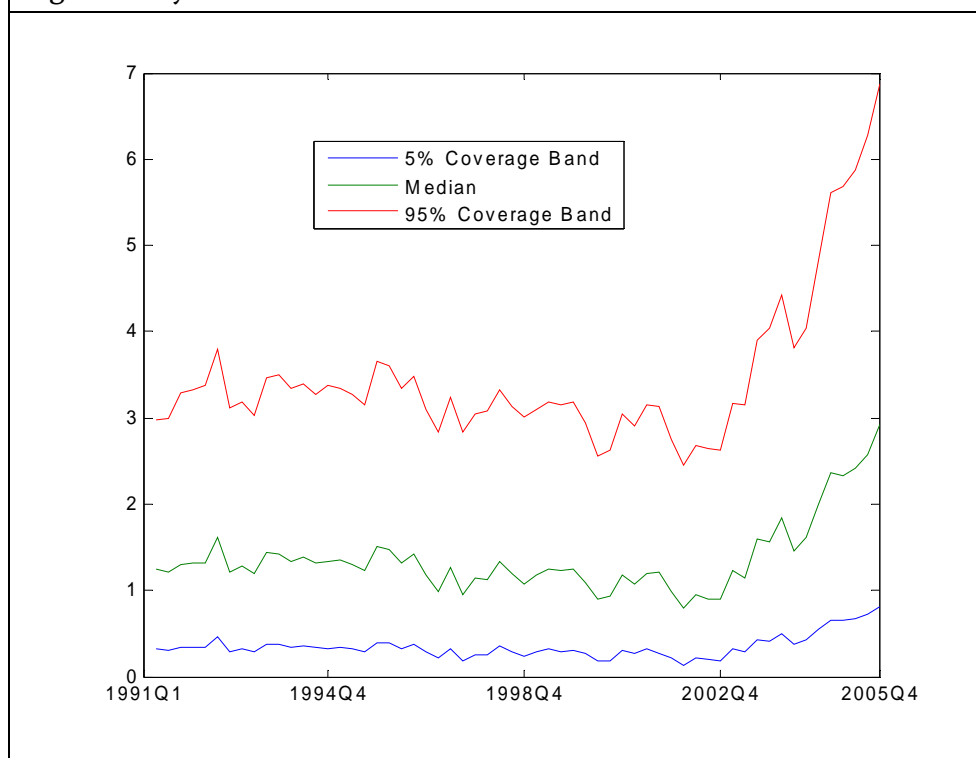
Figure 3: Dynamic Common Factor

Figure 4 shows the country-specific factors together with the associated 95th confidence intervals. There does not seem to be a pattern of declining mean absolute output gap revisions, other than for Finland and Australia. But even in these cases this pattern is not statistically significant. In some countries, like the US, Italy and Ireland, on the other hand, mean absolute output gap revisions seem to increase over time. In most other countries, there does not appear to be a clear trend towards an increase or decrease in the mean absolute output gap revision over time.

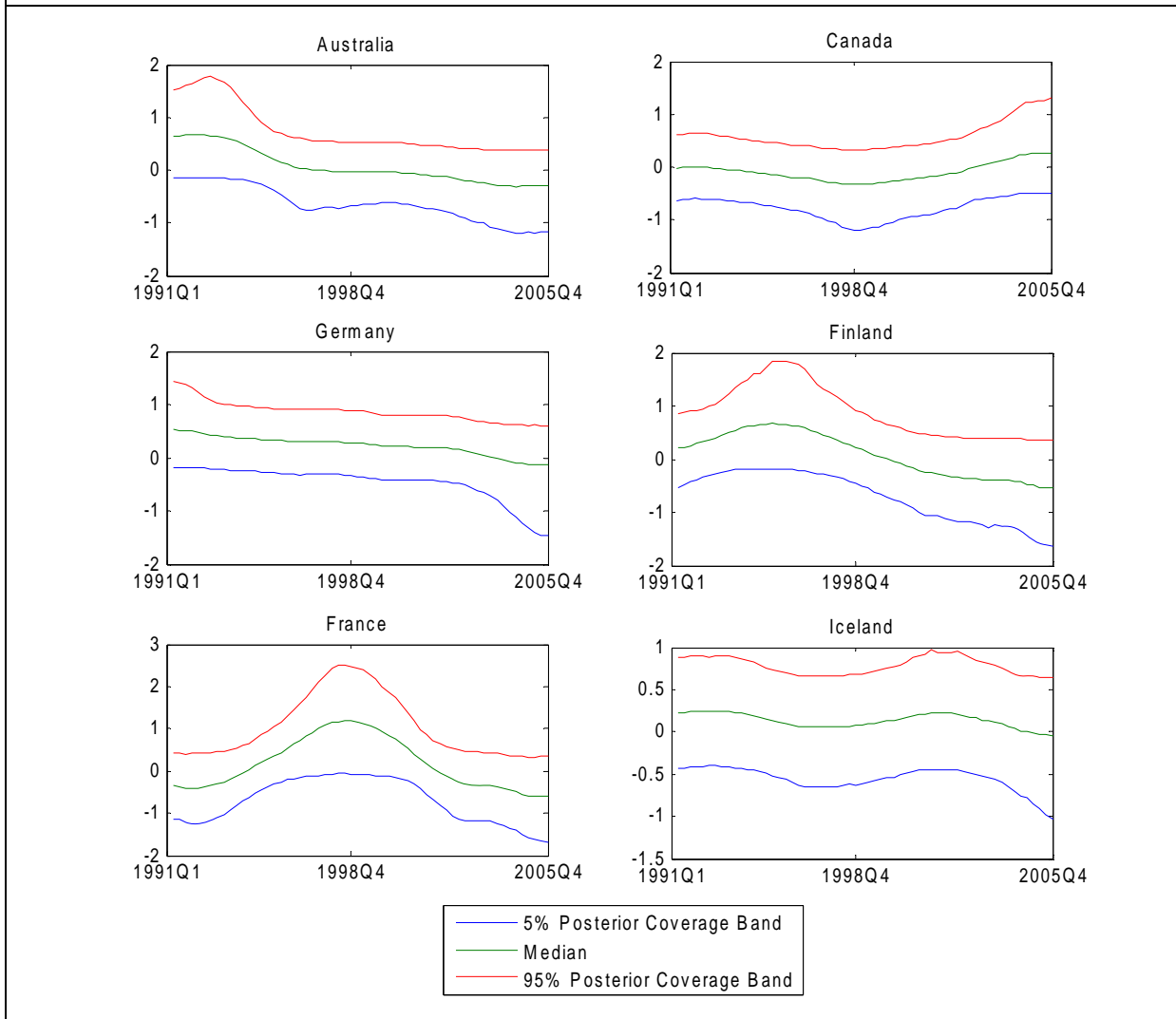Figure 4: Time-varying mean coefficients

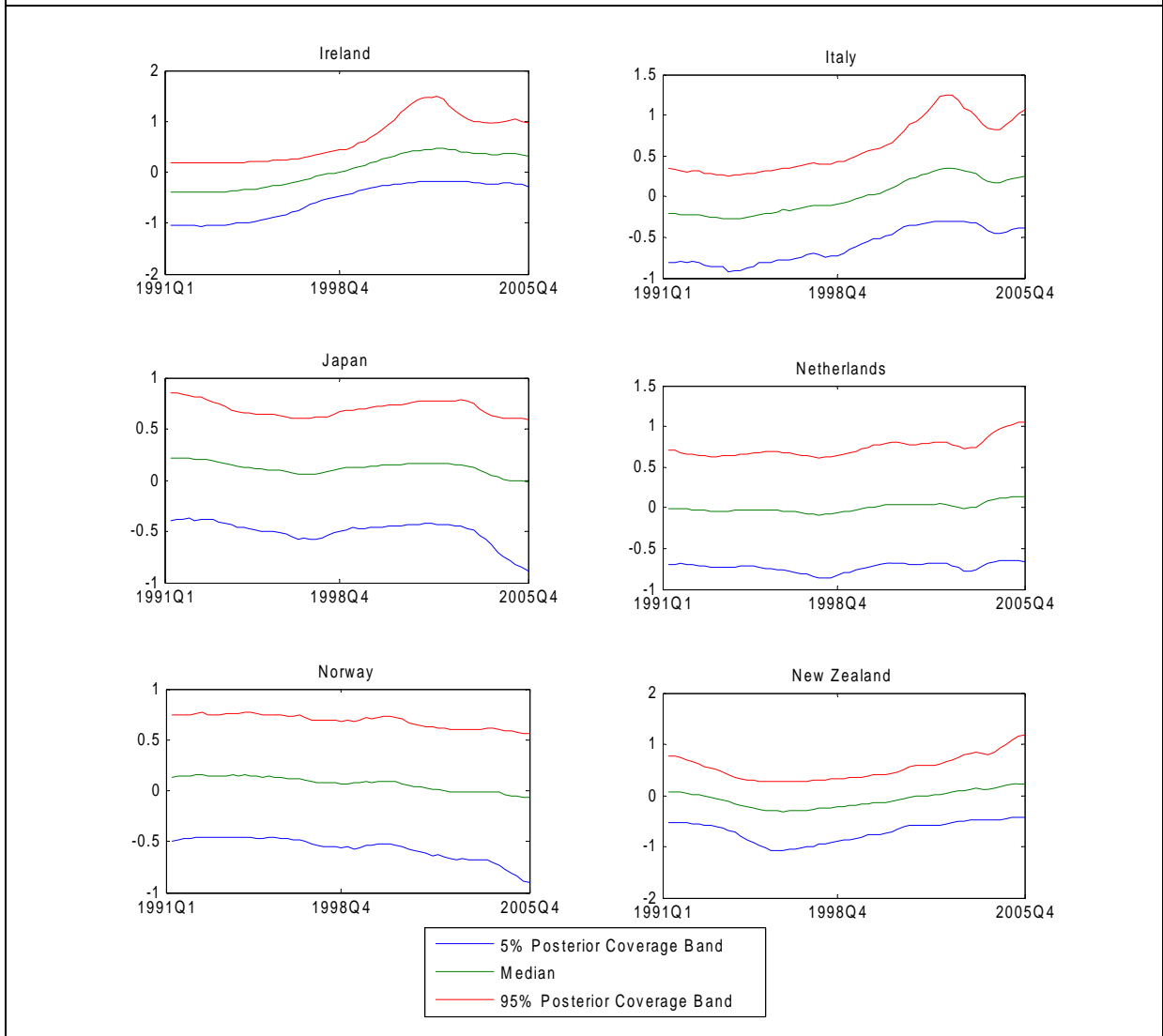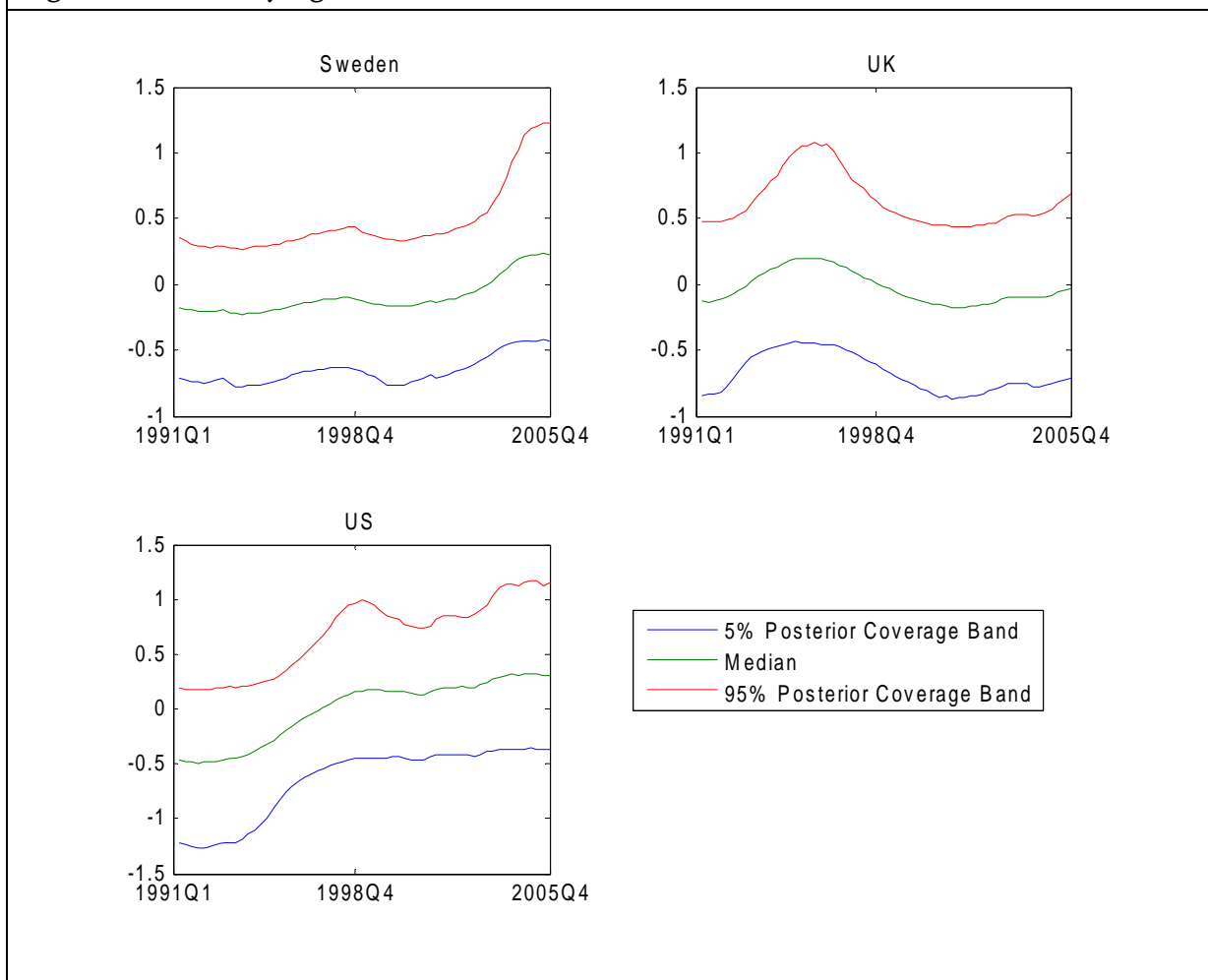Figure 4: Time-varying mean coefficients - continued

Figure 4: Time-varying mean coefficients - continued

4.3 Mincer-Zarnowitz Regressions

Table 2 presents estimates of equation (9) for two different time periods. For the sample as whole, an F test rejects the null-hypothesis that $\alpha_i = \beta = 0$, suggesting that revisions are predictable.

Table 2

	1991Q – 2005Q4	1991Q1 – 1999Q4	2000Q1 – 2005Q4
Constant	0.13	0.17	0.27
(std. error)	*(0.03)*	*(0.03)*	*(0.06)*
Prelim	-0.27	-0.20	-0.47
(std. error)	*(0.01)*	*(0.01)*	*(0.04)*
F	40.61	57.48	20.14
(p-value)	*(0.00)*	*(0.00)*	*(0.00)*
Adjusted R^2	0.40	0.61	0.44

For the sample up to 2000Q1, the adjusted R^2 of .61 suggests that roughly 60% of the revision can be explained by omitted information available at the time the preliminary estimate, with the rest due to information only available later. Repeating the same exercise for the sample after 2000Q1 suggests that only 44% of the revision can be explained by omitted contemporaneous information at the time of the preliminary output gap release. One may thus conclude that the fraction of output gap revisions as a result of noise appears to have fallen over time.

As before, we also re-estimate this regression over a rolling window of five years to assess how the split between news and noise has changed over time. The results are shown in Figure 5; the 1995 value, for example, would correspond to the adjusted R^2 and F-statistic of the Mincer-Zarnowitz regression for the period 1991Q1 to 1994Q4. Although the F-Statistic has fallen over time, the null-hypothesis that $\alpha_i = \beta = 0$ is still rejected at the 5% level at each point in time, suggesting that revisions are always predictable. The R^2 falls from about .7 to .4 at the end of the sample, confirming the earlier result that the proportion of noise in the preliminary GDP estimates has consistently fallen over time.

Figure 5: Adjusted R-square and F-statistics for five-year rolling Mincer-Zarnowitz regressions
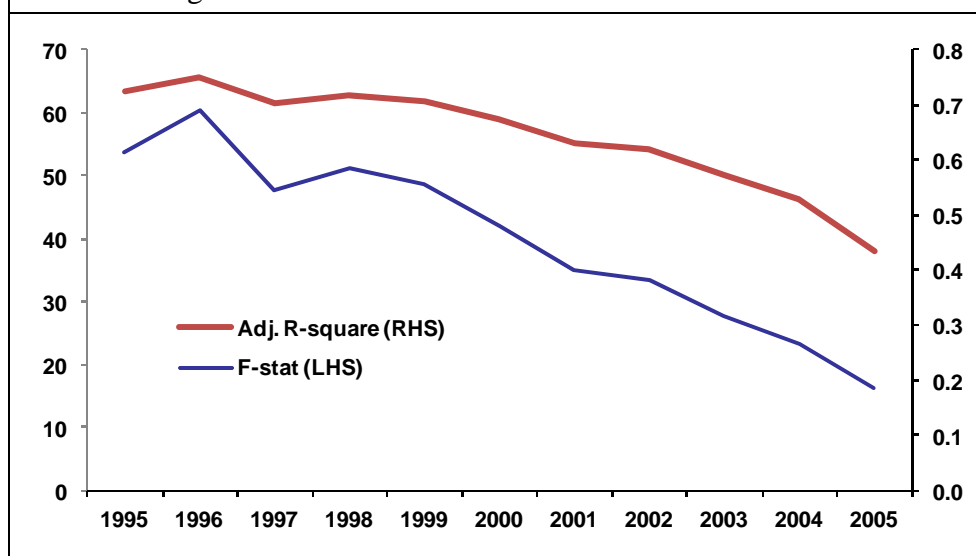
5. Conclusion

A large body of research has shown that real-time estimates of the output gap tend to be unreliable. Greater awareness of this problem and increases in the resources devoted to a possible solution may have contributed to greater accuracy of real-time estimates of the output gap. Similarly, advances in our capability to process ever greater amounts of information in real time, as a result of technological innovation, should have led to a reduction of data noise as a result of omitted contemporaneous information. In this paper, we examine if either the size or composition of revisions in the output gap estimates produced by the OECD have changed over time.

We exploit an OECD database on output gap revisions in 15 countries from 1991Q1 to 2005Q4 to answer this question. We use both a simple panel-data model and a more sophisticated dynamic common factor model with country specific factors to examine whether the mean of absolute output gap revisions has changed over time. Furthermore, we use the Mincer-Zarnowitz regression approach to assess if the composition of revisions has changed. We find evidence that the absolute level of output gap revisions have increased over time. We also find that the fraction of output gap revisions due to omitted contemporaneous information (data noise) has declined. This

suggests that while economists did become better at real-time data processing, this failed to translate into a reduction in the absolute size of output gap revisions, possibly due to the difficulty in separating the trend from the cycle in economic data.

The size of the output gaps in OECD countries is frequently cited as one of the main reasons behind the loose monetary policy implemented following the global financial crisis of 2008. Understanding how well this concept is measured and whether real-time estimation has improved over time is therefore an important policy question. In this paper we found that the composition, but not the size, of output gap revisions has changed over time. Future research should aim to shed light on why this is the case and what can be done to reduce the size of measurement errors of this important variable.

Appendix A

Dynamic Common Factor model - Estimation

Estimation with Gibbs sampling permits us to break the estimation down into several steps, which reduces the difficulty of implementation drastically. For instance, if the unobserved dynamic common factor in equation (4) would be known, then the estimation of the factor loadings, γ_i, would involve a simple OLS regression. Similarly, if the factor loadings are known, then the estimation of the unobserved factors only involves the application of Kalman filter to the state space form of the model in equation (6). Finally, given knowledge of the dynamic common factor, the estimation of the autoregressive parameter in equation (4) can be performed through a simple regression of the lagged factor on itself. For this purpose we employ a Gibbs sampling algorithm that approximates the posterior distribution and describe each step of the algorithm below.

Step 1 - Estimation of the factor-loadings on all other parameters

Conditional on a draw of W_t, we draw the factor loadings γ_i and the covariance matrix R_i. With knowledge of all the other parameters we can estimate each factor loading γ_i via OLS regressions separately. The posterior densities which we use to achieve this are:

$$\gamma_{i,t} \mid Y_{i,t}, W_t, G, R_i \quad \sim \mathrm{N}(\gamma_{i,t}^*, \Delta_i^*) \tag{9}$$

where $\gamma_{i,t}^* = (w_t{}'w_t)^{-1} w_t{}'Y_{i,t}$ and $\Delta_i^* = (w_t{}'w_t)^{-1}R_i$.

Step 2 - Estimation of the dynamic common factor and the time-varying means

We can now obtain an estimate of W_t with the Bayesian variant of the Kalman filter.[9] We assume W_t to be latent and unobservable. We draw W_t conditional on all other parameters from

$$W_T \mid Y_T, \mathrm{H}, G, R_i \quad \sim \mathrm{N}(W^*_{T\mid T, Y_T, \mathrm{H}, G}, \mathrm{P}^*_{T\mid T, Y_T, \mathrm{H}, G}) \tag{10}$$

$$W_t \mid Y_t, \mathrm{H}, G, R_i \quad \sim \mathrm{N}(W^*_{t\mid t, Y_t, \mathrm{H}, G}, \mathrm{P}^*_{t\mid t, Y_t, \mathrm{H}, G}) \tag{11}$$

We first iterate the Kalman filter forward through the sample, in order to calculate $W^*_{T\mid T, Y_T, \mathrm{H}, G, R_i} = E(W_T \mid Y_T, \mathrm{H}, G, R_i)$ and the associated variance-covariance matrix $\mathrm{P}^*_{T\mid T, Y_T, \mathrm{H}, G, R_i} = \mathrm{Cov}(W_T \mid Y_T, \mathrm{H}, G, R_i)$ at the end of the sample, namely time period T. The calculation of these parameters permits sampling from the posterior distribution in (9). We then use the last observation as an initial condition and iterate the Kalman filter backwards through the sample W_t from the posterior distribution in (10) at each point in time.

Step 3 – Estimation of φ, R_i and Q_i

We draw the variance-covariance matrix R_i from:

$$R_i \quad \sim \quad IG(\tfrac{\delta_1}{2}, \tfrac{z_1}{2}) \ , \tag{12}$$

where z_1 is the number of time-series observations and $\delta_1 = (Y_{i,t} - HW_t)'(Y_{i,t} - HW_t)$. The prior on R_i is therefore non-informative. The AR coefficient φ is obtained through a standard regression of w_t on its own lagged value and the coefficients are sampled from a normal distribution. We only retain draws with roots inside the unit circle. G is set to 1 in order to identify the scale of the model. The posterior density in this case is:

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[9] See Carter and Kohn (1994) for derivation and further description.

$$\varphi \mid Y_{i,t}, W_t, G \quad \sim N(\varphi_{i,t}^*, \Delta_i^*) \ , \tag{13}$$

where $\gamma_{i,t}^* = (w_{t-1}'w_{t-1})^{-1} w_{t-1}'w_t$ and $\Delta_i^* = (w_{t-1}'w_{t-1})^{-1}$. Subsequently, we construct the vector of innovations associated with each country-specific factor separately and draw the associated variances, Q_i, from the following inverse gamma distribution as in Del-Negro and Otrok (2008).

$$Q_i \ \sim \ IG(\tfrac{\delta_2}{2}, \tfrac{z_2}{2}) \ , \tag{14}$$

where $z_1 = T + T_1$, where T is the number of observations and $T_1 = .25 * T$, as in Del-Negro and Otrok (2008) and $\delta_1 = (\mu_{i,t} - \mu_{i,t-1})'(\mu_{i,t} - \mu_{i,t-1}) + \alpha^2 K$.

Step 4 - Go to step 1

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