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Discussion Paper No. 37

Disaggregating the international business cycle

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Disaggregating the international business cycle

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Abstract

This paper investigates the international business cycle with new sector level data on hours and output for Canada, Germany, France, Italy, the United Kingdom and the United States from 1992 Q1 to 2011 Q3. We estimate a Bayesian dynamic common factor model on this disaggregate data to decompose the quarterly growth rates of output, hours worked and labour productivity into contributions from global, country, sector and idiosyncratic factors. During the 'Great Recession' our results suggest that the global factor became the most important determinant of output, hours and labour productivity growth. Before the 'Great Recession', on the other hand, the global factor was not very important; country and idiosyncratic factors were the dominant influences on output, hours and productivity; sector factors never matter very much.

Keywords: Labour productivity, international business cycles, dynamic common factor model.

JEL classification: F44.

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1. Introduction

Are the origins of business cycles global, country or sector-specific in nature? This paper presents new data on output and hours in eight sectors, which together sum to the corresponding macroeconomic aggregates, for Canada, Germany, France, Italy, the UK and the US (the G6) from 1992Q1 to 2011Q3. We estimate Bayesian dynamic common factor models with these data to decompose the quarterly growth rates of output, hours worked and labour productivity[1] into the contributions from global, country and sector factors. This provides an indication of the sources of co-movement in output, hours worked and labour productivity in the countries we study.

Most previous work has focused on either macroeconomic aggregates or components of industrial production to understand the origins of aggregate fluctuations across countries at quarterly frequency. Gregory, Head and Raynauld (1997) estimate a dynamic common factor model on the quarterly growth rates of consumption, investment and output to decompose these series into a common, which they refer to as a 'world', factor and country business cycle factors for the G-7. Kose, Otrok and Whiteman (2008) use an identical model, estimated with Bayesian methods over three different time periods, to understand how individual countries' exposure to the world common factor has changed over time.[2] These studies conclude that the country and world factors explain most of the variance in aggregate output of each G-7 country. But since output growth is partially correlated with investment and consumption growth by construction, it is difficult to know whether the estimated factors reflect economic behaviour or simply accounting identities, an issue highlighted by the static factor analysis presented by Stone (1947). Similarly, correlations in aggregate data could be the result of aggregation bias.[3]

[1] Cross-country heterogeneity in labour markets implies that only productivity in terms of hours, as oppose to heads, is an internationally comparable measure (See Ohanian and Raffo, 2011 for an extensive discussion).
[2] See also Crucini, Kose and Otrok (2011) who use this methodology to extract common and country-specific factors for the G-7 for 4 different productivity measures, including labour productivity in terms of hours.
[3] The idea that time series which follow stationary auto-regressive processes become substantially more persistent when aggregated was first documented in Granger (1980). If this aggregation bias is correlated across macroeconomic aggregates, common factors estimated from these aggregates will be biased as well (See Pesaran and Chudik, 2011 for a review of aggregation issues).

An alternative strand of work attempts to establish the sources of business cycles across countries with disaggregated industrial production data instead. In particular, Stockman (1987) and Costello (1993) disentangle the contribution of country and industry-specific effects from cross-country quarterly output and productivity growth rates by industry, respectively. Similarly, Norrbin and Schlagenhauf (1996) estimate a dynamic common factor model to decompose quarterly growth rates of the components of industrial production into global, industry and country factors. Despite different methodologies, these studies typically find that while industry factors are important, country shocks are the more important drivers of these variables. However, as industrial production makes up only about one-fifth of the average G6 GDP, it is unclear if these conclusions apply to the whole economy.

Finally, previous work that examined quarterly sectoral data, which cover the whole economy, focused only on the US. Norrbin and Schlagenhauf (1988) use a dynamic common factor model to decompose US quarterly employment growth by region and sector into national, sector, regional, and idiosyncratic factors. They also find that the national factor is the most important driver of US employment growth. But to our knowledge, we are the first to explore business cycle patterns using disaggregated quarterly sectoral data to estimate global, sector and country factors from sector output, hours and labour productivity growth rates for the G6.

The lack of previous disaggregate cross-country work is probably due to the absence of internationally comparable quarterly data, especially for the US; we construct these for this purpose. This higher level of disaggregation should mitigate possible econometric bias associated with estimating common factors from aggregate variables. Our findings will nevertheless be applicable to the economy as a whole, given that our sector series sum to their macroeconomic counterparts. The work closest to ours is that by Karadimitropoulou and Leon-Ledesma (2012) who estimate a dynamic common factor model of the annual output growth of thirty sectors, that make up real GDP, in the G-7 up until 2004. In contrast to their work, the quarterly frequency of our data decreases the possibility of temporal aggregation bias[4] and increases the likelihood of capturing the co-movement, which economists typically consider to reflect business cycles. Furthermore, our sample period allows us to examine these variables, and in particular labour productivity growth, during the 'Great Recession' of 2008-2009, a topic of substantial current interest in policy circles. In sum, we make two separate contributions to the aforementioned body of work: First, we construct a new dataset for output, hours and employment for 8 sectors which are comparable across the G6. Secondly, we use these data to study the sources of aggregate fluctuations with a dynamic common factor model examining separately the periods both before and including the 'Great Recession'.

We decompose the quarterly growth rates of output, hours and labour productivity in terms of both the level and the variance. The former analysis allows us to infer the relative importance of the estimated factors *at any given point in time*, while the latter indicates their importance *on average*. For comparability to previous studies, we aggregate the models' output to the country level with weights based on the relative size of each economic sector within total output for that country. In terms of the level, the global factor is the most important determinant of aggregate output, hours and labour productivity growth during the 'Great Recession'. Variance decompositions confirm this for output, but the country and idiosyncratic factors explain the largest fraction of the variance of hours and productivity, growth. We also estimate the model on data up to 2007Q2. In this case, the level and variance decompositions lead to the same conclusions. As found in previous work, the country, closely followed by the idiosyncratic (country sector), component explains the largest fraction of output, hours and labour productivity growth, in line with results of the sector-level study by Norrbin and Schlagenhauf (1988) for the US. In both sample periods, idiosyncratic factors make up around 40% of the variance of labour productivity in the UK. Hughes and Saleheen (2012) document that, in comparison to other countries, the UK's productivity growth following the 'Great Recession' has been particularly weak relative to pre-crisis growth rates. Our findings therefore suggest that a sector level explanation is necessary to explain this phenomenon.

[4] See Sims (1971) and Geweke (1978) for a detailed discussion of this issue in econometrics.

Taken at face value, these findings have important implications for macroeconomic modelling: During normal times, the country, closely followed by the idiosyncratic, component explains the largest fraction of the variance in output, hours and labour productivity growth. These findings are consistent with the proposition that aggregate, country-specific, productivity shocks are important determinants of national business cycles. Yet the quantitative significance of idiosyncratic factors suggest exploration of country sector-specific shocks (Long and Plosser, 1983) as an avenue for future research aimed at understanding the business cycle in normal times. Explanations for business cycle co-movement during the 'Great Recession' should probably focus on the international dimension instead, as in Perri and Quadrini (2011). The rest of this paper is structured as follows. Section 2 describes the data and section 3 the empirical methodology. Section 4 presents the results and Section 5 concludes.

2. Data

We construct a new dataset containing quarterly estimates of sectoral output, employment and hours worked for France, Germany, Italy, Canada, the UK and the US (the G6)[5] from 1992Q1 to 2011Q3. As a result of the inherent heterogeneity in labour market institutions across countries, we express labour productivity in terms of hours, instead of heads (See Ohanian and Raffo, 2012 for a discussion). As explained below, employment is used to interpolate hours only for the UK. We exclude the agricultural sector from our analysis for three reasons: there are some large level shifts in the output series[6] for several countries which distort the growth rates, quarterly hours and employment series for US agriculture are not available, and conceptually it is difficult to apportion annual data across the quarters when output is highly seasonal. The sector series for each country therefore sum to the corresponding macroeconomic aggregate excluding the agricultural sector.

[5] A lot of previous work has focused on investigating international co-movement in output and productivity among the G7. Despite our best efforts, we were unfortunately not able to obtain data for Japan.
[6] For example, seasonally adjusted agricultural output in Germany reportedly fell by 25% in 1994 Q1, and increased by over 30% in both 2004 Q1 and 2007 Q1.

For France, Germany, Italy and the UK, most of the output, employment and hours data by sector are available from Eurostat. For Canada, data are available from Statistics Canada and for the US they are available from the Bureau of Labour Statistics (BLS) and the Bureau of Economic Analysis (BEA); however, for both countries the sectoral data are constructed using the North American Industry Classification System (NAICS) which is not directly comparable to the NACE[7] classification system used by Eurostat. Since four out of the six countries considered are constructed with reference to NACE and only two using NAICS, we chose to adjust the Canadian, and US data, to be in line with the European definitions. All of the sectors we use are listed in Table 1, and the sectoral letter codes, a feature of NACE, are explained in that table. It is not possible to obtain separate figures for Finance & Insurance Activities (NACE category K) and Real Estate Activities (NACE category L) for the US and Canada at quarterly frequency. To construct comparable data, we therefore create the Finance, Insurance & Real Estate activities category, by summing NACE categories K and L, for European countries.

Production of quarterly data on employment and hours (CANSIM tables: 282-0088 and 282-0092 respectively) for Canada requires several series to be split and re-allocated. Forestry, fishing, mining, quarrying, oil & gas is split into forestry & fishing and mining, quarrying, oil & gas, with the latter allocated to production (BE). This is based on information on the shares of the sub-sectors that is available for the US, and we assume that these proportions (1/3rd and 2/3rd) apply to Canada as well.[8] This technique is also applied to split Information, culture & recreation into information & communication (J) and culture & recreation, with the latter allocated to other services (R-U[9]). An additional complication is the relatively short time series of output figures, which only start in 1997. We apply the growth rates of the corresponding components of GDP at factor cost (CANSIM table: 379-0008) to the levels of GDP from 1997 to create series that begin in 1992 Q1.

[7] NACE is the acronym for “Nomenclature statistique des activités économiques dans la Communauté européenne”.
[8] Using the US is not ideal, but it seems likely that the industries between the US and Canada are more similar than their European counterparts.
[9] The other component of R-U, Other Services (SIC 81) is already available. This is added to culture & recreation to obtain R-U.

UK employment data from 1992 are available from Eurostat, but, like the Canadian data, output figures by sector are currently available from only 1997 onwards. Output figures for 1992 to 1997 are created using the GVA chained volume indices which were available prior to the release of the Blue Book 2011. GVA indices were not available for every NACE sector; however, indices for sub-sectors can be combined to create new GVA indices[10] which are NACE consistent by taking into account the relative weights of the sub-sectors given by the GVA current price series. For the UK, only whole economy hours (Office for National Statistics (ONS) code: YBUS) are available at a quarterly frequency. We therefore estimate quarterly hours by sector based on the growth rates of quarterly employment by sector, subject to the constraints that sectoral hours sum to the economy wide total in each quarter and that quarterly sectoral hours sum to the corresponding annual total, extending the least-squares method of Mitchell, Smith, Weale, Wright and Salazar (2005), itself a generalisation of Chow and Lin's (1971) approach, to take account of spatial as well as temporal constraints. See appendix B for more details.

Obtaining sectoral data for the US at a quarterly frequency is a notable challenge. Employment figures (excluding agriculture) are available from the BLS and these can be easily put into a NACE consistent format. Quarterly hours figures (excluding agriculture) for employees are also available from the BLS[11], and, as they note, these "account for almost ninety percent of hours worked"; hence, we simply scale up the hours to create sectoral total hours series. US statistical agencies do not provide quarterly estimates of output by sector. But the BEA does provide quarterly estimates of nominal national income without capital adjustment by sector and we can use these along with quarterly

[10] An index for Trade, repairs, transport, accommodation & food (G-I) was created by weighting together Wholesale (ONS code: GDQC), Hotels (ONS code: GDQD) and Transport (ONS code: GDQF). An index for Public administration, education & health (O-Q) was created by weighting together Public administration (ONS code: GDQO), Education (ONS code: GDQP) and Health (ONS code: GDQQ). We also assume that Information & communication (J) is adequately represented by Communication (ONS code: GDQG) and Other service activities (R-U) is proxied by Other social and personal services, private households with employees and extra-territorial organisations (ONS code: GDQR).

[11] More detail is available from: http://www.bls.gov/lpc/hoursdatainfo.htm

whole economy GDP excluding agriculture and annual GVA[12] by sector to create quarterly estimates of output by sector – this is analogous to the approach we followed to obtain quarterly hours estimates for the UK. Before the least-squares technique can be applied, the US data must be converted from the NAICS to NACE and then deflated. The quarterly national income data prior to 2001 are based on the 1987 Standard Industrial Classification (SIC), while the data from 2001 onwards are based on the 2002 NAICS. This results in a discontinuity in the sectoral levels; hence, to create a consistent series, we adjust the data prior to 2001 using the average ratio of the old and new classifications, which is calculated using the overlap of the new and old annual income data from 1998 to 2000. Finally, we deflate the sectoral income series using a variety of seasonally adjusted deflators.[13]

Data limitations make an international comparison of industry sectors using quarterly data somewhat involved. The steps outlined above offer a novel method to construct internationally comparable data in a consistent manner.

Descriptive statistics summarising the final series are shown in table 1 below. These are presented for the period from 1992Q2 to 2007Q2, so as to offer a picture of the six economies before the start of the recent economic crisis. In all countries, output grew the fastest in the information and communication sector, consistent with the presence of an 'information revolution' in these countries during this time period. There is clearly a lot of heterogeneity in sectoral labour productivity and output growth rates between sectors and countries. This suggests that taking this into account could be important when attempting to disentangle the various sources underlying business cycle fluctuations.

[12] Note that US GVA sums across sectors to US GDP and hence is equivalent to US sector level real GDP.

[13] Production (B-E) uses the Producer Price Index: Industrial Commodities (BLS code: PPIIDC), which is seasonally adjusted using X12. Trade, repairs, transport, accommodation & food (G-I) is deflated by CPI Transport (FRED code: CUSR0000SAT). Information & communication (J) uses CPI Information & information processing (FRED codes: CUUR0000SAE21, CUUS0000SAE21), which is seasonally adjusted using X12. Financial, insurance & real estate activities (K) is deflated using CPI Financial services (FRED code: CUUR0000SEGD05), which is seasonally adjusted using X12. Prof., scientific etc (M-N) uses the services deflator (BEA table 1.1.9). Public administration, education & health (O-Q) is deflated using the government consumption deflator (BEA table 1.1.9). Other service activities (R-U) uses CPI Other services (FRED code: CUSR0000SAS367). Unless stated otherwise, seasonality adjustments were already undertaken by the statistical agency.

Table 1: Data Summary, Annualised Growth Rates 1992Q2 to 2007Q2

	Output		Hours		Productivity (Hours)	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
	B-E : Industry					
Canada	2.0	5.0	-0.1	5.4	2.1	5.2
France	1.9	3.1	-1.9	1.7	3.7	2.7
Germany	1.0	6.5	-2.4	5.3	3.5	7.1
Italy	1.2	4.7	-0.5	3.5	1.7	5.1
United Kingdom	0.6	3.3	-3.1	4.1	3.8	4.3
United States	2.9	5.4	-1.3	3.7	4.2	4.6
	F : Construction					
Canada	1.9	7.3	1.9	9.4	0.1	8.6
France	0.4	4.2	-0.2	3.4	0.6	2.9
Germany	-1.2	11.8	-1.2	10.8	0.1	9.7
Italy	0.9	6.4	1.8	7.6	-0.9	8.5
United Kingdom	1.1	6.0	-0.7	6.9	1.8	7.1
United States	0.6	5.6	2.0	6.1	-1.4	6.0
	G-I : Wholesale and retail trade, transport, accomodation and food services					
Canada	3.2	4.5	1.3	3.1	1.9	4.5
France	2.4	2.7	0.3	1.6	2.1	2.7
Germany	1.6	5.1	-0.3	2.5	1.9	5.8
Italy	1.6	3.7	0.2	4.4	1.5	5.9
United Kingdom	2.4	3.5	0.3	3.1	2.2	4.1
United States	4.2	3.7	0.7	2.4	3.5	3.8
	J : Information and communication					
Canada	5.4	3.9	1.9	7.6	3.6	8.1
France	4.7	3.9	1.4	2.5	3.3	3.9
Germany	4.5	10.6	1.1	3.8	3.5	11.4
Italy	5.3	9.1	2.4	4.6	2.9	9.6
United Kingdom	8.4	10.4	-1.1	7.1	9.5	10.5
United States	5.8	5.9	0.7	5.6	5.1	8.2
	K : Financial, insurance and Real Estate activities					
Canada	3.2	2.0	1.3	4.9	1.9	4.9
France	1.9	2.1	-0.1	1.9	2.0	3.2
Germany	2.2	4.2	-0.6	2.2	2.3	4.5
Italy	1.2	3.6	0.6	2.4	0.7	4.15
United Kingdom	4.5	4.0	1.4	2.6	3.1	5.3
United States	3.5	3.0	1.6	2.6	1.9	3.5

	M-N : Professional, scientific and technical activities					
Canada	4.2	4.3	3.9	6.0	0.4	6.1
France	2.3	3.3	2.6	3.4	-0.3	3.9
Germany	2.9	5.9	3.3	3.0	-0.4	5.0
Italy	2.3	9.9	3.8	5.7	-1.4	9.7
United Kingdom	5.7	7.7	2.9	3.4	2.8	6.9
United States	3.5	3.6	3.0	3.8	0.5	3.7
	O-Q : Government					
Canada	1.1	2.8	1.4	3.6	-0.3	3.8
France	1.1	1.2	0.7	1.9	0.4	1.8
Germany	1.6	2.9	0.5	1.7	1.1	3.0
Italy	0.9	1.8	0.1	2.0	0.8	2.8
United Kingdom	2.1	2.8	1.5	2.9	0.7	3.6
United States	1.5	2.7	2.0	2.7	-0.5	3.4
	R-U : Arts, entertainment and recreation and other services					
Canada	2.3	2.9	1.1	4.6	1.3	5.4
France	2.9	3.1	1.8	1.9	1.1	3.6
Germany	1.0	4.1	0.9	3.0	0.0	3.6
Italy	1.0	4.6	1.4	9.0	-0.2	9.9
United Kingdom	2.8	7.5	-0.4	15.6	3.7	16.5
United States	0.8	6.2	1.6	2.3	-0.8	5.9

Note: Figures in table are on data up until 2007Q2 to avoid potential bias from including the 'Great Recession' period. Output figures for the US and hours for the UK were constructed with the technique described in appendix B.

3. Methodology

Our aim is to study sources of macroeconomic fluctuations with cross-country sectoral data. For this purpose, we use a dynamic common factor model, which allows us to decompose the level and variance of sector output, hours and productivity growth rates into contributions from a global factor, common to all series, sector factors, common to each sector across countries, and country factors, common to all sectors within a country. To assess whether some factors have been more important during the 'Great Recession' than on average, we report both level and variance decompositions.

3.1 The Dynamic Common Factor Model

Dynamic common factor methods have been widely used in previous work to study international business cycles. Gregory, Head and Raynauld (1997) use a dynamic common factor model to extract a common factor, which they refer to as the 'world business cycle', from G-7 growth rates of consumption, investment and output. Kose, Otrok and Whiteman (2003) use annual growth rates of these three variables to identify a world business cycle in 60 countries covering 7 regions of the world, while Kose, Otrok and Whiteman (2008) use a similar technique to study the evolution of G-7 business cycles. Finally, Crucini, Kose and Otrok (2011) estimate common and country factors from many different macroeconomic aggregates, including productivity. More recently, Del Negro and Otrok (2012) introduce time-varying coefficients and stochastic volatility into the standard dynamic common factor model to account for these features of the data. They apply their model to the real GDP growth rates of 19 OECD countries. We follow this general approach to extract a global factor, common to all series, sector factors, common to each sector across countries, and country factors, common to all sectors within a country from time series on output, hours and labour productivity growth by sector. The residual movement is attributed to the idiosyncratic factor.

The period prior to the recent economic crisis is typically referred to as the 'Great Moderation', since, as documented by Stock and Watson (2005), the volatility of economic shocks declined across a number of countries since approximately 1984. This has clearly changed with the onset of the 'Great Recession'. Indeed, in more recent work, Stock and Watson (2012) use a dynamic common factor model to document that for the US, the variance of the errors terms has indeed increased again with the 'Great Recession'. Modelling the variances as constant, in a world where they are time-varying might result in the estimate of the factor compensating for this misspecification (Cogley and Sargent, 2005). This in turn would affect the interpretation of our results.[14] To avoid

[14] In preliminary estimations, with a model that assumed fixed variances, the estimate of the factor during periods of greater volatility was indeed larger.

this source of possible bias, it therefore seems important to permit the variances of the error terms to vary over time.[15]

The model we implement is thus the following:

$$Y_{i,j,t} = \beta_{i,j}^{G} w_t^{G} + \beta_{i,j}^{C} w_t^{C} + \beta_{i,j}^{S} w_t^{S} + e_{i,j,t} \tag{1}$$

where $Y_{i,j,t}$ is the growth rate of either output, hours and labour productivity in sector j in country i at time t. w_t^G is the 'global' factor, in common to all countries and sectors. w_t^S is a sector factor, in common to all sectors j across countries, while w_t^C is a country factor, in common to all sectors j in country i. $\beta_{i,j}^{G}$, $\beta_{i,j}^{S}$ and $\beta_{i,j}^{C}$ are the corresponding factor loadings. $e_{i,j,t}$ will be referred to as the idiosyncratic factor throughout. The first three factors are each assumed to evolve according to an auto-regressive process of order one:

$$w_t^f = \varphi^f w_{t-1}^f + \sqrt{e^{\ln h_{f,t}}} v_{f,t} \qquad v_{f,t} \sim N(0,1) \tag{2}$$

where $f = \{G, S, C\}$. The variance of the error term evolves according to a geometric random walk, modelled in log terms to ensure that all variances are always positive:

$$\ln h_{f,t} = \ln h_{f,t-1} + \mu_{f,t} \qquad \mu_{f,t} \sim N\left(0, \omega_f\right) \tag{3}$$

In other words, we are allowing for stochastic volatility in the innovations of the common factor. $\mu_{f,t}$ is distributed with a normal distribution with variance ω_f. The error term of the measurement equation (idiosyncratic factor) is assumed to be serially correlated, again with the innovations following a random walk.

$$e_{i,j,t} = \rho_i e_{i,j,t-1} + \sqrt{e^{\ln h_{i,j,t}}} v_{i,j,t} \qquad v_{i,t} \sim N(0,1) \tag{4}$$

$$\ln h_{i,j,t} = \ln h_{i,j,t-1} + \mu_{i,j,t} \qquad \mu_{i,j,t} \sim N\left(0, \omega_{i,j}\right) \tag{5}$$

$$E\left[v_{i,t} v_{j,t}\right] = 0,\ E\left[\mu_{i,t} \mu_{j,t}\right] = 0 \ \forall\, i \neq j\ , \tag{6}$$

[15] An alternative way to address structural change, associated with the onset of 'Great Recession' period, would be to allow for time-varying factor loadings. This would however increase the computational burden necessary to estimate the model substantially and require much stronger priors. Furthermore, Del Negro and Otrok (2012) find little evidence for such time variation, in a dynamic common factor model with both stochastic volatility and time-varying factor-loadings, during the transition from the 1970s to the 'Great Moderation' period. We therefore choose to only model stochastic volatility instead.

The final assumption that error terms are uncorrelated across equations permits us to draw the stochastic volatility terms equation by equation.

3.1.1 The Dynamic Common Factor Model – Identification

From a purely statistical point of view, the above model is subject to two distinct identification problems. The scales of the factors are not identified and the factor loadings are, as with traditional factor analysis, defined only up to an orthogonal transformation.

For notational simplicity, we will refer to equation (1) in state space form from now on:

$$Y_{i,j,t} = \beta W_t + e_{i,j,t} \tag{7}$$

where $W_t = [w_t^G \; w_t^{S1} \cdots w_t^{SJ} \quad w_t^{C1} \cdots w_t^{CI}]$, $J(I)$ is the number of sectors (countries) and

$$\beta = \begin{pmatrix} \beta_{1,1}^G & \beta_{1,1}^S & 0 & 0 & \beta_{1,1}^C & 0 & 0 \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots & \vdots \\ \beta_{1,J}^G & 0 & 0 & \beta_{1,J}^S & \beta_{1,J}^C & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_{I,1}^G & \beta_{1,1}^S & 0 & 0 & 0 & 0 & \beta_{I,1}^C \\ \vdots & 0 & \ddots & 0 & \vdots & \vdots & \vdots \\ \beta_{I,J}^G & 0 & 0 & \beta_{1,J}^S & 0 & 0 & \beta_{I,J}^C \end{pmatrix}$$

The model is subject to the traditional indeterminance problem associated with factor analysis (Harvey (1993)). For any $k \; x \; k$ orthogonal matrix F there exists an equivalent specification such that the rotations $\beta^* = F\beta$ and $W_t^* = FW_t$ produce the same distribution for Y_t as in the original model. We impose restrictions on the matrix of factors, W_t, to address this problem.

First, there is a question of scale. One can multiply the matrix of factor loadings, β, by a constant d for all i, which gives $\hat{\beta} = d\beta$. We can also divide the factor by d, which yields $\widehat{W_t} = \frac{W_t}{d}$. The scale of the model $\hat{\beta}\widehat{W_t}$ is thus observationally equivalent to the scale of the model βW_t. In order to address this problem, we follow the approach presented in Del Negro and Otrok (2012) and set the initial condition of the stochastic volatility term associated with each factor, $h_{f,0}$, to 1.

Even then a choice remains as to the sign of W_t. We follow Del Negro and Otrok (2012) and impose some of the factor loadings to be positive. To identify the sign of the common factor, we restrict the first factor loading on the common factor in the first country and sector to be positive, which is the production sector in Germany.[16] The sign of each sector factor is identified by imposing a positive loading on each sector factor for Germany. Finally, to identify the sign of each country factor, we impose a positive sign on the associated factor loading for the production sector in each country.

From an economic point of view, we interpret the factor common to all countries and sectors as the world business cycle. The factors associated with each sector, can be interpreted as world sectoral factors, while the factors that are common to all industries in each country can be interpreted as reflecting country specific developments.

3.1.2 The Dynamic Common Factor Model - Implementation

Dynamic factor models can be estimated with maximum likelihood methods (Gregory, Head and Raynauld (1997)). But if the model is complex, because of the presence of stochastic volatility terms for example, estimating the joint density directly by maximising the likelihood function may be difficult. One can, alternatively, use the forward filter, backward smoother introduced in Carter and Kohn (1994) to estimate the model via Gibbs sampling. In this particular application, Gibbs sampling permits us to break down the estimation of this complex model into several stages, which greatly reduces the difficulty of estimation. We use the first 12 observations to set priors for the variance matrix governing stochastic volatility meaning that all model output starts in 1995Q3.[17] Following previous work, we measure the variables relative to their means, and standardise the variance of each series to unity ahead of any econometric analysis. Details of the Gibbs algorithm we use to approximate the posterior, together with information on the priors, are presented in appendix A.

[16] Previous work, such as Kose, Otrok and Prasad (2003), Kose, Otrok and Whiteman (2008) or Del Negro and Otrok (2012) typically impose this restriction on the US. As described in the data section, our sector output data for the US is constructed based on US income data and hence may contain some noise. We therefore choose Germany instead.

[17] In particular

Testing for convergence

We replicate the above algorithm 50,000 times with Gibbs sampling and discard the first 40,000 replications as burn-in, keeping only every 10th draw in order to reduce auto-correlation among the draws. We then obtain the parameter estimates of the posterior distribution from the last 1,000 replications by taking the median and constructing 68% posterior coverage bands around it.[18] We follow previous work and try various length of the iterative process. The results do not change, whether we replicate the model 50,000 times and retain 10,000 draws or replicate it 10,000 times and retain the final 1,000 draws for inference.

4. Results

4.1 Dynamic Common Factor results

Previous work has typically shown the estimated common factors. However, since we have a large number of factors and these are latent variables, we choose to report aggregate measures instead. In particular, all of the result reported in the subsequent figures and tables are based on country level aggregates, which were obtained by aggregating the sector level variables with their relative weight in total output, defined as the sum of output in all 8 sectors, for that country.[19] While most previous work focuses solely on variance decompositions, which indicate the importance of a factor *on average*, we are also interested in assessing if factors differ in their relative importance *over time*. This can be done by providing level decompositions as well.

Figure 1 decomposes the actual data into the level contributions from the median global, the country, the sector and idiosyncratic factors. The 68% posterior coverage bands, together with the actual data outturns, corresponding to each figure shown in the main text are presented in appendix C. Clearly, the global factor explains most of the output growth contraction associated with the 'Great Recession' for most countries with

[18] The choice of this particular posterior coverage band interval follows recent work that estimates dynamic common factor models with Bayesian methods. See for example Mumtaz and Surico (2012) or Kose, Otrok and Whiteman (2003).
[19] We use average weights computed from 1992Q1 to 2011Q3.

the exception of the UK and the US, where the country and idiosyncratic factors appear to have been of some significance during this period as well. During ‘normal times’, that is before the ‘Great Recession’, the country, closely followed by the idiosyncratic, factor appears to be the most important determinant of aggregate output growth.

Figure 2 repeats this analysis for hours worked. The idiosyncratic factor seems important for both France and Canada. In France, this could be a result of the introduction of the thirty-five hour week in 2000. While large firms had to follow this change immediately, there was a gestation period of two years for smaller firms. If the composition of any sector is skewed towards either type of firm, then this type of institutional change will not be picked up by either the global, the sector or country factor, but the idiosyncratic factor. For the other countries, the general impression shown is that country factors account for most of the variation in hours worked, although the global factor was most important during the ‘Great Recession’.

Figure 3 shows the levels decomposition for labour productivity in terms of hours. During the global financial crisis, the global factor appears to be the most important determinant of labour productivity. With the exception of Germany, both the idiosyncratic (country-specific sector) and country factors are important determinants of labour productivity in all other countries.

The picture shown in Figures 2 and 3 conforms, in general terms, but albeit with exceptions noted above, to the idea that both labour supply and productivity are determined in national labour markets. Although output growth is the sum of growth in hours worked and growth in productivity, the factor loadings on each factor may be different in the hours worked decomposition from those in the productivity decomposition. For example, the country factor could have a positive contribution to both hours worked and labour productivity. But if the corresponding factor-loadings are equal in magnitude and of opposite sign, the country factor would make no contribution at all to output growth.

Figure 1: Contributions to Output Growth by Country

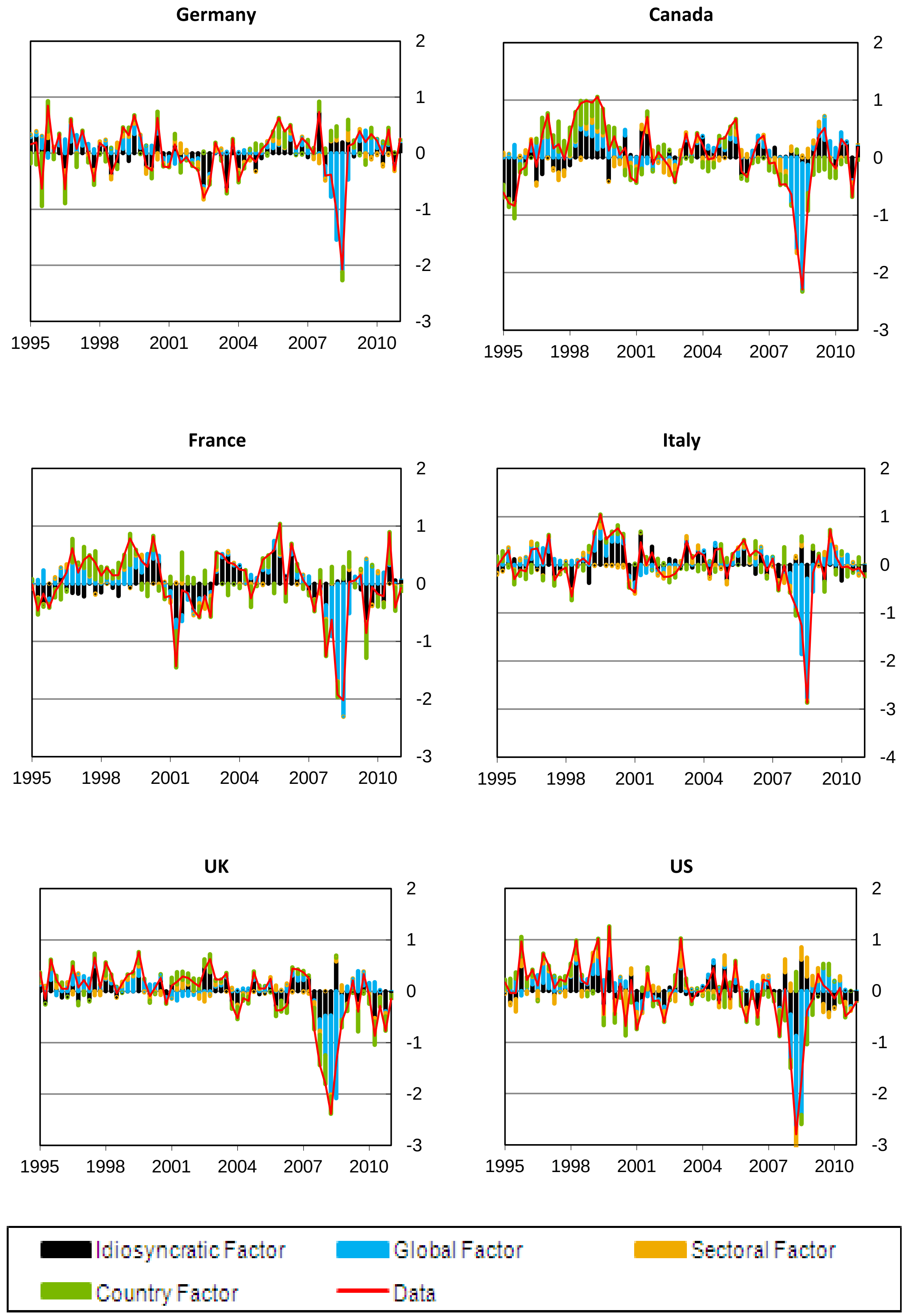


Figure 2: Contributions to Growth of Hours Worked by Country

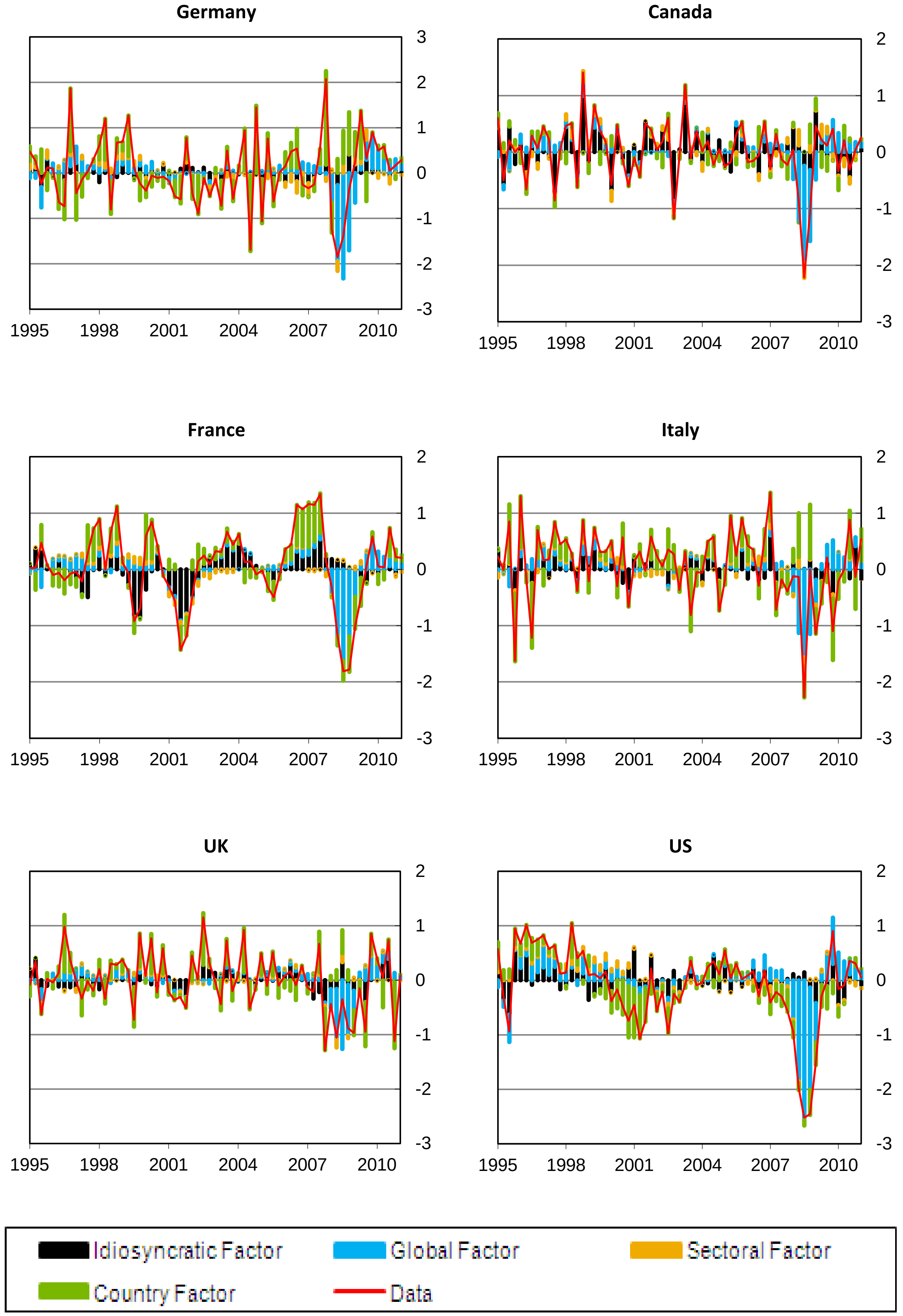
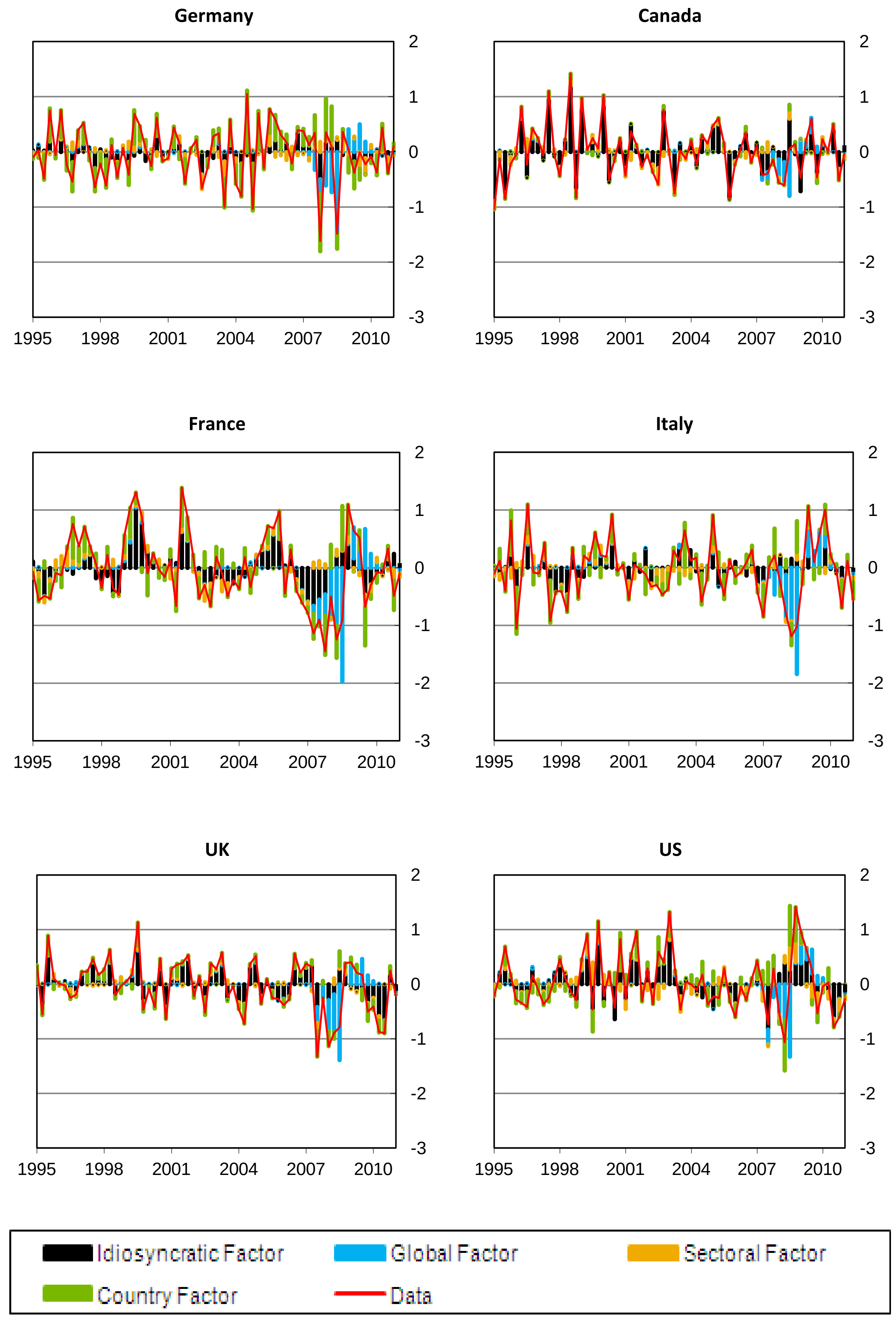


Figure 3: Contributions to Labour Productivity by Country



4.2 Variance decomposition

Most previous studies report variance decompositions at the aggregate level to assess the quantitative significance of their common factors. To maintain comparability between our findings and these, we have also calculated variance decompositions that characterize the fraction of the variance of each sector specific variable that is attributable to either the global, the country, the sector or the idiosyncratic factor. As before, we have aggregated these numbers to country aggregates using the relative weight of each sector in total output. Following Norrbin and Schlagenhauf (1988), we use the following formula to calculate weighted variance decompositions:

$$G_i'V(Y_i)G_i = G_i'\beta_i^G{}'V(W_i^G)\beta_i^G G_i + G_i'\beta_i^C{}'V(W_i^C)\beta_i^C G_i + G_i'\beta_i^S{}'V(W_i^S)\beta_i^S G_i + G_i'V(e_i)G_i$$

where G_i is a $(Jx1)$ vector of each sectors' weights relative to total output for country i. $V(.)$ is the variance operator. Y_i is a matrix with all of the sectors for country i in the columns and the time series in rows. Similarly, W_i^G, W_i^C and W_i^S are matrices containing the global, country and sector factors in the columns, with the time series in the rows, for all sectors in country i. The corresponding matrices containing the factor loadings are β_i^G, β_i^C and β_i^S, respectively. The contribution of the global factor to the aggregate variance of country i can then be obtained as: $\frac{G_i'\beta_i^G{}'V(W_i^G)\beta_i^G G_i}{G_i'V(Y_i)G_i}$. These calculations are reported in table 2 for output growth, hours worked and labour productivity per hour.

For output growth, our results suggest that the global factor explains the greatest fraction of the country level variance. This is different from conclusions reached by previous studies. Gregory, Head and Raynaud (1997) who use macroeconomic aggregates to quantify the fraction explained by the world and country-specific common factors for the G-7, with a dynamic common factor model, find that during the 1970 to 1993 period, the country-specific factor explains on average the largest fraction, 47%, of the variance of output. A similar exercise on a more recent sample from 1986Q3 to 2003Q4 with a Bayesian Dynamic common factor model by Kose, Otrok and Whiteman (2008) attributes 43% of the variance of output growth to the country-specific factor, verifying this conclusion. Norbin and Schlagenhauf (1996) use a dynamic common factor model to decompose growth of various industries within the production sector into common, country-specific and industry specific factors. They also find that, with 34.4%, the country-specific factor explains the largest share of the variance of industrial production. One explanation of the difference may be that the most volatile period for output was the period of the recent recession and this was a global phenomenon accounted for by the global factor. This is explored further in section 4.3.

The visual impression gained from Figure 2 is that country influences are the main driver of hours worked. Indeed, table 2 shows that the country factor accounts for more than half of the overall variation in Germany, France, Italy and the UK, with the global factor being more important than the country factor only in Canada and the US.

For labour productivity, Table 2 suggests that the idiosyncratic and country factor explain most of the variance. To our knowledge, the only other study that applies Bayesian dynamic common factor analysis to labour productivity for the G-7 is by Crucini, Kose and Otrok (2011). They find that for labour productivity measured in terms of hours in the manufacturing sector, the global factor explains a significantly greater fraction of the variance than the country-specific factor, the opposite of our results.

A general observation is that international sector, in contrast to domestic sector (idiosyncratic), factors make little contribution to variance in output, hours or productivity. This could be either because the transmission of sector specific shocks abroad occurs with a lag, rather than contemporaneously, in which case the idiosyncratic factor will pick them up or just that technology shocks are largely national in forms. Equally, to the extent that variations in output growth are driven by variations in demand, these do not seem to be international, but industry-specific phenomena.

Table 2 – Variance Decompositions by Country

Output Growth

	Global Factor			Country Factor			Sector Factors			Idiosyncratic Factors		
	16%	50%	84%	16%	50%	84%	16%	50%	84%	16%	50%	84%
Germany	0.43	0.49	0.54	0.16	0.22	0.28	0.04	0.06	0.07	0.20	0.23	0.26
Canada	0.37	0.44	0.51	0.14	0.22	0.31	0.05	0.06	0.08	0.23	0.27	0.31
France	0.39	0.45	0.51	0.23	0.27	0.33	0.01	0.02	0.02	0.23	0.25	0.28
Italy	0.56	0.61	0.66	0.07	0.10	0.15	0.03	0.04	0.05	0.21	0.24	0.27
UK	0.31	0.40	0.49	0.18	0.29	0.38	0.02	0.03	0.05	0.24	0.28	0.33
US	0.44	0.51	0.57	0.07	0.12	0.17	0.11	0.14	0.16	0.20	0.23	0.27
Average	0.42	0.48	0.55	0.14	0.20	0.27	0.04	0.06	0.07	0.22	0.25	0.29

Growth of Hours Worked

	Global Factor			Country Factor			Sector Factors			Idiosyncratic Factors		
	16%	50%	84%	16%	50%	84%	16%	50%	84%	16%	50%	84%
Germany	0.25	0.31	0.36	0.55	0.60	0.65	0.02	0.03	0.04	0.05	0.06	0.07
Canada	0.37	0.42	0.47	0.08	0.12	0.16	0.06	0.09	0.11	0.33	0.37	0.42
France	0.23	0.31	0.38	0.33	0.41	0.49	0.01	0.02	0.02	0.23	0.26	0.30
Italy	0.16	0.21	0.25	0.56	0.61	0.65	0.01	0.02	0.03	0.15	0.17	0.18
UK	0.14	0.19	0.25	0.54	0.60	0.65	0.01	0.02	0.02	0.17	0.19	0.21
US	0.49	0.56	0.62	0.20	0.26	0.33	0.01	0.02	0.03	0.14	0.16	0.18
Average	0.27	0.33	0.39	0.38	0.43	0.49	0.02	0.03	0.04	0.18	0.20	0.23

Labour Productivity Growth (Hours)

	Global Factor			Country Factor			Sector Factors			Idiosyncratic Factors		
	16%	50%	84%	16%	50%	84%	16%	50%	84%	16%	50%	84%
Germany	0.12	0.17	0.22	0.55	0.60	0.65	0.05	0.06	0.07	0.16	0.18	0.20
Canada	0.06	0.09	0.12	0.01	0.11	0.33	0.06	0.09	0.12	0.51	0.69	0.78
France	0.23	0.29	0.35	0.33	0.38	0.43	0.04	0.05	0.06	0.25	0.28	0.31
Italy	0.21	0.26	0.31	0.37	0.43	0.47	0.03	0.05	0.07	0.24	0.26	0.29
UK	0.15	0.20	0.26	0.22	0.32	0.40	0.01	0.03	0.05	0.39	0.44	0.53
US	0.11	0.16	0.22	0.19	0.38	0.45	0.09	0.12	0.17	0.29	0.33	0.43
Average	0.15	0.19	0.25	0.28	0.37	0.46	0.05	0.07	0.09	0.31	0.36	0.42

4.3 Robustness

Like any statistical model, dynamic common factor models have the tendency to maximise the fit around the most volatile part of a given time series. And despite standardising the variance of each series to unity prior to estimation and allowing for stochastic volatility, one could, of course, still argue that our findings are contaminated by the presence of the ‘Great Recession’ in our sample, a global shock to economic activity unparalleled since the Second World War.[20] To explore to which extent this is the case, we re-estimated our model up until 2007Q2, so as to avoid the risk that differences from previous studies stem from the inclusion of the ‘Great Recession’ in our sample. For this sample, the conclusions from the level and variance decompositions were identical. This implies that the factors with the most explanatory power *at any point in time*, also have the most explanatory power *on average*. In what follows we therefore comment only on the variance decompositions shown in table 3. In contrast to table 2, the global factor does not explain much of the variance of any of these series now, which should not be surprising given the absence of the ‘Great Recession’. For output growth, the average fraction of the variance explained by the global, country and idiosyncratic factor is similar to the magnitudes reported in previous studies based on macroeconomic aggregates (See Gregory, Head and Raynauld, 1997; Kose, Otrok and Whiteman, 2008) or components of industrial production (Norrbin and Schlagenhauf, 1996).[21] This is consistent with the findings from the sector level analysis for the US presented in Norrbin and Schlagenhauf (1988). The role of the country factor in hours worked is even more enhanced, contributing to more than half of the variance everywhere except in Canada. For labour productivity, the role of the country factors is also enhanced, accounting for more of the variance than the idiosyncratic factors in all but the UK. Indeed, the UK stands out in Tables 2 and 3, as the one country where around 40% of labour productivity is accounted for by the idiosyncratic factor, even when estimated on the sample excluding the ‘Great Recession’. In light of the current UK policy debate on stagnating labour productivity (Miles, 2012), our findings suggest that any credible explanation of this phenomenon will need to focus on differences between sectors.

[20] To a certain extent this conclusion could be preliminary, as Chiu and Wieladek (2012) provide evidence to suggest that real GDP outturns among the G7 might be subsequently revised.

[21] Kose, Otrok and Whiteman (2008) report that their global, country and idiosyncratic factors explain 24.5%, 43% and 32%, respectively. The corresponding figures for the world, country and idiosyncratic factor, reported in Norrbin and Schlagenhauf (1996) average to 15.7%, 34.3% and 40.2%, respectively.

Table 3 – Variance Decompositions by Country – Pre-Crisis Sample

Output Growth

	Global Factor			Country Factor			Sector Factors			Idiosyncratic Factors		
	16%	50%	84%	16%	50%	84%	16%	50%	84%	16%	50%	84%
Germany	0.18	0.27	0.35	0.34	0.42	0.49	0.03	0.05	0.07	0.22	0.27	0.32
Canada	0.20	0.28	0.36	0.26	0.34	0.43	0.14	0.18	0.22	0.16	0.20	0.24
France	0.07	0.13	0.20	0.38	0.44	0.50	0.05	0.07	0.10	0.31	0.35	0.39
Italy	0.05	0.10	0.17	0.27	0.35	0.43	0.13	0.17	0.22	0.32	0.37	0.43
UK	0.00	0.01	0.03	0.13	0.21	0.28	0.10	0.14	0.17	0.56	0.64	0.71
US	0.00	0.01	0.04	0.54	0.61	0.66	0.03	0.05	0.07	0.27	0.32	0.37
Average	0.08	0.13	0.19	0.32	0.39	0.47	0.08	0.11	0.14	0.31	0.36	0.41

Growth of Hours Worked

	Global Factor			Country Factor			Sector Factors			Idiosyncratic Factors		
	16%	50%	84%	16%	50%	84%	16%	50%	84%	16%	50%	84%
Germany	0.07	0.12	0.17	0.75	0.80	0.84	0.01	0.02	0.03	0.05	0.06	0.07
Canada	0.07	0.12	0.18	0.22	0.31	0.40	0.07	0.11	0.15	0.37	0.45	0.53
France	0.03	0.05	0.08	0.44	0.51	0.57	0.02	0.03	0.05	0.35	0.40	0.47
Italy	0.13	0.19	0.25	0.46	0.53	0.59	0.01	0.02	0.04	0.22	0.26	0.29
UK	0.00	0.02	0.05	0.66	0.70	0.74	0.02	0.04	0.07	0.19	0.23	0.27
US	0.00	0.00	0.01	0.67	0.72	0.77	0.01	0.02	0.03	0.21	0.25	0.29
Average	0.05	0.08	0.12	0.53	0.59	0.65	0.03	0.04	0.06	0.23	0.27	0.32

Labour Productivity Growth (Hours)

	Global Factor			Country Factor			Sector Factors			Idiosyncratic Factors		
	16%	50%	84%	16%	50%	84%	16%	50%	84%	16%	50%	84%
Germany	0.09	0.14	0.20	0.58	0.63	0.68	0.04	0.05	0.07	0.14	0.16	0.19
Canada	0.07	0.12	0.17	0.30	0.43	0.52	0.07	0.09	0.12	0.30	0.36	0.45
France	0.17	0.24	0.31	0.37	0.43	0.49	0.04	0.05	0.07	0.25	0.28	0.31
Italy	0.11	0.16	0.22	0.41	0.47	0.52	0.02	0.03	0.05	0.29	0.33	0.38
UK	0.04	0.09	0.14	0.30	0.40	0.48	0.07	0.09	0.13	0.35	0.41	0.50
US	0.03	0.05	0.09	0.54	0.59	0.63	0.03	0.05	0.07	0.26	0.30	0.34
Average	0.09	0.13	0.19	0.42	0.49	0.56	0.05	0.06	0.09	0.26	0.31	0.36

5. Conclusion

Understanding aggregate fluctuations is a central goal of macroeconomics. This paper investigates the structure of business cycle fluctuations with a new sector level dataset on output and hours for Canada, Germany, France, Italy, the UK and the US from 1992Q1 to 2011Q3. We estimate a Bayesian dynamic common factor model on this data to decompose sectoral output, hours and productivity growth rates into contributions from a global, sector and country factor. The resulting level (variance) decomposition allows us to examine the importance of a factor at any point in time (on average).

The level decomposition suggests that during the 'Great Recession', the global factor explains most of the changes in output, hours and labour productivity growth. Variance decompositions confirm this for output, but the country factor explains the largest fraction of the variance for hours and productivity growth. For pre-crisis data, the conclusions from variance and level decomposition results coincide. In this case the country, closely followed by the idiosyncratic, factor explains the largest share of these variables on average, consistent with previous work on macroeconomic aggregates and sub-components of industrial production. Regardless of sample, the UK is markedly different from other countries, with around 40% of productivity explained by the idiosyncratic factor - pointing to the need to develop a sector-specific explanation of the continuing stagnation of UK productivity.

Our results suggest that in normal times, the country factor is an important determinant of the business cycle, consistent with the assumption of aggregate, country-specific productivity shocks as the main drivers of the business cycle in standard RBC models. But they also highlight the quantitative importance of idiosyncratic (country-level sector) factors, which suggests that the introduction of a number of sectors into the standard model, as in Long and Plosser (1983), to improve our understanding of the business cycle in 'normal times' is thus a fruitful avenue for future research. Conversely, understanding the 'Great Recession' seems to require models with strong international propagation mechanisms, such as in Perri and Quadrini (2011).

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Appendix A

Dynamic Common Factor model - Estimation

Estimation with Gibbs sampling permits us to break the estimation down into several steps, reducing the difficulty of implementation drastically. For instance, if the unobserved global common factor in equation (1) would be known, then the estimation of the associated factor loadings, $\beta^{g}_{i,j}$, would only involve a simple OLS regression. Similarly, if the factor loadings and the time-varying variances are known, then the estimation of the unobserved factors only involves the application of the Kalman filter to the state space form of the model in equation (7). Furthermore, given the knowledge of the error terms in equation (2) and (4), one can estimate the stochastic volatility component by applying the single move Metropolis-Hastings procedure provided in Jacquier, Polson and Rossi (2004) to equations (3) and (5). We thus estimate the model with Metropolis-Hastings within Gibbs sampling and describe the algorithm below.

Step 1 - Estimation of the factor-loadings

Conditional on a draw of the corresponding global, w_t^G, country-specific, w_t^C and sector, w_t^S, common factor, we draw the three factor loadings $\beta^G_{i,j}$, $\beta^C_{i,j}$, $\beta^S_{i,j}$ and the associated covariance matrix. With knowledge of all the other parameters we can draw these parameters with OLS equation by equation. The posterior densities which we use to achieve this are:

$$\beta_{i,j} \mid Y_{i,j,t}, W_t, h_{i,t}, h_{f,t}, \rho_{i,j} \quad \sim \mathrm{N}(\beta^*, \Delta^*_{i,j}) \tag{A1}$$

where $\beta^* = (\ \Delta_0^{-1} + w_{\mathrm{t}}^{*\prime} w_{\mathrm{t}}^{*})^{-1}(\Delta_0^{-1}\,\beta_0 + w_{\mathrm{t}}^{*\prime} Y^*_{i,j,t})$ and $\Delta^*_{i,j} = (\Delta_0^{-1} +\ w_{\mathrm{t}}^{*\prime} w_{\mathrm{t}}^{*})^{-1}$, where $w_{\mathrm{t}}^{*} = \frac{\mathrm{w_t}}{h_{i,j,t}^{\frac{1}{2}}} - \rho_{i,j}\frac{\mathrm{w_{t-1}}}{h_{i,j,t}^{\frac{1}{2}}}$ and $Y^*_{i,j,t} = \frac{Y_{i,j,t} - \rho_{i,j} Y_{i,j,t-1}}{h_{i,j,t}^{\frac{1}{2}}}$. Note that $w_t = [w_t^G\ w_t^C\ w_t^S]$ and $\beta_{i,j} = [\beta^G_{i,j}\ \ \beta^C_{i,j}\ \ \beta^S_{i,j}\ ]$. β_0 and Δ_0^{-1} are priors on the means and variances of these coefficients, which are set to 0 and 1, respectively.

Step 2 - Estimation of the dynamic common factors

We can now obtain an estimate of W_t with the forward filter, backward smoother.[22] We draw the unobservable factor W_t conditional on all other parameters from

$$W_T \mid Y_T, \beta, PF, H_t, HF_t \quad \sim \mathrm{N}(W^*_{T|T,Y_T,\beta,PF,H_t,HF_t}, \mathrm{P}^*_{T|T,Y_T,\beta,PF,H_t,HF_t}) \tag{A2}$$

$$W_t \mid Y_t, \beta, PF, H_t, HF_t \quad \sim \mathrm{N}(W^*_{t|\mathrm{t},Y_t,\beta,PF,H_t,HF_t}, \mathrm{P}^*_{t|t,Y_t,\beta,PF,H_t,HF_t}) \tag{A3}$$

Where W_t and β are defined in section 3.1.1 and each element in Y_t is expressed as $Y_{i,j,t} - \rho_{i,j} Y_{i,j,t-1}$. PF, H_t, HF_t are matrices with φ_f, $h_{i,j,t}$, $h_{f,t}$ on the diagonals and zero otherwise. We first iterate the Kalman filter forward through the sample, in order to calculate $W^*_{T|T,Y_T,\mathrm{H},G,R_i} = E(W_T \mid Y_T, \beta, PF, H_t, HF_t)$ and the associated variance-covariance matrix $\mathrm{P}^*_{T|T,Y_T,\mathrm{H},G,R_i} = \mathrm{Cov}(W_T \mid Y_T, \beta, PF, H_t, HF_t)$ at the end of the sample, namely time period *T*. The calculation of these parameters permits sampling from the posterior distribution in (A2). We then use the last observation as an initial condition and iterate the Kalman filter backwards through the sample and draw W_t from the posterior distribution in (A3) at each point in time.

Step 3 - Estimation of the stochastic volatility components

We draw each individual stochastic volatility component associated with serially correlated error term, $\ln h_{i,j,t}$, and each dynamic common factor, $\ln h_{f,t}$, equation by equation following the procedure outlined in Jacquier, Polson and Rossi (2004).

Step 4 – Estimation of $\omega_{i,j}$, ω_f and $\rho_{i,j}$, φ_f

We draw $\omega_{i,j}$ from an inverse Gamma distribution:

$$\omega_{i,j} \;\sim\; IG\left(\frac{\delta_0+\delta_2}{2}, \frac{z_0+z_2}{2}\right) \tag{A4}$$

[22] See Carter and Kohn (1994) for derivation and further description.

Where $\delta_2 = (\ln h_{i,j,t} - \ln h_{i,j,t-1})'(\ln h_{i,j,t} - \ln h_{i,j,t-1})$ and z_2 is the number of time-series observations. Similarly, ω_f is drawn from

$$\omega_f \sim IG(\frac{\delta_0+\delta_3}{2}, \frac{z_0+z_3}{2}) \tag{A5}$$

Where $\delta_3 = (\ln h_{f,t} - \ln h_{f,t-1})'(\ln h_{f,t} - \ln h_{f,t-1})$ and z_3 is the number of time-series observations. δ_0 and z_0 are priors. To obtain these, we first estimate an identical model with principal components on the whole sample. Using the first 12 observations, we then estimate AR(1) regressions on both the implied error terms and principal components. The logarithm of the squared standard deviation of the residual from those regressions provides the corresponding prior for δ_0 with z_0 set to 10 to reflect the uncertainty around this prior. The AR coefficient φ_f is obtained through a standard regression of the associated factor on its own lagged value and the coefficients are sampled from a normal distribution. We only retain draws with roots inside the unit circle. The posterior density in this case is:

$$\varphi_f \mid W_t, \quad \sim \mathrm{N}(\varphi_f^*, \Delta_i^*) \tag{A6}$$

where $\varphi_f^* = (\Delta_0^{-1} + w_{f,t-1}^*{}' w_{f,t-1}^*)^{-1}(\Delta_0^{-1}\varphi_f^0 + w_{f,t-1}^*{}' w_{f,t}^*)$ and $\Delta_i^* = (\Delta_0^{-1} + w_{f,t-1}^*{}' w_{f,t-1}^*)^{-1}$, where $w_{f,t}^* = \frac{w_{f,t}}{h_{f,t}^{\frac{1}{2}}}$ and φ_f^0 and Δ_0^{-1} are the prior mean and variance, which are set to 0 and 1, respectively. Similarly, the individual $\rho_{i,j}$'s are sampled from

$$\rho_{i,j} \mid e_{i,j,t} \sim \mathrm{N}(\rho_{i,j}^*, \Delta_{i,j}^*) \tag{A7}$$

where $\rho_{i,j}^* = (\Delta_0^{-1} + e_{i,j,t-1}^*{}' e_{i,t-1}^*)^{-1}(\Delta_0^{-1}\rho_{i,j,}^0 + e_{i,j,t-1}^*{}' e_{i,j,t}^*)$ and $\Delta_{i,j}^* = (\Delta_0^{-1} + e_{i,j,t-1}^*{}' e_{i,j,t-1}^*)^{-1}$, where $e_{i,j,t}^* = \frac{e_{i,j,t}}{h_{i,j,t}^{\frac{1}{2}}}$. $\rho_{i,j}^0$ and Δ_0^{-1} are the corresponding prior mean and variance, which are set to 0 and 1, respectively.

Step 5 - Go to step 1

Appendix B

Users of quarterly data often have to address the problem of interpolating annual flow data. A range of ways in which indicator variables could be used to address the problem was set out by Friedman (1962). Chow and Lin (1971) proposed a method, still widely used, in which a linear relationship is assumed between the level of the variable in question and the appropriate indicator, with least squares adjustments then being used to ensure that the levels of the quarterly flow data thus generated are consistent with the annual total.

Mitchell, Smith, Weale, Wright and Salazar (2005) set out the solution to the analogous problem when the residuals of the relationship between the indicator and the interpoland are serially correlated and also addressed the point that it may be more natural to consider logarithmic rather than linear relationships. The distinction between that faced in most interpolation problems and that of concern here, is that normally the only constraints are that quarterly interpolands should be consistent with annual totals. Here quarterly sectoral interpolands need to be consistent with annual sectoral totals. But quarterly sectoral interpolands also need to be consistent with quarterly economy-wide data. While there are, as a consequence, more constraints than arise in a conventional interpolation problem, the difference is nevertheless, not one of substance.

y_{it} denotes the quarterly interpoland for sector i in quarter t and z_{it} is an initial estimate generated by use of an indicator variable, x_{it}. Y_{iT} denotes the known annual aggregate for sector i in year T and H_t is the economy-wide total for quarter t. It is assumed that the annual estimates generated by aggregating the economy-wide quarterly totals are consistent with the annual economy-wide estimates generated by aggregating the annual sectoral data. The notation $t \in$ T is used to indicate that quarter t is one of the quarters of year T.

It is assumed that the relationship between the interpoland and the indicator is expressed in first log differences, with x_{it} expressed in a way which already takes account of the relevant transformation

$$\Delta \log y_{it} = \alpha_i \Delta x_{it} + \beta_i + u_{it} \qquad \text{(B1)}$$

First of all it is necessary to estimate equation (B1). The equation cannot be estimated as it stands because the quarterly first differences for the dependent variable are not observed. However, after making a second-order approximation, the parameters can be estimated. Write $y_{it} = \frac{Y_T}{4} + \varepsilon_{it}, \sum_t \varepsilon_{it} = 0 \ (t \in T)$ Then

$$\log y_{it} \approx \log \frac{Y_T}{4} + \varepsilon_{it}$$

$\log \frac{Y_T}{4} - \log \frac{Y_{T-1}}{4} \approx \sum_{t \in T} \log y_{it} - \sum_{t \in T-1} \log y_{it} = \alpha_i \sum_{t \in T} \Delta^4 x_{it} + 16\beta_i + u_{i,4T} + 2u_{i,4T-1} + 3u_{i,4T-2} + 4u_{i,4T-3} + 3u_{i,4T-4} + 2u_{i,4T-5} + u_{i,4T-6}$

It should be noted that the approximation is second order rather than first order because $\sum_t \varepsilon_{it} = 0$, since the ε_{it} are deviations relative to the quarterly mean.

The estimation then proceeds in two stages. First of all WLS regression can be used to estimate equation (B1). Secondly, with $\widehat{\alpha_i}$ and $\widehat{\beta_i}$ the fitted values of α_i and β_i we then solve the constrained least squares problem

$Min \sum_i \sum_t u_{it}^2$ such that $\sum_{t \in T} y_{it} = Y_{iT}$ and $\sum_i y_{it} = H_t$

to produce estimates of the y_{it}. Notwithstanding the large number of constraints, this is typically quickly solved using a Gauß-Newton routine. We apply this procedure twice. First, we use this procedure to obtain our estimate of quarterly UK hours by sector, with $\Delta \log y_{it}$ as the growth rate of UK hours and Δx_{it} as the growth rate of UK employment in sector i at time t. Y_{iT} and H_T are the annual total hours in sector i and quarterly whole economy UK hours, respectively. US output by sector is obtained from US real income by sector in an analogous way.

Appendix C

Figure C1: Predicted versus Actual Output Growth

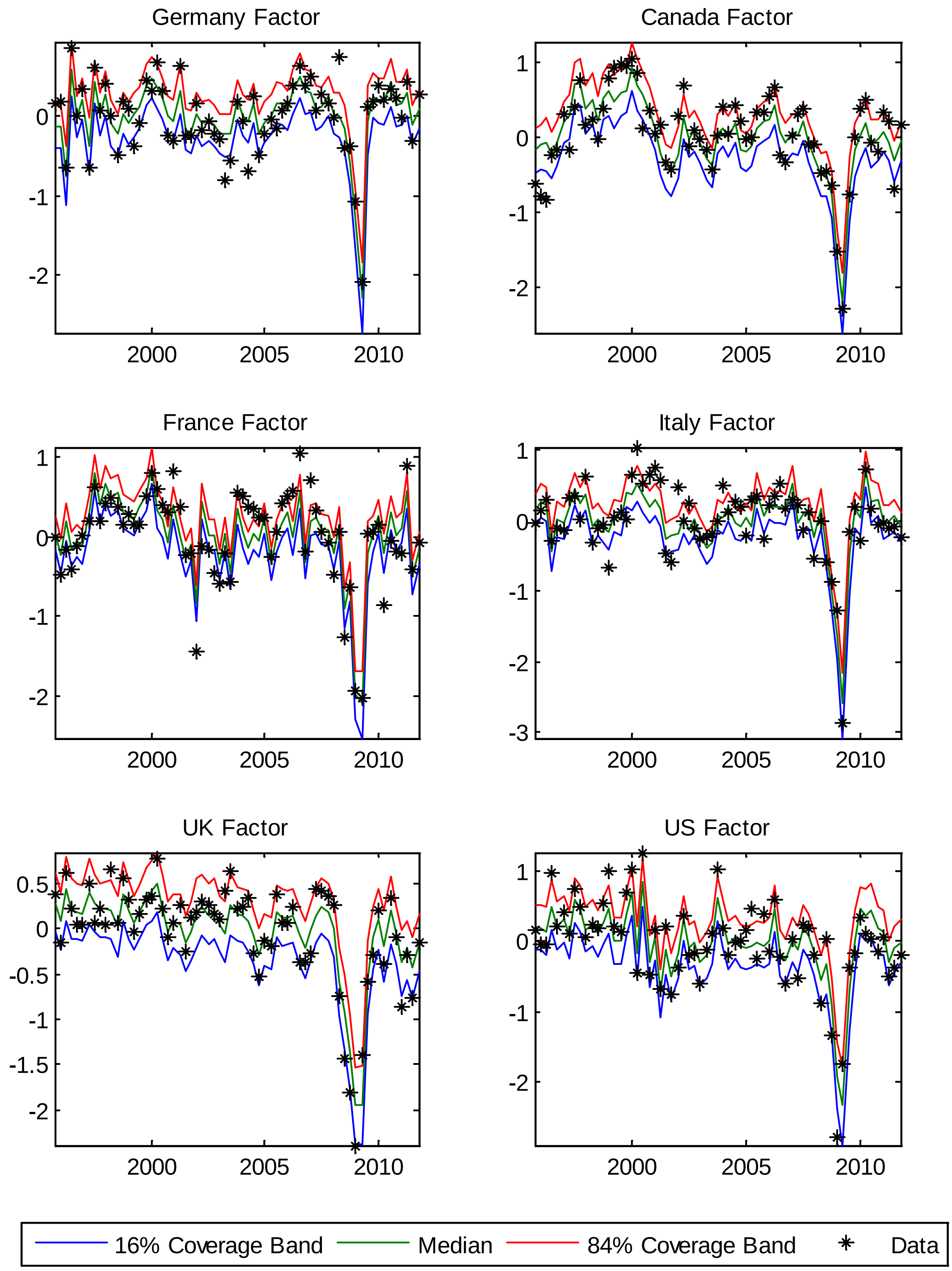

Figure C2: Predicted versus Actual Growth of Hours Worked

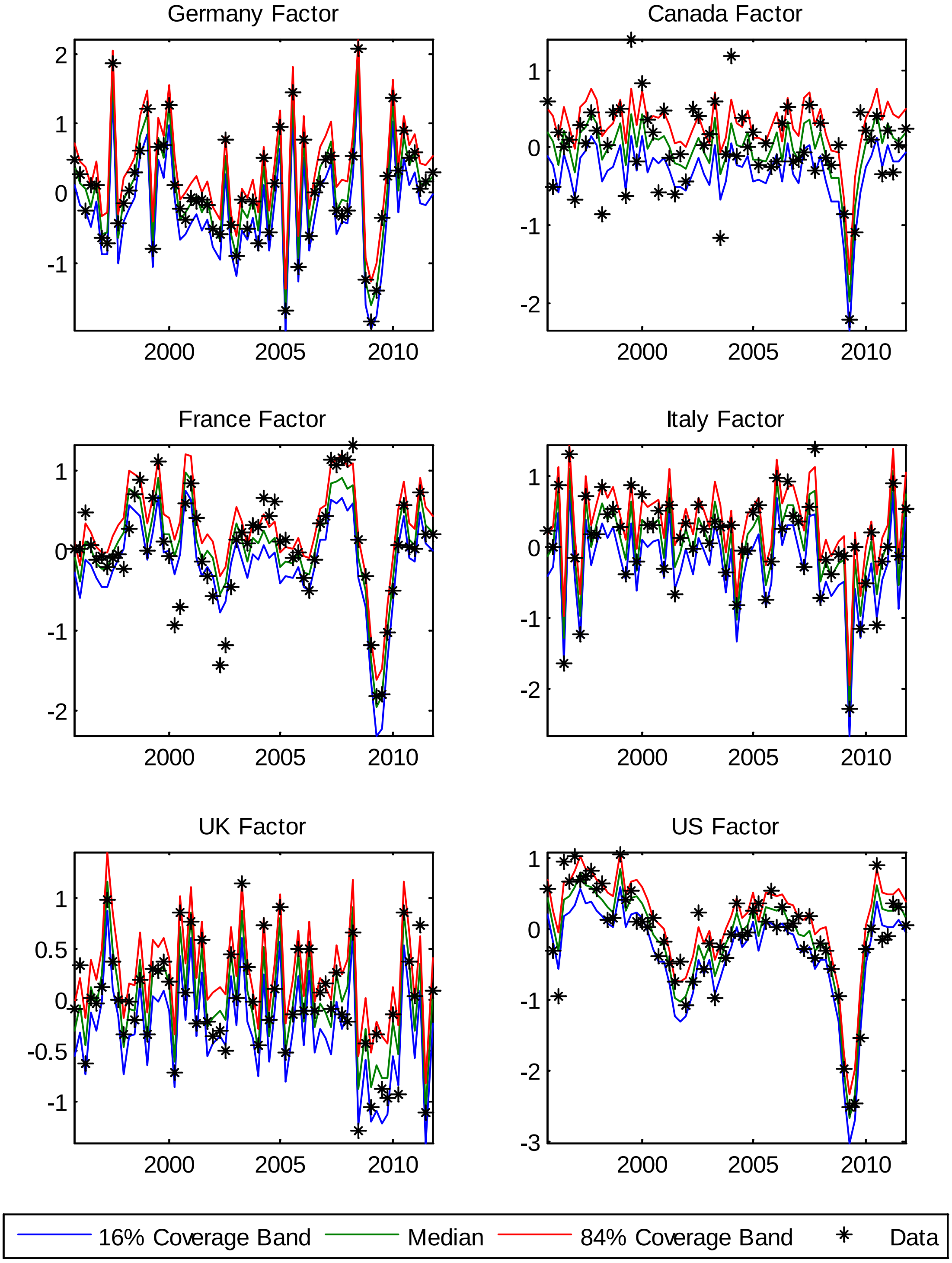

Figure C3: Predicted versus Actual Labour Productivity Growth

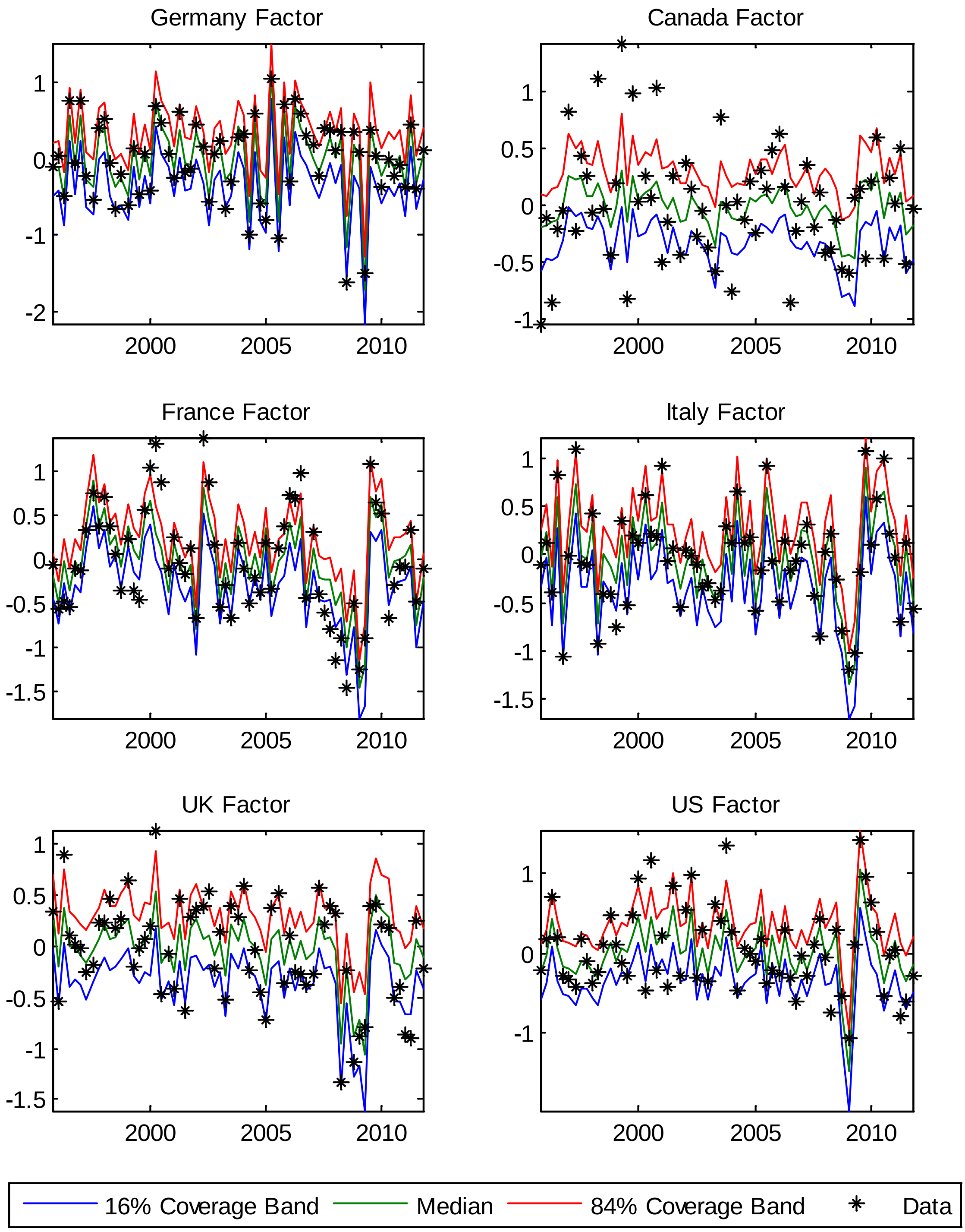

Figure C4: Predicted versus Actual Output Growth – Pre 2007Q2 data

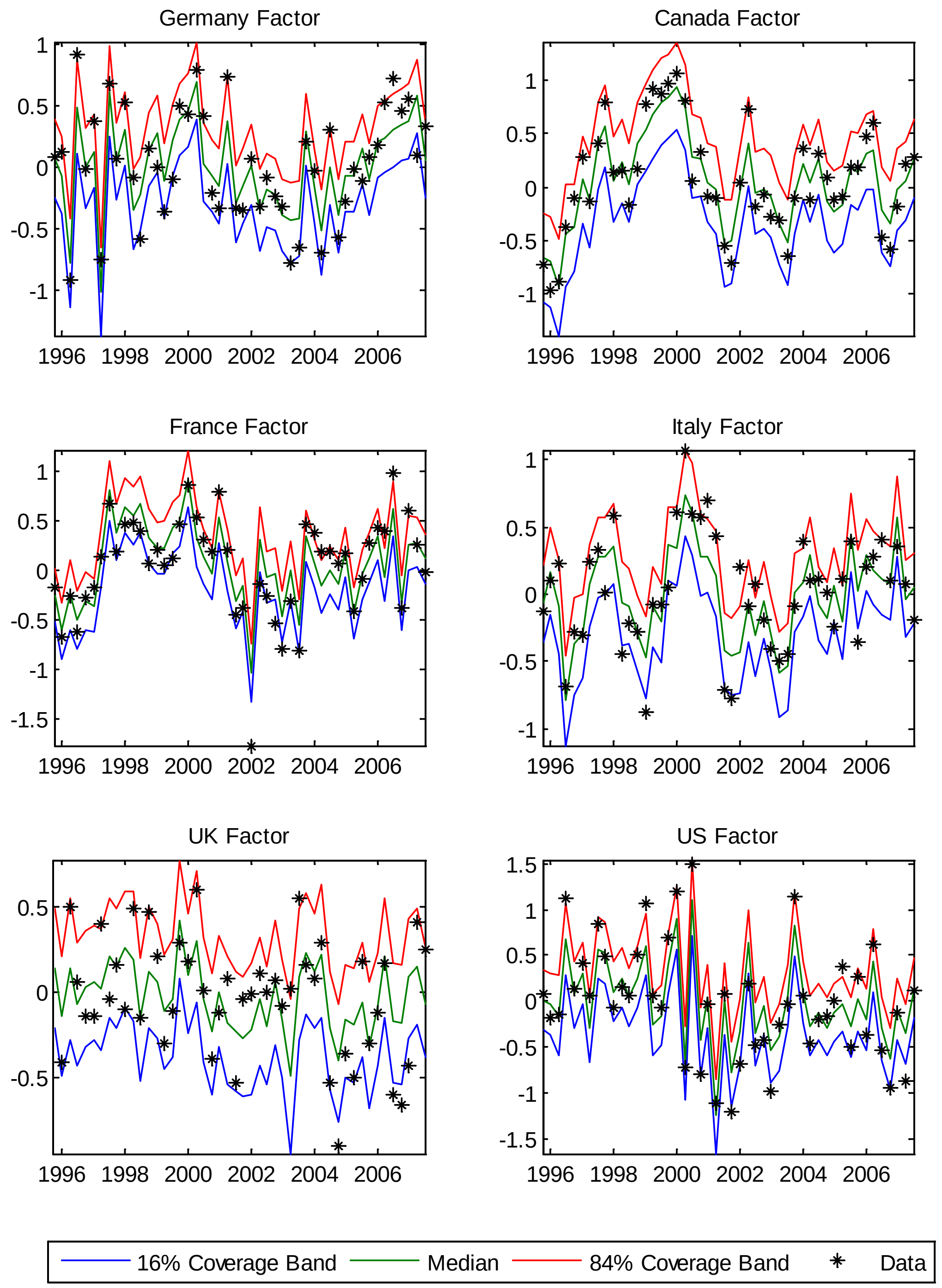

Figure C5: Predicted versus Actual Growth of Hours Worked – Pre 2007Q3 data

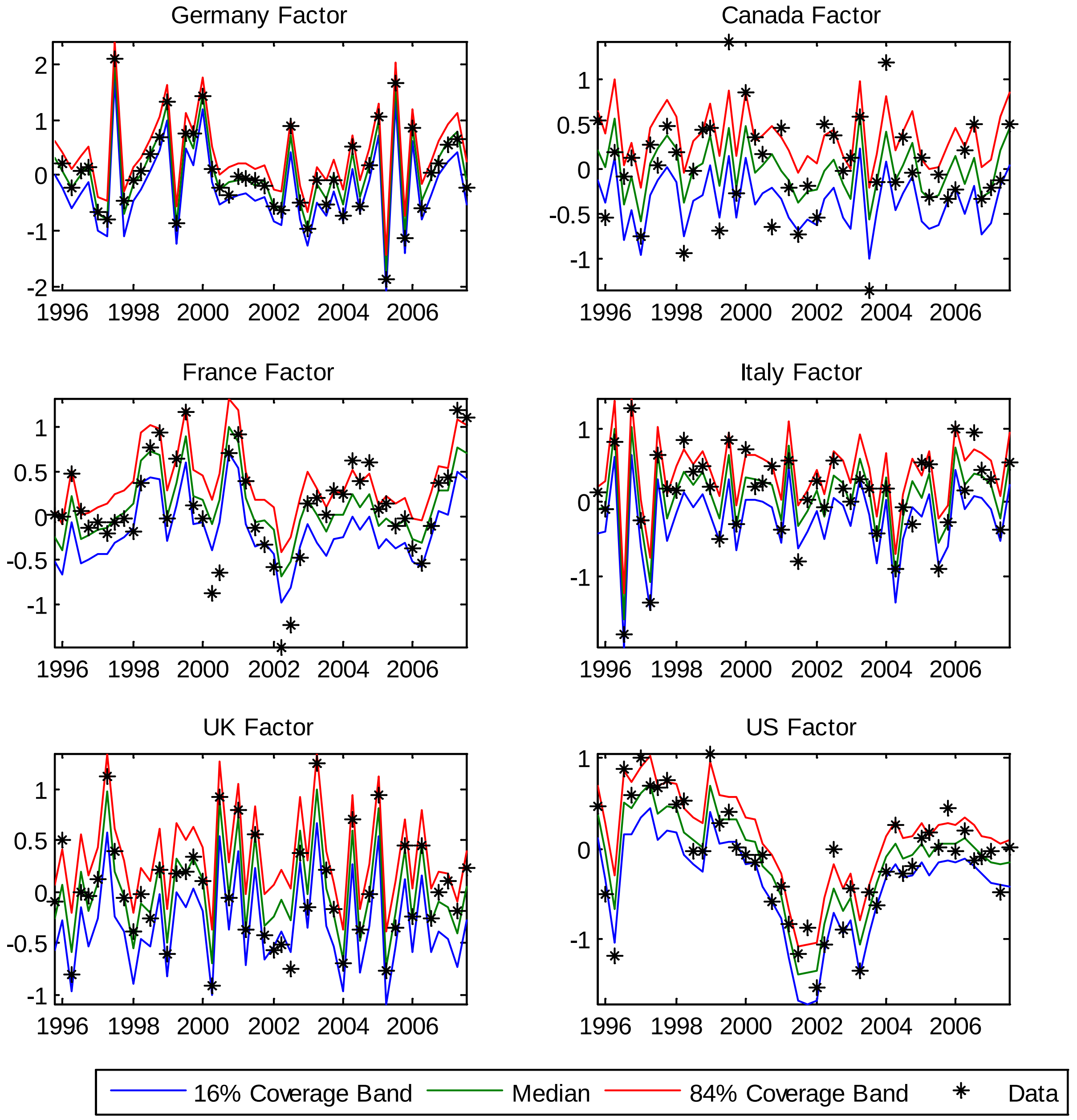

Figure C6: Predicted versus Actual Labour Productivity Growth – Pre 2007Q2 data