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Discussion Paper No. 51

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Ben Broadbent,[1] Federico Di Pace,[2] Thomas Drechsel,[3]
Richard Harrison[4] and Silvana Tenreyro[5]

Abstract

The UK economy has experienced significant macroeconomic adjustments following the 2016 referendum on its withdrawal from the European Union. This paper develops and estimates a small open economy model with tradable and non-tradable sectors to characterise these adjustments. We demonstrate that many of the effects of the referendum result can be conceptualised as news about a future slowdown in productivity growth in the tradable sector. Simulations show that the responses of the model economy to such news are consistent with key patterns in UK data. While overall economic growth slows, an immediate permanent fall in the relative price of non-tradable output (the real exchange rate) induces a temporary 'sweet spot' for tradable producers before the slowdown in tradable sector productivity associated with Brexit occurs. Resources are reallocated towards the tradable sector, tradable output growth rises and net exports increase. These developments reverse after the productivity decline in the tradable sector materialises. The negative news about tradable sector productivity also leads to a decline in domestic interest rates relative to world interest rates and to a reduction in investment growth, while employment remains relatively stable. As a by-product of our analysis, we provide a quantitative analysis of the UK business cycle.

Key words: Brexit, small open economy, productivity, tradable sector, UK economy.

JEL classification: E13, E32, F17, F47, O16.

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(1) Bank of England and Centre for Macroeconomics
(2) Bank of England. Email: federico.dipace@bankofengland.co.uk
(3) London School of Economics and Centre for Macroeconomics
(4) Bank of England and Centre for Macroeconomics. Email: richard.harrison@bankofengland.co.uk
(5) Bank of England, Centre for Macroeconomics, CEPR and London School of Economics

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External MPC Unit, Bank of England, Threadneedle Street, London, EC2R 8AH

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1. Introduction

In the momentous referendum on 23 June 2016, voters decided that the United Kingdom (UK) should leave the European Union (EU). While many of the details regarding the UK's ultimate withdrawal ('Brexit') are still highly uncertain, the aftermath of the referendum has been characterized by significant macroeconomic adjustments in the UK economy. UK economic activity has slowed relative to its long-run trend. Growth in the tradable sector has remained resilient in comparison to the non-tradable sector. The British pound has been subject to a pronounced depreciation (and with it the relative price of non-tradable goods). Exports have been growing robustly. At the same time, UK interest rates have declined relative to their world (US) counterpart and investment fell materially, while employment remained resilient. This paper documents these empirical patterns in UK macroeconomic data and demonstrates that they are consistent with what economic theory predicts for the effects of an anticipated productivity growth slowdown in the UK's tradable sector.

Our analysis is motivated by the remarks of Broadbent (2017*b*), who conjectured that market participants may have interpreted the consequences of the Brexit vote as a future slowdown in the tradable sector, prompting the depreciation of sterling following the referendum. We formalize and assess this idea through the lens of a quantitative two-sector small open economy model estimated using UK macroeconomic data. Our model allows us to characterize how firms and households respond to news about future productivity in the tradable sector by shifting resources across expenditure components, sectors and time. We demonstrate that the macroeconomic dynamics triggered by the news about a disruption in the tradable sector are consistent with the broad patterns in the data following the referendum. While the effects of the referendum encompass a variety of economic channels, our analysis provides an explicit formal framework to interpret some of the macroeconomic mechanisms at play in the face of Brexit.

The paper proceeds in four steps. First, we document a number of stylized facts about UK economy in the period following the 2016 referendum, making use of a novel quarterly macroeconomic data set in which we construct key variables separately for the tradable and non-tradable sectors.[1] The stylized facts describe growth, exchange rate and interest rate dynamics following the Brexit vote. Second, we introduce a two-sector small open economy (SOE) real business cycle model which is composed of tradable and non-tradable sectors. The SOE framework can encompass differential trend growth rates across these sectors under restrictions on preferences and technology. Introducing these differential trends allows us to conduct the relevant experiments. Third, we estimate the model at business cycle frequencies using the newly constructed data set. Our estimation strategy enables us to pin down not only the structural parameters, using relevant information contained in the data, but also the initial steady state around which we simulate Brexit news scenarios.[2] Fourth, we use the model to conduct simulation experiments which are designed to shed light on the economic mechanics

[1]The construction of this novel UK macro data set involves classifying industry data at the 2-digit level into tradable and non-tradable sectors over the period 1997-2015. We construct gross value added, labor productivity as well as relative prices for the tradable and non-tradable sector.

[2]An important feature of our estimation strategy is that we introduce a methodology using ratios to circumvent issues stemming from implicit price deflators in the aggregation of industry-level data.

that generated key patterns in the UK economy following the referendum. At the heart of our analysis is a baseline experiment that assesses the economic impact of *news* that the growth rate of TFP in the tradable sector will be persistently (though not permanently) low. We assume that the fall in TFP growth takes places 11 quarters after it is announced, consistent with the broad contours of the legislative process for EU withdrawal implemented following the referendum.

The model mechanism works as follows. The news about Brexit – conceptualized as an anticipated, persistent decline in the growth rate of TFP in the UK tradable sector – generates a temporary boom in tradable production. This short-run expansion in the tradable sector is driven by the response of the relative price of non-tradable output (an 'internal' real exchange rate) which jumps down when the future TFP growth weakness is revealed. Consequently, there is an opportunity to sell tradable output at a temporarily higher relative price before tradable productivity actually falls, a temporary "sweet spot" for producers of tradable output (Broadbent, 2017*b*,*a*). This generates the reallocation of capital and labor towards the tradable sector, a rise in tradable output growth and an increase in net exports, all of which reverse after the news about the TFP decline in the tradable sector realize. The Brexit news also have important effects on interest rates. In the model, we calculate interest rates that are indexed to tradable goods and to non-tradable goods, respectively. This permits consideration of relative interest rate developments, in particular domestic relative to world interest rates. Following the tradable productivity news shock, the real interest rate on bonds denominated in non-tradable output falls sharply in the short run. Once productivity growth in the tradable sector actually falls, production of tradable output becomes relatively unproductive, prompting a reversal of the inter-sectoral resource flows towards the non-tradable sector. This generates persistent and hump-shaped rise in the real return on non-tradable denominated bonds over the longer-term. The tradable bond rate displays a small but very persistent decline, so that the spread between domestic and foreign rates declines. In addition, the news triggers a material fall in investment, while employment remains resilient.

These patterns of adjustment are in line with the stylized facts for the post-referendum period. As a consequence our broad finding is thus that the macroeconomic response to a disruption in tradable productivity mimics the adjustments following the Brexit vote. Our analysis provides an explicit comprehensive general equilibrium characterization of the effects of news about weaker tradable TFP growth, an intuitive way to conceptualize the referendum outcome through the lens of an SOE model.

A by-product of our exercise is a systematic quantitative analysis of the UK business cycle. In addition to the Brexit experiments we use our model to provide a variety of variance decompositions for UK macroeconomic time series. These decomposition serve as a model-based interpretation of the UK economic developments in the past three decades by characterizing the primitive sources of cyclical fluctuations.

Our work is related to several strands of research. First, there has been a surge in papers exploring the impact of Brexit on the UK economy and beyond, from a variety of angles.[3] This research studies the effects of Brexit on long-run trade (Dhingra et al., 2017; Sampson, 2017),

[3]There are also various studies that focus on the reasons for the outcome of the referendum rather than its economic impact. See for example Becker et al. (2017), Fetzer (2018) and further references provided in these papers.

foreign direct investment (McGrattan and Waddle, 2018) and financial market volatility and stock returns (Davies and Studnicka, 2018). Existing papers have also focused on uncertainty about the final UK-EU trade arrangement in a general equilibrium setting (Steinberg, 2017), the role of uncertainty shocks using the Decision Maker Panel (Bloom et al., 2018; Faccini and Palombo, 2019) and the extent of exchange rate pass-through following the referendum (Forbes et al., 2018).[4] Born et al. (2018) apply a synthetic control method to study the effects of Brexit on UK growth. Our work contributes to the analysis of the referendum impact by providing a novel interpretation of the aggregate UK economy's response to the Brexit news. We highlight that a shock to expectations about productivity in the tradable sector successfully matches the patterns observed in macroeconomic data after the Brexit vote. This is complementary to studying other aspects of Brexit and mechanism through which the Brexit news leads to economic adjustments in the economy as a whole.

Second, our paper relates to research on the role of economic news in business cycles analysis more generally, see in particular Beaudry and Portier (2006), Jaimovich and Rebelo (2009) and Schmitt–Grohe and Uribe (2012). Our paper contributes to the literature that studies the role of news in a open economy setting (Siena, 2014; Kamber et al., 2017) and in multi-sector business cycle models (Gortz and Tsoukalas, 2018; Vukotić, 2018). News shocks in our setting are meant to capture valuable information about the future relationships with the European Union and the structural composition of the UK economy.

Third, we contribute to the broader SOE literature in macroeconomics, which builds upon the classic work of Mendoza (1991). In particular, we depart from the recent contribution of Aguiar and Gopinath (2007), Drechsel and Tenreyro (2018) and others by allowing for TFP growth differentials between a tradable and a non-tradable sector.[5] While these papers have focused on emerging economies, we demonstrate that shocks to trend productivity are a useful modeling device also for advanced economies and show that their role is quantitatively important in the UK.

Fourth, our paper relates to other work that has undertaken a serious calibration of models featuring tradable and non-tradable sectors, such as De Gregorio et al. (1994), Betts and Kehoe (2006) and Lombardo and Ravenna (2012). To the best of our knowledge, we are the first ones to do so using data for the UK. We follow Lombardo and Ravenna (2012) in allocating 2-digit SIC industry level data into a tradable and non-tradable categories, and then use detailed industry-level Gross Value Added (GVA) data to construct time series aggregates following the standard national accounts chain-linking methodology used by the Office of National Statistics (ONS). The same industry classifications are used to construct time-series for total hours as well as the underlying labor productivity data, which are used as input in the estimation of the

[4] In particular the trade literature features many more studies that are helpful to analyze Brexit and its effects. See for example Erceg et al. (2018) for an analysis of the short-run macroeconomic effects of specific trade policies such as tariffs, and Caldara et al. (2019) for a recent paper on trade policy uncertainty.

[5] Other contributions to the broader SOE literature include, but are not limited to, Kose (2002), Garcia-Cicco et al. (2010), Mendoza (2010), Fernandez-Villaverde et al. (2011), Guerron-Quintana (2013), Naoussi and Tripier (2013), Akinci (2013), Hevia (2014), Seoane (2016), Kulish and Rees (2017). The idea of incorporating differential trend growth rates in technologies across sectors in business cycle models also relates to the literature that has studied investment-specific technology shocks alongside shocks to TFP. See in particular Greenwood et al. (2000) and Justiniano et al. (2011).

model.

The remainder of the paper is structured as follows. Section 2 documents some of the key stylized facts about the UK economy following the referendum. Section 3 introduces our two-sector small open economy model. Section 4 presents the data, discusses the results of our estimation to pin down the structural parameters and the initial steady state. Section 5, which forms the core of our analysis, considers the baseline and alternative Brexit scenarios and provides a comprehensive description of the results. As a by-product of our analysis, Section 6 presents a quantitative analysis of the UK business cycle. Section 7 concludes.

2. UK Macroeconomic Adjustments after the Brexit Vote

This section documents key stylized facts about the UK economy following the 2016 Brexit referendum. Some of these facts are based on a novel quarterly macroeconomic data set for the UK, which we build by constructing data series for the tradable and non-tradable sectors separately. To do so, we classify industry data at the 2-digit level into tradable and non-tradable sectors over the period 1997-2015. Detailed information on the construction of the data is provided in Section 4.1.

Figure 1 shows a collection of key UK macroeconomic time series for the years 2010 to 2018. In each panel, the vertical line indicates the date of the referendum, 23 June 2016. Panels A and B are intended to show the change in aggregate UK growth relative to pre-referendum trends and expectations. Panel A is from Vlieghe (2019) and plots the deviation of UK GDP from a 'no Brexit' counterfactual constructed using a synthetic control based on a pool of other countries' GDP.[6] A marked decline is visible, indicating that growth slowed after the referendum relative to what might have been expected in the absence of Brexit. Panel B provides an alternative perspective on this effect and shows that the IMF revised down its UK GDP growth forecasts following the referendum.

Panel C shows a decomposition of gross value added into tradable and non-tradable sectors. It is clear that the two sectors show a parallel trend prior to the referendum, after which there is a sharp break in the growth rate for the non-tradable sector. Panel D presents the relative price of non tradable to tradable output together with the real effective exchange rate (REER). As we will show in the exposition of our two-sector business cycle model, these concepts are closely related. It is evident that the UK real exchange rate drops sharply after the outcome of the referendum.

Panel E plots exports and the trade balance, both measured as a percentage of GDP. While the patterns in this panel are less stark, it suggests that UK trade developed relatively robustly following the Brexit vote. Panels F and G show the evolution of aggregate factors of production. While total investment weakened following the referendum, total labor input (measured relative to labor force participation) has continued to increase. Panel H shows ten-year zero coupon yields for the United Kingdom and the United States. These yields closely track each other prior to the Brexit vote but a spread opens up thereafter. UK yields have remained persistently below

[6] We thank Jan Vlieghe and Rodrigo Guimaraes for sharing this data. See also Born et al. (2018) for an application of this methodology to UK GDP.

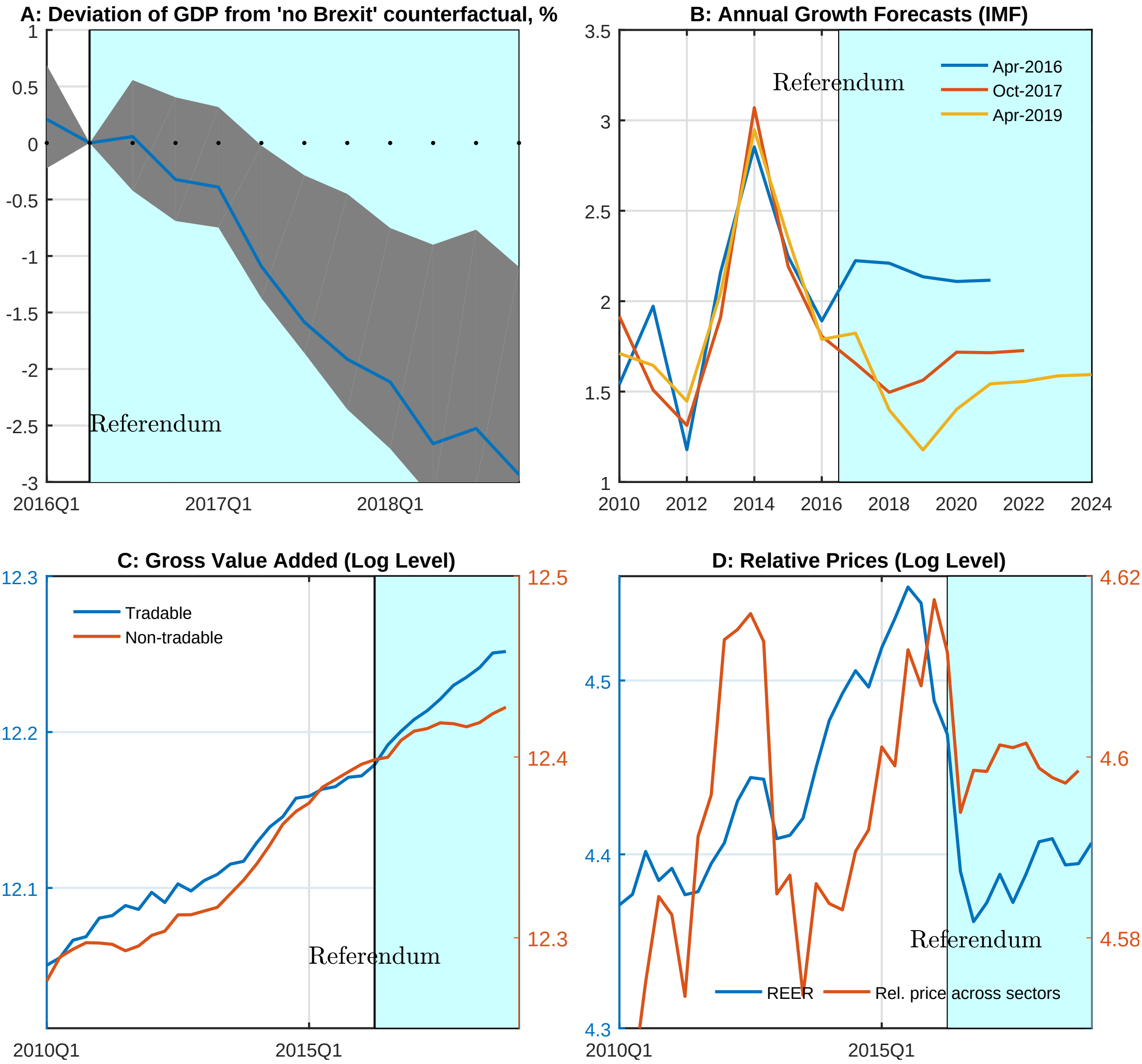

Figure 1: *Adjustments of the UK economy following the Brexit Vote*

their US counterpart in the aftermath of the referendum. Omitting inflation risk and term premia considerations, this pattern is already indicative of a mechanism by which market participants may have perceived a fall in productivity in the UK relative to the US.

In summary, while many of the details regarding the United Kingdom's ultimate withdrawal are still highly uncertain, the aftermath of the referendum has been characterized by significant macroeconomic adjustments in the UK economy. UK economic activity has slowed relative to its long-run trend. Growth in the tradable sector has remained resilient relative to the non-tradable sector. The British pound has been subject to a pronounced depreciation (and with it the relative price of non-tradable goods). Exports have been growing robustly. At the same time, UK interest rates have decline relative to their world (US) counterpart. Our model will tell a coherent and consistent story that jointly explains these facts.

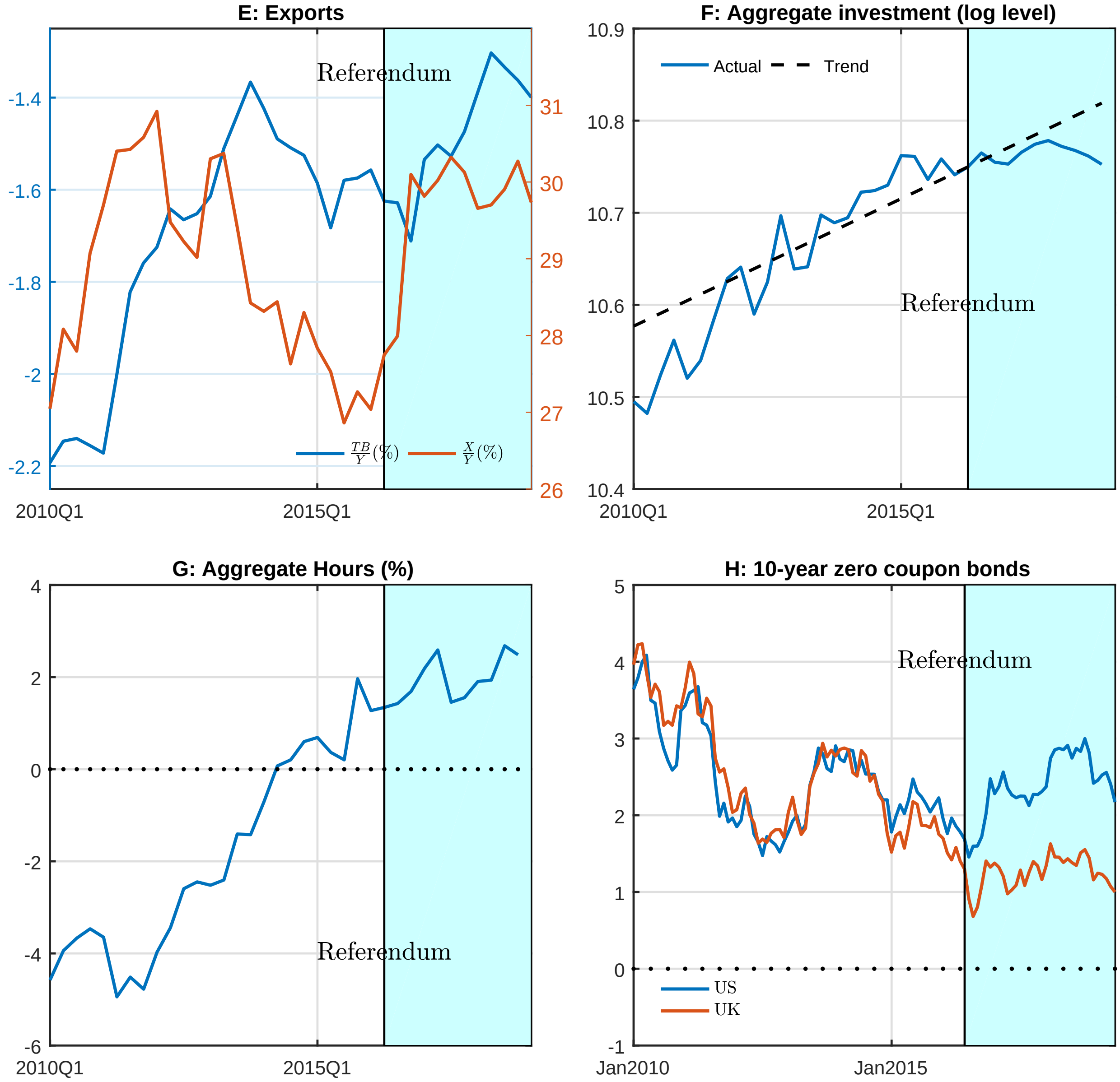

Figure 1 (continued): *Adjustments of the UK economy following the Brexit Vote (cont.)*

3. The Model

The setting is a real small open economy model featuring a tradable (T) and a non-tradable (N) sector. As in Drechsel and Tenreyro (2018), permanent deviations in the levels of sectoral labour-augmenting productivity from their trends are permitted. At time t, each sector grows at its own rate, denoted by g_{Tt} and g_{Nt}. The domestic economy is small in the sense that the world real interest rate is exogenous and the rest of the world absorbs any domestic trade surplus (or supplies any deficit) entirely elastically. Bonds are denominated in terms of tradable and non-tradable goods, the latter being held by domestic households only. The main implication is that the uncovered interest rate parity (UIP) relationship is a 'within economy' concept in the model. More precisely, it takes the form of a no-arbitrage condition between bonds denominated in tradable and non-tradable units, of which only the former is internationally traded. Following Schmitt-Grohe and Uribe (2003), we close the model with a debt elastic premium on external borrowing.

The presence of two stochastic trends in the model implies that different variables grow at different rates along the balanced growth path. To aid exposition, we use lower case letters to denote stationary variables and upper case letters to denote variables that contain a stochastic trend.

3.1. The Firms' Problems

Firms in both sectors combine labor and physical capital using a Cobb-Douglas production technology to produce final output.[7] Physical capital is sector-specific and previously accumulated capital cannot be reallocated across sectors. Labor is sector-specific.[8] Formally, the representative firm in sector $M = \{T, N\}$ produces a final good Y_{Mt} by combining capital K_{Mt} and labor n_{Mt} according to

$$Y_{Mt} = a_{Mt} K_{Mt}^{\alpha_M} (X_{Mt} n_{Mt})^{1-\alpha_M}. \tag{1}$$

Here, a_{Mt} denotes a stationary TFP shock and X_{Tt} the non-stationary component of labor-augmenting productivity. The temporary TFP process in sector M responds to the following process:

$$\ln a_{Mt} = \varrho_M^a \ln a_{Mt-1} + \varepsilon_{Mt}^a, \quad \text{with} \quad \varepsilon_{Mt}^a \sim \mathbb{N}\left(0, \varsigma_M^a\right). \tag{2}$$

where ϱ_M^a is the persistence of the (temporary) sectoral TFP process and ς_M^a its dispersion. The growth rate of sectoral labor-augmenting productivity is defined as

$$g_{Mt} = \frac{X_{Mt}}{X_{Mt-1}}, \tag{3}$$

and follows an autoregressive process of the form:

$$\ln\left(g_{Mt}/\bar{g}_M\right) = \varrho_M^g \ln\left(g_{Mt-1}/\bar{g}_M\right) + \varepsilon_M^g, \quad \text{with} \quad \mathbb{N}\left(0, \varsigma_{Mt}^g\right), \tag{4}$$

where ϱ_M^g is the persistence of the sectoral labour-augmenting productivity shock and ς_{MT}^g its dispersion. The process g_{Mt} captures transitory changes to the growth rate of labor-augmenting productivity in sector M, such that the *level* of productivity is permanently affected. $\bar{g}_M$ denotes the steady state value of the growth rate in sector M. Firms in sector $M = \{T, N\}$ rent both capital and labor services in competitive factor markets at rental rate r_{Mt}^k and real wage w_{Mt}, respectively. Profits are given by

$$Y_{Tt} - W_{Tt} n_{Tt} - r_{Tt}^k K_{Tt} \tag{5}$$

in the tradable sector and

$$P_t Y_{Nt} - W_{Nt} n_{Nt} - P_t r_{Nt}^k K_{Nt} \tag{6}$$

[7]The specification of the production function ensures the existence of balanced growth.

[8]Assuming that labor and capital are freely mobile is likely to generate extreme, and less realistic, inter-sectoral reallocation over the short-run. Under such arrangement, households would supply homogeneous labor, where the relative labor demand would help determining the optimal sectoral allocation.

in the non-tradable sector. Under the assumption of perfect competition, firms make zero profits. The variable P_t denotes the relative price of the non-tradable goods. This price can be interpreted as an 'internal' measure of the real exchange rate. From a conceptual point of view, this interpretation goes back to the work of Samuelson (1964) and Balassa (1964), who have studied international productivity differences and their implications for relative international price levels, that is, for real exchange rates.[9]

3.2. The Household's Problem

From the perspective of the representative household, while tradable and non-tradable consumption are assumed to be gross complements, the consumption of home tradable goods and their foreign counterpart can be perfectly substituted (the law of one price for tradable goods holds). As is standard in the Small Open Economy literature, we specify the period utility function of the representative household in line with Greenwood et al. (1988). In this instance, we scale the disutility of labor supply by tradable labour-augmenting productivity to ensure that both consumption and labor elements of the utility function grow at the same rate along the balanced growth path.[10] The functional form of period utility is given by

$$\mathcal{U}_t\left(C_t, X_{Tt-1}, X_{Nt-1}, n_{Tt}, n_{Nt}\right) = \frac{\left[C_t - X_{Tt-1}\omega^{-1}\left(\theta_T n_{Tt}^{\omega} + \theta_N n_{Nt}^{\omega}\right)\right]^{1-\gamma}}{1-\gamma}, \tag{7}$$

where C_t is a CES aggregator that combines tradable and non-tradable consumption (denoted by C_{Tt} and C_{Nt})

$$C_t = \left[\zeta^{1-\sigma} C_{Tt}^{\sigma} + (1-\zeta)^{1-\sigma}\left(\frac{X_{Tt-1}}{X_{Nt-1}} C_{Nt}\right)^{\sigma}\right]^{\frac{1}{\sigma}}, \tag{8}$$

$\gamma > 1$ the inter-temporal elasticity of substitution and $\eta = 1/\left(1-\sigma\right)$ the elasticity of substitution between tradable and non-tradable consumption.[11] The representative household seeks to maximize the life-time utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \nu_t \beta^t \frac{\left[C_t - X_{Tt-1}\omega^{-1}\left(\theta_T n_{Tt}^{\omega} + \theta_N n_{Nt}^{\omega}\right)\right]^{1-\gamma}}{1-\gamma}, \tag{9}$$

subject to following budget constraint (expressed in tradable units)

$$\begin{aligned} &C_{Tt} + P_t C_{Nt} + B_t^* + P_t B_t + \frac{\phi_T}{2}\left(\frac{K_{Tt+1}}{K_{Tt}} - \bar{g}_T\right)^2 K_{Tt} + P_t \frac{\phi_N}{2}\left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N\right)^2 K_{Nt} + I_{Tt} \\ &+ P_t I_{Nt} + P_t Y_{Nt}\frac{s}{y} s_t = r_{Tt}^k K_{Tt} + P_t r_{Nt}^k K_{Nt} + W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \frac{B_{t+1}^*}{1+r_t^*} + P_t \frac{B_{t+1}}{1+r_t}. \end{aligned} \tag{10}$$

[9]In fact, the *Harrod-Balassa-Samuelson effect* is the empirically observed tendency for countries with stronger productivity in tradable goods relative to non-tradable goods to have higher prices levels overall. The basic mechanics of this effect feature in our model, where a weakness in productivity growth in the tradable sector introduces a fall in the domestic price level.

[10]For more details on detrending, see Appendix A.2.

[11]Note that X_{Tt-1} and X_{Nt-1} enter the utility function to ensure balanced growth. The parameters θ_T and θ_N will allow us to pin down the relative quantities of labor used in the two sectors.

In what follows, we describe the notation and the underlying assumptions. $\beta \in [0,1)$ denotes the subjective discount factor, θ_M the disutility of labour in sector M and ω elasticity of labor supply. The variable ν_t denotes a risk-premium shock given by:

$$\ln \nu_t = \varrho_\nu \ln \nu_{t-1} + \varepsilon_{\nu t} \quad \text{with} \quad \varepsilon_{\nu t} \sim \mathbb{N}\left(0, \varsigma_\nu\right), \tag{11}$$

where ϱ_ν denote the persistence of the discount factor shock and ς_ν its dispersion.

Sectoral physical capital depreciates at the rate δ_M, and its accumulation is subject to sector-specific adjustment costs, where ϕ_M is the parameter that controls how costly capital adjustment is in sector M. Physical investment (I_{Mt}) responds to the following law of motion:

$$K_{Mt+1} = (1 - \delta_M) K_{Mt} + I_{Mt}. \tag{12}$$

One important aspect of the budget constraint is the presence of two different assets, B_t^* and B_t with corresponding interest rates r_t^* and r_t. These are risk-free bonds that pay one unit of tradable goods and non-tradable goods in the following period, respectively. They can be thought of as bonds that are indexed to different types of inflation rates in practice. While a bond that pays tradable units – a standard ingredient of SOE models – allows the economy to achieve a trade balance that is different from zero, the bond that pays non-tradable units remains in zero net supply. Introducing it allows us to determine its interest rate r_t, which will move differently from r_t^*. This feature of the model in turn permits us to analyze relative interest rate developments, shedding some light on how "domestic" relative to "world" interest rates move in response to the Brexit news. This is motivated by the different movement of UK and US rates observed in the data, as shown in Section 2.

The variable s_t is a government expenditure shock, which can be thought of as a broader aggregate demand shifter, and which follows

$$\ln s_t = \varrho_s \ln s_{t-1} + \varepsilon_{st} \quad \text{with} \quad \varepsilon_{st} \sim \mathbb{N}\left(0, \varsigma_s\right), \tag{13}$$

where ϱ_s denotes the persistence of the government expenditure shocks and ς_s its dispersion. The ratio s/y is the steady state share of government expenditure to non-tradable output.

Given preferences, the relative price of the aggregate consumption bundle (in terms of tradable units) is

$$P_t^c = \left[\zeta + (1-\zeta)\left(\frac{X_{Nt-1}}{X_{Tt-1}} P_t\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}. \tag{14}$$

Note that, given the specification of preferences, P_t^c is a stationary variable. The interest rate on the foreign (tradable) bond is given by

$$r_t^* = \bar{r}^* + \psi\left(e^{B_{t+1}^*/X_{Tt} - \bar{b}^*} - 1\right) + (e^{\mu_t - 1} - 1), \tag{15}$$

where r^* denotes the world interest rate, $\bar{r}$ is the steady state value of the foreign interest rate, and the term $\psi\left(e^{B_{t+1}^*/X_{Tt} - \bar{b}^*} - 1\right)$ the country risk premium, which is increasing in the amount of foreign debt. The latter assumption follows Schmitt-Grohe and Uribe (2003) and ensures a stationary solution of the model after detrending.[12] Finally, the term $(e^{\mu_t-1}-1)$ captures an foreign interest rate shock, which follows

$$\ln \mu_t = \varrho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t} \quad \text{with} \quad \varepsilon_{\mu t} \sim \mathbb{N}_t\left(0, \varsigma_\mu\right), \tag{16}$$

where ϱ_μ denote the persistence of the shock and ς_ν its dispersion.

3.3. Resource Constraints

The market clearing conditions are

$$Y_{Tt} = C_{Tt} + I_{Tt} + \frac{\phi_T}{2}\left(\frac{K_{Tt+1}}{K_{Tt}} - \bar{g}_T\right)^2 + TB_t \tag{17}$$

in the tradable sector and

$$Y_{Nt} = C_{Nt} + I_{Nt} + \frac{s}{y} Y_{Nt} s_t + \frac{\phi_N}{2}\left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N\right)^2 \tag{18}$$

in the non-tradable sector. We define the trade balance as

$$TB_t = B_t^* - \frac{B_{t+1}^*}{1 + r_t^*}. \tag{19}$$

The model exhibits two stochastic trends and is de-trended to characterize a stationary equilibrium. Following Aguiar and Gopinath (2007), Garcia-Cicco et al. (2010) and Drechsel and Tenreyro (2018), we divide the sectoral variables by the corresponding technology level $X_{M,t-1}$. The then calculate the deterministic steady state of the model.[13]

4. Estimation Strategy

The primary goal of this section is to estimate the structural parameters of the model to pin down the initial steady state from which the Brexit experiments are conducted. A secondary goal is to assess the quantitative contribution of structural shocks to the variance of economic fluctuations in the UK economy. To that end, we estimate the stochastic processes under the assumption that disturbances are unanticipated (and omit anticipated shocks, or news shocks, as in Beaudry and Portier (2006)). We exploit the variability at business cycle frequencies to estimate a subset of the model parameters by combining different sources of information.

Brexit is a unique and unprecedented event, which is likely to have a long lasting impact on the UK economy. Section 5, which forms the core of the analysis in this paper, models the Brexit shock as an anticipated zero probability event that affects the future economic structure. Since at business cycle frequencies it is very difficult to extract information about the impact of the Brexit referendum on the UK economy, we estimate the model up to the quarter of the EU

[12] As we discuss in Section 5.3 and show formally in Appendix E, the conclusions we draw in this paper are robust to alternative assumptions to ensure the model's stationary solution. Assuming an endogenous discount factor as proposed by Schmitt-Grohe and Uribe (2003) yields similar results.

[13] For more details on the model's de-trending, refer to Appendix A.2. Appendix A.3 explicitly describes how we calculate the deterministic steady state.

referendum (2016Q2) and simulate the impact of Brexit from this date on.[14] While more recent data may contain information about the effects of the referendum on economic outcomes, our estimation procedure does not allow us to selectively switch on news shocks from the quarter after the referendum.

Following An and Schorfheide (2007), the model is estimated using Bayesian techniques. This approach requires a) calibrating a selected number of the structural parameters to match key macroeconomic relationships, b) choosing the prior distributions of the structural parameters, c) selecting the shock processes and d) using the information contained in aggregate time-series data to compute the posterior distributions of the structural parameters.[15] Two issues with this approach, given the underlying modeling structure, are, first, that the choice of the time-series and, second, structural shocks is far from trivial and that parameter identification can be problematic.[16] The approach we take in selecting the structural shocks is rather conservative in that we focus on fundamental shocks that are widely accepted in the literature. We also have a relatively low number of structural parameters due to the parsimonious structure of the model.

4.1. Data

We estimate the model using aggregate UK time-series data from 1987Q3 to 2016Q2, a period during which the UK was a full member of the EU (after having joined the European Economic Community in 1973Q1). A novelty of this paper is that a) we construct time-series data for tradable and non-tradable Gross Value Added (GVA) and labor productivity and b) we use the shares of consumption and investment to GDP as observable variables in order to preserve as much information as possible and to avoid contaminating the time-series with noise arising from aggregation.

Following Lombardo and Ravenna (2012), we classify the low level GVA aggregates (detailed GVA data) into tradable and non-tradable sectors to construct new time-series data. We construct annual time-series data for the period 1997-2015 (rather than taking a snapshot) to rule out that any given sector switches classification from one year to another. This way we obtain a representative classification for the entire sample period. We chain-link ONS detailed industry-level GVA (2-digit) data using the standard national accounts methodology employed by the ONS. We construct time-series data on tradable and non-tradable total hours by adding up (detailed) total hours data under the same industry classification. We then compute time-series aggregates of sectoral labor productivities by taking the ratio between sectoral GVA and total hours.[17] Having aggregated detailed GVA data (from 1990Q1), we calculate the relative price of non-tradable goods by dividing the resulting implicit price deflators. Since nominal industry-level GVA (2-digit) data only starts in 1997Q1, the span of the implicit price deflators is shorter than that of the real sectoral GVA series.

[14] For an approach that attempts to extract information using asset pricing data, see for example Davies and Studnicka (2018).

[15] A difference with the well-known model of Smets and Wouters (2007) is that we deliberately introduce two sectoral stochastic trends rather than a single aggregate deterministic trend in the TFP process.

[16] See in particular Den Haan and Drechsel (2018) and Beltran and Draper (2018) for recent contributions as well as Komunjer and Ng (2011).

[17] The appendix contains additional information regarding the construction of the time-series aggregates.

As observable variables for the model estimation, we use the following set of transformed time-series: the quarterly growth rates of sectoral labor productivity (available from 1994Q1), the quarterly growth rate of the relative price of non-tradable goods (only available from 1997Q1), the quarterly growth rate of the real effective exchange rate, total hours (demeaned) and the ratios of nominal consumption, investment and trade balance to GDP (available from 1987Q3). The data series are chosen to add informational content to the estimation of the posterior distributions of the structural parameters. We make use of the Kalman filter to handle missing observations in the time-series of the sectoral labor productivities and the relative price of non-tradable goods. In the estimation step, we introduce measurement errors for each of the constructed observable variables.[18]

4.2. Mapping the model to observable variables

Selecting and constructing observables to estimate our model poses two key challenges. The first one entails the use of implicit price deflators to obtain real quantities and the second one entails defining the real effective exchange rate in the model. We provide a discussion on these challenges in turn.

Model consistent consumption and investment can be computed by deflating the nominal consumption and investment by the tradable GDP implicit price deflator. However, since the resulting GVA deflators exhibits significant amount of noise (and are only available from 1997Q1), using them to calculate model consistent aggregates would imply discarding useful information and relying on the use of additional measurement errors. To circumvent this issue, we propose to use the ratios of nominal aggregates, rather than the growth rate of real quantities, as observable variables. We therefore construct a set of model variables and then map them to the data. To estimate the structural parameters more precisely, our procedures requires that the values of the steady state ratios implied by the model match the averages in the data.[19]

There are two exchange rates concepts in the model: a) the relative price of non-tradables vis-a-vis tradables (an 'internal exchange rate') and b) the relative price of aggregate home consumption with respect to its foreign equivalent (an 'external exchange rate'). In the data, the internal exchange rate is calculated using the implicit price deflators.[20] Mapping the external real exchange rate measures (across model and data) requires making an assumption about the rest of the world. First, preferences in the rest of the world are assumed to be the same as those in the home economy. Second, at business cycle frequencies, we further assume that stochastic trends of the tradable sectors at home and abroad are cointegrated. We define the real effective exchange rate as:

$$\mathcal{Q}_t = \frac{\mathcal{E}_t \mathcal{P}_t^c}{\mathcal{P}_t^{c,*}},$$

[18]The presence of noise is due to the following two reasons: a) aggregation of detailed industry level data inevitably gives rise to measurement errors and b) although the growth rates of GVA and GDP are highly correlated, the measures of GVA and GDP are not equivalent. Note in particular that a) industry-level data on total hours is available at less disaggregated level relative to GVA data (so some judgment is applied) and b) GVA data is used as proxy for T and N final output.

[19]In Appendix A.3 we derive two alternative ways of pinning down the same steady state (an algebraic steady state used for simulation purposes and numerical steady state used for estimation purposes).

[20]We introduce measurement errors and adopt the Kalman filter to extrapolate the missing values of the relative price of non-tradable goods.

where $\mathcal{E}_t$ denotes the nominal exchange rate, $\mathcal{P}_t^c$ the nominal price level of the home consumption bundle and $\mathcal{P}_t^{c,*}$ its foreign equivalent. Under the Law of One Price (LOOP), it follows that $\mathcal{P}_{Tt}^*/\mathcal{E}_t = \mathcal{P}_{Tt}$ and that

$$\mathcal{Q}_t = \frac{P_t^c}{P_t^{c,*}} = \frac{P_t^c}{\xi_t}.$$

where ξ_t captures exogenous movements in foreign prices ($P_t^{c,*}$) and is governed by the following stochastic process

$$\ln \xi_t = \varrho_\xi \ln \xi_{t-1} + \varepsilon_{\xi t} \quad \text{with} \quad \varepsilon_{\xi t} \sim \mathbb{N}\left(0, \varsigma_\xi\right). \tag{20}$$

This shock is meant to capture variation in the exchange rate that arises from unspecified shocks originating in the rest of the world. We emphasize that the exchange rate is an endogenous object. The exogenous shock to it will play a minor role and can be interpreted as a persistent measurement error.[21] By exploiting this additional relationship, we can bring more information to the estimation in order to pin down key structural parameters more precisely.

4.3. Calibration and Priors

One period in the model corresponds to one quarter in the data. We calibrate a number of structural parameters by targeting key macroeconomic relationships, and estimate the remaining parameters using Bayesian methods. We set σ to -0.5, which corresponds to an elasticity of substitution equal to $\eta = \frac{1}{1-\sigma} = 0.67$, within the range of estimates in the literature.[22] The chosen value gives rise to gross complementarity across consumption aggregates; a feature that helps generating unconditional co-movement across sectoral outputs at business cycle frequencies. The depreciation rates are assumed to be equal across sectors. The chosen values are low ($\delta_M = 0.0065$) in order to match the sample average of the ratio of nominal investment to GDP (18.12%). In line with the data, we choose θ_N and θ_T to equally distribute hours worked across sectors.

Using ONS data we calculate the nominal shares of government expenditure and trade balance to GDP for the period 1987Q3 until 2016Q2. The values of the ratios are $\frac{s}{y} = 0.184$ and $\frac{tb}{y} = -0.015$ respectively. These sample averages determine the values of both $\frac{s}{y_N}$ and $\frac{tb}{y_T}$, which are then used for simulation purposes. We calculate $\bar{g}_T$ and $\bar{g}_N$ directly from the data. The discount factor (β) is set to match a quarterly foreign real interest rate of 1%. Finally, the elasticity of the foreign interest rate with respect to debt (ψ) is set to a small (and positive) number (5×10^{-6}) following the small open economy literature (see e.g. Schmitt-Grohe and Uribe (2003)). The model calibration is summarized in Table 1.

[21] Finally, we follow the approach used by the BIS to construct a time-series of the real effective exchange rate from the data.

[22] There is a wide range of values for the elasticity of substitution between tradable and non-tradable goods. Mendoza (1991) and Corsetti et al. (2008) set the value to 0.75. While Dotsey and Duarte (2008) choose a value for this elasticity of 0.5. Stockman and Tesar (1995) and Rabanal and Tuesta (2013) estimate it to be 0.44 and 0.13 respectively.

Table 1: *Calibrated parameter values*

Parameter	**Description**	**Source**	**Period**	**Value**
σ	preference parameter related to IES	mid-range estimate		-0.5
δ_M	capital share in M sector	ONS	$1987-2016$	$i/y = 0.181$
ϕ_N	capital adjustment cost in N			4
θ_T	disutility of labor in T	ONS and own calculations	$1994-2016$	$n_T/n = 0.5$
θ_N	disutility of labor in N	ONS and own calculations	$1994-2016$	$n_N/n = 0.5$
$\frac{s}{y}$	government exp. over GDP	own calculations	$1994-2016$	0.184
$\frac{tb}{y}$	trade Balance over GDP	own calculations	$1994-2016$	-0.015
$\bar{g}_T$	trend quarterly growth rate of labor productivity in T	ONS and own calculations	$1990-2016$	annual $\bar{g}_T = 1.83\%$
$\bar{g}_N$	trend quarterly growth rate of labor productivity in N	ONS and own calculations	$1990-2016$	annual $\bar{g}_N = 1.02\%$
β	discount factor			$r^* = 0.01$
ψ	debt-elasticity of interest rate premium			5×10^{-6}

Table 2: *Prior information and mean posterior estimates*

					90% HPDI	
	Description	Distribution	Mode	Mean	Lower	Upper
	Structural parameters					
$c_T/\mathcal{C}$	share T consumption	Gaussian	0.59	0.59	0.57	0.61
ω	elasticity of labor supply	Gaussian	1.99	1.99	1.85	2.13
α_T	capital share in T	Gaussian	0.31	0.31	0.30	0.32
α_N	capital share in N	Gaussian	0.25	0.25	0.24	0.26
ϕ_T	capital adjustment cost in T	Gaussian	9.65	9.65	8.45	10.85
	Shocks					
ς_N^g	st.dev. of TFP growth shock in N	Inv. Gamma	0.014	0.014	0.012	0.016
ς_T^g	st.dev. of TFP growth shock in T	Inv. Gamma	0.014	0.014	0.012	0.016
ς_s	st.dev. of government expenditure shock	Inv. Gamma	0.036	0.036	0.031	0.04
ς_μ	st.dev. of foreign interest rate shock	Inv. Gamma	0.01	0.01	0.009	0.011
ς_ν	st.dev. of risk-premium shock	Inv. Gamma	0.035	0.036	0.03	0.042
ς_T^a	st.dev. of TFP level shock in T	Inv. Gamma	0.013	0.013	0.011	0.015
ς_N^a	st.dev. of TFP level shock in N	Inv. Gamma	0.013	0.013	0.011	0.012
ς_ξ	st.dev. of labor supply shock in N	Inv. Gamma	0.026	0.026	0.022	0.029
ϱ_N^g	persistence of TFP growth shock in N	Beta	0.23	0.25	0.07	0.43
ϱ_T^g	persistence of TFP growth shock in T	Beta	0.12	0.15	0.04	0.25
ϱ_s	persistence of government expenditure shock	Beta	0.88	0.86	0.79	0.94
ϱ_μ	persistence of foreign interest rate shock	Beta	0.03	0.04	0.01	0.08
ϱ_ν	persistence of risk-premium shock	Beta	0.94	0.93	0.88	0.98
ϱ_N^a	persistence of TFP level shock in N	Beta	0.80	0.75	0.58	0.93
ϱ_T^a	persistence of TFP level shock in T	Beta	0.97	0.97	0.95	0.99
ϱ_ξ	persistence of exchange rate shock	Beta	0.95	0.94	0.91	0.99

Table 2: *Prior information and mean posterior estimates*

	Description	Distribution	Mode	Mean	90% HPDI	
					Lower	Upper
		Measurement errors				
ι_N	labor productivity in N	Inv. Gamma	0.013	0.013	0.011	0.015
ι_T	labor productivity in T	Inv. Gamma	0.014	0.014	0.012	0.016
ι_P	relative price	Inv. Gamma	0.014	0.015	0.012	0.017

The locations of the prior means of the structural parameters largely correspond to those in Smets and Wouters (2007) (see Table 2). Using the ONS supply-and-use tables for the period 1997-2014/5, we compute the annual shares of tradables into aggregate consumption and then pin down the value of the parameter ζ that targets the sample average ($c_T/\mathcal{C} = 0.59$).[23] The prior mean of ζ is set to match this sample average. We also calculate the sample means of the sectoral capital shares to be $\alpha_T = 0.315$ and $\alpha_N = 0.245$ in the tradable and non-tradable sectors. Since the values are biased downwards (and they are not representative of the entire sample), we center the prior means around the sample averages and then compute their posterior distributions.[24] Note that we set the value of the investment adjustment cost parameter in N to $\phi_N = 4$, in line with the chosen prior mean for sector T.

The posterior mean of the elasticity of labor supply, ω, is estimated to be 1.99, which is in line with standard values used in the literature. The mean estimate of the investment adjustment cost in the tradable sector is relatively higher (9.65) than found in related studies. However, this value is plausible given the low value of the sectoral depreciation rates. Absent adjustment costs, low depreciation rates would tend to generate larger investment flows than observed in the data. The discount factor, the temporary sectoral TFP, government expenditure and the foreign price are estimated to be highly persistent stochastic processes ($\rho_\nu = 0.93$, $\rho^a_N = 0.75$, $\rho^a_T = 0.97$, $\rho_s = 0.85$ and $\rho_\xi = 0.95$ respectively). A common finding in most models featuring stochastic trends in the TFP process is that the estimated persistence of the growth shocks is relatively low ($\rho^g_N = 0.25$ and $\rho^g_T = 0.15$). The foreign interest rate shocks displays very little persistence ($\rho_\mu = 0.04$). The posterior mean of the standard deviation of measurement errors (denoted by ι) for sectoral labor productivities and the relative price of non-tradable goods are similar and statistically different from zero. The estimation results are detailed in Table 2.

5. Main Results: a stylized Brexit scenario

In this section we present a stylized Brexit scenario, which focuses on the prospects for productivity growth in the tradable sector. Broadbent (2017*b*) argues that the effects of greater trade

[23]We $\mathcal{C} = c_T + p \cdot c_N$ denotes aggregate consumption expressed in terms of tradables.

[24]Setting the right priors for the capital shares is very important not only because they affect the value of the depreciation rate that matches the investment to GDP ratio but also because they influence the estimated value of adjustment costs. In addition, both capital shares and the adjustment cost parameters are key parameters for understanding the dynamics of the returns on bonds denominated in tradable and non-tradable units.

frictions may mimic many of the effects of a fall in tradable sector productivity. More broadly, the empirical links between openness and TFP growth have been widely studied (see, for example, Edwards, 1998). Our model is well-suited to studying the economy-wide effects of productivity changes. Naturally, our scenario abstracts from a wide range of potential effects and many of the other implications of Brexit are better suited to alternative frameworks.[25]

5.1. Effects on tradable sector productivity

Brexit is modeled as a structural shift in the economy, exhibiting a prolonged period of historically weak tradable sector productivity growth. Specifically, we study an anticipated fall in tradable sector productivity growth. The shock to growth is persistent, but ultimately temporary. There is a permanent effect on the *level* of tradable sector productivity, but growth eventually recovers to the initial steady-state growth rate. To implement this assumption, we replace the exogenous process determining the growth rate of tradable sector productivity in the estimated version of the model (described in Section 3). While that estimated process captures the business cycle movements in tradable sector productivity during the period of EU membership, it is less suitable for analyzing a structural change of the type we are investigating.

In our scenario, the growth rate of labour-augmenting productivity in the tradable sector, g_{Tt}, is determined by the following equations:

$$\begin{aligned}\ln\left(g_{Tt}\right) &= \varrho_T^g \ln\left(g_{Tt-1}\right) + \left(1-\varrho_T^g\right)\ln\left(\widetilde{g}_{Tt}\right), \\ \ln\left(\widetilde{g}_{T,t}\right) &= \widetilde{\varrho}_T^g \ln\left(\widetilde{g}_{Tt-1}\right) + \left(1-\widetilde{\varrho}_T^g\right)\ln\left(\bar{g}_T\right) + \varepsilon_{Tt}^g.\end{aligned}$$

where $\widetilde{\varrho}_T^g > \varrho_T^g$ so that that $\widetilde{g}_{Tt}$ represents the persistent component of tradable sector productivity growth: g_{Tt} converges on $\widetilde{g}_{T,t}$. We set $\widetilde{\varrho}_T^g = 0.95$ and $\varrho_T^g = 0.8$. This implies that the initial fall in tradable sector productivity growth is gradual and that the total reduction in the *level* of tradable productivity is complete after about 30 years.

We calibrate the scale of the shock with reference to existing studies of the potential effects of Brexit on trade. There are many different estimates of the potential effect, in part because there is a wide range of possible eventual trading arrangements between the United Kingdom and European Union. We use existing estimates of the effects (relative to remaining in the European Union) of moving to trading arrangements governed by World Trade Organisation (WTO) rules. This is not because we believe this is the most likely outcome. Instead, this focus is useful because it allows a clearer comparison between existing estimates, since the underlying assumptions about the eventual trading arrangements are more consistent across studies.

Table 3 summarizes recent estimates. The estimated long-term reduction in UK trade from moving to WTO rules covers a wide range, from 10% to almost 30%. The corresponding reductions in GDP are estimated to range between 3% and 11%. We calibrate our experiment so that trade falls by 10% in the long-run, in line with the smaller estimates of the effects of moving to WTO rules. The results of our experiment could therefore be regarded either as a lower bound estimate of a transition to WTO rules or as a simulation of transition to a relatively closer trading

[25]For example, gravity models have been widely used to study the effects of trade frictions on the pattern of trade in the long run (Dhingra et al., 2017).

Study	Estimated *reduction* in trade, %	Estimated *reduction* in GDP, %
Ebell and Warren (2016)	21–29	2.7–3.7
IMF (2018)		5.2–7.8
Kierzenkowski et al. (2016)	10–20	2.7–7.5
UK Government (2018)	13–18	6.3–10.7

Table 3: *Estimates of long-run effects of WTO trading arrangements on UK trade and GDP*

relationship with the European Union.[26] Our simulation outputs could be scaled up (by a factor of 2–3) to provide a range for the potential effects of transition to WTO trading arrangements.

The experiment is configured so that the future reduction in tradable productivity is fully anticipated. The economy starts in steady state in period 0 (where a period is a quarter of a year). In quarter 1, it is revealed that there will be a persistent reduction in tradable productivity growth from quarter 11 onward. This anticipation horizon broadly mimics the planned timeline for EU exit following the referendum.[27]

Our assumptions abstract from two important aspects of the Brexit process. First, in our model there is no uncertainty about the extent of the reduction in tradable sector TFP.[28] Second, there is no uncertainty about the timing of the fall in productivity. Such uncertainty could have direct effects on spending. Although, to be sure, consumption growth appears to have remained largely unaffected by uncertainty. Moreover, the timing of effects on productivity is unclear: the effect of uncertainty on investment decisions could lower productivity before the actual Brexit date (though the effect on medium-term productivity may be comparatively small, if the uncertainty is relatively short lived).

5.2. Results

Figure 2 presents our main result. The anticipated fall in tradable productivity growth leads to an immediate fall in the relative price of non-tradable output. This encourages a near-term reallocation of resources towards the tradable sector and an export boom. In the longer-term, resources are reallocated towards the non-tradable sector.

Unpacking our main result in Figure 2 reveals the key forces that underpin it. Panel A shows the assumed trajectory of tradable sector labour-augmenting productivity growth. During the anticipation phase, the shaded area between quarters 1 and 10, tradable sector growth is unchanged from the baseline steady state (black dashed line). In quarter 11, productivity growth falls for several quarters before starting to recover gradually to the initial steady state. The cumulative effect of the shock is a permanent reduction in the level of tradable sector productivity of around 10%.

The permanent reduction in tradable sector productivity leads to a permanent fall in the

[26] That is because studies of the effects of moving to WTO rules typically generate larger estimated effects on trade and GDP relative to moving to other trading arrangements (which imply a closer trading relationship with the European Union). See, for example, UK Government (2018).

[27] The referendum was held on 23 June 2016. The UK government triggered Article 50 of the Lisbon treaty on 30 March 2017, with the United Kingdom's membership of the European Union to end within two years of that date. The end date of the UK's EU membership was subsequently postponed as the negotiation process developed.

[28] Steinberg (2017) presents an analysis of Brexit uncertainty and finds that uncertainty plays a relatively small role.

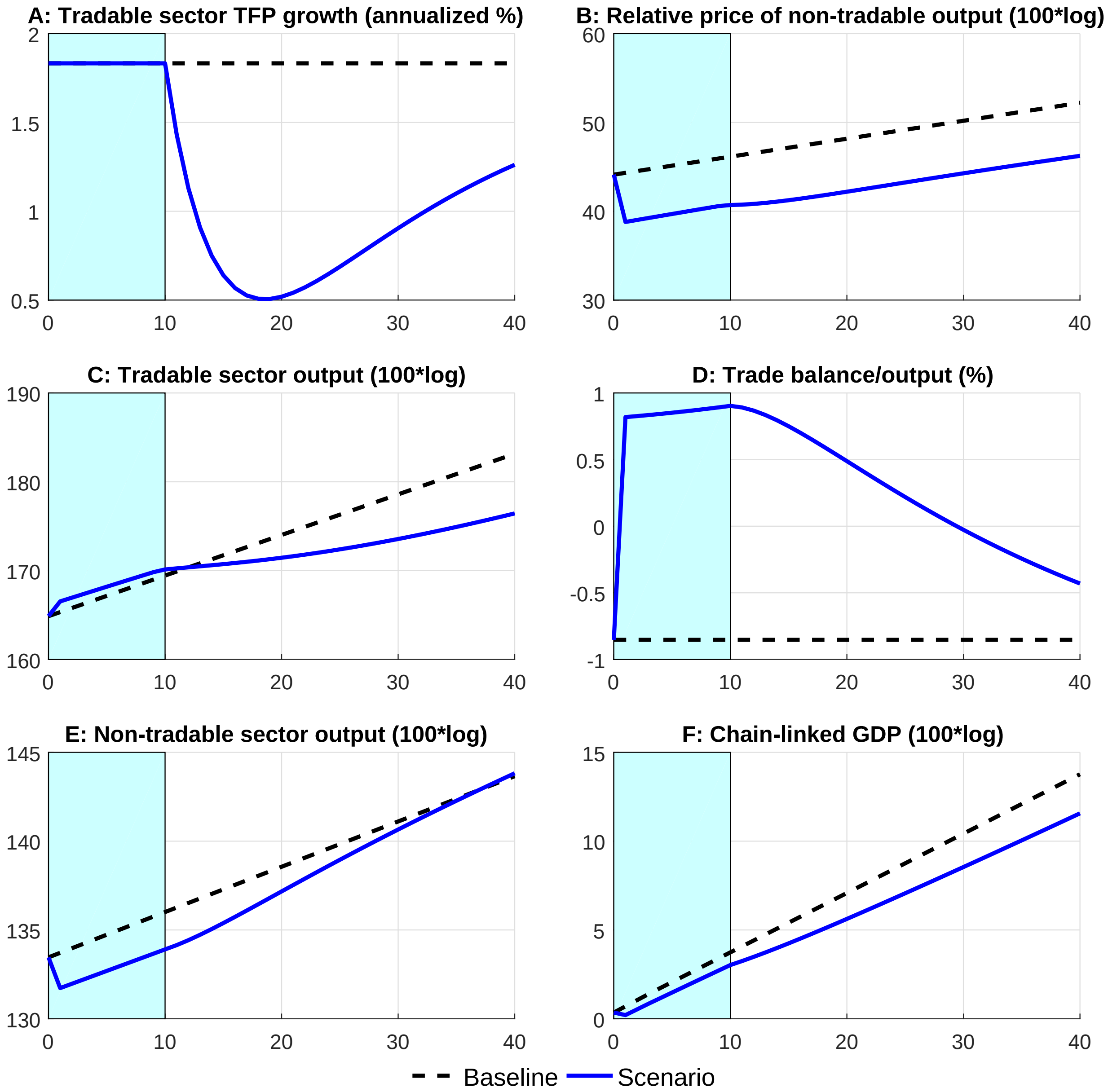

Figure 2: *Headline responses to the tradable TFP growth scenario*

relative price of *non-tradable* output, since it will become relatively more efficient to produce. Panel B shows that the price of non-tradable output falls immediately, even before tradable productivity growth has changed.

During the anticipation phase, tradable goods are relatively profitable to produce because tradable productivity growth has not yet begun to fall. As shown in panels C and D, this effect encourages production of tradable goods and exports in the near term. Once tradable sector productivity falls, however, the incentives to produce tradable goods decline and output and the trade balance fall in the longer term.[29]

Unsurprisingly, the profile of non-tradable output is the mirror image of tradable output, as shown in panel E. During the anticipation phase, non-tradable output is relatively unprofitable and output declines. Once tradable sector productivity falls, non-tradable output becomes

[29]Panel D plots the ratio of the trade balance, TB_t, to the total value of output, $Y_{Tt} + P_t Y_{Nt}$.

relatively profitable and output increases in the longer term. Eventually non-tradable output converges back to the pre-shock trajectory, since the balanced growth path for the non-tradable sector is unaffected by the change in tradable sector TFP.[30]

The net effect of the opposing forces on the tradable and non-tradable sectors gives rise to a muted response of GDP (panel F).[31] The initial response of GDP is small, but the effect builds over time. The long-run level of GDP is around 3% lower. This is towards the smaller end of the range of estimates in Table 3. That is consistent with the fact that the scale of the shock we study generates a relatively small reduction in trade, compared to the studies cited.

To further explore the sectoral implications of the scenario, Figure 3 focuses on factors of production and rates of return. The inter-sectoral reallocation is consistent with the main mechanism underpinning our results: during the anticipation phase, the tradable sector becomes relatively profitable but this effect is reversed once tradable sector productivity actually falls. Panels A and B show that labor moves from the non-tradable sector to the tradable sector during the anticipation period, to support increased production of tradable goods. Overall, total employment rises during the anticipation phase. This pattern starts to reverse once tradable TFP growth actually falls.

Panel C shows that investment in the tradable sector falls abruptly before slowly converging to a new, lower, level. Investment prospects in the tradable sector are dominated by the longer-term outlook for TFP. In contrast, panel D shows that, while non-tradable investment initially falls, it subsequently rises above the baseline path. In aggregate, there is a significant near-term fall in investment while employment increases. Taken together, the responses can be seen as an economy-wide shift from capital towards labor, a phenomenon highlighted by many commentators.[32]

Panels E and F show the real bond returns in both sectors. Overall, the movements in rates of return are relatively small, given the scale of the output effects combined with the financial openness of the UK economy. The small decline in the tradable bond rate is driven entirely by the debt elastic premium.[33] The return on bonds denominated in non-tradable output falls during the anticipation phase, before rising above the steady-state level. These dynamics reflect the behavior of the marginal product of capital, which falls in the near term because returns to production in the non-tradable sector are temporarily lower.[34]

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[30] This convergence occurs over a longer horizon than shown in Figure 2 because asset and capital stocks take a very long time to fully adjust to the shock, which is itself persistent.

[31] The chain-linked GDP growth rate is computed as:

$$g_t^{GDP} = \omega_{T,t} \frac{y_{Tt}}{y_{Tt-1}} g_{Tt-1} + (1 - \omega_{T,t}) \frac{y_{Nt}}{y_{Nt-1}} g_{Nt-1},$$

where $\omega_{T,t}$ is computed as a one-year rolling average of the expenditure share on tradable goods, $\frac{y_{Tt}}{y_{Tt}+p_t y_{Nt}}$. This approximates a national accounts treatment, though abstracts from annual re-basing.

[32] These observations suggest that the source of the shock matters for the pattern of sectoral reallocations. Though a formal comparison is beyond the scope of this paper, it is instructive to compare the simulation with the behavior of the UK economy following the depreciation of sterling associated with the UK's exit from the Exchange Rate Mechanism in 1992. The period following that depreciation saw a significant investment boom, more apparent in tradables than non-tradables. Our simulation does not have these properties, because the depreciation is the result of the anticipation of a negative shock that depresses the returns on investment.

[33] The near term rise in exports reduces foreign debt and hence the tradable bond rate. By calibrating ψ to be extremely small, the effect on the tradable bond rate is restricted to a few basis points.

[34] The relative price of non-tradable output falls immediately, before tradable sector productivity actually falls.

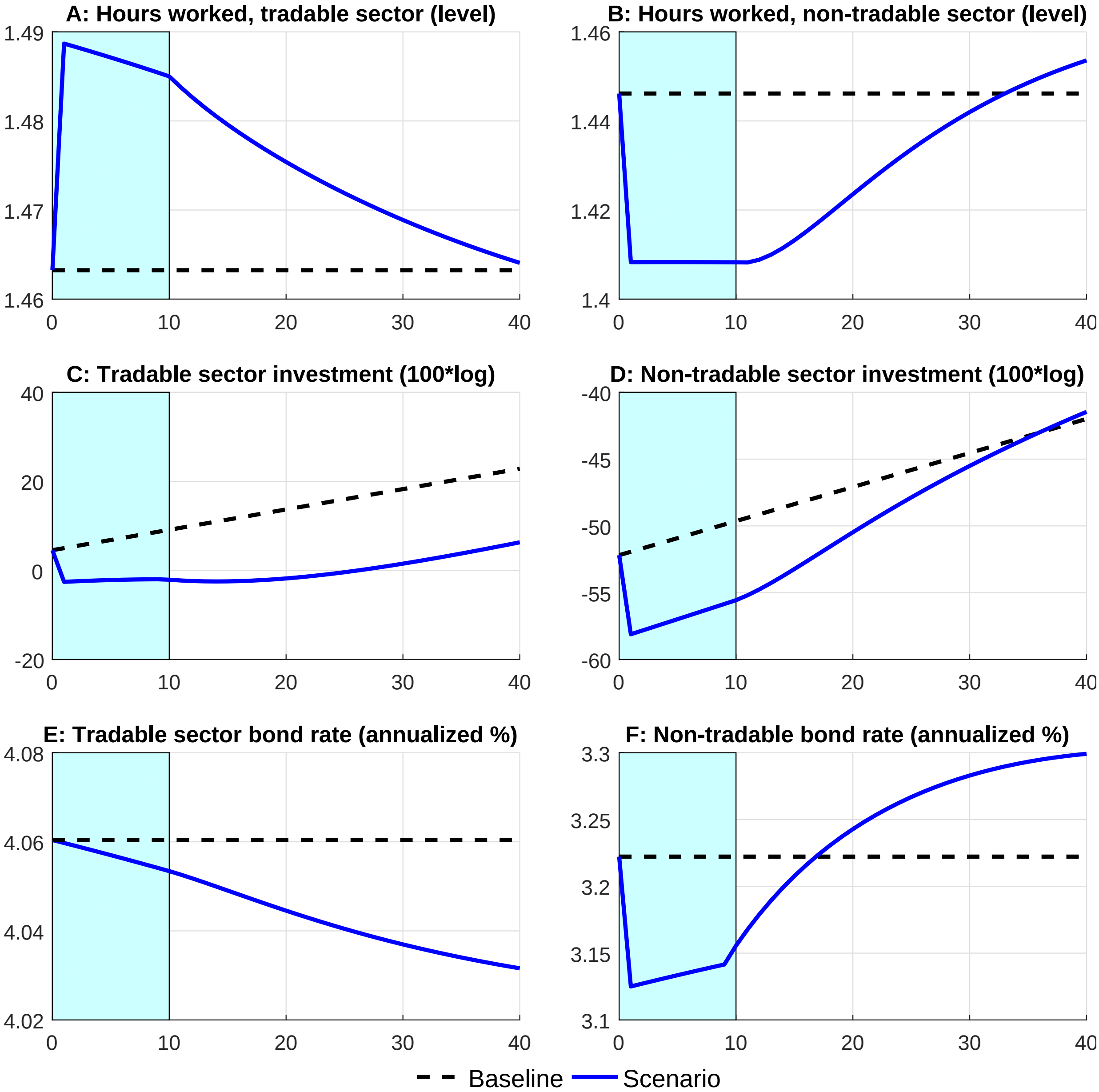

Figure 3: *Sectoral responses to the tradable productivity growth scenario*

Although the Brexit process is still underway, the simulation results are broadly consistent with the macroeconomic dynamics of the UK economy since the EU referendum. In particular, the data reviewed in Section 2 (Figure 1) display many similar patterns.

Figure 1 (panel D) shows a sharp decline in both the relative price of non-tradable output and the real effective exchange rate around the referendum date. Moreover, both series have remained persistently below the levels observed immediately before the referendum, consistent with our simulation. Interestingly, this happened both through the depreciation of sterling and through a protracted adjustment in the price inflation of non-tradables (e.g., rents and house price inflation have fallen markedly since the referendum). Panel C of Figure 1 shows a marked slowdown in GVA growth in the non-tradable sector and a mild acceleration in the tradeable sector, again consistent with our simulation results. The simulation also implies that the news of Brexit triggers a fall in interest rates, on both traded and non-traded bonds. This is broadly consistent with the sharp decline in UK long-term government bond yields following the referendum (Figure 1 panel F), though the fall in the data is larger and more protracted. The comparison with the data is complicated by the range of factors affecting government bond yields that are omitted from the model.[35]

The simulation also predicts a temporary boom in exports, consistent with the UK's relatively strong export performance following the referendum. Figure 1 (panel E) shows a marked pickup in the ratios of net trade and exports to GDP following the referendum. While the movement in the trade balance in the data is relatively modest, the pickup in the export to GDP ratio is around 2 percentage points, similar to the response of the trade balance to GDP ratio in our simulation.[36] The volatility in the trade data make it difficult to draw strong conclusions about the extent to which the simulations match the post-referendum data. However, according to the latest vintage of data, calendar-year export growth in 2017 was 5.6%, which is substantially above the Bank of England's (pre-referendum) May 2016 *Inflation Report* forecast of 1.25% (see Bank of England, 2016, Table 5.E, page 34).

The simulation predicts a long-run reduction of GDP of around 3%, relative to the baseline path. As noted in Section 2, forecasts of UK GDP growth were revised down following the referendum result. For example, the IMF forecasts shown in Panel B of Figure 1 were reduced by roughly 0.5% per year following the referendum, amounting to a reduction in the level of UK GDP (relative to the pre-referendum forecast) of around 2.5% over the five-year forecast horizon. Panel A of Figure 1 suggests that UK GDP in 2018Q4 was between 1% and 3% below the 'no Brexit' counterfactual.[37] Moreover, as our simulation predicts, there was a substantial fall in UK investment following the referendum result (Figure 1, Panel F). Estimates presented by Carney (2019) suggest that the effects of the referendum result may have reduced UK business investment by around 25%.[38] At the same time, employment has been strong (Figure 1, Panel G), consistent with the pick-up in total hours in our simulation. One caveat in comparing our results with these studies, however, is that they may be capturing many effects that we abstract from in our simulation.

While it is too early to carry out a rigorous test of the model's predictions, our tentative conclusion is that it matches the broad contours of UK macroeconomic performance since the referendum.[39]

5.3. Robustness

Appendix C shows that our main results are robust with respect to three important assumptions.

[35]The model abstracts entirely from nominal prices and risk. The ideal data series for comparison purposes would be a short-term risk free real interest rate, but constructing reliable measures of such rates is challenging.

[36]The model abstracts from gross trade flows (differentiated imports and exports) making it less straightforward to map from model concepts to the data. Mechanically, the fact that the trade balance increases by less than exports suggests that imports rose following the referendum. In the absence of strong expenditure switching effects, the value of imports may increase because of the higher price of imports associated with the depreciation of sterling.

[37]Born et al. (2018) provide an estimate of 2%, using a similar methodology.

[38]Carney (2019, Chart 12) shows that UK business investment in 2018 was roughly 25% below the level normally observed at that stage in the business cycle. The EU referendum marks the point at which the recovery in business investment following the 2008 recession deviates from the recovery patterns following previous UK recessions.

[39]Evaluating the model's predictions for sectoral variables is complicated by data limitations and the associated difficulty in producing a counterfactual (no Brexit) baseline.

First, the macroeconomic responses are qualitatively similar for a range of plausible assumptions about the timing of the decline in tradable sector productivity growth. Appendix C.1 reports results for cases in which the fall in tradable sector TFP growth is anticipated to occur 5 quarters and 13 quarters in the future, alongside the baseline assumption of 9 quarters. These variants have predictable effects on the responses to the scenario in the very near term: in particular, the timing of reversals in inter-sectoral allocation changes with the timing of the decline in tradable sector productivity growth. However, the dominant force underpinning the scenario is the long-run decline in the level of tradable sector productivity. Since the long-run decline is independent of the timing of the productivity growth reduction, the results from the variants considered are very similar.

Second, the responses are robust to the assumption that the level of tradable sector productivity falls more sharply than the baseline case. Again, this reflects the fact that the dominant force is the effect on the long-run level of tradable sector productivity. Holding the scale of this effect constant, a faster decline in tradable sector productivity has relatively little effect on the dynamic responses, even in the near term.[40]

Third, the responses are not sensitive to the assumption used to close the model, that is, to ensure a determinate return to the steady-state net foreign asset position (see the discussion below equation (15)). Our baseline model assumes that this is achieved by the presence of a debt-elastic premium on foreign borrowing. Even though the elasticity of the premium with respect to borrowing (ψ) is parameterized to a very small value, the baseline scenario generates a small decline in the tradable bond rate (Figure 3, panel E). Appendix C.3 demonstrates that this effect does not have important implications for rates of return or other macroeconomic variables. The responses to the scenario are almost identical in a variant of the model in which the tradable bond rate is fixed and a determinate net foreign asset position is achieved by the assumption that there is constant growth in the population of infinitely-lived households.

6. Decomposing the UK Business Cycle

What is the relative contribution of different structural disturbances to business cycle fluctuations in the UK? While the main purpose of our model is to conduct Brexit experiments, we can apply it to provide a number of variance decompositions for UK macroeconomic time series. These decomposition serve as a model-based interpretation of the primitive sources of UK macroeconomic developments in the past three decades.

We begin by showing infinite horizon forecast error variance decompositions based on the posterior mean estimates. Table 4 presents the proportion of variation in the 8 observable variables that can be accounted for by a particular structural shock. The table also provides the same model-based decomposition for interest rates that is implied by the estimates.

According to the estimated model, while 24% of the fluctuations in total hours is accounted for by permanent innovations to TFP in both sectors, 69% of the observed variation is due to temporary tradable TFP innovations. Furthermore, we find that sectoral innovations to

[40] The sharper decline in tradable sector productivity growth is generated by reducing the value of $\tilde{\varrho}_T^g$. Appendix C.2 provides the details.

TFP explain the bulk of the variation in sectoral labor productivities, with permanent sectoral TFP shocks contributing to over 40% and temporary TFP shocks to around 55% of the overall variance. Although a large part of the variation in the exchange rate is accounted for by foreign price shocks (84%), permanent productivity innovations play some role, explaining around 12% of the variability. The table also shows that around half of the fluctuations in the ratio of investment to GDP is due to government expenditure and foreign interest rate shocks, 23% of the ratio of consumption to GDP to risk-premium shocks and 19% of trade balance to GDP ratio to risk-premium and foreign price shocks. A large part of the cyclical fluctuations in these ratios are explained by tradable TFP shocks (with non-tradable TFP playing a smaller role). Unsurprisingly, we find that, through the lens of the model, innovations to non-tradable TFP contribute to explaining much of the cyclical movement in the returns on non-tradable bonds (of each 44% is attributed to the permanent component and 11% to the temporary component). Government expenditure and foreign interest rate disturbances play a non-negligible role in explaining the variance of r (around 25% and 13%). Almost all of the cyclical fluctuations of the returns of tradable bonds is attributed to foreign interest rate shocks, which is a direct mechanical consequence of the small open economy assumption.

Table 4: *Variance decompositions of observable variables and interest rates (%)*

Variable	ς_N^g	ς_T^g	ς_s	ς_ν	ς_μ	ς_ξ	ς_N^a	ς_T^a
N	8.72	15.56	0.51	0.75	0.16	0.00	5.37	68.92
$d\ln\left(\frac{Y_N}{N_N}\right)$	41.85	2.00	0.18	0.02	1.02	0.00	54.70	0.23
$d\ln\left(\frac{Y_T}{N_T}\right)$	1.21	39.30	0.14	0.01	0.58	0.00	0.04	58.71
$d\ln(Q)$	5.25	6.49	0.58	0.06	3.23	83.48	0.18	0.73
$d\ln(P)$	12.03	31.98	6.85	0.70	37.83	0.00	2.05	8.55
$\frac{I}{GDP}$	5.63	33.06	25.21	4.45	20.64	0.00	4.40	6.60
$\frac{C}{GDP}$	13.64	25.78	1.51	22.86	0.88	0.00	2.28	33.05
$\frac{TB}{GDP}$	8.92	38.63	0.96	7.54	11.43	0.00	0.58	31.93
r^*	0.01	0.04	0.00	0.00	99.91	0.00	0.00	0.03
r	43.87	1.22	25.31	2.41	12.83	0.00	11.12	3.24

Notes. Forecast error variance decomposition (at infinite horizon) of the 8 observable variables used for estimating the model, as well as for the interest rates computed. Decompositions are computed at the posterior mean estimates.

More importantly, we turn to constructing *historical* variance decompositions that break down the historical variation of UK data into the relative contribution of the different shocks. This exercise consists of running several counterfactuals where we compute the proportion of the variable predicted by the model as we feed the individual filtered shocks. In the presentation of the results here we focus on two of the observables, which are the growth rates of labor productivity in the non-tradable and tradable sectors, respectively. The historical variance decompositions of the remaining observables used to estimate the model are presented in Appendix D. It should also be noted that we focus on the post-2000 period below, but similar decompositions can be presented for the full estimation sample since 1987.

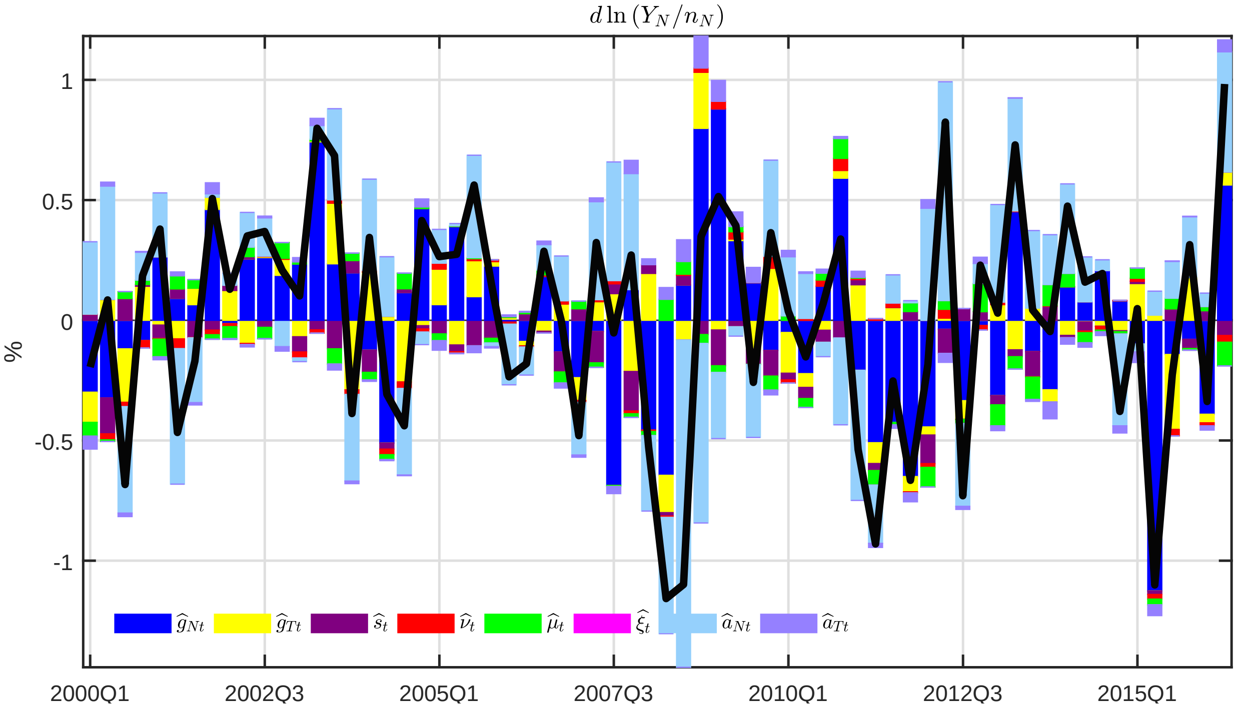

Figure 4: *Historical decomposition of labor productivity in N sector*

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

Figures 4 and 5 present the historical decompositions for the quarterly growth rate of sectoral labor productivities from 2000Q1 until 2016Q2. This allows us to analyze the impact of different shocks on tradable and non-tradable labor productivities leading up to the Brexit referendum. The black line displays the filtered time series data (excluding measurement errors) and the bars denote the contribution of different shocks to the movements in the series at any given point in time. According to the estimated model, over the last decade and a half, transitory and permanent TFP innovations both explain large part of the variation in quarterly sectoral labor productivity growth in the UK. In particular, Figure 5 shows that the model interprets the fall in tradable sector TFP at the onset of the financial crisis as a large contraction in both components of sectoral TFP. Interestingly, the slow and gradual recovery of tradable labor productivity after the Great Recession is largely due to the weakness of the permanent component of tradable sector TFP. As is visible in Figure 4, such shocks to the growth rate of productivity also play an important role in the non-tradable sector after the crisis.

The importance of shocks to the trends in productivity is a finding that warrants particular emphasis. In their seminal contribution Aguiar and Gopinath (2007) ("the cycle is the trend") emphasize that shocks to productivity growth rate are a key drivers of business cycles in emerging economies, where macroeconomic fluctuations can be large and persistent in a way that temporary shocks cannot fully account for the variation. Our findings for the UK highlight

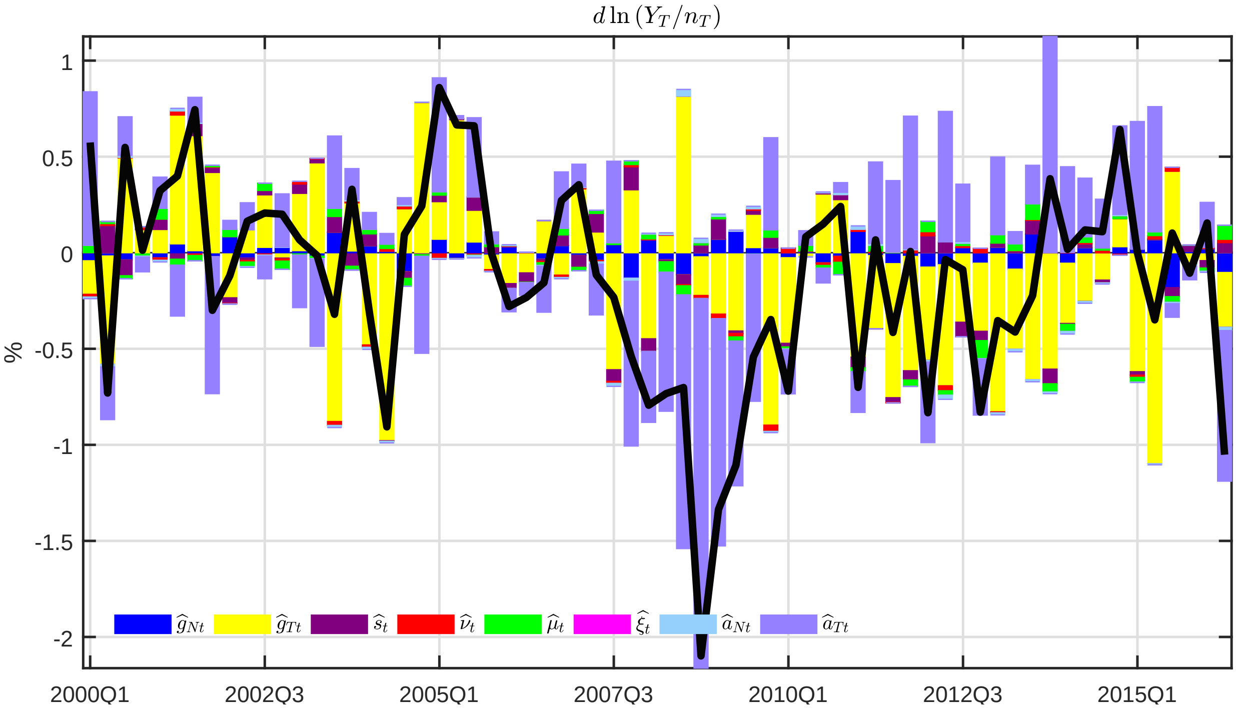

Figure 5: *Historical decomposition of labor productivity in T sector*

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

that a model estimated on data from an *advanced* economy also attributes an important role to these shocks. We conjecture that the importance of permanent productivity variation in the post-2000 sample reveals some important trend changes in the UK that the model picks up. This is particularly interesting in light of the debate on the productivity decline in the UK, something that commentators have dubbed the "productivity puzzle". Pessoa and Van Reenen (2014), Barnett et al. (2014) and Tenreyro (2018) provide more details on this ongoing debate.[41] As a by-product of our main analysis on Brexit, the results above thus highlight the potential usefulness of our methodology in economies that experience important trend changes that may vary across sectors.

7. Conclusions

We develop a quantitative model to study the economic impact of the Brexit referendum on the UK economy. We examine theoretical experiments which we show are consistent with UK macroeconomic data in the aftermath of the referendum. Negative news to tradable TFP growth – our way to conceptualize Brexit in the model framework – have a long-lasting negative effect on the price of non-tradable goods (the 'internal exchange rate'), create a "sweet spot" for exporters during the period leading up to EU withdrawal, triggering resource reallocation towards the tradable sector, with a subsequent reversal after EU withdrawal. The negative news also give

[41] See also Antolin-Diaz et al. (2017) for some econometric estimates of changes in long-run growth rates across the G7 economies.

rise to a gradual fall in the rate of return of tradable bonds and a sharper fall in the return of bonds denominated in terms of non-tradable output. Furthermore, it creates a material fall in investment, with a relatively insensitive response in employment, an unprecedented combination in UK historical data. While Brexit encompasses a variety of economic mechanisms, this paper aims to provide a concise framework in which macroeconomic adjustments to this momentous historical event can be interpreted.

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Appendix A contains details on the derivation of the model. A.2 describes the de-trending of the model and the stationary system of equations. A.3.1 contains the algebraic calculation of the steady state used for simulation purposes. The steady state used to estimate the model can be found in A.3.2. We ensure that the steady state values resulting from the estimation exercise are the same as those used to conduct the simulation exercises. Appendix B contains details on the data construction. Appendix C examines the sensitivity of the Brexit simulations to alternative assumptions about the timing of the shock (C.1) and the speed of the fall in LAP (C.2). Appendix D provides additional historical decompositions. Appendix E lays out the alternative model with population growth.

A. Model Details

A.1. First order conditions

Given technological constraints (as in equation (1)), the optimality conditions of firms are:

$$r^k_{Tt} = \alpha_T a_{Tt} K_{Tt}^{\alpha_T - 1} (X_{Tt} n_{Tt})^{1-\alpha_T}, \tag{21}$$

$$W_{Tt} = (1-\alpha_T)\, a_{Tt} K_{Tt}^{\alpha_T} (X_{Tt} n_{Tt})^{-\alpha_T} X_{Tt}, \tag{22}$$

$$r^k_{Nt} = \alpha_N a_{Nt} K_{Nt}^{\alpha_N - 1} (X_{Nt} n_{Nt})^{1-\alpha_N}, \tag{23}$$

and

$$W_{Nt} = P_t\,(1-\alpha_N)\, a_{Nt} K_{Nt}^{\alpha_N} (X_{Nt} n_{Nt})^{-\alpha_N} X_{Nt}. \tag{24}$$

These conditions simply state that sectoral factors of productions are paid their marginal products.

The household's optimality conditions with respect to C_{Tt}, C_{Nt}, n_{Tt}, n_{Nt}, K_{Tt+1}, K_{Nt+1}, B^*_{t+1}, B_{t+1} are:

$$\left[C_t - X_{Tt-1}\omega^{-1}\epsilon_t\left(\theta_T n_{Tt}^{\omega} + \theta_N n_{Nt}^{\omega}\right)\right]^{-\gamma}\left(\frac{C_{Tt}}{\zeta C_t}\right)^{\sigma-1} = X_{Tt-1}^{-\gamma}\lambda_t, \tag{25}$$

$$\left[C_t - X_{Tt-1}\omega^{-1}\epsilon_t\left(\theta_T n_{Tt}^{\omega} + \theta_N n_{Nt}^{\omega}\right)\right]^{-\gamma}\left[\frac{C_{Nt}}{(1-\zeta)\,C_t}\frac{X_{Tt-1}}{X_{Nt-1}}\right]^{\sigma-1}\frac{X_{Tt-1}}{X_{Nt-1}} = X_{Tt-1}^{-\gamma}\lambda_t P_t, \tag{26}$$

$$\left[C_t - X_{Tt-1}\omega^{-1}\epsilon_t\left(\theta_T n_{Tt}^{\omega} + \theta_N n_{Nt}^{\omega}\right)\right]^{-\gamma}\theta_T X_{Tt-1}\epsilon_t n_{Tt}^{\omega_1 - 1} = X_{Tt-1}^{-\gamma}\lambda_t W_{Tt}, \tag{27}$$

$$\left[C_t - X_{Tt-1}\omega^{-1}\epsilon_t\left(\theta_T n_{Tt}^{\omega} + \theta_N n_{Nt}^{\omega}\right)\right]^{-\gamma}\theta_N X_{Tt-1}\epsilon_t n_{Nt}^{\omega - 1} = X_{Tt-1}^{-\gamma}\lambda_t W_{Nt}, \tag{28}$$

$$\begin{aligned} X_{Tt-1}^{-\gamma}\lambda_t\nu_t\left[1+\phi_T\left(\frac{K_{Tt+1}}{K_{Tt}} - \bar{g}_T\right)\right] &= X_{Tt}^{-\gamma}\beta\mathbb{E}_t\lambda_{t+1}\nu_{t+1}\Bigg[r^k_{Tt+1} + (1-\delta) + \\ \phi_T\left(\frac{K_{Tt+2}}{K_{Tt+1}} - \bar{g}_T\right)\frac{K_{Tt+2}}{K_{Tt+1}} - \frac{\phi_T}{2}\left(\frac{K_{Tt+2}}{K_{Tt+1}} - \bar{g}_T\right)^2&\Bigg], \end{aligned} \tag{29}$$

$$X_{Tt-1}^{-\gamma}\lambda_t\nu_t P_t\left[1+\phi_N\left(\frac{K_{Nt+1}}{K_{Nt}}-\bar{g}_N\right)\right]=X_{Tt}^{-\gamma}\beta\mathbb{E}_t\lambda_{t+1}\nu_{t+1}P_{t+1}\left[r_{Nt+1}^k+(1-\delta)+\right.$$
$$\left.\phi_N\left(\frac{K_{Nt+2}}{K_{Nt+1}}-\bar{g}_N\right)\frac{K_{Nt+2}}{K_{Nt+1}}-\frac{\phi_N}{2}\left(\frac{K_{Nt+2}}{K_{Nt+1}}-\bar{g}_N\right)^2\right], \tag{30}$$

$$X_{Tt-1}^{-\gamma}\lambda_t\nu_t=X_{Tt}^{-\gamma}\beta\left(1+r_t^*\right)\mathbb{E}_t\lambda_{t+1}\nu_{t+1}, \tag{31}$$

and

$$X_{Tt-1}^{-\gamma}\lambda_t\nu_t P_t=X_{Tt}^{-\gamma}\beta\left(1+r_t\right)\mathbb{E}_t\lambda_{t+1}\nu_{t+1}P_{t+1}. \tag{32}$$

Equations (25)-(26) pin down the optimal tradable and non-tradable consumption choices, equations (28)-(27) state the labor supply choices as increasing functions of sectoral wages, (29)-(30) the Euler equations associated to sectoral physical capital and equations (31)-(32) the Euler equations for bonds. Note that, by taking the ratio between (26)-(25), we can work out the relative price of non-tradable output as an internal exchange rate.

A.2. Stationary Equilibrium

We now proceed to characterise the stationary equilibrium by introducing "lower-case" variables, denoting the detrended counterparts of non-stationary variables. Define $c_t=\frac{C_t}{X_{Tt-1}}, c_{Tt}=\frac{C_{Tt}}{X_{Tt-1}}, c_{Nt}=\frac{C_{Nt}}{X_{Nt-1}}, K_{Tt}=\frac{K_{Tt}}{X_{Tt-1}}, K_{Nt}=\frac{K_{Nt}}{X_{Nt-1}}, p_t=P_t\frac{X_{Nt-1}}{X_{Tt-1}}$.

The household first order conditions in normalized forms become

$$c_t=\left[\zeta^{1-\sigma}c_{Tt}^{\sigma}+(1-\zeta)^{1-\sigma}\left(c_{Nt}\right)^{\sigma}\right]^{\frac{1}{\sigma}}, \tag{33}$$

$$\left(c_t-\epsilon_t\left(\frac{\theta_T}{\omega}n_{Tt}^{\omega}+\frac{\theta_N}{\omega}n_{Nt}^{\omega}\right)\right)^{-\gamma}\left(\frac{c_{Tt}}{\zeta c_t}\right)^{\sigma-1}=\lambda_t, \tag{34}$$

$$\left(c_t-\epsilon_t\left(\frac{\theta_T}{\omega}n_{Tt}^{\omega}+\frac{\theta_N}{\omega}n_{Nt}^{\omega}\right)\right)^{-\gamma}\left(\frac{c_{Nt}}{(1-\zeta)\,c_t}\right)^{\sigma-1}=p_t\lambda_t, \tag{35}$$

$$\left(c_t-\epsilon_t\left(\frac{\theta_T}{\omega}n_{Tt}^{\omega}+\frac{\theta_N}{\omega}n_{Nt}^{\omega}\right)\right)^{-\gamma}\epsilon_t\theta_T n_{Tt}^{\omega-1}=\lambda_t w_{Tt}, \tag{36}$$

$$\left(c_t-\epsilon_t\left(\frac{\theta_T}{\omega}n_{Tt}^{\omega}+\frac{\theta_N}{\omega}n_{Nt}^{\omega}\right)\right)^{-\gamma}\epsilon_t\theta_N n_{Nt}^{\omega-1}=\lambda_t w_{Nt}, \tag{37}$$

$$\lambda_t\nu_t\left[1+\phi_T\left(\frac{k_{Tt+1}}{k_{Tt}}g_{Tt}-\bar{g}_T\right)\right]=\beta g_{Tt}^{-\gamma}\mathbb{E}_t\left\{\lambda_{t+1}\nu_{t+1}\left[r_{Tt+1}^k+(1-\delta)\right.\right.$$
$$\left.\left.+\phi_T\left(\frac{k_{Tt+2}}{k_{Tt+1}}g_{Tt+1}-\bar{g}_T\right)\frac{k_{Tt+2}}{k_{Tt+1}}-\frac{\phi_T}{2}\left(\frac{k_{Tt+2}}{k_{Tt+1}}g_{Tt+1}-\bar{g}_T\right)^2\right]\right\}, \tag{38}$$

$$\lambda_t\nu_t p_t\left[1+\phi_N\left(\frac{k_{Nt+1}}{k_{Nt}}g_{Nt}-g_N\right)\right]=\beta\frac{g_{Tt}^{1-\gamma}}{g_{Nt}}\mathbb{E}_t\left\{\lambda_{t+1}\nu_{t+1}p_{t+1}\left[r_{Nt+1}^k+(1-\delta)\right.\right.$$
$$\left.\left.+\phi_N\left(\frac{k_{Nt+2}}{k_{Nt+1}}g_{Nt+1}-\bar{g}_N\right)\frac{k_{Nt+2}}{k_{Nt+1}}-\frac{\phi_N}{2}\left(\frac{k_{Nt+2}}{k_{Nt+1}}g_{Nt+1}-\bar{g}_N\right)^2\right]\right\}, \tag{39}$$

$$\lambda_t \nu_t = \beta \left(1 + r_t^*\right) g_{Tt}^{-\gamma} \mathbb{E}_t \lambda_{t+1} \nu_{t+1},, \tag{40}$$

and

$$\lambda_t \nu_t p_t = \beta \left(1 + r_t\right) \frac{g_{Tt}^{1-\gamma}}{g_{Nt}} \mathbb{E}_t p_{t+1} \lambda_{t+1} \nu_{t+1}. \tag{41}$$

The firms' first order conditions become

$$r_{Tt}^k = \alpha_T a_{Tt} k_{Tt}^{\alpha_T - 1} (n_{Tt} g_{Tt})^{1-\alpha_T}, \tag{42}$$

$$w_{Tt} = (1 - \alpha_T) a_{Tt} k_{Tt}^{\alpha_T} (n_{Tt})^{-\alpha_T} g_{Tt}^{1-\alpha_T}, \tag{43}$$

$$r_{Nt}^k = \alpha_N a_{Nt} k_{Nt}^{\alpha_N - 1} (n_{Nt} g_{Nt})^{1-\alpha_N} \tag{44}$$

and

$$w_{Nt} = p_t (1 - \alpha_N) a_{Nt} k_{Nt}^{\alpha_N} n_{Nt}^{-\alpha_N} g_{Nt}^{1-\alpha_N}. \tag{45}$$

The normalized constraints are

$$y_{Tt} = c_{Tt} + i_{Tt} + \frac{\phi_T}{2} \left(\frac{k_{Tt+1}}{k_{Tt}} g_{Tt} - \bar{g}_T \right)^2 + tb_t, \tag{46}$$

$$y_{Nt} \left(1 - \frac{s}{y} s_t\right) = c_{Nt} + i_{Nt} + \frac{\phi_N}{2} \left(\frac{k_{Nt+1}}{k_{Nt}} g_{Tt} - \bar{g}_N \right)^2, \tag{47}$$

$$i_{Tt} = k_{Tt+1} g_{Tt} - \left(1 - \delta\right) k_{Tt}, \tag{48}$$

$$i_{Nt} = k_{Nt+1} g_{Nt} - \left(1 - \delta\right) k_{Nt}, \tag{49}$$

$$y_{Tt} = a_{Tt} k_{Tt}^{\alpha_T} g_{Tt}^{1-\alpha_T} n_{Tt}^{1-\alpha_T}, \tag{50}$$

$$y_{Nt} = a_{Nt} k_{Nt}^{\alpha_N} g_{Nt}^{1-\alpha_N} n_{Nt}^{1-\alpha_N} \tag{51}$$

and

$$tb_t = b_t^* - \frac{b_{t+1}^*}{1 + r_t^*} g_{Tt}. \tag{52}$$

A.3. Steady State

A.3.1 Analytical Derivation of the Steady State (for Brexit simulation purposes)

We remove time subscripts from the equations to compute the steady state values. From equations (38)-(41), it follows that

$$\beta = \frac{1}{(1+r^*)\,\bar{g}_T^{-\gamma}}, \tag{53}$$

$$r = \frac{g_N}{\beta \bar{g}_T^{1-\gamma}} - 1, \tag{54}$$

$$r_T^k = \frac{1}{\beta \bar{g}_T^{-\gamma}} - (1-\delta), \tag{55}$$

$$r_N^k = \frac{\bar{g}_N}{\beta \bar{g}_T^{1-\gamma}} - (1-\delta). \tag{56}$$

From the rental rates of capital –equations (42) and (44)–, we recover the sectoral capital to labour ratios

$$\frac{k_T}{n_T} = \left(\frac{r_T^k}{\alpha_T}\right)^{\frac{1}{\alpha_T - 1}} \bar{g}_T, \tag{57}$$

$$\frac{k_N}{n_N} = \left(\frac{r_N^k}{\alpha_N}\right)^{\frac{1}{\alpha_N - 1}} \bar{g}_N. \tag{58}$$

Steady state wages can be calculated from equations (43) and (45) as

$$w_T = (1-\alpha_T)\,\bar{g}_T^{1-\alpha_T}\left(\frac{k_T}{n_T}\right)^{\alpha_T} \tag{59}$$

and

$$\frac{w_N}{p} = (1-\alpha_N)\,\bar{g}_N^{1-\alpha_N}\left(\frac{k_N}{n_N}\right)^{\alpha_N}. \tag{60}$$

By calibrating $\frac{s}{y_N}$, and using equations (47) and (51), we can express consumption in N as being linear in n_N,

$$c_N = \left\{\left(\frac{k_N}{n_N}\right)^{\alpha_N} \bar{g}_N^{1-\alpha_N}\left(1 - \frac{s}{y_N}\right) - \left[\bar{g}_N - (1-\delta)\right]\frac{k_N}{n_N}\right\} n_N = A_N n_N. \tag{61}$$

By calibrating the ratio $\frac{tb}{y}$ and using equation (46) and (50), we can express consumption in T in terms of n_N,

$$c_T = \left\{\left(\frac{k_T}{n_T}\right)^{\alpha_T} \bar{g}_T^{1-\alpha_T}\left(1 - \frac{tb}{y}\right) - \left[\bar{g}_T - (1-\delta)\right]\frac{k_T}{n_T}\right\} n_T = A_T n_T. \tag{62}$$

Dividing (35) by (34), we can get an expression for p. We use (61) and (62) to get the ratio of sectoral hours,

$$p = \left[\frac{\zeta c_N}{(1-\zeta)\, c_T}\right]^{\sigma-1} = \left[\frac{\zeta A_N n_N}{(1-\zeta)\, A_T n_T}\right]^{\sigma-1} \Rightarrow \frac{n_N}{n_T} = p^{\frac{1}{\sigma-1}} \frac{(1-\zeta)\, A_T}{\zeta A_N}. \tag{63}$$

We divide equation (37) by equation (36) and substitute for n_N/n_T as above to get

$$\frac{w_N}{p w_T} = \frac{1}{p}\frac{\theta_N}{\theta_T}\left(\frac{n_N}{n_T}\right)^{\omega-1} = \frac{1}{p}\frac{\theta_N}{\theta_T}\left(p^{\frac{1}{\sigma-1}}\frac{(1-\zeta)\, A_T}{\zeta A_N}\right)^{\omega-1} \Rightarrow p = \left[\frac{p w_T}{w_N}\frac{\theta_N}{\theta_T}\left(\frac{(1-\zeta)\, A_T}{\zeta A_N}\right)^{\omega-1}\right]^{\frac{1-\sigma}{\omega-\sigma}}. \tag{64}$$

We divide equation (33) by c_T to obtain the ratio

$$\frac{c}{c_T} = \left(\zeta^{1-\sigma} + (1-\zeta)^{1-\sigma}\left(\frac{c_N}{c_T}\right)^{\sigma}\right)^{\frac{1}{\sigma}}. \tag{65}$$

Finally, we divide equation (36) by equation (34) and substitute for $\frac{c}{c_T}$ to obtain n_T

$$\left[\frac{w_T}{\theta_T}\left(\frac{c_T}{\zeta c}\right)^{\sigma-1}\right]^{\frac{1}{\omega-1}} = n_T. \tag{66}$$

Once the value of n_T is pinned down, the remaining algebra is simple.

A.3.2 Numerical Computation of Steady State (for estimation purposes)

The steady state values of β, r, r_T^k, r_N^k, $\frac{k_T}{n_T}$ and $\frac{k_N}{n_N}$ are given by equations (53)-(58). Given the values of sectoral hours (n_N and n_T), we can compute the steady state values of sectoral physical capital

$$k_N = \frac{k_N}{n_N} n_N, \tag{67}$$

and

$$k_T = \frac{k_T}{n_T} n_T. \tag{68}$$

Sectoral outputs are therefore

$$y_N = k_N^{\alpha_N} \left(n_M \bar{g}_N\right)^{1-\alpha_N} \tag{69}$$

and

$$y_T = k_T^{\alpha_N} \left(n_T \bar{g}_T\right)^{1-\alpha_T}. \tag{70}$$

Given the ratios, s/y, tb/y and $c_T/\mathcal{C}$, we solve numerically for p, c_N, c_T and y .

$$\begin{aligned} c_N &+ \frac{s}{y}\frac{y}{p} = y_N \left\{ 1 - \left[\bar{g}_N - (1-\delta_N) \right] \frac{\alpha_N}{r_N^k} \right\} && (71) \\ c_T &+ \frac{tb}{y} y = y_T \left\{ 1 - \left[\bar{g}_T - (1-\delta_T) \right] \frac{\alpha_T}{r_T^k} \right\} && (72) \\ p c_N &= c_T \frac{1 - \frac{c_T}{\mathcal{C}}}{\frac{c_T}{\mathcal{C}}} && (73) \\ y &= y_T + p y_N. && (74) \end{aligned}$$

We then compute the constants which allow us to exactly match the analytic steady state in the previous section as follows:

$$\frac{s}{y_N} = \frac{\frac{s}{y}}{p y_N}, \tag{75}$$

$$\frac{tb}{y_T} = \frac{tb}{y}\frac{y}{y_T}, \tag{76}$$

$$\zeta = \frac{p^{\frac{1}{\sigma-1}}}{p^{\frac{1}{\sigma-1}} + \frac{c_N}{c_T}}, \tag{77}$$

$$\theta_N = \frac{\frac{w_N}{p}\left(1-\zeta\right)^{1-\sigma}\left(\frac{c_N}{c}\right)^{\sigma-1}}{n_N^{\omega-1}} \tag{78}$$

and

$$\theta_T = \frac{w_T \zeta^{1-\sigma}\left(\frac{c_T}{c}\right)^{\sigma-1}}{n_T^{\omega-1}}, \tag{79}$$

where w_N and w_T are given by (59) and (60).

B. Data Construction

We use ONS detailed supply and use tables for 1997 − 2015 to calculate, for each 2-digit SIC industry, a tradability index at basic prices (the ratio of exports plus imports to gross output). Exports denote exports of domestic output only (i.e. excluding re-exports of imported goods). as in Lombardo and Ravenna (2012), we define a sector as 'tradable' if more than 10% of its total supply is traded using the 2-digit SIC industry level classification. This threshold is *arbitrary* but it coincides with those suggested by De Gregorio et al. (1994) and Betts and Kehoe (2006). This approach yields a sensible classification of industries into tradable/non-tradable sectors.[42] Figure 6 shows the industry classification that yields from using the 10% cut-off. In particular, around 0.54 of aggregate GVA is classifies as non-tradable and the remaining as tradable. Service industries tend to have lower ratios (although there are many exceptions), and manufacturing industries higher ratios. However, according to this classification, the financial services industry is deemed as a tradable sector.

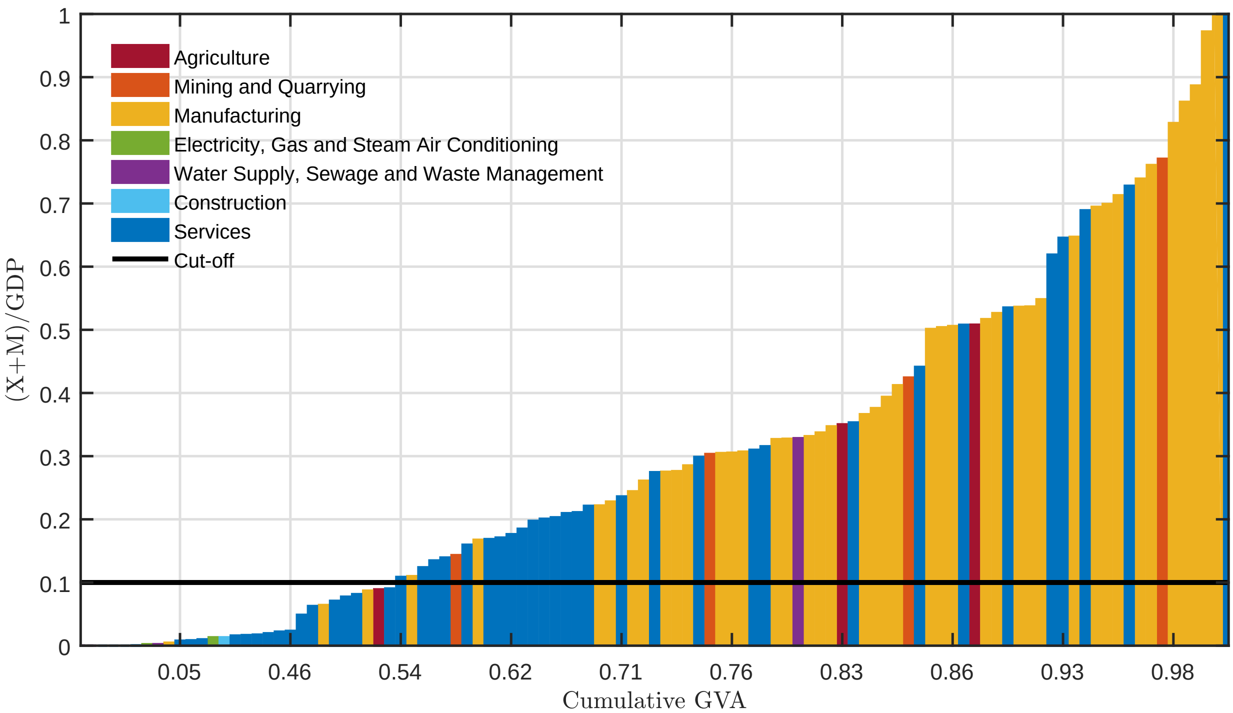

Figure 6: *Industry classification using 2016 Supply and Use Tables*

After classifying each of the 114 industries into the tradable and non-tradable categories, we add the consumption expenditure of households and non-profit institutions serving households. We then divide the sectoral expenditure by aggregate consumption for the years 1997 to 2015 to calculate the share of tradable consumption into total consumption. We then compute the sample mean and retrieve a value of 0.59. As shown in the Figure 7, this share is rather constant over time. This value is in line with the estimate for the UK in Lombardo and Ravenna (2012), who calculated a value of 0.64 (based on 2000 − 2005 data) and with a previous internal Bank's estimate of 0.5 − 0.6. Using this arbitrary threshold, we also find that around half of the economy

[42] An alternative definition is proposed by De Gregorio et al. (1994) that classify a sector as 'tradable' if 10% of its total supply is exported.

by GVA can be classified as tradable. It is worth noting that the share of tradable output in aggregate output is lower than the tradable share in aggregate consumption because non-tradable services, such as construction, public administration and defense and compulsory social security services, have a much higher weight in output than in household consumption.

The **factor shares** are computed using the supply and use tables from 1997 to 2014. Following Goodridge et al. (2018), we use partial appropriation of self-employed income to labour income. This assumes that a fraction of self-employed income accruing to labour income. The labour share in sector i in year t is then defined as the sum of compensation of employees and the fraction of self-employed income accrues to labour divided by total GVA (at basic prices). In computing the labour share in the N sector, we exclude imputed rents as they tend to bias the estimates. The capital share is residually determined as one minus the labour share. The sample means of the capital shares are 0.315 and 0.245 in the T and N sectors sector respectively. Note that assigning self-employed income to labour income tends to increase the values of the labour shares. Figure 7 shows the evolution of the consumption share of T goods into aggregate consumption and the labour shares in the T and N sectors respectively.

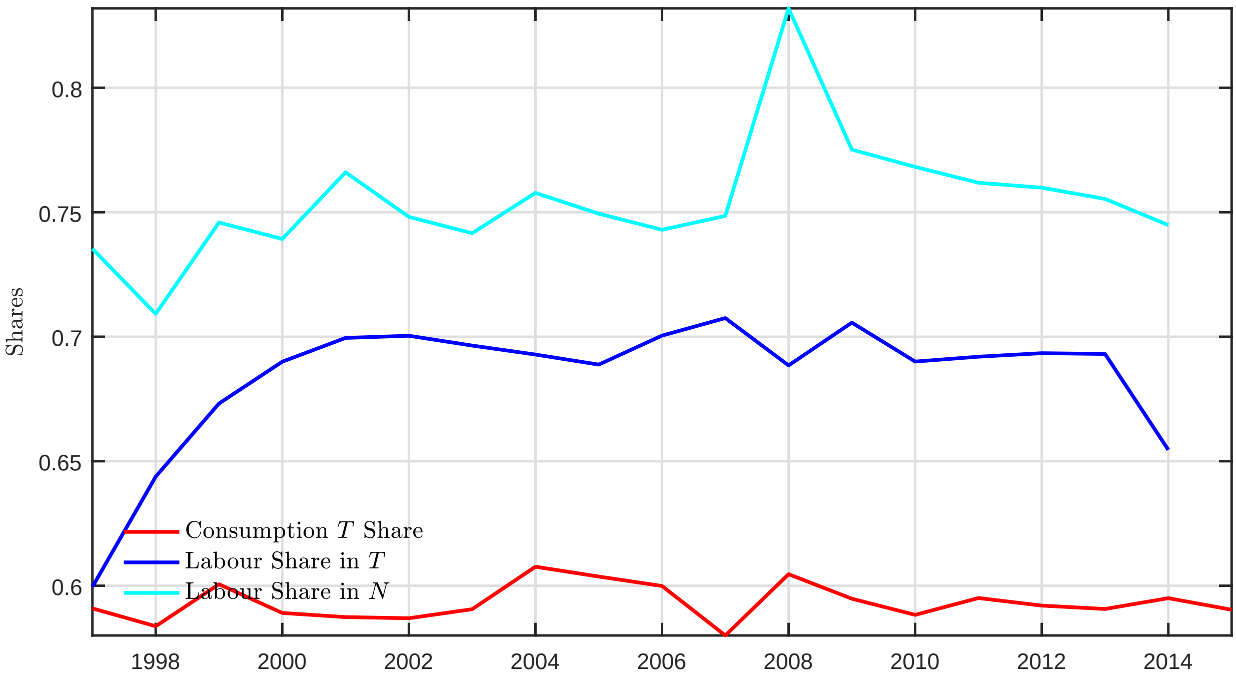

Figure 7: *Labour Shares in T and N and Consumption T Shares*

Once we have classified each 2-digit industries into the tradable or the non-tradable category, we use the associated ONS detailed industry-level GVA data to construct a time-series for tradable output consistent with aggregate GVA, by aggregating GVA over the set of industries in each category, using ONS's standard national accounts chain-linking methodology. The resulting time-series for GVA growth are shown in Figure 8. The growth rates of GVA output do not display significant differences across the two categories since total hours in the non-tradable sector display an upward trend.

We also construct tradable and non-tradable total hours, using the published industry hours

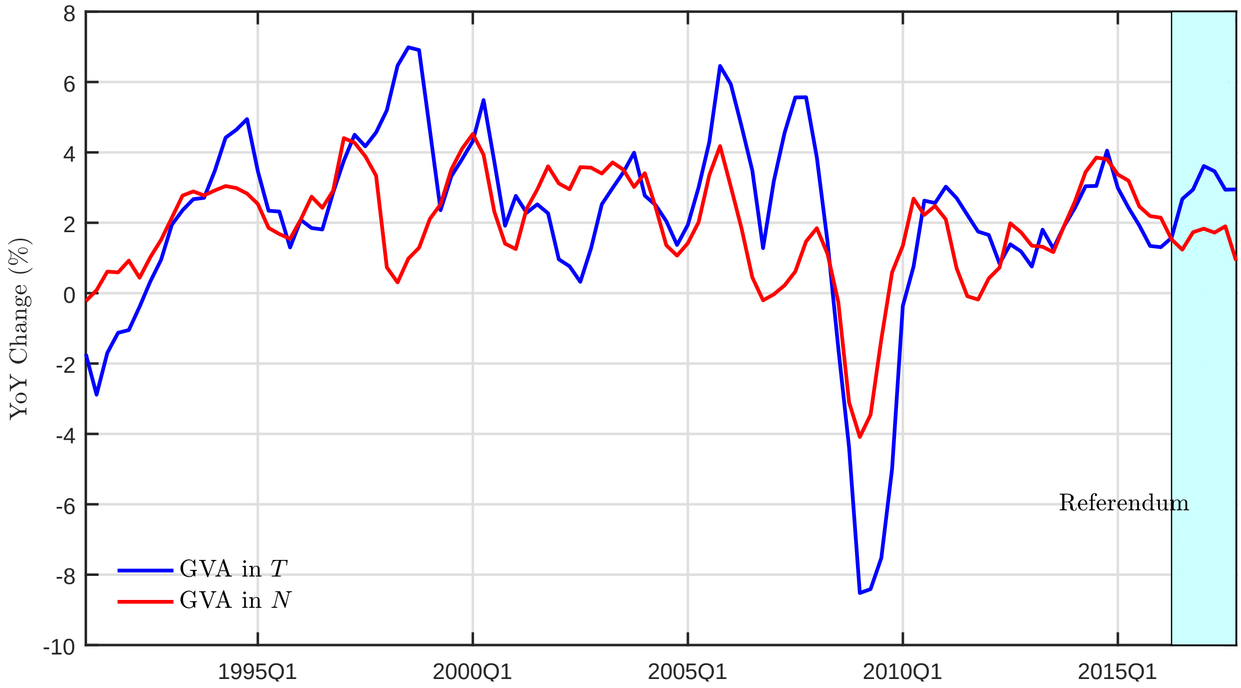

Figure 8: *Gross Value Added in T and N*

data underlying ONS labor productivity estimates, together with our classification of industries.[43] Hours worked by sector are measured following Tenreyro (2018). We then compute the average labor productivity growth rate in each sector from 1994-2017. Over this period labor productivity growth in the tradable sector averaged about 1.8% on an annual basis.

[43]The data on hours are available at a slightly higher level of aggregation than 2-digit level; therefore, we need to make a judgement about the tradability of each grouping of 2-digit industries in the hours data, based on the tradability of the underlying 2-digit industries.

C. Sensitivity experiments

This appendix presents results from alternative variations in the baseline model and/or scenario presented in Section 5.

C.1. The anticipation horizon

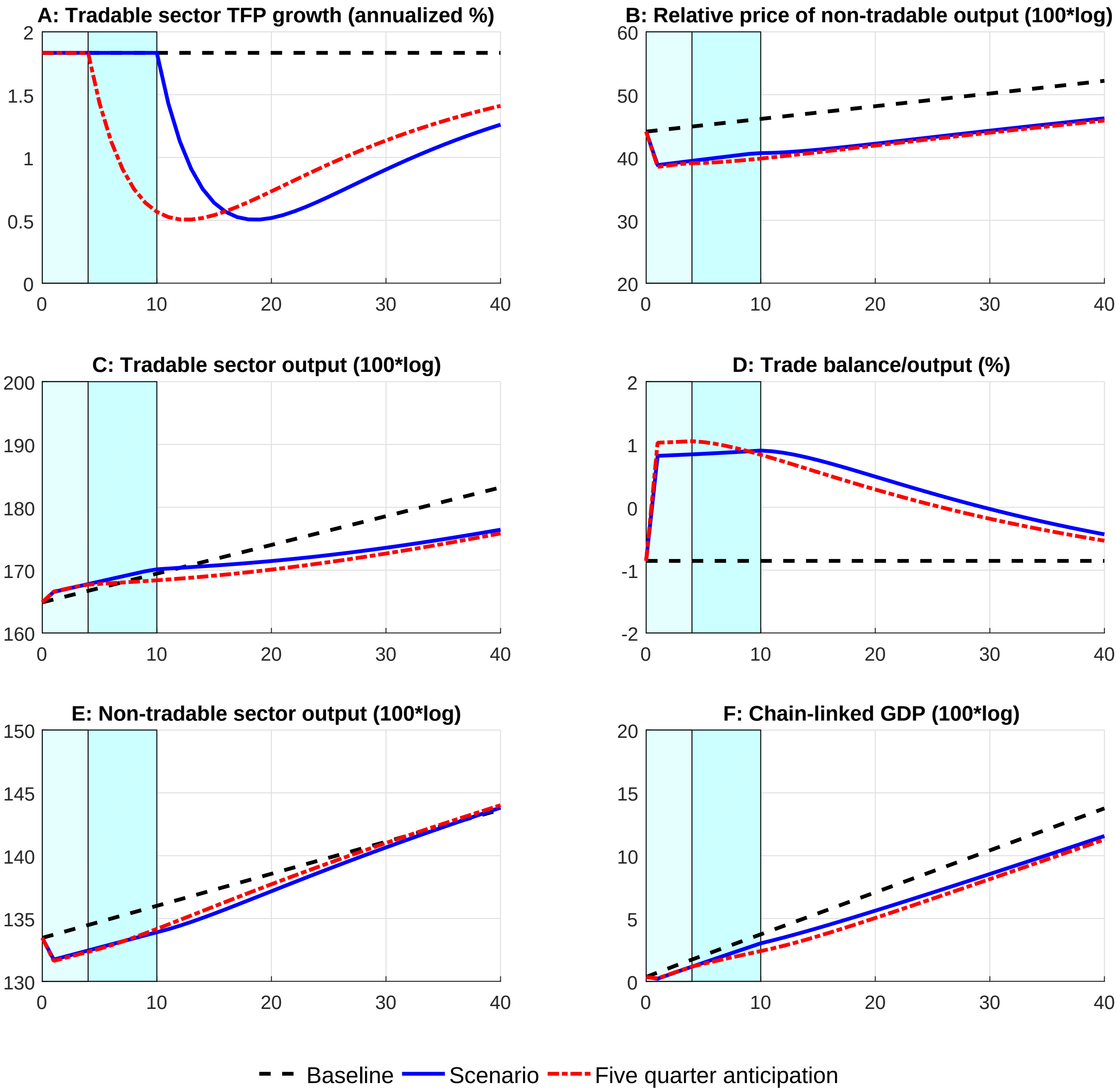

Figure 9: *Headline responses to the tradable TFP growth scenario*

Figures 9 and 10 show the results of the baseline experiment (solid blue lines) and a variant in which the decline in tradable sector productivity growth starts after five quarters (dot-dashed red lines). The long-run effects of the shock are the same, but alternative timing assumptions are likely to affect short-term dynamics.

Unsurprisingly, the dynamics of the alternative assumption are slightly different when the decline in productivity growth occurs earlier. The initial export boom is less long-lived and requires a larger reallocation of labor to deliver higher tradable sector output. The switch in

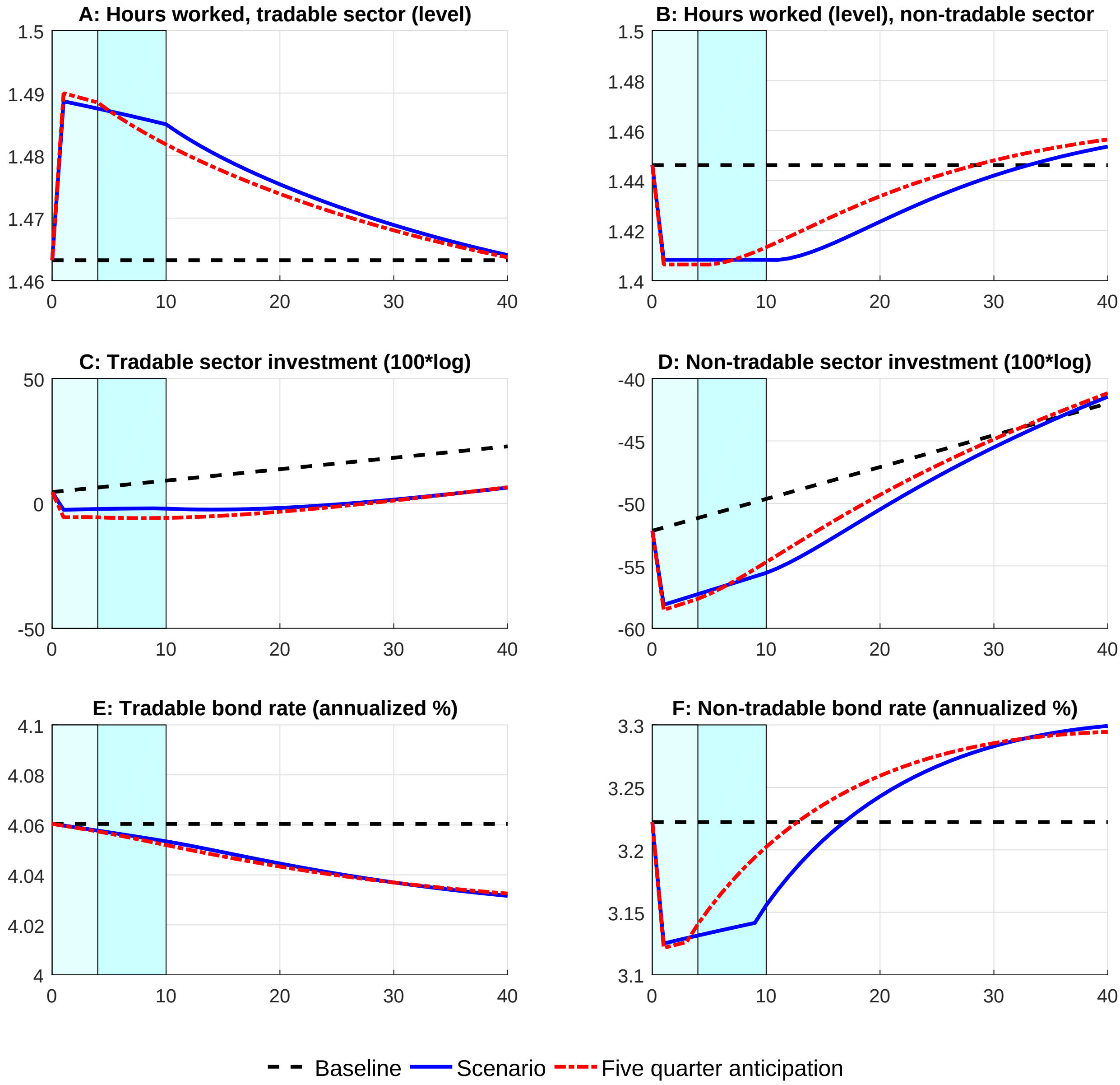

Figure 10: *Sectoral responses to the tradable TFP growth scenario*

factor flows (from the tradable sector to the non-tradable sector) occurs earlier, commensurate with the earlier reduction in tradable sector productivity growth.

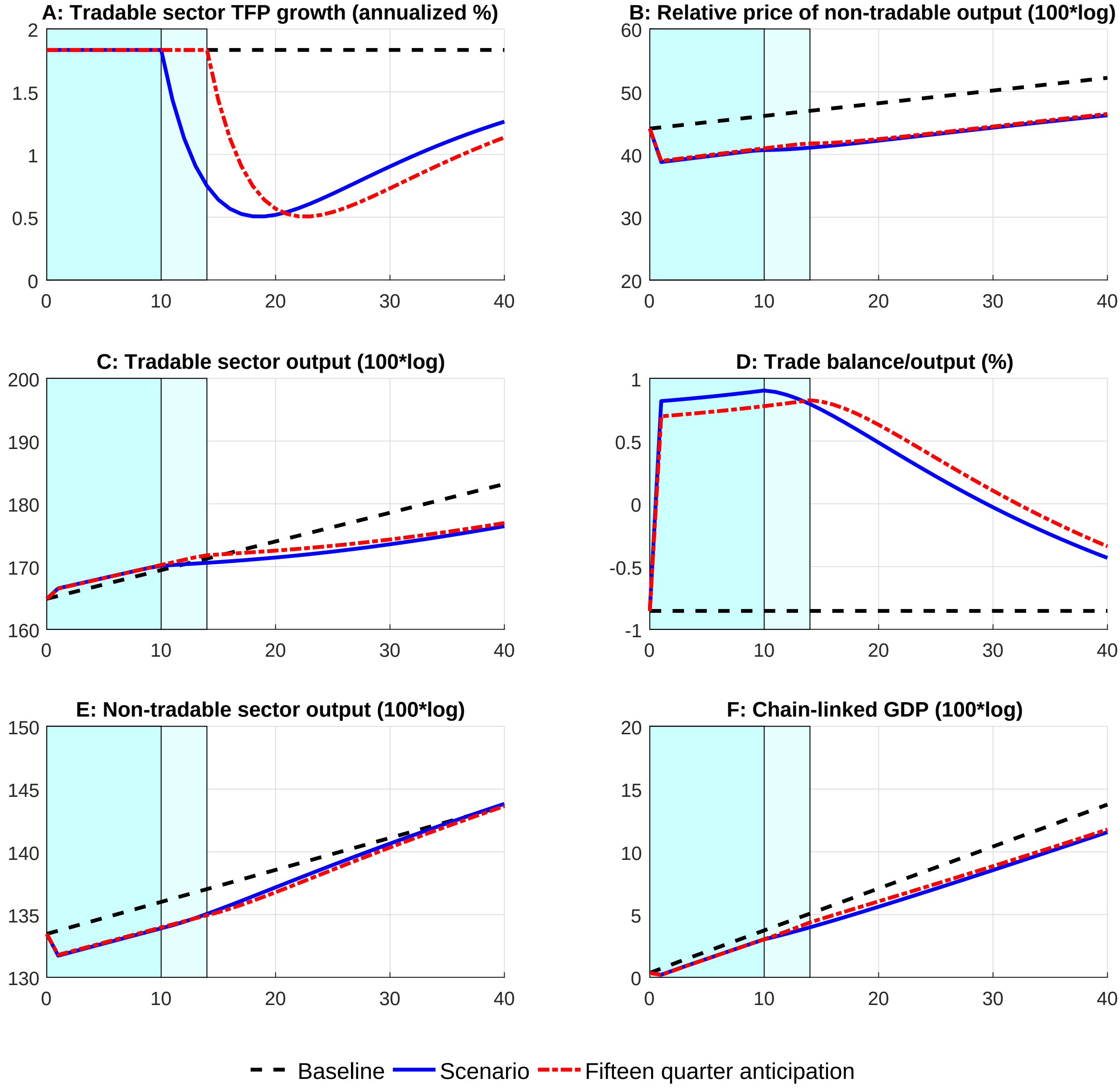

Figure 11: *Headline responses to the tradable TFP growth scenario*

Figures 11 and 12 show the results of the baseline experiment (solid blue lines) and a variant in which the decline in tradable sector productivity growth is delayed for fifteen quarters (dot-dashed red lines).

The relative effect of a longer anticipation horizon is, unsurprisingly, the opposite of the previous case of a shorter anticipation horizon. The adjustment dynamics are more protracted and the near-term reallocation of labor during the anticipation horizon is more muted relative to the baseline case. With a longer anticipation horizon, tradable sector investment falls by less, as the reduction in tradable sector productivity occurs in the more distant future.

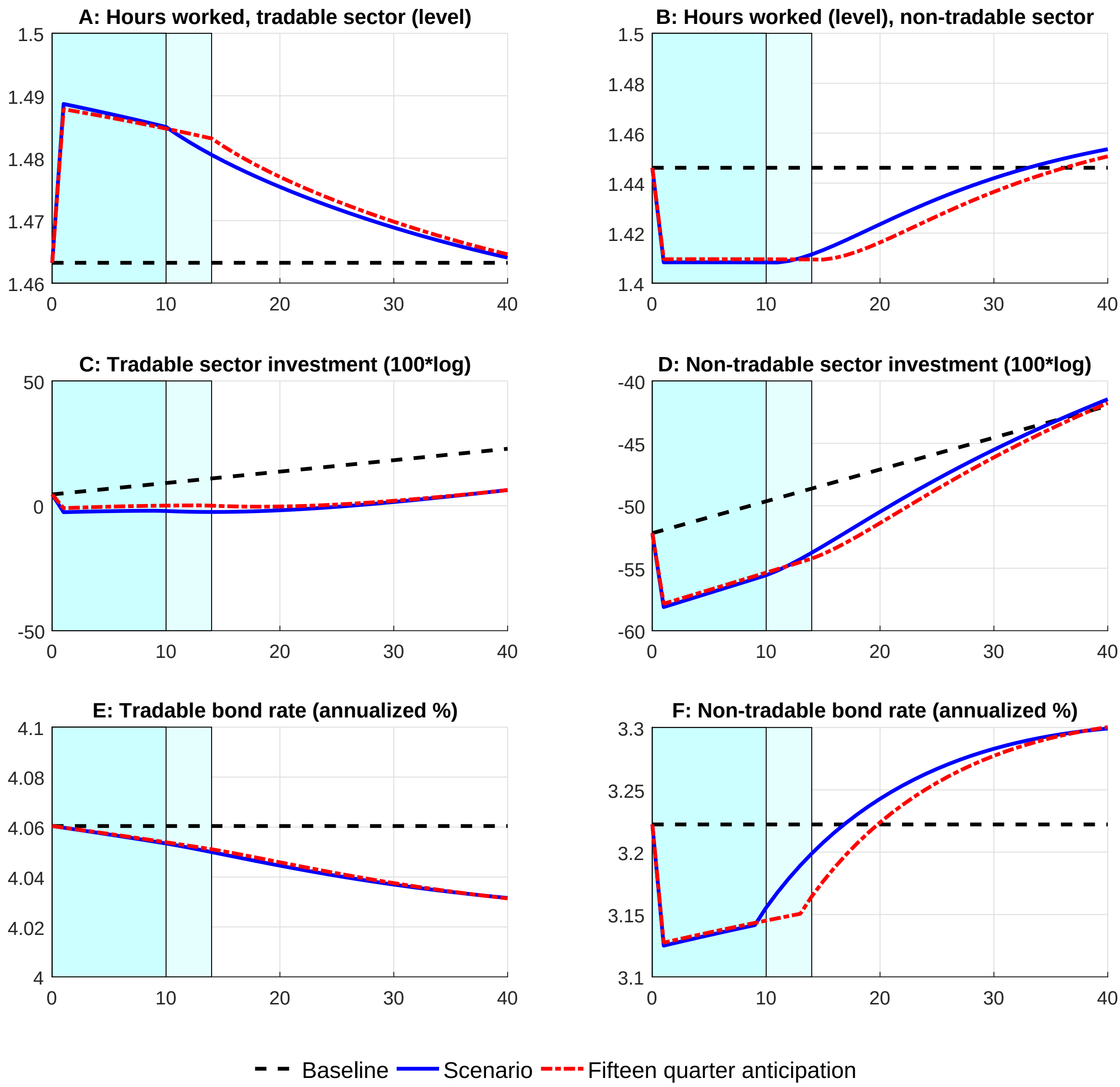

Figure 12: *Sectoral responses to the tradable TFP growth scenario*

C.2. Faster fall in tradable sector TFP

Figures 13 and 14 show the results of the baseline scenario (solid blue lines) alongside a case in which the decline in tradable sector productivity growth occurs more rapidly (red dot-dashed lines). The alternative scenario is constructed by assuming that the parameter controlling the persistent component of tradable sector productivity growth is set to $\tilde{\varrho}_T^g = 0.9$ (compared with the baseline assumption of 0.95).

The alternative scenario implies that tradable sector productivity reaches its new, lower, level in roughly half the time of the baseline scenario. The scale of the productivity growth shock is roughly doubled to ensure that the long-run effect on tradable sector productivity is identical to the baseline scenario.

Unsurprisingly, the dynamic responses to the more rapid productivity growth shock variant are somewhat faster in some cases. However, the broad contours of the macroeconomic responses

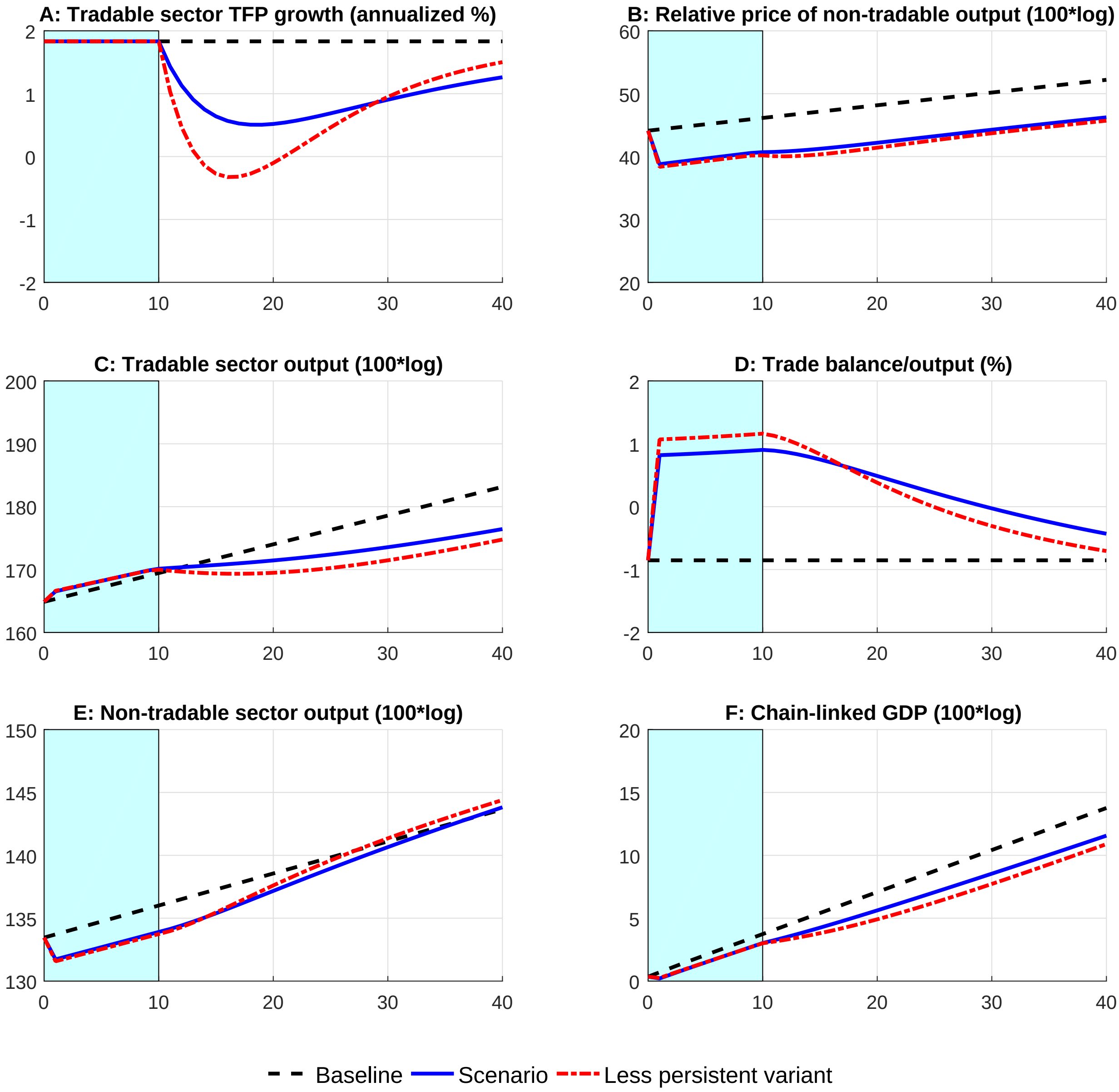

Figure 13: *Headline responses to the tradable TFP growth scenario*

are very similar in both cases. This demonstrates that the dominant effect is the anticipation of permanently lower tradable sector productivity in the *long run*. This effect drives the key relative price in the model: the impact effect on the relative price of non-tradable output is very similar (Figure 13, panel B).

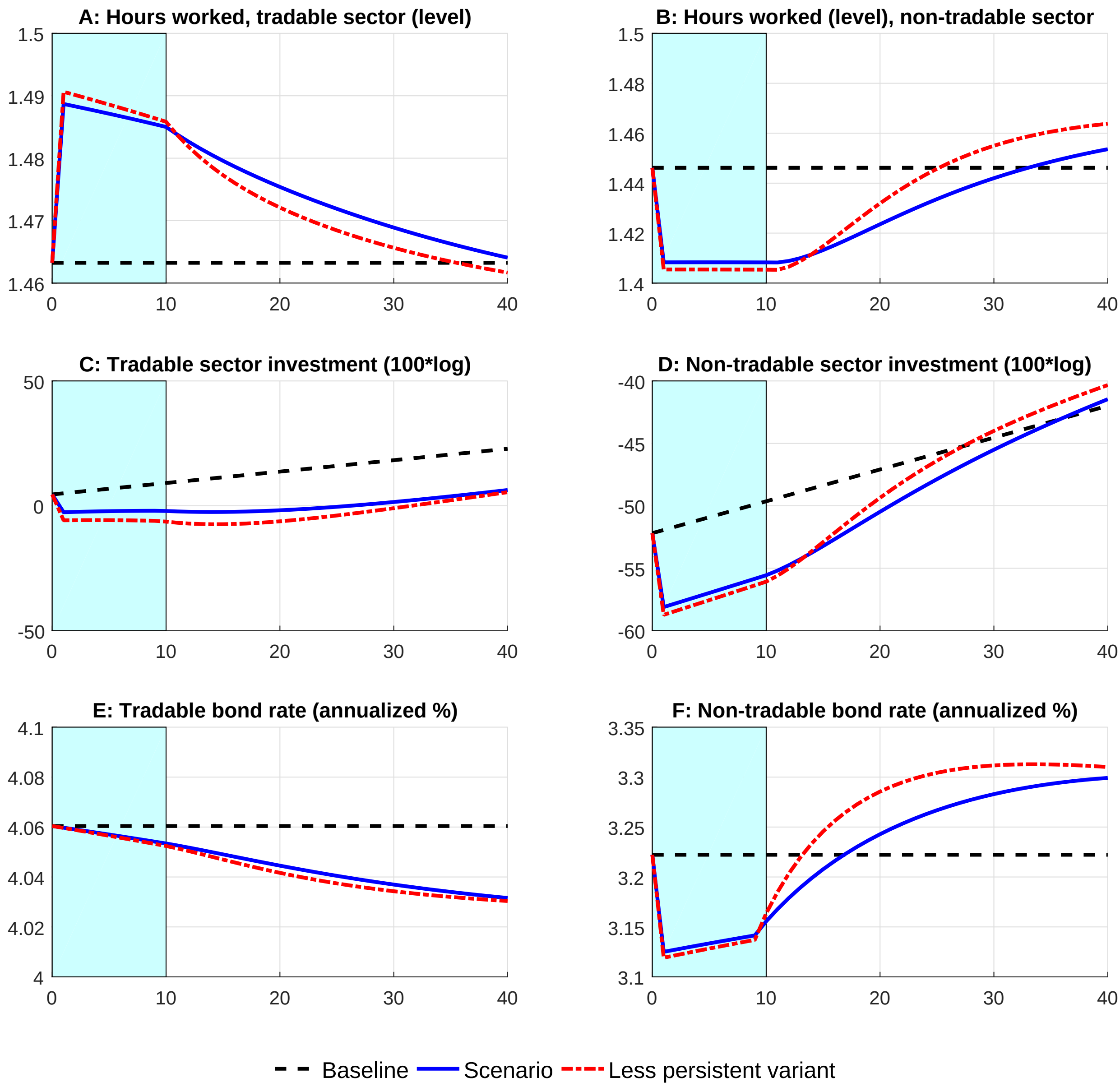

Figure 14: *Sectoral responses to the tradable TFP growth scenario*

C.3. Population growth variant

Figures 15 and 16 show results for a variant of the model that incorporates population growth. A derivation of this variant is presented in Appendix E. However, the innovation compared with the baseline model is straightforward. In the variant, we assume that households are infinitely lived, but that new households are born each period. The population growth rate is constant. Individual households have identical preferences to those that we have assumed in previous versions of the model. So their first order conditions identical to the baseline model.

However, population growth means that aggregate consumption is not characterized by the same Euler equation as the one that holds for each individual household. This is because new households are born with no financial wealth. Accounting for the heterogeneity in financial wealth delivers an aggregate consumption equation that depends on the distribution of wealth. The simple population structure implies that the distribution of wealth can be summarized by

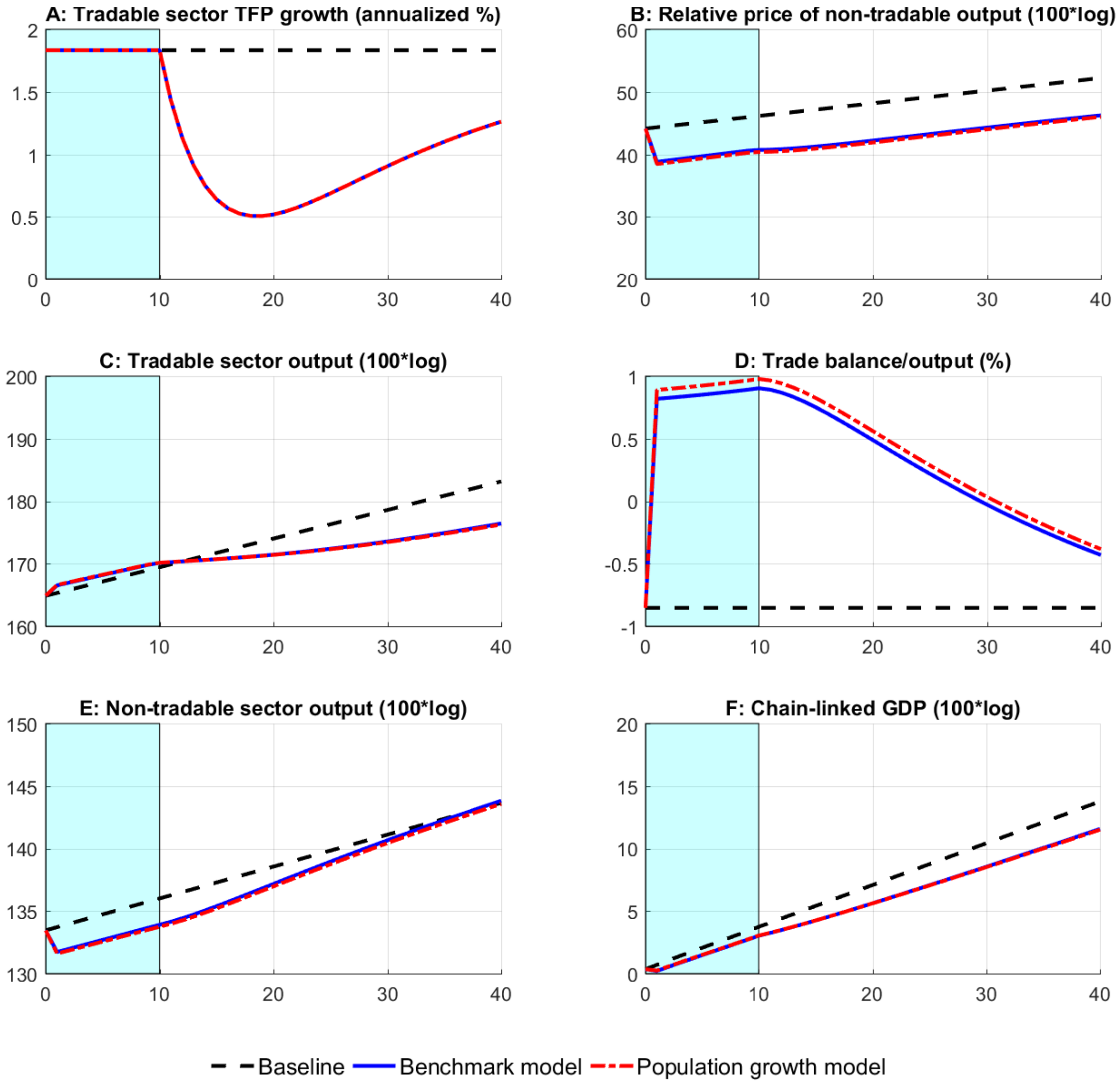

Figure 15: *Headline responses to the tradable TFP growth scenario*

aggregate stocks of wealth (ultimately, the stock of foreign debt).

The dependence of the aggregate consumption Euler equation on wealth means that the steady state net foreign asset position is pinned down, even if the economy may freely borrow and lend at a fixed world interest rate. The steady state NFA position is pinned down by the (im)patience of domestic agents relative to the (growth adjusted) world real interest rate.

While population growth is just a device to close the model, rather than a plausible model of demographics, we set the constant population growth rate to be consistent with 0.5% annual population growth (broadly consistent with 1997-2016 UK data).

The results show that the macroeconomic dynamics are virtually identical in the two variants of the model, despite the fact that the tradable bond rate remains fixed (at the exogenous world real interest rate) in the population growth variant. The slight differences in relative bond rates generate small differences in the returns to capital across the two variants, with minor implications for the dynamic responses of hours and investment. However, the broad contours

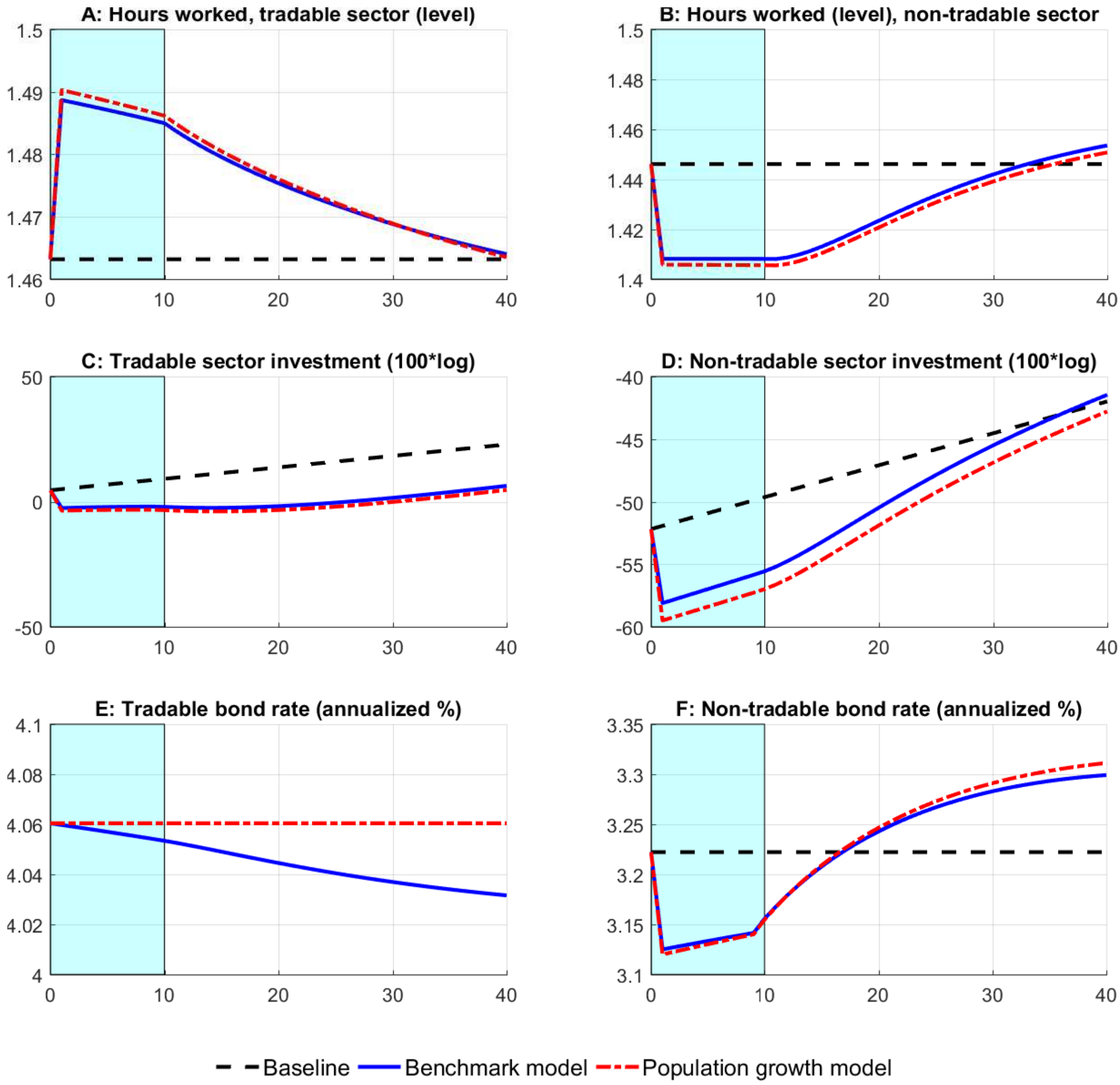

Figure 16: *Sectoral responses to the tradable TFP growth scenario*

of the simulation are very similar in the two variants.

D. Additional Historical Decompositions

This appendix presents historical variance decompositions for the various observables used to estimate the model. This complements the analysis in Section 6 of the paper, where we have analyzed in detail the decompositions for labor productivity in the tradable and non-tradable sector, respectively and have stressed the importance of shocks to the trend component in TFP. Figure 17 shows that temporary tradable TFP disturbances have played a major role in explaining the sharp contraction in total hours since the onset of the financial crisis.

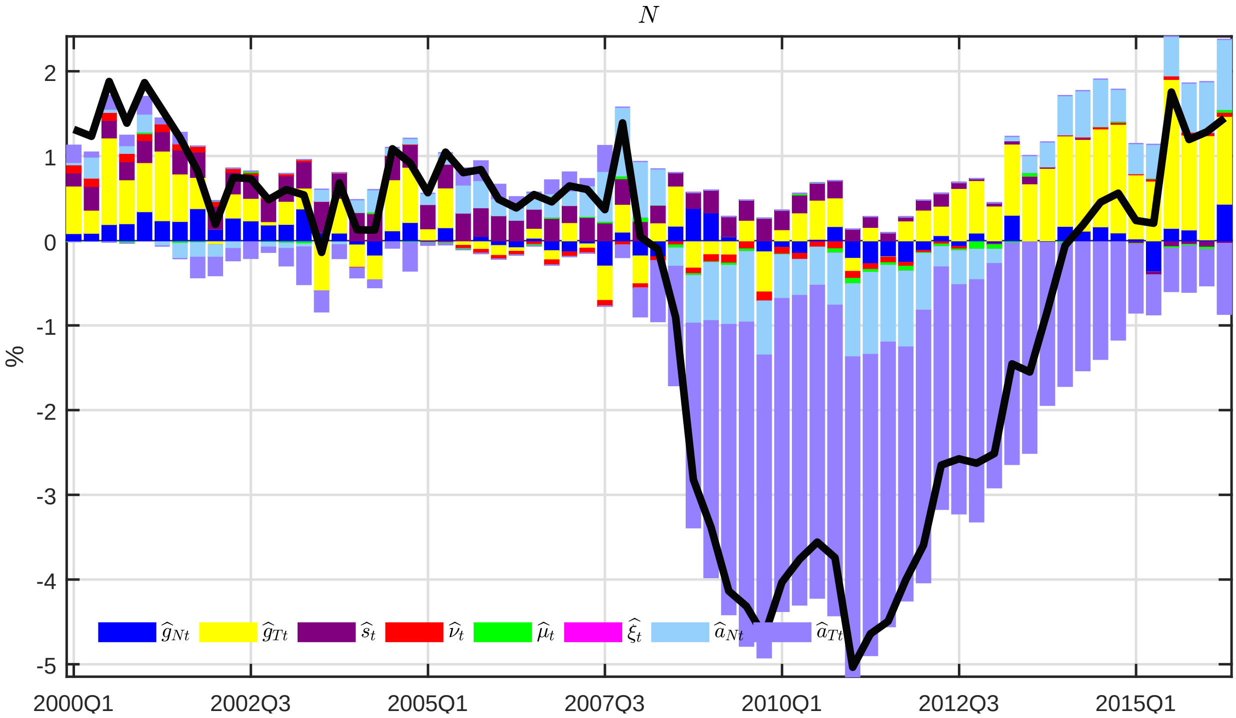

Figure 17: *Historical decomposition of total hours*

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable over time. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

Figures 18-20 report the historical decomposition of consumption, investment and trade balance to GDP ratios. We observe that 'demand' type shocks, such as government expenditure, risk-premium and foreign interest rate shocks, contribute to explaining a large part of the variation in the expenditure shares over time. The Figures show that, over the recent past, tradable TFP growth shocks, together with the potential crowding out of government expenditure, have contributed to a weakening in the UK investment prospects. We observe that, in the run-up to the financial crisis, the consumption boom, which was largely financed through the contraction of external debt, can be largely attributed to a series of negative risk-premium shocks. After the financial crisis, the rise in the risk-premium depresses consumption and reduces the accumulation of external debt that is raised to finance it (trade balance reverts on average to its steady state value).

Figures 21-22 illustrate the decomposition of the relative price of non-tradable goods and

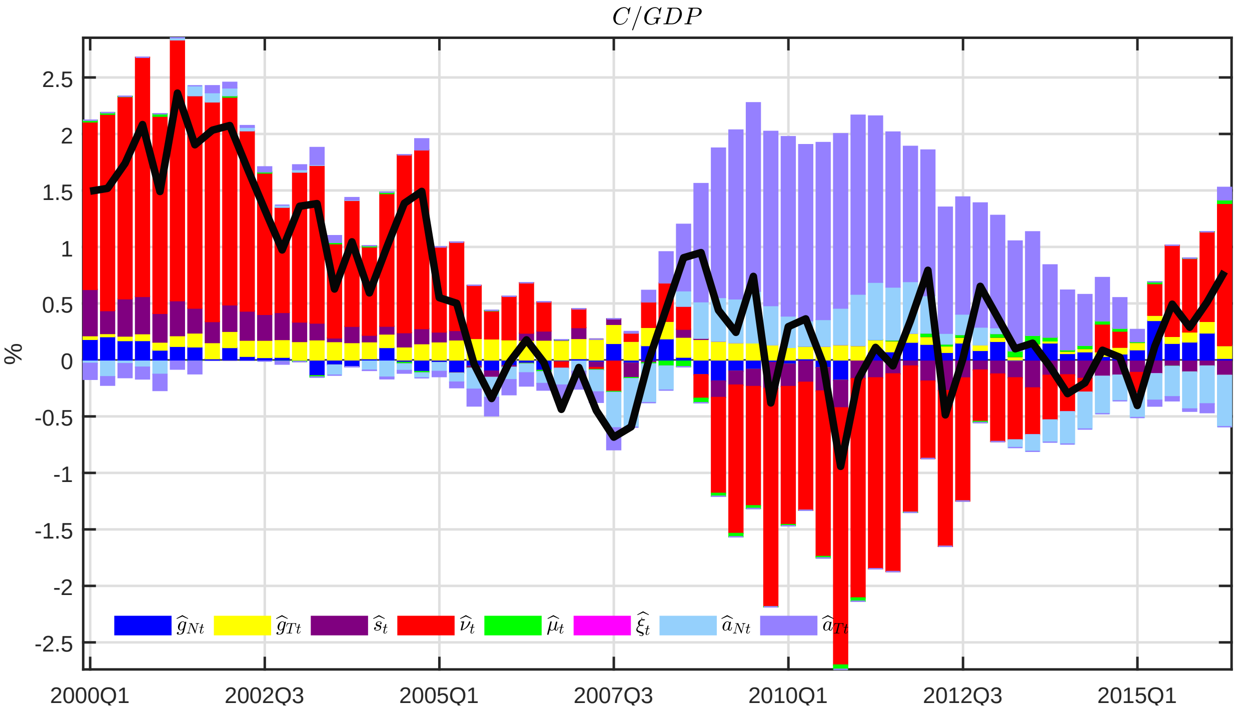

Figure 18: *Historical decomposition of* C/Y

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

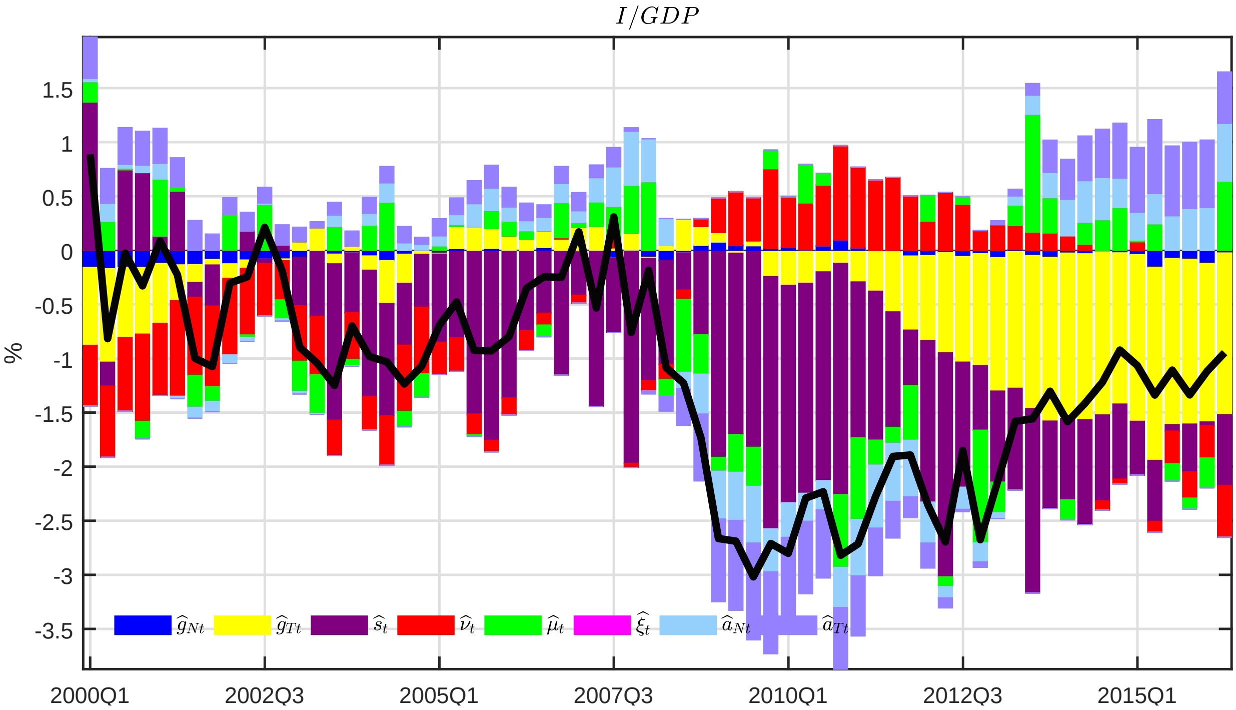

Figure 19: *Historical decomposition of* I/Y

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

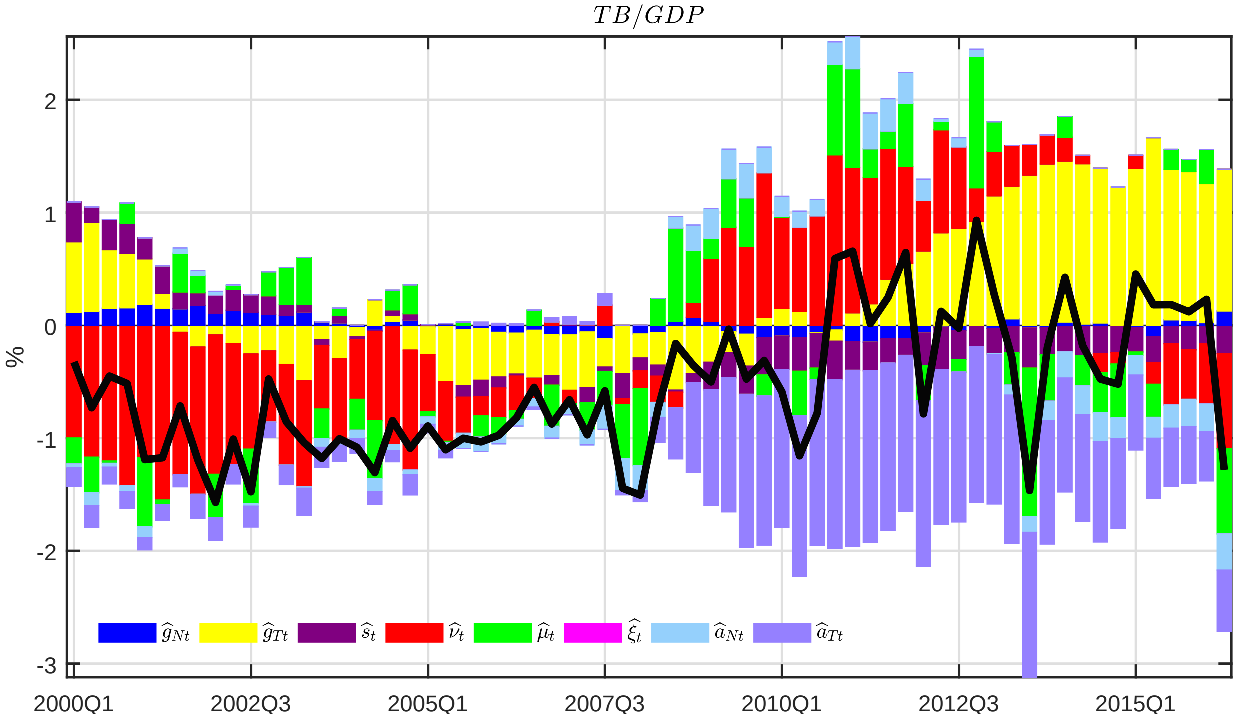

Figure 20: *Historical decomposition of TB/Y*

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

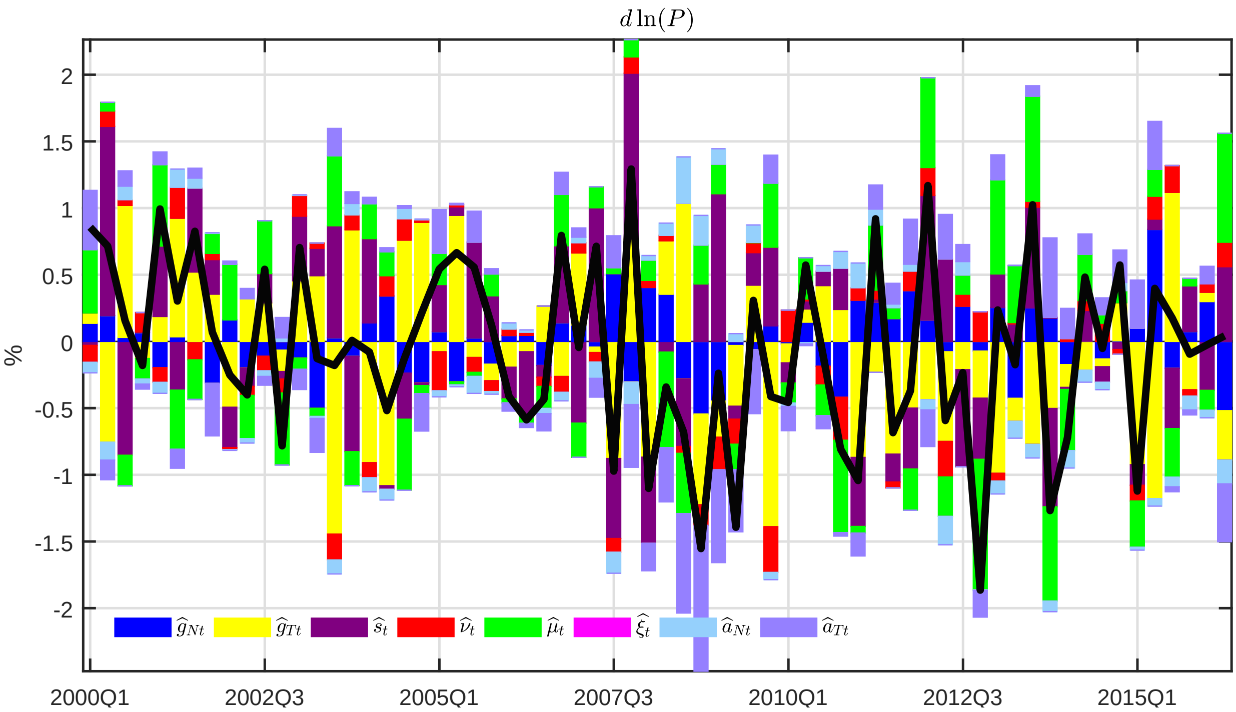

Figure 21: *Historical decomposition of P*

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

the real effective exchange rate over time. We note that, while sectoral TFP innovations help to explain a large proportion of the cyclical fluctuations in the quarterly growth rate of the relative price of non-tradable goods (P), they tend to play a much smaller role at explaining the movements in the real effective exchange rate. Since P is a relative price, it is not surprising that shocks that can be interpreted as purely sectoral shocks, such as government expenditure and the foreign interest rate shocks, turn out to be important sources of variation. We observe that up to the referendum date, fluctuations in the real effective exchange rate ($\mathcal{Q}$) are largely exogenous. These disturbances can be interpreted as an increase in TFP differentials between the UK and the rest of the world.

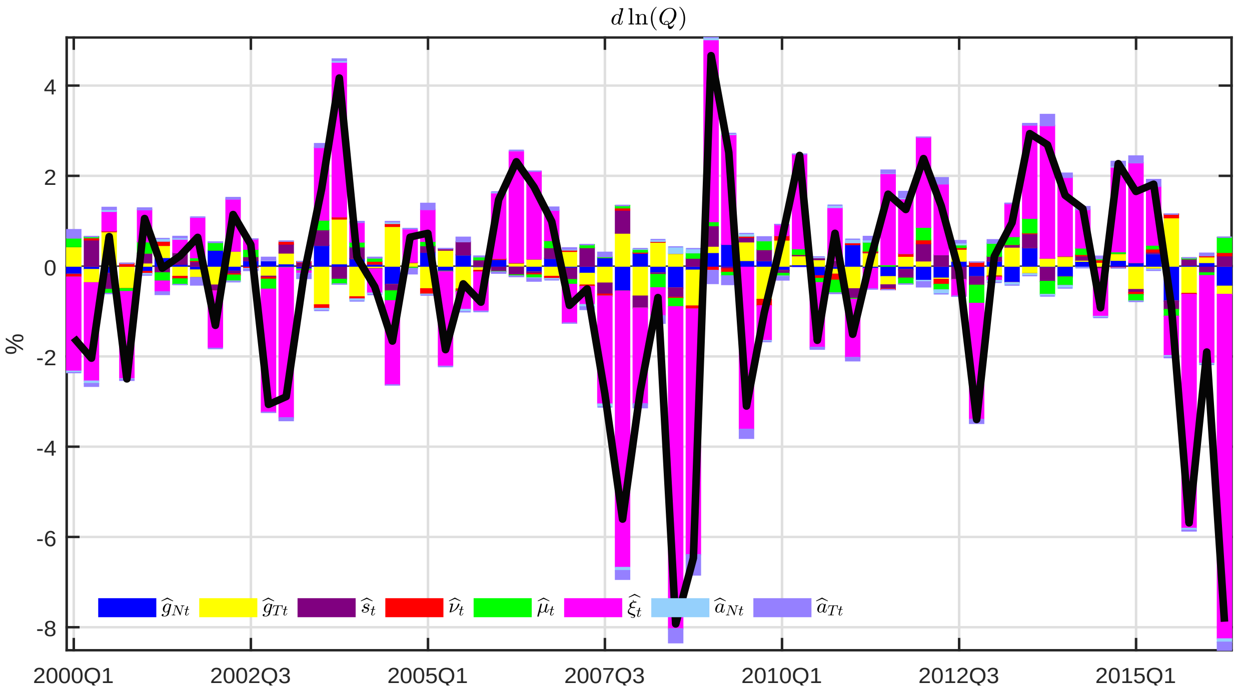

Figure 22: *Historical decomposition of REER*

Notes. The black line is the actual realization of the variable over time. The bars denote the contribution of each shock to the movements in the variable. Business cycle fluctuations are driven by disturbances to non-tradable TFP growth (g_{Nt}), tradable TFP growth (g_{Tt}), government expenditure (s_t), risk-premium (ν_t), foreign price (ξ_t), temporary non-tradable TFP (a_{Nt}) and temporary tradable TFP (a_{Tt}).

E. Model with population growth

E.1. Overview

This variant of the model involves a small adjustment to the household sector. The model is essentially an open economy variant of the Weil (1989, 1991) model with GHH preferences.[44]

In this variant, we assume that households are infinitely lived, but that new households are born each period. The population growth rate is constant. Individual households have identical preferences to those in the baseline model. So their first order conditions identical to the ones derived in the main text. However, population growth means that aggregate consumption is no-longer characterized by the same Euler equation as the one that holds for each individual household. This is because new households are born with no financial wealth and accounting for the heterogeneity in financial wealth delivers an aggregate consumption equation that depends on the distribution of wealth. The simple population structure implies that the distribution of wealth can be summarized by aggregate stocks of wealth (ultimately, in our model, the stock of foreign debt).

The dependence of the aggregate consumption Euler equation on wealth means that the steady state net foreign asset position is pinned down, even if the economy may freely borrow and lend at a fixed (tradeable-good denominated) interest rate. The steady-state net foreign asset position is pinned down by the (im)patience of domestic agents relative to the (growth adjusted) world real interest rate.

E.2. Households

The number of households alive in period t is Z_t. Population evolves according to:

$$Z_{t+1} = (1+\vartheta) Z_t \tag{80}$$

As before, to convert the model into stationary units aggregate quantities must be detrended by sectoral growth rates. However, the introduction of population growth means that the quantities of interest in this variant are measured in *per capita* terms. This means that the detrending factors for quantities (though not prices must also account for population growth). With this in mind, sectoral growth rates for this variant are defined as:

$$g_{Mt} = (1+\vartheta) \frac{X_{Mt}}{X_{Mt-1}}, \tag{81}$$

for $M = \{N, T\}$.

A household born in period s maximizes the following utility function:

$$\sum_{t=s}^{\infty} \beta^{t-s} \frac{\left[C_t^s - X_{Tt-1}\omega^{-1}\left(\theta_T \left(n_{Tt}^s\right)^{\omega} + \theta_N \left(n_{Nt}^s\right)^{\omega}\right)\right]^{1-\gamma}}{1-\gamma}$$

[44] A closed economy version of the Weil model with GHH preferences is analyzed by Ireland (2005), which also forms a guide for our approach.

so that the preferences of an individual household are identical to those in the baseline model. The s superscript indexes the date of birth of the household.

The household budget constraint, denominated in traded goods, is given by:

$$P_t^c C_t^s + B_t^{*,s} + P_t B_t^s = W_{Tt} n_{Tt}^s + W_{Nt} n_{Nt}^s + \Pi_t + \frac{B_{t+1}^{*,s}}{1+r_t^*} + P_t \frac{B_{t+1}^s}{1+r_t} \tag{82}$$

where it is assumed that households are born with no financial wealth or debt, so that $B_s^s = B^{*,s} = 0$. As in previous derivations, positive values of B and B^* represent *debt*.

Relative to the baseline model, two adjustments are made to facilitate the subsequent derivation. First, the budget constraint is written in terms of the total consumption bundle, incorporating the price of consumption in terms of tradable output, P_t^c. Second, households are assumed to receive lump sum profits (allocated from both tradable and non-tradable firms) denoted by Π. These profits are distributed equally to all households (including newborns). This means that households do not own the capital stock in this model. Instead, firms are assumed to own the capital stock, discussed below.

Writing the budget constraint in terms of the aggregate consumption bundle simplifies the derivations of the consumption function considerably. The total expenditure on consumption satisfies:

$$P_t^c C_t = P_t C_{Nt} + C_{Tt} \tag{83}$$

and the consumption bundle (as in the baseline model) is given by

$$C_t = \left[\zeta^{1-\sigma} C_{Tt}^{\sigma} + (1-\zeta)^{1-\sigma} \left(\frac{X_{Tt-1}}{X_{Nt-1}} C_{Nt}\right)^{\sigma}\right]^{\frac{1}{\sigma}} \tag{84}$$

The allocation of consumption between tradable and non-tradable consumption is a static problem and the optimality conditions imply:

$$\left[\frac{C_{Nt}}{C_{Tt}} \frac{\zeta}{1-\zeta} \frac{X_{Tt-1}}{X_{Nt-1}}\right]^{\sigma-1} \frac{X_{Tt-1}}{X_{Nt-1}} = P_t \tag{85}$$

which is the ratio of the first two first order conditions for the household in the previous derivation.

These equations provide solutions for C_{Nt}, C_{Tt}, P_t^c given a solution for C_t. The rest of the subsection derives a representation of the aggregate consumption function (and hence a solution for C_t).

The first order conditions for the household can be written as:

$$\left[C_t^s - X_{Tt-1}\omega^{-1}\left(\theta_T\left(n_{Tt}^s\right)^{\omega} + \theta_N\left(n_{Nt}^s\right)^{\omega}\right)\right]^{-\gamma} = \frac{\beta\left(1+r_t^*\right)P_t^c}{P_{t+1}^c} \times \left[C_{t+1}^s - X_{Tt}\omega^{-1}\left(\theta_T\left(n_{Tt+1}^s\right)^{\omega} + \theta_N\left(n_{Nt+1}^s\right)^{\omega}\right)\right]^{-\gamma}$$

$$1+r_t = \left(1+r_t^*\right)\frac{P_t}{P_{t+1}}$$

$$X_{Tt-1}\theta_T\left(n_{Tt}^s\right)^{\omega-1} = \frac{W_{Tt}}{P_t^c}$$

$$X_{Tt-1}\theta_N\left(n_{Nt}^s\right)^{\omega-1} = \frac{W_{Nt}}{P_t^c}$$

The household's inter-temporal budget constraint is:

$$\begin{aligned}\sum_{j=0}^{\infty}\mathcal{D}_{t+j}P_{t+j}^c C_{t+j} = & \sum_{j=0}^{\infty}\mathcal{D}_{t+j}\left(W_{Tt+j}n_{Tt+j}^s + W_{Nt+j}n_{Nt+j}^s + \Pi_{t+j}\right) \\ & -\sum_{j=0}^{\infty}\mathcal{D}_{t+j}\left(\frac{P_{t+j}}{P_{t+j+1}\left(1+r_{t+j}\right)} - \frac{1}{1+r_{t+j}^*}\right)P_{t+j+1}B_{t+j+1}^s - B_t^{*,s} - P_t B_t^s\end{aligned}$$

where the discount factor satisfies

$$\mathcal{D}_{t+j} \equiv \begin{cases} \frac{\mathcal{D}_{t+j-1}}{1+r_{t+j-1}^*} & \text{for } j \geq 1 \\ 1 & \text{for } j = 0 \end{cases}$$

and the usual transversality condition

$$\lim_{j\to\infty}\mathcal{D}_{t+j+1}\frac{B_{t+j+1}^{*,s}}{1+r_{t+j}^*} = 0$$

has been applied.

The inter-temporal budget constraint says that the present value of consumption expenditures equals non-financial wealth (the top line on the right hand side), net of expected debt revaluation effects and existing debts (second line).[45]

The first order condition for asset allocations implies that expected debt revaluations are zero in all future periods, so that the inter-temporal budget constraint is:

$$\sum_{j=0}^{\infty}\mathcal{D}_{t+j}P_{t+j}^c C_{t+j} = \sum_{j=0}^{\infty}\mathcal{D}_{t+j}\left(W_{Tt+j}n_{Tt+j}^s + W_{Nt+j}n_{Nt+j}^s + \Pi_{t+j}\right) - B_t^{*,s} - P_t B_t^s$$

[45]The debt revaluation effects measure the difference between the expected returns on non-tradable and tradable bonds, given that the tradable bond rate is chosen to value the intertemporal resource constraint.

A household's non-financial wealth is given by:

$$\Omega_t^s \equiv \sum_{j=0}^{\infty} \mathcal{D}_{t+j} \left(W_{Tt+j} n_{Tt+j}^s + W_{Nt+j} n_{Nt+j}^s + \Pi_{t+j} \right)$$
$$= W_{Tt} n_{Tt}^s + W_{Nt} n_{Nt}^s + \Pi_t + \frac{1}{1+r_t^*} \Omega_{t+1}^s$$

where the first line is a definition and the second line exploits the properties of the discount factor and employs a transversality condition.[46]

The household's first order conditions for labour supply demonstrate an important result: labour supply is determined entirely by aggregate conditions (productivity, wages and prices). This means that all households will supply the same labour, independently of their consumption. As a result the non-financial wealth of all households is identical and given by:

$$\Omega_t = W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \Pi_t + \frac{1}{1+r_t^*} \Omega_{t+1} \tag{86}$$

where

$$n_{Tt} = \left(\frac{W_{Tt}}{\theta_T X_{T,t-1} P_t^c} \right)^{\frac{1}{\omega-1}} \tag{87}$$
$$n_{Nt} = \left(\frac{W_{Nt}}{\theta_N X_{T,t-1} P_t^c} \right)^{\frac{1}{\omega-1}} \tag{88}$$

To simplify the Euler equation, define the disutility of labour supply as:

$$\mathcal{N}_t^s \equiv X_{Tt-1} \omega^{-1} \left(\theta_T \left(n_{Tt}^s \right)^{\omega} + \theta_N \left(n_{Nt}^s \right)^{\omega} \right)$$
$$= X_{Tt-1} \omega^{-1} \left(\theta_T \left(\frac{W_{Tt}}{\theta_T X_{T,t-1} P_t^c} \right)^{\frac{\omega}{\omega-1}} + \theta_N \left(\frac{W_{Nt}}{\theta_N X_{T,t-1} P_t^c} \right)^{\frac{\omega}{\omega-1}} \right) \tag{89}$$

where the second line substitutes for the equilibrium levels of labour supply. Once again, the disutility of labour supply is identical for all households: so $\mathcal{N}_t^s = \mathcal{N}_t, \forall s$.

This means that the Euler equation can be written as:

$$C_{t+1}^s - \mathcal{N}_{t+1} = \beta^{\frac{1}{\gamma}} \left(1 + r_t\right)^{\frac{1}{\gamma}} \left(\frac{P_t^c}{P_{t+1}^c} \right)^{\frac{1}{\gamma}} \left[C_t^s - \mathcal{N}_t \right]$$

which implies that

$$P_{t+1}^c C_{t+1}^s - P_{t+1}^c \mathcal{N}_{t+1} = \beta^{\frac{1}{\gamma}} \left(1 + r_t\right)^{\frac{1}{\gamma}} \left(\frac{P_t^c}{P_{t+1}^c} \right)^{\frac{1}{\gamma}-1} \left[P_t^c C_t^s - P_t^c \mathcal{N}_t \right]$$

[46]Specifically that human wealth does not grow faster than the interest rate: $\lim_{j\to\infty} \left(1 + r_{t+j}\right)^{-1} \Omega_{t+j+1}^s = 0$.

Iterating the Euler equation forward implies that

$$P^c_{t+j}C^s_{t+j} = P^c_{t+j}\mathcal{N}_{t+j} + \beta^{\frac{j}{\gamma}}\mathcal{D}^{\frac{-1}{\gamma}}_{t+j}\left(\frac{P^c_t}{P^c_{t+j}}\right)^{\frac{1}{\gamma}-1}\left[P^c_tC^s_t - P^c_t\mathcal{N}_t\right]$$

Using this expression in the household's inter-temporal budget constraint gives:

$$\begin{aligned}\Omega_t - P_tB^s_t - B^{*,s}_t &= \sum_{j=0}^{\infty}\mathcal{D}_{t+j}\left[P^c_{t+j}\mathcal{N}_{t+j} + \beta^{\frac{j}{\gamma}}\mathcal{D}^{\frac{-1}{\gamma}}_{t+j}\left(\frac{P^c_t}{P^c_{t+j}}\right)^{\frac{1}{\gamma}-1}\left(P^c_tC^s_t - P^c_t\mathcal{N}_t\right)\right]\\ &= \left(P^c_tC^s_t - P^c_t\mathcal{N}_t\right)\sum_{j=0}^{\infty}\beta^{\frac{j}{\gamma}}\mathcal{D}^{1-\frac{1}{\gamma}}_{t+j}\left(\frac{P^c_t}{P^c_{t+j}}\right)^{\frac{1}{\gamma}-1} + \sum_{j=0}^{\infty}\mathcal{D}_{t+j}P^c_{t+j}\mathcal{N}_{t+j}\end{aligned}$$

This implies that the household's consumption function can be written as:

$$P^c_tC^s_t = P^c_t\mathcal{N}_t + \Psi^{-1}_t\widetilde{\Omega}_t - \Psi^{-1}_t\left(P_tB^s_t + B^{*,s}_t\right)$$

where Ψ_t is the inverse of the marginal propensity to consume and $\widetilde{\Omega}_t$ is adjusted non-financial wealth, given respectively by:

$$\Psi_t = 1 + \beta^{\frac{1}{\gamma}}\left(\frac{P^c_{t+1}}{P^c_t\left(1+r^*_t\right)}\right)^{\frac{\gamma-1}{\gamma}}\Psi_{t+1} \tag{90}$$

$$\widetilde{\Omega}_t = W_{Tt}n_{Tt} + W_{Nt}n_{Nt} + \Pi_t - P^c_t\mathcal{N}_t + \frac{1}{1+r^*_t}\widetilde{\Omega}_{t+1} \tag{91}$$

Aggregation across households is straightforward. The consumption function is an affine function of financial wealth. The non-financial wealth components are common across all households. So the aggregate consumption function is also an affine function of (aggregate) financial wealth.

Aggregate consumption is equal to:

$$P^c_tC^{\text{agg}}_t \equiv Z_{t-1}P^c_tC^o_t + \left(Z_t - Z_{t-1}\right)P^c_tC^n_t$$

where C^o_t and C^n_t are *per capita* consumption levels of 'old' households (i.e., those alive in period $t-1$) and newborn households respectively.

Since newborn households enter the model with no financial wealth, we have:

$$C^n_t = P^c_t\mathcal{N}_t + \Psi^{-1}_t\widetilde{\Omega}_t$$

The consumption functions of all old agents are affine in financial wealth/debt, so

$$C^o_t = P^c_t\mathcal{N}_t + \Psi^{-1}_t\widetilde{\Omega}_t - \Psi^{-1}_t\left(P_tB_t + B^*_t\right)$$

where B_t and B^*_t are *per capita* debt stocks (since the date t 'old' households represent the entire population in period $t-1$).

This implies that:

$$P_t^c C_t^{\text{agg}} = Z_t \left(P_t^c \mathcal{N}_t + \Psi_t^{-1} \widetilde{\Omega}_t \right) - Z_{t-1} \Psi_t^{-1} \left(P_t B_t + B_t^* \right)$$

and dividing both sides by Z_t gives the *per capita* consumption function:

$$P_t^c C_t = P_t^c \mathcal{N}_t + \Psi_t^{-1} \widetilde{\Omega}_t - \Psi_t^{-1} \frac{P_t B_t + B_t^*}{1 + \vartheta} \tag{92}$$

where C, B and B^* are *per capita* consumption and debt stocks.

Note that the case of log utility, $\gamma = 1$, implies that the expression for Ψ_t simplifies substantially to:

$$\Psi_t = (1 - \beta)^{-1}, \forall t$$

which implies that the consumption function under log utility is

$$P_t^c C_t = P_t^c \mathcal{N}_t + (1 - \beta) \widetilde{\Omega}_t - \frac{1 - \beta}{1 + \vartheta} \left(P_t B_t + B_t^* \right)$$

The aggregate household budget constraint is given by:

$$P_t^c C_t + \frac{P_t B_t + B_t^*}{1 + \vartheta} = W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \Pi_t + P_t \frac{B_{t+1}}{1 + r_t} + \frac{B_{t+1}^*}{1 + r_t^*} \tag{93}$$

reflecting the same logic as above.[47]

We can eliminate non-financial wealth from the consumption function to derive an aggregate Euler equation. Rearranging the consumption function gives:

$$\widetilde{\Omega}_t = \Psi_t \left(P_t^c C_t - P_t^c \mathcal{N}_t \right) + \frac{P_t B_t + B_t^*}{1 + \vartheta}$$

which we can substitute into the difference equation for $\widetilde{\Omega}$ to give:

$$\begin{aligned} \Psi_t \left(P_t^c C_t - P_t^c \mathcal{N}_t \right) + \frac{P_t B_t + B_t^*}{1 + \vartheta} &= W_{Tt} n_{Tt} + W_{Nt} n_{Nt} + \Pi_t - P_t^c \mathcal{N}_t \\ &\quad + \frac{1}{1 + r_t^*} \left(\Psi_{t+1} \left(P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1} \right) + \frac{P_{t+1} B_{t+1} + B_{t+1}^*}{1 + \vartheta} \right) \end{aligned}$$

Using the aggregate budget constraint to substitute for $\frac{P_t B_t + B_t^*}{1 + \vartheta}$ gives:

$$\begin{aligned} \Psi_t \left(P_t^c C_t - P_t^c \mathcal{N}_t \right) + P_t \frac{B_{t+1}}{1 + r_t} + \frac{B_{t+1}^*}{1 + r_t^*} &= P_t^c C_t - P_t^c \mathcal{N}_t \\ &\quad + \frac{1}{1 + r_t^*} \left(\Psi_{t+1} \left(P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1} \right) + \frac{P_{t+1} B_{t+1} + B_{t+1}^*}{1 + \vartheta} \right) \end{aligned}$$

[47] The household budget constraint holds for all households, but newborns have no initial financial wealth/debt: $B_t^t = B_t^{*,t} = 0$. So the per capita value of previously accumulated debt is equal to the per capita value of debt held last period, divided by the change in population.

which can be rearranged to give:[48]

$$\left(\Psi_t - 1\right)\left(P_t^c C_t - P_t^c \mathcal{N}_t\right) = \frac{\Psi_{t+1}}{1+r_t}\left(P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1}\right) - \frac{\vartheta}{1+\vartheta}\frac{P_{t+1}B_{t+1} + B_{t+1}^*}{1+r_t^*}$$

Finally, noting from (90) that $\Psi_t - 1 = \beta^{\frac{1}{\gamma}}\left(\frac{P_{t+1}^c}{P_t^c(1+r_t)}\right)^{\frac{\gamma-1}{\gamma}}\Psi_{t+1}$, gives

$$\begin{aligned} P_t^c C_t - P_t^c \mathcal{N}_t = & \left(\frac{P_{t+1}^c}{P_t^c}\right)^{\frac{1-\gamma}{\gamma}}\left(\beta\left(1+r_t\right)\right)^{-\frac{1}{\gamma}}\left(P_{t+1}^c C_{t+1} - P_{t+1}^c \mathcal{N}_{t+1}\right) \\ & - \frac{\vartheta}{1+\vartheta}\left(\Psi_t - 1\right)^{-1}\frac{P_{t+1}B_{t+1} + B_{t+1}^*}{1+r_t^*} \end{aligned}$$

This demonstrates that the aggregate Euler equation depends on total asset holdings. In the case of no population growth, $\vartheta = 0$, the aggregate and individual household Euler equations coincide.

E.3. Firms

Firms maximize dividends over an infinite horizon and distribute them lump sum to households.

The non-tradable firm maximizes the present discounted value of dividend payments (expressed in units of *tradable* output):

$$\max \sum_{i=0}^{\infty}\lambda_{t,t+i}\left[\begin{array}{l} P_{t+i}a_{Nt+i}K_{Nt+i}^{\alpha_N}(X_{Nt+i}n_{Nt+i})^{1-\alpha_N} - P_{t+i}\frac{\phi_N}{2}\left(\frac{K_{Nt+i+1}}{K_{Nt+i}} - \bar{g}_N\right)^2 K_{Nt+i} \\ -P_{t+i}I_{Nt+i} - w_{Nt+i}n_{Nt+i} \end{array}\right]$$

subject to: $K_{Nt+i+1} = \left(1-\delta_N\right)K_{Nt+i} + I_{Nt+i}$

where $\lambda_{t,t+1}$ is a (compound) discount factor (discussed below) and the term in brackets is the per-period dividend.

Substituting for I_{Nt+1} implies that the firm maximises:

$$\max \sum_{i=0}^{\infty}\lambda_{t,t+i}\left[\begin{array}{l} P_{t+i}a_{Nt+i}K_{Nt+i}^{\alpha_N}(X_{Nt+i}n_{Nt+i})^{1-\alpha_N} - P_{t+i}\frac{\phi_N}{2}\left(\frac{K_{Nt+i+1}}{K_{Nt+i}} - \bar{g}_N\right)^2 K_{Nt+i} \\ -P_{t+i}K_{Nt+i+1} + \left(1-\delta_N\right)P_{t+i}K_{Nt+i} - w_{Nt+i}n_{Nt+i} \end{array}\right]$$

The first order conditions are:

$$0 = W_{Nt} - P_t\left(1-\alpha_N\right)a_{Nt}K_{Nt}^{\alpha_N}(X_{Nt}n_{Nt})^{-\alpha_N}X_{Nt} \tag{94}$$

$$\begin{aligned} 0 = & -\lambda_{t,t}P_t\left(1+\phi_N\left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N\right)\right) \\ & + \lambda_{t,t+1}P_{t+1}\left(\begin{array}{c} \alpha_N a_{Nt+1}K_{Nt+1}^{\alpha_N - 1}(X_{Nt+1}n_{Nt+1})^{1-\alpha_N} + \left(1-\delta_N\right) \\ -\frac{\phi_N}{2}\left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N\right)^2 + \phi_N\left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N\right)\frac{K_{Nt+2}}{K_{Nt+1}} \end{array}\right) \end{aligned} \tag{95}$$

The first order condition for labor is identical to the baseline model. The second equation

[48] The no-arbitrage condition for asset returns implies that $\frac{P_t}{1+r_t} = \frac{P_{t+1}}{1+r_t^*}$, which allows us to collect terms in both types of bonds.

looks slightly different because of the different ownership structure (ie firms are now assumed to own the capital stock).

The first order condition for capital can be written as:

$$\alpha_N a_{Nt+1} K_{Nt+1}^{\alpha_N - 1} (X_{Nt+1} n_{Nt+1})^{1-\alpha_N} + 1 - \delta_N = \frac{\lambda_{t,t} P_t}{\lambda_{t,t+1} P_{t+1}} \left(1 + \phi_N \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N\right)\right) + \frac{\phi_N}{2}\left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N\right)^2 - \phi_N \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N\right)\frac{K_{Nt+2}}{K_{Nt+1}}$$

The left hand side is the return on capital, net of depreciation, measured in units of non-tradable output. The right hand side captures the inter-temporal cost of substituting non-tradable output across time and adjustment costs. The right hand side depends only on the ratio of discount factors between time periods. This is the same for all households and is given by the inverse of the rate of return on intermediary bonds. This means that the first order condition for non-tradable capital is:

$$\alpha_N a_{Nt+1} K_{Nt+1}^{\alpha_N - 1} (X_{Nt+1} n_{Nt+1})^{1-\alpha_N} + 1 - \delta_N = \frac{P_t (1 + r_t^*)}{P_{t+1}} \left(1 + \phi_N \left(\frac{K_{Nt+1}}{K_{Nt}} - \bar{g}_N\right)\right) + \frac{\phi_N}{2}\left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N\right)^2 - \phi_N \left(\frac{K_{Nt+2}}{K_{Nt+1}} - \bar{g}_N\right)\frac{K_{Nt+2}}{K_{Nt+1}}$$

The tradable firm solves an isomorphic problem. So the labor demand equation is the same as the baseline model. The first order condition for capital is:

$$\alpha_T a_{Tt+1} K_{Tt+1}^{\alpha_T - 1} (X_{Tt+1} n_{Tt+1})^{1-\alpha_T} + 1 - \delta_T = (1 + r_t^*) \left(1 + \phi_T \left(\frac{K_{Tt+1}}{K_{Tt}} - \bar{g}_T\right)\right) + \frac{\phi_T}{2}\left(\frac{K_{Tt+2}}{K_{Tt+1}} - \bar{g}_T\right)^2 - \phi_T \left(\frac{K_{Tt+2}}{K_{Tt+1}} - \bar{g}_T\right)\frac{K_{Tt+2}}{K_{Tt+1}}$$

E.4. Market clearing

The market clearing conditions are the same as in previous derivations. Substituting into the *per capita* budget constraint gives:

$$\frac{B_t^*}{1+\vartheta} = TB_t + \frac{B_{t+1}^*}{1+r_t^*}$$

where we impose that non-tradable bonds are in zero net supply.

E.5. Stationary units

The redefinition of sector-specific growth rates to include deterministic population growth rates means that the transformations into stationary units in the baseline model continue to hold in almost all cases. The main exception is the consumption Euler equation and household budget constraints, which we have written in per capita terms, but without adjusting for non-stationary tradable productivity. Adjusting the Euler equation for productivity gives:

$$X_{Tt-1}\left(P_t^c c_t - P_t^c n_t\right) = \left(\frac{P_{t+1}^c}{P_t^c}\right)^{\frac{1-\gamma}{\gamma}} \left(\beta\left(1+r_t^*\right)\right)^{-\frac{1}{\gamma}} X_{Tt}\left(P_{t+1}^c c_{t+1} - P_{t+1}^c n_{t+1}\right) - \frac{\vartheta X_{Tt}}{1+\vartheta}\left(\Psi_t - 1\right)^{-1}\frac{P_{t+1}b_{t+1} + b_{t+1}^*}{1+r_t^*}$$

where lower case letters denote stationary units (as in the baseline model), which in this variant means adjusted for both productivity and population.

In particular,

$$n_t = X_{Tt}^{-1}\mathcal{N}_t = \omega^{-1}\left(\theta_T\left(\frac{w_{Tt}}{\theta_T p_t^c}\right)^{\frac{\omega}{\omega-1}} + \theta_N\left(\frac{w_{Nt}}{\theta_N p_t^c}\right)^{\frac{\omega}{\omega-1}}\right)$$

These considerations means that the Euler equation in stationary units is given by:[49]

$$p_t^c\left(c_t - n_t\right) = \left(\frac{p_{t+1}^c}{p_t^c}\right)^{\frac{1-\gamma}{\gamma}}\left(\beta\left(1+r_t^*\right)\right)^{-\frac{1}{\gamma}}\frac{g_{Tt}}{1+\vartheta}p_{t+1}^c\left(c_{t+1} - n_{t+1}\right) - \frac{\vartheta g_{Tt}}{1+\vartheta}\left(\Psi_t - 1\right)^{-1}\frac{P_{t+1}b_{t+1} + b_{t+1}^*}{1+r_t^*}$$

Similar arguments apply to the flow budget constraint:

$$\frac{X_{Tt-1}b_t^*}{1+\vartheta} = X_{Tt-1}tb_t + \frac{X_{Tt}b_{t+1}^*}{1+r_t^*}$$

so that

$$\frac{b_t^*}{1+\vartheta} = tb_t + \frac{g_{Tt}b_{t+1}^*}{\left(1+\vartheta\right)\left(1+r_t^*\right)}$$

E.6. Steady state

In steady state (also imposing market clearing) the Euler equation implies:

$$p^c\left(c - n\right)\left[1 - \left(\beta\left(1+r^*\right)\right)^{-\frac{1}{\gamma}}\frac{\bar{g}_T}{1+\vartheta}\right] = -\frac{\vartheta\bar{g}_T}{1+\vartheta}\frac{b^*}{\left(\Psi - 1\right)\left(1+r^*\right)}$$

Under the assumption that $c > n$ (so that marginal utility is positive in steady state), this expression reveals that the sign of the economy's foreign debt position depends on the relative patience of households. Specifically, it depends on the size of the discount factor β in relation to the (productivity) growth adjusted real interest rate. In the particular case in which $\beta = \beta_0 \equiv \left(\frac{1+\vartheta}{\bar{g}_T}\right)^{-\gamma}\frac{1}{1+r^*}$, the economy will hold no foreign debt or assets (and the trade balance will be zero) in steady state. If the economy is relatively less patient (so $\beta < \beta_0$) then the economy will be a net debtor, with $b^* > 0$, in steady state. Conversely, if households are more patient, then the economy will hold foreign bonds in steady state ($b^* < 0$ means that *debt* is negative and the economy holds positive assets).

[49]Recall that P_t^c is cointegrated with tradable productivity, so $p_t^c = P_t^c$.

These observations mean that we can calibrate β to deliver a desired steady-state foreign debt position (conditional on the values of the other model parameters). First note that the steady-state (inverse) marginal propensity to consume is given by:

$$\Psi = \frac{1}{1 - \beta^{\frac{1}{\gamma}} \left(1 + r^*\right)^{\frac{1}{\gamma} - 1}}$$

Plugging this into the steady-state Euler equation and rearranging gives:

$$\begin{aligned}
-\frac{\vartheta \bar{g}_T}{1+\vartheta} \frac{b^*}{p^c \left(c - n\right)\left(1 + r^*\right)} &= \left(\frac{1}{1 - \beta^{\frac{1}{\gamma}} \left(1 + r^*\right)^{\frac{1}{\gamma} - 1}} - 1\right) \left[1 - \left(\beta \left(1 + r^*\right)\right)^{-\frac{1}{\gamma}} \frac{\bar{g}_T}{1 + \vartheta}\right] \\
&= \frac{\beta^{\frac{1}{\gamma}} \left(1 + r^*\right)^{\frac{1}{\gamma} - 1}}{1 - \beta^{\frac{1}{\gamma}} \left(1 + r^*\right)^{\frac{1}{\gamma} - 1}} \left[1 - \left(\beta \left(1 + r^*\right)\right)^{-\frac{1}{\gamma}} \frac{\bar{g}_T}{1 + \vartheta}\right] \\
&= \frac{\mathcal{B}}{1 + r^* - \mathcal{B}} \left[1 - \mathcal{B}^{-1} \frac{\bar{g}_T}{1 + \vartheta}\right]
\end{aligned}$$

where the final line makes use of the following definition:

$$\mathcal{B} \equiv \beta^{\frac{1}{\gamma}} \left(1 + r^*\right)^{\frac{1}{\gamma}}$$

Rearranging the final equation allows us to solve for $\mathcal{B}$:

$$\mathcal{B} = \frac{\bar{g}_T \left(1 - \frac{\vartheta b^*}{p^c (c - n)}\right)}{\left(1 + \vartheta\right) \left(1 - \frac{\vartheta \bar{g}_T b^*}{(1+\vartheta)(1+r^*) p^c (c - n)}\right)}$$

as a function of other parameters, steady-state allocations and the desired steady-state foreign debt position, b^*.

Finally, we can solve for $\beta = \mathcal{B}^{\gamma} \left(1 + r^*\right)^{-1}$.