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# Assessing the adequacy of CCPs' default resources

Fergus Cumming and Joseph Noss

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Central counterparties (CCPs) maintain financial resources that can absorb losses in the event of their members defaulting. These include initial margin collected from members and default funds designed to absorb losses that exceed the initial margin posted by defaulting members.

This paper proposes a methodology whereby daily data on a CCP's member exposures may be used to form a 'top-down' statistical model of the risk arising from CCPs' exposures to their members. In doing so, it may offer a tool with which CCPs, their members and their regulators, could assess the adequacy of CCPs' total default resources and quantify the trade-off that occurs in the balance of resources between initial margin and default funds. It may also provide a technique to estimate the relative risk borne by clearing members on their CCP default fund contributions.

## Introduction

Central counterparties (CCPs) stand between market participants and mitigate the risk of one (or more) of their members failing to honour their trade settlement obligations. Historically, CCPs have mainly served exchange-traded financial markets. But in recent years, central clearing has extended to over-the-counter (OTC) derivatives. This area of CCPs' business will likely expand further as the G20 objective for all standardised OTC derivatives to be centrally cleared comes into effect.

Central clearing necessarily leads to the concentration of risk in a single entity — the CCP — the failure of which could lead to widespread disruption in financial markets. Regulation has therefore sought to ensure the resilience of CCPs by requiring that they hold sufficient resources to cover the risk of one or more of their members defaulting.<sup>(1)</sup> Such 'default resources' typically take a number of forms. Specifically, CCPs collect collateral (initial margin) from each of their members, which is available to meet the risk of that member failing to meet its individual financial obligations to the CCP. Furthermore, CCPs establish a mutualised 'default fund' (or 'guarantee fund') to which all their members contribute, and which may be called upon to meet losses in the case that they exceed a defaulting member's initial margin. Default funds thus act as a mechanism through which CCPs protect their members against the risk that the CCP's exposure to another member exceeds that member's initial margin. If losses exceed both initial margin and default fund, then any remaining loss would potentially fall on the CCP's members (including through a requirement to top up the default fund) and others in the creditor hierarchy in proportion to their claims on the CCP.<sup>(2)</sup> The distribution of these losses can be made clear *ex ante* through the specification of a loss allocation rule.

This raises the question of what level of financial resources is sufficient to mitigate the risk of member default. For derivatives transactions, this question is complicated by the uncertainty not only as to whether a given clearing member will default, but also the amount the CCP would lose in the event that it did so.

This paper provides a framework in which to assess the adequacy of a CCP's financial resources, whether they are sufficient to meet the risk of its members defaulting, and the frequency with which they may be called upon. Most CCPs determine the size of their default funds through the 'bottom-up' stress testing of individual member positions under extreme but plausible market conditions. But the novel contribution of this work is the 'top-down' statistical approach it takes to measuring risk. We illustrate this approach by constructing some hypothetical daily changes in the value of representative member derivatives positions at a CCP. Techniques from Extreme Value Theory are then used to model

the distribution of extreme changes in the value of these member positions of a magnitude large enough to risk exceeding members' initial margin contributions, and perhaps the CCPs' default fund. This provides an estimate of the frequency with which member exposures will likely exceed a given level of default resources.

In addition, the second part of the analysis estimates the risk of these exposures *crystallising into actual losses* due to member defaults. This allows for a more holistic assessment of the overall risk faced by the CCP than that offered by models of exposure alone. We extract information on the nature of firm defaults — including both their probability and codependence — from the traded prices of structured credit instruments. In doing so, we draw an analogy between a CCP's default fund (which protects against the risk of losses from clearing member default exceeding their initial margin), and the middle ('mezzanine') tranche of a structured credit securitisation — which offers insurance against the risk of the default of an underlying pool of loans reaching a certain level.

The policy implications of this work are broadly threefold. First, it has the potential to provide CCPs, their members and their regulators, with a framework with which to cross-check the adequacy of CCP default resources. Were the methodology operationalised by a CCP, and combined with the superior data and information available to the CCP and its members, it may act as a helpful complement to the risk management tools currently used in the industry.

Second, by considering the distribution of exposures across different clearing members, our approach allows for the comparison of different metrics of default fund adequacy. By comparing the concentration of the CCP's exposures across its members, the methodology can show the size of the default fund necessary to cover the CCP's largest exposures to different numbers of its members, with a given level of confidence. This permits an insight into the suitability and adequacy of regulatory standards based on a requirement to meet the default of the largest one or two clearing members (the 'cover-1' and 'cover-2' requirements set down in international regulatory standards (see Bank for International Settlements (BIS) (2012))). This work therefore also naturally highlights the importance of rigorous stress testing in implementation of the regulatory framework for CCPs.<sup>(3)</sup>

Third, it serves to illustrate and quantify the 'trade-off' faced by the CCP in determining its balance of initial margin and default fund, and the relative risk faced by clearing members. A CCP could, in principle, require its members to post a large

(1) See BIS (2012), 'Principles for financial market infrastructure'.

(2) See Elliott (2013).

(3) In this context, the Bank of England plans to consider the case for developing a regime for concurrent, cross-CCP stress testing that could complement the emerging framework for banks outlined in Bank of England Discussion Paper (2013).

amount of margin, reducing the risk to be mutualised by, and hence the size of, its default fund. Conversely, a smaller amount of initial margin would require a large default fund to recognise the risk of exposures exceeding initial margin. This trade-off has been examined from a theoretical perspective in previous literature,<sup>(1)</sup> but this work offers a methodology for formal quantification that could help inform choices regarding the level of, and balance between, the two resources (initial margin and default fund) within CCPs.

The paper is structured as follows. Section 1 briefly describes the role of a CCP, and the resources it has available to mitigate the risk of its members defaulting. A model of a CCP's exposures to its members, and how large these could grow under stressed market conditions, is introduced in Section 2. Section 3 extends this to a model of clearing member default, and by doing so provides an estimate of the frequency with which such exposures might crystallise into losses. A final section concludes, and suggests further directions in which this work could be used to inform current regulatory policy debates.

## 1 Background: the role of a CCP and its risk management mechanisms

A CCP mitigates risk in a financial transaction by interposing itself between counterparties. The counterparties' original bilateral transaction is replaced by two new transactions established by each with the CCP. The result is a simplified network of transactions between the CCP and its clearing members, with the CCP at its centre (Figure 1).

Day-to-day, and in the absence of a clearing member defaulting, the CCP is not exposed to market risk on its members' cleared trades: any decrease in the value of its claim on one member is matched exactly by an increase in the value of its claim on another member. Members make and receive 'variation margin' payments to and from the CCP commensurate with the amount owed by (to) them on their trades. These have the effect of 'resetting' a CCP's exposure to its members to zero once a day, or sometimes more frequently.

A CCP is, however, exposed to the risk of clearing members defaulting after the market has moved in the CCP's favour, leaving it owed variation margin from the defaulting member. It would also be exposure to future market movements over the lifetime of the underlying exposure. In order to return to having a 'matched book', it will seek to 'close out' the defaulted position, for example by entering into off-setting/hedging transactions with, or by auctioning the defaulting member's positions to, its non-defaulting clearing members. If market prices move against the CCP during this 'close-out period', it will incur further costs that it would hope to cover using the margin of the defaulter or, if necessary, the default fund.

As noted above, the CCP's primary protection against the risk of member default is the *initial margin* that it collects from its each member. The size of the initial margin requirement is set with the aim of ensuring that it is large enough to, with a high degree of confidence, cover the potential increase in the value of exposures to a member that the CCP may incur between the last successful variation margin payment from that member, and the point at which the CCP successfully hedges or auctions the defaulting clearing member's position and returns to having a matched book.

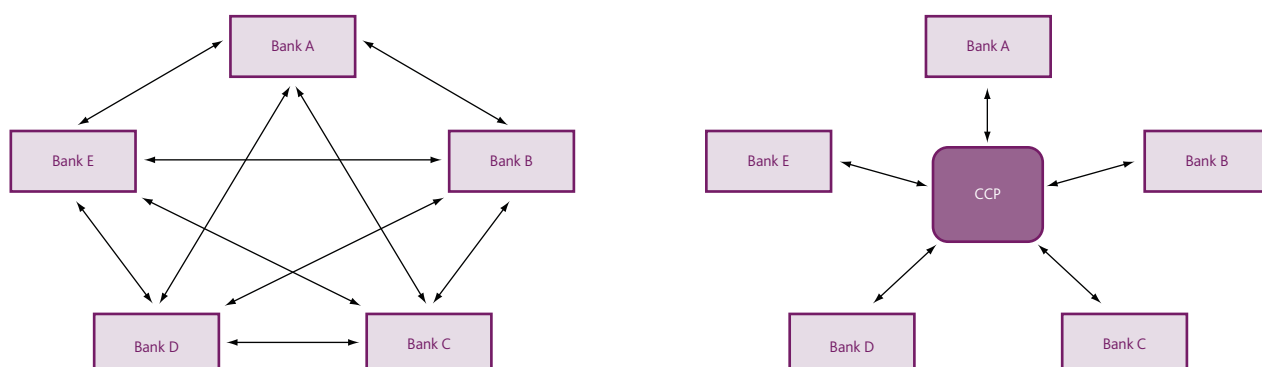
CCPs also mitigate risk by mutualising between their members the risk that exposures exceed these initial margin amounts. The CCP maintains a default fund — a pot of money (contributed, in the most part, by the members themselves) — for this purpose.<sup>(2)</sup>

This raises questions as to the level of initial margin and default fund that are sufficient to absorb the 'tail risk' of extreme market movements during the close-out period. The internationally agreed Principles for financial market infrastructure (BIS (2012)), prescribe that the default resources should be sufficient to cover the one or two largest clearing member exposures in 'extreme but plausible' market

(1) See, for example, Haene and Sturm (2009) and Nahai-Williamson *et al* (2013).

(2) A CCP also faces risk from its other functions, for example the holding and reinvestment of its members' collateral. For simplicity, these are ignored here.

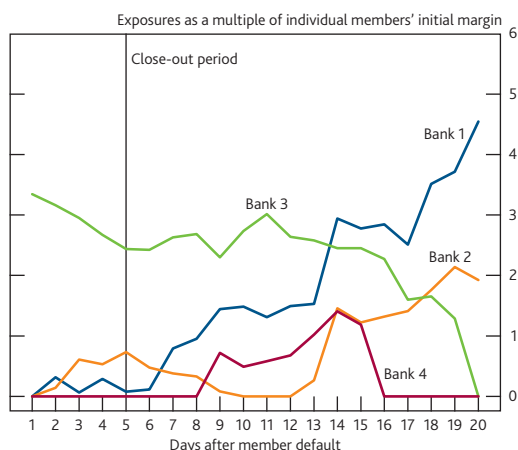
Figure 1 A collection of bilateral exposures is reduced to a smaller number of exposures between the CCP and its members



conditions (a 'cover-1' or 'cover-2' requirement). But calculating the amount of default resource that is sufficient for this is complicated by uncertainty as to how much the CCP would lose in the event that a clearing member with derivative positions of inherently variable value defaulted. It also depends critically on how the CCP identifies 'extreme but plausible' market conditions that are likely to affect the value of its member exposures in the event of member failure.

By way of illustration, **Chart 1** shows a hypothetical time series of a CCP's exposures to four members as a multiple of each member's initial margin, assuming default at time zero. These exposures vary as a function of the market price of the underlying instruments. When this moves in the CCP's favour — that is, in such a way that the CCP is owed money by the (now defaulted) clearing member — it results in a positive exposure of the CCP to that clearing member.<sup>(1)</sup> The exposure is bounded below at zero because the CCP is not exposed to members to whom it owes money.

**Chart 1** The hypothetical evolution of a CCP's exposures to four of its members



Source: Bank of England calculations.

The level of these exposures relative to each member's initial margin determines the loss to be absorbed by the default fund at the point at which the position is closed out. If the ratio is above one, the default fund will be used; so if close out were to occur at day five (vertical black solid line in **Chart 1**), the CCP's exposure to Bank 3 would exceed its initial margin. In contrast, the exposure to Banks 1 and 2 would be sufficiently small to be absorbed by their initial margin. Finally, throughout the assumed five-day close-out period, the CCP has zero exposure to Bank 4 — perhaps because its positions have moved in the bank's favour. Therefore, in this hypothetical example, the adequacy of the CCP's overall default resources depends on whether its exposure to Bank 3 exceeds the sum of Bank 3's initial margin and the CCP's default fund. Were this to be the case, any remaining loss would accrue either to the CCP itself, or be allocated between its members.

Understanding the risk borne by a CCP's default resources, and assessing their adequacy, therefore reduces to assessing the likely changes in member positions between the last successful transfer of variation margin prior to their default and close out, along with the probability that one or more members will default.

## 2 Modelling a CCP's exposure to its members over the period of risk

This section develops a framework for estimating the adequacy of CCPs' default resources. It is 'top-down' and statistical in nature, requiring calibration to the values of clearing member portfolios at the relevant CCP and amounts of initial margin held.

In order to illustrate our approach, we use hypothetical daily changes in the value of fixed clearing member portfolios at a CCP with approximately 20 clearing members. We then construct exposure data based on plausible changes in the value of these members' portfolios. A practical application of our approach to analyse a particular CCP could replace this indicative data with data providing a more accurate read of the likely change in its exposures to members in the event of their default. One obvious candidate might be data showing the historic observed changes in the value of the portfolio for all clearing members that are part of a particular default waterfall. Another alternative might be the likely changes in the value of member portfolios in a default scenario, as modelled by the CCP. Eitherway, our analysis aims to indicate a plausible shape of the results, rather than an estimation of the expected outcome for any given CCP.

**Chart 2** shows the distribution of changes in exposures of a CCP to a single member in excess of initial margin (in terms of its standard deviation), based on our indicative data. This distribution is 'fat tailed', containing a higher incidence of extreme changes in exposure compared with that under, for example, a normal distribution. This is salient to the problem at hand, because it is the 'tail risk' of large increases in exposure in the event of member failure that would cause the CCP to face losses in excess of its default resources.

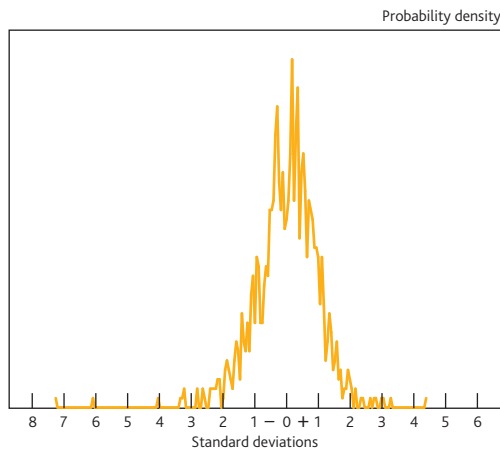
Assessing the adequacy of a CCP's default resources therefore requires an assessment of the distribution of its exposures to its members between default and close out.

But this assessment of risk is, by its very nature, fraught with difficulty. Historical data collected by CCPs will likely contain little information on the likelihood and size of extreme increases in exposure that are of a magnitude sufficient to

(1) Note that exposure does not arise from market price changes in the opposite direction, as these lead to the CCP owing money to — rather than being owed by — the defaulted member.

exhaust member initial margin, let alone the default fund. This is because such market movements are, by definition, rare. What is required is a means of assessing the risk of large changes in member positions, despite this lack of historical experience.

**Chart 2** Indicative distribution of daily changes in a CCP’s exposures to a single clearing member in excess of that member’s initial margin

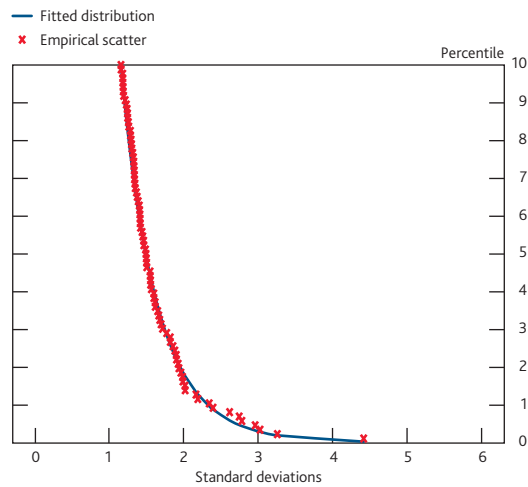


Source: Bank of England calculations.

One way to assess this ‘tail risk’ of large changes in member positions is to fit a probability distribution function to it. Such a function mathematically represents the likelihood of changes in exposure of varying degrees of rarity, including those sufficient to exhaust the initial margin or default fund following a default. This technique is taken from Heller and Vause (2012) and uses the result that (under certain regularity conditions, which are satisfied here) such tail observations adhere to a particular functional form known as the ‘Generalised Pareto’ distribution function. This function is fitted to the tail of the distribution of changes in the CCP’s exposure to each of its members, in excess of the relevant member’s initial margin. For each clearing member, this amounts to estimating the two parameters of the distribution that best fit the observed data — one of which controls the ‘position’ of the distribution, and the other of which controls the ‘shape’ of its tail, or its propensity to take values far from the centre of the distribution. This is illustrated in **Chart 3**, in which the red crosses show the largest 10% of observations of that member’s approximate exposures, and the blue line represents the fitted distribution function. This provides an estimated distribution of ‘tail’ movements in member exposures that are rarely observed in practice.<sup>(1)</sup>

Having used this technique to estimate the probability distribution of the change in exposures in excess of initial margin to each of the CCP members, we look at the *interdependence of exposures across different members* — that is the tendency of a large exposure to one member to be associated with a large exposure to another. This

**Chart 3** Fitting a distribution to the tail (upper 10% of) observations from the distribution in Chart 2



Source: Bank of England calculations.

interdependence informs the sum of exposures across all members, and hence the severities of risk to which the CCP’s default resources are exposed. It is estimated by fitting a ‘copula function’ to the observed joint coincidence of exposures across members contained within the data. The combination of the marginal distribution of changes in the CCP’s exposures to each member and this copula function allows us to construct the ‘joint probability distribution function’ that gives the probability of any combination of exposures across members occurring simultaneously. Further details of this methodology are given in Box 1.

The CCP’s exposure across all members, in excess of their initial margin, is then simulated by making random draws from this joint probability distribution function. **Chart 4** shows the distribution of the CCP’s largest two clearing member exposures — in excess of the relevant members’ hypothetical initial margin. This is presented as a multiple of average initial margin contributions observed in the data. Note that this distribution takes value zero in just over 80% of simulated scenarios, implying that it is roughly a fifth of scenarios that the largest two exposures exceed those members’ initial margin.<sup>(2)</sup> The remainder of the distribution illustrates the degree by which they do so.

(1) Two points are worthy of note here. First, our selection of the edge of the tail is arbitrary; with more data, we could use the standard technique of quartile-quartile plot analysis to determine whether 10% is indeed a reasonable threshold. Second, even if clearing member portfolios change relatively slowly, large changes in variation margin may be dominated by changes due to portfolio composition rather than those due to market moves in the value of pre-existing and already-margined trades. This concern justifies our use of the term ‘approximate’ in the illustrations presented.

(2) This is roughly to be expected given that CCPs typically require members to post initial margin sufficient to cover all but the largest 1% of the CCP’s likely future exposures to them. Given this requirement, and assuming that it exactly binds, the members with the two largest exposures in our data would be expected to exceed their initial margin in  $1 - 0.99^{19} = 17\%$  of cases.



## Box 1 Fitting a distribution to the daily changes in a CCP's exposures to its members

This box gives more detail on the methodology used in Section 2 to estimate the distribution of changes in exposures to each CCP member.

The approach is based on a result in Extreme Value Theory that establishes that, subject to certain regularity conditions (all of which are met here), the distribution of the extreme points of a statistical distribution — above or below a certain threshold — follows a particular functional form. Specifically, these rare observations follow a Generalised Pareto distribution, a flexible statistical distribution that subsumes a number of distributions in a single parametric form. Significantly, it is 'heavy tailed', in that it places significantly more weight on rare outcomes than more standard statistical distributions, including the normal. It is based largely on the work of Frey and McNeil (2000), and has been used elsewhere in the literature (for example, see Heller and Vause (2012)).

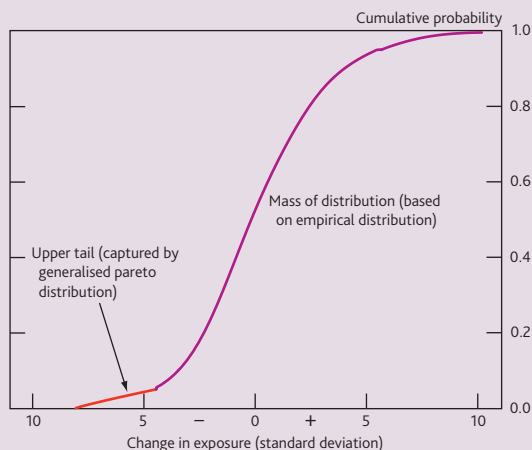
The methodology proceeds as follows. First, the observed changes in exposure to each member is divided into:

1. The 'mass' of the distribution — that is, outcomes occurring with 80% probability, between the 10th and 90th percentiles. To this, a non-parametric 'empirical density function' is fitted (magenta line in **Chart A**), which exactly captures the shape of the distribution of the observed data. Such empirical density functions are ideal for modelling the centre of the distribution where there are plenty of observations.
2. To the upper 'tail' of the distribution — that is, outcomes above the 90th percentile — a Generalised Pareto distribution is fitted (red line in **Chart A**).<sup>(1)</sup> This captures the frequency of tail events that are, by definition, relatively rare. The parameters of this distribution are fitted using a maximum likelihood procedure (see Bouye *et al* (2000)).

This approach therefore combines the advantage of both empirical and parametric statistical techniques to capture the observed distribution of exposure. It incorporates information on adverse changes in exposure that are particularly important for the calculation. This allows for the veracity of historical information to be retained, whilst also allowing for more accurate estimation of the likelihood of changes in member position that are rarely observed in practice.

Second, the distributions of the CCP's exposure to each individual member are combined into an estimate of the joint distribution of exposures to all members simultaneously. This

**Chart A** Using extreme value theory to model the distribution of clearing member variation margin flows



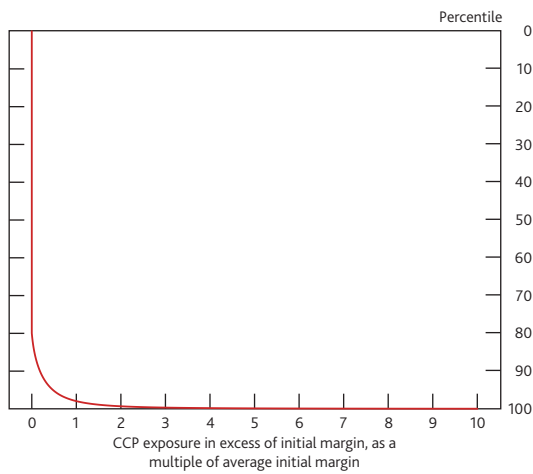
Source: Bank of England calculations.

is achieved using a copula function which 'links' each individual distribution together, based on their observed dependence. Parameters of the copula determine the shape of this joint distribution. Following Heller and Vause (2012), we assume a 't' distributed copula, a generalisation of that based on the normal distribution but which places greater weight on their being more extreme correlation of values further from the centre of the distribution.

The third step of the procedure is to simulate values from the fitted joint distribution. This amounts to picking draws from this joint distribution of member exposures, and is achieved simply by drawing uniformly distributed random numbers on the interval  $[0, 1]$ , and passing them through the inverse of the joint distribution function (see Embrechts, Lindskog and McNeil (2003) for details).

(1) This choice of the 90th percentile is somewhat arbitrary. 'Attaching' the Generalised Pareto distribution to a higher percentile allows the tail of the distribution to be captured more faithfully, but at the cost of fitting the distribution to fewer data points; conversely, a lower choice of percentile increased the available number of data points but captures less information as to the most extreme values. The choice of the 90th percentile could be varied were this methodology to be operationalised; though the results presented here did not appear sensitive to this assumption.

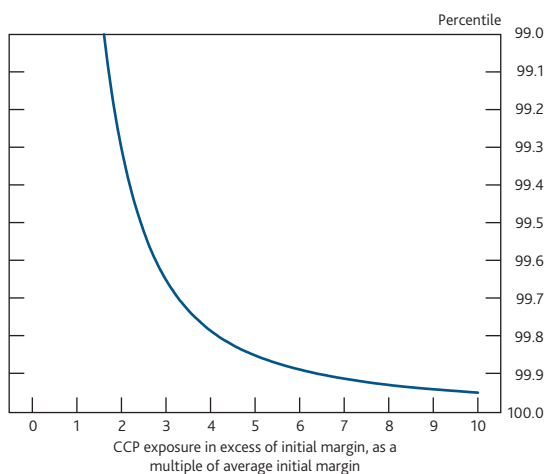
**Chart 4** The distribution of the CCP’s two largest member exposures — in excess of those members’ initial margin — over a one-day period of risk



Source: Bank of England calculations.

The right-hand tail of the distribution in **Chart 4** offers an insight into the severest exposures to which the CCP is subjected, and hence the adequacy of its total default resources. This is illustrated in **Chart 5**, which shows the distribution of the largest 1% of approximate exposures in the same distribution as in **Chart 4**. With knowledge of the CCP’s total default resources, it would also be possible to estimate the likelihood of this exposure being sufficient to exceed both the default fund and the CCP’s equity, thus meaning that losses would fall on participants’ trade exposures to CCP.

**Chart 5** The largest 1% of approximate exposures in excess of initial margin in the distribution shown in **Chart 4**



Source: Bank of England calculations.

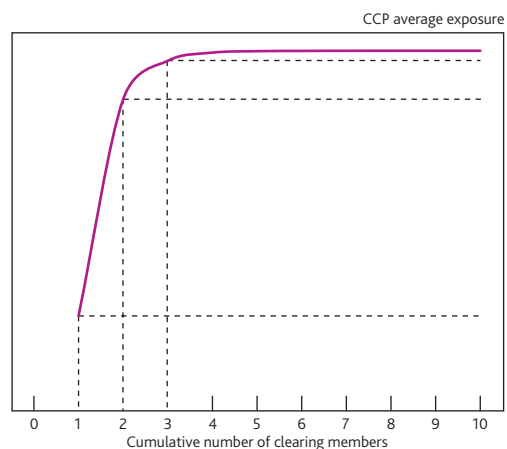
Note that this approach implicitly assumes that the entirety of the CCP’s exposure to the defaulting member is ‘closed out’ on a single day. In reality, it may be that the defaulting member’s position is ‘closed out’ over a longer time frame. Indeed, standards require CCPs clearing OTC derivatives to assume a

close-out period of at least five days. Were this methodology to be operationalised, it would be reasonably straightforward to adapt the current approach to allow close-out periods of any length.

These results are informative as to the relative allocation of risk between the default fund and the residual possibility that losses fall on the CCP or its members through losses on their trade exposure to the CCP. The CCP will typically be insolvent once the entirety of its default resources, including initial margin, default fund and its own equity have been exhausted (unless it has rules that allocate losses across participants, for example, through variation margin haircutting). The distributions in **Charts 4** and **5** could also provide information on the relative risk borne by members on these trade exposures.

Finally, this methodology can potentially also be used to assess the appropriateness of the cover-1 or cover-2 requirement, as a metric for default fund adequacy. This is because it allows the CCP’s exposure to be calculated across any number of members, not just those giving rise to the one or two largest exposures respectively. To this end, **Chart 6** shows the approximate exposure of the CCP, in excess of initial margin, across different numbers of members, averaged across simulations — a ‘cover *n*’ requirement, as it were. In our hypothetical data, exposure is very concentrated within the largest clearing members: the marginal contribution of the next largest exposure is sharply diminishing in the number of members. Indeed, the additional exposure captured by taking the top three members as opposed to the top two members is small. Almost all of the exposure is captured by looking across the largest four members.

**Chart 6** The CCP’s approximate exposure across its largest ‘*n*’ member exposures



Source: Bank of England calculations.

Like any model-based approach, this methodology is subject to a number of caveats. In particular, there may be considerable uncertainty around the results unless a long time

series of data is available to calibrate extreme tail events. Results are also sensitive to the parameter estimates used in the calibration of the Generalised Pareto functional form applied to the shape of each distribution of changes in exposures — rendering the results vulnerable to possible model error. Were the model to be used as an operational risk assessment tool by CCPs, these shortcomings would need to be addressed. However, the approach does yield a methodology capable of assessing the risk of extreme events of which there is, by definition, limited historical experience.

### 3 Adding an estimate of the risk of clearing member default

The previous section offered a model of a CCP’s exposures to its members, its statistical distribution, and an estimation of the ‘tail risk’ of this exposure exceeding the CCP’s default resources. Such a risk would only crystallise — and the CCP would only incur loss — in the case that not only did the CCP experience such an exposure, *but that the members to whom the CCP was so exposed also defaulted*. Put another way, large exposures do not in themselves lead to a CCP’s inability to meet its obligations to its participants, unless they are accompanied by actual member default.

A more comprehensive view of the adequacy of a CCP’s default resources, and the risk to which they are exposed, can therefore be constructed by making an assessment of the likelihood of member default. In doing so, it is crucial to account for the *correlation* between clearing member defaults. This is because it is likely that only the default of multiple clearing members will generate losses of a magnitude sufficient to exhaust the CCP’s default fund. But, like the extreme changes in clearing members’ exposures described in Section 2, the risk of multiple clearing member defaults is hard to quantify, since it has so rarely crystallised.

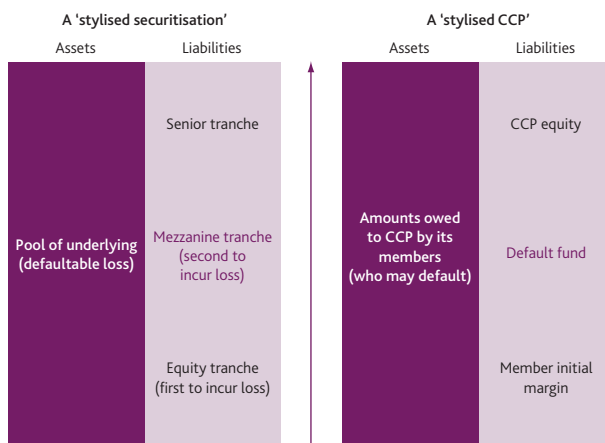
One way to estimate the correlation between bank defaults is to use the information contained in the premia payable on structured credit products. These are financial securities created by collecting a portfolio of defaultable assets — such as mortgages, corporate bonds or derivatives (such as credit default swaps) that replicate their payoffs — and issuing multiple ‘tranches’ of security that represent claims of different seniorities against these portfolios. The relative seniority of a tranche determines the order in which it accrues losses incurred by the default of the underlying credits. If any of the assets in the portfolio default during the life of the structured credit product, the resulting losses accrue first to the junior tranches, and then to the mezzanine and senior tranches only if losses reach a sufficient magnitude. The probability and correlation with which firms default therefore determine the relative value of structured credit tranches, and the premia that must be paid to investors for bearing the risk on the relevant tranche.

The premia paid on different tranches of structured credit products therefore yield information as to market participants’ perceptions of the nature of the underlying risk of default. These tranche premia reflect not only the probability of each underlying asset defaulting, but also perceptions as to the correlation of these defaults, as it is only when a large number of firms default together that the more senior tranches will bear loss.

The approach used here is to calibrate a model of default correlation to the prices of tranches of structured credit, infer the correlation of underlying default, and assume the default of clearing members is correlated in the same way. This model is taken from Noss (2010). A summary is contained in Box 2.

To further illustrate the approach, note the (partial) analogy, first drawn by Murphy (2012), between a CCP’s initial margin and default fund, and the junior and mezzanine tranches of a securitisation. In the same way that the mezzanine tranche of the securitisation pays the bearer a premium in return for their facing the risk of defaults on the underlying pool of credits reaching a given magnitude, a CCP’s default fund protects against the risk of the amounts owed to the CCP by clearing members exceeding their initial margin. This analogy is illustrated in Figure 2.<sup>(1)</sup>

**Figure 2** A simple analogy: the risk faced by a CCP’s default fund is similar to that borne by mezzanine tranche of a structured credit securitisation



The model is calibrated to the premia paid on securitisation tranches taken from the iTraxx index, a set of structured credit instruments based on credit default swaps written on the debt of 125 European firms.<sup>(2)</sup> The index consists firstly of a ‘main index’ that tracks the average credit default swap premia on the underlying firms. This enables the calculation of

(1) This analogy is, however, imperfect. For example, a CCP members’ initial margin only absorbs losses of the defaulting member whereas the equity tranche of a securitisation absorbs losses from any of the underlying defaultable assets.  
 (2) For a general introduction to the index and its technicalities, see Markit (2008).

**Box 2**

**Using the premia payable on tranches of structured credit instruments to estimate the correlation of default of underlying firms**

This box describes the methodology used in Section 3 to estimate the nature and magnitude of the default risk posed by firms underlying structured credit indices. Properly modelling the distribution of defaults and, in particular, their codependence and hence correlation, is crucial to fitting the traded values of structured credit tranches. Full details are contained in Noss (2010).

Early attempts to model codependence of defaults used a 'Gaussian Copula model', based on the normal, distribution, to attempt to capture the correlation between firms' defaults.<sup>(1)</sup> This has the advantage of simplicity, and represents the structure of this interdependence in a single correlation parameter. However, it has the significant drawback of giving insufficient weight to the 'tail event' of multiple firms defaulting together.

The framework used here is instead based on a 'gamma distribution' — a statistical distribution that is a generalisation of the normal distribution, but which places more weight on adverse tail events. The model also includes the possibility of a 'catastrophe' state that allows the extreme codependence of default that goes beyond that found under any standard statistical distribution. By doing so, it is able to capture the possibility of extreme dependence between defaults and therefore more successful in matching the traded premia of structured credit products.

The approach consists of allowing the default of each firm to be determined by a 'state variable',  $X_i$ , where  $i$  varies across firms. Each firm defaults if  $X_i$  is less than some constant  $\theta$ , equivalent to the firm's probability of default (in this case common across all firms and calibrated to the main iTraxx index).

Each  $X_i$  is the sum of the two terms. The first is a 'global' term, which is common to all firms and determines their risk of simultaneous default. The second is an idiosyncratic shock that affects only that firm. The balance in the magnitudes of the two terms affects the degree to which defaults are correlated, with greater weight on the global term making defaults more correlated, as their state variables are more alike.

The value of each firm's state variable  $X_i$  is defined as:

$$X_i = \underbrace{K I_{\{c>0\}} + \Gamma_g(\phi y) I_{\{c=0\}}}_{\text{Global term}} + \underbrace{\Gamma_i((1-\phi)y)}_{\text{idiosyncratic term}}$$

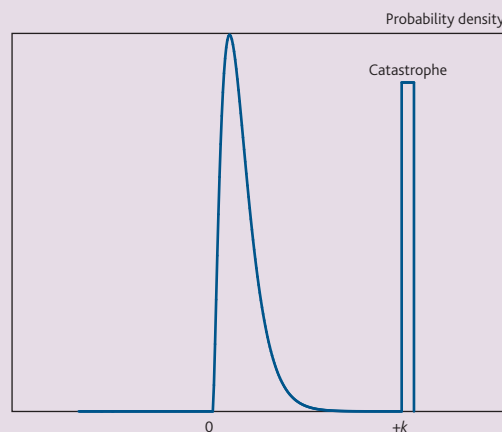
where  $\Gamma_g$  and  $\Gamma_i$  are gamma distributed random variables, with shape parameters  $\phi y$  and  $(1-\phi)y$  respectively. The properties of the gamma distribution mean that the sum of the global and idiosyncratic terms is distributed with shape parameter  $y$  — the sum of its constituents. This parameter can be thought of as controlling for the 'heavy tailedness' of the joint distribution — that is the probability of defaults displaying a higher codependence than that given by the Gaussian distribution. The parameter  $\phi$  is therefore analogous to the correlation parameter in a standard Gaussian model, and controls the linear correlation between defaults.

Parameter  $c$  controls the occurrence of the catastrophe state in which the global factor takes an (arbitrarily) high value,  $K$ . The prevalence of this state is determined via a 'Poisson process',<sup>(2)</sup> which takes integer values greater than or equal to zero with a probability controlled by an 'intensity parameter'  $\lambda$ , with higher values increasing the probability of this state occurring.

The resulting codependence of the set of  $\{X_i\}$  across all firms — and their joint probability distribution — is controlled by three parameters,  $\phi$ ,  $y$  and  $\lambda$ . The distribution of the global factor, common across all firms, is shown in **Chart A**. Note the 'lump' of probability density in the tail of the distribution, which allows for the 'catastrophic' probability of a high number of firms defaulting simultaneously.

For any given choice of parameters, tranche premia are generated using a numerical approach described in Li and Liang (2005). The parameters are then varied to obtain the optimal fit to observed tranche premia.

**Chart A** The probability density of the global factor

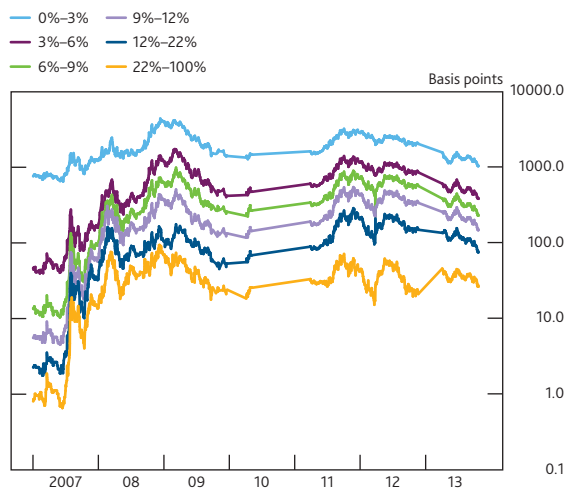


Source: Bank of England calculations.

(1) See, Li (2000).  
 (2) For details, see Cox and Miller (1965).

the average probability of default.<sup>(1)</sup> It also includes a number of tranches, each of which absorb the risk of a certain proportion of the underlying firms defaulting, with these proportions set at 0%–3% for the junior tranche; 3%–7%, 7%–10%, 10%–15% and 15%–30% for the four ‘mezzanine’ tranches, and 30%–100% of firms for the senior tranche. Times series of tranche premia are shown in **Chart 7**.

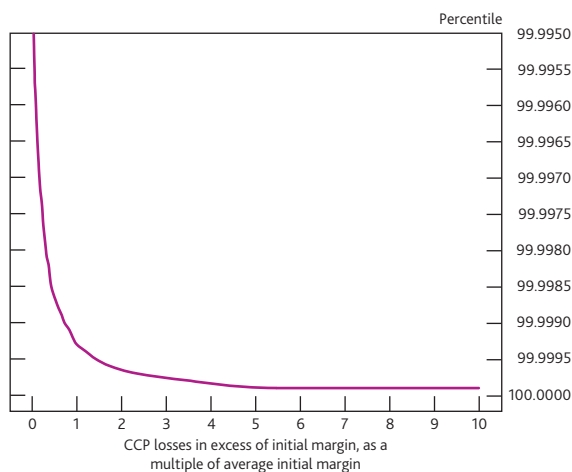
**Chart 7** The premia on the tranches of the iTraxx structured credit index



Source: JPMorgan.

Given this model of default correlation for financial firms, it is possible to simulate the frequency with which different numbers and combinations of clearing members default. Combining this with the indicative distribution of CCP exposures (in Section 2), leads to an estimate of the distribution of total expected losses faced by the CCP across each of the simulated periods of risk. **Chart 8** shows the distribution of total losses, in excess of the defaulting members’ initial margin, over a one-day holding period. Analogous to the results in **Chart 4**, only on the worst

**Chart 8** The distribution of total losses — in excess of members’ initial margin



Source: Bank of England calculations.

0.00663% of days do losses exceed the defaulters’ initial margin based on our indicative exposure data. In addition, if the default fund were equal to members’ average initial margin contributions, the CCP would be expected to exhaust its default fund only once every 550 years.

So, taken at face value, this model suggests that the risk of the CCP’s exposures to its members exceeding its default resources and *actually crystallising into losses*, is, unsurprisingly, degrees of magnitude smaller than the risk of the exposure itself. This is to be expected, given that clearing member defaults are rare.

However, it may also underline an important caveat to the methodology. Currently, the approach implicitly assumes that the distribution of clearing member default is independent of the distribution of clearing member exposures; that is, that default of a given clearing member is equally likely across all the simulated scenarios for the CCP’s exposure to that clearing member. In reality, it is possible that the risk of clearing member default increases in scenarios where the market has moved against them, leading that clearing member to record losses on their cleared portfolio. These are exactly the states of the world in which the CCP’s exposure to its members is larger-than-average. Such ‘wrong-way risk’ is well documented in derivatives markets,<sup>(2)</sup> and has the effect of increasing the risk that larger CCP exposures crystallise into losses. The risk of losses in excess of default resources calculated here is likely therefore to be an underestimate of that which would prevail in the presence of wrong-way risk.

Other shortcomings of the methodology come through its use of the traded market prices of structured credit products to draw inference as to the nature of the default risk of the CCP’s clearing members. First, the nature of firm default represented by the iTraxx structured credit index may not match that of the CCP’s members. This is partly because the 125 members of the index differ from the CCP’s members. And, this problem notwithstanding, the probability and correlation of default inferred from the iTraxx may not provide a faithful representation of that pertaining even to its underlying firms. For example, there are a variety of other factors, such as market liquidity, which can distort the prices of such financial instruments, meaning that their prices are an unreliable guide to their future payoffs. Such liquidity effects were a particular problem during the recent financial crisis, when uncertainty regarding the future of securitisation caused structured credit products to trade at a significant discount to their ‘fair value’ implied by the prospects of their underlying firms (see Noss (2010)). In addition, expectations of default estimated from derivatives in this way will also be those of a ‘risk neutral’ investor. In the likely case that investors are averse to risk, the

(1) For details see, for example, Hull (2005).

(2) See, for example, Gregory (2010).

market-implied probability of 'bad states of the world' — that is correlated defaults — materialising, will be overstated. These problems combine to mean that the probability and correlation of defaults derived from these data may well overstate the true probability of default.

But, like the model of exposure in Section 2, these caveats may be an acceptable price to pay for a model that estimates the risk of a phenomenon observed as rarely as the simultaneous default of clearing members. This structured credit index offers at least some insight into the nature of this particular tail risk and, despite its imperfections, may be a useful proxy for the risk of multiple clearing member default.

## 4 Conclusion

This paper presents a methodology for a 'top-down' statistical assessment of the risk of a CCPs' member exposures. At its core lies a model based on extreme value theory that aims to estimate the risk of large changes in member exposure, of the sort that pose risk to the adequacy of the CCP's default resources. This is illustrated using indicative data on a CCP's member exposures. The power of the approach lies in its ability to offer a summary 'ready reckoner' of the risk of exposures exceeding a CCP's initial margin and default fund and the adequacy of these default resources. It may also have implications for CCP regulators' judgement as to the relative risk faced by clearing members on their default fund contributions and their trade exposures to CCPs.

It is not without caveats, some of which could be partially overcome in future work to operationalise the methodology. First, like any model-based approach, its results are sensitive to its underlying parameter estimates — both those governing the distribution of exposures, and the model of default used to extract the probability of concurrent clearing member default

from the traded prices of structured credit instruments. These are calibrated to a (necessarily) finite sample of data, and, in the latter case, are compounded by the usual problems faced when extracting information from financial market prices, including that of distortions caused by premia for investor risk aversion and market illiquidity.

Second, another issue is the sensitivity of the results to the parameters of the chosen Generalised Pareto distribution and copula that governs the distribution of clearing member defaults. An important robustness check would be to examine the stability of this calibration over time. Alternatively, conservative versions of these parameters could be chosen. For example, the parameters underlying the Generalised Pareto distribution of extreme changes in exposures, or the correlations embodied in the copula that are calibrated to the data, could be 'shocked', and the corresponding effect on the results — such as the frequency with which default resources are exhausted — measured. A high sensitivity to such parameters would alert users to the dependence of the results on potentially inaccurate calibrations. In addition, such an exercise could also serve as a form of 'top-down' stress testing, allowing regulators to examine the resilience of the CCP in the face of changing market conditions.

Further work could also seek to address the issue of wrong-way risk arising from the risk that the CCP's exposure to its members is positively correlated with their risk of default. Accounting for wrong-way risk is compounded by the limited history of clearing member defaults, and hence limited data as to how exposures are correlated with the deterioration in clearing member credit worthiness. However, a growing literature offers methodologies through which the behaviour of exposures can be adjusted to reflect the risk of counterparty default (see, for example, Levy and Levin (1999)), and could be incorporated into further work.

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