BANK OF ENGLAND

Financial Stability Paper No. 19 – April 2013

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We thank David Barr, Paul Bedford, Matthew Dive, Louise Carter, Edwin Schooling Latter and Graham Young from the Bank of England, Richard Spooner from the Financial Services Authority and Thomas Nellen from the Swiss National Bank for their helpful comments. Any remaining errors are our own.

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The views expressed in this paper are those of the authors, and are not necessarily those of the Bank of England. This paper was finalised on 25 April 2013.

ISSN 1754–4262

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Central counterparties and their financial resources — a numerical approach

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New regulatory standards require central counterparties (CCPs) to have robust processes in place to mitigate their counterparty credit risk exposures. At the same time, the standards allow CCPs to tailor their risk management models. This paper considers how CCPs can optimally determine the relative mix of initial margin and default fund contributions in a stylised setting, by balancing the costs of default resources with the expected losses they protect against. Where members are of good credit quality and the probability of experiencing losses is low, the loss-mutualising properties of the default fund are favoured over the defaulter-pays properties of initial margin. Significant tail risks in the markets cleared by the CCP further favour the use of the default fund as a cost-effective insurance against potentially large losses. By contrast, when members are more likely to default or extreme losses are unlikely, the CCP has incentives to maximise the defaulter-pays collateral it takes, and the benefits of the loss-mutualising default fund are reduced. Our numerical results support the recognition that CCPs should have some discretion over how they set the optimal level and composition of their default resources, based on the specific risks of the markets and portfolios that they clear. Our results also show that changes in collateral costs and capital requirements can have a significant impact on a CCP's optimal risk management choices.

Introduction

Central counterparties (CCPs) play a key role in the financial system. They interpose themselves between counterparties to financial market trades, becoming the buyer to every seller and the seller to every buyer (a process called 'novation'). By acting as central counterparty, a CCP can significantly reduce participants' counterparty credit risk through multilateral netting of trades.

Through novation, the CCP takes on counterparty credit risk to its members. Should one of those members default, the CCP will find itself with an unbalanced set of positions or obligations, which it will either need to transfer to other, healthy members, or 'close out' by buying hedging positions in the market and either retaining them to expiry or effectively cancelling the trades. This process may carry a 'replacement cost' if the market moves against the CCP.

In order to mitigate the risks that arise from novating trades between clearing members, CCPs typically rely on both a set of rules (defining membership criteria and default management procedures) and financial resources, including variation margin (VM), initial margin (IM) and default fund (DF) contributions. VM is generally cash paid to or received from members daily to offset profits or losses on their mark-to-market exposures; this prevents large exposures between the CCP and its members from building up. IM is collateral lodged by a clearing member to protect the CCP against potential losses should that member default in normal market conditions (at which point it would stop making VM payments). All members generally also contribute to a mutualised default fund, which acts as an extra loss-absorption mechanism should one or more members default in 'extreme but plausible' market conditions. In this case, the CCP could be exposed to losses greater than those offset by the defaulting members' IM. This paper considers how CCPs can optimally determine the relative mix of IM and DF contributions in a stylised setting.

The paper is organised as follows. Section 1 summarises the paper's results and its contribution to the debate on CCP risk management. Section 2 describes a simple quantitative model for determining the optimal allocation of a CCP's default resources, and explains how our stylised model abstracts from several factors that would affect the expected losses that might fall on a real CCP and its members. Section 3 presents the model's key findings. Section 4 presents the results of an extension to the model in which the tail risks faced by the CCP are increased. Section 5 concludes. Technical details are given in the annexes.

1 An overview of the paper

Central clearing through CCPs has expanded in recent years, driven by regulatory changes and with the active support of market participants.(1) At their 2009 Pittsburgh Summit the G20 leaders stated that 'all standardized OTC derivatives contracts should be traded on exchanges or electronic trading platforms, where appropriate, and cleared through central counterparties by end-2012 at the latest.'(2) Since 2006, the volume of centrally cleared interest rate swaps, as measured by outstanding notional, has increased from US$25 trillion to over US$150 trillion.(3) Outstanding notional in centrally cleared credit default swaps has risen from zero in 2008 to US$2.6 trillion in June 2012.(4) Membership of CCPs meanwhile has risen in recent years, with one large UK CCP's membership increasing from 117 members in 2008 to 168 in March 2013.(5) Given their vital and growing role in the financial system, it is critical that CCPs have adequate financial resources.

CCPs are highly regulated entities, and supervisors regularly assess the adequacy of their resources. As a minimum, CCPs are expected to adhere to international standards, in the form of the CPSS-IOSCO Principles for financial market infrastructures (PFMIs) (see Section 5 for more detail).(6) Consistent with the PFMIs, European legislation (EMIR) sets minimum standards for IM, and requires that a systemically important CCP's total pre-funded available financial resources should be sufficient to 'enable the CCP to withstand the default of at least the two clearing members to which it has the largest exposures under extreme but plausible market conditions.'(7) Thus, regulatory requirements for CCP risk management affect the choice between IM and DF.

In addition, capital requirements for the users of a CCP could have an indirect impact on the CCP's resource allocation. Until recently, banking supervisors applied a zero risk weight to collateral lodged with CCPs in all forms.(8) Henceforth, the Basel rules on risk weights applied to banks' exposures to CCPs require that IM lodged at major CCPs will be subject to a risk weight of 2%, equating to a 0.16% capital charge,(9) while a non-zero risk weight will apply to DF contributions.(10) As such,

(1) The International Swaps and Derivatives Association (ISDA) and industry participants, for example, have committed to global supervisors to improve over the counter (OTC) market infrastructure and reduce risk, in part through expanding central clearing.
(2) www.g20.utoronto.ca/2009/2009communique0925.html.
(3) 'Highlights of the BIS international statistics', *BIS Quarterly Review*, June 2011, available at www.bis.org/publ/qtrpdf/r_qt1106b.pdf; www.lchclearnet.com/swaps/volumes/. Outstanding amounts are halved for CCPs to represent what the corresponding bilateral outstanding notional would be.
(4) 'OTC derivatives statistics at end-June 2012', BIS, available at www.bis.org/publ/otc_hy1211.pdf. Outstanding amounts are halved for CCPs to represent what the corresponding bilateral outstanding notional would be.
(5) Membership data from the ECB Statistical Data Warehouse and www.lchclearnet.com.
(6) 'Principles for financial market infrastructures', CPSS-IOSCO, available at www.bis.org/publ/cpss101a.pdf.
(7) Regulation (EU) No. 648/2012 of the European Parliament and of the Council, of 4 July 2012, on OTC derivatives, central counterparties and trade repositories.
(8) 'International convergence of capital measurement and capital standards', Basel Committee on Banking Supervision, available at www.bis.org/publ/bcbs128.pdf.
(9) If the collateral is held at a third party and thus cannot be lost in the event of a CCP insolvency, it attracts no capital charge.
(10) 'Capital requirements for bank exposures to central counterparties', Basel Committee on Banking Supervision, July 2012, available at www.bis.org/publ/bcbs227.pdf.

regulation may alter CCPs' incentives in setting the balance of collateral between IM and DF because of the different costs they impose on clearing members. Clearly, requiring banks to hold capital against their exposures to CCPs will improve the resilience of those banks (by design), which in turn makes it less likely that the CCP's members will default; this benefit of capital charges is not considered in our model.

The regulatory standards described above will determine the minimum loss-absorbing capacity of a CCP's total financial resources. They do however allow CCPs some discretion in tailoring their risk management choices. In particular, they enable CCPs to determine the balance between IM and DF that suits their risk appetite, providing IM is sufficient to meet the 99% confidence level requirement (Box 1). Relative reliance on IM and DF is summarised for several major CCPs in **Table A**.

Table A Total IM and DF held at several major CCPs at end-2011

CCP	Total initial margin	Total DF size
Eurex	€50.5 billion	€1.1 billion
ICE Clear Europe	US$13.7 billion	US$2.9 billion
ICE Clear US	US$976 million	US$48 million
ICE Clear Credit	US$8.6 billion	US$5.3 billion
CME Group	US$92.5 billion	US$4.5 billion

Sources: CME Group *Annual Report 2011*, Eurex Clearing *Annual Report 2011* and ICE *Annual Report 2011*.

This paper does not seek to determine the actual level of financial resources that CCPs would choose (as these are in large part determined by regulatory requirements). Rather, it discusses why it is appropriate that CCPs have some discretion over how they set the optimal composition of their default resources, based on the specific risks of the markets and portfolios that they clear, and how these risk characteristics affect the level of total resources that the CCP will collect. To do this, we introduce a model that investigates the impact of a number of factors on a CCP's optimal choice of resources, in the absence of regulatory requirements.

The paper finds that a mutualised DF provides better protection to the CCP when the probability of a member default is low and/or asset price volatility is high. In contrast, IM is the preferred resource when member default probability is high and/or asset price volatility is low. The model also shows how changing capital charges on banks' exposures to a CCP shifts the optimal allocation towards the collateral type with the lower cost. IM is particularly disincentivised when it is charged at a higher rate than DF contributions (a scenario that will not occur in reality). Finally, the simulation results show that an increase in the opportunity cost of collateral leads to greater reliance on DF contributions, as might be expected.

The paper makes a contribution to the small, but growing academic literature on central counterparty risk management. Early papers focused on the margining problem (eg Knott and Mills (2002) and Baer, France and Moser (2004)). Baer, France and Moser (2004) for example model a clearing house that internalises the potential and actual costs to its members of participation, and seeks to minimise these costs. Their model suggests that a clearing house's optimal level of margin falls as the funding cost increases, a result supported by empirical data. Haene and Sturm (2009) consider the optimal balance of margin and default contributions for a user-owned CCP that seeks to maximise the joint utility of its members. They model the optimal choice in terms of the relative cost of collateral and the probability of one member defaulting, subject to an external (regulatory) total resource requirement. This ensures that the CCP will not default on its obligations to non-defaulting participants. They find that under these conditions, establishing a default fund is always optimal, and in some cases the DF is the only resource needed. Carter, Creighton and Manning (2009) consider a similar optimisation approach, but model different ownership structures. Pirrong (2011) provides a survey of CCP risk models and discusses the implications of CCP risk management choices for market liquidity and systemic risk in general. Monnet and Nellen (2012) examine how collateralisation and loss mutualisation affect the incentives to clear either bilaterally or centrally where counterparties have a two-sided limited commitment, ie they may strategically default. They find that loss mutualisation through the default fund becomes more important both as collateral costs increase, and as the volatility of the cleared market increases.

The current paper models the loss function for a representative surviving member, using an approach in which the number of possible member defaults is limited only by the number of members and the total optimal resources are unconstrained by an external requirement. It then minimises the expected costs to such a member of participating in the CCP, by minimising the sum of the representative member's loss function and collateral costs. This minimisation is performed numerically for reasons discussed in Annex 2. Similarly to Haene and Sturm, we consider only the minimisation of collateral costs and expected losses from member defaults, implicitly assuming that members are risk neutral. A key feature of the present paper is that it can derive the optimal size and repartition of IM and DF simultaneously. By not placing external constraints on the total resources held, the model allows us to explore the relative intrinsic risk mitigation properties of IM and DF.

2 Modelling the CCP

This section explains the main features of the numerical simulation model. This requires the following:

Box 1
How a CCP manages its financial resources — a brief overview

As part of their financial resources, CCPs collect both initial margin (IM) and default fund (DF) contributions from their members to cover costs that may arise through a member defaulting.

IM is usually calibrated to absorb the potential losses to the CCP due to adverse price movements arising in 'normal market conditions' when replacing a defaulting member's positions. To calculate IM, CCPs determine the potential future exposures they wish to protect against, based on past price movements over a predetermined time period. The CPSS-IOSCO Principles for financial market infrastructures require that initial margin meets at least 99% of expected price movements against the CCP's position and state that the intended number of days of coverage should reflect the expected time required to liquidate positions, thus factoring in variations in liquidity between different products.

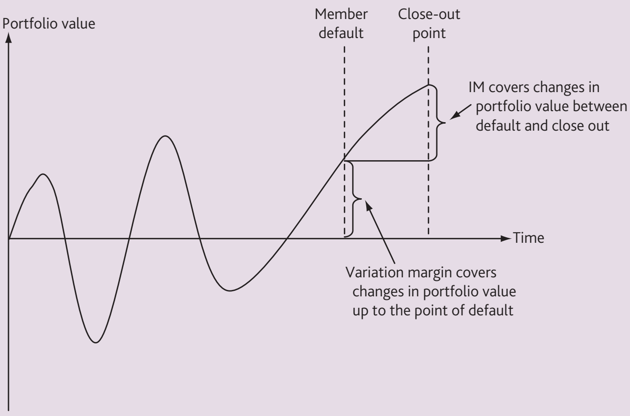
A DF provides the CCP and its members with additional financial resources. It can be used if a participant defaults and the CCP faces higher-than-expected losses, which could be due to large and unforeseen price movements, breakdowns in correlations between assets, or a longer-than-expected liquidation period.

For a given level of protection, initial margin is less collateral efficient than the default fund, since IM can only be used to absorb losses arising from the default of the member posting it and thus has no loss-mutualising capacity. DF on the other hand can absorb losses arising from *any* member's default.

CCPs typically use stress scenarios to estimate the potential additional replacement losses they face — and as a result how much DF collateral is required — in the case of one or more members defaulting, under 'extreme but plausible' market conditions. CCPs run these stress tests on all of their members' portfolios to verify whether their resources are sufficient to cover losses arising under the assumed stress scenarios.

The actual replacement cost to the CCP in the event of member default will depend on the extent to which prices move against its positions. Since CCPs typically request variation margin (VM) to cover changes in the Net Present Value (NPV) of trades on a regular basis, the replacement cost is limited to the price change between the last time variation margin was collected and the point at which the CCP liquidates a defaulting member's position (eg the intraday price change if variation margin is collected daily and the CCP closes out the member's positions on the day of default). This is illustrated in **Figure A**.

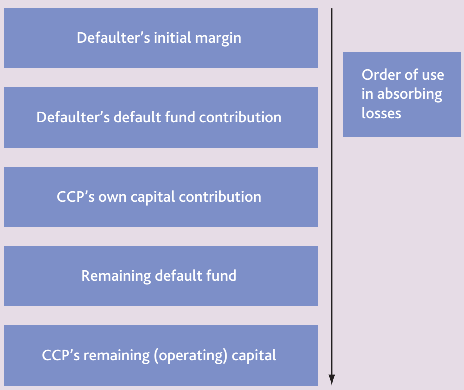
Figure A Illustration of how variation margin and initial margin are used to protect the CCP against replacement costs



When dealing with a member default, a CCP will typically use the defaulting firm's collateral first (its initial margin and default fund contributions). Should these resources not be sufficient, the CCP can then meet further replacement costs by using the surviving firms' default fund contributions, introducing a cost of risk mutualisation to surviving members. CCPs also often have the right to assess non-defaulting members to make additional contributions to the default fund.

Finally, a CCP can use its capital if the default fund is exhausted. The order in which the CCP uses its resources in the event of member default is known as the 'default waterfall'; **Figure B** shows a schematic representation of a typical CCP default waterfall. A CCP can put its own capital higher up in the waterfall, so some of the CCP's own resources are used before the remaining default fund is tapped. This is referred to as 'skin in the game', as it ensures that the CCP has sufficient incentives to calculate conservative margin requirements. It is a legal requirement under EMIR.

Figure B Schematic of a CCP's default waterfall



From a surviving member's perspective, another member's default could lead to several potential costs arising: i) the cost of risk mutualisation through DF contributions; ii) the cost associated with losing equity in the CCP; and iii) any additional costs, for example losses on in-the-money positions at the CCP, arising if the CCP's default resources are insufficient.

Several CCPs have 'loss-allocation' rules which determine how losses that have exhausted the CCP's default resources will be allocated in an orderly manner. The effects of loss-allocation rules on the optimal allocation of default resources are not considered in this paper.

- A description of the CCP, its members, and the market it serves (Section 2.1);
- A description of the default process (Section 2.2);
- A description of members' losses in the case of a CCP default (Section 2.3);
- A description of the relative properties of IM and DF in protecting the CCP and its members from losses (Section 2.4).

An intuitive explanation of the CCP's approach to risk management is given in Box 1, while the technical detail underpinning the model is described in Annexes 1–3.

2.1 Defining the CCP, its members and the market it serves

To model our CCP and the market in which it operates, we have to make a number of simplifying assumptions and define the key parameters that will characterise the costs and benefits associated with the CCP's default resources.

We model a CCP consisting of n direct members, which we assume is owned by those members, who each contribute equity k to the CCP's total capital K. We assume that the CCP has only direct clearing members, ie no members provide clearing services to clients. We model the CCP's capital as the last resource in the default waterfall instead of splitting it into two components — this gives a simpler default waterfall structure than the one depicted in Box 1, and does not materially affect our results. We set this parameter exogenously, reflecting the fact that CCPs' equity contributions to the default waterfall are generally constant (in contrast to IM and DF).

Members post collateral to the CCP in the form of IM and DF; this collateral carries a fractional opportunity cost, $c > 0$, reflecting the lost return on collateral that could be invested elsewhere. We treat c as a fixed percentage of the collateral posted, which is independent of the level of resources required.

We assign all members a probability of default q, which determines the probability that the CCP will face losses.(1) For simplicity, we assume that members have evenly distributed long and short market positions of equal size, on portfolios with an initial notional value of 1. We model an underlying market with Normally distributed symmetric returns with volatility σ, such that for any market move, $n/2$ members will be in-the-money (ITM) and hold a net credit position, and the other $n/2$ will be out-of-the-money (OTM) and hold a net debit position (Box 2). All members thus post the same initial margin y and default fund contribution z. While any member may enter bankruptcy, only OTM members will default on their immediate obligations to the CCP and so we treat ITM members as surviving members in our model.

So the CCP's default resources consist of members' IM, DF and equity contributions. If one or more members default and the CCP's resources are insufficient to absorb the losses, our CCP becomes insolvent. We do not include loss-allocation rules in our model, which could in practice allow the CCP to allocate losses to members in an orderly way and continue operations. We also do not consider the existence of a recovery and resolution regime, which could facilitate an orderly winding down or restructuring of the CCP which could be a less costly alternative to liquidation. Without having loss-allocation rules or a recovery and resolution regime in place, a CCP insolvency could inflict significant disruption on the financial markets, carrying a 'systemic' cost(2) — we therefore include a parameter s which imposes an additional cost on members in situations where the CCP defaults. This cost is a fixed, exogenous one-off loss imposed equally on all members at the point of CCP default. In future, the model could be extended to endogenise this 'systemic cost', perhaps by increasing the likelihood of large price moves after the CCP defaults, reflecting the market disruption that would likely occur in reality.(3)

Finally, we consider the effect of extra costs on members' default resource contributions in the form of regulatory capital charges. These may differ for IM and DF reflecting their different risk characteristics; these costs are characterised by the parameters d_{IM} and d_{DF}, the capital charges on default

(1) In practice, CCPs collect IM against the *loss given default* of each member, and DF against stressed losses *given default* of the one or two members to which the CCP is most exposed. We choose to model the CCP's expected losses across all members as this allows us to investigate the intrinsic risk-mitigating properties of IM and DF, and how these are affected by external variables.
(2) See for example Tucker (2011).
(3) The authors thank Thomas Nellen for this idea.

Box 2
In-the-money and out-of-the-money members

At any point in time, a member's position *vis à vis* the CCP may be in-the-money (ITM) or out-of-the money (OTM):

- ITM: the market value has moved such that the CCP owes the member money.
- OTM: the market value has moved such that the member owes the CCP money.

The CCP pays and receives variation margin to and from members daily in order to reduce mark-to-market exposures to zero; essentially the CCP distributes money from OTM to ITM members. If an OTM member defaults, the CCP still needs to meet payment obligations to members with ITM positions, but no longer receives the full amount from OTM members; the shortfall will be funded by the defaulter's initial margin and default fund contribution, and then the mutualised default fund and CCP equity if necessary (Box 1).

When the CCP's default fund resources and equity are exhausted in our model, it can no longer meet its obligations to members with ITM positions. In that case, ITM members will face losses defined by a) the number of surviving OTM members who remain to fulfil their economic obligations; and b) the magnitude of the ITM members' positions. The larger the ITM members' positions, the higher their losses due to their counterparty defaulting.

In contrast, non-defaulting OTM members' losses extend only to their default fund contributions; their obligations to the CCP are extinguished upon the CCP's default.

resources, and c_C, the cost to banks of holding regulatory capital.

In Section 4 we extend the model to consider the effects of using a simple price distribution with 'fat tails', in which the probability of large price movements is greater than those predicted by Normally distributed returns. Comparison between results for the Normal and fat-tailed price distributions can provide an insight into how different price behaviour in different markets will affect the optimal allocation of resources.

2.2 The default process
The primary losses resulting from a member default arise from the replacement cost to the CCP of hedging, transferring or closing out its exposed positions. It is these losses that the CCP's default resources are designed to absorb. As explained in **Figure B** in Box 1, the defaulter's own IM and DF will absorb losses first; any further losses will fall on the mutualised default fund and the CCP's capital. In our model, the CCP's capital is the final loss-absorbing resource, and once it is exhausted the CCP itself defaults.

Since we do not include loss-allocation rules, once the CCP defaults in our model, we assume that a liquidator steps in and closes out all open positions in the market. It then transfers funds from surviving OTM members to ITM members *pro rata*, such that all ITM members make a loss depending on the number of OTM members who have defaulted. We assume that surviving OTM members will fulfil their obligations in full, but that liquidation may carry a cost $a \geq 0$. This means that ITM members will face losses beyond those on their DF contributions should the CCP default. This is discussed more fully in Box 2.

The potential losses to the members are thus a function of the probability of members defaulting, the volatility of the underlying market in which the CCP operates, and the quantity and type of collateral that the CCP has collected from its members to absorb these losses. We assume that losses will not fall on surviving members' initial margin, as the symmetry in member positions means that ITM exposures will be large enough to absorb all losses.

In considering the costs of default to the CCP and its members arising from how the CCP manages member defaults, we make an important assumption that the CCP will want to minimise costs for surviving members rather than all members; and that members will *ex ante* assume their own survival in weighing up the costs and benefits of posting default resources. This leads us to introduce a 'cost of mutualisation', which essentially represents a preference of members to accept the 50% probability (*ex ante*) that they will have ITM positions and thus lose money if the CCP defaults (and 50% that they will have OTM positions and thus lose no money), rather than accepting with 100% probability the losses that would fall on their default fund contribution for the same default scenario. In other words, members acknowledge the possibility of losses arising through their participation in the CCP, and seek to minimise these losses *ex ante*.

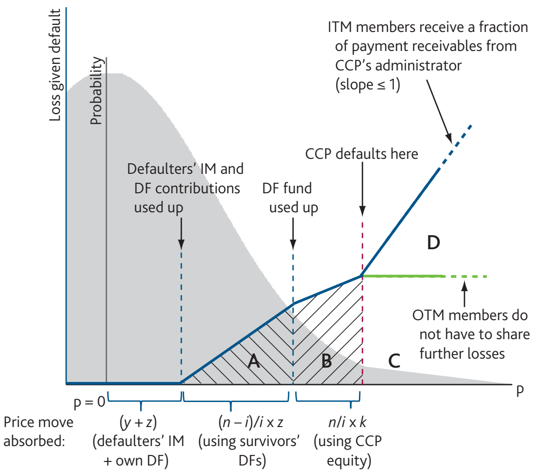
In aggregate of course, mutualisation carries no cost, as it simply redistributes losses from ITM members and defaulting OTM members to surviving OTM members; but in reality, CCP members are unwilling to accept full risk mutualisation due to concerns over moral hazard and the uncertain liabilities that mutualisation entails. Without this perceived cost, it would be optimal for CCPs to hold all their default resources in a mutualised default fund, due to its greater loss-absorbing power relative to initial margin. This 'cost' is further discussed in Section 2.4 and Annex 1.

2.3 Understanding members' losses

We now describe the losses borne by individual members in more detail. Annex 2 provides the mathematical detail on the loss functions for OTM and ITM members, respectively. The remainder of this section gives an intuitive explanation.

Figure 1 below shows how both price changes and member defaults affect surviving members' losses as a function of their IM and DF contributions.

Figure 1 Potential loss of OTM and ITM members for different price moves (p) and number of defaulters (i), showing how IM and DF absorb losses



The sequence of events is as follows:

- Price moves of up to $(y + z)$ will be absorbed by the defaulting members' own IM and DF contributions.
- Next, the DF contributions of surviving members will be used. There are $(n - i)$ survivors each contributing z to the DF, which has to absorb losses split between i defaulting members; so the maximum loss that can be absorbed by the survivors' default fund contributions is for an additional price move indicated by area A in the figure.
- Once the DF is exhausted, all members' equity contributions are used up to absorb losses from the i defaulting members, as indicated by area B in the figure.
- Beyond this point, all of the CCP's default resources have been exhausted and further losses will fall directly on surviving members.
- So OTM members bear no further losses (C) while ITM members continue to face higher losses as price increases further (D). ITM members' losses can be at most the entire gain in the value of their portfolio (ie a slope of 1 in the figure); any payments received from surviving OTM members will reduce the rate at which ITM members bear further losses as the price continues to increase (ie a slope of less than 1 in the figure).

2.4 The loss-absorbing mechanisms of IM and DF

In considering the optimal allocation of default resources between IM and DF, it is useful to describe how they respectively absorb losses, and distribute those losses among ITM and OTM members. We do this by considering the two extreme cases in which the CCP collects a set amount of resources as *either* IM or DF (with the same collateral costs attached to each), under the simplifying assumption that $a = 0$, $k = 0$, and $s = 0$ (such that OTM members face no losses after the DF is exhausted, and ITM members' losses are no greater than the losses on payments due from OTM defaulters). In the next section, we will relax these assumptions.

Figure 2 shows the expected losses experienced by surviving members when the **CCP collects only IM**. When members default in this scenario, surviving members will suffer no losses as long as the cost to the CCP of replacing the defaulters' positions is smaller than the defaulting members' IM. Once the losses exceed IM however, the CCP will default. Beyond this point, ITM members suffer losses for the reasons discussed above. The expected loss for OTM members in this scenario is always zero.

Figure 2 Loss given default where the CCP collects only IM

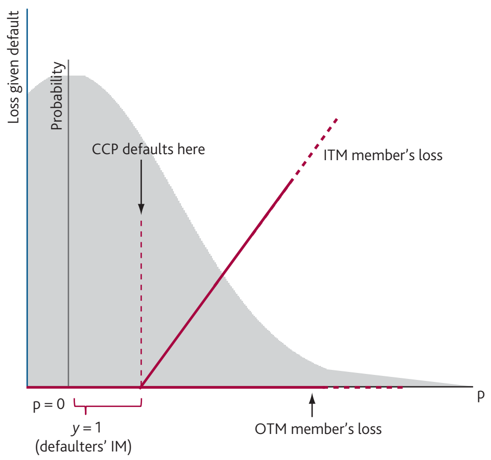


Figure 3 shows the alternative scenario where the CCP collects the same amount of resources per member in the form of **DF contributions only**. Now, the defaulting members' own contributions to the DF are initially used by the CCP to meet replacement costs, and so provide a similar function to IM. But once the defaulting members' DF contributions are exhausted, the CCP will meet further losses by using the surviving members' DF contributions. The CCP does not default until the surviving members' default fund contributions

are also insufficient to absorb the losses of the *i* defaulting members. Both ITM and OTM members suffer losses when DF is used as a default resource. As before ITM members' losses increase as the price rises, while OTM members' losses are capped at their individual DF contribution.

Figure 3 Loss given default where the CCP collects only DF

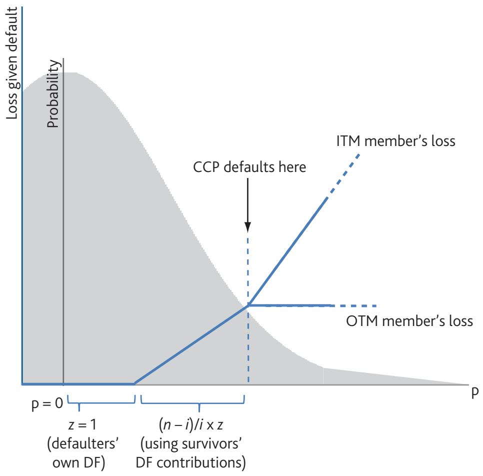


Figure 4 combines the two scenarios, and shows that with IM alone, the CCP goes bankrupt when the failed members' IM contributions are exhausted. With DF contributions, the CCP does not default until DF posted by all members is exhausted. So DF contributions are better at protecting the CCP against insolvency, which is clearly shown by **Figure 4**.

Figure 4 Comparison of the loss given defaults for the collection of only DF or IM

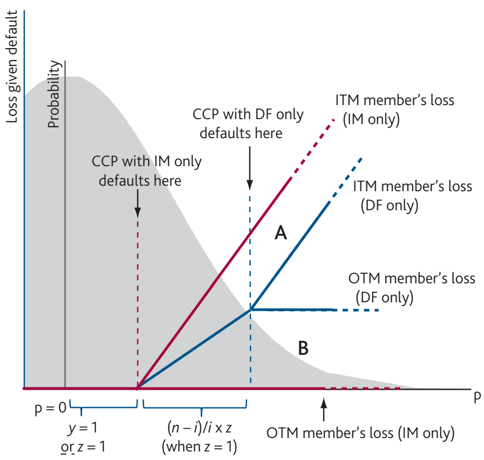


Figure 4 also shows that cost mutualisation benefits ITM members since it reduces their losses relative to the case where only IM is collected (area A). But it is more costly for OTM members since they now face a positive cost of mutualisation (area B). In other words, DF contributions facilitate income transfer from OTM members to ITM members.

Still staying with **Figure 4**, the additional cost of mutualisation to each surviving OTM member is larger than the additional benefit of mutualisation to each ITM member, relative to the case where only IM is collected (ie area B will be larger than area A for a given price move). This is because in our model, given one or more OTM members defaulting, the number of ITM members must be larger than the number of surviving OTM members (since we start with an equal number of each).

So comparing the simplified scenarios considered in this section, it follows that IM will be marginally preferred by every member as a protection against another member default. This follows from the fact that *ex ante* each member has an equal probability of being ITM or OTM (as discussed in Section 2.2), and from our cost assumptions. Once we relax the assumption that OTM members face no further losses after the DF is exhausted, and that there is no extra administration cost of default to ITM members, this preference for IM no longer always holds.

It is easy to see why additional systemic costs or losses of equity will incentivise the CCP to allocate resources to the default fund, as CCP default is then costly for all members. To see why the *administration* cost is important, consider its effect on the losses in **Figure 4**. A non-zero administration cost on VM payments from surviving OTM to ITM members would increase the rate of losses for ITM members as the price increases post-CCP default. The size of area A relative to area B would also increase, making mutualisation more beneficial to ITM members. When the administration cost increases above a certain level, the benefit to ITM members of mutualisation will become greater than the cost of mutualisation to OTM members, and the optimal allocation of default resources will begin to include DF contributions.

2.5 Abstractions from a real-life CCP

We conclude this section by briefly explaining how the CCP in our model differs from a real-life CCP. The main abstractions we make are the following:

- We do not model the possible correlation between the losses made by members on their position at the CCP and their own default probability. In reality, large losses on a given position could trigger a member default. Instead, in our model, we treat a member default as exogenous and examine the effects of a range of default probabilities and values of potential losses on CCP losses. Similarly, the effects of member defaults on market volatility are not included in the model.

- We do not consider loss-allocation rules for distributing losses once the default fund is exhausted; nor do we allow for resolution mechanisms which may facilitate the orderly allocation of losses and any other actions to restructure or wind-down a failed CCP. Related, we implicitly assume that surviving members' losses will not exceed the money owed to them on their ITM positions, so survivors' IM will not generally bear losses. In reality, it is possible for the costs of a member or CCP failure to cause losses beyond those on survivors' market positions. CCP loss-allocation rules are described in detail in Elliott (2013).

- The clearing members in our model do not provide clearing services to clients. Client clearing will naturally affect the incentives and costs of the CCP and its participants, for example because clients do not contribute to the CCP default fund. Our stylised approach in which our CCP is owned by homogeneous users is not able to account for such effects.

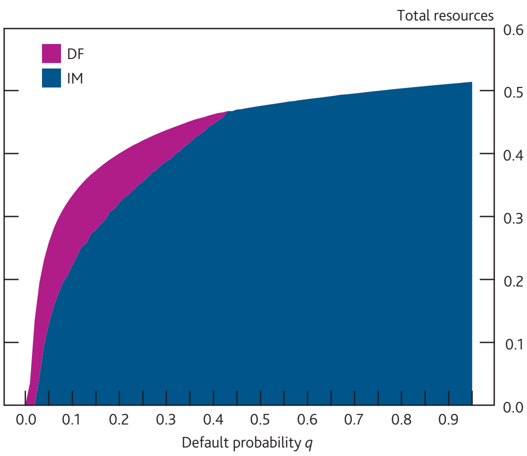
3 Numerical results — main findings

In the next section, we optimally determine the balance between IM and DF for the full range of parameters. We consider the effects of varying six parameters within ranges detailed in Annex 3.

Simulation 1: effect of the probability of member default on the optimal allocation

Figure 5 depicts the effect of varying members' probability of default q while maintaining other parameters constant. The amount of resources is given relative to the initial portfolio value of 1 (so an amount of resources of 0.5 units is equal to a margin and default fund contribution of 50% of the initial portfolio value). The key message is an intuitive one: as the risk of members defaulting increases, our model CCP will manage the risk via initial margin rather than default fund contributions.

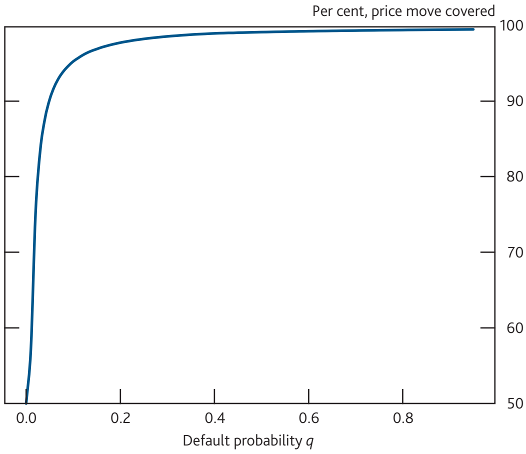
Figure 5 Impact of increasing probability of member default on the optimal size and allocation of default resources



This result arises because as it becomes more likely that members default, the cost of mutualisation for surviving members increases, as explained in Sections 2.3 and 2.4. On the other hand, when q is small, DF is preferred to IM because the cost of mutualisation is relatively small compared with the cost of the CCP becoming insolvent and the reduction in collateral costs achieved through using DF.

Note that the size of optimal total CCP resources increases rapidly as q increases from small values. It is trivial that the optimal level of CCP resources is zero if members are risk-free entities. But once the members become even only slightly risky, the optimal size quickly increases. On the other hand, the optimal level of total resources is nearly flat when q is large, as the marginal benefit from holding more collateral becomes comparable to the marginal cost. This is illustrated by **Figure 6**, which shows that for default probabilities above approximately 40%, the optimal level of IM already provides coverage of price moves to a 99% confidence level, beyond which further marginal benefits are relatively small.(1)

Figure 6 Corresponding coverage percentile of defaulter-pays resources (ie member's IM + DF contribution)



Simulation 2: effect of the volatility of the portfolio on the optimal allocation

In our model, the volatility of the portfolio represents the market risk managed by the CCP. **Figures 7a** and **7b** show that as market risk increases, it is optimal for the CCP to collect more resources, both in a low (**Figure 7a**) and a high (**Figure 7b**) member default probability scenario (for a given level of q, c and a).

Figure 7a shows that when the probability of member default is low, the optimal amounts of both default fund contributions and initial margin increase with market risk. The proportion of optimal total resources consisting of DF increases as larger price moves become increasingly likely, driven by the lower collateral costs associated with DF for a given level of loss-absorbency.

(1) Zero collateral still provides coverage against 50% of possible price moves as the member will be ITM 50% of the time and the CCP will face no losses if it defaults.

Figure 7 Impact of increasing asset price volatility on the optimal size and allocation of default resources for default probabilities of a) $q = 0.05$ and b) $q = 0.25$

Figure 7a

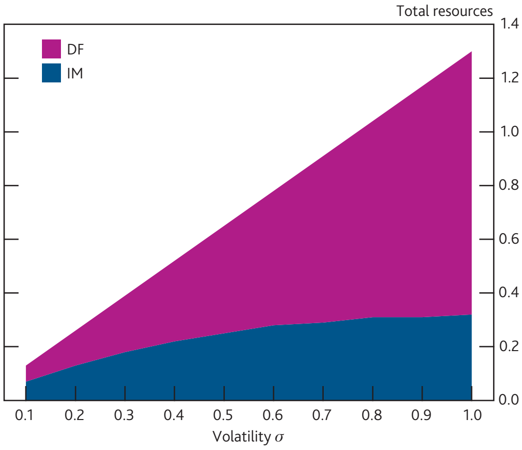


Figure 7b

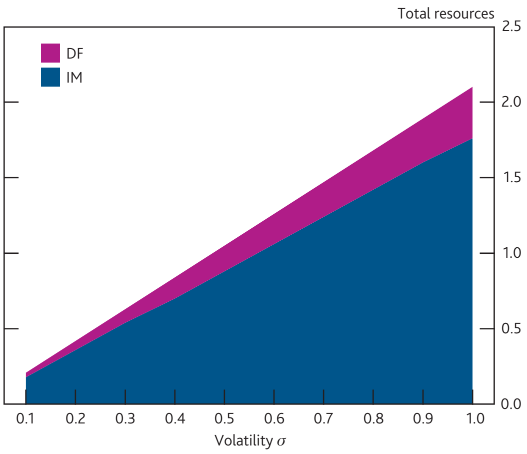


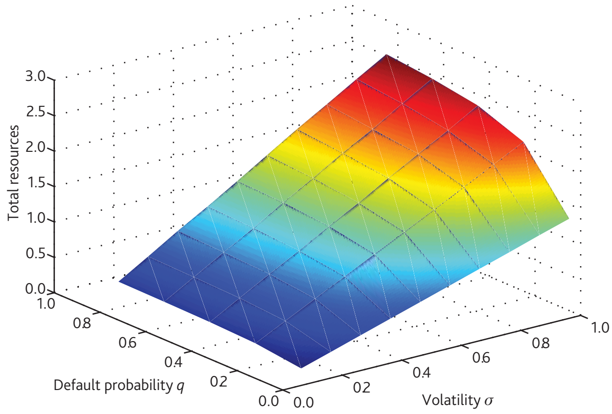
Figure 7b depicts the optimal allocation of collateral when the probability of a member defaulting is high (25%). In this case, the optimal allocation consists mainly of initial margin, and the total level of optimal resources for a given volatility is higher, consistent with the results of simulation 1. **Figure 7c** (below) illustrates more clearly how the optimal level of total resources varies with both volatility and default probability.

Simulation 3: effect of the cost of collateral on the optimal allocation

As the opportunity cost of collateral increases, it becomes relatively more expensive for members to protect against credit risk. In these cases it makes sense for members to insure only against adverse events with a higher likelihood of materialising.

Figure 8 shows how increasing collateral cost affects the optimal level of CCP default resources. When members have a default probability of 5% (**Figure 8a**), increases in collateral cost have a relatively large impact on both the size and allocation of total CCP resources. A fourfold increase in the

Figure 7c Impact of increasing asset price volatility and member default probability on the optimal size of total member default resources



opportunity cost of collateral from 25 basis points to 100 basis points reduces the optimal size of default resources by about 50%. Increasing collateral cost also drives a redistribution of resources from IM to DF to the extent that for a cost of 175 basis points, the optimal allocation of total default resources consists solely of DF. The total level of DF increases up to a cost of 100 basis points, before falling as costs increase further.

This redistribution arises due to the higher loss-absorbing ability of DF. When expected losses are held constant and the cost of collateral is increased, the CCP could either: reduce DF and IM by the same proportion, in which case it would face higher expected losses; or allocate more resources (both in absolute and relative terms) to DF, in which case the total loss-absorbing capacity of the CCP's default resources can be maintained even though the total resource level falls. Once the collateral costs are high enough, the only way to optimise costs is to protect against a lower level of losses, and so the level of DF falls once it becomes the sole default resource.

When the probability of members defaulting is high ($q = 25\%$), increases in collateral cost have a smaller impact on the optimal size and allocation of total resources than when $q = 5\%$, as the potential cost of member defaults remains the dominant incentive and the superior loss-absorption of DF relative to IM becomes less pronounced (**Figure 8b**). For the same reasons, IM is still the dominant contributor to total default resources when the collateral cost has increased from 25 basis points to 350 basis points.

Table B illustrates further that the sensitivity of the optimal level of collateral to the cost of collateral decreases as q increases. Increasing volatility for fixed q on the other hand does not significantly affect the dependence of total resources on collateral cost within the resolution of our simulations, as shown in **Table C**.

Figure 8 Impact of increasing opportunity cost of collateral on the optimal size and allocation of default resources for default probabilities of a) $q = 0.05$ and b) $q = 0.25$

Figure 8a

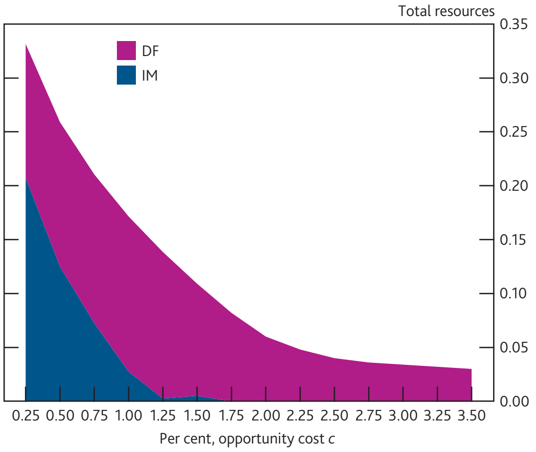


Figure 8b

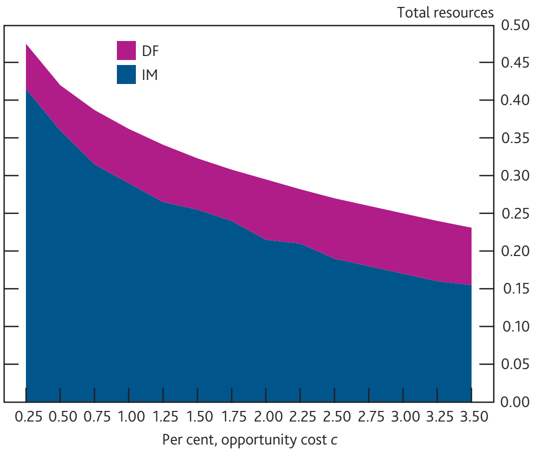


Table B Ratio of total default resources at collateral costs of c = 100 basis points and c = 200 basis points to total default resources at a collateral cost of c = 25 basis points, for different values of q and constant $\sigma = 0.2$

q	0.05	0.1	0.2	0.3	0.4	0.5
Ratios of c						
100 basis points/ 25 basis points	52%	66%	74%	78%	79%	80%
200 basis points/ 25 basis points	18%	44%	59%	64%	68%	68%

Table C Ratio of total default resources at a collateral cost of c = 100 basis points to total default resources at a collateral cost of c = 25 basis points for different values of σ, for q = 5% and q = 50%

σ	0.2	0.4	0.6	0.8	1
q = 5%	52%	52%	52%	52%	52%
q = 50%	80%	80%	79%	81%	80%

In reality, minimum regulatory standards would limit the ability of a CCP to reduce its default resources significantly, so the results of our simulations do not describe what a CCP could do in reality. These standards are motivated by the large systemic costs that would result from a CCP default, which we can (in part) introduce to the simulation through our systemic cost parameter 's'. Setting s to a large, non-zero value reduces the rate of reduction in optimal total resources with increasing collateral cost as one would expect (not shown here). At the same time as having to meet regulatory minima, CCPs' own risk tolerance would also likely mean that they would not be prepared to reduce IM and DF resources below a certain level.

Simulation 4: effect of capital charges on the optimal allocation

Capital charges have a similar effect on the optimal allocation of collateral as the opportunity cost, as one would expect. **Figures 9a** and **9b** show that where a single charge is applied to both DF contributions and IM, the effect is to reduce total

Figure 9 Impact on the optimal size and allocation of default resources of an equal capital charge on IM and DF, for member default probabilities of a) $q = 0.05$ and b) $q = 0.25$

Figure 9a

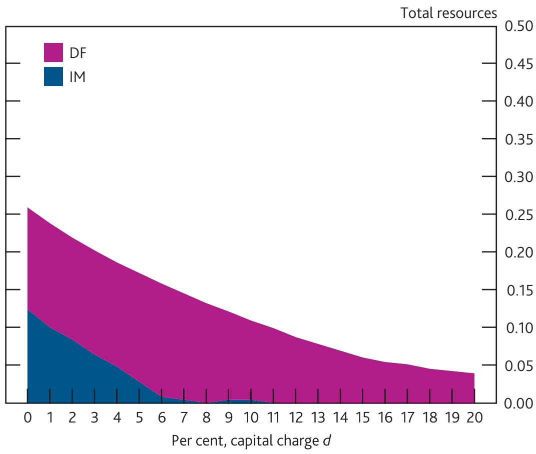
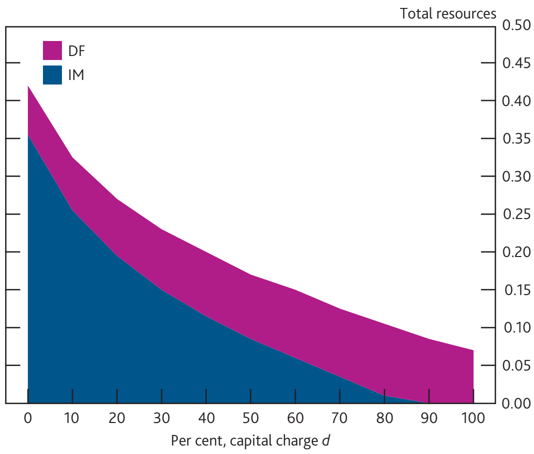


Figure 9b



resource levels and to favour holding collateral solely in the form of DF contributions for a sufficiently high charge.

Next we consider the effects of levying differing capital charges on IM and DF. **Figure 10** shows the impact of varying the capital charge on IM for a fixed DF capital charge, and a member default probability of $q = 5\%$. It is immediately obvious that levying any capital charge on IM greater than that on DF contributions drastically changes the incentives within our model, leading to an optimal collateral composition that consists almost solely of DF contributions.

Figure 10 Impact on the optimal size and allocation of default resources of a capital charge on IM, for a DF charge fixed at a) 0% and b) 0.16% (ie a 2% risk weight), and a default probability of $q = 0.05$

Figure 10a

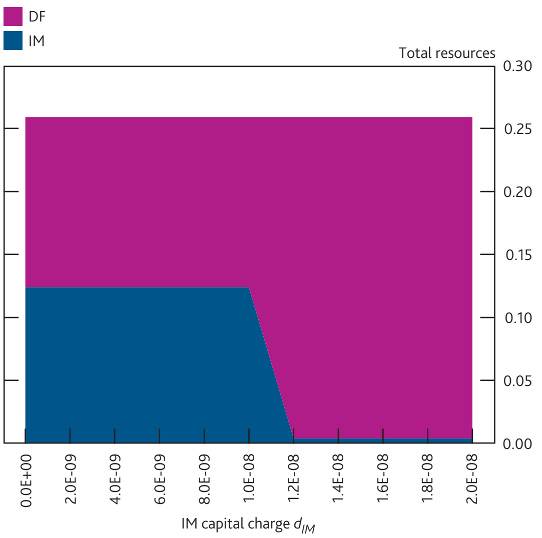
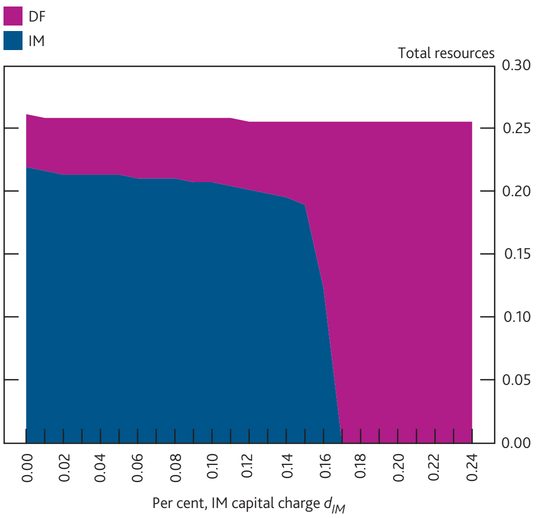


Figure 10b



There are two major factors that contribute to this change. First, a member's own IM and DF contributions provide equal protection to other members against losses arising from that member's default, so applying an asymmetric charge shifts the optimal allocation towards the cheaper form of collateral. Second, surviving members' IM resources do not provide protection against losses caused by other members defaulting; so unless the extra mutualisation cost of holding more DF outweighs the cost to surviving members of the capital charge on IM, it will also become less attractive for surviving members to hold IM.

This effect of the capital charge on IM is also observed for higher values of default probability (not shown here). This suggests that in our internalised cost/benefit model, levying a capital charge on IM greater than that applied to DF contributions will always lead to an optimal collateral pool consisting mainly of DF contributions.

The effect when a capital charge is applied to DF larger than that on IM is much less extreme (**Figure 11**); a non-zero DF contribution persists for capital charges up to around 19% (for $d_{IM} = 0$). This is because DF contributions can absorb more

Figure 11 Impact on the optimal size and allocation of default resources of a capital charge on DF, for an IM charge fixed at a) 0% and b) 0.16%, and a default probability of $q = 0.05$

Figure 11a

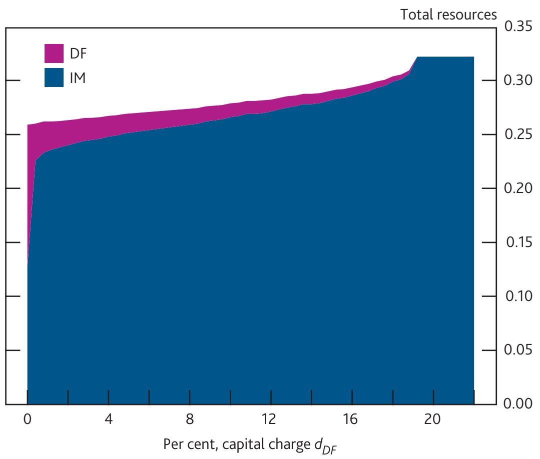
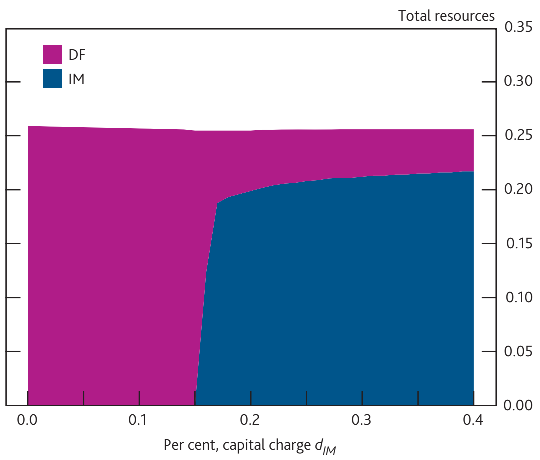


Figure 11b



loss than an equivalent level of IM through their loss mutualisation property, and so it takes a higher capital charge to offset this benefit than in the case of IM. The mutualising benefit of DF also means that as the optimal DF contribution falls, IM necessarily increases in order to maintain the overall level of loss-absorption provided by the CCP's total resources (as far as collateral cost constraints allow). In reality, as members hold more capital against their DF contributions, they become more resilient to those DF contributions being utilised by the CCP; this benefit however is not captured in our model.

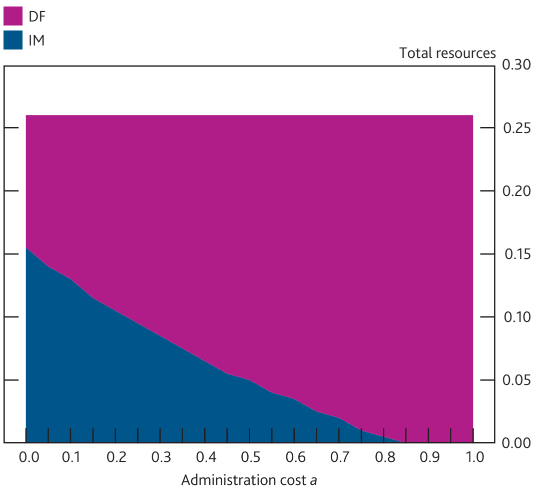
Our simulated results suggest that different capital charges on IM and DF will change the optimal allocation of default resources significantly; in the other simulations presented here, a slightly higher charge on DF than IM would change the allocation for a given set of parameter values to be much more dependent on IM (and *vice versa*). Again, in reality there are limits to how much a CCP would either wish to or be able to change its allocation of default resources, but these results show that within these constraints, capital charges on default resources may have an effect on CCPs' risk management choices.

Simulation 5: effect of an additional administration cost of CCP default on the optimal allocation

The variable a captures the administration cost of a CCP defaulting: ITM members recover a proportion $(1 - a)$ of the remaining debt owed by surviving OTM members. If a is non-zero, ITM members make a further loss when the CCP defaults, as the CCP administrator keeps some of the funds transferred to them from OTM members.

From **Figure 12** we can see that the introduction of this additional expected loss has little effect on the total resources collected from each member by the CCP in our baseline model (where $q = 5\%$ and $\sigma = 20\%$); the same is true when we model higher default probabilities or price volatility (not shown). The main impact of increasing post-default costs is that a substitution effect occurs between IM and DF, since DF is more effective at preventing a CCP default than IM due to its loss-mutualising properties. The more that is likely to be lost on ITM positions if the CCP defaults, the greater the optimal relative contribution of DF, because the expected cost of mutualisation decreases relative to the expected benefit of reducing the likelihood of the CCP defaulting.

Figure 12 Impact on optimal size and allocation of default resources of increasing administration cost following a CCP default



Simulation 6: Effect of a systemic cost of CCP default on the optimal allocation

Figures 13a and **13b** show the effects of increasing 'systemic cost' on optimal collateral levels in our model. It is immediately apparent that, consistent with our previous results, increasing the cost of a CCP default incentivises DF over IM due to its increased loss-absorbing capabilities. Also consistent with the results of the other simulations presented,

Figure 13 Impact on optimal size and allocation of default resources of introducing a further 'systemic' cost of default for member default probabilities of a) $q = 0.05$ and b) $q = 0.25$

Figure 13a

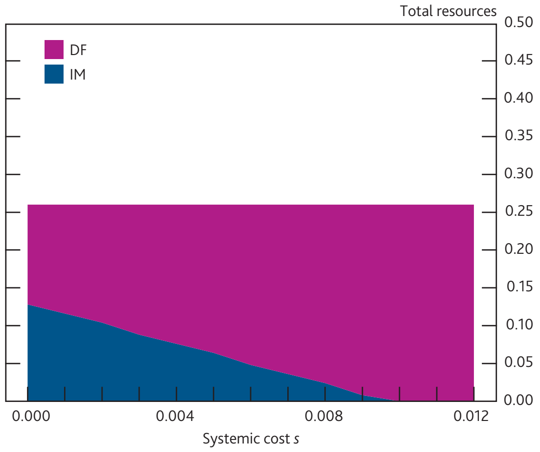


Figure 13b

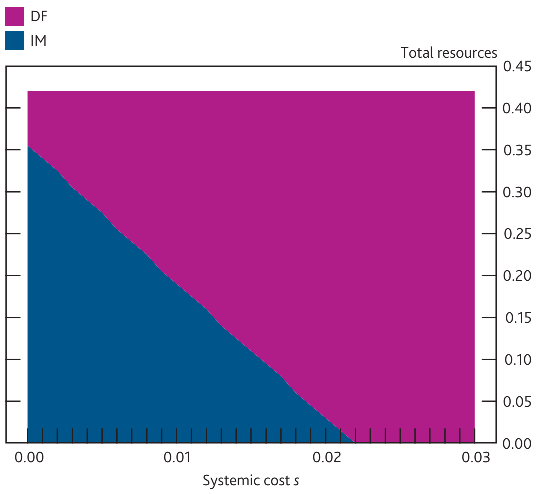


Figure 13b shows that when the probability of member defaults is higher, IM contributions to total resources are larger than for small default probability, at least for small systemic costs. For a high enough systemic cost of default however, the better loss-absorbency of DF is still favoured as the overall cost to members due to the potential cost of systemic 'spillover' is higher than the probable cost of mutualisation.

We also find that, in our model, a very large relative systemic cost of default (even of the order of 1,000 times the members' initial portfolio values) makes almost no difference to the optimal level of CCP resources. This is because our model chooses optimal resource levels which cover losses to a very high confidence level, such that the probability of a CCP default is orders of magnitude smaller than the resulting costs, unless they are inflated to extreme levels.

4 An extension to the model: the effects of a fat-tailed price distribution

The previous results were for a CCP operating in a market in which changes in asset prices are described by the Normal distribution. It is however well known that such a distribution has low tail risk relative to the empirical behaviour observed in some financial markets.(1)

In order to investigate how higher underlying tail risks might affect the optimal size and allocation of IM and DF, the above simulations were repeated using a symmetric Student-t distribution of asset price moves with seven degrees of freedom,(2) scaled to our Normal distribution with a volatility of $\sigma = 0.2$. The Normal and Student-t distributions are shown in **Figure 14**. The Student-t clearly has fatter tails, and so provides a useful point of comparison to the results above.

Figure 14 Comparison between a Normal distribution and a Student-t distribution with seven degrees of freedom

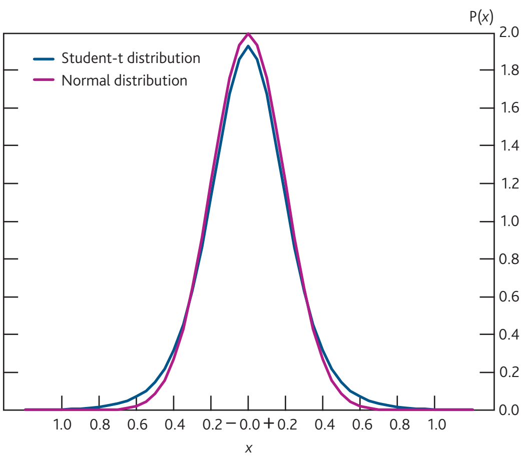
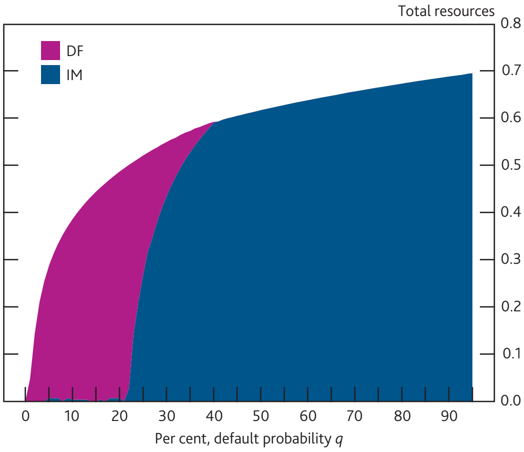


Figure 15 shows how the optimal size and allocation of default resources changes with the probability of member default when the potential loss to the CCP follows a fatter-tailed distribution. At low default probabilities, DF contributes more for a fat-tailed distribution than for the Normal distribution. This is because a fatter-tailed distribution produces larger expected losses for a given default probability, and extra DF is a less costly way to mitigate this additional potential loss than IM (in terms of collateral demand). For higher default probabilities where total resources consist mostly of IM, the quantum of total resources is larger than that for the case where prices are governed by the Normal distribution. This is an intuitive result: fat tails in the price distribution increase the size of expected losses in the case of member defaults, and so a higher level of default resources is held against these losses.

Figure 15 The impact on the optimal size and allocation of default resources of increasing member default probability, with a fat-tailed price move distribution



The effect of larger tail risks on the optimal size and allocation of default resources in the face of external costs is shown in **Figures 16–18**. **Figure 16** shows how a capital charge on IM affects the optimal size and allocation of default resources, for the case where there is no capital charge on DF. This simulation is carried out for a default probability of $q = 25\%$, since at $q = 5\%$ the optimal level of IM is close to zero — as can be seen in **Figure 15** — and so the effects of a capital charge on IM will not be visible.

The result is similar to that in simulation 4 (where the underlying price distribution was Normal): a minimal additional capital charge on IM shifts the optimal allocation completely towards DF.

(1) See eg Embrechts, McNeil and Frey (2005).
(2) The number of degrees of freedom was arbitrarily chosen to produce a distribution similar to the Normal around the mean, but with noticeably more pronounced tails. Average degrees of freedom of five (ie distributions with fatter tails than that used here) have been observed for S&P 500 and FTSE 100 stock returns, see eg Zumbach (2006) and Stoyanov *et al* (2011).

Figure 16 The impact on the optimal size and allocation of default resources of an increasing capital charge on initial margin, with a fat-tailed price move distribution (for q = 0.25)

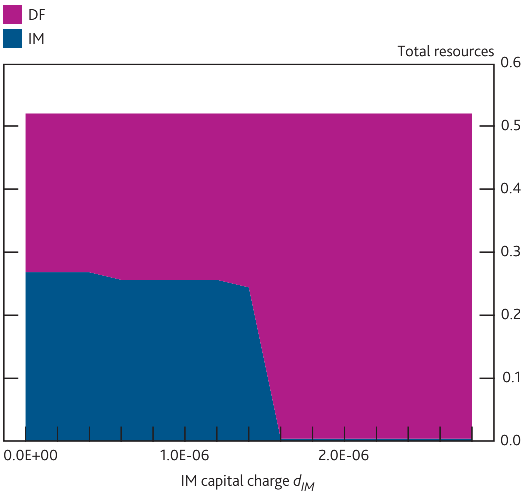


Figure 17 The impact on the optimal size and allocation of default resources of an increasing capital charge on default fund contributions, with a fat-tailed price move distribution (for q = 0.05)

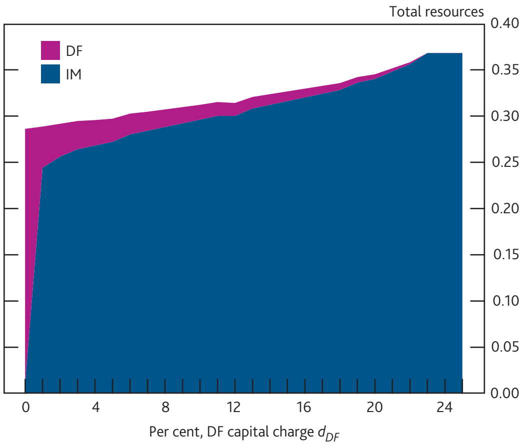


Figure 18 The impact on the optimal size and allocation of default resources of an increasing capital charge on both IM and DF, with a fat-tailed price move distribution (for q = 0.25)

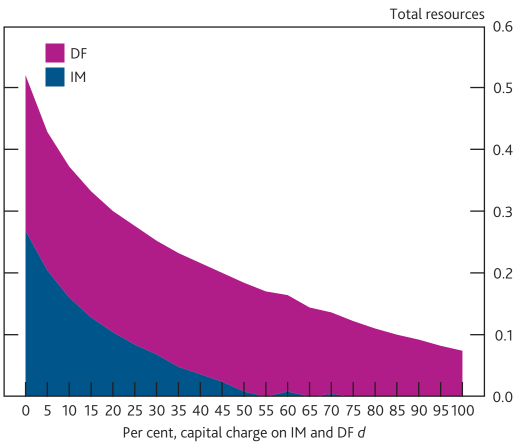
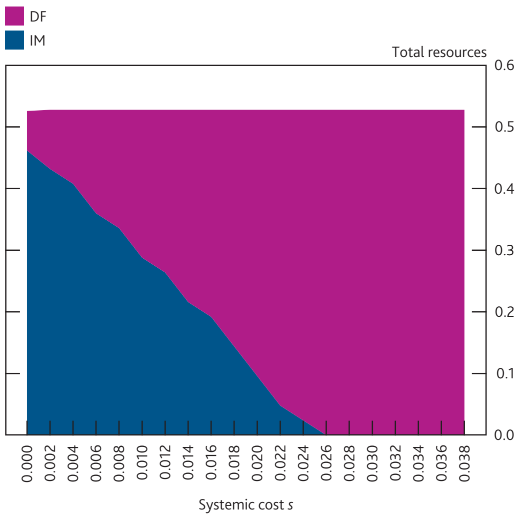


Figure 17 shows that when a capital charge is applied to DF contributions, IM becomes the sole contributor to total resources above a charge of around 23%. This is a little higher than the 19% charge which results in zero DF contribution in simulation 4, reflecting the extra benefit of DF where tail risks are greater. The amount of IM held also increases more markedly as DF falls off in this simulation as the potential losses it has to cover are larger for a fatter-tailed price distribution.

Figure 18 shows the impact on the CCP's optimal size and allocation of default resources of a capital charge applied equally to IM and DF, for q = 25%. The behaviour is similar to that in simulation 4.

Finally, **Figure 19** shows that modelling an additional systemic cost of CCP default shifts the optimal allocation of total resources towards DF contributions more quickly where tail risks are more significant. It was also found that the quantum of total resources slowly increases with increasing systemic cost, with a systemic cost of 50 times the portfolio value leading to an increase in optimal total resources of approximately 6.5% (not shown).

Figure 19 Effects of a 'systemic' cost of default for a member default probability of q = 0.25 with a fat-tailed asset price distribution



The comparison between the results of this section and those of the previous section demonstrates that the behaviour of the underlying market is an important factor in how a CCP will optimally distribute its resources. Larger tail risks are likely to skew the allocation towards DF as the expected cost of defaults increases, due to its higher loss-absorbing capacity; the total resources held by a CCP also naturally increase with greater tail risk.

The current model cannot account for asymmetric market risk due to its construction; nor does it include jump-to-default risks such as those inherent to credit default swap contracts and bonds, for example (although jump-to-default can be thought of as an extreme tail risk, so the results for the fat-tailed distribution still have some relevance to products with this risk characteristic). The results presented here are intended to illustrate generally how different market-based and external factors may influence the potential costs and benefits of IM and DF, with a view to informing the policy debate on the relative merits of each.

5 Conclusions and policy considerations

The Principles for financial market infrastructures (PFMIs), published by CPSS and IOSCO in April 2012, require a CCP to set aside sufficient resources to cover its current and potential future exposures to each participant (Principle 4). As part of this, a CCP should establish margin levels commensurate with the risks and particular attributes of each product, portfolio, and market it serves (Principle 6).

In addition, a CCP that is involved in activities with a more complex risk profile or that is systemically important in multiple jurisdictions should maintain additional financial resources to cover a wide range of potential stress scenarios that should include, but not be limited to, the default of the two participants and their affiliates that would potentially cause the largest aggregate credit exposure for the CCP in extreme but plausible market conditions (Principle 4). See Box 3 for more detail.

Together, the new regulatory standards require CCPs to have robust processes to monitor and mitigate their credit risk exposures. At the same time, they allow CCPs to tailor their risk management models, in particular giving them discretion on the precise balance between reliance on IM and DF, providing IM is sufficient to meet the 99% confidence level requirement. Our simulation results support the recognition in the Principles that CCPs should have discretion over how they set the optimal level and composition of their default resources, based on the specific risks of the markets and portfolios that they clear.

First, in our model a CCP's optimal risk management choices depend on the risk characteristics of the market(s) served by the CCP. Our results suggest that CCPs may choose to rely more on DF (survivors-pay) than IM (defaulter-pays) resources for markets in which asset price volatility is higher than those in which prices are less volatile. This result follows from the lower collateral demands of DF relative to IM. The level of tail risk in markets is also a key driver of how CCPs allocate their

Box 3
CPSS-IOSCO Principles for financial market infrastructures relevant to CCP margin and default fund resources

Principle 4: credit risk

'An FMI should effectively measure, monitor, and manage its credit exposures to participants and those arising from its payment, clearing, and settlement processes. An FMI should maintain sufficient financial resources to cover its credit exposure to each participant fully with a high degree of confidence. In addition, a CCP that is involved in activities with a more-complex risk profile or that is systemically important in multiple jurisdictions should maintain additional financial resources sufficient to cover a wide range of potential stress scenarios that should include, but not be limited to, the default of the two participants and their affiliates that would potentially cause the largest aggregate credit exposure to the CCP in extreme but plausible market conditions. All other CCPs should maintain additional financial resources sufficient to cover a wide range of potential stress scenarios that should include, but not be limited to, the default of the participant and its affiliates that would potentially cause the largest aggregate credit exposure to the CCP in extreme but plausible market conditions.'

Principle 4 requires that CCPs' stress tests should include peak historic volatilities, potential changes in market liquidity, and other market and credit events which are 'extreme but plausible' including a change in correlations between different products. While it is recognised that it is not feasible to cover all tail risks, the standard specified in this Principle is designed to cover a conservative portion of the tail risk to CCPs. It is also specified that CCPs must take into account the specific risks inherent in the products they clear, such as jump-to-default price changes or correlations between a product's value and potential participant defaults.

Principle 6: margin

'A CCP should cover its credit exposures to its participants for all products through an effective margin system that is risk-based and regularly reviewed.'

Principle 6 states that a CCP should establish margin levels 'commensurate with the risks and particular attributes of each product, portfolio, and market it serves.' In the event of a member defaulting, IM should be sufficient to cover the CCP's potential future exposure to participants in the interval between the last margin collection and the close-out of the defaulter's positions. IM should meet a single-tailed confidence level of at least 99% with respect to the distribution of this potential future exposure.

resources. In our model, CCPs clearing products with a higher probability of extreme price moves should rely more on DF than those clearing products for which extreme price moves are less likely.

The risks associated with a particular product can vary over time, as market volatility changes or extreme price moves become more likely due to a financial crisis, for example. In our model, the optimal response for a CCP would be to adjust its level and allocation of resources accordingly, calling more collateral from members in times of market uncertainty and relaxing margin and default fund requirements in more stable periods.

Hence our model produces a strong tendency to procyclicality in resource requirements. In times of increasing volatility, our CCP would call more collateral, increasing liquidity pressures on their participants and potentially the markets they clear, or causing participants to reduce their exposures to the CCP, driving further volatility. In reality, CCPs face a delicate trade-off between responding to changing market conditions, and avoiding creating undue pressure on their participants and on market liquidity via margin calls. To reduce the procyclicality that arises from using current and recent market conditions to set default resource levels, CCPs will be required under EMIR regulations to consider stressed market conditions when setting the level of their IM. CCPs also take into account periods of market stress in setting the level of their DF. These features are not captured in our simulation model.

Second, our simulations show that CCPs' margin practices should also take into account the credit quality of their members. As the probability of a member default increases, IM becomes increasingly preferable to DF as a default resource. This suggests that a change in access policy (eg lowering capital thresholds) which results in a change in the credit profile of the CCP's members should be reflected in its risk management processes.

Note that our model does not address the question of heterogeneity in member credit quality, and whether members with lower credit quality should be subject to higher margin requirements than members within the same CCP with a higher credit quality; or indeed whether they should even be eligible for membership of the CCP.

Third, our simulations show that the CCP's optimal choices are influenced by the cost of collateralisation and by the capital requirements faced by its members. So in our model, as the cost of collateral increases, CCPs choose to rely increasingly on DF. The extent to which collateral costs affect the CCP's incentives depends on likelihood of losses materialising (ie the member default probability). When losses are likely to be realised, increasing collateral costs leads to a smaller reduction in total optimal resources than when losses are unlikely to materialise.

Changes in capital charges also have strong effects in our stylised model. For example, increasing the capital charge on IM, keeping the charge on DF contributions constant, immediately reduces the incentives of the CCP and its members to rely on a defaulter-pays model. Likewise, increasing the capital charge on DF contributions, keeping the charge on IM constant, leads to a (less extreme) move away from the survivors-pay model. In other words, in our model, changes in regulatory capital requirements on clearing members have a significant impact on the CCP's optimal risk management choices. It is the existence of financial incentives such as these that make a compelling case for regulators to consider minimum requirements on CCPs with regard to the level and allocation of their default resources, while giving CCPs enough discretion to manage the risks they face appropriately.

To conclude, it is the role of both CCPs and their regulators to ensure that CCPs do not compromise the robustness of their risk management tools in response to increasing external costs. The Principles and forthcoming legislation such as EMIR will help to ensure that such compromises are not made, while allowing and encouraging CCPs to tailor their counterparty credit risk models to best manage the risks specific to each product that they clear.

Annex 1
Setting up the numerical model — defining the CCP, its members and the market it serves

The CCP: We model a user-owned CCP with n direct members.(1) By modelling a user-owned CCP we ensure that the members' and CCP's interests are automatically aligned and that all members, regardless of their market positions, stand to lose from the default of the CCP. As part of its default resources, the CCP holds capital K, composed entirely of equally sized member contributions k, to reflect the fact that CCPs contribute some of their own capital to their default resources. We determine this capital level exogenously and hold it constant, reflecting the fact that CCPs in practice do not use their equity level as a dynamic default resource, in contrast to their initial margin and default fund levels.

We assume that the CCP is the only one in its economy and clears one representative portfolio of products; and that membership of the CCP is a given (ie our members cannot choose to terminate their membership of the CCP). These simplifications mean that we ignore competition between CCPs, and assume that the benefits of central clearing outweigh the potential costs to our CCP's members (or alternatively that central clearing is mandated).

Because members constitute the whole universe in our model (ie facing all losses and pledging all available collateral), our user-owned CCP internalises both the costs and benefits of collateralisation faced by all its members. Its primary incentive then is to minimise costs to its members, including in the case of one or more members defaulting. This could theoretically be achieved by holding infinite collateral. In practice, as well as the fact that there is a finite amount of collateral in the system, holding that collateral is not free (as some assets acceptable as collateral give a lower return than other assets, or because the same collateral could be used elsewhere to support business generating a greater return); so there is an upper bound of expected losses that should be covered by collateral. This upper bound is determined when the marginal benefit of collateral (further reduction of expected losses) and its marginal cost (increased cost of collateral) are equalised. So we introduce an exogenous opportunity cost of collateral $c > 0$ in our model, which represents the difference between the return paid by the CCP and the return that could be generated elsewhere by members. We treat this cost as being independent of the amount of collateral required.

This interplay between the cost of collateral and the benefit it brings in reducing potential losses turns out to be crucial in determining the optimal level of default resources the CCP should hold.

Members: For simplicity, we assume that members have evenly distributed long and short market positions of equal size, on portfolios with an initial notional value of 1; we therefore require that there be an even number of members. The sizes of initial margin y and default fund contribution z are thus symmetric across all members. Given any price move, $n/2$ members will be in-the-money (ITM) and hold a net credit position, and the other $n/2$ will be out-of-the-money (OTM) and hold a net debit position.

We assign all members the same probability of default q. This implicitly assumes that when one member defaults, the default probabilities of surviving members do not change; this assumption could be relaxed in future work.

Mathematically of course, members have a $(1 – q)$ probability of surviving; rational members would try to extract benefit from their own default in addition to trying to minimise the costs of other members defaulting. But this is not consistent with real life incentives; market participants are unlikely to value the benefits that the DF would bring in the case that they default on their obligations to the CCP. So the assumption of survival is necessary to introduce the aversion to free-riding and moral hazard that CCP members exhibit in reality.

We do not include the effects that IM and DF may have on members' default probabilities and risk-taking behaviour. Since IM and DF are costly, in reality they may incentivise members to hold less risky positions which may in turn reduce their default probability. This incentive effect has been investigated by Haene and Sturm (2009), for example; and Monnet and Nellen (2012) endogenise this effect in a model on clearing with two-sided limited commitment.

CCP resources: The CCP's total resources in our model are finite and composed of members' initial margin (IM), default fund (DF) contributions and equity contributions. Only IM contributions from defaulting members can be used to absorb losses; we assume that losses will fall in members' market exposures and that surviving members cannot lose their IM if the CCP becomes insolvent. Variation margin is implicitly included by setting the current portfolio value to zero. We do not explicitly distinguish further loss-sharing arrangements such as rights of assessments from paid up default fund contributions, or consider the use of loss-allocation rules.(2) The total quantum of resources available to the CCP in the event of i members defaulting is therefore equal to $iy + nz + nk$, where y is each member's initial margin contribution, z is each

(1) The model could be extended to consider a non user owned CCP, in which case the interaction between members' incentives and the CCP's incentives would have to be quantified.
(2) Rights of assessment entail the CCP requiring its members to commit to replenishing the default fund a specified number of times if it is exhausted in the event of member default(s).

member's default fund contribution and k is each member's equity holding in the CCP. Once these resources are exhausted, our CCP becomes insolvent.

Other costs: A further cost considered in the model is the capital charges that may be applied to members' IM and DF contributions by regulators. We thus have a cost ($c + d_{IM} \times c_c$) for initial margin and a cost ($c + d_{DF} \times c_c$) for default fund contributions, where d_{IM} and d_{DF} are the capital charges on IM and DF respectively, and c_c is the cost to banks of holding capital, which we fix at 10%. The values we use for these are informed by the Basel rules on risk weights applied to banks' exposures to CCPs. The Basel rules require that IM will be charged at either 0% or 0.16%, while the charge on DF contributions is expected to be variable, with a floor of 0.16%.(1) In this paper, we also explore the case in which d_{DF} is smaller than d_{IM} to see what the effects would be if IM were to become more costly for members than DF in terms of capital requirements.

We do not include the cost to the members of holding equity in the CCP, since, as a constant exogenous parameter, it does not affect the optimisation problem in our model. It is worth noting that the capital charge on equity holdings is generally 100%, so that there is no cost incentive for a CCP or its members to include equity in the default waterfall in preference to IM or DF.

We also consider the fact that in reality, CCPs are not isolated from the market in which they operate, and a CCP default would likely have a significant impact on the financial system beyond the direct effect on its surviving members. To illustrate this in our model, we introduce a constant parameter s which acts as a proxy for the 'systemic cost' of a CCP default, expressed as a further loss suffered by all members.

The market: Next, we turn to the market in which the CCP operates. We model the range of possible price movements over a (fixed) notional liquidation period as a probability distribution f(p), which follows a Normal distribution N $(0, \sigma)$. By adopting such an approach, we ensure that: i) possible price movements are not bounded by a specific scenario; and ii) possible outcomes are weighted by their probability of occurring. We set the current price (p) to be zero without loss of generality.

In Section 4 we further consider the effects of using a simple price distribution with 'fat tails', in which the probability of large price movements is greater than that under a Normal distribution. Comparison between simulation results for the Normal and fat-tailed price distributions can provide an insight into how different price behaviour in different markets will affect the optimal allocation of resources.

The default process and its costs

Having defined the workings of the CCP and the market in which it operates, we now describe its potential losses in the case of member default(s) and the further losses to surviving members thereafter. We make the assumption that the CCP and its members will want to minimise costs for surviving members rather than all members. This is motivated by the observation that members will assume their own survival in weighing the costs and benefits of posting collateral to the CCP and the potential losses caused by member defaults.

It is customary to refer to the DF as a 'survivors-pay' resource, and to IM as a 'defaulter-pays' resource. In addition, we introduce the concept of 'cost of mutualisation,' defined as the difference between the losses that would be incurred on an individual member's DF contribution and the losses he would incur on an outstanding ITM position if the CCP should default. This 'cost of mutualisation' is a key driver behind the results of our modelling exercise, and arises directly from the assumption that members will make decisions predicated on their own survival, as described above. Without this assumption, the aggregate cost of mutualisation is zero as it simply represents a redistribution of losses (from ITM and defaulting OTM to surviving OTM members), and DF will always be preferred to IM due to its greater loss-absorbing capacity (all else being equal). In reality, the perceived cost of mutualisation through the DF also reflects other factors, such as a lack of visibility of other members' exposures to the CCP, as well as a lack of control over the risk management decisions taken by the CCP. These are not modelled in the current paper.

Default by members — or in an extreme case by the CCP — can generate many different types of losses. These include losses on DF contributions, losses on in-the-money positions and also operational costs, legal costs and reputational costs. In this model, we focus on first-order losses, ie the replacement cost of closing out defaulting members' positions.(2) The potential replacement cost is a function of both counterparty credit risk and market risk, and increases in expectation as the number of defaulting members and the volatility of prices increase. The CCP's potential losses thus depend on parameters such as the probability of a member defaulting, the distribution of asset price moves, and the quantity and type of collateral (IM or DF) it has collected from members.

Member losses: Following insolvency, we assume that the CCP's liquidator closes out all positions. The liquidator will have a claim on surviving OTM members, and owe money to ITM members. We assume that the surviving OTM members

(1) 'Capital requirements for bank exposures to central counterparties', Basel Committee on Banking Supervision, available at www.bis.org/publ/bcbs227.pdf.
(2) We consider legal costs, the operational costs to the CCP of hedging or transferring positions, and the potential costs of liquidating its default resources to be second-order losses.

pay in full and that the CCP transfers their funds to the ITM members *pro rata* (who we assume have equal seniority over the CCP's assets). This process will in the first instance produce a loss for ITM members, as any money owed by defaulting OTM members will be lost, diluting the money recovered by ITM members. We further assume that this liquidation process could carry a cost, $a \geq 0$, which is fixed exogenously (to reflect higher-order losses such as potential fees charged by the liquidator or the effect of fire sales on the liquidation value of the portfolio of positions). Surviving ITM members may thus face losses over and above their default fund contributions and the losses on money owed from defaulting OTM members in the case of CCP insolvency.

If ITM members enter bankruptcy, they do not (immediately) default on their obligations to the CCP as they do not owe any money. We make the simplifying assumption that the administrators of any bankrupt ITM members will claim full payment on their open positions, such that the bankruptcy of an ITM member has no impact on the economic obligations in our model. We thus treat all ITM members as surviving members.

Annex 2
Setting up the numerical model — the loss functions

Setting up the numerical model involves three main steps:

i. we construct expected loss functions for both ITM members and surviving OTM members arising from their exposure to member (and CCP) defaults through default fund contributions, capital contributions and in-the-money claims on the CCP;

ii. we then define the loss function for a representative non-defaulting member by aggregating the loss functions obtained above; and

iii. finally, we minimise the representative member's expected loss function with respect to initial margin and default fund contributions by taking into account the costs of those contributions through the opportunity cost of collateral, capital charge and cost of capital parameters, c, d and c_C respectively.

Expected loss function if the member is a surviving net debtor (ie out-of-the-money)

The loss function for a surviving net debtor is an algebraic description of the losses for an OTM member as follows:

$$E[loss^{OTM}] =$$

$$\sum_{i=0}^{\frac{n}{2}-1} q^i (1-q^i)^{\frac{n}{2}-i-1} C\left(\frac{n}{2}-1, i\right) \cdot \left\{ \begin{array}{c} \int_{y+z}^{y+z+\frac{n-i}{i}z} \frac{i}{n-i}(p-y-z)\cdot f(p)\cdot dp \\ \int_{y+z+\frac{n-i}{i}z}^{y+z+\frac{n-i}{i}z+\frac{n}{i}k} \left\{\frac{i}{n}\left(p-y-z-\frac{n-i}{i}z\right)\right\}\cdot f(p)\cdot dp \\ \int_{y+z+\frac{n-i}{i}z+\frac{n}{i}k}^{+\infty} (z+k)\cdot f(p)\cdot dp \\ s\cdot\int_{y+z+\frac{n-i}{i}z+\frac{n}{i}k}^{+\infty} f(p)\cdot dp \end{array} \right\} \tag{1}$$

The first expression in the bracket denotes the cost of mutualisation to a surviving OTM member through the DF (area A in **Figure 1**). Because each member has market positions of equal size (normalised to one), the replacement cost is equal to the price change (dp, the distance between p and the origin). As long as dp is smaller than $y + z$, (ie defaulter's own IM and DF), surviving members face no loss. But if dp > $(y + z)$ and the number of defaulters is i, then the CCP uses a fraction $i/(n - i)$ of each survivor's DF contribution to cover each unit of loss in excess of $y + z$. Note that a DF can cover a larger price movement if there is only one member defaulting ($i = 1$), because the CCP can dedicate the whole DF to meet excess losses arising from a defaulting member's position. If several members default, the DF has to cover losses arising from several defaulting members and it runs out much more quickly.

Once the DF is exhausted, the CCP is forced to use its capital (equity) to cover the loss. The second term in the bracket represents this (area B in **Figure 1**). Here i/n of each member's contribution to the CCP's capital is used to cover each unit of losses in excess of $y + nz$; we assume that these losses fall on all members' equity contributions equally.

And once equity runs out, the CCP goes bankrupt. When the CCP defaulting has no systemic impact ($s = 0$), surviving OTM members will service their own obligations but do not need to share any further losses. The loss function for OTM members does not increase even if the price increases further (the third term in the bracket, and area C of the figure). If there is a finite systemic cost to the CCP defaulting, OTM members will face the additional loss expressed by the final term in equation **(1)**.

We noted above that the number of defaulting members has a significant impact on the losses that can be borne by the CCP. This impact is included in the term outside the bracket in equation **(1)**, which calculates the probability that i members default simultaneously. Summing the loss function over the possible scenarios of i member defaults weights the expected losses in each scenario by its probability of occurrence.

We assume that all members are exposed to the risk of default, but here **we assume that only OTM members default**. This assumption does not lose any generality, since whether ITM members default or not does not change any member's loss function. We further assume that at least one OTM member survives, consistent with our condition that the CCP will seek to minimise losses for surviving members only; this is why i is bounded above by $(n/2 - 1)$ in the function. $C(n/2 - 1, i)$ is simply a combination representing the number of ways in which i out of $(n/2 - 1)$ members can default: ie $(n/2 - 1)!/(i!(n/2 - 1 - i)!)$.

Expected loss function if the member is a surviving net creditor (ie in-the-money)

The loss function of surviving ITM members is in large part identical to the one described above for OTM members. But in this case, when ITM members are owed money by the CCP and the CCP goes bankrupt, the recovery rate on the debt owed by the defaulting members becomes lower than one. This is included in the third integral in equation **(2)** and can happen in two ways.

First, we assume that the administrator of the failed CCP receives payments in full from surviving OTM members (ie the fraction $(n/2 - i)/(n/2)$ of total claims by ITM members), but receives no payments from defaulting OTM members.[1] Then ITM members have to write down $1 - (n/2 - i)/(n/2)$ of the uncovered losses, arising directly from OTM member defaults.

(1) The worst-case scenario for a surviving ITM member is that all OTM members default, and so here we sum to $n/2$.

$$E[loss^{ITM}] = \sum_{i=0}^{\frac{n}{2}} q^i (1-q^i)^{\frac{n}{2}-i} C\left(\frac{n}{2}, i\right) \cdot \left\{ \begin{array}{c} \int_{y+z}^{y+z+\frac{n-i}{i}z} \frac{i}{n-i}(p-y-z)\cdot f(p)\cdot dp \\ \int_{y+z+\frac{n-i}{i}z}^{y+z+\frac{n-i}{i}z+\frac{n}{i}k} \left\{\frac{i}{n}\left(p-y-z-\frac{n-i}{i}z\right)\right\}\cdot f(p)\cdot dp \\ \int_{y+z+\frac{n-i}{i}z+\frac{n}{i}k}^{+\infty} \left\{\left(1-a\frac{\frac{n}{2}-i}{\frac{n}{2}}\right)\left(p-y-z-\frac{n-i}{i}z-\frac{n}{i}k\right)+z+k\right\}\cdot f(p)\cdot dp \\ s\cdot\int_{y+z+\frac{n-i}{i}z+\frac{n}{i}k}^{+\infty} f(p)\cdot dp \end{array} \right\} \qquad (2)$$

Second, the CCP liquidation process can carry a cost so that not all monies received from surviving OTM members are paid to ITM members. In our model, we account for this cost with the 'administration cost' *a*. If *a* = 0, the full value of the payments are delivered to ITM members and the slope of the loss function (area C + D in **Figure 1**) becomes *i*/(*n*/2). If *a* = 1, the payments are all lost and the slope is 1.

Our next step is to define a representative loss function which captures the average surviving member's loss. This is simply the sum of the two loss functions above, which captures all possible scenarios in which a surviving member can make a loss.

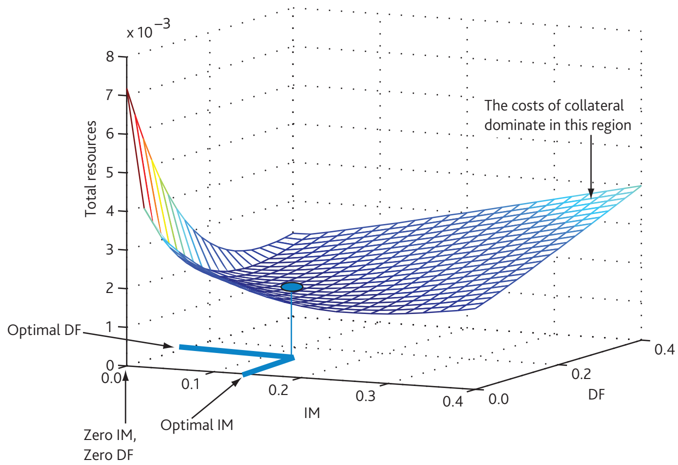
Finally, we need to define the optimisation problem: the optimal allocation and level of resources is determined by offsetting the benefits of loss-absorption by the members' margin and DF contributions (measured by the loss function) against the costs of lodging collateral (and holding regulatory capital). We thus minimise the sum of the representative member's loss function and the cost of collateral and capital, with respect to IM and DF contributions. This is summarised by the following objective function:

$$\min_{y,z}\{E[loss^{ITM}] + E[loss^{OTM}] + (c + d_{IM} \times c_c)y + (c + d_{DF} \times c_c)z\} \qquad (3)$$

We minimise the objective function numerically. Doing so allows a significant degree of flexibility in altering or adding costs to the model and is significantly more straightforward than attempting to solve analytically. It also allows us to change the underlying distribution of price changes, for example, and quickly explore the effects of doing so. Our model is thus well suited to various extensions in future work.

This minimisation is depicted graphically in **Figure A1**, which shows how the sum of expected losses and collateral costs varies with IM and DF. The minimum of this two-dimensional function defines the optimal amounts of IM and DF.

Figure A1 Minimisation of the representative member's expected loss function with respect to IM (*y*) and DF (*z*)



In order to have confidence in the robustness of our numerical results, we use a high granularity in our IM and DF increments. The results in this paper are generally for simulations using a 100 × 100 matrix of IM and DF values over the smallest range necessary to capture the full distribution of resources in a given simulation. Where the optimal levels changed slowly within a narrow range (such as in **Figure 11a**), the resolution was increased around the range of interest. Simulations were also repeated at lower and higher resolutions, producing very similar qualitative and quantitative results. The only noticeable sensitivity to resolution was found to occur where the optimal level of IM was close to zero; in such cases, the resolution determined whether a zero or small non-zero optimal value was found, accounting for the small fluctuations in the amount of IM in **Figure 15** for low values of *q*.

Annex 3
Setting up the numerical model — model inputs

The outputs of the simulations (comparative statics) are obtained by defining both 'baseline' values for the parameters when these are fixed and a 'variation range' when the parameters are varying. These are summarised in **Table A1**.

Table A1 Summary: value of variables when fixed or floating

	Variable	Value of the parameter when fixed	Range of variations for analysis
q	Default probability	5% and 25%	1% to 95%
n	Number of members	20	–
σ	Volatility	0.2	0.1 to 1
c	Collateral cost	50 basis points	25 to 350 basis points
k	Equity contribution	0.1%	–
a	Administration cost	10%	0% to 100%
d_{IM}	Capital charge on IM	0% and 0.16%	0% to 100%
d_{DF}	Capital charge on DF	0% and 0.16%	0% to 100%
s	Systemic cost to members of CCP default	0	0% to 100% of portfolio value

The number of members n is arbitrarily fixed to 20. Varying the number of members has a limited effect in the model because of our assumption of equal probability of default amongst members. However, if this assumption was removed it would be of interest to vary n (this could be done in future work).

We set the probability of default q equal to 5% per member when the parameter is fixed. This value is chosen as it ensures that there is a significant probability that one of the 20 members will default,(1) but further, simultaneous defaults are relatively unlikely (but still possible). Where we model a high probability of member default, we set q equal to 25%, meaning that multiple defaults become more probable. The variation range of q is defined over the interval 1% to 95%, where the maximum is an unrealistic number for reference.

The volatility of the asset prices σ is set to 0.2 when it is a fixed parameter. This corresponds to a 68% likelihood (one standard deviation) that price movements will be at most 20% up or down for our Normal price distribution, since the value of our members' positions is initially set to 1. This calibration value is chosen to be a rather conservative estimate of the likely changes in asset value over a conservative close-out period. The variation range of σ is bounded between 0.1 and 1.(2) The value zero is excluded because no collateral would be required in the absence of market risk. The maximum value is high but arbitrary.

The cost of collateral c is arbitrarily set equal to 50 basis points when fixed. In practice, CCPs have implemented different remuneration policies for their members' collateral, so it is difficult to estimate a representative average cost. For instance, a CCP can charge for holding collateral but pay interest on cash. DF contributions are also sometimes remunerated at a higher rate than margin. For simplicity, we assume that both IM and DF collateral carry the same cost, and 50 basis points is chosen as the opportunity cost of collateral in the baseline model. When we study the effect of changing the cost of collateral, we vary c between 25 basis points and 350 basis points. The value zero is excluded from the interval (infinite collateralisation would then obviously be optimal). The upper bound is arbitrary. In our baseline model, the total collateral cost $(y + z) \times c$ increases linearly, but the rationale and effect of introducing a quadratic cost function could be considered in future work.

The capital contribution by members (k) is fixed at 0.001 units of capital per unit of market position. This low value is chosen to reflect the fact that CCPs often have only very thin layers of equity relative to IM and DF. We do not model the effects of varying k since for a fully user-owned CCP, equity contributions from members are similar to DF contributions in that they ensure that all members will bear losses: increasing k essentially increases the level of mutualisation. We choose a non-zero value of k to ensure that all members will always experience some loss from the CCP defaulting in our model, even if the optimal level of DF is zero.

The administration cost a for ITM members of the CCP's default, which represents the proportion of debt from surviving OTM members that is not passed on to ITM members via the CCP's administrators, is set to 10% when fixed, and varied between 0% and 100% when variable. The value of 10% is chosen arbitrarily to introduce some extra cost to ITM members of the CCP defaulting, a likely outcome in the event of a real CCP default. The effect of a larger fixed administration cost can be inferred from the simulation in which the cost is varied from 0% to 100%.

We fix the capital charges on IM and DF to be 0% for most simulations, as we are primarily interested in how the optimal allocation of IM and DF depends on their endogenous rather than exogenous differences. In the simulations in which we investigate the effects of varying the capital charges on IM and DF, we vary one between 0% and 100%, and fix the other either to 0% or to 0.16%. We use these two fixed values in order to investigate whether there is a crossover in behaviour as the capital charge on one form of default resource increases above that on the other. The range of variation is not meant to accurately represent the expected range of real capital charges, but is chosen to illustrate the possible effects of additional (regulatory) costs on collateral.

(1) An individual member default probability of 5% corresponds to a 38% probability that exactly one out of 20 members will default (or a 19% probability that exactly one member will default and be OTM).
(2) Lower volatilities (of order 1%) do not change the qualitative results of our model.

Finally, in general we fix the systemic cost variable *s* equal to 0, so that the optimal level and allocation of resources is driven purely by the CCP's members' direct internal costs and benefits. When we do include the effects of an external systemic cost, it is varied between 0 and 1 with the upper bound meaning that members lose the entire initial value of their portfolio in addition to other costs arising from the CCP default. This is chosen purely for illustrative purposes, as it proved to be a large enough range to capture the qualitative impact of our systemic cost parameter. It would be interesting to extend the model in future so that the systemic cost is endogenised; for example, the change in price p could be conditioned on the CCP default, such that our model includes fire sale conditions; this in turn would affect the potential gains and losses of ITM and OTM members differently, representing the conflicting incentives that might occur during a CCP insolvency.(1)

Robustness and sensitivities

Although the simulations presented herein were performed with other parameters fixed to 'baseline' levels, we also performed further numerical simulations for different values of these fixed parameters and observed that the qualitative dependence of optimal default resources on our model parameters was consistent across a wide range of initial conditions (with the initial level/allocation of resources determined by the fixed parameters).

We thus have confidence in the robustness of our results regarding how each of our parameters affects the optimal allocation of IM and DF.

Quantitatively, our results are sensitive to the cost of exogenously imposed capital charges, fixed at zero for most simulations. As **Figures 10** and **11** show, differences in the charges on IM and DF can have a significant effect on the composition of default resources of our CCP. This highlights how exogenous factors may have a significant impact on our model CCP's incentives.

The sensitivity of our results to the fixed cost of equity is limited; an order of magnitude increase in members' equity costs leads to a non-zero DF contribution persisting for higher levels of member default probability, for example, but does not significantly change the absolute levels of IM, DF or total member resources. Similarly, an order of magnitude decrease in equity has little effect on the results.

Finally, if equity contributions *k*, the administration cost *a* and the systemic cost of CCP default are set to zero, then the optimal composition of default resources in our model consists solely of IM. Under these conditions, OTM members face no losses unless they contribute DF, while ITM members face no losses following a CCP default beyond the shortfall in payments owed by defaulted OTM members; then the cost of mutualisation we have introduced means that the optimal solution involves no loss mutualisation, as discussed in Section 2 and Annex 1.

(1) The authors thank Thomas Nellen for this idea.

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