Bank of England Museum

THE FUTURE OF MONEY

The Future of Money

PACK 3

Future Methods of Payment

An education resource for students aged 11-14.

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About the Future of Money exhibition

This resource collection is designed to accompany The Future of Money exhibition at the Bank of England Museum. The resources explore the links between the exhibition and a range of mathematical ideas, using exhibition objects and themes as a starting point for discussion and mathematical problem-solving. These activities will work in the classroom or at home and are designed for students aged 11-14.

The resources contain supporting notes for teachers, images from the exhibition and student activity sheets. There are five resource packs, each focusing on a different exhibition theme:

Pack 1: What is Money?

Topics covered include: compound measures; units of measurement; problem solving; probability

Pack 2: Futureproofing Today's Systems

Topics include: data collection and questionnaires; analysing data; bar charts; pie charts.

Pack 3: Future Methods of Payment

Topics include: prime numbers and their properties; divisibility rules; sampling methods; calculating percentages

Pack 4: Education, Environment, Sustainability

Topics include: 3D shapes; adding, subtracting and dividing with decimals; problem solving

Pack 5: Data and Privacy

Topics include: sequences and patterns; problem solving; inverse operations; division with remainders

Whichever activities your students complete, we'd love to see the results, so please share them with @boemuseum **#TheFutureofMoney**

Object in focus: Bitcoin miner

Mathematics curriculum topics: Prime numbers and their properties; products of prime factors; divisibility rules

The Future of Money exhibition explores how we might pay for things in the future and looks at things like cryptocurrency. Bitcoin is an example of a cryptocurrency.

Bitcoins are produced by complex computer processes known as ‘mining’. This is mostly completed using powerful servers but can also be done on a smaller scale using compact, dedicated ‘mining’ devices like the one depicted above.

The technology that produces Bitcoins and other cryptocurrencies is closely related to cryptography and so has a variety of mathematical themes at its root. A fundamental part of both cryptography (hiding and coding information) and cryptocurrencies are prime numbers.

Student activity 1: Cryptography and prime numbers

Modern cryptography (and therefore mining cryptocurrencies) relies on a type of number that has intrigued mathematicians for thousands of years: prime numbers.

All whole numbers have factors. The factors of 6 are 1, 2, 3 and 6 because 6 occurs in the multiplication tables for 1, 2, 3 and 6.

Numbers that have exactly two factors are called **prime numbers.**

Task 1

Question 1: List all the factors of the following numbers:

a) 7 b) 8 c) 9 d) 10 e) 11 f) 12 g) 13

Question 2:

Which of the numbers in question 1 are prime numbers?

Question 3:

How many prime numbers are there between 1 and 20?

Student activity 1: Answers and supporting notes for teachers

Task 1

Question 1: List all the factors of the following numbers:

a) 1, 7 b) 1, 2, 4, 8 c) 1, 3, 9 d) 1, 2, 5, 10 e) 1, 11 f) 1, 2, 3, 4, 6, 12

g) 1, 13

Question 2.

7, 11 & 13.

Note that these numbers all have exactly two factors, and that these factors are 1 and the number itself.

Question 3.

There are eight prime numbers between 1 and 20: 2, 3, 5, 7, 11, 13, 17 & 19.

Student activity 2: Finding prime numbers

In this task you are going to use the grid to identify prime numbers and remove any non-prime numbers. Before you start, cross out the number 1 on your number grid: this isn't prime!

Task 1

a) Find the lowest number that hasn't been crossed out yet: **this must be prime, so circle it!**
b) Any multiple of the number you've just circled cannot be prime (because it has that number as a factor, plus 1 and itself at least!), so **cross out every multiple of the number you've just circled**.
c) Go back to step **a**.

Repeat these steps until every number in your grid has either been crossed out or circled. The numbers that are circled are prime numbers!

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

You can complete this activity on any size grid to find all of the prime numbers between 1 and the largest number.

Student activity 2: Answers and supporting notes for teachers

Task 1

This process for identifying prime numbers is commonly known as *the Sieve of Eratosthenes* and has been used for more than 2,000 years.

Once complete, if you have used a grid of numbers from 1 - 100, the numbers that are circled should be 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

Potential questions for further research:

What is the largest prime number discovered so far? When was it discovered? How often are new prime numbers discovered? Why is finding new prime numbers so difficult?

Student activity 3: Prime numbers and their properties

This activity looks at prime numbers and their properties, some of which make them useful for use in cryptography and the development of cryptocurrencies.

Task 1

The following numbers are the result of multiplying **two prime numbers** together. Can you work out which numbers have been multiplied?

a) 10 b) 21 c) 18 d) 77

e) 85 f) 143 g) 57 h) 323

Task 2

How many different answers can you find for each part of question 1? Compare your answers to those of other people in your class: are they all the same?

Task 3

The following numbers are the result of multiplying **three prime numbers** together. Which ones?

a) 30 b) 70 c) 105 d) 715

e) 595 f) 182 g) 455 h) 289

Task 4

How many different answers can you find for each part of question 1? Compare your answers to those of other people in your class: are they all the same?

Student activity 3: Answers and supporting notes for teachers

Task 1

a) 2 × 5 b) 3 × 7 c) 2 × 9 d) 7 × 11

e) 5 × 17 f) 11 × 13 g) 3 × 19 h) 17 × 19

Task 2

Students should find that there is only one possible answer for each question.

Ask which were the easiest to find, and which were the most difficult. Why?

Task 3

a) 2 × 3 × 5 b) 2 × 5 × 7 c) 3 × 5 × 7 d) 5 × 11 × 13

e) 5 × 7 × 17 f) 2 × 7 × 13 g) 4 × 7 × 13 h) 17 × 17

Ask which were the easiest to find, and which were the most difficult. Can students explain this?

There is an opportunity here for a discussion about discovering which prime numbers have been used by thinking about divisibility rules: for example, a, b and f are even, so one of the prime numbers must be 2.

The digits in a and c sum to a multiple of 3, so one of the prime numbers must be 3.

A, b, c, d, e and g all end in 0 or 5 so one of the prime numbers must be 5.

H introduces the idea that the same prime number can be used more than once.

Task 4

Students should find that there is only one possible answer for each question.

Student activity 4: Encryption algorithms

"RSA" is an encryption algorithm named after its creators, Rivest, Shamir, and Adleman, who described it in 1977. RSA, like many other encryption algorithms, begins with choosing two prime numbers (we could call them p and q). These are multiplied together (we could call the result n). The value n is used as part of a "public key", which means that it is widely known.

If you can find out what someone used for p and q you could recreate the entire process using the same algorithm (the algorithm is complicated but not a secret and is described in various places online).

For example, if $n = 6$, what must p and q have been?

As p and q are both prime and $p \times q = 6$, then p and q must be 2 and 3 (it doesn't matter which is which).

Task 1

In the following questions, each number has been calculated by multiplying together two prime numbers (p and q).

For each of the following values for n, work out what p and q must be:

a) $n = 15$ b) $n = 35$ c) $n = 55$ d) $n = 221$ e) $n = 589$

Task 2

What if $n = 279{,}850{,}309$? Can you work out what p and q must be?

Student activity 4: Answers and supporting notes for teachers

Students do not need to know how RSA and related encryption techniques work, but it is helpful to introduce the idea to set the scene:

"RSA" is an encryption algorithm named after its creators, Rivest, Shamir, and Adleman, who described it in 1977. Like many other encryption algorithms, it begins with choosing two prime numbers (we could call them ***p*** and ***q***) and multiplying them together (we could call the result ***n***). The value ***n*** is used as part of a "public key", which means that it is widely known.

Note: Systems like RSA use an algorithm to generate two sets of numbers from the starting prime numbers: these sets of numbers are called the "public key" and the "private key". The idea is that anyone can use the public key to encrypt a message, but only the individual it is intended for can use their private key to decrypt it.

Task 1

a) 3 and 5 b) 5 and 7 c) 5 and 11 d) 13 and 17 e) 19 and 31

Task 2

This is difficult to work out and most students are unlikely to find an answer. This should be used more as a discussion point around what might help to make it easier to find the answer. Possible discussion prompts are:

- Can we narrow down the range of numbers we're looking in? (e.g. ***n*** isn't even so the largest prime must be less than half of ***n***; similarly it can be quickly discovered that it isn't divisible by 3 or 5, so the largest prime must be less than ⅓ and then ⅕ of ***n*** respectively; if both primes are a similar size then they will be relatively close to the square root of ***n***)
- Is there a quick way to find the factors of a number? (No, not really!)
- Would a list of known prime numbers help? (We could pick pairs from the list and see how close we can get to ***n*** by multiplying them)
- Could technology help? (We could make a really big multiplication grid with all prime numbers, then search the grid for ***n***)
- How big would such a grid need to be?

Whilst RSA is rarely used in cryptocurrency systems today, the principles behind it are similar and these are what allow it to be a system that is both anonymous and secure.

Central Bank Digital Currency (CBDC)

Mathematics curriculum topics: Collecting data; sampling methods; calculating percentages

Object in focus: Anti-CBDC flashnote

CBDC stands for Central Bank Digital Currency and is the name given to a digital currency that is issued by a central bank (such as the Bank of England). The Bank of England is exploring the idea of a CBDC for the UK, known as the Digital Pound. If it was introduced, it would provide a new way for households and businesses to pay for things.

Some people are opposed to a CBDC due to fears that it might be used to track or control what users spend their money on, but ensuring privacy will be a core part of the development of a future Digital Pound by the Bank of England.

The object depicted above is a ‘flashnote’, a type of advertising made to look like a banknote, which in this instance is promoting the use of cash.

Student activity 5: Consultation and sampling

When considering the introduction of something like a Central Bank Digital Currency, or Digital Pound, it is important that everyone has an opportunity to have their say, including asking questions and voicing concerns, but is it realistic to ask *everyone* their opinion?

In February 2023 the Bank of England and HM Treasury led a consultation to explore people's views about the proposed model of the Digital Pound called "The Digital Pound: A new form of money for households and businesses?":

The consultation received more than 50,000 responses. The publication is available at: **https://www.bankofengland.co.uk/paper/2024/responses-to-the-digital-pound-consultation-paper**

At the time of the consultation the UK population was 67,000,000.

Task 1

a) What percentage of the UK population responded to the questionnaire?
b) Why wasn't the entire population included in the survey?
c) Would asking more people make the results more accurate?

Task 2

500 of the people who responded to the questionnaire did so on behalf of organisations.

a) If the number of organisations who took part in the consultation is representative of the number of organisations in the UK, how many organisations are there in the UK?

The table below shows how many of each different type of organisation responded to the survey.

Type of organisation	Number that responded to the survey	Percentage of total	Estimated number of organisations in the UK
Civil society	55		
Technology	50		
Trade body	40		
Consultancy	50		
Financial services	45		
Academic	20		
Consumer group	15		
Other	225		
Total	**500**		

a) Complete the column labelled "percentage of total":
b) Use your answer to task 2 to complete the column labelled "Estimated number of organisations in the UK".

Student activity 5: Answers and supporting notes for teachers

Task 1

a) 0.07% (2dp)
b) It is unrealistic to collect data from the entire population because it would take too much time.
c) Not necessarily: what's important is that the **sample** represents the **population**.

Task 2

a) 500 is 1% of the sample size (50,000), so if it is a representative sample then there should be 0.01 × 67,000,000 = 670,000 organisations in the UK.
b)

Type of organisation	Number that responded to the survey	Percentage of total	Estimated number of organisations in the UK
Civil society	55	11%	73,700
Technology	50	10%	67,000
Trade body	40	8%	53,600
Consultancy	50	10%	67,000
Financial services	45	9%	60,300
Academic	20	4%	26,800
Consumer group	15	3%	20,100
Other	225	45%	301,500
Total	**500**	100%	670,000

Extend this:

- Discuss what sampling strategy may have been used to choose organisations to respond to the survey.
- Imagine your school was asked to choose a representative sample of 5% of its students to respond to a survey about the Digital Pound: How many students would need to be chosen? Explore how you could use different sampling methods (such as random, stratified, systematic sampling) to select students to take part.