The demand for money in the United Kingdom: a further investigation

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The impact of monetary policy on the economy depends on what influences persons or companies to hold more, or less, of their assets in the form of 'money'. As real incomes grow, does the demand for money by persons and companies grow proportionately faster or slower? If the cost of holding money (as measured by the yields to be obtained on competing assets) changes, how much does the demand for money change? The authorities need to have some idea of the answers to questions such as these if they are to operate monetary policy successfully. For example, if the demand for money changes substantially when the cost of holding money changes little, then the authorities would probably need to engineer very large changes in the money stock to produce any significant effect on the real economy.[1]

Economic theory does not provide firm answers to these questions, and economists have therefore undertaken detailed econometric investigation of the relationships between the stock of money and income, wealth, and the prices of other assets. Most of this work has related to the United States economy, however, and much less has been done in the United Kingdom.

Kavanagh and Walters [**5**] demonstrated that over the long run there has been a close relationship between annual observations of the money stock, incomes and interest rates in the United Kingdom. But attempts to gain more detailed knowledge of the determinants of the demand for money in this country have not been as fruitful as has research on U.S. data. Fisher [**2**], Laidler and Parkin [**7**], and Goodhart and Crockett [**3**] have used quarterly data for recent years to try to test the predictability of the demand for money in the short run. All produced closely fitting equations, but estimates of elasticities varied considerably, depending on the precise specification of the equations and on the variables used. The estimates appear to be greatly affected by the time lags assumed in the adjustment of money balances, and further work to unravel this difficult problem of the lag relationships seemed to be called for.

An appendix to this paper describes further econometric research which, it is believed, throws new light on the interrelationships between money, interest rates, and incomes in the United Kingdom. It extends the work of Goodhart and Crockett in two directions. First, there is a more detailed investigation into the time taken for money balances to be adjusted to changes in real income, prices and interest rates; and secondly, the determinants of money holdings by persons and by companies are looked at separately. Each of these approaches yields interesting results, helping to answer some of the questions raised above.

1 These issues were discussed more fully in an article in the June 1970 *Bulletin* [**3**].

References in bold type are listed on page 55.

Empirical results

In most previous work it has usually been assumed either that the full impact of changes in real incomes, prices and interest rates on the demand for money appears immediately, or that the largest effect appears immediately with further effects becoming smaller and smaller as time progresses. In the appendix these assumptions are shown to have been particularly inappropriate when considering the effect of changes in undated gilt-edged yields. A rise in the yield on Consols is shown to have been associated in the current quarter with a *rise* in money holdings, whereas prima facie a rise in interest rates should have made money relatively less attractive to hold and therefore led to a fall in money holdings. However, it is of course not the running yield alone on Consols that is important to an investor, but the yield including any capital gains or losses he expects to make. The apparently perverse association of movements in the yield on Consols and in the money stock in the same quarter probably resulted because investors held 'extrapolative expectations' *i.e.* on seeing a fall in the price of Consols, they expected a further fall and the attraction of the higher running yield was not great enough to overcome this expected capital loss. In the past, the supply of money has increased at such times because the authorities, pursuing their policy of moderating autonomous short-run changes in gilt-edged yields, have been prepared to buy in stock from investors expecting further falls in price. But investors' extrapolative expectations do not appear to have been very long lived; after one quarter the attraction of the higher (or lower) running yield has dominated any further expected fall (or rise) in prices, so that in the long run a rise in the yield on Consols has in fact, in accordance with popular belief, reduced the demand for money. Once the appropriate time-lags were established, the yield on Consols performed markedly better than the three-month local authority rate as an explanation of the total demand for money balances in recent years.

Because movements in real incomes and prices have generally been less erratic than movements in interest rates, it is more difficult to estimate the speed with which changes in these variables have affected the amount of money held. There is also much uncertainty about the scale of the eventual total effect of these changes. These difficulties have been partly resolved by studying separately the demand for money by persons and by companies. Substantial differences between the determinants of the two sectors' money holdings were revealed and the differences were great enough to improve the predictive power of the equations by over 10%.[1] Moreover, a serious statistical problem,[2] which cast doubt on the validity of the method of estimation being used, disappeared when the two sectors' money holdings were split.

1 On the broadest definition of money holdings; no data are available separating persons' and companies' current and deposit accounts.

2 Auto-correlation of the residuals which, in the type of equation being estimated, will lead to inconsistent estimates.

The main difference between the two sectors seems to lie in the speed of their reactions. Persons have changed their money balances more gradually than companies. Although, at any time, the amount of money which persons have wished to hold has depended on real incomes, prices and interest rates in the previous six months, they have not reached this amount immediately – probably because of the cost and inconvenience of switching their wealth from one asset to another – but have made only about 40% of the desired adjustment within one quarter. Companies, on the other hand, appear to have adjusted their money balances rather more quickly to the amounts that they wished to hold.

A further difference between the two sectors is the interest rates which they appear to have regarded as significant for their money balances. Companies seem to have responded to changes in short-term rates (for which the local authority three-month rate is taken as representative) while individuals apparently have been more influenced by long-term rates – represented by the yield on Consols as described earlier. It may be that this result was only obtained because companies hold very few Consols and persons few local authority deposits, so that these particular interest rates are not important to them; it cannot be assumed that the results reported here would hold equally for other short and long-term interest rates. Nevertheless, they do seem to confirm what has generally been believed: that companies have shown a greater and quicker response than persons to changes in interest rates. Moreover, as companies have been responding to changes in more volatile short-term rates and persons to changes in less volatile long-term rates, the variation in companies' money balances as a result of cyclical interest rate changes has been more than twice that of personal balances.

In most theoretical economic discussion, a 1% rise in the price level is held ultimately to lead to a rise of 1% in the demand for money. However, in the tests described here, a smaller effect than this was found for both persons and companies, although for persons the difference was insignificant.

On the direct effect of a 1% rise in real income on the demand for money, there is no generally accepted theoretical view. This research indicates an increase of over 2% in persons' demand for money; for companies, the evidence is much less clear, but suggests a response in excess of 1%. Such a large reaction to changes in real income may seem unlikely to those accustomed to accept the traditional one-for-one relationship, but the results appear to be sufficiently significant for that hypothesis to be rejected with some confidence. One implication of this is that people seem to treat money in the same way as luxury goods – as something of which they like to hold relatively more when their real incomes are higher. If this is so, an apparently rapid rate of monetary expansion might actually be restrictive in its effects when associated with a strong increase in output, because of this apparent preference for holding a considerable proportion of the income earned from the increasing output in money balances, rather than spending or investing it and stimulating yet more output.

The results that have been presented for the personal sector appear to be quite well determined. However, it is clear that further research into companies' money balances is needed. Possibly the type of demand-for-money function estimated in this paper is too highly simplified to capture the subtleties of corporate portfolio management. A number of liquid assets held by companies are close substitutes for money and can be converted quickly into money when required. Nor is money necessary for transactions purposes; a large proportion of company spending is made by increasing bank overdrafts. Therefore the somewhat special position normally held by money in a portfolio may be fairly weak for companies, and it may be possible to predict their demand for money successfully only as part of a larger exercise to predict a wider portfolio.

Conclusions

Although the factors determining the amounts of money held by individuals and companies are, without doubt, more numerous and complex than those discussed here, equations of the types described provide a sufficiently accurate statistical explanation of past movements in the stock of money to be a useful guide for monetary policy. It is already well recognised that the authorities cannot pursue independent objectives for monetary aggregates and for interest rates at the same time; possible policy options must satisfy the private sector's demand for money, and so estimates of demand-for-money equations provide a schedule of the combinations of monetary aggregates and interest rates which the authorities can reasonably pursue at any time – given the current, and recent levels of incomes.

In practice, the authorities do not know the current level of incomes in the economy as a whole; a reasonably comprehensive and reliable picture emerges only some months after the event. Meanwhile, they must grasp at straws in the wind. As interest rates are known from day to day and monthly data on the money stock are received quite quickly, the demand-for-money equations can be applied to discover what level of income would be consistent with the observed interest rates and money stock; this provides an early, if approximate, indicator of movements in income besides those already available.

The possible options for future monetary policy will depend on future incomes; but future incomes themselves will be influenced by monetary policy, though with an uncertain, and perhaps variable, lag. To explore fully the effects of possible monetary policies, the demand-for-money equations would have to be integrated into a larger model capable of predicting the effect of financial factors on output, incomes and prices. None of the major models of the U.K. economy developed so far has succeeded in formalising the role of financial factors to any extent; incorporating the effects of monetary policy in such models rests on the judgment of those who supply data for the models. Consequently it is not yet possible to simulate the effects of alternative monetary policies in the same way as is already done for fiscal policies.

Nevertheless, the estimates of demand-for-money equations presented above can help to clarify the choices facing the monetary authorities. One example of this arises from the suggestion that the personal sector's holdings of money do not respond quickly to changes in interest rates. If this is so, the initial impact of any contraction or expansion of the money stock will fall on companies and financial institutions. Moreover, the slow adjustment of the personal sector's money holdings to changes in interest rates implies that there would have to be sharp fluctuations in interest rates in order to bring about any sudden adjustments in the rate of growth of monetary aggregates. So unless the authorities are prepared to enforce sharp, short-lived changes in monetary conditions their scope for achieving a sudden change in the direction of monetary policy must be limited.

Appendix

Estimation of demand-for-money functions

References in bold type are listed on page 55

The first part of this paper discussed the need for reliable estimates of demand-for-money functions and presented the main results of some further empirical work on such functions. That empirical work is described more fully in this appendix.

Lagged adjustment

It is clear from Fisher's [**2**] and from Goodhart and Crockett's [**3**] results that there are important lags in the demand-for-money function: poor estimates were obtained of equations relating money holdings to income and interest rates in only the current quarter. It is important for the correct interpretation of events and thus for policy prescription to be able to identify any slow adjustment of money balances to changed economic circumstances. No attempt is made here to identify or explain any variability in the lags; although any such variability would also be important to policy it is doubtful whether the data would stand such close examination, and so the investigation was confined to attempting to identify the average lag pattern.

Goodhart and Crockett proposed a stock adjustment model, in which desired money holdings, M*, depend only on the current levels of money income, Y, and an interest rate, R.

$$M^*_t = a_0 + a_1 Y_t + a_2 R_t \qquad (1)$$

Because of the costs, both monetary and non-monetary, incurred in buying and selling assets, money holders will usually delay adjustment of their money balances to their 'desired' level. It is assumed that, in any one quarter, money holders adjust their actual money balances by a fixed proportion, b, of the movement they would need to make to reach their desired money balances.

$$M_t = M_{t-1} + b(M^*_t - M_{t-1}) + u_t \qquad (2)$$

Elimination of M^*_t from these equations leads to the reduced form:

$$M_t = ba_0 + ba_1 Y_t + ba_2 R_t + (1-b) M_{t-1} + u_t \qquad (3)$$

Estimates of (3) were obtained (by ordinary least squares) for three definitions of the money stock and two interest rates.[1] Data for the widest definition of the money stock commonly used, M_3, which includes currency and all resident deposits with U.K. banks, are available only from 1963; data for M_1 and M_2, approximately corresponding to the usual narrow and broad definitions of money applied to the London clearing banks alone, are available for a much longer period and were therefore used as well. The relative importance of these banks in the system has, however, declined in recent years. The short and long interest rates were not included together, although theoretically each has a role to play.[2] Had the two rates been included together, the similarity between them would have hindered rather than helped understanding of the demand for money. For further comments on the data, see [**3**], page 191.

Table A

Variables used	Estimated long-run elasticities of: Nominal income	Interest rate (a)	Rate of adjustment (*b*)	Standard error of estimate (*c*)
M_1;RS	1·47	−1·30	0·032	0·00953
M_2;RS	1·07	−0·42	0·041	0·00663
M_3;RS	1·20	−0·09	0·255	0·00851
M_1;RL	0·99	−0·67	0·124	0·00986
M_2;RL	1·91	−1·55	0·034	0·00598
M_3;RL	1·34	−0·19	0·296	0·00831

(*a*) The interest rate term in the estimated equations was log (1+r), but for comparison with elasticities estimated on log r, the conventional elasticity, measured at the mean value of the interest rate, is given.

(*b*) Defined as is "b" in equation (2).

(*c*) Coefficients of determination are, in all cases, greater than 0·99.

Estimates of the parameters of equation (3), based on the data described in footnote *1*, are shown in Table A. They suggest an income-elasticity of rather over unity; the estimated interest-elasticity varies considerably; and the estimated rate of adjustment is very slow indeed (and not significantly different from zero[3]). At the estimated rates, it would take around five years to make half the long-run adjustment to a permanent change in income or interest rates; for M_3, this 'median lag' is estimated to be six or seven months.

Equation (3) was also estimated with nominal income split into real income, Q, and the price level, P.

Table B

Variables used	Estimated long-run elasticities of: Real income	Price	Interest rate	Rate of adjustment	Standard error of estimate
M_1;RS	−2·38	17·33	−9·52	0·005	0·00958
M_2;RS	1·28	0·90	−0·44	0·040	0·00669
M_3;RS	2·47	0·51	−0·12	0·321	0·00804
M_1;RL	0·79	1·36	−0·79	0·108	0·00994
M_2;RL	1·35	2·31	−1·46	0·037	0·00602
M_3;RL	1·81	1·02	−0·18	0·343	0·00832

1 The precise variables used were:

M_1: *Currency and net current account deposits of the London clearing banks* (quarterly average of mid-monthly observations), seasonally adjusted, £ millions.

M_2: *Currency and net deposits of London clearing banks* (quarterly average of mid-monthly observations), seasonally adjusted, £ millions.

M_3: *Currency and net deposits of U.K. residents with the U.K. banking sector* (end-quarter figures), adjusted seasonally and for day-of-the-week variations, £ millions.

Y: Average of the three official estimates of *gross domestic product at factor cost*, separately derived from output data, expenditure data and income data, seasonally adjusted, £ millions. (Before 1958 it was possible to take the average of only the income and expenditure-based estimates.)

RS: 1+the *interest rate on 3-month local authority deposits.*

RL: 1+the *interest rate on 2½% Consolidated Stock*, an undated government bond.

All variables are cast in logarithmic form, so as to yield functions with constant elasticities. The estimates presented in this paper are based on a revised version of Goodhart and Crockett's data. Functions for M_1 and M_2 were estimated for 1956:1 to 1969:4; and M_3 for 1964:1 to 1970:4.

It should be noted that the definitions of M_1 and M_2 differ from those in Table 12 of the annex.

2 See, for example, Laidler [**6**].

3 Estimated standard errors of the estimated rates of adjustment range from 0·05 to 0·09. Standard errors of the long-run elasticities have not been calculated.

In only one case is the fit of the equation, as measured by the standard error of estimate, improved. Great confidence cannot be attached to separate estimates of price and real income-elasticities, for the real income and price variables are too similar for different effects on money balances to be estimated. The highest rates of inflation have roughly coincided with the periods of most rapid real growth, to produce a simple correlation coefficient of 0·99 between prices and real incomes.

However, equations such as (3) are reduced forms not only of stock adjustment models as above, but also, in particular, of adaptive expectations models. Friedman and others have postulated that the demand for money depends on permanent, or expected, income, which is defined to be a geometrically weighted average of past levels of income. A model which postulates that money holders adapt to permanent income, Y^*, and to a similarly defined permanent interest rate, R^*, with the same geometric weighting, *viz.*

$$M_t = a_0 + a_1 Y^*_t + a_2 R^*_t + v_t \tag{4}$$

$$Y^*_t = bY_t + b(1-b)Y_{t-1} + b(1-b)^2 Y_{t-2} + \dots \tag{5}$$

$$R^*_t = bR_t + b(1-b)R_{t-1} + b(1-b)^2 R_{t-2} + \dots \tag{6}$$

also has (3) as its reduced form.

Different geometric lags

But why should the rates of decay used to define the permanent variables be equal? Indeed, it has been usual to propose that money holders react slowly to income, but only to the current values of interest rates. On the other hand, it seems likely that individuals are more rapidly aware of changes in their incomes than of changes in interest rates. If the slow rate of adjustment of money balances has any cause other than pure stock adjustment, as in equations (1) and (2), then the speeds of adjustment to incomes and interest rates may differ. To admit the possibility that they do, an extended version of the above model was tested. Nominal income was split into real income and prices, and different rates of adjustment were allowed for real income, prices and the interest rate. There are then seven structural parameters to be estimated (the long-run elasticity and rate of decay for each of the three exogenous variables, together with a constant), but the reduced form has thirteen variables:

$$\begin{aligned} M_t = {} & c_0 + c_1 Q_t + c_2 Q_{t-1} + c_3 Q_{t-2} + c_4 P_t + c_5 P_{t-1} + c_6 P_{t-2} + c_7 R_t \\ & + c_8 R_{t-1} + c_9 R_{t-2} + c_{10} M_{t-1} + c_{11} M_{t-2} + c_{12} M_{t-3} + w_t \end{aligned} \tag{7}$$

The thirteen coefficients, c_0 to c_{12}, are non-linear functions of the seven structural parameters. Consequently, the parameters must be estimated iteratively. The results of such an estimation are presented in Table C.[1] In some cases, the maximum maximorum gives particularly implausible estimates; the more plausible local maxima are then shown in the table.[2]

1 Assuming that the stochastic term, w, has an independent normal distribution, maximum likelihood estimates of (7) were obtained. The residual sum of squares, as a function of the structural parameters, was minimised by Powell's method [8].
The estimates in Tables C and D were made on Goodhart and Crockett's original data, and not on the revised data used elsewhere in this paper. In view of the extremely poor results obtained from the original data, it was not thought worthwhile to re-estimate. The data period for M_1 and M_2 is 1956:1 to 1969:3, and for M_3, 1963:4 to 1969:3.

2 The greater maxima discovered were (set out as in Table C):

M_2;RS	0·10	0·15	0·41	0·67	1·04	−0·03	0·00684
	(0·15)	(0·12)	(0·18)	(0·30)	(0·32)	(0·01)	
M_2;RL	1·28	0·17	0·01	0·11	0·63	1·00	0·00697
	(0·37)	(0·31)	(0·03)	(0·06)	(0·45)	(0·45)	
M_3;RL	−0·29	0·52	9·03	1·45	0·48	−0·01	0·01116
	(0·18)	(1·16)	(128·79)	(0·27)	(0·33)	(0·08)	

Table C

Variables used	Estimated long-run elasticities of: Q	P	R	Estimated rates of adjustment: (*a*) Q	P	R	Standard error of estimate
M_1;RS	0·93 (0·19)	0·22 (0·14)	−0·10 (0·05)	0·28 (0·13)	1·21 (0·19)	0·37 (0·15)	0·01038
M_2;RS	1·09 (0·28)	0·14 (0·09)	−0·03 (0·02)	0·08 (0·06)	1·16 (0·21)	0·42 (0·20)	0·00693
M_3;RS	1·10 (14·77)	−0·00 (0·20)	−0·08 (0·06)	0·02 (0·13)	1·40 (0·30)	0·54 (0·25)	0·01070
M_1;RL	0·84 (0·21)	0·34 (0·17)	−0·12 (0·09)	0·37 (0·20)	1·07 (0·21)	0·44 (0·21)	0·01079
M_2;RL	1·05 (0·38)	0·15 (0·11)	−0·03 (0·06)	0·06 (0·06)	1·10 (0·24)	0·46 (0·22)	0·00712
M_3;RL	1·06 (10·69)	0·06 (0·27)	−0·04 (0·13)	0·03 (0·15)	1·26 (0·44)	0·62 (0·43)	0·01215

Note: asymptotic standard errors of the estimated coefficients are shown in brackets.
(*a*) Defined as is "b" in equation (5). This is comparable with the initial rate of adjustment in Tables A and B.

The estimates are poorly determined, and do not accord with *a priori* expectations. Most satisfactory is the estimate of the real income-elasticity at around unity, usually with slow adjustment. But a price-elasticity of around 0·2, all the effect being felt in the current quarter, is hardly credible; few economists would expect a price elasticity greatly different from unity.[1] For interest rates we have the paradox of an estimated elasticity usually not significantly different from zero, but having a quite rapid effect. Moreover, in each of the six cases, the fit of the equation has been worsened[2] by allowing for different rates of adjustment to changes in real incomes, prices and interest rates.[3]

Rational lags

Clearly this specification of the model is erroneous. Such a conclusion is supported by unconstrained estimation (by ordinary least squares) of equation (7), standard errors of which are reproduced in Table D. Imposition of the constraints (*i.e.* forcing the lagged effects of the exogenous variables to decline geometrically) makes the fit of equation (7) significantly worse.[4] But can any economic

Table D

Variables used	Standard error of estimate	Significance of constraints (*a*)
M_1;RS	0·00879	1%
M_2;RS	0·00656	20%
M_3;RS	0·01001	(30%)
M_1;RL	0·00790	0·1%
M_2;RL	0·00585	1%
M_3;RL	0·00974	20%

(a) Significance level at which the hypothesis underlying Table C would be rejected in favour of a hypothesis underlying these results (using an F-test on the standard errors of estimate).

meaning be attached to the unconstrained estimates? Indeed it can; equation (7) is a rational lag equation as used by Jorgenson [**4**]. In a similar way to that in which it is possible to 'unscramble' the estimates of equation (3) into long-run elasticities and a lag profile, it is possible to interpret the estimates of equation (7) into long-run elasticities and lag profiles for each of the three exogenous variables.

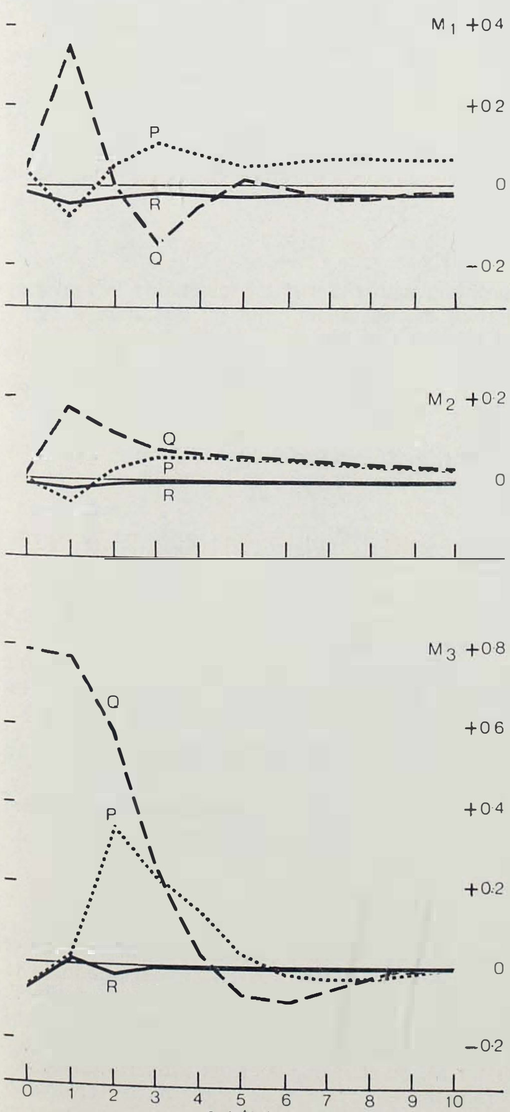
1 It might be suggested that rising prices of goods have produced higher and higher expectations of inflation, thus reducing the attractiveness of holding money; however, any such growing expectations of inflation should be reflected in the nominal interest rate which appears as an independent variable.
2 When compared with estimates of equation (3) on the same data.
3 The assumed auto-regressive nature of the error term has also been changed, but analysis of the residuals of the estimated equations in Tables A and C suggests this change to be unimportant. We shall return later to the problem of auto-regression.
4 The rather low confidence attached to this statement for equations using M_3 results from the unconstrained estimates having only eleven degrees of freedom.

Table E

Variables used	Estimated long-run elasticities of: Real income	Price	Interest rate	Standard error of estimate
M_1;RS	∞	∞	∞	0·00863
M_2;RS	1·00	0·58	−0·10	0·00661
M_3;RS	2·19	0·62	−0·08	0·00857
M_1;RL	0·42	1·79	−0·86	0·00807
M_2;RL	0·55	1·94	−0·75	0·00582
M_3;RL	1·69	1·03	−0·14	0·00770

Chart A

Rational lag profiles[a]
(Local authority rate)



a The chart shows the percentage change estimated to be produced in each monetary aggregate in the ten quarters following a 1% change in real incomes, prices or interest rates.

Returning to the data used in Tables A and B, unconstrained estimates of equation (7) are presented in Table E; the lag profiles are shown in Charts A and B. The long-run elasticities bear a strong resemblance to those in Table B; but the freer form of the lag profiles – involving the loss of eight degrees of freedom – has resulted in improved standard errors of estimate in five of the six equations. However, the real income and price-elasticities still vary considerably about unity; the interest-elasticity also varies, but is, as expected, higher for the long interest rate, and higher for narrower definitions of money. The effect of a change in real income usually appears quite quickly: in three equations (M_3;RS, M_1;RL and M_3;RL) over half the long-run effect has been felt before half the following quarter has elapsed. In each equation, the response to a change in prices is estimated to be slower than to real income. But the most interesting estimate is the lag profile of interest rates. Whereas the reduction in money balances resulting from a rise in the local authority rate starts in the current quarter, holdings of M_1 and of M_3 *rise* in a quarter in which the yield on Consols rises – and holdings of M_2 fall very little.

This paradoxical reaction to a change in the yield on Consols need not imply that money holders are irrational. The return from holding bonds is not wholly represented by their running yield, but will also include capital gains. If investors expect yields to continue moving in the same direction in the future as in the recent past, then a rise in yields will initially cause investors to expect capital losses and consequently they will stay more liquid. Short-run extrapolative expectations will produce a perverse initial reaction to a rise in bond yields, provided that the expected capital losses are strong enough to offset the more attractive running yield on bonds.

But the estimated magnitude of this initial reaction to a change in bond yields also depends on the authorities' policy in the gilt-edged market; believing that extrapolative expectations had a strong influence on the demand for gilt-edged, the Bank of England had usually aimed during this period to reduce autonomous short-run fluctuations in interest rates by buying stock on a weak market, and selling when prices were rising, so that a rise in Consols yield will normally have been associated with the authorities increasing the supply of money by buying in stock.

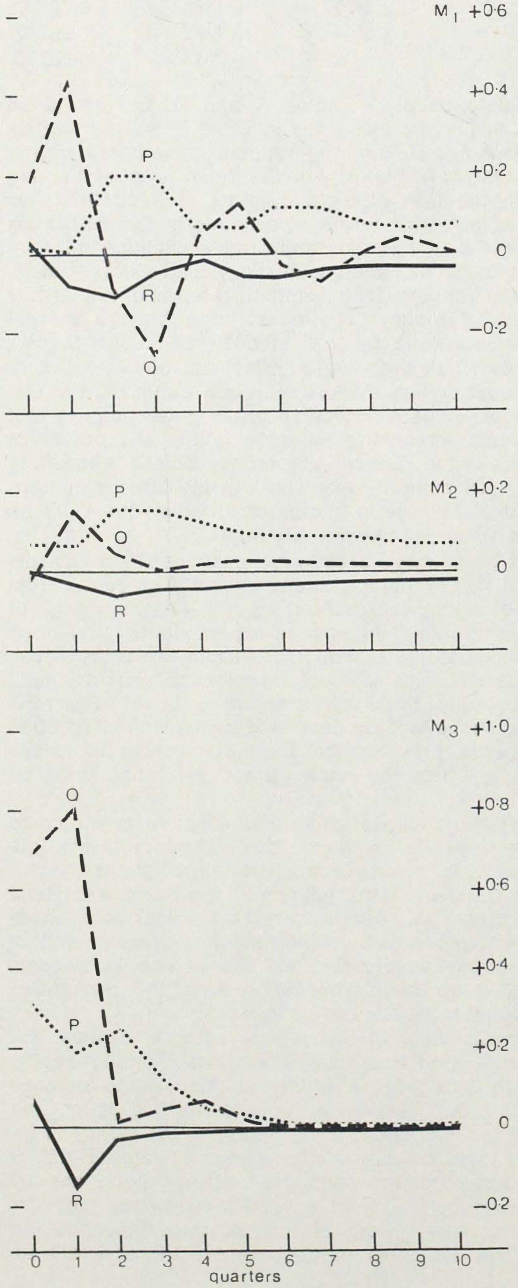
The role of Consols yield in the private sector's demand for money cannot be separated from its role in the authorities' supply function without fully specifying a supply function – and perhaps not even then. Such a task is outside the scope of this paper, and the assumption will be maintained that the supply of money is inelastic with respect to incomes, prices and interest rates. But it is important not to forget that the estimates obtained here, and by other researchers are not strictly of a demand-for-money function; to the extent that incomes, prices or interest rates influence the supply of money, the estimates of the demand function will be biased.

It is clear that forcing the effect of a change in Consols yield to decline geometrically from the current quarter was an important error in the original specification. A comparison of the Consols results of Table E with those of Table B shows the former to be an improvement significant at the 0·1% level[1] for M_1, at the 20% level for M_2 and a non-significant improvement for M_3. On the other hand, the local authority rate results are, on balance, little different when

1 That is, there would be less than one chance in a thousand of finding such a large improvement if the original model were, in fact, correct.

Chart B

Rational lag profiles (Consols yield)



the more liberal rational lag form is permitted (although the M_1 equation is improved at the 5% level, there is no significant improvement for M_2, and the fit is worse for M_3).[1]

Now that the effect of a change in Consols yield has not been forced to decline from the current quarter, for each definition of money the Consols yield provides a much better explanation of the money stock than the local authority rate does. The shorter rate might have been expected to have been better, for surely short-term assets will be closer substitutes for money. Over a long period the local authority rate has moved fairly closely with Bank rate and other short-term rates, and so should pick up substitution between money and other short-term financial assets; the Consols yield has probably moved more closely with expectations of price inflation, and may pick up substitution between money and a much wider set of assets – both longer-term financial assets and real assets.

In the earlier results, there was little difference in the fit of the equations when prices and real incomes were split, or combined as nominal income. In the rational lag estimates, the explanation of M_1 is significantly improved (at the 10% and 5% levels respectively for the short[2] and long rates) by splitting prices and real incomes, but there is no significant improvement in the explanations of M_2 and M_3. The lag profiles and long-run elasticities of interest rates are much the same in the 'combined' estimates as in those in Table E, and the lags and elasticities of nominal income are approximate averages of those for real incomes and prices – the nominal income-elasticity is always near unity. Again, the data do not seem capable of distinguishing different effects of changes in real incomes and prices.

Auto-correlation

An acknowledged problem with rational lag estimates is that they imply auto-correlation of the residuals.[3] Let us postulate a distributed lag model with random errors, e:

$$M_t = \frac{\alpha(E)}{\delta(E)} Q_t + \frac{\beta(E)}{\delta(E)} P_t + \frac{\gamma(E)}{\delta(E)} R_t + e_t \qquad (8)$$

where E is a lag operator, α, β, γ, δ are polynomials in E, and $\delta(E)$ has a unit constant term. Then the rational lag estimate is of the form:

$$M_t = \alpha(E)Q_t + \beta(E)P_t + \gamma(E)R_t + [1-\delta(E)]M_t + \delta(E)e_t \qquad (9)$$

which is, in general, an equation with auto-correlated errors. The presence of lagged dependent variables will ensure that the estimates of the parameters are biased.

An attempt to avoid this source of bias by using Shirley Almon's distributed lag technique [1] did not yield useful estimates. The estimated lag profiles depended greatly on the degree and length of the polynomial to which they were constrained, and the fits of the estimated equations were significantly worse than of those in Table D.

We can gain some insight into the seriousness of the problem by searching for first-order auto-correlation while estimating (9).

All variables were transformed as:

$$x_t \rightarrow x_t - \rho x_{t-1}$$

and ρ was varied so as to minimise the residual sum of squares of the estimated form (9). A quite coarse search proved adequate to determine the approximate optimal values of ρ given in Table F; the residual sums of squares described clear U-shaped functions of ρ. There was no evidence of first-order auto-correlation in the estimates for M_1, or for M_2 on the long interest rate. However, the matter should not rest there, for one or both of the estimated

1 Fisher [2], using a similar bond yield, found a positive interest-elasticity when estimating a demand-for-money function in first-difference form. This would be expected, given the above results.

2 Although the lag profile does converge in this case if prices and real income are not split.

3 That is, that the error in any quarter depends to some extent on the errors in earlier quarters.

coefficients of M_{t-2} and M_{t-3} are significantly different from zero in each of these equations; a test should be performed for higher orders of auto-correlation.

Table F

Variables used	Estimate of ρ	Standard error of estimate (*a*)	Significance of $\rho \neq o$
M_1;RS	0·0	0·00873	N.S.
M_2;RS	+0·6	0·00649	20%
M_3;RS	−0·6	0·00776	10%
M_1;RL	−0·3	0·00806	N.S.
M_2;RL	+0·3	0·00585	N.S.
M_3;RL	−0·6	0·00763	(30%)

(*a*) Allowing for the loss of an additional degree of freedom in estimating ρ.

The negative auto-correlation displayed by the estimated equations for M_3 is of about the same magnitude as the coefficient of M_{t-1}. If the correct specification is a distributed lag model with random errors (8), then this result would be expected. However, further work on disaggregated data for money holdings of persons and companies throws more light on the nature of the lags.

Money holdings by persons and companies

One major omission in the foregoing analysis is the failure to allow for different money holders reacting by differing amounts and at different speeds to changes in incomes and interest rates. Ideally, one would like to study individual money holders' behaviour. In the absence of data for large numbers of individuals, a far less ambitious task is undertaken here: money holdings of the personal and of the industrial and commercial company sectors are analysed separately. The data used are on the same basis as M_3, of which persons (including unincorporated businesses) hold about 65%, and industrial and commercial companies around 25%; the remaining 10% is held by financial companies and public sector bodies.

Personal sector

The single geometric lag and rational lag models which had been applied to the aggregate money stock were tested on personal money holdings. The same two interest rates were used, but an alternative measure of income was also tried; personal disposable income was tested in addition to gross domestic product, but the results were substantially the same. Although personal income should in theory be more closely related to individuals' desire to hold money for transactions purposes, estimates using G.D.P. are presented below in order to preserve comparability with the aggregate estimates.

The results are much more unequivocal than those for aggregate money holdings. The local authority rate is not a significant determinant of personal money holdings;[1] it may be that the return on some other short-term asset, more widely held by the personal sector, does influence their demand for money. But persons show a similar response to changes in Consols yield as in the aggregate data – a positive (though insignificant) contemporaneous response, followed by the expected negative response in the next quarter.

The number of lagged variables included in the estimated rational lag equation (7) arose originally from its use in constrained form to estimate the triple geometric lag model. In the unconstrained form there is no reason to retain all these variables. In the personal sector estimates, the coefficients of Q_{t-2}, P_{t-1}, P_{t-2}, R_{t-2}, M_{t-2} and M_{t-3} are not significantly different from zero.[2] If these variables are excluded, we are left with quite a simple estimate:

$$MP_t = -4{\cdot}129 + \underset{(0{\cdot}220)}{0{\cdot}457}Q_t + \underset{(0{\cdot}221)}{0{\cdot}396}Q_{t-1} + \underset{(0{\cdot}079)}{0{\cdot}336}P_t - \underset{(0{\cdot}421)}{1{\cdot}686}RL_{t-1} + \underset{(0{\cdot}093)}{0{\cdot}628}MP_{t-1} \qquad (10)$$

$$\bar{R}^2 = 0{\cdot}997; \quad \text{s.e.e.} = 0{\cdot}00670$$

Long-run elasticities: Q, 2·29; P, 0·90; RL, −0·30.

1 For example, in the estimate of (3), it has a t-statistic of only 1·17.
2 Singly, or in combination, at the 20% level.

Furthermore, this equation exhibits no first-order auto-correlation (when tested as earlier). If (10) were an estimate of a distributed lag model in the form of equation (9), this would imply that the errors in the behavioural equation (8) followed an auto-regressive process. But if the lagged dependent variable in (10) results from persons following a stock adjustment process of the form of equation (2), then equation (10) should not display auto-correlation – assuming (2) does not do so. Thus, if we are restricting ourselves to systems with random errors, we can accept a hypothesis that persons adjust according to the stock adjustment equation (2),[1] rather than that (10) is essentially an estimate of the distributed lag model in (8).[2] Moreover, the rate of adjustment of money balances to their desired level is much more credible than in Goodhart and Crockett [**3**]; even after allowing for the dependence of desired money balances on lagged incomes and interest rates, over half of the effect of a continuing shift in any of the three exogenous variables appears within eight months – only six months for a change in real income or in the price level.

Company sector

A similar procedure was followed to that described for persons, but the results are quite different. Firstly, companies respond more strongly to changes in the short than in the long-term interest rate. Secondly, the estimated coefficients of the rational lag forms do not yield plausible lag profiles and elasticities – the price-elasticity is negative. Thirdly, there is serious first-order auto-correlation of around $-0{\cdot}7$, significant at the 1% level. It appears to be the behaviour of companies' money balances which produced some serial correlation in the aggregate equation.

One possible source of difficulty is that G.D.P. may not provide an adequate measure of companies' turnover, or whatever factors are the major determinants of their desire to hold money. The replacement of G.D.P. by Goodhart and Crockett's industrial output series[3] yields much more plausible elasticities and lag profiles; the goodness of fit of the equations is very little changed. Moreover, the coefficients of the lagged dependent variables are no longer significantly different from zero. Again removing lagged variables not significant at the 20% level,[4] we have the equation:

$$MC_t = \underset{(0{\cdot}395)}{1{\cdot}936}Q_t + \underset{(0{\cdot}475)}{0{\cdot}833}Q_{t-1} + \underset{(0{\cdot}188)}{0{\cdot}406}P_t - \underset{(0{\cdot}839)}{4{\cdot}327}RS_t + \underset{(1{\cdot}179)}{0{\cdot}630}RS_{t-1} - \underset{(1{\cdot}106)}{0{\cdot}008}RS_{t-2} - \underset{(0{\cdot}768)}{1{\cdot}934}RS_{t-3} - 4{\cdot}574 \qquad (11)$$

$$\bar{R}^2 = 0{\cdot}983; \quad \text{s.e.e.} = 0{\cdot}01459; \quad \text{D-W} = 1{\cdot}80.$$

Long-run elasticities: Q, 2·77; P, 0·41; RS, −0·36.

However, the coefficients of this equation are not very well determined, and change noticeably if the data period is varied. Using some more recent data with total final expenditure as the income variable, a price-elasticity of unity and a real income-elasticity of 1·3 have been estimated. Despite the doubts which such variability in the estimates must raise, two features consistently emerge; the real income-elasticity is greater than unity, and companies do show some immediate response to a change in the local authority rate.

1 With their *desired* money balances, M*, depending on incomes and interest rates a little time previously.

2 This includes rejection of a permanent income/permanent interest rate model.

3 Real income is measured by the industrial production index; prices by wholesale prices of manufactured products.

4 Except that RS_{t-3} was added as it was found to make a significant improvement to the fit of the equation (RS_{t-4} does not).

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