The note circulation: recent developments

Since the last article in the *Bulletin* on the note circulation, in March 1966, there have been a number of developments. Chief among these are the measures taken by the Bank to moderate the demand for new notes, ·and the introduction of a new series of notes. This article describes these developments and also the changes which have occurred in the note circulation and its composition. It explains too the importance to the Bank of forecasts of the note circulation, a subject which is considered in more detail in the appendix.

The total circulation and its composition have changed markedly since the beginning of 1966. The total value of the circulation has risen by over 80% (see Table A), a somewhat slower rise than consumers' expenditure at current prices. It has occurred at varying speeds depending on changes in the pace of economic growth and in the rate of inflation, but there has been an increase every year. However, despite the growth in the total value of the circulation, the number of notes making up the total has changed very little (see Table B).

The reason for this stability in the number of notes when the total value of the circulation has risen sharply is of course that there are now more higher denomination notes in circulation. Issue of 10s notes, which in 1965 represented 14% of the number of all notes in circulation, ceased in October 1969; there has been a sharp increase in the number of £5 and £10 notes; and a £20 note was introduced in 1970. As a result the average value of all notes in circulation has risen since 1965 from £1·80 to about £3·20. Despite this rise the average is still low compared with most other industrialised countries. In them, the average at the end of last year ranged, for example, from about £4·50—£5 in the United States, Canada, and Japan, through a band between about £6·50 and £9 in the larger EEC countries, up to some £14 in Switzerland. New Zealand, the Republic of Ireland, and Australia, with average values from over £3·50 up to about £5, also use more high-value notes than this country.

The circulation not only varies in the long term; there are also short-term fluctuations caused by the seasonal variation in the demand for notes (most markedly about Christmas). Because the Bank wish if possible to maintain steady production at their Printing Works, seasonal fluctuations are largely met by building up and running down stocks of new and reissuable notes. To do this efficiently requires reasonably accurate forecasts of the short-term demand for notes, and these are obtained by projecting the trend of the weekly figures into the near future and then allowing for seasonal and other effects.

With the short-term fluctuations thus allowed for, production of notes can in theory remain roughly constant throughout the year. However the rate of production which can be maintained clearly depends upon available resources of men and machines; and, as these cannot be greatly varied at all quickly, it is necessary to forecast the quantity of each denomination that will be needed during the next few years. Using data for several years past, a statistical relationship is derived between consumers' expenditure at current prices and the total value of the note circulation. This relationship is then used in conjunction with official forecasts of consumers' expenditure over the next few years to estimate the average value of the note circulation year by year. The final step is to project in a simple way for each denomination the current trends of its share of the total and of its average·life, and thus to estimate the quantity of each denomination that will be required. Further details of these long-term and short-term forecasts are given in the appendix.

The new series of notes referred to at the beginning of the article was announced in March 1968. There were a number of reasons for it. It enabled the Bank to reduce slightly the sizes of the different denominations, and so economise on costs; a generally smaller size of note

facilitated the addition of a £20 note of convenient size at the top end of the range; and improved techniques in design made it possible to introduce greater protection against forgery.

The first note in the new series, the £20, was issued in July 1970. £20 notes had been issued by the Bank previously, but none since 1943. The present £20 note includes on the front a new portrait of the Queen and a vignette depicting St George and the Dragon. On the back the dominant features are a reproduction of the statue of Shakespeare from the Kent Memorial in Westminster Abbey and a vignette of the balcony scene from *Romeo and Juliet*. Demand for £20 notes has grown steadily since 1970, and they now account for over 7% by value of the total circulation.

The second note in the new series, the £5, followed in November 1971. It also includes a portrait of the Queen on the front; the vignette on the front is of a Winged Victory, and on the back the main features are a portrait of the first Duke of Wellington and a vignette depicting a battle scene of the Peninsular War. By May 1973 the new note had achieved a sufficiently wide circulation for the Bank to call in the previous £5 note. This ceased to be legal tender after the end of August 1973, about ten and a half years after it was first issued. But notes of the earlier series continue to be exchangeable at the Head Office of the Bank indefinitely – as do all Bank of England notes.

Also last year the Bank made a small change in the method of printing the new £5 note. Initially part of the 'Wellington' design on the back had been produced by direct plate printing. Last year the Bank began to print the back wholly by offset processes. The design of the note remained unchanged except for the addition of a small letter 'L' in the lower left-hand corner. The change of process enabled the Bank to print in a continuous run and bring existing equipment into use to meet the rising demand for £5 notes.

The new series of notes will be completed by new £10 and £1 designs. The former is likely to be issued shortly, and the £1 will follow in a few years' time.

The new series retains a link with the previous series in that the predominant colour of the design for each denomination remains similar to that of its predecessor. Unity of design and concept within the new series is based on the portrait of the Queen on the front, the depiction of a famous historical figure on the back, and the continuity of designer: all the notes are designed by H. N. Eccleston, ARWS, RE, a member of the Bank's staff.

The second main development since 1966 has been measures to economise on the costs of Bank note production, transport, and destruction. Since the profits of the Bank's Issue Department accrue direct to the Treasury, any economies in costs which the Bank can make benefit the public purse.

The number of new notes issued in recent years is shown in Table C: the figures show a steady growth of demand up to and including 1965. By 1966 the average life of the £1 note had fallen to eight months (from twenty months ten years earlier), and more and more notes were coming back to the Bank for destruction still clean and long before the end of their useful life. The Bank were issuing four times as many notes a head of the population as were issued in comparable countries abroad. The situation was in the Bank's view wasteful of real resources in production and destruction, and entailed excessive handling and transporting of notes within the banking system.

With the co-operation of the clearing banks, the Bank acted to restrain the demand for new notes. They encouraged the public to make greater use of higher denomination notes, and to this end planned the issue of the £20 note. But the chief method whereby the Bank sought economies was to extend the average life of notes by issuing fewer new ones. Thus the clearing banks agreed to proposals from the Bank that, for a few weeks each year, as far as possible no new notes would be issued to the public.

The first of these periods of suspension, lasting two weeks and applying to 10s and £1 notes only, was in June/July 1966. The results were encouraging, and thereafter such periods were instituted at regular intervals, with three of them totalling seven weeks in each year in both 1967 and 1968.

The savings to be achieved both directly, and also indirectly, by increasing, through familiarity, the acceptability of used notes at other times, soon became apparent. Over the ten months covering the first three periods, issues of new 10s and £1 notes were about a quarter below normal, whereas issues of £5 notes, which were not covered by the arrangements, fell by only 4%. There were similar reductions in the number of notes returned for destruction. With the Bank's total costs of note production and destruction running at that time at over £5 million a year, the savings were substantial.

However, after the initial impact of these periods, it soon became apparent that further savings would be needed, for, despite the withdrawal of the 10s note from October 1969, the secular upward trend in the circulation would in due course revive pressure on productive capacity. Consequently in 1969 suspension was lengthened to ten weeks in all, and in 1970 to fourteen weeks, split into four periods. Also in 1970, £5 notes were included for the first time. After two years of fourteen weeks, suspension periods were reduced in 1972 to thirteen weeks, and this remains the practice for the present.

Early this year, during the period of restrictions on electricity consumption, production of new notes was somewhat reduced, and for a while the Bank were obliged to restrict issues of new notes to the clearing banks at times other than the regular periods of suspension; and from the beginning of this year £10 and £20 notes have been included in the arrangements.

A reduction in the issue of new notes and in the destruction of old notes does not, of course, permanently reduce the number of notes in circulation (though there is a temporary impact through movements in banks' tills — see the appendix). The effect of the suspension periods has been to lengthen the life of notes. The life of the £1 note, which had fallen to eight months on average in 1965, rose to ten months in 1966 and then to eleven months in 1970. This occurred despite the continuing fall in the internal purchasing power of the pound, which might have been expected to reduce the note's life (as it became more used as change for higher denominations). If the life of £1 notes had been eight months in 1973 instead of eleven, the Bank would have had to produce about 30% more of them. Similar, though less extensive, savings have been achieved with £5 notes. Total notes produced by the Bank's Printing Works are now running at around six million every working day, but this is over two million a day fewer than the Bank were producing nine years ago.

Undoubtedly the cumulative effect of the measures taken over the past eight years has been some reduction in the former exceptionally high standard of cleanliness of notes in circulation. The Bank have no wish to see the quality of the circulation fall below what is acceptable to the public. But the savings from extending the life of the note in recent years have been considerable; and, despite the measures taken, the average life is still low compared with other industrialised countries.

Table A
Value of notes in circulation

£ millions: *percentage of total circulation in italics*

Last Wednesday in November	10s		£1		£5		£10		£20		Other notes[a]		Total
1953	93	*6*	1,180	*76*	190	*12*					87	*6*	1,550
1954	94	*6*	1,253	*76*	213	*13*					92	*5*	1,652
1955	97	*5*	1,362	*76*	240	*14*					97	*5*	1,796
1956	99	*5*	1,439	*76*	265	*14*					103	*5*	1,906
1957	100	*5*	1,393	*70*	400	*20*					108	*5*	2,001
1958	101	*5*	1,314	*64*	521	*26*					108	*5*	2,044
1959	102	*5*	1,266	*59*	666	*31*					110	*5*	2,144
1960	103	*5*	1,199	*53*	843	*37*					113	*5*	2,258
1961	105	*4*	1,125	*48*	1,001	*43*					113	*5*	2,344
1962	98	*4*	986	*42*	1,138	*49*					108	*5*	2,330
1963	100	*4*	1,052	*43*	1,190	*48*					112	*5*	2,454
1964	103	*4*	1,036	*40*	1,270	*48*	95	*4*			108	*4*	2,612
1965	106	*4*	1,037	*37*	1,388	*50*	140	*5*			108	*4*	2,779
1966	107	*4*	999	*34*	1,523	*52*	190	*6*			108	*4*	2,927
1967	109	*3*	997	*33*	1,586	*52*	239	*8*			115	*4*	3,046
1968	112	*3*	979	*31*	1,685	*53*	276	*9*			116	*4*	3,168
1969	63	*2*	963	*30*	1,789	*55*	311	*9*			121	*4*	3,247
1970	18	*–*	955	*27*	1,988	*57*	353	*10*	59	*2*	127	*4*	3,500
1971	14	*–*	922	*25*	2,139	*58*	380	*10*	125	*3*	126	*4*	3,706
1972	13	*–*	926	*22*	2,407	*58*	479	*11*	213	*5*	149	*4*	4,187
1973	13	*–*	931	*20*	2,530	*55*	648	*14*	319	*7*	173	*4*	4,614
1974[b]	13	*–*	913	*18*	2,883	*56*	775	*15*	381	*7*	190	*4*	5,155

[a] Including notes held by the Bank of England on behalf of customers, for example, for banks of issue in Scotland and Northern Ireland as cover for their excess note issues.

[b] Last Wednesday in October.

Table B
Number of notes in circulation

Millions of notes: *percentage of total number in italics*

Last Wednesday in November	10s		£1		£5		£10		£20		Total
1953	185	*13*	1,180	*84*	38	*3*					1,403
1954	188	*13*	1,253	*84*	43	*3*					1,484
1955	194	*12*	1,362	*85*	48	*3*					1,604
1956	198	*12*	1,439	*85*	53	*3*					1,690
1957	200	*12*	1,393	*83*	80	*5*					1,673
1958	202	*13*	1,314	*81*	104	*6*					1,620
1959	205	*13*	1,266	*79*	133	*8*					1,604
1960	206	*13*	1,199	*76*	169	*11*					1,574
1961	211	*14*	1,125	*73*	200	*13*					1,536
1962	197	*14*	986	*70*	228	*16*					1,411
1963	200	*13*	1,052	*71*	238	*16*					1,490
1964	205	*13*	1,036	*69*	254	*17*	9	*1*			1,504
1965	211	*14*	1,037	*67*	278	*18*	14	*1*			1,540
1966	213	*14*	999	*65*	305	*20*	19	*1*			1,536
1967	219	*14*	997	*64*	317	*20*	24	*2*			1,557
1968	224	*14*	979	*62*	337	*22*	28	*2*			1,568
1969	127	*9*	963	*65*	358	*24*	31	*2*			1,479
1970	35	*2*	955	*67*	398	*28*	35	*3*	3	*–*	1,426
1971	28	*2*	922	*65*	428	*30*	38	*3*	6	*–*	1,422
1972	27	*2*	926	*62*	481	*32*	48	*3*	11	*1*	1,493
1973	26	*2*	931	*60*	506	*33*	65	*4*	16	*1*	1,544
1974[a]	26	*2*	913	*56*	577	*36*	77	*5*	19	*1*	1,612

[a] Last Wednesday in October.

Table C
Number of new notes issued

Millions of notes: *percentage of total number in italics*

Calendar year	10s		£1		£5		£10		£20		Total
1953	327	*26*	883	*72*	21	*2*					1,231
1954	346	*28*	878	*71*	13	*1*					1,237
1955	366	*29*	863	*69*	28	*2*					1,257
1956	387	*29*	917	*68*	34	*3*					1,338
1957	401	*29*	903	*66*	75	*5*					1,379
1958	416	*29*	927	*66*	66	*5*					1,409
1959	430	*29*	983	*65*	86	*6*					1,499
1960	452	*28*	1,063	*65*	112	*7*					1,627
1961	493	*28*	1,120	*64*	143	*8*					1,756
1962	475	*26*	1,221	*66*	148	*8*					1,844
1963	490	*24*	1,320	*66*	199	*10*					2,009
1964	493	*24*	1,379	*66*	194	*9*	12	*1*			2,078
1965	509	*23*	1,482	*67*	223	*10*	8	*–*			2,222
1966	451	*23*	1,237	*64*	229	*12*	11	*1*			1,928
1967	428	*23*	1,181	*63*	249	*13*	12	*1*			1,870
1968	447	*23*	1,220	*62*	280	*14*	14	*1*			1,961
1969	329	*18*	1,139	*64*	297	*17*	16	*1*			1,781
1970			995	*79*	248	*20*	18	*1*	4	*–*	1,265
1971			976	*77*	266	*21*	20	*2*	4	*–*	1,266
1972			998	*73*	332	*24*	30	*2*	6	*1*	1,366
1973			1,074	*70*	406	*26*	44	*3*	8	*1*	1,532

Appendix

The purpose of this appendix is to explain the more technical issues connected with forecasting the demand for notes both in the short term and in the longer term.

Short-term forecasting

The method employed for short-term forecasting is to project the trend of the weekly figures for the next three months and then allow for seasonal and other effects, which are assessed by analysing the historical weekly series. Weekly (Wednesday) figures are available for the value of:

a   total notes in circulation (published in the Bank Return);

b   notes in London clearing banks' tills (reported by the banks); and

c   notes with the public (the difference of a and b).

Each is analysed as follows.

Long-term growth is removed by taking ratios of the original figures to the trend, the latter being estimated by an unweighted thirteen-term moving average.

The basic seasonal variation of the ratios is assumed to be adequately described in terms of twenty-eight harmonic variables. Twenty-two are of the form

$$\left.\begin{matrix} \cos \\ \sin \end{matrix}\right\} \left(\frac{2\pi k t}{365}\right), \; k = 1, 2, ..., 11,$$

where t is the day of the year (i.e. it runs from 1 to 365, 29th February being treated as 28th February), and a further six of the form

$$\left.\begin{matrix} \cos \\ \sin \end{matrix}\right\} \left(\frac{2\pi k' t'}{m'}\right), \; k' = 1, 2, 3,$$

where t' is the day of the month and m' is the number of days in the month. These two sets of variables pick up the within-year and within-month variations respectively. The within-year variables for $k = 12$ are almost identical with the within-month variables for $k' = 1$ and are therefore omitted.

Allowance also has to be made for the effects of bank and public holidays and, since their introduction in 1966, of the periods when the issue of new notes is suspended. The shapes of the humps or dips that these cause are determined by inspection of the data for the dates about Christmas (where the peak is very sharp and the holiday does not occur on a fixed day of the week), for the Wednesdays about the moving holidays, and for the Wednesdays before, during and after the suspensions. To do this, the ratios to trend need first to be adjusted for day-of-year and day-of-month effects. The ratios thus adjusted are plotted against time for several weeks each side of a bank holiday and for the period covering each category of suspension (two weeks, three weeks, or four/five weeks), and dummy variables are constructed which exhibit the indicated pattern of behaviour about each event.

Obviously there is here a chicken and egg problem: for example, in order to determine suspension effects, any overlapping bank holiday effects have first to be removed; and vice versa. Moreover, the trend is an average of seasonally-adjusted, rather than original, figures, and estimation of the trend thus depends upon the prior existence of a seasonally-adjusted series. In practice the seasonal adjustments are updated annually, and the previous year's estimates of the seasonal factors and the bank holiday and suspension effects are used to provide an initial trend, and to disentangle bank holidays from suspensions.

A regression of the ratios against the seasonal and dummy variables is now carried out. From the resulting coefficients, together with the original Wednesday figures and the various bank holiday and suspension patterns, another computer program produces:

a   seasonal factors for each day of the year;

b   adjustments to be applied in the neighbourhood of bank holidays and suspension periods;

c   combined factors (total adjustments) for each Wednesday of the data period and for the next eighteen months;

d   the original series divided by the total adjustments;

e   thirteen-term moving averages of these totally adjusted figures (the 'trend'); and

f   ratios of original figures to trend.

The predicted seasonal factors are used in the short-term forecasting of the note circulation. The trend is projected forward, account being taken of any known future abnormalities, and then multiplied by the total adjustments to produce a forecast of the actual note circulation.

Long-term forecasting

The long-term forecasting depends on three steps:

a estimation of the average value of the note circulation year by year;

b forecasting the shares of each denomination in the total by projecting the current trends; and

c derivation of the average life of each denomination, and forecasting these by projecting the current trends.

a The first step depends upon a statistical relationship between consumers' expenditure at current prices and the total value of the note circulation. This relationship is assumed to be linear on the logarithmic scale, because most economic variables tend to show exponential growth. The form of the model is therefore:

$$\ln y_t = a + b \ln x_t + u_t,$$

where

$$y_t = \text{average value of notes in circulation in year } t,$$

$$x_t = \text{consumers' expenditure in year } t,$$

$$u_t = \text{'unexplained' component in year } t.$$

Annual data from 1963 onwards are used, with y_t being adjusted downwards in the earlier years for the break which occurred in 1970 when the 10s note was withdrawn (the adjustment being an estimate of the 10s notes then replaced by coin). The estimates of this model take into account the autocorrelated structure u_t. Using a path for x_t provided by the official national income forecasts, a path for y_t is derived.

b The second step is to project the current trend of the shares of each denomination in the circulation. This can only be rough and ready, though certain aspects are fairly clear. Thus, the average value per note, which is low by comparison with other industrialised countries, is expected to continue to rise, because the higher denominations which were reintroduced a few years ago are becoming more popular, and also because of inflation.

c The third step entails calculating the average life of each denomination of bank note month by month. This is done in two ways: the average number of each denomination in circulation during the month is divided (1) by the number of each withdrawn from circulation ('paid') during the month and (2) by the number issued during the month. The figure given by method (1) represents the time it would take to withdraw at the present rate the whole of the present circulation of the denomination in question; and that by method (2) represents the time it would take to issue it. Both give estimates of the 'life' of the denomination which in general are different, and the 'true' life is assumed to lie somewhere between them.

The calculations are made as follows. For each denomination, weekly figures are obtained for the number of notes in circulation, notes issued and paid, and notes returned to store for reissue. From these are derived the average number of notes in circulation during the month (which may contain data for four or five weeks), and the corresponding monthly totals of notes issued, notes paid, and notes returned to store for reissue (these being multiplied by a factor so as to adjust to a standard month of 4⅓ weeks). Average circulation, gross issues, and the gross numbers paid all show a seasonal pattern – for example, a large number of notes are issued in November and December, giving rise to a large number in circulation in December and a large number being withdrawn in January. They are therefore all seasonally adjusted. The reissue series for each denomination is very erratic and shows no such seasonal pattern. However it was found that the ratio

$$R_t = \text{reissues (unadjusted)/gross paid (unadjusted)}$$

formed a more stable series, which is then smoothed exponentially by forming the series

$$\bar{R}_t = 0.8\,\bar{R}_{t-1} + 0.2\,R_t.$$

The adjusted figures for reissues are therefore calculated by applying the smoothed ratios $\bar{R}_t$ to the seasonally-adjusted gross paid figures. Figures for net issues and net paid are then obtained by:

$$\text{net issues} = \text{gross issues (seasonally-adjusted)} - \text{reissues (smoothed)},$$

$$\text{net paid} = \text{gross paid (seasonally-adjusted)} - \text{reissues (smoothed)}.$$

For each denomination, the two estimates of the life are calculated as follows:

$$P_t = \text{circulation/net paid},$$

$$Q_t = \text{circulation/net issues}.$$

However, both P_t and Q_t are still somewhat erratic, and, after some experimenting, the following exponential smoothing was found to be satisfactory for all denominations:

$$\ln \bar{P}_t = 0.75 \ln \bar{P}_{t-1} + 0.25 \ln P_t,$$

$$\ln \bar{Q}_t = 0.75 \ln \bar{Q}_{t-1} + 0.25 \ln Q_t.$$

The life of notes in the forecast period is assessed using the $\bar{P}_t$ and $\bar{Q}_t$ series as a guide, but taking other developments into account. For example, although the proportion of £1 notes may be declining, they are increasingly being used as an item of change rather than as a store of value, and thus have a shorter life through being handled more often.

Finally, from the forecasts of the total value of the note circulation, of the contribution of each denomination to the total and of the life of each denomination, the printing requirements for each denomination during each year of the forecast period is estimated.