Yield curves for gilt-edged stocks: a further modification

This is a research article prepared mainly by Oliver Page and J. P. Burman.

Introduction

The present formulation of the yield curve was introduced in the December 1972 issue of the *Bulletin* and, with one or two minor modifications, it performed well over the following two years. At the end of January 1975, however, a dip appeared in the yield curve between four and six years and has subsequently persisted: the curve for 20th March 1975, when the dip was at its greatest, is shown in Chart A. The dip does not appear to be reflected in the yields of actual gilt-edged stocks; in fact, market views are that it would not be possible for the Government to take advantage of the dip by issuing stocks in that maturity band at the yields indicated by the curve. Further investigation has revealed the causes of the dip, and the modification described below has accordingly been made to the method of fitting: this eliminates the dip and should help to prevent similar problems from recurring. However, it is always possible that other anomalies may arise in the future, and the Bank will be prepared to make further modifications if these should prove necessary.

Before explaining how the dip occurred, it may be useful to give a brief summary of the theory on which the construction of the current yield curve is based (a full description was published in the December 1972 and September 1973 issues of the *Bulletin*). Investors are assumed to have expectations about future interest rates up to a point called the 'planning horizon'; beyond this point, uncertainty is so great that investors assume no further changes in interest rates. The time up to the planning horizon is called the 'decision period'. The theory also assumes that the market is divided into segments, in each of which the planning horizons of the many investors are distributed over a fairly narrow range: average expectations about interest rates in each segment may thus be regarded as relating to an average planning horizon. For each segment, the expected returns on a number of stocks over the decision period can be calculated from the value of the expected yield at the planning horizon, and it is postulated that arbitrage should ensure that these returns over the decision period are equal. In the event, it was found desirable to divide the market into two segments – a short market with the planning horizon at one year and a long market with the horizon at four years.

In order to derive the combined yield curve, it was necessary to decide which stocks were relevant to each curve, and the way in which the two curves should be spliced together. The assumption made was that stocks with maturities between one and four years belong to the short curve and stocks longer than eight years to the long curve, while stocks between four and eight years enter the estimation of both the short and long curves, with weights depending on their position in the band (i.e. in accordance with the notion that, in the determination of yields, the relative importance of investors with short horizons diminishes gradually with maturity while that of investors with long horizons increases). The resultant two curves are then spliced using the same weighting system.

An explanation of the dip

There are two factors which, acting together, have produced the dip in the yield curve: the behaviour of a small group of medium-coupon stocks maturing in the mid-eighties, and the weighting system described above. Over the past year or so, the stocks in question (5½% Funding Stock 1982/84, 8½% Treasury Loan 1984/86, 6½% Funding Loan 1985/87 and 7¾% Treasury Loan 1985/88) have consistently yielded much less than would be calculated from the yield curve. It seems likely

Table A

Returns over the decision period in relation to the yield curve, and the size of the dip

The emergence of the dip in February 1975 appears to have been associated with a change in the relationship between the return over the decision period and the yield curve. Before February 1975, the two were within 1% of each other (except in January 1975); subsequently, the gap increased sharply. With the new curve, the gap would on no occasion have been greater than 1%.

Per cent

	Yield curve at 4 years [a]	Return over the holding period to the long horizon		Size of the dip [b]
		Present curve	New curve	
1973 Nov. 15	12·6	12·0	12·4	–
1974 Mar. 21	12·8	13·1	12·9	–
May 16	12·2	12·4	12·3	–
1975 Jan. 23	11·8	10·5	11·4	–
Feb. 13	11·2	8·4	10·5	0·18
Mar. 20	10·4	6·8	9·7	0·37
May 1	11·0	6·8	10·1	0·37
Aug. 28	11·8	9·2	11·3	0·21
Oct. 29	12·5	9·4	12·0	0·30
1976 Jan. 14	10·7	8·5	10·4	0·11
Mar. 8	10·5	7·5	10·1	0·07
May 3	10·9	7·6	10·4	0·26

[a] The average of the two methods; the difference between the two curves was, in fact, never greater than 0·4% at this maturity.

[b] The dip is the difference between the maximum point on the yield curve at about four years and the minimum point at about six years.

that the absence of a high-coupon stock maturing in the mid-eighties has induced gross investors to bid up the prices of medium-coupon stocks.

This small number of stocks has a large impact on the shape of the yield curve because of the weighting system used in the splicing band. Of the stocks used in estimating the long yield curve at maturities less than the mid-eighties stocks, all but three have lives of between four and five years. Although the theory postulates that investors in the long segment of the market have definite views about the appropriate yields for four-year stocks, the method assumes that investors in the short segment dominate in the four to five-year maturity band. Thus, in the estimation of the long curve, weights of between 0 and ⅙ (out of a total of 1) are assigned to the stocks in this band, i.e. they are virtually ignored. In consequence, the return over the decision period to the long horizon is poorly determined and the long curve is not firmly anchored at its left-hand end.

In recent months, because of the low yields on stocks maturing in the mid-eighties, the long curve has been estimated as starting from implausibly low values – implying that, on occasions during 1975, the market was willing to accept returns of less than 7% for the four years up to the horizon (see Table A), i.e. considerably less than the (certain) yield that could be obtained from a stock with four years to run. The operation of splicing the long and short curves into a single yield curve has then produced the anomalous dip.

It might be considered that it would be incorrect for the long investors' expected return to their planning horizon ever to be below the yield on four-year stocks. However, the theory contains a number of simplifying assumptions, e.g. that *all* long investors have planning horizons of *exactly* four years. Thus, all that should be demanded of the parameters of the theory is that their relationships with the theoretically correct ones should be reasonably close. The discrepancy described above between the first two columns of Table A does not appear to meet this criterion.

Remedies

One way of overcoming the above difficulties would be to exclude certain stocks maturing in the mid-eighties from the estimation of the yield curve. However, apart from the arbitrary nature of such a course, fresh problems would be created: the yield curve between five and fifteen years would depend heavily on the choice of stocks to be removed; and there would be difficulties in deciding when the stocks should be reintroduced. Moreover, this approach would not solve the general problem that the shape of the curve can be influenced too strongly by a relatively small number of stocks. On the contrary, the long curve would be even more dependent on particular stocks, because its shape between four and fourteen years would then be effectively determined by no more than five stocks.

The approach which has been adopted is to anchor the long curve more firmly at four years – thus ensuring that small groups of stocks have less influence on its shape at shorter maturities – by using the estimate from the short curve of the yield at four years to fix more firmly long investors' expected return to the long horizon. This has been achieved by including an extra term in the estimation procedure, which progressively penalises the divergence between long investors' expected return over the four years to their planning horizon and the estimate of the four-year yield from the short curve. The underlying assumption is that the short curve at four years is a good (and stable) estimate of yields at that maturity, and that it should therefore be reasonably close to the long investors' views of the expected return up to their horizon. The new estimates of this return are much more stable (see Table A) and never differ from the four-year yield on the short curve by more than 1%.

The new method has been tested for a number of dates over the past two years and appears to work well: the dip is eliminated on each occasion (Chart A shows the new curve for 20th March 1975, when the dip of the present curve was greatest); and for dates before the dip emerged, the difference between the present and new curves is relatively small (Chart B). Also, the parameters derived from the model – especially, as mentioned above, the long investors' return to their horizon – look more reasonable; the movement in the new curve between January and March 1975 (during which the dip first appeared) seems more closely related to movements in the yields of actual stocks than is the case for the present curve; and the fit of the new curve is not significantly worse than the present one.

The cost of government borrowing

The yield curves produced by the Bank not only aid analysis of changes in interest rates over time, but also provide a means of measuring the potential cost of government borrowing at different maturities. The par yield curve, however, understates the potential cost of borrowing because, whenever new funds are in fact being raised through the sale of tap stocks, those stocks will typically be offering a yield somewhat higher than that at the corresponding point on the yield curve. This is a natural consequence of the marketing arrangements for tap stocks, on which market demand is concentrated largely because of the attractive relative yields which from time to time they display. An examination of the differentials between the par curve and yields on tap stocks has shown that, over the last eighteen months, long or short tap stocks, when active, had yields averaging about 3/16% above the new yield curve. When the tap stocks were inactive, their yields were generally below the yield curve. These relationships of tap stock yields to the curve are more strongly observed when the new curve (rather than the old one) is used, and give further support to the superiority of the new formulation.

Conclusion

The modification described above has been adopted for two main reasons. First, it reduces to a plausible size the gap between the estimated return over the decision period to the long horizon and the yields obtainable on stocks with four years to maturity: the previous estimation implied that market operators were willing to accept uncertain returns of 3% to 4% lower than the certain yield on four-year stocks. Secondly, and more practically, one of the essential requirements of any yield curve theory is that it should produce a sensible curve: in this respect, the new method is much to be preferred.

The main effect of the change is to raise the yield curve for maturities between five and fifteen years; for example, the increase at ten years averaged about 5/8% over the last year (see Table B).

Table B
The change in the yield curve at 10 years

Per cent

	Present curve	New curve	Change
1973 Nov. 15	12	12 1/8	+1/8
1974 Mar. 21	13 3/4	13 5/8	−1/8
May 16	13 3/8	13 3/8	—
1975 Jan. 23	14 5/8	14 3/4	+1/8
Feb. 13	13 1/8	13 5/8	+1/2
Mar. 20	11 3/4	12 3/8	+5/8
May 1	13 1/8	13 7/8	+3/4
Aug. 28	13	13 1/2	+1/2
Oct. 29	13 7/8	14 1/2	+5/8
1976 Jan. 14	12 5/8	13 1/8	+1/2
Mar. 8	12 3/4	13 1/4	+1/2
May 3	12 1/2	13 1/8	+5/8

Chart A

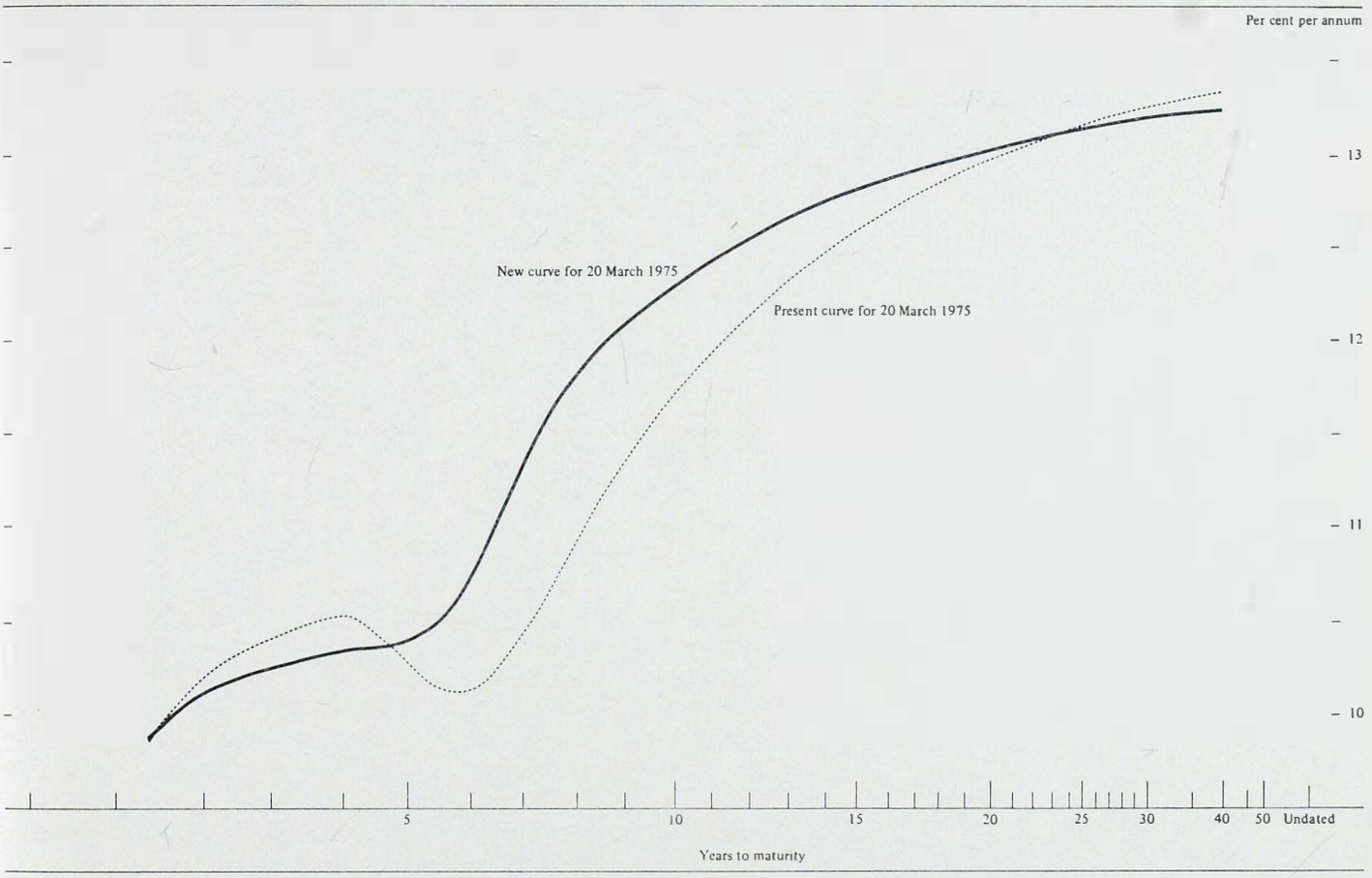

Chart B