A transactions demand for money

This research article was prepared mainly by R. T. Coghlan of the Bank's Economic Intelligence Department. [1] *An earlier version was presented to the Money Study Group in December 1977.*

Summary

For as long as economists have been interested in the operation of the economy as a whole, they have been concerned about the existence, or otherwise, of a stable demand for money. That is to say, they have endeavoured to discover what aspects of the economic situation were likely to affect people's desire to hold money balances and what were the time-lags involved between changes in such economic variables and changes in money balances. Economic theory, and indeed commonsense, suggests that changes in, for example, incomes, prices, and interest rates are very likely to lead people to try to increase or reduce their money balances. In recent years, attempts have been made to estimate such relationships statistically, in the form of 'demand-for-money' equations. The practical objectives of such work have been not only to improve the understanding of the financial side of the economy but also to establish an analytical basis for the operation and interpretation of monetary policy.

A good deal of work in this area has been carried out in the United States over the past twenty or thirty years; and it appeared that a stable relationship between various economic variables and money holdings could be established. In this country, work of this type was hampered until the 1960s by the lack of comprehensive statistics for the various definitions of the money supply. Towards the end of the 1960s, however, economists in this country too seemed able to identify a similar stable relationship for the United Kingdom. Three of the Bank's studies were published in earlier issues of the *Bulletin*: June 1970, March 1972 and September 1974.

After about 1972, however, the situation became less clear, apparently because of the institutional changes which followed the introduction of competition and credit control in September 1971. It is now some years since this change, and a longer and more varied run of data is now available. This article describes work which incorporates this additional information and the conclusions which have emerged. It is confined to relationships affecting the narrow definition of the money stock (M_1), which comprises balances held mainly for transactions purposes.

The results indicate that limited data availability probably precluded the earlier studies from actually being able to identify a demand-for-money function which was stable in a strict statistical sense. Moreover, there seems to be evidence that the equations which had previously been estimated were generally too restrictive in the way in which they treated lags between changes in economic variables and the reaction of the money stock to them: in general, most previous studies assumed that such lags in adjustment were the same for all variables determining the demand for money. The results described in this article, however, suggest that there are in fact reasonable grounds for believing that a stable demand relationship for M_1 can be identified, although the pattern of lags involved is more complex than has generally been considered in the past.

Introduction

This article presents some results for the estimation of the demand for money, narrowly defined (M_1). The objective is to determine whether a stable demand relationship can be estimated, paying particular attention to the lag structure. In recent years, attention in this country has been concentrated on the apparent breakdown since about 1972 of all previous estimated demand-for-money functions. The general view seems to have been that reasonably stable relationships had been established up to about 1972 but that they no longer held good. Furthermore, as is made clear below, the 'breakdown' appeared to be independent of the definition of money employed. This article examines the validity and generality of this argument which, if true, would have important repercussions not only for our understanding of financial markets but also on how the impact of monetary policy should be assessed. In fact, the results described in this article suggest, at least as far as M_1 is concerned, that the picture of stability before 1972 and breakdown afterwards is misleading. Although much of the problem with the previous results stemmed from insufficient variability in the data, the results presented in this article suggest that, in general, the lag assumptions imposed have been too restrictive. Adopting a more flexible approach, it is possible to identify a more complex lag structure, which also appears to have remained stable over time.

As a starting point it is argued that there is no general theory of the demand for money which is applicable regardless of the definition of money adopted. We should expect a different behavioural relationship to apply to the demand for M_1 (essentially transactions balances) from that applying to the demand for M_3, where variations are more likely to be caused by changes in portfolio preferences. Furthermore, while there are certain strong theoretical propositions which should be incorporated into any empirical framework, the question of the existence, length and shape of any lags must remain an empirical one. It is the flexible approach to the estimation of lags that distinguishes this study from most others. The estimation results from the tests are summarised below, and set out in detail in Appendix 1. First, however, the existing empirical evidence for the United Kingdom is briefly surveyed to illustrate the background against which this study was undertaken.

Background

At the end of the 1960s, the evidence seemed to suggest that a stable demand for money, as a function of a few variables, had been identified. The general view was probably best stated by Laidler [2] who claimed that, 'this evidence for Britain certainly points to the existence of a stable demand-for-money function in that economy.

[1] A number of colleagues in the Bank have contributed to this article; in particular, most of the calculations were carried out by J. M. Hoffman and Miss L. M. Smith. Valuable comments and suggestions have also been provided by economists outside the Bank, notably D. F. Hendry and J. Wise.

[2] D. E. W. Laidler, 'The Influence of Money on Economic Activity—A Survey of some Current Problems', *Monetary Theory and Monetary Policy in the 1970s*, edited by G. Clayton, J. C. Gilbert and R. Sedgwick (Oxford University Press, 1971).

For the United States the evidence is overwhelming, and for Britain it is at the very least highly suggestive'. In September 1971, competition and credit control was introduced and, in the years that followed, the demand-for-money functions which had previously been estimated failed to forecast at all accurately. This was interpreted as meaning that the previously reliable, stable demand functions had broken down. [1] All at once it seemed we had moved from a situation in which the demand for money had been reliably and accurately estimated to a new environment where the money stock no longer exhibited these stable characteristics.

It is important to recognise that the earlier demand-for-money studies in the United Kingdom, upon which the conclusion of stability was based, employed a wide variety of definitions of money, interest rates, income, lag structures and estimation periods.[2] There was very little, if any, concern with the actual definition of money employed. The theoretical basis from which empirical tests were developed, and the interpretation of the results obtained, seemed independent of whether money was defined narrowly or broadly.

Evidence that simple demand functions for both M_1 and M_3 had broken down was provided by Artis and Lewis[3] who reported that, 'the standard demand function simply does not fit the experience of 1971 and 1972', adding that 'the forecasting ability of this equation (in common with alternative equations of the same general character that we tested, whether for M_1 or M_3) is quite good for 1971, but disastrously bad for 1972 and the first two quarters of 1973'. Hacche[4], on the other hand, found that the M_1 equation continued the forecast 'fairly satisfactorily' up to 1973. However, this equation did break down when the period was extended further into the 1970s.

These results have encouraged the belief that there has been a *general* breakdown of all empirical demand-for-money equations in the United Kingdom. The failure to estimate a stable demand function employing single equation techniques, whatever the definition of money employed, had important implications for any attempt to explain recent monetary experience. Artis and Lewis claimed, for example, that, because of the breakdown of M_1 demand functions, this instability could not be fully explained by such new features as the growth of interest-bearing deposits or distortions arising from the CD market.

The equations from which the conclusion of a general breakdown was derived were of the simplest kind, assuming the same length of lag on all variables, and it was important to determine whether they had indeed broken down, or whether the estimates were simply not very stable to begin with. This is particularly necessary since the lag restrictions assumed seem unduly restrictive.

Theory

Most previous work in this country appeared to assume that there was a general theory of the demand for money which was applicable regardless of the actual definition of money employed. This is not the approach adopted in this study. Instead it is assumed that the demand for M_1 balances is predominantly determined by transactions, and to some extent precautionary needs, so that speculative motives for holding money are not expected to be important. Modern theories of a transactions demand for money originated in the work of Baumol[5] and Tobin, [6] who adopted an inventory-theoretic approach which resulted in the so-called 'square-root formula'.[7] Acceptance of this basic approach does not, however, necessarily require acceptance of this formula, as that expectation is dependent upon some rather restrictive assumptions. At the individual level, transaction costs, subjective as well as objective, may be so high, relative to the rate of interest and the level of income/transactions, that this type of active cash management is uneconomic. In that case there would be no interest elasticity, and the demand for money would necessarily rise in line with transactions. An implication of this asymmetry is that aggregation from the individual to the total demand for transactions balances has the effect of increasing the income elasticity and reducing the interest elasticity. Furthermore, once uncertainty about future income and expenditure patterns is allowed for, as is likely to be the case for the large institutional money holders, many different elasticities are obtainable depending upon the specific assumptions made.[8] This approach means that we should expect short rates on closely competing assets to be more relevant than long rates on less liquid assets. It also means that measures of expected capital loss, which portfolio theory suggests should importantly affect the demand for money, will not in this case be relevant. Moreover, the 'income' variable should naturally be selected to represent expected transactions requirements. Taking a broader view of portfolio allocation, we might still expect total wealth to be included in the demand specification. If, however, the inventory-theoretic approach is relevant, then wealth is likely to be of secondary importance, and is assumed, for present purposes, to be reflected in the transactions variable employed.

It is generally assumed that the long-run price elasticity of the demand for money should be unity, on the grounds that economic rationality implies the absence of money illusion. However, although it may seem a reasonable

[1] This 'breakdown' refers only to the *estimation* of single equation demand functions, and does not exclude the possibility that a stable, but unidentified, demand function actually exists.

[2] Appendix 3 contains a summary of previous published results.

[3] M. J. Artis and M. K. Lewis, 'The demand for money: stable or unstable?', *The Banker*, March 1974; and 'The Demand for Money in the United Kingdom: 1963–1973', *The Manchester School*, June 1976.

[4] Graham Hacche, 'The demand for money in the United Kingdom: experience since 1971', September 1974 *Bulletin*, page 284.

[5] W. J. Baumol, 'The Transactions Demand for Cash: an Inventory Theoretic Approach', *Quarterly Journal of Economics*, November 1952.

[6] James Tobin, 'The Interest Elasticity of Transactions Demand for Cash', *Review of Economics and Statistics*, August 1956.

[7] This requires the demand elasticities on transactions and the rate of interest (as the opportunity cost of holding transactions balances) to be $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively. There is also expected to be an elasticity of $\frac{1}{2}$ on the 'brokerage fee' (the cost of switching between money and alternative short-term assets). This last influence is generally not measured. under the assumption that the transfer cost remains constant over the estimation period.

[8] A survey of this literature is contained in C. A. E. Goodhart, *Money, Information and Uncertainty*, (Macmillan, London 1975), pages 22–30.

assumption to make, it can be no more than that.[1] We may well expect interest rates to influence the level of investment; that does not relieve us of the necessity to test that assumption.

From an empirical viewpoint, the most important characteristic of M_1 balances is the high probability, firstly, that they have been determined by demand, and, secondly, that simultaneous equation bias is at a minimum; thereby justifying the use of single equation estimation techniques employing M_1 as the dependent variable. M_1 consists of currency and private sector sight deposits with the banking sector, both of which are free from supply constraints.[2] Currency is supplied upon demand, and banks accept all money placed with them on current account, usually without payment of interest. It might be argued that any buffer stocks of liquidity would also be held in the form of money, thereby concealing the true demand relationship. However, if such buffer stocks do exist they would seem more likely to be held in the form of interest-bearing deposits than on current account. As regards the problems of simultaneity, it seems most unlikely that money market interest rates should be determined by the stock of, or change in, M_1 balances. Naturally, if interest rates were varied in an attempt to control M_1, this could introduce simultaneous bias into any attempt to estimate single equation demand functions, but without necessarily changing the underlying relationship. But the authorities have not attempted to control M_1, and single equation estimation seems appropriate to this exercise. This might not be the case when employing other definitions of the money stock.

Most previous demand-for-money studies have included real income only in *per capita* terms, a procedure that proved necessary in order to obtain a significant coefficient. While this might be reasonable for households' money holdings, these are not distinguished separately in the data. Statistics breaking down M_1 balances either between sectors or between interest-bearing and non-interest-bearing deposits have been available only since 1975, and these show that only about two thirds of M_1 balances are held by the personal sector; furthermore this sector includes not only individuals but also unincorporated businesses, non-profit-making bodies and private trusts. Even if we assume households to hold 75% of all personal sector M_1 balances (which is probably on the high side), that still leaves them holding only 50% of total M_1. Deflating aggregate money balances by the number of people, or the number of households, therefore seems to be an arbitrary assumption, and has not been incorporated into this study.

These then are the initial theoretical propositions. Beyond that, the existence and form of any lags must be an empirical question. The existence of lags can be justified in many ways, including the formation of expectations, costs of adjustment, habit preference and lags in the availability of information (or uncertainty about its reliability); they may also simply reflect lags in adjustment in other markets. The actual form of the lag expected is likely to be affected by the rationalisation adopted, and *a priori* there seems little justification for imposing any rigid formula. In fact, most demand-for-money studies have assumed that an identical lag applied to all explanatory variables. The only real attempt to obtain a more illuminating alternative was by Price,[3] who estimated different lags for each of the independent variables. His results were inconclusive (which is probably not surprising given the limited data available at that time), but very interesting. This more flexible approach seems more likely to be correct and it has been generalised in this study. Even if it still turns out that the lags in adjustment are the same for each explanatory variable, at least they will not have been imposed from the outset.

Estimation results

The results from trying to estimate flexible lags are given in Appendix 1, together with estimates from the simple distributed lag models. Simple models refer to those which have imposed *a priori* constraints on the lag weights; the lags have been derived from the standard assumptions of partial adjustment and adaptive expectations, both separately and in combination, and including the implications of imposing various restrictions on long-run parameter values. This represents the approach adopted in the majority of demand-for-money studies in all countries.

There are two main reasons for including the results from estimating equations imposing lag restrictions, even though we have argued in favour of an alternative approach. Firstly, it is within this framework that the argument has been made that a previously stable relationship has broken down. Since the more general approach also gave inconclusive results for the earlier periods dominated by the 1960s, the discovery of a more general relationship for longer periods including the 1970s may be interpreted as a shift in behaviour. It therefore seemed important to establish whether the simpler equations did actually break down. Secondly, given the wide use made of simple distributed lag models, they provide an ideal alternative against which to compare the more flexible approach.[4] Estimating these simple models over a variety of different overlapping data periods[5] does not establish a stable function for the earlier periods. However, as the data period is extended into the second half of the 1970s, the

[1] An argument that is often employed to support this assumption is that a change in the scale of measurement, e.g. a conversion of pounds to dollars, would not change real expenditures or the demand for real balances. This is certainly true, but it is not strictly relevant to this argument. What we are concerned with is a continuous updating of uncertain information, not a single discrete change that is universally accepted. Moreover, even if there is some theoretically correct concept of the price-level which would display a unitary elasticity, it is unlikely to correspond to any of the actual data series available.

[2] For a formal discussion of this question see Artis and Lewis.

[3] L. D. D. Price, 'The demand for money in the United Kingdom: a further investigation', March 1972 *Bulletin*, page 43.

[4] In order to estimate separate lags for each of the variables we have attempted to fit a general form of rational distributed lags, and also to estimate freely the individual lag coefficients. Attempts to estimate the weights employing the Almon technique (Shirley Almon, 'The Distributed Lag between Capital Appropriations and Expenditures', *Econometrica*, January 1965) did not prove particularly successful, with small changes in specification capable of producing quite large changes in the lag pattern. Perhaps this should be expected as there does seem to be some evidence, e.g. T. F. Cargill and R. A. Meyer, 'Some Time and Frequency Domain Distributed Lag Estimators: A Comparative Monte Carlo Study', *Econometrica*, November 1974, that the Almon technique is an unreliable method of approximating an unknown lag distribution.

[5] The first period is for 1964(1) to 1970(4), and this is extended by four quarters at a time up to 1976(4). In addition, equations were estimated for 1965(1) to 1976(4), and 1966(1) to 1976(4).

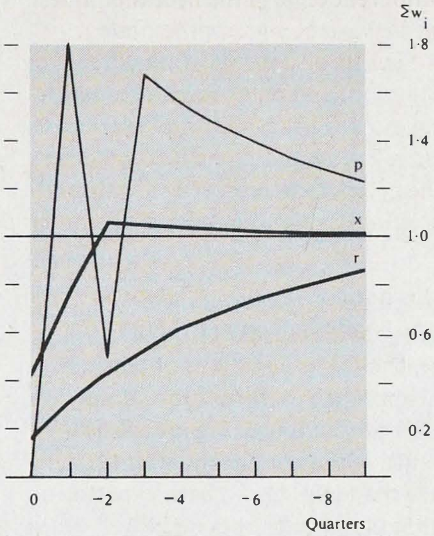
coefficients do become reasonably stable (i.e., they are changed only slightly, if at all, by lengthening or shortening the data period), and also seem economically plausible; in addition, estimates obtained employing quarterly and monthly data are very similar. This evidence could be used to support this approach, employing the argument that the existence of multicollinearity between the explanatory variables concealed the true relationship when the data available were limited to the 1960s and early 1970s. If this argument, in terms of limitations in independent data variation, is the complete explanation, then we would not expect to be able to improve on the best estimates obtained with this model. However, it is also possible that the equations have been mis-specified, and that this provides at least a partial explanation of the results obtained. In either case, it seems difficult to claim that a stable demand function for M_1 had been identified before 1971–72.

This conclusion is supported by the summary of results obtained in previous studies contained in Appendix 3. These results, taken together, do not suggest that stable equations, in any statistical sense, had been identified. A possible reconciliation of the apparent divergence between the earlier claims for stability, and the evidence of empirical studies, may be provided by recognising a qualitative difference in the nature of the evidence. The earlier studies, with only limited data available, obtained equations which had a good statistical fit, and a fairly low interest elasticity. When compared with the more extreme Radcliffian claims of an unstable relationship, and a potentially infinite interest elasticity, it was reasonable to claim that a stable demand function could be estimated. However, when those same equations were examined for internal stability, e.g. by Artis and Lewis, they failed the test. But now the criteria for comparison had changed. Moreover, the traditional M_3 relationship really did seem to break down, and this is likely to have coloured the interpretation of the less obvious evidence for M_1.

Although the evidence would now appear to support the existence of a stable demand-for-money function in the form of the previous simple lag models, the approach still seems too restrictive. In particular, it is unrealistic to impose the requirement that income, prices and interest rates all follow the same lag pattern of adjustment, suggesting that in this respect such equations may have been mis-specified. Rather than accept, or even attempt to 'improve', the statistical performance of these equations, an alternative approach has been adopted: to see if the data itself contained more information on the structure of the relationship, which was being ignored simply because the models that have generally been considered have been too restrictive. Instead of trying to justify very specific, narrowly defined, equations, it would be better to start from a general form and only accept the former if the restrictions they imply on the more general form are found to be satisfied.

This more flexible approach did in fact result in an improvement in explanatory power; see Appendix 1. Furthermore, as might be expected, the resulting lag pattern is quite complex, and is not the same for each of the explanatory variables (see chart). Lags do exist, although they generally seem to be short, and it is easy to see how the simpler distributed lag approximations might provide

Cumulative lag adjustment weights [a]



[a] These weights, derived from the general rational distributed lag approach, represent the *proportion* of the total adjustment to the long-run, equilibrium, value of the elasticities, for each of the explanatory variables, completed by the end of each period. The variables are defined in Appendix 1.

an apparently reasonable explanation; *at least so long as no more realistic alternative was considered.* An encouraging feature of these results is that very similar estimates are obtained employing a general rational distributed lag approach and when the individual lag coefficients are freely estimated. This is particularly true of the short-run coefficients (up to a year), although there is some difference in the estimated long-run behaviour.

The evidence clearly indicates a long-run real income elasticity that is very close to unity, with adjustment essentially completed after six months; a price elasticity of approximately 0.75, with no effect in the current quarter, over-adjustment after one quarter and probably completion of the process inside nine months. The total coefficient, and lag pattern, on the rate of interest is less obvious, but is definitely negative. There would seem to be a geometrically declining distributed lag on the rate of interest, although the possibility that the lag is much shorter cannot be excluded. The long-run elasticity is between -0.30 and -0.13, although in either case the value after six months is the same, at about -0.13. Given the transactions approach adopted, a high income elasticity suggests a numerically low interest elasticity. To this extent, the estimates are consistent, although most economists would expect a higher elasticity on prices and, possibly, a lower elasticity on income. This is because there is generally thought to be no money illusion, and some economies of scale in holdings of transactions balances. A number of tentative explanations can be put forward. For example, a possible explanation of the high income elasticity might be that transactions have been growing relative to expenditure so that the coefficient estimate on total final expenditure over-estimates the 'true' coefficient on transactions. Similarly for prices, it could be that as the price level rises the brokerage cost of transferring between money and alternative short-term assets has declined, so that the price term reflects this relationship, and the estimated elasticity is correspondingly reduced. Another possibility is that the deflator employed in estimation is not the same variable as in individual demand functions. This is always a problem with any attempt to find aggregate, composite variables to approximate individual behaviour. Such *ex post* justifications should be treated with extreme caution, and many more possibilities could probably be suggested. The results obtained, and the ease with which plausible explanations can be provided, however, illustrates the necessity to test even such firmly held theoretical propositions as the assumption of a unitary price elasticity.

Comparing the forecast performance of the main equations (Appendix 2) provides support for the conclusions already reached. The equations obtained employing the general approach perform substantially better than the more simple distributed lag models.[1] The forecast performance of these latter equations is in fact considerably worse than might have been expected on the basis of the estimation results obtained.

The stability of the preferred equations, and their *ex ante* forecasting performance, is highly encouraging; these equations correctly predicted the rapid growth of M_1 during 1977 on the basis of forecasts of income, prices and the rate of interest.[2] Any estimated equation must, to some extent, be specific to the period within which it was estimated, and it is therefore encouraging that the equations should have forecast accurately over 1977, which was completely excluded from the estimation period.

The apparent stability of the equations, and their predictive performance is rather better than might originally have been expected, particularly given the recent growth of interest-bearing deposits within the definition of M_1. Further developments in the financial system are quite likely to result in changes in the estimated relationship. It is hoped, however, that such changes will only take place gradually and not result in an abrupt shift in the estimated parameters. The limitations of any formal statistical relationship should be recognised, and there can be no guarantee that these equations will continue to perform as well in the future. However, as decisions have to be made about an uncertain future on the basis of imperfect information, a structural model of this kind should provide a useful guide in clarifying the various options which may exist.

Conclusions

The first point to emphasise is the relatively short lags in the adjustment of M_1 balances that have been estimated. These contrast sharply with the findings of many previous studies. The approach adopted has been not to impose *a priori* lag restrictions, but rather to allow for differing lag patterns on each of the explanatory variables. It is argued that this is a more illuminating approach and, in this case, has resulted in a better understanding of the short-run adjustment process. If there is a strong relationship between certain variables, this is likely to show up in many different forms, as we have seen. It is only in comparison with some alternative that the results can be judged, and very often the range of alternatives considered has been too narrowly defined.[3]

The wide use of distributed lag models in economics means that these results have potentially important implications over quite a broad area. It may be argued, as in this article, that there has been a lack of consistency in the various approaches adopted, but an added problem, which is not restricted to the demand for money, is that most quarterly data series have been compiled only since 1963. There have therefore been only limited data available for econometric testing. Furthermore, the 1960s may well not have been a very representative period, or one likely to yield strong econometric results. These considerations make it difficult to judge parameter stability over time.

There is no real evidence of a breakdown of the demand for M_1 function, as had been suggested, although the possibility of some shift in the relationship cannot be completely ruled out. However, the recent rapid growth in M_1 has at least been consistent with an apparently stable demand for money which is interest elastic, and the outcome has so far closely followed prior forecasts.

Finally, the preferred equations taken together do seem to have identified an adjustment process which is different for each of the explanatory variables, and is both rapid and quite stable. Furthermore, this behaviour could not be adequately captured by simple partial adjustment and adaptive expectations assumptions. It is a conclusion of this study that these simple models should not represent a starting point but only a *possible* outcome of a wider analysis of the data. Taking these results as a base, it should be possible to develop the analysis and obtain further improvements.

[1] In addition cusum and cusum of squares tests reveal no evidence of a breakdown of the preferred equations: see R. L. Brown, James Durbin and J. M. Evans, 'Techniques for Testing the Constancy of Regression Relationships over Time', *Journal of the Royal Statistical Society, Series B*, vol. 37, no. 2, 1975.

[2] However, all the forecasts contained in Appendix 2 have been made using actual, published, data on these variables.

[3] A similar argument has recently been proposed by A. S. Courakis, 'Serial Correlation and a Bank of England study of the Demand for Money Function: an Exercise in Measurement Without Theory', *University of Oxford:* mimeo., 1977; and D. F. Hendry and G. E. Mizon, 'Serial Correlation as a Convenient Simplification, Not a Nuisance: a Comment on a study of the Demand for Money by the Bank of England', *London School of Economics*, mimeo., January 1978.

Appendix 1

Estimation results

This appendix reports the main results obtained. Although only the most important results have actually been included, because of limitations on space, these should be sufficient to justify the arguments contained in this appendix, and the main text. Most of the other results referred to can be made available upon request.

The traditional model imposing prior lag constraints

The immediate objective is very limited. It is simply to attempt to determine whether, employing a simple 'traditional' model, there is sufficient evidence of stability over the 1960s to justify the conclusion that the statistical relationship had actually broken down in the early 1970s. Certainly when the results (see Appendix 3) are subjected to close scrutiny they reveal a lack of conformity, and no real evidence of stability.

A number of standard models have been estimated with the lags constrained to be the same on each of the independent variables. The results from estimating the best fitting of these equations (equation 2) are given below in Table A. The basic equation of this type incorporates the assumption of a geometrically declining lag on all explanatory variables, and involves the inclusion of the lagged dependent variable, e.g.:

$$m = \lambda a_0 + \lambda a_1 x + \lambda a_2 p - \lambda a_3 r + (1-\lambda)m_1 + u \qquad \dagger(1)$$

where,

m = M_1 definition of money (currency plus current accounts with the banks),

x = for quarterly data: total final expenditure (TFE) at constant 1970 prices; for monthly data: index of industrial production,

p = for quarterly data: price deflator of TFE; for monthly data: price deflator of the retail sales index,

r = local authority three-month rate,

and all variables are measured in natural logarithms (represented by lower case letters). The data employed were the latest available series as at September 1977.[1]

This equation was estimated over a variety of different, overlapping, data periods, employing both quarterly and monthly data. It is only when estimated over the longest periods that the coefficient estimates take approximate values suggested by economic theory, and also, in the end, seem to settle into a fairly stable pattern. However, once the period is shortened, particularly for quarterly data, the coefficients become highly variable, unreasonable, and eventually insignificant. When the estimates do settle down, however, they are very similar for both data frequencies; and perhaps more importantly the lag lengths estimated are very similar.[2] This would seem quite encouraging, since the periods covered by the data are different in each case. The long-run coefficients on income and prices are very similar, and close to unity, but if anything the results suggest that the income elasticity may be slightly higher than the price elasticity.

A possible interpretation of the evidence might be that for the shorter data periods, particularly for quarterly data which is dominated by the 1960s, there is insufficient variation in the data to identify accurately the individual coefficients. (In all there are only fifty-two quarterly[3] and sixty-four monthly observations available.) If multicollinearity is the explanation, then it is clear that the earlier studies had not in fact been able to identify a stable demand-for-money function which later broke down. A Chow test for stability,[4] which splits the period at the end of the second quarter of 1970, provides no support for the view that there has been a structural shift in the equation. The test is, however, not strictly appropriate, given the existence of significant negative first order autocorrelation in the residuals when estimated over periods extending beyond 1972.[5] At the same time, the standard error of the equations also rises considerably.

Correcting for first order autocorrelation results in very similar coefficient estimates; it does not, however, explain the change in the pattern of residuals as the quarterly data period is extended beyond 1972. The simultaneous increase in the standard error could possibly be explained by the greater variability in data series during the 1970s, which, while it improved the coefficient estimates, also increased the error of the equation. Moreover, the greater precision of the coefficient estimates may also have made it easier to identify any mis-specifications, which could account for the worsening of the Durbin-Watson statistic as the period is extended.[6] In which case, while multicollinearity is still present, it does not provide the only explanation for the failure of the earlier studies to identify a stable demand function for narrow money balances.

Estimating this equation with nominal TFE in place of real TFE and the price deflator, produces very similar results. The main difference is that the coefficient on TFE is significant for the earlier periods, but does not seem particularly stable. Imposing a long-run price elasticity of unity, but

† Such an equation may be derived from the assumption of a desired level of nominal money balances, M^*, dependent upon expected interest rate(s), R^e, expected expenditures, X^e (as a proxy for expected transactions requirements), and the expected price level, P^e, where expectations are represented by current values, R, X and P, and there is a partial adjustment of actual money balances, M, to this desired level. Furthermore, this relationship is assumed to be log linear in form, so that,

$$M^* = \frac{A_0 X^{a_1} P^{a_2}}{R^{a_3}} \text{ and, } \frac{M}{M_{-1}} = \left[\frac{M^*}{M_{-1}}\right]^{\lambda} U.$$

Where U is the stochastic error term, generally assumed to have unit mean and constant variance, and λ is the constant coefficient of adjustment.

[1] Most of this work was originally done last summer with slightly different data; these estimates were then updated using revised data.

[2] The more usual discovery is that the lag length estimated increases as data are aggregated over time (see Y. Mundlak, 'Aggregation Over Time in Distributed Lag Models', *International Economic Review*, May 1961.

[3] Indeed the shortest quarterly data period, i.e. up to the end of 1970, contains only twenty-eight observations.

[4] All tests of significance are at the 5% level unless otherwise stated.

[5] All monthly equations exhibited significant negative first order autocorrelation.

[6] It is of course true that the Durbin-Watson statistic is biased towards two as the result of the inclusion of the lagged dependent variable. For some equations, h statistics have also been calculated. However, if we are interested in autocorrelation in the residuals, there seems no obvious reason to restrict attention to the first order, and we have generally tested for autocorrelation up to fourth order.

allowing lagged adjustment,[1] results in an insignificant coefficient on real TFE for the first two periods, and one which is not particularly stable overall; the coefficient is however very stable, at approximately unity, if the constant term is left out.[2] The pattern of results for the equation as a whole is again very much the same. The results suggest that if it is reasonable to impose a long-run price elasticity of unity, the same should also be done for the income term. Doing this should reduce the multicollinearity to a minimum, and as long as the assumption is approximately correct, should improve the efficiency of the estimating procedure.

These results indicate that such an assumption is reasonable even for the earlier periods when the unconstrained estimates suggested very different values. It therefore seems clear that at least part of the problem is associated with multicollinearity. The results however still leave much to be desired, and the deterioration in equation performance, similar to that obtained for the other equations, remains.

Staying within this model framework, it seems possible to argue that the previous equations, reported above, are deficient in not allowing for lags in adjustment and the formation of expectations.

Including simple adaptive expectations of the form,

$$X^e, P^e, R^e = \Pi(X, P, R)_{-i}^{\alpha(1-\alpha)^i}$$

results in the following equation (writing β in place of λ),

$$m = \alpha\beta a_0 + \alpha\beta a_1 x + \alpha\beta a_2 p - \alpha\beta a_3 r + [(1-\alpha) + (1-\beta)]m_{-1} - (1-\alpha)(1-\beta)m_{-2} + u \qquad (2)$$

$$= b_0 + b_1 x + b_2 p - b_3 r + b_4 m_{-1} - b_5 m_{-2} + u$$

where,

$$u = v - (1-\alpha)v_{-1}.$$

The results of estimating such an equation, employing quarterly and monthly data, are given in Table A.

Table A

1 Quarterly equations

Estimation period	Constant	x	p	r	m_{-1}	m_{-2}	$\bar{R}^2$	DW	SE	Long-run elasticities x	Long-run elasticities p	Long-run elasticities r	Average lag
1964(1)—1970(4)	0.335	0.385	0.183	−0.055	0.635	−0.066	0.971	1.949	0.0121	0.893	0.425	−0.128	1.153
	0.16	*1.64*	*1.27*	*2.66*	*3.17*	*0.31*		h = –					
1964(1)—1971(4)	1.552	0.187	0.271	−0.060	0.711	−0.065	0.985	2.108	0.0128	0.528	0.766	−0.169	1.641
	0.72	*0.91*	*1.86*	*3.80*	*3.75*	*0.35*		h = –					
1964(1)—1972(4)	−0.207	0.150	0.119	−0.049	0.822	0.055	0.992	1.937	0.0132	1.220	0.967	−0.398	7.577
	0.10	*0.78*	*0.90*	*3.43*	*4.46*	*0.31*		h = –					
1964(1)—1973(4)	0.893	0.033	0.206	−0.063	0.408	0.477	0.993	1.741	0.0148	0.287	1.791	−0.548	11.843
	0.50	*0.18*	*1.78*	*4.98*	*2.87*	*3.37*		h = 1.852					
1964(1)—1974(4)	1.628	−0.007	0.264	−0.069	0.443	0.403	0.994	1.736	0.0163	−0.045	1.714	−0.448	8.110
	1.64	*0.06*	*4.09*	*5.52*	*3.34*	*2.99*		h = 1.842					
1964(1)—1975(4)	0.523	0.127	0.211	−0.066	0.451	0.375	0.996	1.880	0.0160	0.730	1.213	−0.379	6.902
	0.92	*1.87*	*4.27*	*5.52*	*3.62*	*3.00*		h = 0.823					
1964(1)—1976(4)	0.099	0.209	0.206	−0.065	0.466	0.320	0.997	1.795	0.0165	0.977	0.963	−0.304	5.168
	0.18	*3.57*	*4.06*	*5.52*	*3.72*	*2.61*		h = 1.723					
1965(1)—1976(4)	0.063	0.216	0.208	−0.066	0.457	0.325	0.997	1.769	0.0172	0.991	0.954	−0.303	5.078
	0.11	*3.33*	*3.92*	*5.25*	*3.48*	*2.54*		h = 1.928					

2 Monthly equations

Estimation period	Constant	x	p	r	m_{-1}	m_{-2}	$\bar{R}^2$	DW	SE	Long-run elasticities x	Long-run elasticities p	Long-run elasticities r	Average lag
1971(8)—1975(6)	0.125	0.094	0.074	−0.024	0.639	0.307	0.994	2.127	0.0091	1.741	1.370	−0.444	23.204
	0.37	*1.61*	*2.42*	*3.19*	*4.51*	*2.18*		h = −1.832					
1971(8)—1975(12)	0.166	0.086	0.074	−0.023	0.652	0.293	0.996	2.158	0.0090	1.564	1.345	−0.418	22.509
	0.51	*1.59*	*2.47*	*3.26*	*4.85*	*2.22*		h = −2.801					
1971(8)—1976(6)	0.408	0.087	0.082	−0.019	0.599	0.319	0.997	2.022	0.0093	1.061	1.000	−0.232	15.085
	1.32	*1.60*	*2.70*	*2.96*	*4.71*	*2.59*		h = −0.395					
1971(8)—1976(12)	0.339	0.084	0.081	−0.021	0.584	0.343	0.997	2.102	0.0099	1.151	1.110	−0.288	17.397
	1.05	*1.49*	*2.53*	*3.12*	*4.91*	*2.95*		h = −1.450					

t statistics are given below the coefficient estimates.

[1] Rather than define a relationship for desired *real* balances, as has been done in most studies in the United Kingdom, a nominal demand function is assumed which is homogeneous in prices, i.e.

$$M^* = A_0 X^{a_1} P / R^{a_3}.$$

Combined with the partial adjustment of nominal balances this gives,

$$M = A_0^{\lambda} X^{\lambda a_1} P^{\lambda} R^{-\lambda a_3} M_{-1}^{(1-\lambda)} U$$

which can be rearranged and written as,

$$(m-p) = \lambda a_0 + \lambda a_1 x - \lambda a_2 r + (1-\lambda)(m_{-1} - p) + u.$$

Surprisingly, the only time an estimated equation of this form has been published for the United Kingdom was by Hacche (September 1974 *Bulletin*, page 284), and then only in first differences. This implies that those other demand equations estimated in real terms may all be mis-specified in not allowing for lagged adjustment to price changes. An alternative way to allow for lagged adjustment to price changes is to include the rate of inflation as a separate explanatory variable in an equation with lagged real balances. In this case it is necessary to test whether that is the only reason for the inclusion of inflation, or whether it also has some independent effect. This is because the above equation can be rewritten as,

$$(m-p) = \lambda a_0 + \lambda a_1 x - \lambda a_2 r - (1-\lambda)(p - p_{-1}) + (1-\lambda)(m-p)_{-1} + u$$

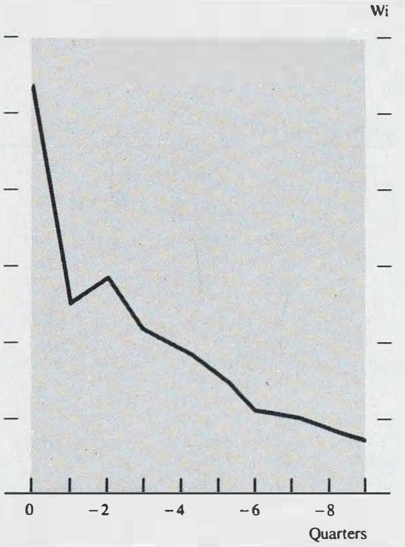
since

$$P^{\lambda} = PP_{-1}^{-(1-\lambda)} \left[\frac{P}{P_{-1}}\right]^{-(1-\lambda)}.$$

[2] It is interesting to note that apparently stable coefficients could have been identified from 1970 if the constant term had been left out. The statistical performance of the equation does, however, deteriorate beyond 1972, and this remains to be explained.

There is a significant improvement in the fit of this equation over the single lag case, although there is the same increase in the standard error of the equation as the period is extended, that was observed previously. There is some reduction in the autocorrelation of the residuals, but it is only occasionally significant. As for the single lag equations, the results from estimating over the longest periods support the hypothesis that the long-run price and income elasticities are very close to unity, with the income elasticity marginally higher. Again the quarterly and monthly coefficient estimates, and average lags, are very similar. The quarterly pattern of lagged adjustment is shown in the chart; over half the adjustment is completed after three quarters and 60% is completed after a year. Furthermore, a Chow test on the quarterly equation does not reject the hypothesis of structural stability.

Chart A
Lag weights



If we were looking for the simplest possible model, then a case could obviously be made in support of this equation, with multicollinearity providing the explanation for the failure of earlier studies to identify this structure. We might, however, have some lingering doubts as to the justification for imposing such a specific lag pattern, as shown in the chart, to all explanatory variables. It was therefore considered necessary to start from the other direction and see whether allowing for separate lag processes on each of the explanatory variables actually reduced to this simple model.

Models with unrestricted lags

The lag patterns considered so far have all been specific, restricted, forms of rational distributed lags, which are any lag structure that can be written as,

$$P(L)H = Q(L)G + R(L)F$$

where $P(L)$, $Q(L)$ and $R(L)$ are polynomials in the lag operator, L, and H, G and F are three variables.

This approach is very flexible and can obviously allow for differential lags on each of the independent variables, e.g.

$$H = \frac{a_0 + a_1(1 - \lambda_1 L)G + a_2(1 - \lambda_2 L)F}{1 - bL - cL^2} + U. \qquad (3)$$

Rational lags of this type were estimated with reasonable success for the United Kingdom by Price.[1] It proved difficult, however, given the data available at that time, to identify individual coefficients accurately, particularly for M_1. The present article develops this approach, taking the view that it is very unlikely that the same lag can be reliably applied to all explanatory variables in an equation.[2] If such an assumption were in fact valid it should be revealed in empirical estimation and not imposed from the outset. An alternative development on this approach has recently been suggested by Hendry and Mizon. They estimated a demand-for-money equation (but not for M_1)[3] that can be interpreted as a specific rational distributed lag formulation,

$$m = b_0 + b_1 x + b_2 x_{-1} + b_3 x_{-4} + b_4 p - b_5 p_{-1} + b_6 p_{-2} + b_7 r - b_7 r_{-1} - b_8 r_{-4} + b_9 m_{-1} - b_{10} m_{-2} + e. \qquad (4)$$

Although the procedure adopted in this article should be general enough to include this formulation, the actual equation Hendry and Mizon estimated was in the equivalent form,

$$\Delta(m - p) = a_0 + a_1 \Delta x + a_2 \Delta r + a_3(m - p)_{-1} - a_4 \Delta p - a_5(m - y)_{-1} - a_6 r_{-4} + a_7 x_{-4} + e. \qquad (5)$$

As a starting point, the most general form likely to occur was estimated:

$$m = a + \sum_{i=0}^{4} (b_i x_{-i} + c_i p_{-i} + d_i r_{-i} + e_i m_{-1-i}) + u. \qquad (6)$$

The best fitting equation estimated over 1964(1)–1976(4), including an insignificant constant term, turned out to be of the form,

$$m = b_0 + b_1 x - b_2 x_{-3} + b_3 p_{-1} - b_4 p_{-2} + b_5 p_{-3} - b_6 p_{-4} - b_7 r + b_8 m_{-1} + u \qquad (7)$$

which follows a pattern very similar to that suggested by the significant coefficient estimates obtained for equation 6. This equation is, however, not particularly stable when estimated over different data periods. This is unfortunate because the statistical fit of the equation, as measured by the

[1] March 1972 *Bulletin*, page 43.

[2] A further possibility, but one that is not examined here, is that the lags in adjustment are not constant and should therefore be endogenised to allow for variable speeds of adjustment. In this respect it might be that the size of any exogenous change, and consequently the size of the adjustment necessary to achieve equilibrium, will directly affect the *speed* of adjustment. For example, we might expect the unusually large changes in the price level that have taken place in recent years to have resulted in a speeding up of the adjustment process. In practice the effect is not so clear-cut, and it is also possible that the associated rapid inflation would have the opposite effect. Again, the existence of such a mechanism must be an empirical question.

[3] The actual definition employed was personal sector M_3 balances. It has, however, been suggested that M_3 may have been, to a large extent, supply determined.

standard error and Durbin–Watson statistic, is very good, and does not deteriorate in the same way as the simpler specifications estimated. The coefficients estimated are very stable when the data period is shortened by excluding the early 1960s, and the fit of the equation represents an improvement over the assumption of the same lag on all explanatory variables.[1] It is arguable that the estimates for the later periods are the more reliable since there is greater variation in the data series over the 1970s than the early 1960s therefore enabling the 'true' coefficients to be more accurately identified. Further evidence in support of the values obtained for the equation estimated over the longest period is provided when the constant is excluded in estimation. In this case, the coefficients are very similar for the full period, but now remain reasonably stable back to the end of 1972. These results are given in Table B, although it is recognised that there is no *a priori* justification for leaving out the constant term, thereby constraining the scale variable to unity.

Table B

1 Linear model

Estimation period	x	x_{-3}	p_{-1}	p_{-2}	p_{-3}	p_{-4}	r	m_{-1}	$\bar{R}^2$	DW	SE	Long-run elasticities: x	p	r
1964(1)—1970(4)	0.618 *2.67*	−0.251 *1.04*	0.729 *1.46*	−1.109 *1.52*	0.875 *1.17*	−0.357 *0.71*	−0.043 *1.91*	0.621 *3.87*	0.971	1.686	0.0121	0.969	0.362	−0.114
1964(1)—1971(4)	0.406 *1.88*	−0.114 *0.46*	0.924 *1.91*	−1.202 *1.71*	0.741 *0.99*	−0.309 *0.60*	−0.059 *2.74*	0.706 *5.72*	0.985	1.875	0.0127	0.989	0.521	−0.200
1964(1)—1972(4)	0.386 *2.13*	−0.259 *1.39*	0.637 *1.52*	−1.182 *1.91*	0.905 *1.33*	−0.237 *0.48*	−0.039 *2.52*	0.876 *13.18*	0.993	1.946	0.0125	1.024	0.997	−0.317
1964(1)—1973(4)	0.380 *2.41*	−0.221 *1.56*	1.157 *3.18*	−2.154 *4.45*	1.706 *2.65*	−0.580 *1.26*	−0.047 *4.33*	0.843 *14.84*	0.995	2.288	0.0134	1.014	0.819	−0.298
1964(1)—1974(4)	0.424 *3.05*	−0.266 *2.07*	1.226 *3.96*	−2.009 *4.16*	1.970 *4.00*	−1.077 *3.11*	−0.047 *4.29*	0.843 *15.23*	0.996	2.067	0.0136	1.006	0.695	−0.299
1964(1)—1975(4)	0.405 *3.89*	−0.247 *2.37*	1.335 *4.89*	−2.127 *4.99*	1.854 *4.13*	−0.951 *3.14*	−0.049 *5.08*	0.843 *17.35*	0.997	2.191	0.0131	1.009	0.708	−0.314
1964(1)—1976(4)	0.379 *3.80*	−0.216 *2.17*	1.326 *6.05*	−2.069 *5.73*	1.675 *4.57*	−0.816 *3.53*	−0.048 *5.16*	0.838 *18.62*	0.998	2.220	0.0128	1.008	0.709	−0.300
1965(1)—1976(4)	0.384 *3.89*	−0.238 *2.31*	1.450 *6.40*	−2.316 *6.23*	1.851 *4.99*	−0.884 *3.84*	−0.051 *5.44*	0.857 *17.38*	0.998	2.174	0.0126	1.018	0.098	−0.356
1966(1)—1976(4)	0.405 *3.85*	−0.233 *2.16*	1.430 *6.00*	−2.282 *5.81*	1.813 *4.60*	−0.836 *3.37*	−0.053 *5.29*	0.830 *14.41*	0.998	2.142	0.0131	1.010	0.732	−0.309

2 First differences

Estimation period	Δx	Δx_{-3}	Δx_{-2}	Δp_{-1}	Δp_{-2}	Δp_{-3}	Δr	Δr_{1}	Δr_{-2}	$\bar{R}^2$	DW	SE	Long-run elasticities: x	p	r
1963(4)—1970(4)	0.588 *2.28*	0.483 *2.14*	0.261 *1.35*	0.585 *1.23*	−0.765 *1.30*	0.344 *0.60*	−0.052 *1.87*	−0.017 *0.62*	−0.027 *0.98*	0.253	2.017	0.0132	1.332	0.164	−0.096
1963(4)—1971(4)	0.441 *1.81*	0.464 *2.04*	0.196 *1.07*	0.891 *1.88*	−0.689 *1.28*	0.251 *0.45*	−0.051 *1.91*	−0.023 *0.84*	−0.038 *1.39*	0.294	2.035	0.0139	1.101	0.453	−0.112
1963(4)—1972(4)	0.488 *2.25*	0.412 *1.79*	0.167 *0.90*	0.709 *1.55*	−0.734 *1.58*	0.684 *1.32*	−0.023 *0.97*	−0.019 *0.73*	−0.037 *1.42*	0.297	1.659	0.0144	1.067	0.659	−0.079
1963(4)—1973(4)	0.360 *1.95*	0.476 *2.76*	0.250 *1.74*	0.865 *2.25*	−0.856 *2.22*	0.594 *1.30*	−0.050 *2.87*	0.001 *0.04*	−0.059 *2.94*	0.513	1.949	0.0142	1.086	0.603	−0.108
1963(4)—1974(4)	0.335 *2.46*	0.442 *3.08*	0.217 *1.66*	0.825 *2.57*	−0.801 *2.57*	0.751 *2.16*	−0.051 *3.00*	−0.002 *0.08*	−0.070 *3.78*	0.630	1.859	0.0140	0.994	0.774	−0.123
1963(4)—1975(4)	0.309 *2.47*	0.408 *3.36*	0.228 *2.00*	1.073 *4.06*	−0.891 *3.25*	0.589 *1.92*	−0.048 *2.91*	−0.012 *0.69*	−0.070 *4.04*	0.652	1.956	0.0137	0.945	0.771	−0.130
1963(4)—1976(4)	0.284 *2.34*	0.419 *3.51*	0.224 *2.03*	0.999 *4.70*	−0.816 *3.30*	0.567 *2.52*	−0.045 *3.08*	−0.018 *1.11*	−0.064 *4.01*	0.658	2.010	0.0135	0.927	0.750	−0.127
1965(1)—1976(4)	0.280 *2.19*	0.449 *3.51*	0.228 *1.82*	1.077 *4.71*	−0.938 *3.47*	0.609 *2.58*	−0.047 *3.02*	−0.021 *1.20*	−0.061 *3.59*	0.667	1.925	0.0139	0.957	0.748	−0.129
1966(1)—1976(4)	0.306 *2.27*	0.490 *3.63*	0.213 *1.63*	1.082 *4.59*	−0.945 *3.41*	0.604 *2.48*	−0.045 *2.75*	−0.026 *1.35*	−0.070 *3.72*	0.673	1.929	0.0142	1.009	0.741	−0.141

It is clear that inclusion of the constant term results in considerable instability in the other coefficients. When the constant is appoximately zero, the other estimated coefficients appear very stable, and the further the estimated constant is from zero the less believable (in terms of economic theory) are the coefficient estimates on income and prices. Furthermore, there seems to be some correlation between the sign taken by the constant and the direction of bias in the estimated coefficients on these other variables.[2]

The lag profile of equation 7 estimated over the full period is very interesting. The implied cumulative lag adjustment weights are given in the chart on page 51. The average lags on x and r are 0.494 and 4.882 respectively (that on p is not defined because the overadjustment in the first period results in negative weights thereafter).

[1] It is difficult to compare equation 7 directly with equation 2, since they are non-nested hypotheses. Both are, however, contained in the general form, equation 6, to which they can be compared. Employing a standard F test, equation 7 is not significantly different from the general form (the standard error is actually reduced), whereas the constrained equation is significantly worse.

[2] The larger the constant the lower the long-run income elasticity and the higher the long-run price elasticity. When the estimated constant was 3.300 the income elasticity was −0.654 and that on prices 1.654; when the estimated constant was −1.584 the respective elasticities were 1.404 and 0.091.

Allowing for small errors in estimating the individual coefficients, it is clear that these lags are consistent with quick adjustment on income and prices and a distributed lag on the rate of interest, i.e.,

$$m = \sum_{i=0}^{2} b_i x_{-i} + \sum_{j=1}^{3} c_j p_{-j} + dr^e + v, \tag{8}$$

where,

so that,

$$r^e = \frac{(1-\alpha)r}{(1-\alpha L)}$$

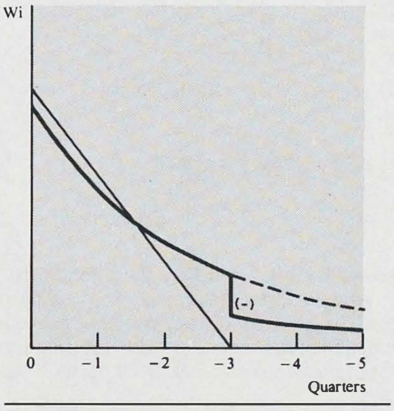
$$m = [1-\alpha L][\sum_{i=0}^{2} b_i x_{-i} + \sum_{j=1}^{3} c_j p_{-j}] + d(1-\alpha)r + \alpha m_{-1} + u. \tag{9}$$

For x,

$$b_0 x + [b_1 - \alpha b_0]x_{-1} + [b_2 - \alpha b_1]x_{-2} - \alpha b_2 x_{-3}$$

which means that as long as $b_1 \approx x b_0$, and $b_2 \approx x b_{-1}$, this formulation would result in zero estimated coefficients on x_{-1} and x_{-2}, and a significant negative coefficient on x_{-3}. A similar argument can also be applied to p, although in this case the correspondence is not so close, and implies slightly larger errors in estimation. This approximation of short lags by means of a distributed lag model is illustrated in the chart.

Chart B
Distributed lag approximation of a two period lag



The next step, in the light of these results, was to estimate a general equation leaving out any lagged money terms to see if short lags could be freely estimated. This revealed strong evidence of first order autocorrelation, and when this was allowed for the coefficient on *rho* was not significantly different from unity, suggesting that perhaps the equation should be estimated in first differences. The results from doing this are reported in Table B, and the best fitting equation conforms closely to the general estimates obtained.[1]

The long-run elasticities, and the short-run pattern of adjustment, are very similar to those given in Table B part 1. The maximum lag on income is two quarters and the maximum lag on prices is three quarters.[2] The coefficient on the current price term is insignificant, and the long-run elasticities are almost exactly the same as those previously estimated. The lag on the rate of interest is obviously considerably shorter, and the total elasticity substantially less, than before. Adding additional lagged values of the rate of interest does result in correctly signed coefficients but they do not significantly improve the fit of the equation.

The results from these two equations should really be considered together. When estimated in first differences, the implied model is essentially short run, and does not yield reasonable long-run properties. On the other hand, the levels equation has more reasonable long-run properties; although when the constant term is included it is less able to identify the separate short-run properties of the equation. The first difference equation suggests that the lags on income and prices really are very short, and, on the basis of this additional evidence, it would seem reasonable to accept those results imposing a zero intercept (in logarithms).[3] The close correspondence between the two sets of results would suggest that there is sufficient data variation to identify fairly accurately the separate coefficient values even in the levels equation.

The results so far have been obtained employing published seasonally-adjusted data. It should, however, be noted that the use of seasonally-adjusted data in regression has recently been questioned by a number of economists. The argument is essentially that while it may be useful to apply seasonal adjustment to a single data series, in order to isolate its underlying trend, this does not justify the separate seasonal adjustment of each variable in a regression. In reply it might be argued that the official seasonal adjustment procedures employed do far more than filter out a fixed-seasonal pattern; they also allow the seasonal pattern to vary over time, and take account of various changes in timing, e.g. of holidays, and in the pattern of tax payments, that otherwise need to be modelled explicitly. The choice between using seasonally-adjusted and unadjusted data is not always clear, and is anyway likely to depend on the particular problem under consideration. In the light of possible biases that may be caused, it seemed desirable also to try estimating demand functions employing non-seasonnally-adjusted data.

We have not done a great deal of work with non-seasonally-adjusted data, but the results obtained so far conform closely to those obtained using seasonally-adjusted data. The long-run elasticities are marginally higher, but the main difference is that all adjustment to any price change takes place after a one quarter lag. This pattern of behaviour is more plausible than that obtained with seasonally-adjusted data, but overall there is little to choose between the two sets of results.

[1] None of these preferred equations exhibited evidence of autocorrelation up to fourth order.
[2] Lags longer than four on prices were tried but never resulted in any significant improvement.
[3] It might be more accurate to impose some constant close to zero. As more data become available it may be possible to identify what this constant should be.

Appendix 2

Forecast performance and stability

Forecasts

The preferred equations, and the best of the constrained lag equations, have been estimated up to end-1975, and end-1976, and used to forecast dynamically up to the third quarter of 1977. Only the quarterly forecasts are given (Table A) together with their percentage forecast errors. For the purposes of this exercise, the equations have been re-estimated employing new data series (as at January 1978) so that the forecasts are consistent with the latest data available at the time of writing. There is therefore also the added requirement that these equations be stable in the face of such data alterations. The forecasts from the preferred levels equation are obtained using the equation estimated without a constant term. This is done because one would not expect the equation including a constant to forecast very well, given that the coefficients seem unreliable when it is estimated up to the end of 1975.[1]

Table A

Quarterly dynamic forecasts: 1976(1)–1977(2) and 1977(1)–1977(2)

Percentage forecast errors are given in italics [(forecast-actual)/actual]

	Actual M_1	[a]		[b]		[c]	
Estimated to 1975							
1976 (1)	17,960	18,345	*2.14*	18,186	*1.26*	17,991	*0.17*
(2)	18,400	19,027	*3.41*	18,639	*1.30*	18,313	*−0.47*
(3)	19,220	19,787	*2.95*	19,481	*1.36*	19,377	*0.82*
(4)	19,140	20,323	*6.18*	19,440	*1.57*	19,370	*1.20*
1977 (1)	19,750	21,250	*7.59*	20,135	*1.95*	20,241	*2.49*
(2)	20,490	22,534	*9.98*	20,783	*1.43*	20,681	*0.93*
(3)	21,860	23,820	*8.97*	21,764	*−0.44*	21,859	—
Estimated to 1976							
1977 (1)	19,750	20,234	*2.45*	19,821	*0.36*	19,979	*1.16*
(2)	20,490	21,384	*4.36*	20,449	*−0.20*	20,446	*−0.21*
(3)	21,860	22,595	*3.36*	21,408	*−2.07*	21,615	*−1.12*

[a] $$m = b_0 + b_1 y - b_2 r + b_3 m_{-1} - b_4 m_{-2} + u.$$

[b] $$m = b_1 x - b_2 x_{-3} + b_3 p_{-1} - b_4 p_{-2} + b_5 p_{-3} - b_6 p_{-4} - b_7 r + b_8 m_{-1} + u.$$

[c] $$\Delta m = b_1 \Delta x + b_2 \Delta x_{-1} + b_3 \Delta x_{-2} + b_4 \Delta p_{-1} - b_5 \Delta p_{-2} + b_6 \Delta p_{-3} - b_7 \Delta r - b_8 \Delta r_{-1} - b_9 \Delta r_{-2} + u.$$

The preferred equations in fact forecast very well, and considerably better than the constrained lag equation. At its worst, this last equation has a forecast error that is 5.4 times its standard error, whereas the errors on the preferred equations are never as much as two standard errors. The first difference equation provides far better dynamic forecasts of the levels of M_1 than might be expected, given that the error structure of the levels will follow a random walk when forecasts are derived from a difference equation with a stationary error. An encouraging feature of the forecasting ability of both the preferred equations is that they are the only ones to predict (at least partially) the downturn in the money stock in the last quarter of 1976.

Stability

The preferred equations have also been tested for stability employing the recursive tests recently developed by Brown, Durbin and Evans. The stability tests employed were the cusum and cusum of squares tests. Although only the results of the latter test are given here it should be noted that the cusum test did not reveal any equation instability.[2]

Very briefly, the cusum of squares test is based upon a statistic derived from residuals of one-period-ahead forecasts. Predictions are made for each observation greater than the number of explanatory variables, and the test statistic (S_t) obtained by summing the square of the prediction residuals (W_t^2) and dividing by the total sum of squares of the prediction residuals, i.e.,

$$S_t = \sum_{k+1}^{t} w_t^2 / \sum_{k+1}^{n} w_t^2$$

$$t = k+1,\ k+2,\ \ldots n$$

where k is the number of explanatory variables, and n is the total number of observations.

The statistic therefore takes values between zero and unity, and under the null hypothesis of stability has the expectation $(t\text{-}k)/(n\text{-}k)$, i.e. that the variance of the prediction is constant. A statistical measure of divergence from stability is provided by drawing a pair of lines parallel to the mean value line, i.e.,

$$S_t^* = \pm c_0 + (t-k)/(n-k),$$

where c_0 is distributed as Pyke's modified Kolmogorov–Smirnov statistic.

[1] We have also made the alternative forecasts, and, as expected, they are very poor. However, when estimated to end-1976 and used to forecast the next three quarters, the equation performs very well. This is not surprising as, for the full period, the constant is approximately zero, and the estimates conform to the pattern expected.

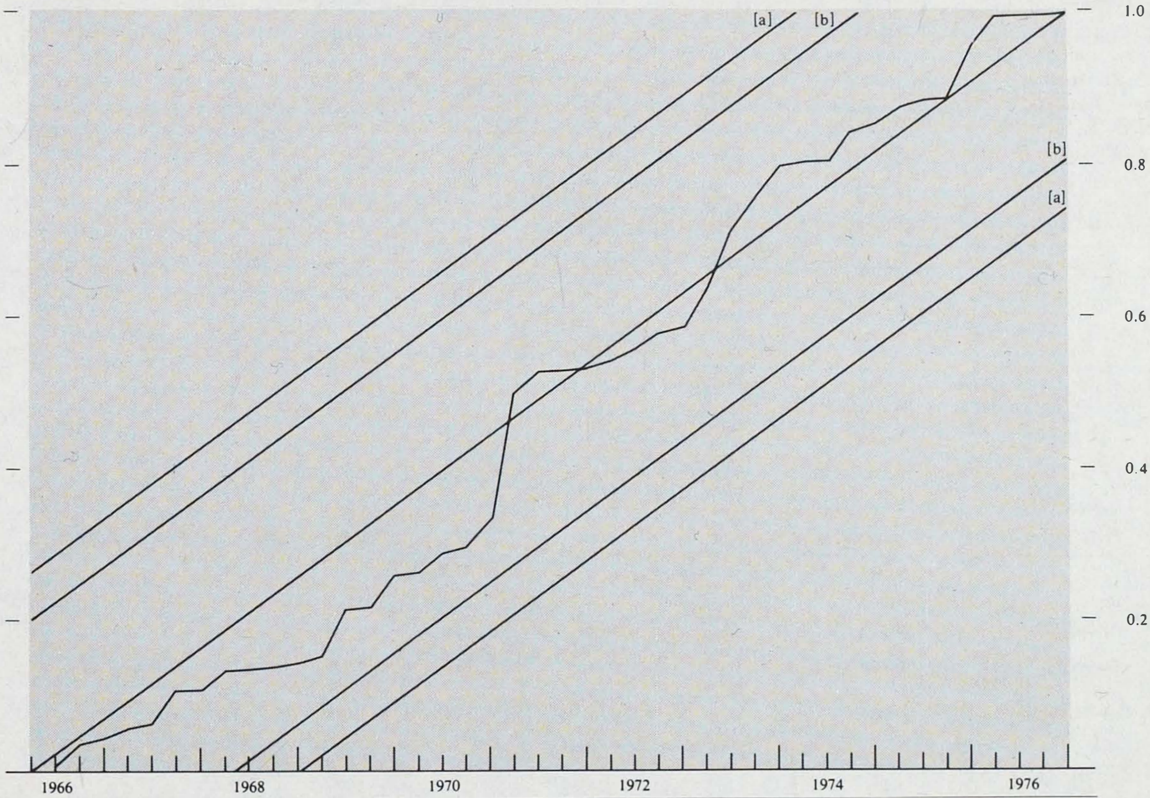
[2] Attention is concentrated on the cusum of squares because K. Garbade, 'Two Methods for Examining the Stability of Regression Coefficients', *Journal of the American Statistical Association*, March 1977, has shown that it is much more likely to reveal a structural change than is the normalised cusum. Examples of the use of the former test include M. S. Kahn, 'The Stability of the Demand-for-Money Function in the United States, 1901–1965', *Journal of Political Economy*, November/December 1974, and C. Adams and M. G. Porter, 'The Stability of the Demand for Money', in *Conference in Applied Economic Research: Papers and Proceedings*, Reserve Bank of Australia, 1976.

For any predetermined level of significance (and Brown *et al* suggest 10% or less), instability is indicated if S_t crosses either of these significance lines. It is probably best not to think of this as a rigorous test and Brown *et al* 'prefer to regard the lines constructed in this way as yardsticks against which to assess the observed sample path rather than providing formal tests of significance'.

The charts show the path of the cusum of squares for the distributed lag equation (including a constant) and the first difference equation respectively. The test lines are drawn at the normal 5% and also the much stricter 20% level. It can be seen that the paths of S_t are very similar, and in neither case is the null hypothesis of stability rejected.

Chart A

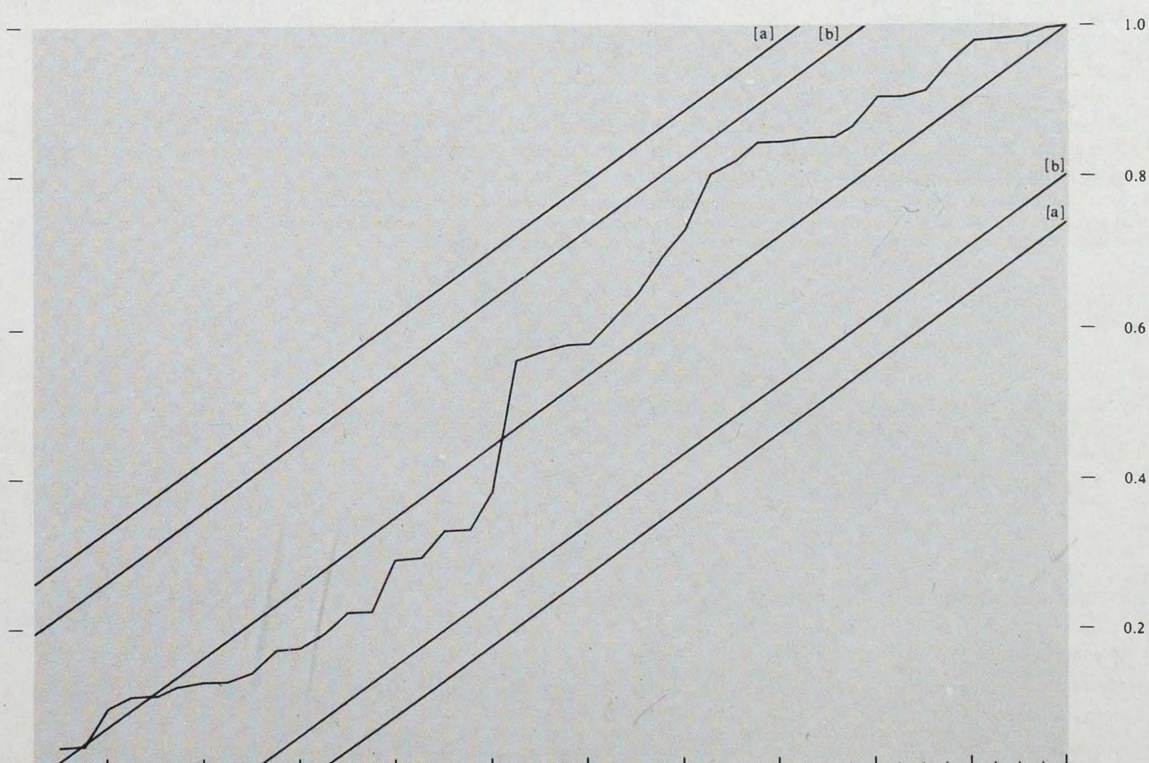
Cusum of squares of recursive residuals, forward recursion, for levels equation



[a] 5% significance line. [b] 20% significance line.

Chart B

Cusum of squares of recursive residuals, forward recursion, for differences equation



[a] 5% significance line. [b] 20% significance line.

Appendix 3

Demand for money in the United Kingdom: summary of results

Author	Data	Money	Interest rate	Interest elasticity[a]	Income variable	Income elasticity[a]	Price
Kavanagh and Walters	Annual: 1880–1961	Broad	Long	−0.31 (−0.22)[b]	GNP	1.149	—
	1926–1961	Broad	Long	−0.50 (−0.25)[b]	GNP		—
Crouch	Quarterly: 1954–1965	LCB Bank Deposits (total)	Long	−0.50	GNP	1.08	—
Fisher	Quarterly: 1955(1)–1967(2)	Narrow	Short	−0.11	PDI	0.686	—
		Broad	Short	—	PDI	0.742	—
		Narrow	Long	−0.30	PDI	0.686	—
		Broad	Long	—	PDI	0.742	—
Goodhart and Crockett	Quarterly: 1955(3)–1969(3)/1963(2)–1969(3)	Narrow	Short	−1.05	GDP	1.25	—
		Narrow	Long	−0.80	GDP	1.09	—
		Broad	Short	−0.09/−0.21[c]	GDP	0.77/1.50[c]	—
		Broad	Long	−0.35/−0.51[c]	GDP	1.09/1.89[c]	—
Laidler and Parkin	Quarterly: 1955(3)–1967(4)	Broad	Short	−0.008	GDP (perm.)	0.68	1*
Laidler	Annual: 1900–1965	Broad	Long	−0.570	GDP	0.795	1*
	1900–1913	Broad	Long	−0.268		1.241	1*
	1920–1938	Broad	Long	−0.448	GPD	0.793	1*
	1946–1965	Broad	Long	−0.739	GDP	0.684	1*
Price	Quarterly: 1964(1)	Broad	Short	—			
	1970(4): *Persons*	Broad	Long	−0.30	GDP	2.29	0.90
	Quarterly: 1964(1)	Broad	Short	−0.36			
	1970(4): *Companies*	Broad	Long	—	GDP	2.77	0.41
Hacche	Quarterly: 1963(4)–1971(3)	Narrow	Short	−0.081	TFE	0.391	1*
			Long	−0.184			
		Broad	Short	−0.091	TFE	0.450	1*
			Long	—			
	Persons	Broad	Short	—	PDI	0.927	1*
			Long	−0.069			
	Companies	Broad	Short	−0.067	TFE	0.511	1*
			Long	−0.197			
	Quarterly: 1963(4)–1972(4)	Narrow	Short	−0.062	TFE	0.697	1*
			Long	−0.206			
		Broad	Short	—	TFE		1*
			Long	—			
	Persons	Broad	Short	—	PDI	1.081	1*
			Long	−0.110			
	Companies	Broad	Short	−0.044	TFE	2.206	1*
			Long	−0.696			
	Quarterly: 1963(4)–1972(4)	Broad	Short	−0.248	TFE	0.995	1*
			Long	—			
			Own rate	0.537			
	Companies	Broad	Short	—	TFE	1.003	1*
			Long	−0.345			
			Own rate	0.568			
Artis and Lewis[d]	Quarterly: 1963(2)–1970(4)	Narrow	Long	−0.26	GDP	0.77	—
	1963(2)–1971(4)	Narrow	Long	−0.39	GDP	0.95	—
	1963(2)–1973(1)	Narrow	Long	−0.66	GDP	1.24	—
	1963(2)–1970(4)	Broad	Long	−0.47	GDP	1.42	—
	1963(2)–1971(4)	Broad	Long	−0.52	GDP	1.48	—
	1963(2)–1973(1)	Broad	Long	−3.00	GDP	4.27	—
Hamburger	Quarterly: 1963(2) 1971(1)	Narrow	Short[e]	−1.20[f]	GDP	0.672	—
			Long[e]	−1.07[f]			

Key

* A long-run unitary price elasticity imposed.
GNP = Gross national product
PDI = Personal disposable income
TFE = Total final expenditure

[a] These are the long-run elasticities.
[b] From the same equation estimated in first differences.
[c] Results for money on a broad definition for both M_2 and M_3 respectively.
[d] Here only the results of the standard approach are summarised, and not the attempts to provide an improved specification.
[e] In this case the short rate is the three-month euro-dollar rate, and the long rate is the dividend-price ratio on ordinary shares.
[f] These are not the interest elasticity, but the elasticity *times* one *plus* the rate of interest *divided* by the rate of interest.