Bank of England notes

Introduction

At intervals since 1961, several articles have been published in the *Bulletin* on various aspects of the note circulation. [1] This is an area of the Bank's work which always arouses interest among the general public, and demand for off-prints of the earlier articles has remained strong over the years. The object of this article is to bring together, in summary form, the information previously published and to bring it up to date by describing developments since the publication of the most recent article, in December 1974.

The Bank have been issuing notes since 1695. At first, notes were issued only in London but from 1826 branches were established in the provinces in order to expand the note circulation, and notes have been issued through the branches ever since. In 1844 the Bank Charter Act, by imposing restrictions on the rights of other banks to issue notes (which eventually led to the discontinuance of such issues), provided the legal foundation for the Bank's present monopoly of note issue in England and Wales. [2]

The 1844 Act also required the Bank to set up a separate Issue Department to handle the note issue, and authorised the department to issue notes against gold to an unlimited extent and against securities (the 'fiduciary' issue) up to £14 million. This figure has been increased by subsequent legislation and currently stands at £1,575 million by virtue of the Currency and Bank Notes Act 1954. The same Act authorised the Treasury to vary this amount at the instance of the Bank. An increase beyond £1,575 million is not permitted to continue for longer than two years without a Treasury Order, laid before Parliament, extending the period for a further two years. This last vestige of Parliamentary control of the note issue survives as a relic of the days when manipulation of the level of notes in circulation was regarded as a means of stimulating or restricting economic activity. Today, the increase in various forms of government debt and the growth of other means of payment (e.g. cheques) has shifted the emphasis of monetary policy away from notes. The Bank stand ready to supply all demands for notes made upon them by the commercial banks, which in turn satisfy the requirements of their customers (and, indirectly, the public at large) for notes; and the Treasury authorise upward or downward movements in the amount of the fiduciary issue, in multiples of £25 million in order to meet expansions and contractions in this demand. It should be pointed out that the whole of the circulation is now fiduciary: the gold holdings of the Issue Department were added to the Exchange Equalisation Account's reserve holdings of gold and foreign currencies in 1939 (except for a token amount which was finally transferred in August 1970). [3]

The separate status of the Issue Department, established in 1844, although as a bookkeeping convention rather than a physical entity in the Bank's organisation, continues to this day. The costs of managing the note issue (printing, safe custody, distribution and destruction) are kept separate from the Bank's other expenses. They are deducted from the income received, in the form of interest plus capital appreciation (or minus capital losses) on the securities held by the Issue Department as backing for the note issue; the whole of the net income remaining is paid over at intervals to the Treasury.

The only other aspect of the management of the note circulation which is not entirely a matter for the Bank is that the denominations of notes issued by the Bank require Treasury approval. The Bank's responsibilities therefore cover the design, printing and distribution of notes, with the aim of maintaining a note circulation sufficient in quantity and in condition to meet the needs of the population while avoiding unnecessary or unreasonable expense.

Design

Bank note design is subject to the same considerations as industrial design generally: the aim is to produce at reasonable cost an article which is functionally fully satisfactory yet pleasant to look at. [4] A design is not necessarily a good one merely because it is beautiful or cheap to manufacture; more important than either is that it should satisfy its functional requirements. For notes, the chief of these is security against counterfeiting.

There is no infallible single security feature. Apart from well-known features such as the watermark and the thread, the security of a note depends also on the combination of specialised design elements (such as hand-engraved portraits and geometric patterns produced by purpose-built machinery) and the use of specialised printing techniques and materials. The artist has to understand and make use of this variety of aids and yet evolve an acceptable design which is fit for mass production.

Security against counterfeiting does not stop at designing a note that will make the forger's task more difficult nor one that will merely ensure that a forgery is obvious to an expert. The real art is to produce a note such that any attempt at forgery is obvious to a member of the public at a casual glance: forgers are caught by the vigilance of the public followed by good police work.

In modern conditions, bank note design can never be static. Facilities for counterfeiting improve over the years and must continually be more than matched by developments in security features and improved printing techniques. To exploit such developments it is necessary to change the designs of notes from time to time.

The new £1 note, first put into circulation in February of this year, was the last of the newly-designed notes announced in 1968 which now form a consistent series with certain features in common. Each of the notes has a portrait of HM The Queen on the front and of a person representing a particular area of achievement in British history on the reverse. The historical portrait is accompanied by an illustrative background design on the back of the note and the same theme is picked up in a 'vignette' on the front. Each note also features a medallion

[1] See the issues for September 1961, page 24; September 1962, page 186; September 1963, page 199; March 1965, page 39; March 1966, page 37; December 1974, page 421.

[2] A restricted right of note issue remains for banks in Scotland and Northern Ireland.

[3] Between 1939 and 1968 the net profits of the Issue Department were also paid over to the Exchange Equalisation Account. Since 1968 they have been credited to the National Loans Fund.

[4] A further consideration is that notes of different denominations should be of different sizes so as to facilitate their recognition by the blind.

representing Britannia, who has long been associated with the Bank.[1] The style of lettering on each note has been chosen as characteristic of the styles in vogue during the historical period the note represents. The details of the designs, and their significance, are as follows:

£1

The portrait is of Sir Isaac Newton, the mathematician and scientist. He is shown sitting under an apple tree, a reference to the famous, but probably apocryphal, story of the falling apple. His reflecting telescope and a prism are on a table at his side and a copy of his *Principia* is open on his knee. The remainder of the back design consists of machine-engraved patterns suggesting the solar system, overlaid by a geometric diagram from the *Principia*. The vignette on the front shows a design comprising a caduceus, the classical symbol of trade, a cornucopia, symbolising plenty, and an olive branch, representing peace. The design is taken from a tradesman's half penny token issued in 1793 with the head of Newton on the reverse. It reflects Newton's association with the coinage as Master of the Royal Mint.

A particular feature of this new £1 note is that it has only one serial number. The space at the top left-hand side of the note has been deliberately left blank for encoding marks which may in future be applied for use in note-sorting equipment.

£5

The portrait is of the first Duke of Wellington. The battle scene is based on a nineteenth century engraving illustrating the Battle of Fuentes de Onoro which took place in 1811, during the Peninsular Campaign. The vignette on the front of the note is of a Winged Victory, a fashionable symbol of military prowess in Wellington's time, based on classical Greek models.

£10

Florence Nightingale appears on the back, together with a scene based on a lithograph of the Crimean War period entitled 'Miss Nightingale at the Barracks Hospital, Scutari'. The lamp shown is of the type included in the original lithograph although it is in fact not of the type known to have been used by Miss Nightingale. The vignette on the front of the note depicts a lily symbol she is said to have used and is based on a water colour attributed to her sister Parthenope.

£20

The portrait is a reproduction of the statue of Shakespeare from the Kent memorial in Westminster Abbey. In this respect, the £20 note differs from the other notes of the series, for which the portraits were specially created from a number of contemporary sources. (It also differs in having a portrait of the Queen as watermark—the watermarks on the other notes are portraits of the historical characters shown on the reverse.) The back design on the £20 note represents the balcony scene from Shakespeare's *Romeo and Juliet*. The vignette on the front is of St George and the Dragon, a motif which has associations with Shakespeare, whose birthday falls on St George's Day, and with money, since it appears on the reverse of gold sovereigns.

Printing

The Bank have printed their notes on their own premises since 1791. The Bank's printing operations outgrew their London premises after the last war and by 1956 were rehoused in a new works, specially designed, at Loughton in Essex. Later, additional premises were built to provide for the examination and destruction by burning of soiled notes withdrawn from circulation. Some 2,000 people are now employed there on bank note production, destruction of old notes, engineering and other services.

The techniques employed in the printing of bank notes combine processes not far removed from those which are normal in commercial printing with special processes normally found only in security printing. Perhaps the most important of the latter is printing by the intaglio process, commonly called 'plate-printing', which reproduces all the subtle variety in width and depth of line that is found in an artist's engraving. Instead of the artist's flat copper plate, however, a chromium-faced electroformed nickel plate attached to a cylinder is squeezed against the paper with the pressure of a metal strip-rolling mill. This printing deliberately overlaps other surface printings in a way which is intended to put difficulties in the way of the counterfeiter.

What may be described as the conventional method of bank note production is to use sheet-fed printing machines which print a number of notes together on a single sheet of paper. A series of separate printing operations is necessary and after each of these a stack of printed sheets is stored until the ink is dry enough for the next operation. When the printing processes are complete the sheets are examined and cut to produce single notes which are then finally sorted, counted and packed. This whole cycle of manufacturing operations takes place over a period of several weeks. This type of bank note printing is in common use throughout the world and is still used by the Bank for some of the present denominations.

The techniques of security printing have, however, advanced significantly in the last twenty years. A major contribution by the Bank has been the development of a printing press in which reels (or 'webs') of paper are converted into sheets of finished notes in one continuous operation. Special inks and heat-drying methods enable the machine—a 'web press'— to apply to the paper, as it passes once through the machine, the sequence of printing processes which in more conventional methods are separated by periods of storage. Machines for printing from reels of paper are common in ordinary commercial applications and it may seem surprising that there should be any difficulty in using them to print bank notes. There are two main reasons. First, the paper, manufactured for toughness and durability, is not as receptive to print as softer commercial papers. Secondly, note design is extremely complex and requires several quite different printing processes; these are deliberately intended to be so far as possible different from commercial counterparts. It was difficult to find a manufacturer who had the range of skills required to design such a composite machine or who was ready to finance the development costs; the Bank, therefore, themselves assembled a team of engineers, physicists and chemists who worked closely with the manufacturers finally selected to build the machines, Masson Scott Thrissell Engineering Ltd, of Bristol. The first notes were produced by the new process, on an experimental machine, in 1960. Four production machines were later installed and the entire production of the former series of £1 notes had been transferred to web presses by 1974. Since then, two additional presses have

[1] The Court of Directors on 30th July 1694 decided that the device for the Bank's Common Seal should be 'Britannia sitting and looking on a Bank of mony'.

been installed, and these are capable of printing notes to a higher specification. The original four machines are being successively converted to match this higher capability. Because of this conversion programme, it has not yet been possible to use the web presses for the new £1 note. The machines already converted, as well as the two new machines, are in use for the production of the £5 denomination. Each machine has a six-man crew and can print approximately one million notes in a fourteen-hour working day, divided into two shifts. The techniques developed by the Bank in this new method of note printing have been made available to bank note printers in other countries, at least four of whom are now printing notes by similar methods.

Despite the sophistication of the printing machinery and the very high levels of skill of the Bank's printers, flaws do occasionally occur in the printing of notes, as in any other mass production process. Because it is the Bank's aim to present a perfect product to the public, all misprinted notes (called 'spoils') have to be identified and extracted. Electronic inspection has been investigated, but no substitute has so far been found for the human eye as a means of spotting deviations from the standard of perfection which the Bank have set, or for the human brain for making the overall judgment as to whether the degree of variation present in a particular print is acceptable within agreed tolerances. The Bank therefore rely on a large force of examiners whose job it is to inspect every sheet of notes which the Bank produce.

Significant advances have, however, been made in automating procedures subsequent to this examination. Note-sorting equipment has been installed which recognises marks placed upon the spoiled notes by the examiners, picks out these pre-marked spoils and automatically groups the issuable notes in packets of either 100 or 50, depending on the denomination. The packets are secured by a welded plastic band. A computer controls the sorting process and provides a printed record of the serial numbers of all notes printed. The packets are grouped into parcels (in quantities of 5,000 notes for £1 and £5, 2,500 for £10 and 1,000 for £20), and the Bank have installed packaging equipment which wraps the parcels in reinforced polythene, labels them and finally seals them in an outer polythene wrap.

Distribution

The finished parcels of notes are despatched from the Bank's Printing Works either to the Issue Office in London or to one of the seven branches which the Bank maintain in the provinces.[1] There they are held in store until they are required to meet drawings by the commercial banks for distribution to their branches. Similarly, the commercial banks pay in surplus notes but, in the interests of maintaining notes in circulation in a satisfactory condition, when notes are returned to them the banks sort them into those which are suitable for reuse and those which have become too dirty or damaged for further circulation. The latter are returned to the Bank for destruction, but reissuable notes are either recirculated by the commercial banks themselves, or, if a temporary surplus of notes has built up, may be paid in to the Bank in parcels which are identifiable as reissuable and which the banks are prepared to draw at a future date to meet their requirements.

Destruction

The notes due for destruction are cancelled by machines which cut out a portion of each note, and sent back to the Printing Works, where they are examined in order to detect the presence of forgeries or notes issued by other note-issuing authorities (i.e. mainly notes issued by the Scottish and Northern Ireland banks) and counted by machine in order to maintain a check on accuracy. They are then destroyed in a complex of incinerators under an automated control system which ensures that after examination and counting the notes are untouched by human hand. Burning is used rather than other forms of disposal, because it guarantees complete destruction which is necessary on security grounds. The ash which is produced is unrecognisable as bank note residue and is dumped by contractors; some use is made of the heat from the incinerators to supplement the heating installation in the building where the examination of soiled notes is carried out.

The scope of the task

The reader may already have formed some impression of the scale of the note issue operation. The statistics shown in Tables A and B provide further evidence, but the size of the task can perhaps best be gauged from the following selection of figures.

During 1977, the Bank manufactured and distributed some 1,700 million new notes. This required an output of nearly 8 million notes every working day from the Printing Works. As well as handling this quantity of new notes, the Issue Office and branches of the Bank took in, and subsequently reissued, some 1,460 million reissuable notes and sent about 1,590 million soiled notes back to the Printing Works for destruction.

Table A
Number of new notes issued by denominations

Millions

Year to end-February	1963	1970	1972	1974	1976	1978
10*s*[a]	478	272	—	—	—	—
£1	1,252	1,122	1,035	981	1,021	990
£5	158	283	282	382	482	545
£10	—	16	21	42	111	133
£20	—	—	4	8	11	23
Total	**1,888**	**1,693**	**1,342**	**1,413**	**1,625**	**1,691**

[a] The 10*s* note ceased to be legal tender in November 1970 but notes still in the hands of the public can be cashed at the Bank.

In July 1978 there were in circulation about 1,880 million notes, with a face value of £8,700 million. This is equivalent to about 34 notes, with a value of £155, for every man, woman and child of the population of the United Kingdom. On average, each note lasts for not much more than a year, so that in a twelve-month period 30 notes have to be printed and issued per head of the population simply to keep the note circulation at its present level.

Table B
Value of notes in circulation by denominations

£ millions

End-February	1963	1970	1972	1974	1976	1978
10*s* [a]	95	42	14	13	13	13
£1	983	949	920	906	831	800
£5	1,124	1,797	2,094	2,513	3,261	3,625
£10[b]	107	315	386	644	1,157	2,172
£20[b]		141	145	325	500	852
Other notes[b]			139	172	280	314
Total	**2,309**	**3,244**	**3,698**	**4,573**	**6,042**	**7,776**

[a] See footnote [a] to Table A above.

[b] The issue of denominations higher than £20 was discontinued in 1943, but such notes are still used internally in the Bank, e.g. on behalf of banks of issue in Scotland and Northern Ireland as cover for their note issues in excess of their permitted fiduciary issues. The issue of £10 and £20 notes was suspended between 1943 and February 1964 and July 1970, respectively.

[1] It is intended also to issue notes through the Bank's agency in Glasgow when its new building is ready, probably in 1979.

This enormous output of notes seems to be a peculiarly British phenomenon. The note circulation in Western Germany is equivalent to about 17 notes per head, and the number of notes printed and issued in a twelve-month period per head of population is only 9. The corresponding figures for France are 25 notes in circulation and 11 printed and issued. In the United States the number of notes in circulation per head is similar to that in the United Kingdom (31) but the number printed and issued is only 15.

It is difficult to account satisfactorily for this disparity. One reason is the greater use made in many other countries of higher denomination notes. For example, the 1,880 million notes in circulation in the United Kingdom have a face value of £8,700 million, giving an average value for each note of something under £5. The average value of a West German note is equivalent to about £16. Whereas the most popular note in the United Kingdom is still the £1 (41% of the number of notes in circulation), in Western Germany the most common note is of DM 100 (roughly equivalent to our £20 note) which represents 35% of the circulation by number. This preference, in the United Kingdom, for low denomination notes has a disproportionate effect on the printing requirement, since it seems to be almost universally true that low denomination notes have shorter lives than higher denomination notes and so have to be replaced more often.

The reason for the United Kingdom's preference for low denomination notes is partly historical. Until 1957, over 75% of the circulation by value was in 10*s* and £1 notes. The old white £5 notes were comparatively little used, and notes of higher denominations had not been issued since the war. The first coloured £5 note was issued in 1957, the £10 note in 1964 and the £20 note not until 1970. Given the conservatism which affects most people's attitudes to money, it is perhaps remarkable that, even in the relatively short space of time since the issue of the higher denomination notes, the proportions of the total circulation represented by the various denominations should have changed as much as they have. Table B shows that since 1963 the proportion of the value of the note circulation represented by £1 notes has declined from 43% to 10%, while £10 and £20 notes, starting from scratch, had taken over 39% of the circulation by 1978. This move towards higher denomination notes has had the effect that while the value of the note circulation has more than trebled over the same period, the number of notes in circulation has gone up by only 30%.

The total number of notes required for issue in a year has actually fallen since 1965. This is partly because of the swing to higher denomination notes, which tend to have longer lives, but it owes much also to the measures taken since 1966, with the co-operation of the banks, to encourage the recirculation of used, but still reasonably clean, notes, thus prolonging their effective lives and reducing the quantity of new notes required to replace them. Reference has already been made to the work of the banks in sorting out and recirculating reissuable notes, and this goes on all the year round. To reinforce its effects, there are certain periods in the year when the Bank issue virtually no new notes at all, and the banking system and the public are therefore supplied with reissued notes only.

Although the number of notes to be printed has thus been reduced since the peak of the 1950s, there has not been a corresponding reduction in the work-load of the Bank's Printing Works. Bank note designs become more complex and therefore more difficult and expensive to print, as each new design succeeds its predecessor. Moreover, the more complex designs tend to be used, naturally enough, for the higher denomination notes. The trend of the last few years has therefore been for very large-scale production of relatively simple notes (such as the old 10*s*) to be succeeded by production on a rather smaller scale of very much more complicated notes, such as the £20.

In order to provide the capacity to produce the quantities of notes required in these changing circumstances, forward planning of some sophistication is essential. A large printing press, such as the web press described above, may take five years from design to operational running, and cost upwards of £1 million. The Bank are therefore concerned constantly to refine their methods of forecasting the future requirement for bank notes over as long a period as possible.

Research has shown a strong and stable relationship between consumers' expenditure and the value of notes in circulation, and this has become the basis for a forecasting model which was first discussed in the December 1974 *Bulletin* and is fully described in the appendix to this article. The model has become an indispensable tool in predicting not only trends in the note circulation over a period of up to five years ahead but also the immediate requirement for the next year.

Appendix

The purpose of this appendix is to explain the more technical issues connected with forecasting the demand for notes both in the short term and in the longer term.

Short-term forecasting

The method employed for short-term forecasting is to project the trend of the weekly figures for the next three months and then allow for seasonal and other effects, which are assessed by analysing the historical weekly series. Weekly (Wednesday) figures are available for the value of:

(a) total notes in circulation (published in the Bank Return);

(b) notes in London clearing banks' tills (reported by the banks); and

(c) notes with the public [(a) *minus* (b)].

Each is analysed as follows.

Long-term growth is removed by taking ratios of the original figures to the trend, the latter being estimated by an unweighted thirteen-term moving average.

The basic seasonal variation of the ratios is assumed to be adequately described in terms of twenty-eight harmonic variables. Twenty-two are of the form:

$$\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} \left(\frac{2\pi kt}{365}\right), k = 1, 2, \ldots, 11,$$

where t is the day of the year (i.e. it runs from 1 to 365, 29th February being treated as 28th February), and a further six of the form:

$$\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\} \left(\frac{2\pi k' t'}{m'}\right), k' = 1, 2, 3,$$

where t' is the day of the month and m' is the number of days in the month. These two sets of variables pick up the within-year and within-month variations respectively. The within-year variables for $k = 12$ are almost identical with the within-month variables for $k' = 1$ and are therefore omitted.

Allowance also has to be made for the effects of bank and public holidays and, since their introduction in 1966, of the periods when the issue of new notes is suspended. The shapes of the humps or dips that these cause are determined by inspection of the data for the dates about Christmas (where the peak is very sharp and the holiday does not occur on a fixed day of the week), for the Wednesdays about the moving holidays, and for the Wednesdays before, during and after the suspensions. To do this, the ratios to trend need first to be adjusted for day-of-year and day-of-month effects. The ratios thus adjusted are plotted against time for several weeks each side of a bank holiday and for the period covering each category of suspension (two weeks, three weeks, or four/five weeks), and dummy variables are constructed which exhibit the indicated pattern of behaviour about each event.

Obviously there is a chicken and egg problem here: for example, in order to determine suspension effects, any overlapping bank holiday effects have first to be removed, and vice versa. Moreover, the trend is an average of seasonally-adjusted, rather than original, figures, and estimation of the trend thus depends upon the prior existence of a seasonally-adjusted series. In practice, the seasonal adjustments are updated annually, and the previous year's estimates of the seasonal factors and the bank holiday and suspension effects are used to provide an initial trend, and to disentangle bank holidays from suspensions.

A regression of the ratios against the seasonal and dummy variables is now carried out. From the resulting coefficients, together with the original Wednesday figures and the various bank holiday and suspension patterns, another computer program produces:

(a) seasonal factors for each date of the year appropriate for when it falls on a Wednesday;

(b) adjustments to be applied in the neighbourhood of bank holidays and suspension periods;

(c) the total factors combining those of (a) and (b) [for each Wednesday of the data period and for the next eighteen months];

(d) the original series divided by the total factors;

(e) thirteen-term moving averages of these totally adjusted figures (the 'trend'); and

(f) ratios of original figures to trend.

A third program produces day-of-week effects from daily data which have been adjusted by the estimated seasonal factors corresponding in each case to Wednesdays having the same date. The program estimates and removes the trend in two parts, a week-to-week trend and an average within-week trend. From this seasonal and trend-free series, the average value of notes in circulation for each day of the week is then estimated, and hence the deviations from Wednesday.

The predicted total factors and day-of-week effects are used in the short-term forecasting of the note circulation. The trend is projected forward, multiplied by the total factors, and the day-of-week effects added. Any known future abnormalities can also be taken into account by altering the total factors.

Long-term forecast

The long-term forecast of the printing requirements of the various denominations of Bank of England notes embraces three component forecasts of:

(a) the average value of the note circulation quarter by quarter;

(b) the share of each denomination in the total; and

(c) the average life of each denomination (a separate exercise described in the following section).

(a) This first step depends on the construction of a statistical relationship between consumers' expenditure at current prices, standing as a proxy for the total value of transactions in which notes might be used, and the total value of the note circulation. The process is split into two steps: the first relates consumers' expenditure to the total of currency in circulation with the public (i.e. including the Northern Ireland and Scottish bank note issues and coin, but excluding notes held in banks' tills) and the second converts total currency in circulation to the total of Bank of England notes held by the public and the banks ('notes in circulation'). This two-stage approach is adopted since the first relationship, in setting aside the arbitrariness of the division between Bank of England notes and other currency, achieves a better level of statistical explanation than one linking consumers' expenditure and notes in circulation directly.

In order to take account of the growth of the circulation and of consumers' expenditure, this first relationship is taken to be linear on a logarithmic scale and of the form:

$$lnN_t = a + b_1 lnCE_t + b_2 lnCE_{t-1} + b_3 lnN_{t-1} + b_4 D1_t + b_5 D2_t + u_t,$$

where N_t = average value of notes and coin with the public in quarter t:
CE_t = consumers' expenditure (at current prices) in quarter t:
$D1_t$ = dummy variable in quarter t for series A £1 notes withdrawal:
$D2_t$ = dummy variable in quarter t for 1971 postal strike: and
$u_t = \alpha u_{t-1} + e_t$ where e_t = 'unexplained' component in quarter t.

Quarterly seasonally-adjusted data from 1957 onwards are used to estimate this equation and, from a path for CE_t provided by the Bank's economic forecasts, a path for N_t can be derived.

The average value of notes and coin with the public is then adjusted for:

(1) notes in London clearing banks' tills:

(2) coin:

(3) Scottish and Irish notes:

(4) the remaining 10*s* notes in circulation:

to estimate the average value of notes in circulation.

(b) The denominational mix is obtained by considering first the average denomination and the spread of the denominations about this average. This latter is defined as the standard deviation about the average denomination.

On the assumption of some underlying stability in the structure of payments, the average denomination is thought to be related to the level of prices. It is in fact forecast using a relationship of the form:

$$\Delta \ln AD_t = c_1 \Delta \ln PC_t + c_2 \Delta \ln AD_{t-1} + e_t,$$

where AD_t = average denomination of notes in circulation at quarter t:
PC_t = price deflator for total personal consumption at quarter t: and
e_t = 'unexplained' component in quarter t.

Making a similar basic assumption, the spread is in turn related to the average denomination, after first differences of the logarithmic data have been taken. The relation is of the form:

$$\Delta \ln SP_t = d_1 \Delta \ln AD_t + d_2 \Delta \ln AD_{t-1} + u_t,$$

where SP_t = spread of notes in circulation in quarter t:
AD_t = average denomination of notes in circulation in quarter t: and
$u_t = \beta u_{t-1} + e_t$ where e_t = 'unexplained' component in quarter t.

In both these equations, quarterly seasonally-adjusted data from 1963 onwards are used. With these two relationships, a path for PC is utilised to provide forecast paths for the average denomination and for the spread of notes in circulation.

The value of each of the four denominations in circulation is then calculated. using the three quantities previously forecast, i.e. the total value, average denomination and spread of notes in circulation,together with an estimate of the increase in the proportion of £20 notes in the total note circulation. This latter estimate is obtained by projecting current trends, bearing in mind that the proportion of £20 notes in circulation is likely to increase.

Finally, from the forecasts of the total value of the note circulation, of the denominational mix and of the average life of each denomination, the printing requirement for each denomination during each half year of the forecast period is estimated.

Life of notes

This exercise entails calculating the average life of each denomination of bank note month by month.

This is done in two ways: the average number of each denomination in circulation during the month is divided:

(1) by the number of each withdrawn from circulation ('paid') during the month: and
(2) by the number of each issued during the month.

The figure given by method (1) represents the time it would take to withdraw at the present rate the whole of the present circulation of the denomination in question: and that by method (2) represents the time it would take to issue it. Both give estimates of the 'life' of the denomination which in general are different, and the 'true' life is assumed to lie somewhere between them.

The calculations are made as follows. For each denomination, weekly figures are obtained for the number of notes in circulation, notes issued and paid, and notes returned to store fit for reissue. From these are derived the average number of notes in circulation during the month (which may contain data for four or five weeks), and the corresponding monthy totals of notes issued, notes paid, and notes returned to store fit for reissue (all these being multiplied by a factor so as to adjust to a standard month of $4\frac{1}{3}$ weeks). Average circulation, gross issue, and the gross number paid all show a seasonal pattern—for example, a large number of notes are issued in November and December, giving rise to a large number in circulation in December and a large number being withdrawn in January. They are therefore all seasonally adjusted. The fit for reissue series for each denomination is very erratic and shows no such seasonal pattern. However, it was found that the ratio:

$$R_t = \text{fit for reissue (unadjusted)/gross paid (unadjusted)}$$

formed a more stable series, which is then smoothed exponentially by forming the series:

$$\bar{R}_t = 0.8\,\bar{R}_{t-1} + 0.2\,R_t.$$

The adjusted figures for reissue are therefore calculated by applying the smoothed ratios $\bar{R}_t$ to the seasonally-adjusted gross paid figures. Figures for net issue and net paid are then obtained by:

$$\text{net issue} = \text{gross issue (seasonally-adjusted)} - \text{reissue (smoothed)};$$
$$\text{net paid} = \text{gross paid (seasonally-adjusted} - \text{reissue (smoothed)}.$$

For each denomination, the two estimates of the life are calculated as follows:

$$P_t = \text{circulation/net paid};$$
$$Q_t = \text{circulation/net issue}.$$

However, both P_t and Q_t are somewhat erratic, and, after some experimenting, the following exponential smoothing was found to be satisfactory for all denominations:

$$\ln \bar{P}_t = 0.75 \ln \bar{P}_{t-1} + 0.25 \ln P_t;$$
$$\ln \bar{Q}_t = 0.75 \ln \bar{Q}_{t-1} + 0.25 \ln Q_t.$$

The average life of notes estimate used in the long-term forecast (described above) is assessed using the $\bar{P}_t$ and $\bar{Q}_t$ series as a guide, but taking other developments into account. For example, £1 notes are increasingly being used as an item of change and thus may have a shorter life through being handled more often.