Yield curves for gilt-edged stocks: an improved model

Revisions have been made to the Bank's model of gilt-edged yields, used (with one minor modification) since 1973. The number of variable parameters has been reduced from eight to seven, and the model is now considerably more stable.

Certain features of the market—notably the way in which coupons and yields are related—have now been more appropriately modelled.

The par yield curve published by the Bank (the estimated yield on stocks standing at par) is derived from a model of gross redemption yields in the gilt-edged market. It was developed nearly ten years ago (see *Bulletin* articles in December 1972, September 1973 and June 1976). This note explains the reasons for another modification of the model, but first gives a brief summary of the theory underlying it.[1]

The model

Investors are assumed to have expectations about future interest rates up to a point called the 'planning horizon'. Beyond this point, their uncertainty is taken to be so great that they cannot form any expectations about further changes in interest rates. Empirical evidence suggests, and the theory assumes, that the market is divided into two segments, in each of which the planning horizons are distributed over a fairly narrow range. Average expectations within each group may be regarded as relating to an average planning horizon.

For each segment of the market, the expected returns on different stocks, held until the common horizon and then sold, can be calculated from the expected yield at the horizon. Arbitrage is assumed to equalise the returns over the holding period. The two average planning horizons have been assumed to be at one and four years.

In order to derive a single par curve, it is necessary to splice together the two segments. It is assumed that stocks maturing between one and four years belong solely to the short market, those longer than eight years entirely to the long market, and stocks in the band between four and eight years enter both markets but with weights depending on their positions in this middle band. The par curves corresponding to the two market segments are spliced together in the same way.

Variation in yields between stocks of the same maturity but different coupons is due mainly to the incidence of tax. High coupon stocks appeal mostly to 'gross' investors (those who are indifferent between capital gain and income because they pay tax on neither, or on both at similar rates). Lower coupon stocks appeal more to 'net' investors (those who pay tax primarily on income).

But another factor which influences differences in yields between high and low coupons is the bullishness or bearishness of expectations. When prices are expected to rise, low coupon stocks are more attractive because of their greater volatility (ie they are expected to rise in price proportionately more than high coupon stocks). The tax effect is therefore reinforced. Conversely, if prices are expected to fall, high coupon stocks are preferred and the tax effect is reduced (see December 1972 *Bulletin*, page 474).

In the model, these effects are treated as purely due to tax. The observed price of a stock is assumed to be a weighted average of the prices that are necessary to attract gross and net investors, the weights depending on coupon and maturity. The relative weight of net investors decreases steadily as the coupon increases up to a certain level and thereafter remains steady. For coupons higher than this level (which will vary with maturity), differences in gross redemption yield are small (even, at times, zero), and the region where this occurs is called the 'gross zone', because gross investors dominate in setting market prices. In the gross zone, at a given maturity, price is usually linearly related to coupon. Below the gross zone, the price-coupon relationship is curvilinear, and is described as the coupon curvature effect.[2] The gross zone was introduced into the model in 1973, and was regarded then as little more than a mathematical device. However, empirical evidence of its existence and size has since accumulated.

Similarly, at each horizon, the expected price is assumed to be a weighted average of prices for gross and net investors. This produces a spread of yields with respect to coupon at the horizon.

In order to clarify the link between coupon, yield and tax, it is helpful to consider the yields on two stocks with different coupons. At low tax rates, the higher coupon will have the higher net yield but, as the tax rate increases, the yields come closer together and eventually cross over, so that the

(1) A fuller account of the research described here can be obtained from the Mathematical Techniques Group, Financial Statistics Division, Bank of England, London, EC2R 8AH. A tape of the computer program is also available.

(2) See R S Clarkson, 'A mathematical model for the gilt-edged market'. *Transactions of the Faculty of Actuaries*, vol 36, part 2 (1978), pages 85–160 (with discussion).

higher coupon then has the lower yield. The tax rate at the cross-over point defines an 'effective tax rate' for this pair of stocks. (This is a shorthand description of the combined tax and volatility effects.) Such effective tax rates are represented in the model by the weights given to gross and net investors. The effective tax rate appears to be quite low among high coupon stocks, usually averaging under 10 per cent for coupons in the 11%–15% range. In comparing a low coupon with a medium or high coupon stock, the effective tax rate depends on the particular coupons chosen: for example, comparing 5% coupons with 13% coupons, the effective tax rate is between 30 and 45 per cent.

Recent experience

In September and October 1981, the peak in the par curve at around five years began to display signs of instability. At this time, yields were well above the highest coupons in the market, so there were no stocks standing close to par. Consequently the yield on a par stock could only be estimated by extrapolation. It was decided that the upper limit for one of the model parameters should be reduced in order to stabilise the peak of the curve at a slightly lower level than would otherwise have been estimated. This change was accomplished with, on average, little change in the goodness of fit of the model, suggesting that it had too many parameters. Research was therefore undertaken to see if the effect of coupon on yield could be explained more simply, without altering the basic expectation structure of the model, and—most important—to provide more stable estimates of the par curve in a wide variety of circumstances.

The time was, in any case, ripe for reconsideration of the model. The number of conventional stocks has doubled since 1974. There are now no significant gaps in the distribution of maturities, and there is a good range of high coupons. Moreover, what were then considered high coupons are nowadays regarded as medium coupons.

A revised model

The previous model had eight variable parameters—four to describe interest rate expectations and four for relative weights of gross and net investors in the two segments. There was also a ninth parameter, representing the risk premium—an addition to the return expected by investors in the case of longer-dated stocks, because of their greater price volatility. However, it has never been possible to measure the risk premium separately; a higher premium is offset by a lower value of another parameter, giving virtually identical calculated stock yields and par curve. Hitherto, a conventional one percentage point premium has been added to the return on undated stocks, with smaller proportionate additions on dated stocks. As it plays no useful role, this ninth parameter has now been dropped, which simplifies the model.

Three other main changes and two minor ones have been made.

Hitherto, for convenience, 'net' investors have been assumed to be those paying income tax at the standard rate. This is clearly unrealistic but, when the model was set up, it was found that changing the tax rate had virtually no effect. This is no longer true, and the model now contains three effective tax rates as parameters, instead of the four relative weights of gross and net investors. These are:

- the effective tax rate on low coupon stocks in the short-dated market;
- the effective tax rate on low coupon stocks in the long-dated market;
- the effective tax rate on high coupon stocks (the 'gross zone') in both segments of the market.

A crucial part of the coupon-yield structure is how to define the boundary of the 'gross zone'. Empirical evidence has accumulated that this boundary is always below the par curve; also it must tend to zero coupon for the undated stocks. Changes have therefore been made to the definition, especially for downward-sloping curves. The relative weights of gross and net investors have also been redefined, so as to make them vary more smoothly with coupon.

At various times in the past, the extent of the splicing band (the band over which the two segments of the curve are spliced) has been re-examined, but not found to be critical. However, over the last year, with the emergence of downward-sloping long-dated curves, the model has fitted less well at around ten years' maturity. This was still true after the changes to the tax parameters. The fit around ten years has been improved by changing the splicing band from 4–8 years to 5–10 years, and by altering the long planning horizon from four to five years. It is difficult to believe that four years represents an average planning horizon (given annual analysis of portfolio performance), and this interpretation is even more implausible if the parameter is increased to five years. What the data are indicating, therefore, is that a structural change in the par curve occurs at around five years. This change may perhaps be connected with the fact that the discount houses confine themselves almost entirely to stocks below five years. Similarly, the bulk of banks' holdings mature within ten years, and this may possibly be linked with the end of the splicing band.

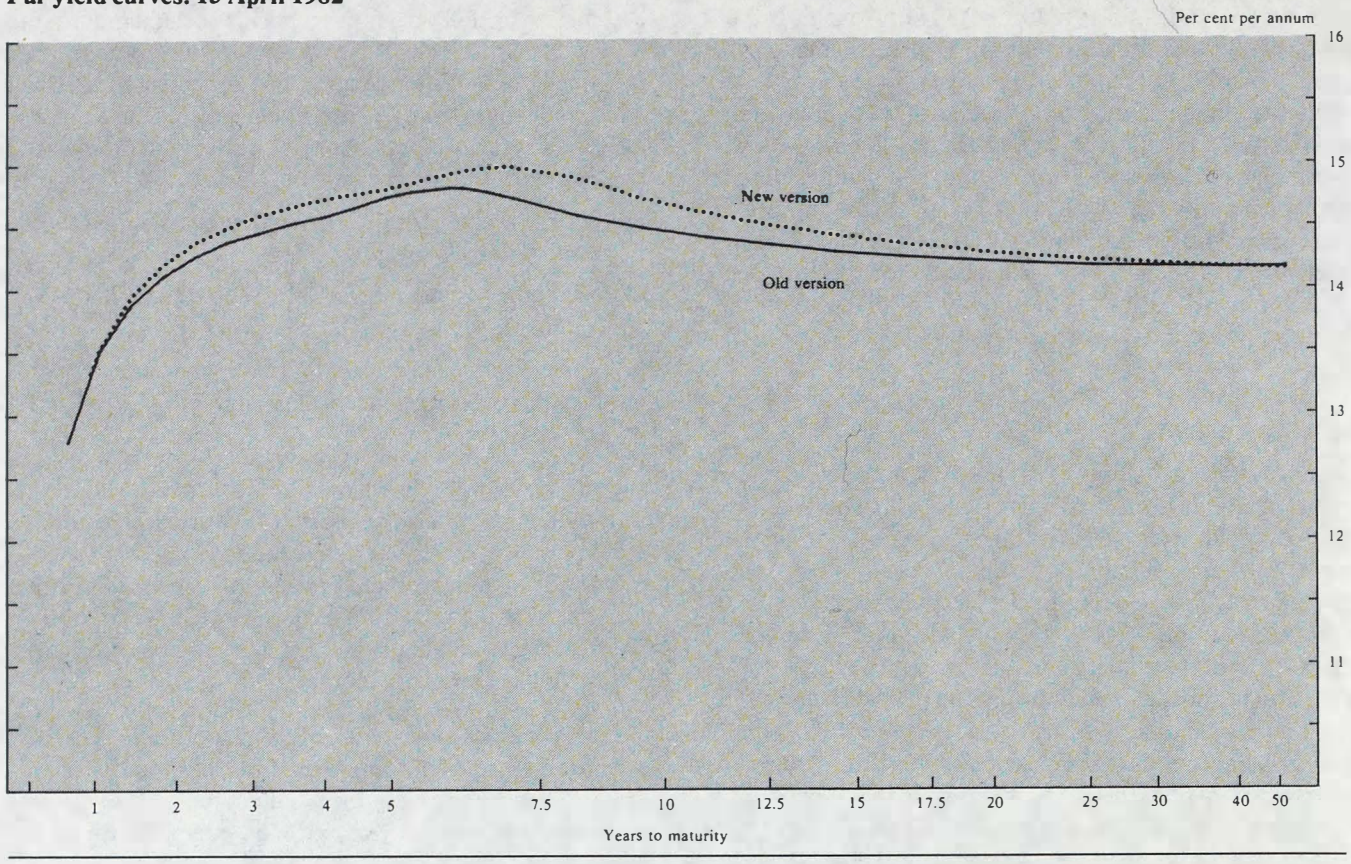
Two minor changes have been made. The calculated yield for the shortest stock (under one year) has been altered to allow for the incidence of capital gains tax, which should put net investors in virtually the same position as gross investors. Also, a slight change has been made to the method of splicing the short and long curves together.

A chart compares the old and new par curves for 15 April 1982.

Results

Some actual capital-income (C–I) diagrams are shown for two dates, similar to the theoretical ones given in the 1973 article. These relate the running yield on a stock (coupon divided by price) to the capital gain/loss from holding it to

Par yield curves: 15 April 1982



maturity. Low coupons are in the upper left region, high coupons in the lower right. The continuous curve shows the trade-off between capital gain and running yield predicted by the revised model at that maturity. (The model postulates that investors will, on average, be indifferent between combinations of capital gain and income given by points on this line, which is called the indifference line.) The par yield equals the running yield at the intersection of this line with the line of zero capital gain. The plots of individual stocks reflect prices based on their actual yields, but corrected for maturity.

In equilibrium, the indifference lines should be straight (and the effective tax rate constant), or concave towards the origin. It is quite difficult to detect by inspection where the curvature starts, since it is not very marked. The indifference lines become steeper with increasing maturity. Detection of coupon curvature becomes impossible beyond twenty years, both because of the steepness of the curve and the sparseness of stocks. The less steep the indifference line at a given maturity, the smaller the extra capital gain required to offset a given loss in running yield, ie the higher the effective tax rate.

A stock significantly to the left of, and below, the indifference line has a higher price than predicted by the model. But its deviation may be due to slope—ie if the par curve slopes upwards and the stock is of shorter maturity than the indifference line, or if the curve slopes downwards and the stock is longer. No correction is made in the charts for the slope of the yield curve between the maturity of a stock and that of the indifference line, because this correction is sometimes difficult to estimate from the yields on the actual stocks.

The first set of C–I diagrams relates to October 1977 when, after a long bull market, some high coupon stocks (in the 15–20 year range) were standing substantially above par. These are so far to the right of the indifference line that it looks as if the market was not in switching equilibrium: for example, at 20 years a switch out of a 9% into a combination of $6\frac{3}{4}$% and 12% stocks would have improved both capital gain and running yield.

The second set of diagrams relates to October 1981, when the state of affairs was quite the opposite—that is, all stocks were standing below par. The par curve therefore had to be estimated by extrapolation. The lower tax rates than in 1977 are reflected in the much steeper indifference lines (allowing for the different vertical scales).

The table compares the new model with the old one at selected dates over the last eight years, under widely differing market conditions. Although the overall fit is not, on average, significantly changed, the seven parameters of the new model are much better determined than the eight parameters of the old one, and the model is therefore more stable. At all dates except two, the estimated tax rates are reasonable—mostly under 12 per cent in the gross zone and between 30 and 50 per cent for low coupons. The high tax rate for high coupon stocks in October 1977 indicates exceptional yield differences; these stocks (as already noted)

were standing substantially above par. They were not popular with investors, perhaps because of the accounting problems of writing off the capital loss, or perhaps because investors were looking for a continuation of the bull market and preferred the higher volatility of lower coupons. The high coupon tax rate in January 1976 is not, however, of much significance; par yields were around 12%–14%, but there were very few stocks with coupons over 10%, so the estimated tax rate of 29 per cent is subject to a wide margin of error.

To sum up, the new model removes the instability displayed by the five-year yields estimated on the old model in September–October 1981, and avoids the need for *ad hoc* adjustments.

Effect of revisions on the model

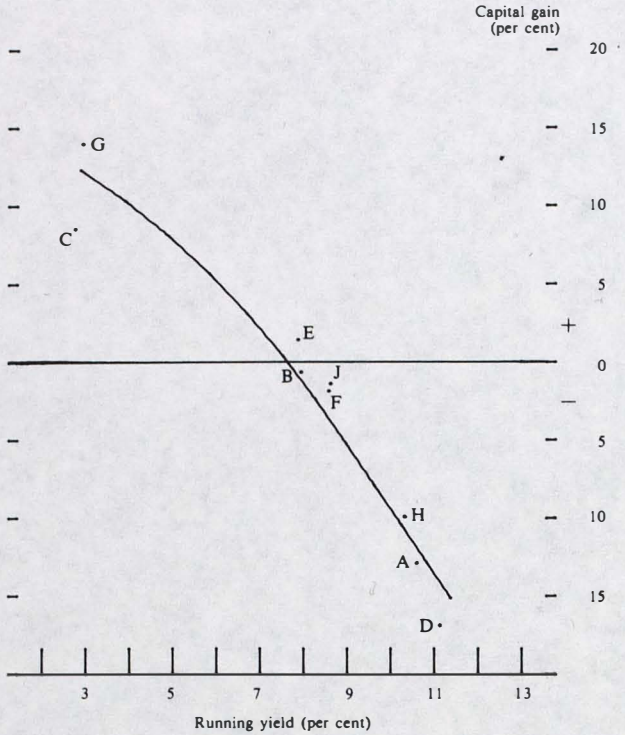
	Coefficient of determination $\bar{R}^2$(a)		Effective tax rates in new model (per cent)		
	Old model	New model	Low coupon		High coupon
			Short	Long	
4 Jan. 74	0.911	0.894	34.9	37.0	—
30 Dec. 74	0.971	0.953	36.5	46.0	2.3
9 Jan. 76	0.941	0.934	46.4	42.2	28.1
3 Oct. 77	0.967	0.964	71.7	55.6	25.3
4 Jan. 80	0.919	0.904	33.2	49.6	8.9
9 Jan. 81	0.897	0.913	34.8	51.7	8.0
30 June 81	0.897	0.940	36.1	49.8	11.7
25 Aug. 81	0.899	0.921	41.0	53.3	5.5
26 Oct. 81	0.888	0.887	36.9	49.1	9.9
5 Jan. 82	0.906	0.915	41.5	51.0	10.1
15 Apr. 82	0.897	0.920	34.5	39.3	16.3

(a) Measures the proportion of the overall variation in yields that is explained by the model.

Capital-income diagrams at 3 October 1977

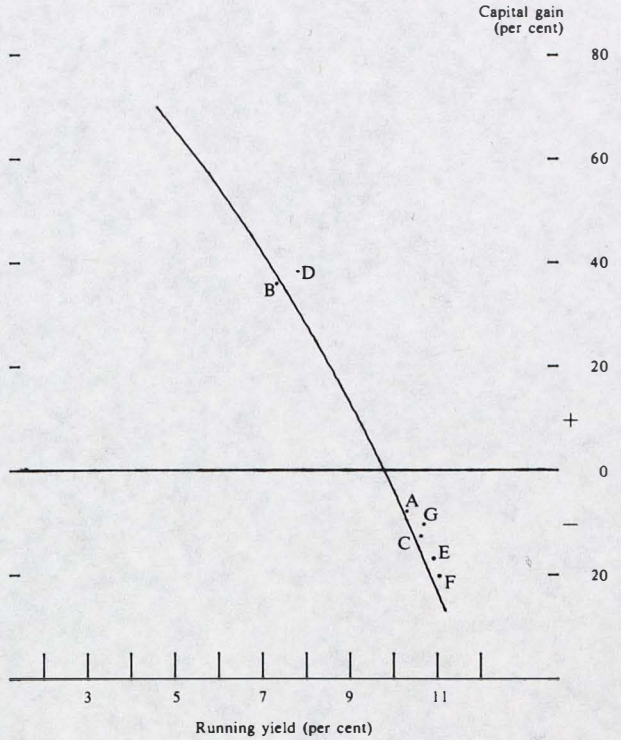
Calculated indifference lines compared with selected individual stocks

5 years maturity



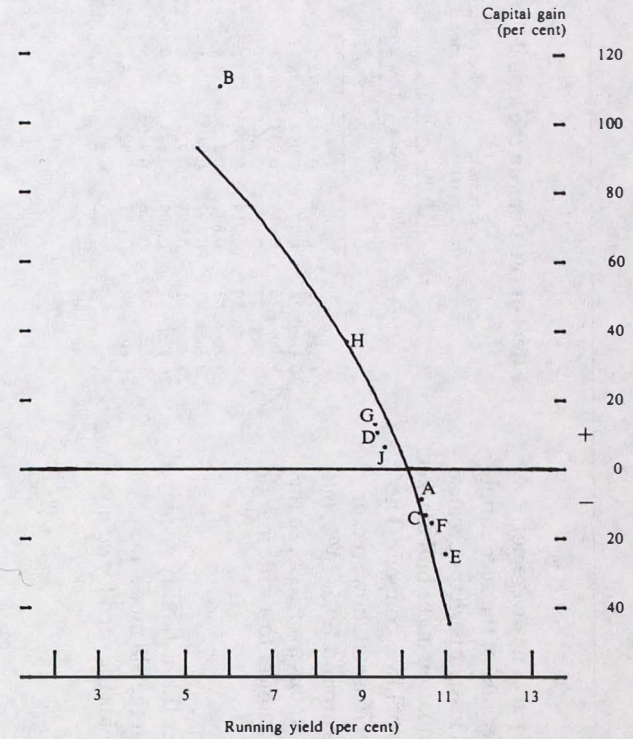
	Coupon	Actual maturity	Gross redemption yield (per cent)
A	12.75	Nov. 1981	9.00
B	8.50	Jan. 1982	8.34
C	3.00	Feb. 1982	4.79
D	14.00	Mar. 1982	8.87
E	8.25	July 1982	8.60
F	9.25	Sept. 1982	8.77
G	3.00	Feb. 1983	5.87
H	12.00	Mar. 1983	9.20
J	9.25	July 1983	8.90

15 years maturity



	Coupon	Actual maturity	Gross redemption yield (per cent)
A	11.75	Jan. 1991	10.59
B	5.75	Apr. 1991	9.00
C	12.75	Jan. 1992	10.77
D	6.00	Sept. 1993	9.50
E	13.75	Nov. 1993	10.94
F	14.50	Mar. 1994	10.99
G	12.50	Aug. 1994	10.91

20 years maturity

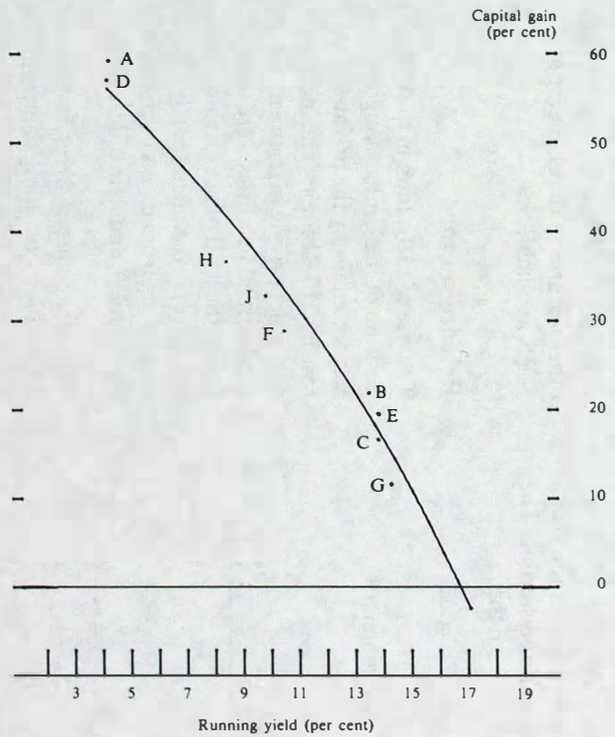


	Coupon	Actual maturity	Gross redemption yield (per cent)
A	12.00	Jan. 1995	10.82
B	3.00	May 1995	8.51
C	12.75	Nov. 1995	10.86
D	9.00	Mar. 1996	10.12
E	15.25	May 1996	11.15
F	13.25	May 1996	10.95
G	8.75	Sept. 1997	10.10
H	6.75	May 1998	9.85
J	9.50	Jan. 1999	10.22

Capital-income diagrams at 26 October 1981

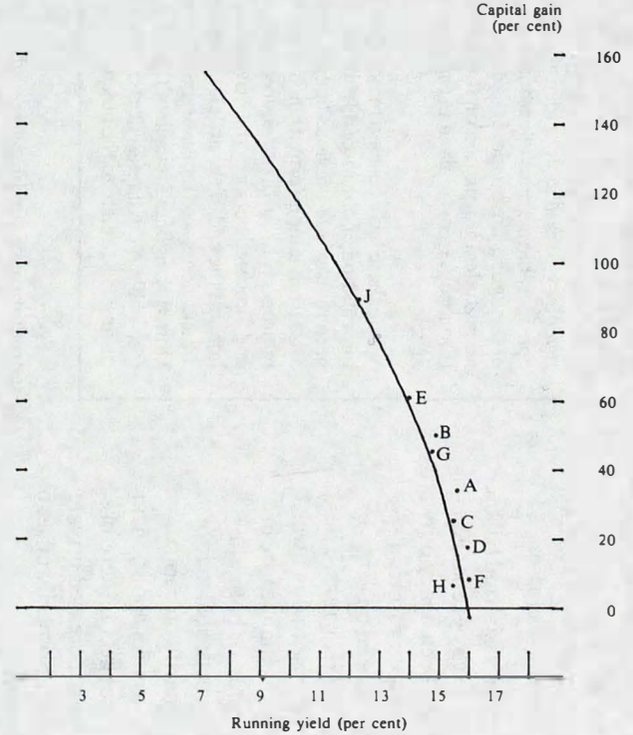
Calculated indifference lines compared with selected individual stocks

5 years maturity



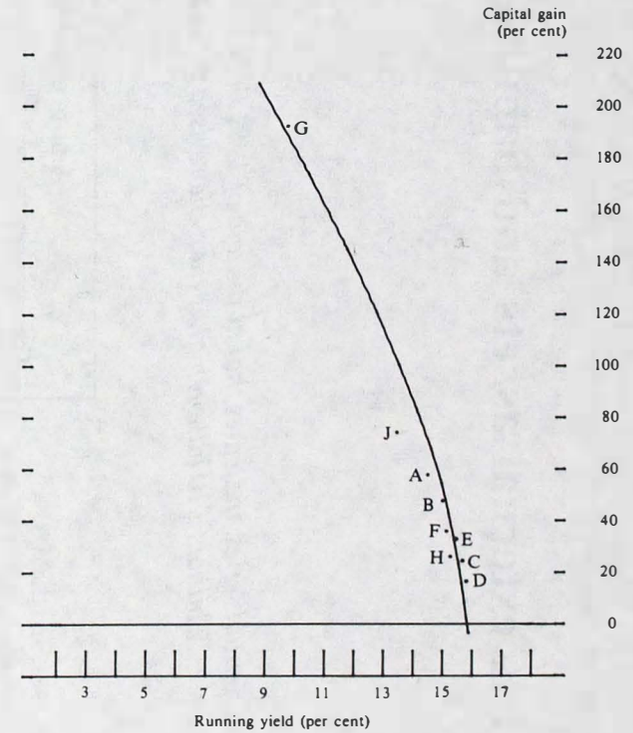
	Coupon	Actual maturity	Gross redemption yield (per cent)
A	3.00	May 1985	13.47
B	11.50	July 1985	16.97
C	12.25	Nov. 1985	16.55
D	3.00	May 1986	13.16
E	12.00	June 1986	16.99
F	8.50	July 1986	15.02
G	13.25	Jan. 1987	16.35
H	6.50	May 1987	14.18
J	7.75	Jan. 1988	14.93

15 years maturity



	Coupon	Actual maturity	Gross redemption yield (per cent)
A	12.00	Jan. 1995	16.66
B	10.25	July 1995	16.24
C	12.75	Nov. 1995	16.41
D	14.00	Jan. 1996	16.74
E	9.00	Mar. 1996	15.60
F	15.25	May 1996	16.64
G	10.50	Feb. 1997	16.05
H	15.00	Oct. 1997	16.09
J	6.75	May 1998	14.58

20 years maturity



	Coupon	Actual maturity	Gross redemption yield (per cent)
A	9.50	Jan. 1999	15.47
B	10.50	May 1999	15.87
C	13.00	July 2000	16.35
D	14.00	May 2001	16.43
E	12.00	Jan. 2002	16.20
F	11.50	Mar. 2004	15.91
G	3.50	July 2004	12.57
H	12.50	Nov. 2005	15.97
J	8.00	Oct. 2006	14.63